Calculating Solar Sodium Abundance

Matt Lastovka, William Wu, Cassie Moats, Sam Eckart, Kolya Larson

28 March 2023

1 Introduction

An important extension on the study of stars and exoplanets is made by analysing absorption lines from light passing through their atmospheres. These absorption lines are indicators of significant atomic or molecular presence that have effectively consumed the light of discrete wavelengths throughout a continuous spectrum. The usefulness of this behavior lies in that we can reverse engineer the location of absorption lines to determine the composition of a star's photosphere, or, for instance, an exoplanet's atmosphere.

On top of learning *what* atoms or molecules are present, we can also determine *how much* of that species is present in terms of number and column densities, as well as stellar abundance. During this experiment, we focused specifically on two prominent sodium absorption lines (sodium D-lines) from the sun's photosphere [1].

2 Methods

2.1 Calculating Equivalent Width and Number Density

To determine the equivalent width, we need to match the area of a rectangle with height (flux) equal to that of the continuum emission, with the area of the absorption spectral line. To do this in a more mathematically abstract way, we employ the following equation for equivalent width [2],

$$W_\lambda = \int \frac{F_c - F_s}{F_c}\, d\lambda \tag{1}$$

Here, F_c is the continuum intensity and F_s is the spectral absorption line intensity, which can be read directly from the data table used in this experiment and is visualized below.

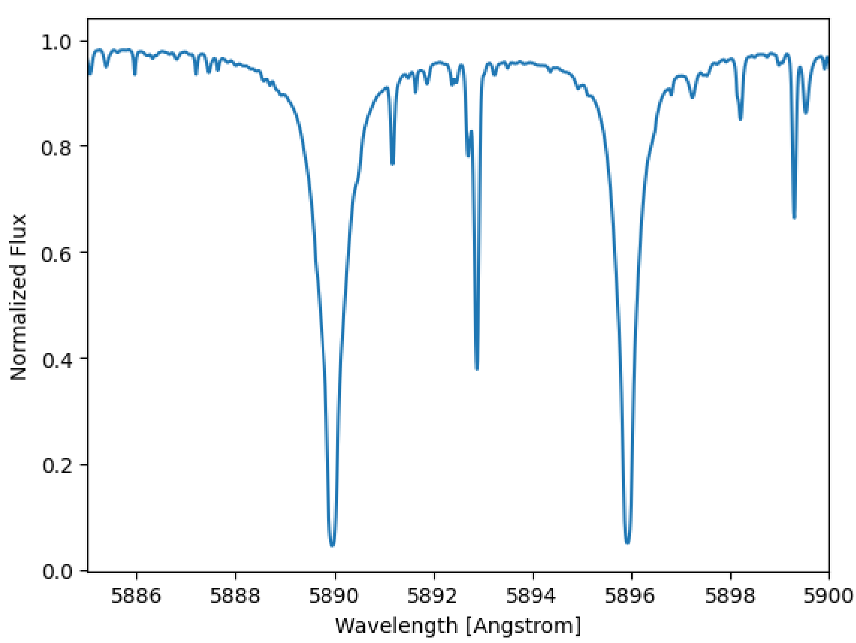

Figure 1: Normalized flux vs wavelength graph for the sodium doublet.

With the equivalent width, we can look at the curve of growth for the sun in order to obtain a point and recover the number density from the expression governing the x-axis,

$$\log\left(\mathrm{Nf}\left(\frac{\lambda}{5000\text{Å}}\right)\right)$$

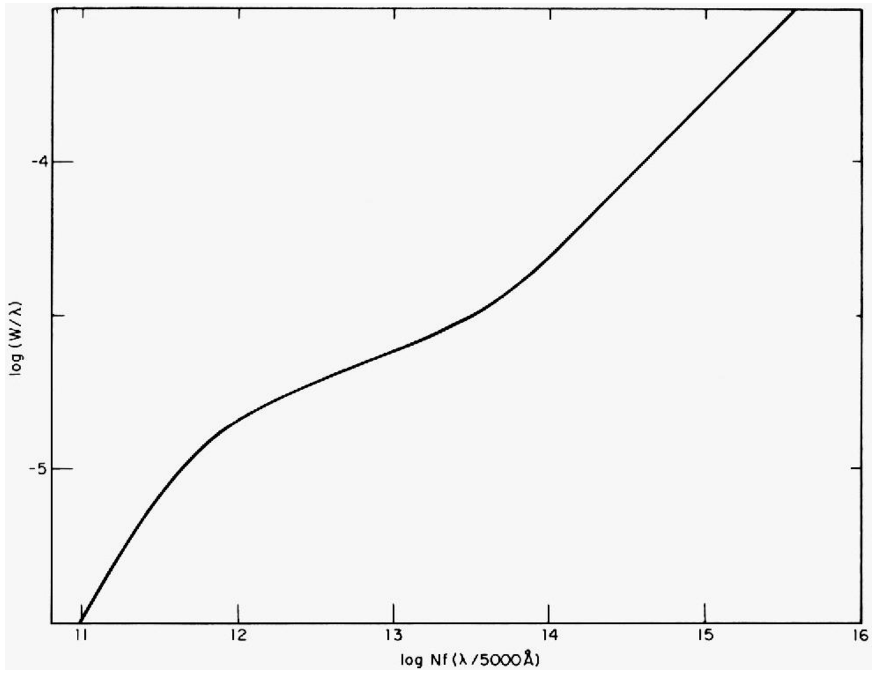

Figure 2: Plot of the curve of growth for the Sun.

2.2 Calculating Excited to Ground State Ratio

To determine the ratio of ground state atoms to excited state atoms for the sodium doublet, we make use of the *Boltzmann Equation* [3]:

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp\left(-\frac{E_2 - E_1}{kT}\right) \tag{2}$$

Within this equation, we begin with the subscripts 1 and 2, which correspond to the 3s and 3p states respectively. N is the number density of the atoms in the state, g is the number of unique individual states that are degenerate in energy, E is the energy of that state, k is the Boltzmann constant, and T is taken to be the effective temperature of the sun.

2.3 Calculating Ionized to Neutral Sodium Ratio

To determine the ratio of ionized sodium atoms to neutral sodium atoms, we make use of the *Saha Equation* [3]:

$$\frac{\mathrm{Na_{II}}}{\mathrm{Na_{I}}} = \frac{2kT}{P_e}\frac{Z_{II}}{Z_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{\frac{3}{2}} \exp\left(-\frac{\chi}{kT}\right) \tag{3}$$

In this equation, we have the number density of Na_{II} and Na_I, k and T again as the Boltzmann constant and effective temperature of the sun respectively, Z_{II} and Z_I as the partition functions, P_e as the electron pressure, h as Planck's constant, and χ as the ionization energy.

2.4 Calculating Total Column Density

To calculate the total column density, we must incorporate the previous two results in the following equation,

$$N_{\mathrm{Na}} = N_1 \times \left(1 + \frac{N_2}{N_1}\right) \times \left(1 + \frac{Na_{II}}{Na_I}\right) \tag{4}$$

The first term is the value obtained from the curve of growth, and the final two terms are what incorporate the Excited to Ground state and Ionized to Neutral state ratios. This equation returns the total number of sodium atoms, which is equivalent to the total column density.

2.5 Calculating Sodium Abundance Relative to Hydrogen

Finally, we want to find the abundance of sodium relative to hydrogen. We can do this in two different ways – a simplified physics version, and a more complex astronomy version.

The physics version is simply the ratio between the calculated total sodium count and supplied total hydrogen count,

$$\frac{\mathrm{Na}}{\mathrm{H}} = \frac{N_{\mathrm{Na}}}{N_{\mathrm{H}}} \tag{5}$$

From here, we use it as an input for the astronomy version of the calculation.

$$\frac{\text{Na}}{\text{H}} = 12 + \log\left(\frac{N_{\text{Na}}}{N_{\text{H}}}\right) \tag{6}$$

3 Results

With all of the theory provided, we can now run through the results we determined throughout the experiment. Starting with the Equivalent Width. By selecting a domain which the entire left-side spectral absorption line inhabits, we can manually integrate over the relevant parts of the data set with python. The result of this integration is $W_\lambda = 0.835\text{Å}$. We then apply the y-axis expression from the curve of growth using the true D-line wavelength, $\lambda = 5890\text{Å}$ to find

$$\log\left(\frac{W_\lambda}{\lambda}\right) = \log\left(\frac{0.835}{5890}\right) = -3.851 \tag{7}$$

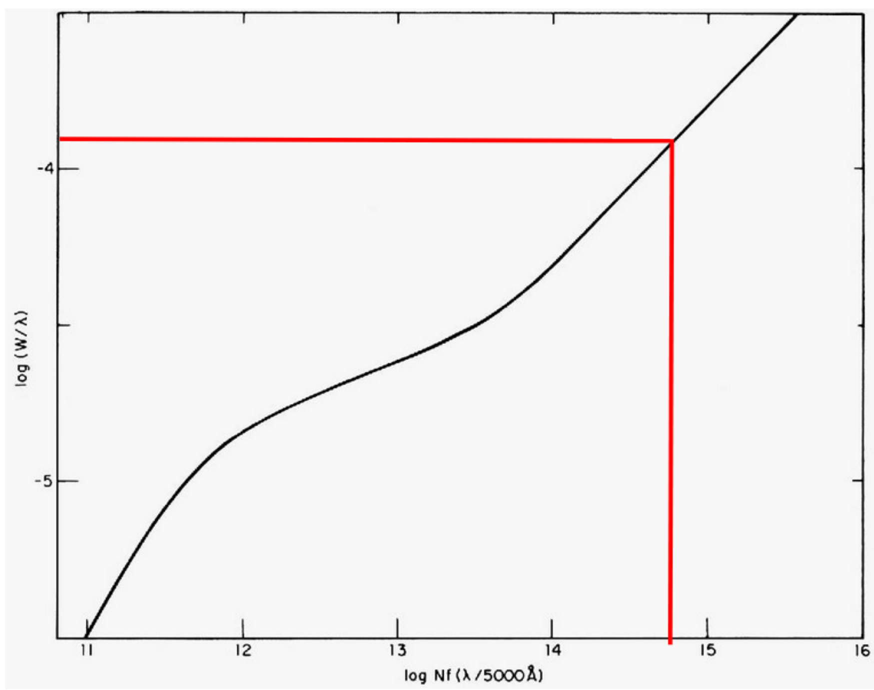

Figure 3: Sun's curve of growth graph with the point of interest clearly marked with red lines.

From this, we locate the point that corresponds to this y-value on the curve of growth. We find it to correspond with an x-value of roughly 14.8, which we can then obtain the number density from,

$$\log\left(\text{Nf}\left(\frac{\lambda}{5000\text{Å}}\right)\right) = 14.8$$

Assuming f = 0.65, we then find,

$$N = \frac{10^{14.8}}{0.65\left(\frac{5890}{5000}\right)} = 8.24 \cdot 10^{14}\text{atoms/cm}^2 \tag{8}$$

With this, we can move on to the ratio of the excited to ground state atoms. We use that there are 6 possible 3p states, $g_1 = 6$ and 2 possible 3s states $g_2 = 2$, as well as the ground state energy, $E_1 = 0\text{eV}$, and excited state energy, $E_2 = 2.11\text{eV}$ to satisfy the unknowns of equation [2]. Assuming $T = T_{\text{eff}} = 5780\text{K}$ for the sun and $k = 8.617 \cdot 10^{-5}\frac{\text{eV}}{\text{K}}$ we find the ratio between the excited and ground state atoms,

$$\frac{N_2}{N_1} = \frac{2}{6}\exp\left(-\frac{2.11 - 0}{(8.617 \cdot 10^{-5})(5780)}\right) = 0.00482 \tag{9}$$

The next step is to determine the ratio of ionized to neutral atoms. We start with determining the unknowns in equation [3]. We take $Z_{II} = 1$ and $Z_I = 2.4$ from the partition function, $P_e = n_e kT = 1.0\frac{\text{N}}{\text{m}^2}$ as the electron pressure, and $\chi = 5.1\text{eV}$ as the ionization energy. We must use the previous values for k and T, and also define the electron mass, $m_e = 9.1 \cdot 10^{-31}\text{kg}$, and Planck constant, $h = 4.136 \cdot 10^{-15}\frac{\text{eV}}{\text{Hz}}$. So,

$$\frac{\text{Na}_{\text{II}}}{\text{Na}_{\text{I}}} = 2517.951 \tag{10}$$

Which gives us the ratio between the ionized and neutral state sodium atoms. Determining the total column density of sodium atoms is now an easy task, as we have determined all of its unknown values. We take $N_1 = 8.24 \cdot 10^{14}$, $\frac{N_2}{N_1} = 0.00482$, and $\frac{\mathrm{Na_{II}}}{\mathrm{Na_I}} = 2517.951$ and use them as inputs for equation [4],

$$N_{\mathrm{Na}} = 2.086 \cdot 10^{18} \text{atoms} \tag{11}$$

Finally, we can discuss the solar abundance of sodium with respect to hydrogen. The simple physics version is easy enough to compute, using $N_{\mathrm{Na}} = 2.086 \cdot 10^{18}$ and $N_{\mathrm{H}} = 6.6 \cdot 10^{23}$ in equation [5], we find,

$$\frac{\mathrm{Na}}{\mathrm{H}} = 3.16 \cdot 10^{-6} \tag{12}$$

which we use to find the galactic astronomer's solar abundance value from equation [6]

$$\frac{\mathrm{Na}}{\mathrm{H}} = 6.4997 \tag{13}$$

Unfortunately, this is our first and only opportunity to check our work against a documented value. The accepted galactic astronomer's solar abundance value for Na with respect to H is 6.30 [4]. Obviously, our result of 6.5 is quite close to this, and certainly reassuring that all of the steps made in this complex calculation align with the accepted methodology. The percent error on our result with respect to the accepted value is given,

$$\left| \frac{6.4997 - 6.30}{6.30} \right| \cdot 100 = 3.17\% \tag{14}$$

One final value we can compute is the stellar astronomer's sodium abundance. To do this, we compare the calculated and literature galactic astronomer's abundance values for the sun with the following equation,

$$[\mathrm{N_a/H}] = \log \left(\frac{(\mathrm{N_a/H})_{\mathrm{calculated}}}{(\mathrm{N_a/H})_{\mathrm{literature}}} \right) \tag{15}$$

which gives us,

$$[\mathrm{N_a/H}] = \log \left(\frac{6.4997}{6.3} \right) = 0.0136 \tag{16}$$

This is a pretty good result, considering it's true value should be 0. Being off by such a small amount is similarly reassuring to the percent error we found earlier.

4 Conclusion

We did not go through this work for other spectral lines, although, this same method could be used to determine *any* atomic or molecular presence in the sun's photosphere, given that you can associate spectral lines with that particular atom or molecule. The small error (3.17%) lends itself as evidence that the procedure we went through is associated with the accepted measurements of the solar abundances for sodium. Furthermore, this same technique could be used to determine the composition of other stars, and by determining the abundance of heavy elements in those stars, we can begin to make judgements about the internal structure of its exoplanets such as the core mass fraction. All of this is to say, we can accurately measure the presence of specific atoms from great distances, which allows us to narrow down the definition of an exoplanet's habitability, or determine characteristics of a distant host star to compare it to our own.

5 References

[1] UCI RWFSodium Powerpoint
[2] Stahler, Steven; Palla, Francesco (2004). *The Formation of Stars*
[3] PSU Boltzmann and Saha Writeup
[4] Palme, Lodders, Jones, Solar System Abundances of the Elements

6 Contribution Statement

Cassie Moats and Kolya Larson collaboratively completed the data analysis on the sodium absorption lines, producing all of the results presented in this paper. Matt Lastovka wrote and gave the presentation. Sam Eckart wrote the written report.