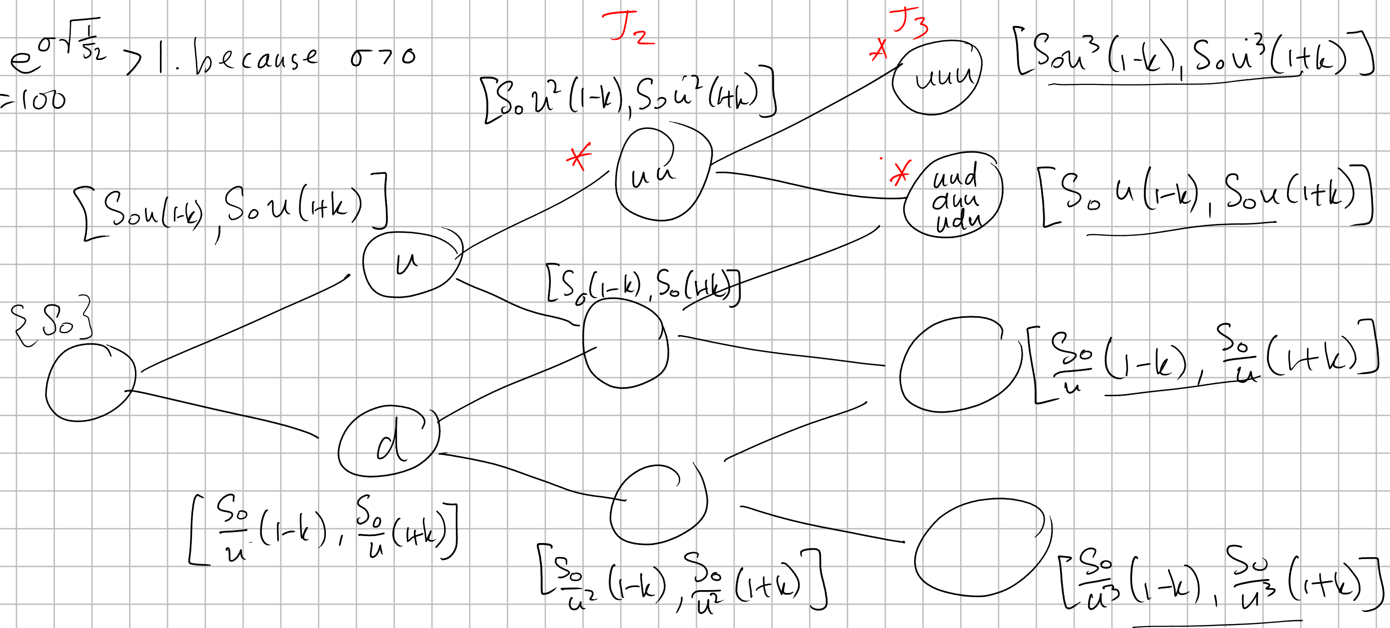
$u = e^{\sigma\sqrt{\frac{1}{52}}} > 1$. because $\sigma > 0$
$S_0 = 100$



$C_0 = 0$
$C_1 = 0$
$C_2 = 0$
$C_3 = (-100, 1)\,\mathbb{1}_{\{S_3 > 100\}} = \begin{cases} (-100,1) & \text{if } S_3 > 100 \\ (0,0) & \text{if } S_3 \le 100 \end{cases}$ $\quad (S_3 = S_0 u^k$ where $k = -3,-1,1,3)$

Look at node uuu: $S_0u^3 > S_0 = 100$ so

$$C_3^{uuu} = (-100, 1).$$

Construction 6.1 in the paper:

$$X^{uuu} = -C_0 - C_1 - C_2 - C_3^{uuu} = (100, -1)$$

J_3^{uuu} is a function $\mathbb{R} \to \mathbb{R} \cup \{\infty\}$ defined as

$$J_3^{uuu}(x) = \begin{cases} \underline{100 - x} & \text{if } x \in [S_0u^3(1-k), S_0u^3(1+k)] \\ \infty & \text{if } x \in \mathbb{R} \setminus [S_0u^3(1-k), S_0u^3(1+k)] \end{cases}$$

similarly

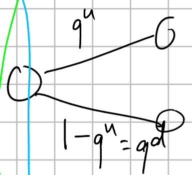
$$J_3^{uud}(x) = \begin{cases} 100 - x & \text{if } x \in [S_0u(1-k), S_0u(1+k)] \\ \infty & \text{if } x \in \mathbb{R} \setminus [S_0u(1-k), S_0u(1+k)] \end{cases}$$

So now to construct J_2 at node uu we write $f_2^{uu}: \mathbb{R} \to \mathbb{R} \cup \{\infty\}$

$$f_2^{uu}(x) = \inf \; q^u \Big(\underbrace{J_3^{uuu}(x^u)}_{f_1(x^u)} + a_2 \ln \underbrace{\frac{q^u}{P_2^{uuu}}}_{g_1(q^u) = q^u x} \Big) + (1-q^u)\Big(\underbrace{J_3^{uud}(x^d)}_{f_2(x^d)} + a_2 \ln \underbrace{\frac{1-q^u}{P_2^{uud}}}_{g_2(1-q^u)}\Big)$$

$P_2^{uud} = 1 - P_2^{uuu}$

f



subject to $\quad q^u + (1-q^u) = 1$

$q^u \in [0,1]$

$x^u \in \operatorname{dom} J_3^{uuu} \iff J_3^{uuu}(x) < \infty$

this equivalence does not always hold in earlier steps $\to \iff x^u \in [S_0u^3(1-k), S_0u^3(1+k)]$

$x^d \in \operatorname{dom} J_3^{uud} \iff \cdots \iff x^d \in [S_0u(1-k), S_0u(1+k)]$

$q^u x^u + (1-q^u)x^d = x.$

~ - if we take this away then we are computing the convex hull of J_3^{uud} and J_3^{uuu}

In Appendix A of the paper:
$m = 2$
(A.2)

$f_2^{uu}(x)$ is finite for all $x \in \operatorname{conv}\{[S_0u^3(1-k), S_0u^3(1+k)], [S_0u(1-k), S_0u(1+k)]\}$

we restrict it to current bid-ask interval:

$$J_2^{uu}(x) = \begin{cases} f_2^{uu}(x) & \text{if } x \in [S_0u^2(1-k), S_0u^2(1+k)) \\ \infty & \text{otherwise.} \end{cases}$$

Prop A.4 $\to$ the inf is attained, we can replace it by min

Section A2 $\to$ piecewise linear approximation to f_2^{uu} / f.