The Application and Misapplication of Factor Analysis in Marketing Research

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The use of factor analysis as a method for examining the dimensional structure of data is contrasted with its frequent misapplication as a tool for identifying clusters and segments. Procedures for determining when a data set is appropriate for factoring, for determining the number of factors to extract, and for rotation are discussed.

# The Application and Misapplication of Factor Analysis in Marketing Research

Factor analysis is one of the more widely used procedures in the market researcher's arsenal of analytic tools. Despite its wide-scale usage, factor analysis is not a universally popular technique and has been the subject of no small amount of criticism (Ehrenberg 1968; Ehrenberg and Goodhart 1976). Both the popularity and the criticism of the method can be traced to the availability of computer programs that are easy to use and require little knowledge of the underlying theory or methodology of factor analysis. This rather blind use of factor analysis is a principal cause of the dissatisfaction with the technique.

So widespread are current misconceptions about factor analysis in the marketing community that even its defenders and some prominent reviewers perpetuate misinformation. Many reviewers have discussed the use of factor analysis as a clustering procedure, at best an extreme perversion of the method. The alleged subjectivity of the technique is part of the folklore of the discipline. In a field where precedent often provides justification for action, misinformation about and misapplication of factor analysis tend to be perpetuated. The intent of this article is to review selected issues involved in the use of factor analysis.

## *WHAT IS FACTOR ANALYSIS?*

Factor analysis is a multivariate statistical technique that is concerned with the identification of structure within a set of observed variables. Its appropriate use involves the study of interrelationships among variables in an effort to find a new set of variables, fewer in number than the original variables, which express that which is common among the original variables. Factor analysis establishes dimensions within the data and serves as a data reduction technique. Three general functions may be served by factor analysis.

1. The number of variables for further research can be minimized while the amount of information in the analysis is maximized. The original set of variables can be reduced to a small set which accounts for most of the variance of the initial set.
2. When the amount of data is so large as to be beyond comprehension, factor analysis can be used to search data for qualitative and quantitative distinctions.
3. If a domain is hypothesized to have certain qualitative and quantitative distinctions, factor analysis can test this hypothesis. Thus, if a researcher has an *a priori* hypothesis about the number of dimensions or factors underlying a set of data, this hypothesis can be submitted to a statistical test.

Much of the expressed dissatisfaction with factor analysis can be attributed to its use for purposes other than those stated above.

## *COMMENTS ON THE INTERPRETATION OF FACTORS*

A basic assumption of the factor analyst is that major differences found in everyday human relationships become part of the language of the culture. At one level factor analysis is concerned with how people use the language, its words, concepts, etc. and the empirical relationships within the language. The underlying assumption is that these empirical relationships within the language will reveal something about human behavior on another level. A factor is a qualita-

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tive dimension, a coordinate axis. It defines the way in which entities differ much as the length of an object or the flavor of a product defines a qualitative dimension on which objects may or may not differ. A factor does not indicate how much different various entities are, just as knowing that length is an important physical dimension does not indicate how much longer one object is than another. Quantitative differences may very well be important, but the identification of a particular factor does not provide that information. Factor analysis provides a dimensional structure for data; it indicates the important *qualities* present in the data. This definition of factor analysis is clearly in keeping with both Thurstone's (1942, p. 69–70) and Cattell's (1978, p. 44–6). It also has a bearing on at least two common misconceptions about factors.

Factors are not clusters. The confusion between factors and clusters seems to have arisen because a cluster is "more concrete, immediately evident, and easier to understand than a factor" (Cattell 1978, p. 45). Clusters are defined by relatively contiguous points in space and are not the axes that are factors. The criterion for admission to a cluster is much more arbitrary than is the definition of a factor. There are no well-established rules for the definition of a cluster as there are for a factor. As a result, clusters tend to be weaker theoretical constructs than are factors. Cattell (1978) provides a systematic treatment of the distinction. It is sufficient here to point out that there will almost always be more clusters than factors though a cluster may be close to a particular factor. Because factor analysis provides a set of coordinate axes, numerous clusters can be defined by the position of entities on any one dimension and by various combinations of dimensions. It is also possible to obtain meaningful factors in the absence of clusters. Such a circumstance would occur where the subject coordinates are homogeneously distributed in factor space. The practice of interpreting an otherwise successful factor analysis in terms of clusters of products, or clusters of television shows, misses the point of factor analysis. Such clusters could probably be identified more readily by other means (Ehrenberg and Goodhart 1976).

Few factor analytic studies in the marketing literature have provided such an interpretation. Ehrenberg and Goodhart (1976) give three examples of marketing applications of factor analysis. In two of the examples, one involving purchase frequency of ready-to-eat cereals and the other involving television viewing habits, an attempt is made to interpret the factors as clusters. Ziff (1974) and Pernica (1974) describe studies using factor analyses of psychographic data as a means for identifying market segments, i.e., clusters of persons similar with respect to certain attributes. In each case what the factor analysis has actually done is to identify the important dimensions by which people or products *may* be differentiated. The determination of actionable market segments requires analysis beond factoring. Horton (1979) provides an example of a more appropriate use of factor analysis. Starting with a large number of personality and demographic variables, he used factor analysis to reduce the data to six basic dimensions. These six dimensions, rather than the original set of variables, were then related to product usage by means of discriminant and canonical variate analyses. Thus, factor analysis provided a means for reducing the number of variables in the study without great loss of information and served to identify the important qualitative distinctions in the data.

Also related to the notion that factors are underlying dimensions involving an examination of the use of language (through there may be applications in which language is less important) is the problem of differences among factors across populations. Differences in factors have been found between samples taken from different cultures, between samples of children and adults, and between samples of men and samples of women. In each case it is reasonable to assume that the individuals who make up the samples use language somewhat differently. Thus, when language differences are suspected to be present, separate factor analyses may be warranted. In most marketing applications they are not. There is no reason to presume that the user of beer (at least when sober) will use the language in a manner different from that of the nonuser. The underlying dimensions should be the same. In fact, when different factors emerge analysis must stop. Apples and oranges cannot be compared beyond mere description. The marketing researcher should hope to find the same factors so that comparisons of magnitude on the same factor can be made through the use of factor scores.

## *MODES OF FACTORS ANALYSIS*

Most market researchers are familiar with two modes of factor analysis, the *R* technique whereby the relationships among items or variables are examined, and the *Q* technique whereby the relationships among persons or observations are examined. Several other modes of analysis with which market researchers may be less familiar may be useful to the thoughtful researcher. Table 1 is a description of the several modes of factor analysis. Though not well known to market researchers, the *S* and *T* techniques appear particularly suited for the analysis of purchase incidence and message recall data. The *P* and *O* techniques might be useful for analyzing the life cycle of a particular product class or promotional campaign. Cattell and Adelson (1951) and Cattell (1953) give examples of the use of *P* technique for analyzing changes in demographic and economic characteristics of a nation over time.

All of the various modes of factor analysis provide information about the dimensional structure of data.

**Table 1**
**MODES OF FACTOR ANALYSIS**

| *Technique* | *Factors are loaded by* | *Indices of association are computed across* | *Data are collected on* |
|---|---|---|---|
| R | Variables | Persons | one occasion |
| Q | Persons | Variables | one occasion |
| S | Persons | Occasions | one variable |
| T | Occasions | Persons | one variable |
| P | Variables | Occasions | one person |
| O | Occasions | Variables | one person |

The use of factor analysis as a means for defining types, or clusters, has led to much of the criticism of the method and, in some cases, the outright rejection of factor analysis (Ehrenberg 1968). *Q* factor analysis is often a principal tool employed in market segmentation studies. Its use has continued despite repeated criticisms and its disappointingly poor record of reliability (Wells and Sheth 1974). The use of *Q* factor analysis for the identification of segments or types is one of the paramount abuses of factor analysis.

Applications of *Q* factor analysis as a clustering tool have involved at least two major misunderstandings. Both might have been avoided by a reading of the original literature of factor analysis rather than the superficial summaries found throughout the literature. The essential concepts of *Q* factor analysis can be found in Burt's early work (1917, 1937). The issues have recently been restated by Cattell (1978, p. 326–7).

> The two main problems in *Q* technique . . . concern a) the need for introducing sampling methods with variables, and b) the realization that though it correlates people it is not a method for finding types but for finding dimensions. Further, as in all transpose pairs, the dimensions from *Q* technique are systematically related to, and with proper conditions, identical with, those from the corresponding *R* technique analysis. . . .
>
> As regards *dimensions*, *Q* technique tells us nothing we do not know from *R* technique . . . . The choice of *R* or its transpose (or any other technique and its transpose) is therefore not a matter of end goal but of convenience and of the ease of meeting statistical requirements. . . .
>
> . . . Rogers' index of similarity of two people (or an ideal and a real person) by *Q*-sort overlooked that the correlation coefficient is blind to differences of level in their profiles, while other "*Q* typists" have overlooked that any different choice of variables could lead to a radical difference of correlation between two people and of subsequent sorting.

Thurstone takes a similar position (1942, p. 558).

When an analyst uses *Q* factor analysis to define types or segments, each dimension defines a segment. Figure 1 provides three examples of the types of distortions this approach may bring about. Figure 1A is a two-dimensional space. The axes represent two factors identified by *Q* technique. The points represent the vector termini of the subjects in the two-dimensional space. Two very obvious segments are present. These segments differ in terms of the direction and level of association with Factor I. A *Q* factor analysis used for typing would define two segments, one associated with Factor I and one associated with Factor II. However, Factor II makes no appreciable contribution to the differentiation of the segments. Any attempt to classify subjects on the basis of their relationship to Factor II would distort the actual clusters present in the data. In addition, if only one factor were retained in the *Q* factor analysis it would suggest only one segment. Indeed, factor analysis could not recover the two segments in the data.

Figure 1B is a set of four clusters in two-dimensional space. Again, the axes are the factors defined by *Q* technique and the points are the termini of the subject vectors in person space. All four clusters are uniquely defined. A *Q* factor analysis used for typing would distort the nature of the segments. Because a cluster interpretation of *Q* factor analysis would require a segment to correspond to each factor, no more than two segments could be identified. Rotation of the axes as is customary in factor analysis would still result in only two clusters. After rotation clusters 1 and 4 would be indistinguishable and cluster 3 would be divided between cluster 2 and cluster 1, 4.

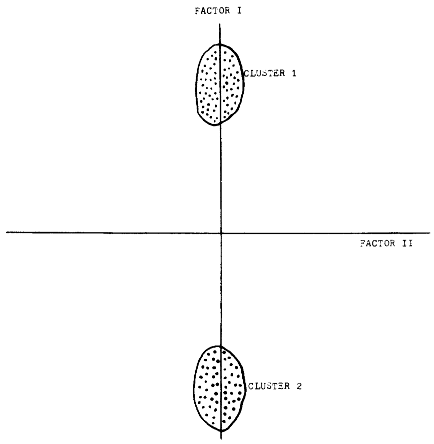
Figure 1C is only one cluster in two dimensions. However, defining clusters as axes would cause the single cluster to be divided in two on the basis of the factor loadings of the subjects. Thus, the analyst might be led to believe a unitary market was composed of two segments.

Some researchers (Levin 1965; Overall and Lett 1972; Tucker 1972) have argued that the factors obtained from *Q* analysis might be interpretable as ideal or pure types. This approach to the definition of *Q* factors would be acceptable if most individuals relate primarily to only one ideal type or if one is willing to ignore all subjects who do not load on only one factor. In most marketing applications it is unlikely that many subjects will load on a single factor. In such cases the discussion of ideal types tells the marketer little about the nature of the market segments. Subjects will belong to a different type for each dimension found.

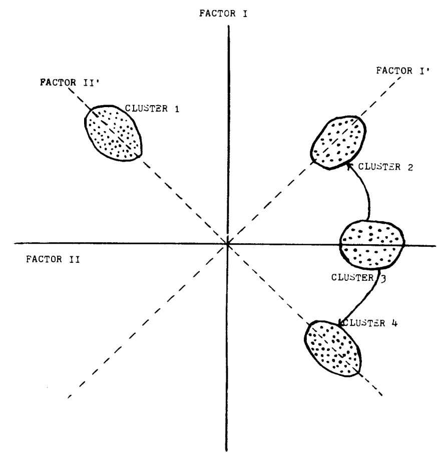
The only appropriate use of *Q* factor analysis is as the transpose of *R* factor analysis. When more variables than observations are present, *Q* factor analysis will result in a more stable factor solution than *R* factor analysis (Gorsuch 1974). The reason is that the standard error of a correlation is a function of the sample size. Cliff and Hamburger's (1967) results suggest that the standard errors of the elements of

**Figure 1**

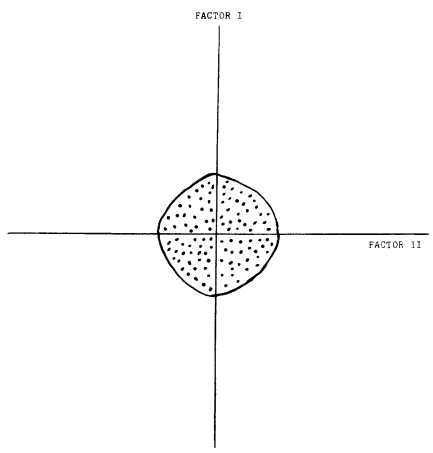
**A. Two dimensions—two clusters**




**B. Two dimensions—four clusters (dotted lines represent possible transformation of axes reducing clusters to two)**




**C. Two dimensions—one cluster**




a factor loading matrix are even greater than those of the ordinary correlation coefficients. Further, statistical independence of correlation coefficients in an *R* analysis can be obtained only when the number of subjects is greater than the number of variables. Thus, when faced with a circumstance involving more variables than subjects, an analyst can compute factors by using the *Q* technique. The solution will be more reliable than that obtained with the *R* technique. In addition, many current computer software packages will not provide a solution to an *R* factor analysis if the number of variables is greater than the number of subjects. Both Gorsuch (1974) and Cattell (1978) hold that the use of *Q* factor analysis is justified only when there are considerably fewer observations than variables. Otherwise, *R* factor analysis should be used to maximize the stability of the solution.

The relationship between the *R* mode of factor analysis and the *Q* mode has been recognized by some marketing researchers (Johnson 1970) and is often capitalized on in the computation of a *Q* factor analysis. The most commonly used computer programs for *Q* factor analysis actually begin with the *R* matrix. The rationale for this approach lies in the computational limitations of computers and the need for a matrix composed of independent correlation coefficients. No computer in use today can decompose a correlation matrix of several thousand individuals; the smaller *R* matrix is used instead. In addition, the decomposition of a *Q* matrix based on fewer variables than subjects presents a more fundamental problem of decomposition. In such a case the correlation coefficients in the *Q* matrix are not independent of one another. To obtain a unique factor solution the *R* matrix is used. The Eckhart-Young decomposition procedure (1936) can be employed to find $Q$ factors by first decomposing the $R$ matrix. The components of the $R$ matrix can then be used to determine the $Q$ factors. Yet, this is just the case in which the $R$ mode would provide a more stable result. One reason for the observed unreliability of the results of $Q$ factor analysis is the failure to use it only when the number of variables is greater than the number of observations. Thus, the use of $Q$ factor analysis with large numbers of subjects and small numbers of variables is questionable at best.

In fact there is a well-known, if complex, relationship between the factors obtained from a $R$ analysis and those obtained from a $Q$ analysis (Burt 1937; Cattell 1966a, 1978). If an unrotated factor solution is obtained from the cross-products matrix derived from the original raw score matrix, the $R$ and $Q$ techniques will produce the same factors (Cattell 1966a). If a correlation matrix is used, the first unrotated factor obtained from the $R$ technique is missing in the $Q$ technique and vice versa. The first factor obtained in $Q$ analysis is a "species" factor, that is, a general factor related to the average characteristics of the subjects. If physical measures were involved this first factor would tell only that people have longer arms than fingers. The first general factor obtained from the $R$ technique is a general size factor related to the average differences among subjects. Various transformations have been suggested to eliminate these general factors and to make $R$ and $Q$ analyses of correlation matrices equivalent. Though none of these approaches appears to completely solve the problem, the use of a double-centered matrix whereby all the means and variances of the variables and all the means and variances of the subjects are brought to the same value offers some hope.

Factors obtained from either mode are interpreted the same way, as dimensions. The factors obtained in the $R$ mode are dimensions along which attributes may differ; the factors of $Q$ analysis are dimensions along which persons may differ. Aside from the first factor the unrotated factors of $Q$ and $R$ techniques should be identical. Thus, the choice of which mode to use is a tactical decision dictated by the relationship of attributes to subjects.

Other problems with $Q$ factor analysis make it less desirable than $R$ factor analysis in most cases. For example, in significance testing one would assume a random sampling of variables from a population of variables. Because this assumption would be very difficult to defend, significance tests do not generally apply to $Q$ factor analysis. When the definition of a variable is arbitrary in terms of direction of scoring, the direction selected can affect the resulting correlation coefficient. Cohen (1968, 1969) provides an example in which one of five variables is reversed in scoring and the correlation coefficient between individuals shifts from .67 to $-.37$. This effect is not found in the $R$ technique because an individual cannot be reversed. Cohen (1968, 1969) suggests a method for addressing this problem. An additional problem associated with the use of $Q$ factor analysis with large numbers of observations is related to the rotation procedure. It is customary to rotate the eigenvectors obtained from the factoring procedure. Given current computer limitations, no more than about 10 $Q$ factors can be rotated when several thousand observations are involved. It is reasonable to assume that more than 10 dimensions are present in many data sets. The arbitrary limitation on rotation can have severe repercussions on the results of a factor analysis. Evidence that rotation of too few factors seriously distorts the final solution is presented hereafter.

A note on the use of factor analysis for defining types would not be complete without at least a mention of the problems associated with using covariance and correlation indices for that purpose. The computation of a covariance eliminates differences associated with the means of variables or observations, whereas the correlation coefficient eliminates differences attributable to both the mean and the dispersion of the variables or observations. The information contained in the correlation coefficient describes only the extent to which variables or observations vary together and the shape of their distributions. Information about mean and variance differences among individuals would be particularly important in a clustering application. Though it may be reasonable to assume that the mean and the variance of a *variable* are arbitrary, this assumption is probably not valid with respect to individuals. Computing correlation coefficients among individuals thus removes potentially meaningful information. A more appropriate measure of individual similarity is the Mahalanobis generalized distance, $D^2$, which incorporates information about mean and variance differences among individuals.

Related to the correlation problem is the common practice of using ipsative data as the starting point for a clustering procedure. Ipsative data contain information about intraindividual differences only. Interindividual differences, which would be of primary interest in market segmentation, are eliminated. Ipsative measures for a given variable are not comparable from one individual to another. Thus, a score of three on one variable for one person may not have the same meaning as a score of three on the same variable for another person. Ipsative data may arise as a result of data collection procedures, as in $Q$ sorts, or through an arithmetic transformation. Indeed, $Q$ factor analysis implicitly ipsatizes data, thus removing any mean differences among the individuals. The transformation differs from subject to subject so that comparisons among subjects are meaningless. Any data set in which the rows add up to the same value would be said to be ipsative. A comprehensive treatment of the problems posed by ipsative measures is given by Clemans (1956).

The foregoing discussion should not be taken to mean that there is no place in segmentation for factor analysis. Factor analysis, properly employed, may play an important and perhaps essential role in the definition of types. Its role is to identify the basic dimensions of a data set. These dimensions can then be used for further analyses aimed at identifying segments. Gorsuch (1974) and Cattell, Coulter, and Tsujioka (1966) provide excellent overviews of the search for types and the place of factor analysis in defining clusters.

## *TYPES OF FACTOR ANALYSIS*

Two very general types of factor analysis can be defined by the intended purpose of the analysis. In those cases where the underlying dimensions of a data set are unknown, exploratory factor analysis is appropriate. That type of analysis is most common in marketing research applications. Though exploratory factor analysis is an important tool, all too often it is applied *post hoc* to a set of data that was collected without regard to the assumptions and requirements of factor analysis. Previous factor analytic research is often ignored and factors are given new names by each researcher. Ehrenberg (1968, p. 58) has criticized the use of factor analysis for that reason.

> Factor analysis is in fact generally regarded as an exploratory technique, to be used when one happens to know nothing about the subject matter.

Factor analysis need not be merely a data reduction tool or a means for exploratory data analysis. Indeed, it is a particularly appropriate procedure for theory building. Relationships can be hypothesized and tested by use of confirmatory factor analysis, a procedure developed to allow the testing of hypotheses about the structure of a data set. This second type of factor analysis, confirmatory analysis, has been virtually ignored by marketing researchers. The most common of the confirmatory factor analytic procedures is maximum likelihood factor analysis (Jöreskog 1971). This procedure is an attempt to estimate population parameters from sample statistics. Thus, in using the procedure one seeks to provide generalizability from a sample of individuals to a population of individuals (in exploratory procedures one assumes the entire population of interest is represented in the analysis). The procedure is concerned with estimating the population correlation matrix under the assumption that the variables come from a designated number of factors. The procedure has built-in tests of statistical significance for the number of factors.

Anderson, Engledow, and Becker (1979) provide an excellent example of the use of confirmatory factor analysis in a marketing application. These researchers used maximum likelihood factor analysis to test a hypothesis about the underlying dimensions of a set of attitudinal and behavioral data. The hypothesis was generated from an exploratory factor analysis of a data set collected some years earlier.

A second example of the use of factor analysis for hypothesis testing can be found in an article by Jones and Siller (1978). Although these authors did not use the maximum likelihood procedure, they hypothesized that several dimensions related to the content of a daily newspaper could be found in audience exposure data. The authors were able to confirm their prior expectations by using a factor analytic procedure.

Any researcher embarking on the use of factor analysis is confronted with a bewildering array of techniques. Principal components, principal factors, alpha analysis, and maximum likelihood analysis are but a few of the types of factor analysis the researcher may encounter. An argument often advanced by the critics of factor analysis is that the choice of technique is crucial to the final result. Fortunately, the empirical evidence comparing the several types of factor analysis does not support this conclusion (Browne 1968; Gorsuch 1974; Harris and Harris 1971; Tucker, Koopman, and Linn 1969).

An important difference among the several procedures is the nature of the value placed in the diagonal of the covariance or correlation matrix. This issue is actually a problem of communality estimation. Communality estimation is an important theoretical issue and the interested reader can refer to Gorsuch (1974) or Cattell (1978) for a discussion of the problem. Fortunately, the communality issue is of less practical concern for the typical marketing researcher. Studies by Tucker, Koopman, and Linn (1969), Harris and Harris (1971), and Browne (1968) have demonstrated that practically any technique other than the multiple-group analysis (an early development in factor analysis, seldom used today) will lead to the same interpretations. This is nearly always the case when the number of variables is moderately large and the analysis contains virtually no variables expected to have low communalities, e.g., less than .4. The principal components procedure, which involves using ones in the diagonals of the correlation matrix, does tend to produce somewhat inflated loadings in comparison with the other procedures but otherwise yields similar results. When communalities are high there are virtually no differences among the procedures. Thus, the subjective choice of procedure ultimately has little bearing on the results of an analysis.

## *WHEN IS A DATA SET APPROPRIATE FOR FACTOR ANALYSIS?*

Even when the goal of the researcher is to identify dimensions within a set of data, factor analysis may not be appropriate. The determination of the appropriateness of factor analysis for a particular data set is not always a simple matter. The factors obtained from an analysis are often readily interpretable and intuitively appealing. Armstrong and Soelberg (1968) and Shaycroft (1970) show that an ostensibly acceptable factor structure can be obtained through the application of factor analysis to a correlation matrix based on random normal deviates. Few computer programs provide any test of the appropriateness of a matrix for factoring and even those that do then go on to produce a solution which the researcher may try to interpret.

There are several very useful methods for determining whether a factor analysis should be applied to a set of data. Some are relatively simple; others require some computation. Two of the simplest procedures for determining the appropriateness of a matrix for factoring are the examination of the correlation matrix and the plotting of the latent roots obtained from matrix decomposition. An examination of communality estimates may also be instructive. Though none of these procedures may be definitive, they may provide some important clues. If the correlation coefficients are small thoughout the matrix, factoring may be inappropriate. Factor analysis is concerned with the homogeneity of items. A pattern of low correlations indicates a heterogeneous set of items.

A plot of the latent roots obtained from a factoring procedure should ordinarily contain at least one sharp break. This break may represent the point where residual factors are separated from the "true" factors (details of this procedure are given hereafter). If a plot of the original, unrotated roots results in a continuous, unbroken line, whether straight or curved, factoring may be inappropriate. An examination of the communality estimates should reveal moderate to large communalities. Consistently small values may be an indication that factor analysis is inappropriate.

Another procedure for determining the appropriateness of a correlation matrix for factoring is an inspection of the off-diagonal elements of the anti-image covariance or correlation matrix. The anti-image of a variable is that part which is unique, i.e., cannot be predicted from the other variables. The work of Kaiser (1963) suggests that when the factor-analytic model is appropriate for a set of data, the matrix of the covariances or correlations of the unique parts of the variables should approach a diagonal. Even in the absence of a factor-analytic package that routinely produces the anti-image matrix, such a matrix can be readily obtained by $\mathbf{S}^2\mathbf{R}^{-1}\mathbf{S}^2$, where $\mathbf{R}^{-1}$ is the inverse of the correlation matrix and the diagonal matrix $\mathbf{S}^2$ is $(\text{diag } \mathbf{R}^{-1})^{-1}$. If the anti-image matrix does have many nonzero off-diagonal entries, the correlation matrix is not appropriate for factoring. Psychometric theory also suggests that the inverse of the correlation matrix, $\mathbf{R}^{-1}$, should be near diagonal if the matrix is appropriate for factoring (Dziuban and Shirkey 1974).

A widely programmed statistical test of appropriateness is Bartlett's test of sphericity (1950, 1951). Tobias and Carlson (1969) recommend that the test be applied prior to factor analysis. It is computed by the formula:

$$-\left[(N-1)-\left(\frac{2P+5}{6}\right)\right]\text{Log}_e|R|$$

where $N$ is the sample size, $P$ is the number of variables, and $|R|$ is the determinant of the correlation matrix. The statistic is approximately distributed as a chi square with $\frac{1}{2}P(P-1)$ degrees of freedom. The hypothesis tested is that the correlation matrix came from a population of variables that are independent. Rejection of the hypothesis is an indication that the data are appropriate for factor analysis. One problem with this procedure has been demonstrated by Knapp and Swoyer (1967): for sample sizes greater than or equal to 200, variables equal to 10, and a .05 level of significance, one is virtually certain to reject the independence hypothesis when the intercorrelations among the variables are as low as .09. Thus, though failure to reject is a clear sign of inappropriateness, rejection of the independence hypothesis may fail to indicate appropriateness.

A final test of the appropriateness of a matrix for factoring is a Kaiser-Meyer-Olkin measure of sampling adequacy, MSA (Kaiser 1970). This measure appears to have considerable utility and has recently been incorporated into some statistical software packages (Dixon 1975). Although it involves additional computation the MSA may be the best of the methods currently available. A measure can be obtained for both the correlation matrix as a whole and for each variable separately. The overall MSA is computed as:

$$\text{MSA} = \frac{\sum\sum_{j\neq k} r_{jk}^2}{\sum\sum_{j\neq k} r_{jk}^2 + \sum\sum_{j\neq k} q_{jk}^2}$$

where $q_{jk}^2$ is the square of the off-diagonal elements of the anti-image correlation matrix and $r_{jk}^2$ is the square of the off-diagonal elements of the original correlations. For each variable the formula is:

$$\text{MSA}_j = \frac{\sum_{k\neq j} r_{jk}^2}{\sum_{k\neq j} r_{jk}^2 + \sum_{k\neq j} q_{jk}^2}.$$

The MSA provides a measure of the extent to which the variables belong together and are thus appropriate for factor analysis. Kaiser and Rice (1974) give the following calibration of the MSA.

.90+—marvelous
.80+—meritorious
.70+—middling
.60+—mediocre
.50+—miserable
Below .50 —unacceptable

Indiscriminate application of factor analysis to data sets is both unwarranted and misleading. The use of one, or preferably several, of the preceding techniques may prevent the analyst from factor analyzing data that are inappropriate for that purpose. The analyst can then search for alternative methods for answering questions about the data.

## *THE NUMBER OF FACTORS PROBLEM, REVISITED*

Perhaps no problem has generated more controversy and misunderstanding than the number of factors problem. Ehrenberg and Goodhart (1976) discuss the "relatively arbitrary decision of how many factors are extracted." Careful examination of the literature of factor analysis indicates that the criteria for ceasing to extract factors are both well established and objective. Even when the analyst has no idea about what to expect from a data set, several very useful stopping rules are available.

Bartlett's test (1950, 1951) is one of the most widely used statistical rules for determining the number of factors to extract. It has been incorporated into a number of factor-analytic packages. Gorsuch (1973, 1974) has reviewed this procedure, which tests the hypothesis that the correlation matrix (or subsequent residual correlation matrices) does not depart from an identity matrix except by chance. His conclusion is that this test is useful only for those situations in which the *complete* components model is used. Further, it is applicable only to *R* analysis and serves only to indicate the *maximum* number of factors that could be extracted with confidence. Gorsuch presents data to support his position and concludes that Bartlett's test should not be applied routinely when one is attempting a factor analysis.

Even more widely used than Bartlett's test is the well-known roots criterion. This procedure, which stops the extraction process when all factors with eigenvalues greater than 1.0 have been removed, is an almost universal default criterion built into most computer programs. The rule is widely misunderstood. First, the rule involves the assumption that population correlations are being considered and that it represents the minimum number of factors in such cases. The introduction of error can substantially change the accuracy of the measure. Second, the roots $\geq 1.0$ criterion holds only when unities are in the diagonal of the correlation matrix being factored (Guttman 1954). When squared multiple correlations are in the diagonal, the criterion should be roots greater than 0.0. The rationale for the rule lies in the psychometric proof that the reliability of the factor scores in association with factors that have roots smaller than 1.0 in the case of unities in the diagonal, or 0.0 when squared multiple correlations are in the diagonal, is negative. Because a negative reliability is an absurdity such factors would be meaningless. Though the roots criterion provides an indication of the minimum number of factors, many researchers have used it to determine both the minimum and maximum number of factors.

Horn (1963, 1965) and Hakstian and Muller (1973) criticize this procedure on theoretical grounds, but the most damaging testimony about the roots criterion comes from Monté Carlo work with the technique.

Linn (1968) and Tucker, Koopman, and Linn (1969) show that when a sample correlation matrix is being factored, the roots criterion produces an exorbitant number of factors. Gorsuch (1974) concludes that the roots criterion is at best an approximate procedure. The roots criterion appears to be most accurate when the number of variables is small to moderate and the communalities are high. When large numbers of variables are involved (e.g., >40), the roots criterion seems particularly inaccurate.

Horn (1965) offers a useful if complex and time-consuming approach to the determination of the number factors. Horn suggests the construction of a series of random numbers matrices with characteristics similar to those of the data matrix of interest, e.g., the same dimensions. These random numbers matrices are factored and the roots obtained are plotted. The roots obtained from the data matrix are also plotted and the point where the roots of the data matrix intersect the roots of the random numbers matrices is taken as the point for stopping the factoring. The approach has much to justify it and seems to work well. The time and cost of constructing random numbers matrices have limited its use, however.

A simpler approach is suggested by Cattell (1966b). Cattell's scree test involves the plotting of the roots obtained from decomposition of the correlation or covariance matrix. A large break in the plot of the roots is taken to indicate the point where factoring should stop. The procedure is relatively simple to apply. A straight edge is laid across the bottom portion of the roots to see where they form an approximately straight line. The point where the factors curve above the straight line gives the number of factors, the last factor being the one whose eigenvalue immediately precedes the straight line. Strong support for the efficacy of the scree test is provided by Tucker, Koopman, and Linn (1969), Cattell and Dickman (1962), Cattell and Sullivan (1962), Cattell and Gorsuch (1963), and Cattell and Jaspers (1967). The most definitive study to date of the scree test is that of Cattell and Vogelmann (1977). The authors provide a very complete set of guidelines for the application of the test and report that even relatively naïve individuals are able to obtain reliable and dependable judgments of the number of factors. Because their data involved data sets with known factor structures and included some "tricky" solutions, this finding suggests that the determination of the number of factors is not a subjective decision if one follows the procedure and has a modicum of training. Cattell and Vogelmann also report the scree test to be more accurate than the roots criterion. Woods (1976) recently used the rules for the scree test given by Cattell and Vogelmann (1977) and Cattell (1978) to develop a computer program which automates the test. Such a routine should facilitate the reliability of factor solutions obtained by investigators.

Most authorities in the field recommend a combination of approaches for determining the number of factors to extract (Cattell 1978; Gorsuch 1974; Harman 1976). The use of the roots criterion and the scree test appears to provide an effective means for determining the number of factors.

Several researchers have sought to examine the effects of extracting too few or too many factors on the stability of the factors after rotation (Dingman, Miller, and Eyman 1964; Howard and Gordon 1963; Keil and Wrigley 1960). The findings of these studies suggest that over-factoring by one or two factors has less severe consequences for the final solution than does taking too few factors. Cattell (1952) has long recommended the extraction of an extra factor or two on the grounds that the extra factors become residual factors upon rotation and their presence improves the interpretation of the results. Too many factors will result in factor splitting, however. Having too few factors in the solution appears to seriously distort the rotated solution.

## *THE ROTATION PROBLEM*

It is perhaps fortuitous that the most common rotation procedure, VARIMAX, has been shown to be among the best orthogonal rotation procedures (Dielman, Cattell, and Wagner 1972; Gorsuch 1974). Equally fortuitous are the findings (Dielman, Cattell, and Wagner 1972; Gorsuch 1970; Horn 1963) that the basic solutions provided by most rotational programs result in the same factors. Thus, the rotation employed should have relatively little impact on the interpretation of results.

Several points should be made about rotation. First, VARIMAX, or any of the procedures based on it (e.g., PROMAX, Harris-Kaiser), should not be used when there is a theoretical expectation of a general factor (Gorsuch 1974). Because VARIMAX serves to spread variance evenly among factors, it will distort any general factor in the data. QUARTIMAX (Carroll 1953) is probably the orthogonal rotation procedure of choice when a general factor is expected.

Many marketing researchers do not seem well acquainted with the aim of rotation. Thurstone (1942) sought not only interpretable factors, but also the most parsimonious solution when he proposed the use of simple structure as a criterion for the "goodness" of a factor solution. Interpretation of factors is important certainly, but most marketing researchers seem to stop at the point of meaningful factors; they may not have found the most parsimonious set of factors. Thurstone (1942, p. 335) provides five criteria for evaluating a solution for simple structure.

1. Each variable should have at least one zero loading.
2. Each factor should have a set of linearly independent variables whose factor loadings are zero.
3. For every pair of factors, there should be several variables whose loadings are zero for one factor but not for the other.
4. For every pair of factors, a large proportion of the variables should have zero loadings on both factors whenever more than about four factors are extracted.
5. For every pair of factors, there should be only a small number of variables with non-zero loadings on both.

Cattell (1952) has been a leading proponent of the notion of simple structure. He proposed an additional criterion for simple structure, the hyperplane count. The hyperplane count consists of the number of essentially zero loadings on a factor or set of factors. In practice, the hyperplane count is usually defined as the total number of loading coefficients between $\pm .10$. Such a criterion can be easily programmed for use with a computer and may serve as a means for deciding what rotated solution to use.

The marketing literature, though filled with factor analytic reports, provides few examples of oblique rotations. The orthogonal rotation dominates despite the strong likelihood that correlated factors and hierarchical factor solutions are intuitively attractive and theoretically justified in many marketing applications. The careful researcher should almost invariably perform both an orthogonal and an oblique rotation, particularly in exploratory work. These solutions can be compared to identify the simpler structure and to determine whether the oblique rotation produces a marked increase in the hyperplane count. Oblique solutions have been found particularly useful in the theory building of other disciplines (e.g., psychology, sociology, regional science, biology), and are likely to play a significant role in the development of any theory of consumer behavior.

## *A NOTE ON THE CORRELATION COEFFICIENT*

Most factor analytic studies in marketing start with the correlation matrix. The Pearson product moment correlation coefficient is certainly the most frequently reported measure of association in the marketing literature. It is also poorly understood. Carroll (1961) presents a general discussion of the correlation coefficient and Horn (1973) and Nunnally (1978) also offer useful treatments of the topic. The discussion here is restricted to problems the Pearson $r$ may pose for factor analysis.

Correlation coefficients, and factors derived from such coefficients, can be influenced by departures from normality, departures from linearity, departures from homoskedasticity, restrictions of range, and the form of the distribution regardless of normality. The first three problems are probably not serious unless the departure is great (Nunnally 1978) and even then alternative procedures may be available (e.g., nonlinear factor analysis, data transformation prior to factor analysis). The latter two problems are much more serious.

The restriction of the range that may occur when sampling procedures are biased with respect to one or more of the variables in the analysis can result in spuriously low correlation coefficients. To the extent that all coefficients are equally affected by the restriction of range, the results of a factor analysis will not be affected appreciably. The problem occurs when some of the variables suffer a restriction of range and others do not. Sampling is an important component of well-planned factor analytic research and serves to prevent the restriction of range of variables in many cases.

A more insidious problem lies in the shape of the distributions of the variables involved in the analysis. If the distributions of variables are shaped differently from one another, the size of the correlation is restricted. Fortunately, these effects are slight in studies of continuous variables (Nunnally 1978). Unfortunately, most of the data available to market researchers are discrete. Carroll (1961) points out that any Pearson product moment correlation coefficient computed between discrete variables will, in part, be a function of the marginal distributions of the variables. The extreme case that illustrates the problem is the use of dichotomous data. Dichotomous data in the form of user-nonuser/agree-disagree information are commonly used in marketing research. The correlation between such variables will range between $\pm 1.0$ only when both variables have the same marginal distributions. It should be pointed out that the restriction on the coefficient is due to the *difference* in the distributions. A perfect correlation can be obtained when variables have the same marginal distributions regardless of what those distributions look like. To illustrate the problem consider two items involving the use or nonuse of a particular product. If 50% of the respondents are users of one product and 20% are users of a second product, the correlation coefficient cannot be greater than .50. Also, because it is not possible for the distribution shape of a dichotomous variable to be the same as that of a continuous variable or nondichotomous discrete variable, a perfect correlation cannot exist between such variables. For example, the maximum size of the correlation between a dichotomous variable and a normally distributed variable is about .80, which occurs when the $p$ value of the dichotomous variable is .50. The further $p$ deviates from .50, the greater the restriction on the obtained correlation. Though this problem is most severe with dichotomous variables, it is also serious for other types of discrete variables. The problem generally becomes less severe as the number of event classes for each discrete variable increases.

Because factor analysis involves a correlation matrix composed of many variables, and factors are defined by the relative magnitudes of the various correlation coefficients, it is possible to obtain factors that are based on the marginal distributions of variables rather than an underlying associative relationship. These spurious method factors are of no interest to the market researcher. In some cases, these factors may even yield a plausible marketing interpretation if the analyst does not realize what has transpired.

Avoiding this distribution problem requires a careful examination of the marginal distributions of the variables proposed for factor analysis. Meaningful factor analysis may not be possible in many cases where the variables are a mixture of dichotomous, nondichotomous discrete, and continuous variables. The best recommendation appears to be to avoid variables that are extremely skewed.

One suggestion for addressing the distribution problem in dichotomous data is to use the *G*-coefficient (Holley 1966; Holley and Guilford 1964). To compute the *G*-index a hypothetical observation that is the exact opposite of each of the original observations is added to the matrix. This extended matrix is then submitted to factor analysis. Because all variables now have the same marginal distributions, no method factors should emerge. No such procedure has been developed for the case involving a mixture of dichotomous and nondichotomous variables. In such cases the researcher might dichotomize all variables, compute a *G*-index, and proceed with the factor analysis.

A second approach to the distribution problem has been suggested by Peters and Van Voorhis (1940) and elaborated by Martin (1978). This approach involves the use of correction factors which take into account the differences in scaling. These correction factors, derived by Peters and Van Voorhis, may be useful for removing the effects of the distribution of responses from the correlation coefficients prior to factor analysis. Martin (1978) provides a brief example of the use of the procedure. To date little work has examined the effects the corrected correlation coefficients would have on the subsequent use of multivariate procedures.

## *SUMMARY AND CONCLUSION*

The preceding discussion is an attempt to demonstrate that much of the criticism of factor analysis is based on a misunderstanding and misapplication of the technique. Factor analysis is not a clustering technique. Rather it seeks to establish dimensions within the data. This is true regardless of the particular mode, i.e., *R* or *Q*, employed.

Factor analysis need not be the subjective exercise it is often accused of being. By examining the raw data and the correlation matrix, one can determine those instances in which factor analysis is appropriate. Examining the marginal distributions of discrete data and computing the measure of sampling adequacy are the minimum steps a researcher should take before initiating a factor analysis. The type of analysis or rotation does not appear to be critical to the final solution in most situations but the extraction of too few or too many factors may have a dramatic effect on the outcome of the analysis. Use of the roots criterion *and* the scree test can provide a very reliable and consistent indication of the number of factors to extract, however.

The ultimate goal of any factor analysis should be the identification of not only interpretable factors but also simple structure. Simple structure may often be better after an oblique rotation and researchers would do well to use oblique rotations more often.

Factor analysis is not a procedure for every season. The preceding discussion identifies when it might be most useful.

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