February 1, 2013

Recap

Prepositions and Formulas of Prepositions (Compound Prepositions)

- Formula example, (P||Q)&R
- There 16 binary connectors (4 or 5 are mostly used)and 1 unary connector

Precedence Order

Number	Name	Symbol
1.	Not	$\neg$
2.	Or	$\vee$
3.	And	$\wedge$
4	Implied	$\rightarrow$
5	Double implication	$\leftrightarrow$

- **Tautology** - always true
- **Contradiction** - always false

Equivalence

- Two formulas, F1 $\equiv$ F2 if their truth values are equivalent

De Morgan's Rule

- $$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$
- $$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$
- $$P \rightarrow Q \equiv \neg P \vee Q$$

> *This is the same because if P is false, then !P is true, then Q could be either true or false, it does not matter. Only one or the other matters as it is an **OR** statement. If !P is false, Q could still be true.*

Identities of Equivalence

$$P \vee F \equiv P$$

$$P \vee T \equiv T$$

$$P \wedge F \equiv F$$

$$P \wedge T \equiv P$$

$$P \vee Q \equiv Q \vee P$$

$$P \wedge Q \equiv Q \wedge P$$

$$P1 \wedge P2 \vee P3 \ldots \vee PK$$

- Any order can be used because of distrubution

$$P \wedge !P \equiv F$$

$$P \land \neg P \equiv T$$

$$(P \to Q) \equiv \neg Q \to \neg P$$

$$((P \land Q) \to R) \equiv P \to (Q \to R)$$

Left Side

1. $$\neg(P \land Q) \lor R$$
2. $$\neg P \lor \neg Q \lor R$$

Right Side

1. $$P \to (\neg Q \lor R)$$
2. $$\neg P \lor \neg Q \lor R$$

$$\neg(P_1 \lor P_2) \equiv \neg P_1 \land \neg P_2$$

Suppose we have 3

$$\neg(P_1 \lor P_2 \lor P_3) \equiv \neg P_1 \land \neg P_2 \land P_3$$

What if we have any number?

$$\neg(P_1 \lor P_2 \ldots \lor P_m) \equiv \neg P_1 \land \neg P_2 \ldots \land P_m$$

Functional Completeness

- **Functional Complete** - Set of operators is functionally complete if every formula can be expressable

$$\land, \lor, \neg$$

- **Lemma** - every formula can be expressed as a *disjunction* of *conjunctions*
  - This is called **disjunctive normal form** (DNF)

$$(P_1 \land \neg P_2 \land P_3) \lor (\neg P_1 \land P_3) \land (P_1 \land \neg P_3) \land (\ldots)$$

$$F \equiv (Q_1 \vee Q_2) \rightarrow \neg Q_3$$

Q_1	Q_2	Q_3	**F**	**Expression**
0	0	0	1	$\neg Q_1 \wedge \neg Q_2 \wedge \neg Q_3$
0	0	1	1	$\neg Q_1 \wedge \neg Q_2 \wedge Q_3$
0	1	0	1	$\neg Q_1 \wedge Q_2 \wedge Q_3$
0	1	1	0	Nothing
1	0	0	1	$Q_1 \wedge \neg Q_2 \wedge \neg Q3$
1	0	1	0	Nothing
1	1	0	1	$Q_1 \wedge Q_2 \wedge \neg Q_3$
1	1	1	0	Nothing

Compute DNF of

$$P \rightarrow Q$$

P	**Q**	$P \rightarrow Q$	**Expression**
1	1	1	$(P \wedge Q) \vee (\neg P \wedge Q) \vee (\neg P \vee \neg Q)$
1	0	0	Nothing
0	1	1	Nothing
0	0	1	Nothing