Exoplanet Detection Methods

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1 Introduction and Motivation

Exoplanet detection is a vast and varied field with multiple detection techniques. The type of technique used for detection impacts the occurrence rate of different types of planets. An important aspect, therefore, of exoplanet research, is a thorough understanding of possible detection techniques and comparisons between them.

2 Methods

To gain a better understanding of the limitations and benefits of various exoplanet detection methods, we evaluated each method for the case of a Jupiter-like planet orbiting a Sun-like star. The planet detection techniques we assessed were: radial velocity, transit, direct imaging, and astrometry. For each method, the mathematical origins, chosen parameters, and limitations are explored below.

2.1 Radial Velocity

One method of exoplanet detection is radial velocity. This technique utilizes the relation between the orbital position of a planet and its radial velocity [1]. Radial velocity detection results from an observable periodic change in the stellar radial velocity measured through Doppler shift in the observed stellar spectrum [2]. Another way of thinking of this is observing when the exoplanet causes its host star to "wobble," which changes the color of observable light. This method results in finding a minimum planetary mass. To evaluate this method, we find the period, P, and the amplitude of the radial velocity signal, K. We use Kepler's Third law to calculate P, with the assumption of a circular orbit [2]:

$$P = 2\pi\sqrt{\frac{a_J^3}{GM_\odot}} \tag{1}$$

where a_J is the semi-major axis, G is the gravitational constant, and $M_\odot$ is the mass of the host star. We calculate K using the following equation with the assumption that the planet is being viewed exactly head-on which gives the inclination, i, as 90 degrees [2]:

$$K = \frac{M_J}{M_\odot}\sqrt{\frac{GM_\odot}{a_J}}\sin(i) \tag{2}$$

where M_J is the mass of the Jupiter-like planet.

2.2 Transit

The next method we evaluated of exoplanet detection is transit. Transit detection is utilized to find when a planet passes directly between a star and an observer, dimming the star's light by a measurable amount [2]. This method was also evaluated with the assumption of viewing the system edge-on and the exoplanet having a circular orbit. The transit depth, f, is proportional to the square ratio of the Radius of the Jupiter-like planet, R_J, to the radius of the Sun-like star, $R_\odot$ [2]:

$$f = \frac{R_J}{R_\odot}^2 \tag{3}$$

The transit duration, t, is how long it takes for the planet to cross the star. This is found with [2]

$$t = \frac{2R_\odot}{v_J} \tag{4}$$

where v_J is the velocity of the Jupiter-like planet. Assuming a circular orbit, this then allows for the calculation of v_J [2]:

$$v_J = \sqrt{\frac{GM_\odot}{a_J}} \tag{5}$$

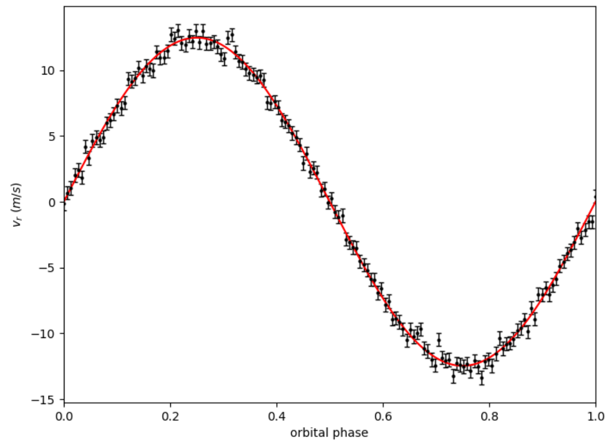

Figure 1: The RV signal for a Jupiter-like planet orbiting a Sun-like star. We plot the radial velocity over the phase of the planet's orbit. The amplitude of the signal is $K = 12.1$ m/s. The period of the signal is $P = 11.9$ yr. The noise is characteristic of state-of-the-art RV data.

2.3 Direct Imaging

The direct imaging technique for exoplanet detection is at the most basic level, directly capturing pictures of exoplanets. To take these pictures, the glare from the light of the host star an exoplanet orbits must be removed. We calculate the fraction of light that the exoplanet reflects, f [2]:

$$f_{reflect} = \frac{\pi R_J^2}{4\pi a_J^2} A \tag{6}$$

where A is the albedo, which is an expression of a surface's ability to reflect light (here the surface is the exoplanet.) We also calculate the ratio between the thermal emission of the planet and the star, as calculated by the Plank function [2].

$$f_{emission} = \left(\frac{R_p}{R_*}\right)^2 \frac{\exp(h\nu/k_B T_*) - 1}{\exp(h\nu/k_B T_p) - 1} \tag{7}$$

where T_* and T_p are the surface temperatures of the star and planet, respectively, and ν is the frequency of light we observe at. The total contrast is the sum of these two fractions [2].

$$f = f_{emission} + f_{reflect} \tag{8}$$

For direct imaging, we also need to be able to resolve the planet as a separate object from the star. This depends on the wavelength of the light we observe (λ) and the aperture of the telescope with which we observe it (D) in the following way [2]:

$$\theta \sim 1.22 \frac{\lambda}{D} \tag{9}$$

2.4 Astrometry

Astrometry relates to the wobble of a star in space caused by the orbit of a planet. This manifests as observable motion of the stellar photocenter in the sky [2]. The definition of the center of mass is [2]

$$M_p a = M_* a_* \tag{10}$$

The astrometric wobble of the star is given by the value a_*. At a distance d, the angular size θ of this wobble is therefore given by [2]

$$\theta = \left(\frac{M_p}{M_\odot}\right) \frac{a}{d} \tag{11}$$

3 Results

We calculate the detection signals for a Jupiter-sized planet orbiting a Sun-like star. For radial velocity, we assume that the system is edge-on ($i = 90°$), and therefore the $\sin(i)$ term in Eq. 2 is equal to 1. We also assume that the planet is on a circular orbit, which makes the RV curve sinusoidal. Given these assumptions, we would observe an RV amplitude of $K = 12.5$ m/s and a period of $P = 11.9$ yr. A plot of a sample RV light curve is shown in Fig. 1. We compare this to the state-of-the-art RV amplitude, which is 0.5 m/s, which is smaller than the signal we calculate. This indicates that a Jupiter-like planet would be detectable using radial velocity.

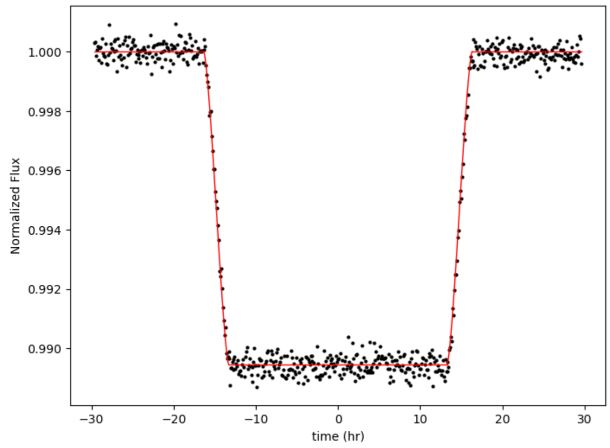

Figure 2: A transit signal for Jupiter-like planet orbiting a Sun-like star. The y-axis of this plot is the normalized flux of the star, with 1 equalling the flux of the star when the planet is not transiting. The transit depth here is $f = 0.01$ and the transit duration is $t = 29.6$ hr. The noise is characteristic of state-of-the-art transit data.

For the transit method, we again assume the planet is on a circular orbit and that $i = 90°$. This allows us to use the equations from Section 2.2 to calculate transit depth and duration. We calculate that the transit depth is $f = 0.01$ and the transit duration is $t = 29.6$ hr. A plot of one of these transits is shown in Fig. 2. This transit depth is larger than the transit depths of other planets discovered, such as Kepler 42c [7], indicating that a Jupiter-like planet would be detectable with the transit method.

For direct imaging, we calculate the total contrast between the planet and the star. We use an albedo of $A = 0.503$ [3], the surface temperature of the star to be $T_* = 5780$ K [4], and the surface temperature of the planet to be $T_p = -110\,°\mathrm{C}$ [5]. Observing at a wavelength of 3.8μm (an observable wavelength using the state-of-the-art Gemini Planet Imager [6]), we calculate a total contrast of $f \sim 10^{-9}$. The current state-of-the-art contrast is $\sim 10^{-7}$, which is larger than the contrast we calculate. To see if a Jupiter-like would even be resolvable from the star, we use Eq. 9, $\lambda = 3.8\mu$m, a telescope diameter of 8.1 meters, and an orbital radius of 5.2029 AU, to find that our current technology would be able to resolve a Jupiter-like planet if we observed it in a system less than 44 pc away. The typical distance at which we observe exoplanets is about 400 pc. This and the low contrast make a Jupiter-like planet unlikely to be detected by direct imaging.

For the astrometry method, we again assume the planet is on a circular orbit and we assume that we view the system face-on ($i = 0°$). We use Eq. 10 to solve for a_* and find that the star would wobble on a circular orbit with a radius approximately equal to the radius of the star. If we observe this system at a distance of 400 PC (the typical distance we observe exoplanets), the angular size of this wobble is $\sim 20\mu$as. The best instruments can currently only detect as low as $\sim 100\mu$as, indicating that we are unable to detect a Jupiter-like planet with astrometry.

Fig. 3 shows all known exoplanets. The data was taken from the NASA Exoplanet Archive (NEA) on 26 January 2023. It plots the mass and radius of each planet against its semimajor axis and orbital period. The figure also shows the detection method used to discover each planet. We also plot the Solar System planets for reference. Finally, we find the detection limits for the radial velocity, transit, and direct imaging methods, and add those to the plot. We use the state-of-the-art detection limits for each case as described above, and the equations in Section 2 to obtain expressions for the mass and radius of the planet versus its semimajor axis and orbital period. We also use the mass-radius relationships described in [8]. We see that the interior, rocky planets are undetectable by any of the methods. We see that Uranus and Neptune would be barely detectable by transits, but undetectable with RV or direct imaging. Finally, we see that Jupiter and Saturn are detectable using transit and RV, but likely undetectable using direct imaging. This is consistent with the calculations for a Jupiter-like planet above.

4 Discussion/Conclusion

We conclude that different methods of exoplanet detection have different levels of effectiveness in different parameter spaces. Each detection limit leads to detection biases that must be accounted for when examining the demographics of exoplanets. For example, RV and transit are able to detect large numbers of massive planets on short orbits but are unable to detect small planets

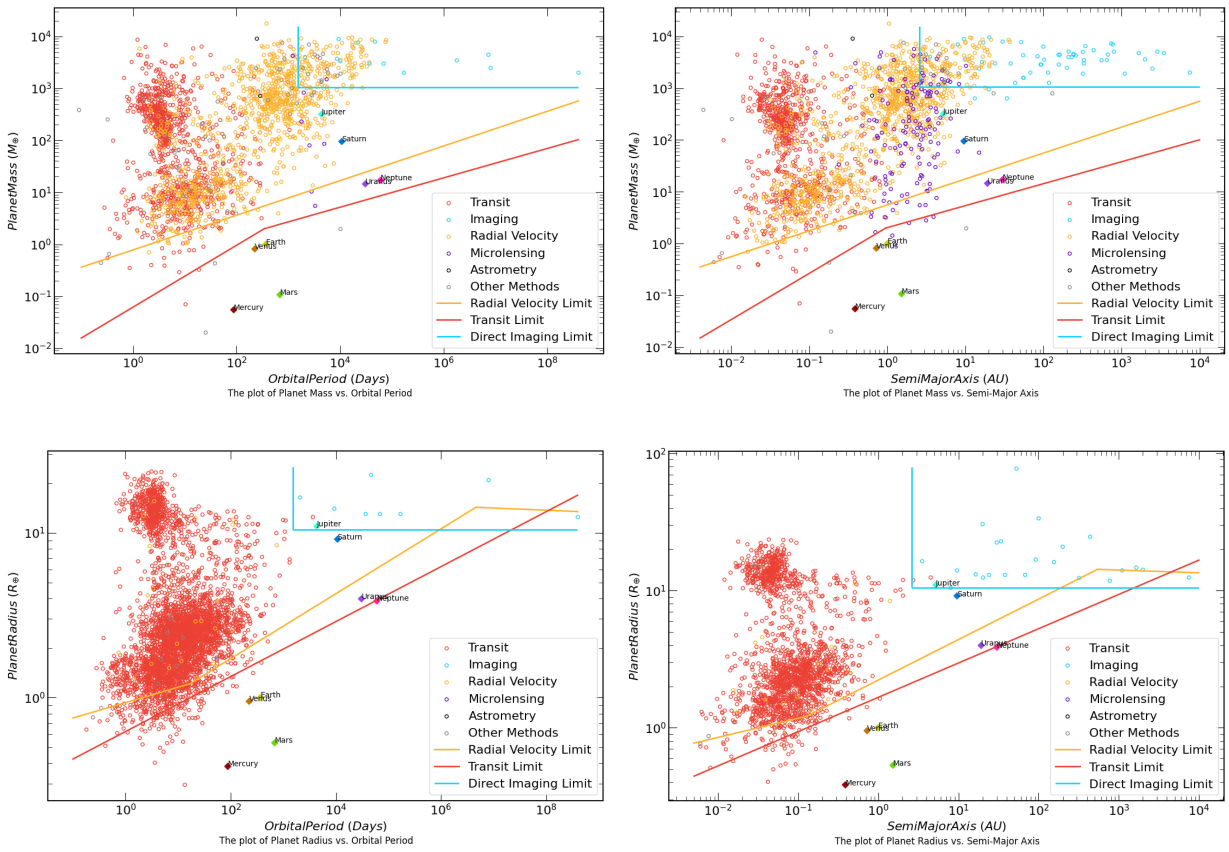

Figure 3: This figure shows known exoplanets plotted by mass and radius vs semimajor axis and orbital period. The colors of each data point indicate the method that was used to detect each planet. The lines show the current detection limits of each method. For reference, we indicate the positions of the Solar System planets.

near the orbit of Earth. These plots can, however, shed light on the evolutionary processes of planets. For example, the existence of the radius gap between $6 - 9R_\oplus$ tells us about the process of run-away gas accretion that occurs when an Earth-sized protoplanet accretes mass onto it to create a giant planet. The existence of this gap indicates that this process takes only a short time, like a few thousand years.

We also conclude that a Jupiter-sized planet orbiting a Sun-like star would be detectable using our current technology. In particular, the RV and transit signals of Jupiter are well larger than the state-of-the-art limits. For direct imaging, the contrast between Jupiter and the Sun seems to be large enough for it to be detected, but it would not be possible to fully resolve Jupiter as a separate object using our current telescope technology. It is possible that observing at different wavelengths or with a larger telescope may make Jupiter detectable with direct imaging. The astrometric signal for Jupiter, however, would only be on the order of 10μas, much smaller than what our current technology could effectively resolve, meaning that Jupiter would be undetectable using astrometry.

5 References

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