Digital Technologies and Value Creation

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Learning objectives of today

Goals:

- Understand what optimization is and where it appears
- Understand the concepts of objective, decision variables and constraints
- Understand that linear programming is a particular type of optimization

How will we do this?

- Examples of where optimization occurs in real-life
- Worked examples for linear programming

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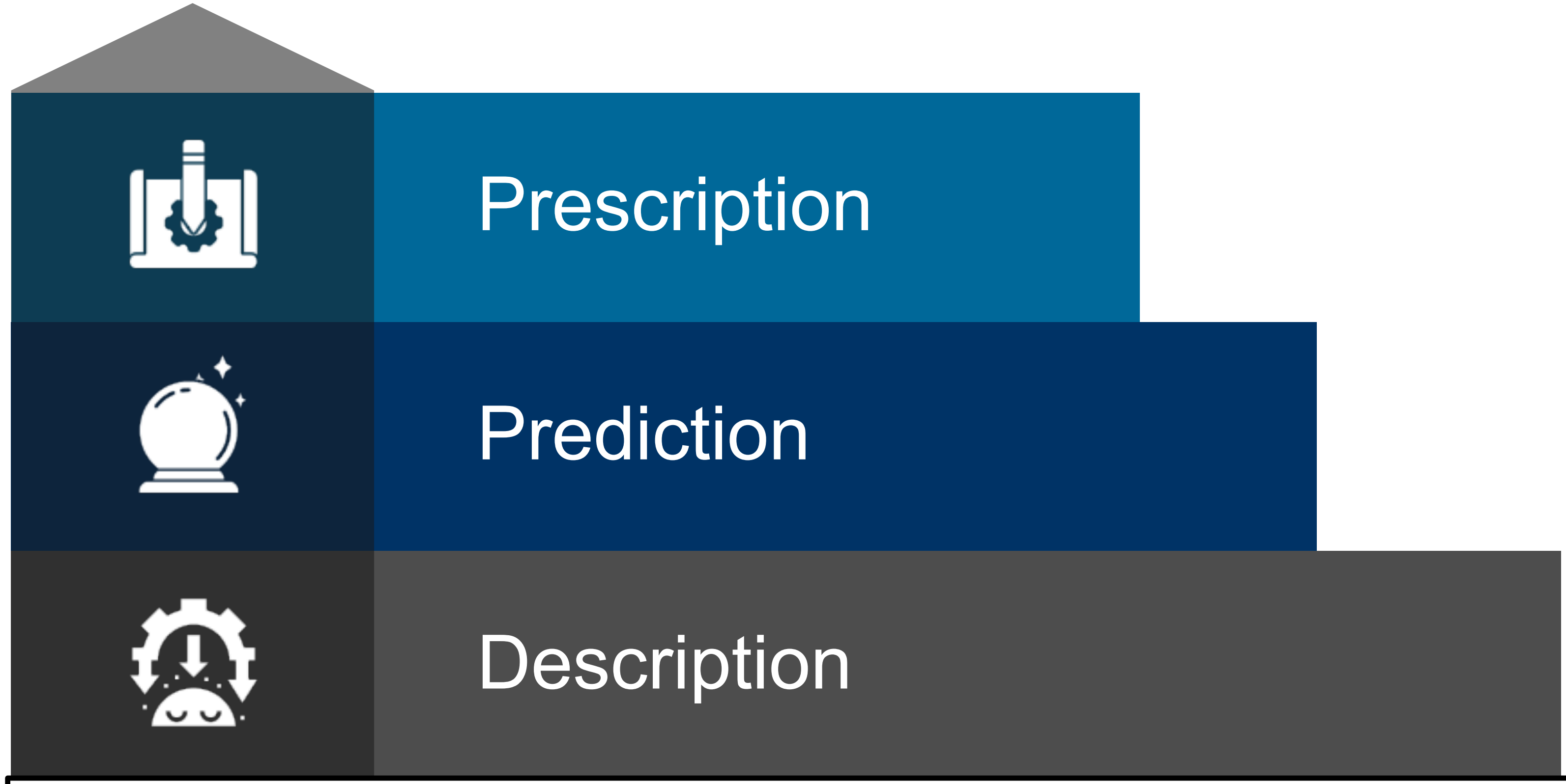
Introduction to optimization

Reaching the top of the analytics hierarchy



Experimentation, optimization

Building a solid foundation: Gathering data (internally, externally), cleaning, pre-processing

What is optimization?

- Optimization = making the most out of any situation

- You routinely solve optimization problems in your everyday life:

What to eat at lunch?

- Maximize healthiness
- Or maximize pleasure?

- Don’t want to eat same as yesterday
- Vegetarian
- Not eat something I could spill on myself

How to organize my week?

- Maximize long periods with nothing to do

- Certain amount of homework to be done
- No work on Saturday evenings/Sundays
- Need to eat (and sleep?)

Building blocks of optimization

All these examples have three commonalities: these are the building blocks of optimization.

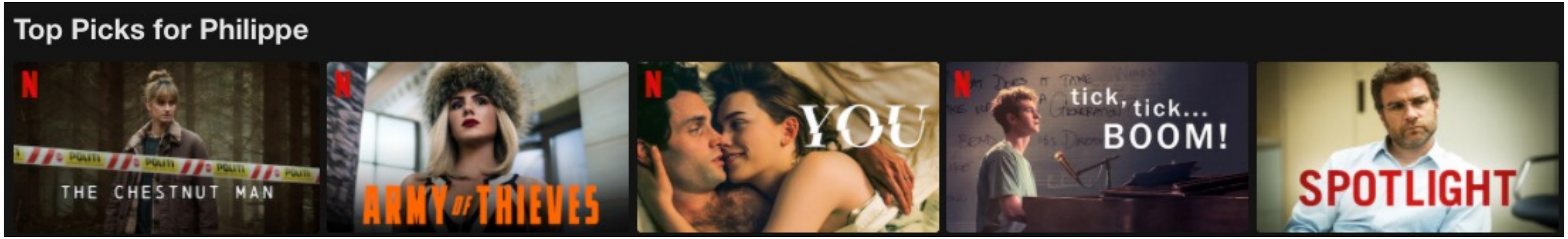
Decision variables	What you have control over to change, the alternatives you choose among many
Objective	Quantity that you are trying to maximize or minimize
Constraints	Restrictions on what your decision variables can be

Always, always start with **decision variables. Always**.

Optimization in companies

Used routinely in businesses:

NETFLIX



Netflix's recommendation system

- Maximize the chance that I will click on one of the movies suggested

- Can't show me movies I've already seen
- Maximum of 10 movies to show

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Optimization in companies

Used routinely in businesses:

Amazon package delivery

- Minimize the cost to deliver my package

- Cannot take more than a couple of days
- Cannot go through warehouses that are full
- Must arrive at the time I specified

Optimization in companies

Many other examples:

- **Finance**: how to invest money into stocks to maximize profit
- **Revenue management and pricing**: how to price tickets or goods so as to maximize occupancy
- **Resource allocation:** how to allocate goods (e.g., rooms, staff, etc.) or other resources (e.g., budgets) while respecting everyone's constraints
  - Marketing: how much budget to allocate to each campaign?
  - Organizational design: how much budget to allocate to different development programs?
- **Routing:** shortest path to take (e.g., Google Maps)
- **Facility location:** where to construct a new facility? Which facility to close?
- And many more…

A typical optimization problem

A distribution example

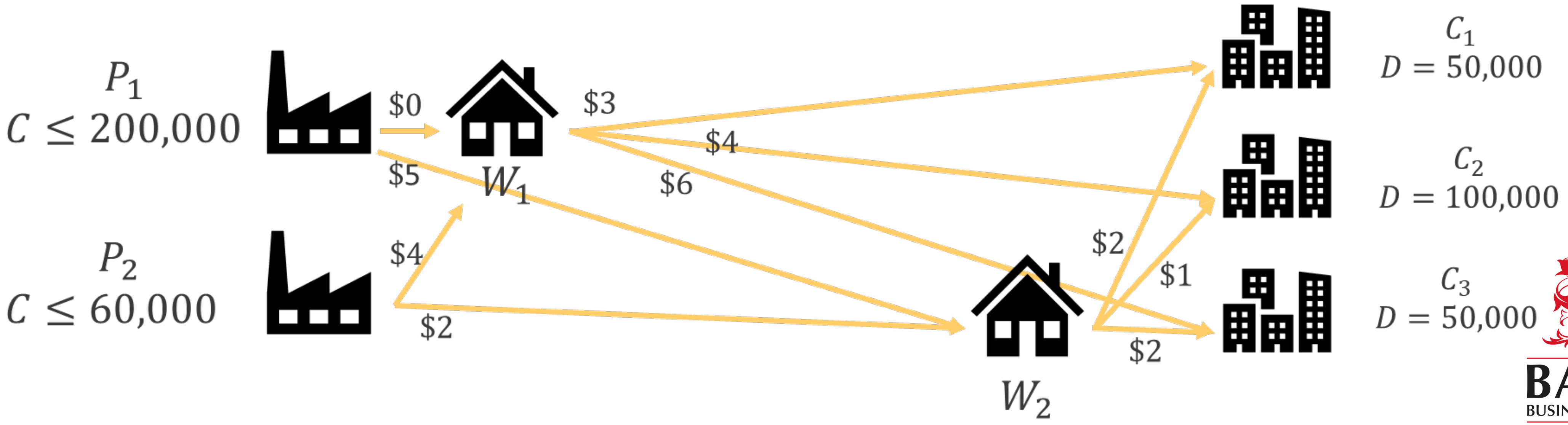
We direct the logistics for a firm that produces and manages

- A single product
- Two plants: P1 with capacity 200,000 units and P2 with capacity 60,000. The manufacturing costs are identical.
- Two warehouses with equal holding costs: W1 close to production, W2 close to the customers

A distribution example

We direct the logistics for a firm that produces and manages

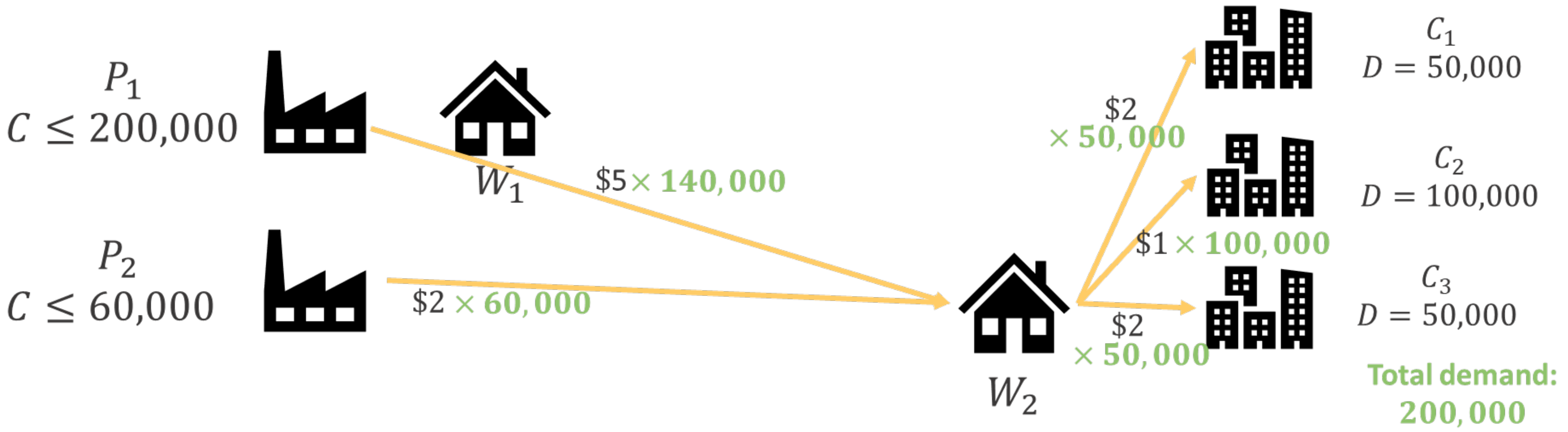
- A single product
- Two plants: P1 with capacity 200,000 units and P2 with capacity 60,000. The manufacturing costs are identical.
- Two warehouses with equal holding costs: W1 close to production, W2 close to the customers
- Three customer regions C1, C2, C3 with demands 50,000, 100,000, and 50,000 units



The big question

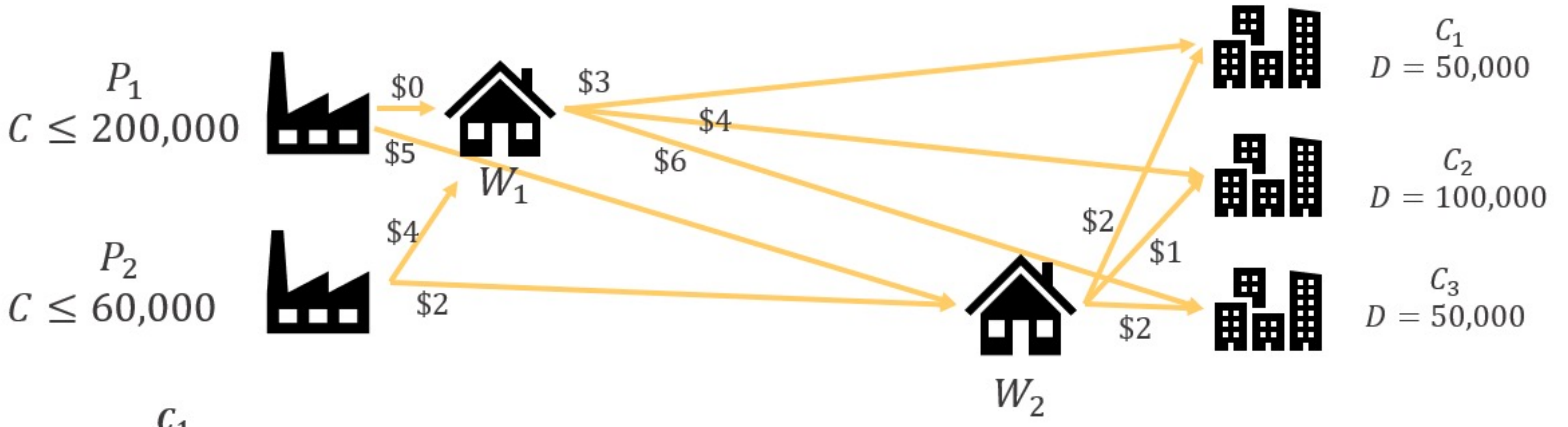
What is the minimum total cost to fulfill the demand?

Idea 1: Ship everything to the warehouse closest to the market (low dispatch costs from the warehouse)



Total cost of this strategy: $1,120,000

Idea 2: dispatch based on min-to-market cost



C_1

$P_1 \to W_1 \to C_1$	3\$
$P_1 \to W_2 \to C_1$	7\$
$P_2 \to W_1 \to C_1$	7\$
$P_2 \to W_2 \to C_1$	4\$

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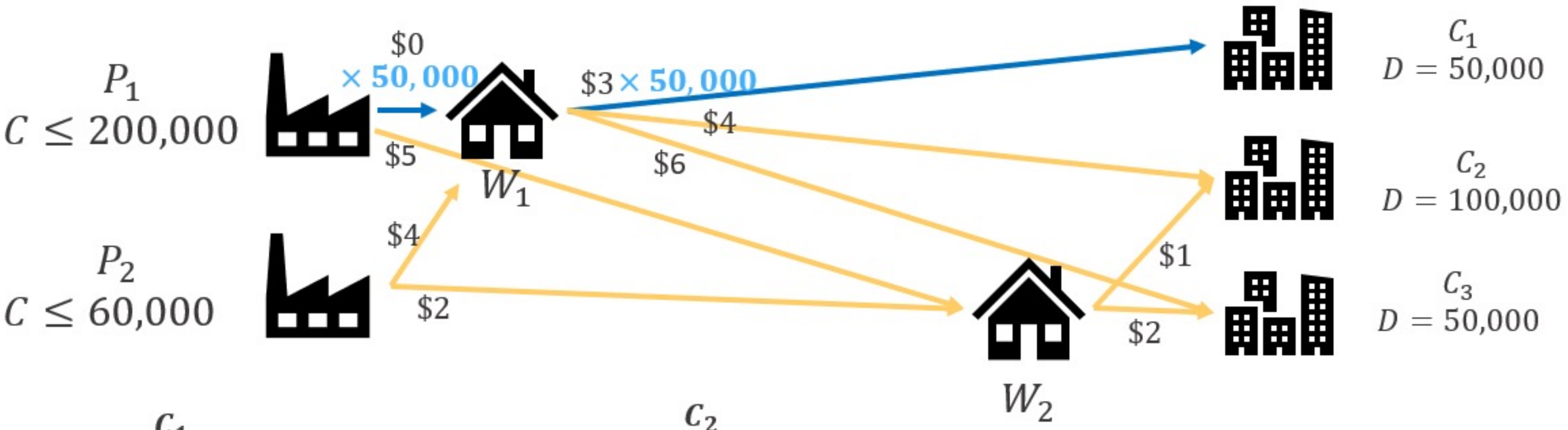
Some intuitive solutions

Idea 2: dispatch based on min-to-market cost



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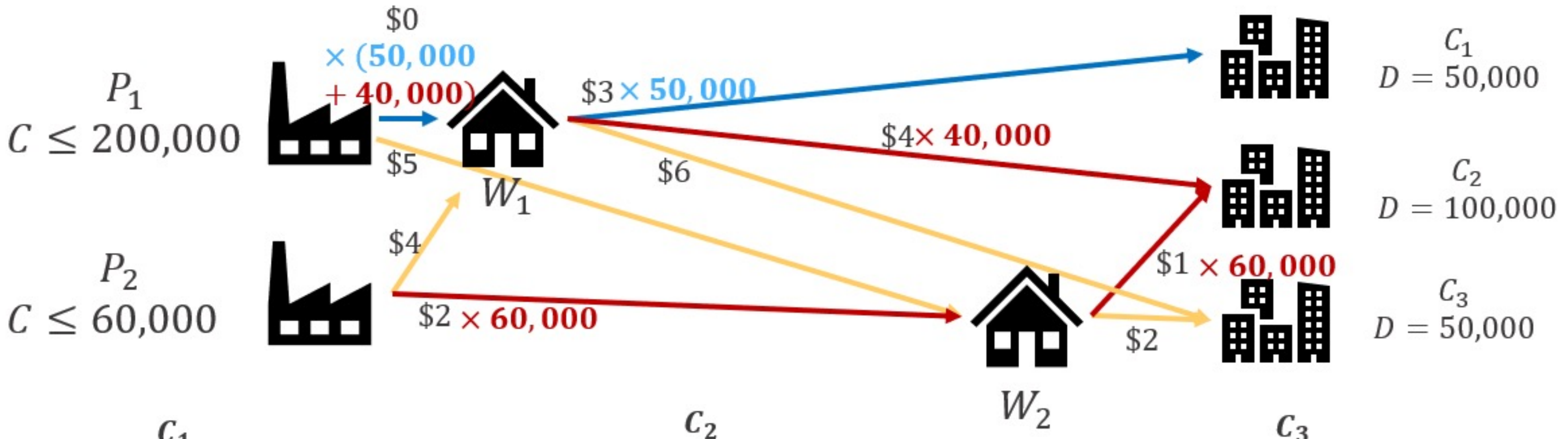
$P_1 \to W_2 \to C_1$	7$
$P_2 \to W_1 \to C_1$	7$
$P_2 \to W_2 \to C_1$	4$

C_2

$P_1 \to W_1 \to C_2$	4$
$P_1 \to W_2 \to C_2$	6$
$P_2 \to W_1 \to C_2$	8$
$P_2 \to W_2 \to C_2$	3$

Idea 2: dispatch based on min-to-market cost



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C_3

$P_1 \to W_1 \to C_3$	6$
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$P_2 \to W_2 \to C_3$	4$

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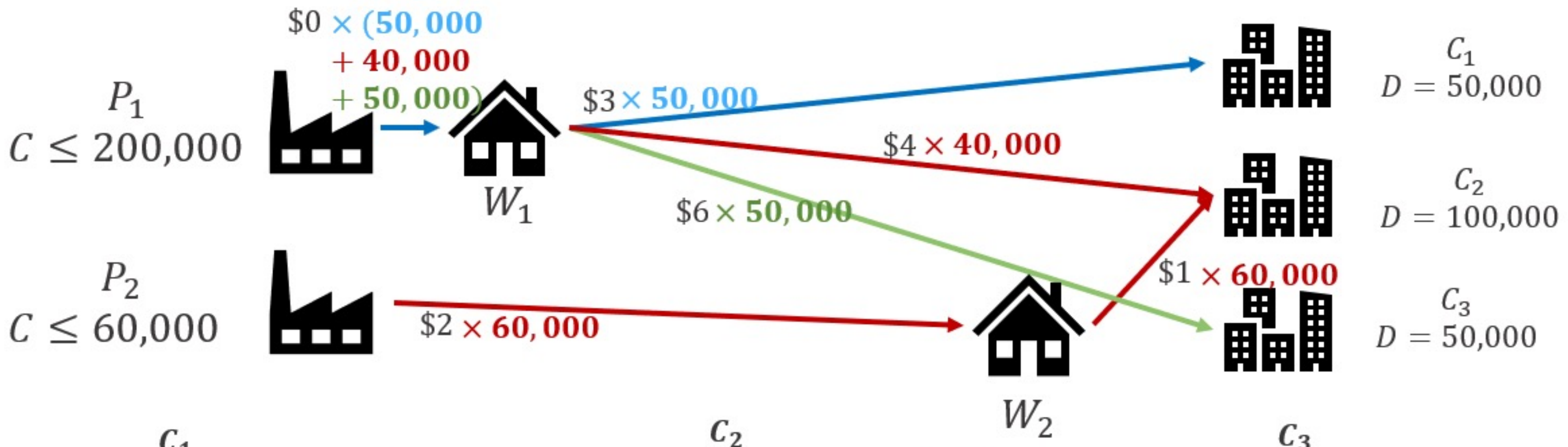
Idea 2: dispatch based on min-to-market cost



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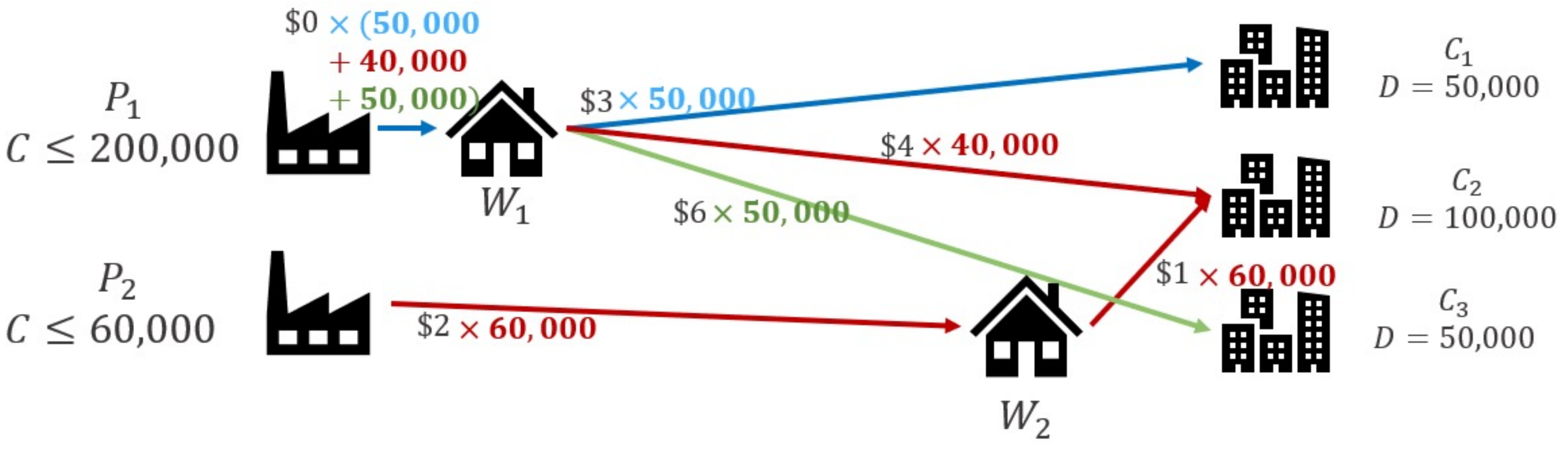
Idea 2: dispatch based on min-to-market cost



Total cost of this strategy: $790,000

Better than before. Can we do even better?

Yes! Optimal allocation gives us $740,000. How to find it?
First step: formulate the problem.

Modeling the problem

Decision variables: how much we ship from one place to another.

- Plants to warehouses: $x_{P_1 \to W_1}, x_{P1 \to W_2}, x_{P_2 \to W_1}, x_{P_2 \to W_2}$
- Warehouses to customers: $x_{W_1 \to C_1}, x_{W_1 \to C_2}, x_{W_1 \to C_3}, x_{W_2 \to C_1}, x_{W_2 \to C_2}, x_{W_2 \to C_3}$

Constraints: in=out, demand satisfied, capacity.

- In=out at W_1: $x_{P_1 \to W_1} + x_{P_2 \to W_1} = x_{W_1 \to C_1} + x_{W_1 \to C_2} + x_{W_1 \to C_3}$
- In=out at W_2: $x_{P_1 \to W_2} + x_{P_2 \to W_2} = x_{W_2 \to C_1} + x_{W_2 \to C_2} + x_{W_2 \to C_3}$
- Capacity: $x_{P_1 \to W_1} + x_{P_1 \to W_2} \leq 200{,}000$ and $x_{P_2 \to W_1} + x_{P_2 \to W_2} \leq 60{,}000$
- Demand satisfied: $x_{W_1 \to C_1} + x_{W_2 \to C_1} \geq 50{,}000$ and $x_{W_1 \to C_2} + x_{W_2 \to C_2} \geq 100{,}000$ and $x_{W_1 \to C_3} + x_{W_2 \to C_3} \geq 50{,}000$
- Quantities are nonnegative: $x_{P_1 \to W_1} \geq 0, x_{P1 \to W_2} \geq 0, \ldots$

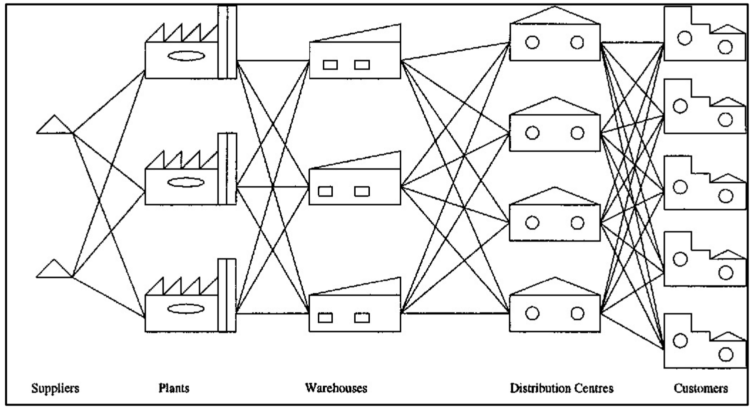
Objective: minimize cost

- $0x_{P_1 \to W_1} + 5x_{P1 \to W_2} + 4x_{P_2 \to W_1} + 2x_{P2 \to W_2} + 3x_{W_1 \to C_1} + 4x_{W_1 \to C_2} + 6x_{W_1 \to C_3} + 2x_{W_2 \to C_1} + x_{W_2 \to C_2} + 2x_{W_2 \to C_3}$

Solving optimization problems

We modeled it – now how to solve it?

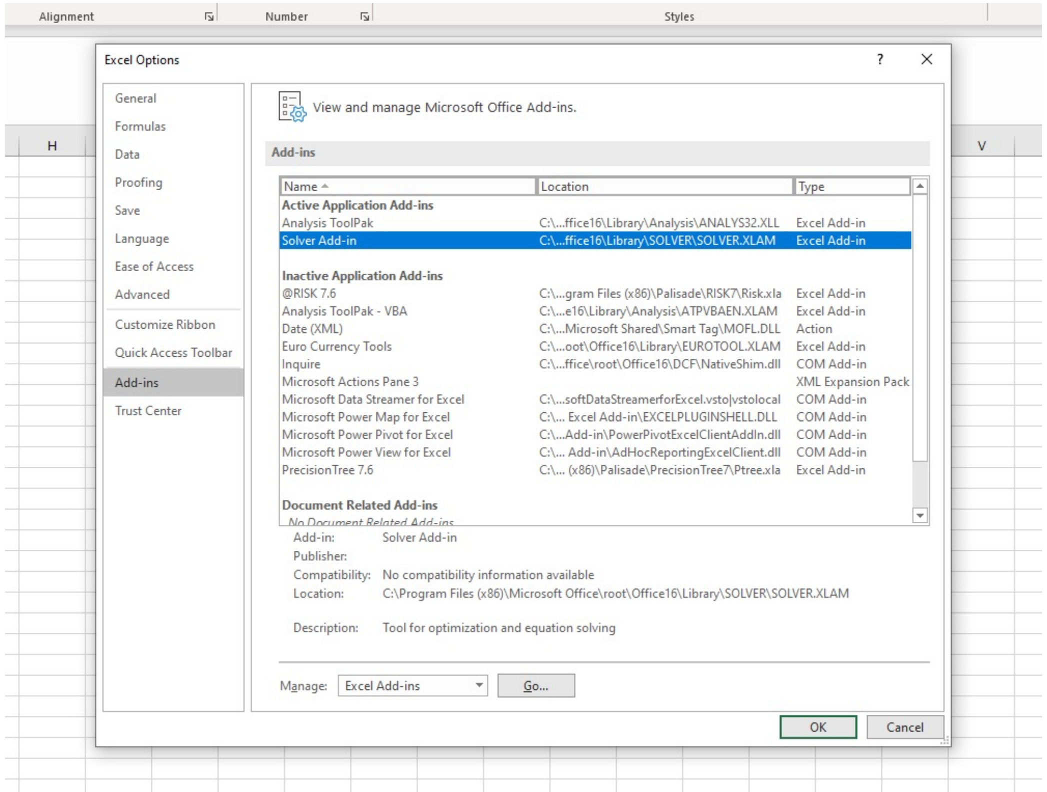
- Could try to solve this manually, plugging all possible solutions in
- Would take a long time – and this is a small problem (only 10 decision variables) What about this?



- Thankfully, we have automated ways of solving this: requires a “solver”.

Solvers

- Any solver takes an optimization problem as input in a “standard” form and outputs a solution.
- We will use the Excel solver (visual, easy to use)
- But of course, there are many other solvers (also in Python!)



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Solving our problem with the Excel solver

Linear programming and the Excel Solver

Optimality of the solution

- In what we've seen so far, there is (at least) one optimal solution.
- But other things could happen:

$$\begin{gathered}\min_{x,y} x+y \\ s.t.\, 0 \le x \le 1 \\ 0 \le y \le 1 \\ x+y \ge 3\end{gathered}$$

$$\begin{gathered}\min_{x,y} x+y \\ s.t.\ x \le 1 \\ y \le 1 \\ x+2y \le 0\end{gathered}$$

What happens in these cases?

Infeasible problem
(no x, y can be found such that the constraints are true)

Unbounded problem
(we can take x, y as small as we want, and it still works)

In summary: only three cases can happen when solving an optimization problem: **optimal, infeasible, unbounded.**

Optimality in the solver

How do we know which category we fall in in **Solver**?

The Objective Cell values do not converge.

Solver can make the Objective Cell as large (or small when minimizing) as it wants.

Unbounded problem

Solver could not find a feasible solution.

Solver can not find a point for which all Constraints are satisfied.

Infeasible problem

Solver found a solution. All Constraints and optimality conditions are satisfied.

When the GRG engine is used, Solver has found at least a local optimal solution. When Simplex LP is used, this means Solver has found a global optimal solution.

Optimal problem

Pros and cons of Solver

Pros:

- Very “visual” to solve
- Uses software that all companies are familiar with
- Integrates well into a presentation
- Faster to set up if you have a small dataset

Cons:

- Very limited in terms of size (200 variables, 100 constraints)
- Can be quite buggy/crashes
- Actual “solver” in the background not very good
- Slower to set up if you have a very large dataset

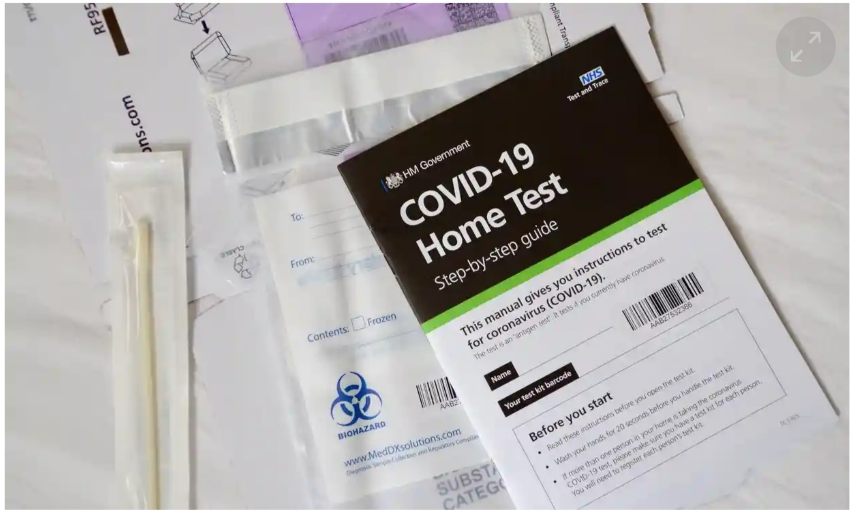
⇒ This is where more advanced solvers come into play. Usually, we interface them through Python and other programming languages (through an API ☺)

Limitations of Excel can have real consequences

Covid: how Excel may have caused loss of 16,000 test results in England

Public Health England data error blamed on limitations of Microsoft spreadsheet

- Coronavirus - latest updates
- See all our coronavirus coverage



▲ More than 50,000 potentially infectious people may have been missed by contact tracers after 15,841 positive tests were left off the daily figures. Photograph: Simon Leigh/Alamy

A million-row limit on Microsoft's Excel spreadsheet software may have led to Public Health England misplacing nearly 16,000 Covid test results, it is understood.

The data error, which led to 15,841 positive tests being left off the official

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What drives solving complexity?

- Not all optimization problems can be solved easily
- Many factors drive solving time and quality of solution obtained:
  - Underlying algorithm used in solver
  - Number of variables/number of constraints
  - **Most importantly:** the constraint and objective “types”

- Depending on the type of constraints/objective, it could take infinitely long to get a solution

Importance of linear programming

Linear programming

A linear program only has **constraints that are linear combinations of the decision variables**, i.e.,

$$a_0 + a_1x_1 + \cdots + a_nx_n \leq 0$$
$$a_0 + a_1x_1 + \cdots + a_nx_n \geq 0$$
$$a_0 + a_1x_1 + \cdots + a_nx_n = 0$$

where $x_1, \ldots, x_n$ are the decision variables.

A linear program only has an **objective that is linear**, i.e.:

$$c_0 + c_1x_1 + \cdots + c_nx_n$$

We know how to solve linear programs **really fast** and we can guarantee that the solution we obtain is **correct.**

Try it yourself

The situation

A company makes two products (X and Y) using two machines (A and B).

Each unit of X that is produced requires 50 minutes processing time on machine A and 30 minutes processing time on machine B. Each unit of Y that is produced requires 24 minutes processing time on machine A and 33 minutes processing time on machine B.

At the start of the current week there are 30 units of X and 90 units of Y in stock. Available processing time on machine A is forecast to be 40 hours and on machine B is forecast to be 35 hours.

The demand for X in the current week is forecast to be 75 units and for Y is forecast to be 95 units. Company policy is to maximize the combined sum of the units of X and the units of Y in stock at the end of the week, while meeting demand.

The task

1. What are the decision variables?
2. What are the constraints?
3. What is the objective?
4. Write down the corresponding optimization problem
5. Solve the problem using the Excel Solver!

See you in class!