SMM641 Revenue Management and Pricing - Problem Set 1
Mattheus Lim Jen Sern

Question 1: (30 points)
A café receives 50 croissants daily. During the morning hours, the croissants can be sold individually for £1.00. The croissants can also be used to make sandwiches for lunch time. Regardless of the other ingredients in a sandwich, each croissant brings £1.50 if used for a sandwich. Suppose that the demand in the morning for individual croissants is Poisson with mean 50 and the demand for croissant sandwiches during lunch is Poisson with mean 20.

(a) Suppose that the café does not reserve any croissants for lunch time and serves customers based on a first-come first-served (FCFS) basis. Compute the café's expected daily revenue.

- Expected daily revenue (FCFS) = £51.39

(b) How many croissants should the café reserve to be used for lunchtime sandwiches in order to maximise their expected daily revenue?

- Optimal Protection Level to maximise Expected Daily Revenue = 18

(c) What is the expected daily revenue from this protection (reserve) policy? What is the percent improvement compared to the expected daily revenue from the FCFS allocation that you computed in part (a).

- Expected Daily Revenue with an Optimal Protection Level of 18 = £57.61
- % improvement compared to Expected Daily Revenue (FCFS) = 12.11%

(d) Explore how the allocation decision changes with changes in the expected demand for sandwiches, the revenues that each croissant brings as an individual breakfast item or as a sandwich, and the number of croissants the café receives daily. For example, is it better to protect more croissants for lunch time if the demand for lunch time is higher?

- The following table represents the change in protection level when expected demand, revenue of croissant **(CR)** and sandwich **(SW)**, and daily number of croissants varies:

Variation	Mean Demand Croissant (MDCR)	Mean Demand Sandwich (MDSW)	Croissant Price (CRP)	Sandwich Price (SWP)	Croissant Capacity (CRC)	Optimal Protection Level	Optimal Expected Revenue
Increase MDCR	70	20	1	1.5	50	18	57.6
Decrease MDCR	30	20	1	1.5	50	18	56.13
Increase MDSW	50	30	1	1.5	50	28	62.05
Decrease MDSW	50	10	1	1.5	50	9	53.16
Increase CRP	50	20	2	1.5	50	0	98.57
Increase SWP	50	20	1	2.5	50	21	75.66
Increase CRC	50	20	1	1.5	70	18	75.59
Decrease CRC	50	20	1	1.5	30	18	37.61

- Therefore, the cafe should protect more croissants for lunch if demand for lunch time increases because it would provide higher optimal expected revenue.

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Question 2: (40 points) (Please limit your response to 200 words.)
Consider the Airline Network Example we discussed in class, in which an airline is operating two flights, one from Dublin to London, and the other from London to Edinburgh.

(a) Please explain in words what the optimal revenue obtained by dynamic programming, i.e., the value function for the state (100,120,300), represents.

- It represents the optimal expected revenue at stage 300, given that the flight capacities for Dublin to London, and London to Edinburgh are 100 and 120 respectively. The total expected revenue is maximised by comparing the expected revenue of the current period against next period using probabilities of arrival and fares for all scenarios (including no arrival).

(b) Consider the structure of the optimal acceptance decision for product 2 at t=100 periods to go (available in the slides). Please explain in words the main insights you gain from this figure.

- When Leg 2 capacity is limited and Leg 1 increases in capacity, we reserve the capacity to sell Product 3 since it would generate higher revenue
- When Leg 2 and Leg 1 capacities are high, we keep selling Product 2

(c) Consider the structure of the optimal acceptance decision for product 3 at t=100 periods to go (available in the slides). Please explain in words the main insights you gain from this figure.

- When Leg 2 and Leg 1 capacities are approximately > 40, we keep selling Product 3
- When Leg 2 and Leg 1 capacities are approximately < 40, we don't sell Product 3 since selling them individually will bring higher revenue

(d) Suppose the company is considering adding a new product, which will be a flight from Dublin to London at a premium fare for £200, for which it estimates that the per-period arrival probability will be 1/20. Please explain briefly - how would you modify the dynamic program we have discussed in class to accommodate this new product?

- The optimal value function would have 2 changes:
    1. The total expectation will have to account for the new product by adding this equation:
       $+ \ 1/20 \ * \ max\{200 \ + \ V(x_1 - 1, x_2, t - 1), V(x_1, x_2, \ t - 1)\}$
    2. The probability of no arrival changes from 1/10 to 1/20

(e) Explain in words why the first-come-first-serve allocation brings a lower revenue than the optimal allocation found through dynamic programming.

- The FCFS approach always sells at each stage hence it doesn't account for the potential profit in the next stage if resources were retained in the current stage.

Question 2 total words = 198

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<u>Question 3:</u> (30 points) (Please limit your response to 200 words.)
Identify a relatable setting either based on a service/operation on campus, locally, or elsewhere, in which any of the concepts and methodologies we have learned so far can potentially improve the service provider’s objective (e.g., maximising revenue)

Please describe the setting clearly and how you would model/analyse the setting. You do not need to work on the analysis itself or provide any results.

Please make sure to try to pick a setting that allows a sufficiently rich variation and extension to the examples we have covered in class, that is, your analysis should not be a very simple variation of the examples we have discussed in class but be challenging and require you to think creatively.

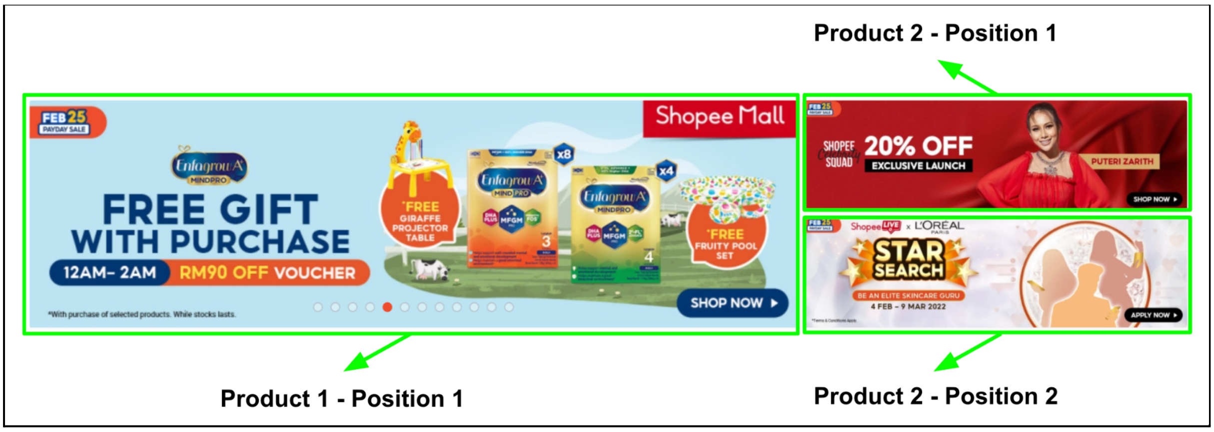
If you’d like, and if the setting is relevant and enables a sufficient depth of analysis, you will have the option to expand on your idea for your projects as well.

<u>Solution to Question 3:</u>

- An e-commerce platform like Shopee (https://www.shopee.com.my) can maximise revenue of marketing services by applying dynamic programming to the prices of banner advertisement slots during campaign periods (i.e. singles day, black friday etc). Moreover, capacity allocation can be controlled to increase the value of banners by intentionally restricting supply, hence creating more demand.

- Prices can be significantly increased during peak periods of campaigns (i.e. double digit dates like 11.11 and 12.12) since the value of the advertisement would directly translate to much higher traffic and hence better conversion. Protection levels can be applied on the banners with the best exposures (Position 1) for special brand launches (i.e. Adidas launching an official store on Shopee). This would create a good relationship with the brand and increase revenue in future marketing campaigns with the brand.

- Variables to model the setting and context on advertisement banner:
  - Resource: Banner advertisement slots
  - Products:
    - Product 1 - Primary Rotating Banner (Position 1 = best exposure)
    - Product 2 - Secondary Static Banner



  - Period: Days
  - Demand: Average sale during peak days, campaign and non-campaign periods

Question 3 total words = 182