Magnetic Field Model Equations

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1 T89

1.1 Tail Current Sheet

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\begin{align}
B_x &= A_1 B_{x1} + A_2 B_{x2} + A_{16} B_{x3} + A_{17} B_{x4}, \tag{1}\\
B_y &= A_1 B_{y1} + A_2 B_{y2} + A_{16} B_{y3} + A_{17} B_{y4}, \tag{2}\\
B_z &= A_1 B_{z1} + A_2 B_{z2} + A_{16} B_{z3} + A_{17} B_{z4}, \tag{3}
\end{align}
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\begin{align}
B_{x3} &= B_{x1}\psi^2 \tag{4}\\
B_{x4} &= B_{x2}\psi^2 \tag{5}\\
B_{y3} &= B_{y1}\psi^2 \tag{6}\\
B_{y4} &= B_{y2}\psi^2 \tag{7}\\
B_{z3} &= B_{z1}\psi^2 \tag{8}\\
B_{z4} &= B_{z2}\psi^2 \tag{9}
\end{align}
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\begin{align}
B_x &= (A_1 + A_{16}\psi^2)B_{x1} + (A_2 + A_{17}\psi^2)B_{x2} \tag{10}\\
B_y &= (A_1 + A_{16}\psi^2)B_{y1} + (A_2 + A_{17}\psi^2)B_{y2} \tag{11}\\
B_z &= (A_1 + A_{16}\psi^2)B_{z1} + (A_2 + A_{17}\psi^2)B_{z2} \tag{12}
\end{align}
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\begin{align}
B_{x1} =& B_x^{(C1)}\cos\psi + B_z^{(C1)}\sin\psi \tag{13}\\
B_{x2} =& B_x^{(C2)}\cos\psi + B_z^{(C2)}\sin\psi \tag{14}\\
B_{y1} =& Q_T^{(C1)} y z_r \tag{15}\\
B_{y2} =& Q_T^{(C2)} y z_r \tag{16}\\
B_{z1} =& B_z^{(C1)}\cos\psi - B_x^{(C1)}\sin\psi \tag{17}\\
B_{z2} =& B_z^{(C2)}\cos\psi - B_x^{(C2)}\sin\psi \tag{18}
\end{align}
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\begin{aligned}
Q_T =& \frac{W(x,y)}{\xi_T S_T}\left[\frac{C_1}{S_T + a_T + \xi_T} + \frac{C_2}{S_T^2}\right] && (19)\\
=& Q_T^{(C1)} C_1 + Q_T^{(C2)} C_2 && (20)\\
Q_T^{(C1)} =& \frac{W(x,y)}{\xi_T S_T (S_T + a_T + \xi_T)} && (21)\\
Q_T^{(C2)} =& \frac{W(x,y)}{\xi_T S_T^3} && (22)\\
S_T =& \sqrt{\rho^2 + (a_T + \xi_T)^2} && (23)\\
\xi_T =& \sqrt{z_r^2 + D_T^2} && (24)\\
z_r =& z - z_s(x,y,\psi) && (25)\\
z_s(x,y,\psi) =& 0.5 \tan\psi (x + R_c - \sqrt{(x+R_c)^2 + 16}) - G \sin\psi \cdot y^4 (y^4 + L_y^4)^{-1} && (26)\\
\frac{\partial z_s}{x} =& 0.5 \tan\psi \left[1 - \frac{x + R_c}{\sqrt{(x+R_c)^2 + 16}}\right] && (27)\\
\frac{\partial z_s}{\partial y} =& \frac{4Gy^3 L_y^4 \sin\psi}{(y^4 + L_y^4)^2} && (28)\\
D_T =& D_0 + \delta y^2 + \gamma_T h_T(x) [+\gamma_1 h_1(x) \text{ not included}] && (29)\\
\frac{\partial D_T}{\partial x} =& \frac{\gamma_T L_T^2}{2(x^2 + L_T^2)^{2/3}} && (30)\\
\frac{\partial D_T}{\partial y} =& 2\delta y && (31)
\end{aligned}
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\begin{aligned}
B_x^{(C1)} &= Q_T^{(C1)} x z_r && (32)\\
B_x^{(C2)} &= Q_T^{(C1)} x z_r && (33)
\end{aligned}
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B_z^{(C1)} = \frac{W(x,y)}{S_T} + \frac{x\frac{\partial W}{\partial x} + y\frac{\partial W}{\partial y}}{S_T + a_T + \xi_T} + B_x^{(C1)} \frac{\partial z_s}{\partial x} + B_y^{(C1)} \frac{\partial z_s}{\partial_y} + Q_T^{(C1)} D_T \left(x\frac{\partial D_T}{\partial x} + y\frac{\partial D_T}{\partial y}\right) \tag{34}
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B_z^{(C2)} = \frac{W(x,y)(a_T + \xi_T)}{S_T^3} + \frac{x\frac{\partial W}{\partial x} + y\frac{\partial W}{\partial y}}{S_T(S_T + a_T + \xi_T)} + B_x^{(C2)} \frac{\partial z_s}{\partial x} + B_y^{(C2)} \frac{\partial z_s}{\partial_y} + Q_T^{(C2)} D_T \left(x\frac{\partial D_T}{\partial x} + y\frac{\partial D_T}{\partial y}\right) \tag{35}
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References