SMM 641
Revenue Management and Pricing

Week 1: Course Overview and Introduction to RMP

Quantity Based Revenue Management Part 1

Optimal Booking Limits and Protection Levels for Two-Fare Classes

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Introduction to Revenue Management (RM)

How should companies set and adjust their prices in order to maximize profitability?

Pricing decisions are:

- commonplace
- complex
- critical determinants of profitability

Introduction to Revenue Management (RM)

Very few companies have the right prices in place:

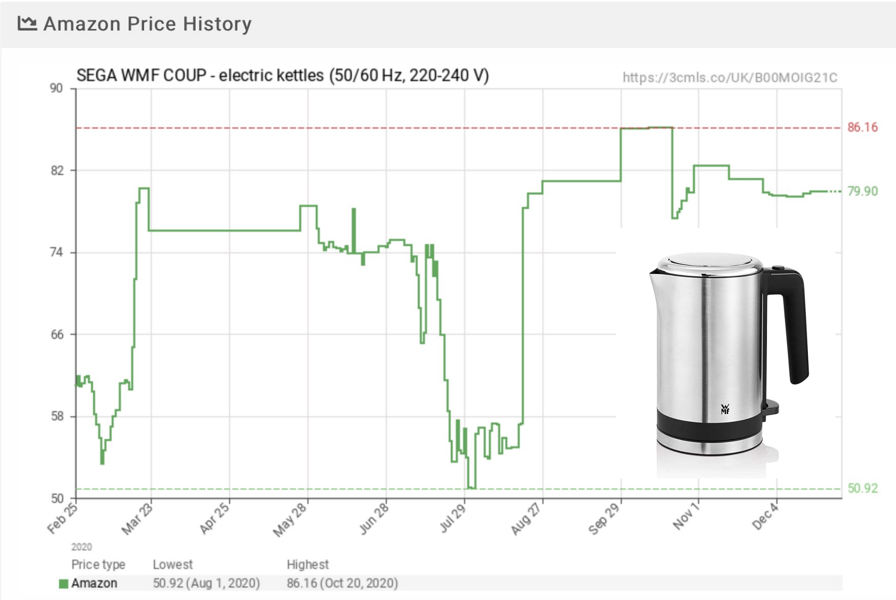
- for all products
- to all customers
- through all channels
- all the time

Revenue Management (RM) uses probabilistic modeling and data-driven optimization techniques to determine how firms should make tactical (rapidly adjusted) capacity allocation and pricing decisions for perishable assets in order to maximize revenues from investments in capacity/inventory.

Example Applications

- Hotels – Marriott:
  - https://www.youtube.com/watch?v=b2zg81CSZ64
- Cruise Ship - Carnival UK
  - https://www.youtube.com/watch?v=M_jYlwENAPc
- Transportation – Seattle Parking
  - https://www.youtube.com/watch?v=Y4tKz3ZU7vM
- Electricity – Victorian Energy
  - https://www.youtube.com/watch?v=LqOio2FKk0U
- Transportation – Uber
  - https://www.youtube.com/watch?v=6JZ6yjJprok
- Accommodation – Airbnb
  - https://www.youtube.com/watch?v=iWrSlwnT8zM
- Entertainment – Disney:
  - https://www.youtube.com/watch?v=KPZ94deQP0M
- Ski Ticketing - The SkiArena Andermatt-Sedrun, Switzerland
  - https://www.youtube.com/watch?v=WEW3Ip5SdkU
- Airlines:
  - https://youtu.be/-oJIJ5oo5AM?t=89

Example Applications – Online Retailing



Our Road Map

- Quantity Based Revenue Management
  - Single Resource Revenue Management with Two-Fare Classes
  - Overbooking
  - Single Resource Revenue Management with Multiple-Fare Classes
  - Network Revenue Management (Multi Resource, Multi Product)
- Revenue Management Under Customer Choice
  - Basic Price Optimization
  - Segmentation and Price Differentiation for Constrained Supply
  - Choice Modelling
  - Peak-Period Pricing
  - Market Basket Analysis and Assortment Optimization
- Dynamic Pricing and Markdown Optimization
- Dynamic Pricing with Demand Learning

Methodologies

- The course will expose you to several fundamental methodologies and concepts related to operations research, analytics, and statistics, including:
  - Linear Programming
  - (Stochastic) Dynamic Programming
  - Nonlinear Optimization
  - Choice Modelling
  - Markov Chain Demand Models
  - Exploration vs Exploitation - Multi-Armed Bandit Models

Lecture Structure

- Face-to-Face Session (to the extent permitted and possible)
  - Mondays 1:00pm - 2:50pm
  - Main discussion on the week's content, exercises, cases.
- Asynchronous Content
  - Code supplements

When can I meet my instructor?

- When/where can I meet my instructor?
  - Optional Online Office hours:
    - TBD (e.g., Mondays 3:00pm - 3:30pm)
    - Online through Zoom
    - For connection instructions, please see the module page on Moodle
  - E-mail:
    - oben.ceryan@city.ac.uk
    - Please put "SMM641" in the subject line.

      e.g. SMM641: My dog ate my coursework!

Module Materials

Required:

- Lecture slides.
  - Lecture slides will be posted to Moodle.
- Software.
  - The course will involve extensive use of software. We will mainly use R to illustrate the fundamental concepts. I suggest you install R Studio or register for a free account on R Studio Cloud at http://rstudio.cloud
    - (You can however also choose to work with any of your other preferred programming languages as well such as Python, Matlab, Julia, etc. to implement the methods and algorithms we will be covering. If you prefer, you can also utilize JupyterLab's (web) development environment for most of these languages.)
  - I strongly recommend you bring your computers to each session.

Module Materials

Optional Reading:

- Phillip, R.L. (2021) *Pricing and Revenue Optimization,* Stanford University Press. (recommended reading, but we will cover topics in more technical depth)
- Chapman, C. and McDonnell Feit, E. (2019) *R for Marketing Research and Analytics*, Springer (recommended supplementary reading)
- Talluri, K.T. and Van Ryzin, G.J. (2008) *Theory and Practice of Revenue Management*, New York: Springer. (optional, overly detailed and specialized focus on some topics but has a good industry profiles section)
  - All are available as e-books through the Library.

Module Website

- The Module website is accessible through:
  - https://moodle.city.ac.uk
- Please make sure to check the module website regularly for
  - Announcements,
  - Class slides
  - Videos
  - Additional supplementary materials (e.g., R codes)

Assessment

- Coursework Problem Sets (30%):
  - The problem sets will reinforce the material taught in class and will also help guide you towards the project topic.
- Coursework Project (70%):
  - Additional details on the project description and guidelines on expected deliverables will be provided towards the middle of the term.
  - A brief presentation of the projects will be scheduled for the last session.

Academic Honesty

- Ethics - Academic honesty lifts all of us to a higher level.
- Please review carefully the Student Handbook for academic policies.
- Please note that students are not permitted or authorized to post or redistribute any of the module materials.

How do I do well in this course?

- Before Each Class:
  - Study the lecture slides before each class
  - Try to work on the examples on your own
  - Be prepared to answer or ask questions
- During Each Class:
  - Participate in class discussions and exercises.
- After Each Class:
  - Carefully go through the class notes and materials
  - Always keep up with the topics, do not hesitate to stop by during office hours or to email to seek help regarding any of the topics

Today

- Quantity Based Revenue Management
  - Single Resource Revenue Management with Two-Fare Classes
  - Single Resource Revenue Management with Multiple-Fare Classes
  - Network Revenue Management (Multi Resource, Multi Product)
  - Incorporating Additional Consumer Behavior, Overbooking
- Revenue Management Under Customer Choice
  - Basic Price Optimization
  - Segmentation and Price Differentiation for Constrained Supply
  - Choice Modelling
  - Peak-Period Pricing
  - Assortment Optimization
- Dynamic Pricing with Demand Learning
- Markdown Optimization, Market Basket Analysis, Fairness

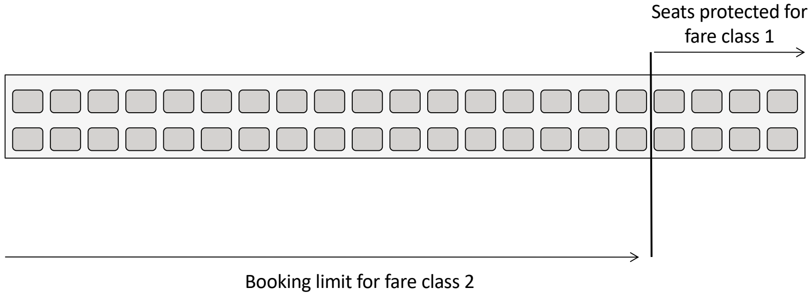
Quantity-Based Revenue Management

- Common characteristics of related settings:
  - There exists a fixed supply of perishable capacity to sell
  - Customers buy in advance and they all claim their product/service simultaneously at a fixed time in the future
  - There exists a set of fare classes, each of which has a predetermined price
  - The fare classes can be made available or unavailable over time.
- Examples: Hotels, Airlines, Cruise, Car Rental, Events

Booking control

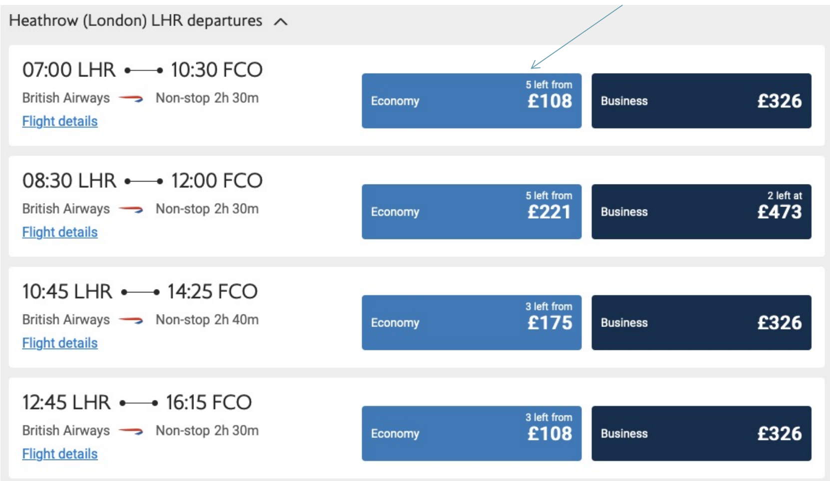
Whenever a "booking request arrives," an accept / reject decision is made based on these booking limits and the bookings to date.

Fare 2 < Fare 1



Booking Limits

This fare class closes after 5 more bookings



Single Resource Two Fare Class Setting

- Single capacity/resource, e.g., seats on a plane
  - Unsold capacity does not have any value
- Two Fare classes:
  - High fare class: pays p_1
  - Low fare class: pays $p_2 < p_1$
  - Low fare class arrives first, followed by the high fare class.
  - Demands for the low and high fare class are random with known distributions and are independent of each other.
- The objective is to maximize the total expected revenue from all fare classes by choosing how many seats to allocate for the fare classes.

Optimal booking limits with two fare classes

Suppose the plane has a capacity of C. There are two fare classes: class 2 (discount passengers) arrive first and pay p_2; class 1 (full fare passengers) arrive afterwards and pay p_1.

Period-to-go	2		1
Fare	p_2	$\leq$	p_1
Booking limit	b	$\leq$	C
Demand	D_2		D_1
# bookings to date	$x_2 = \min(b, D_2)$		
			$x_1 = \min(C - x_2, D_1)$

The question is: what is the optimal value of b, the booking limit for class 2 (discount passengers)?

Example: Single Resource Two Fare Class Setting

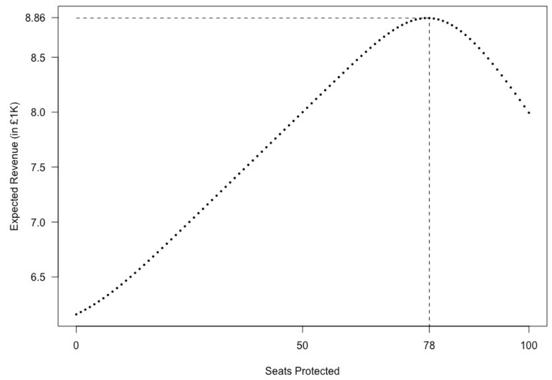
- Suppose that:
  - Capacity=100
  - $p_1 = 100$
  - $p_2 = 60$
  - D_1 is a Poisson random variable with mean 80.
  - D_2 is a Poisson random variable with mean 100.

Example: Single Resource Two Fare Class Setting

```
# A NUMERICAL EXAMPLE USING POISSON DISTRIBUTION
mL=100          # Mean Demand for Low-Fare, Poisson
mH=80           # Mean Demand for High-Fare, Poisson
pL=60           # Price for Low-Fare
pH=100          # Price for Low-Fare
capacity=100    # Capacity
ExpRevenue=rep(0,capacity+1)
for (i in 1:(capacity+1)){
    protect=i-1
    availforLowFare=capacity-protect;
    ExpRevenue[i]=0;
    for(dL in 0:200){
        soldLowFare=min(availforLowFare,dL)
        remainforHighFare=capacity-soldLowFare
        for(dH in 0:200){
            soldHighFare=min(remainforHighFare,dH)
            RevenueThisIter=pL*soldLowFare+pH*soldHighFare
            ExpRevenue[i]=ExpRevenue[i]+
                RevenueThisIter*dpois(dL,mL)*dpois(dH,mH)
        }
    }
}
Protectindexbest = which(ExpRevenue == max(ExpRevenue))
ProtectBest=Protectindexbest-1
OptimalExpRevenue=max(ExpRevenue)
print(paste("The Optimal Protection Level for High-Fare Demand:", ProtectBest))
```

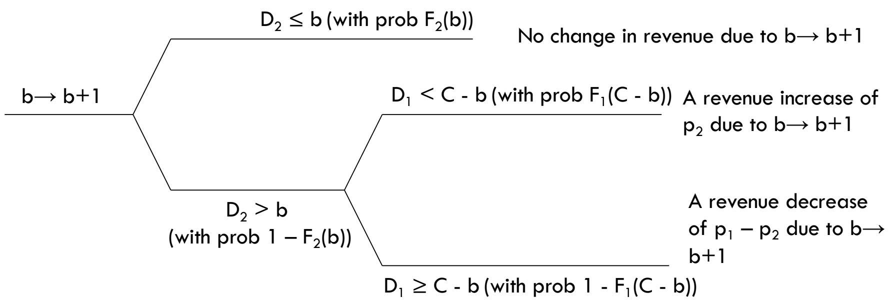
Example: Single Resource Two Fare Class Setting

- Optimal protection level for high fare is 78.
- Optimal booking limit for low fare is 22 (i.e., 100 - 78).



Optimal booking limits with two fare classes

- Let F_1 be the distribution of D_1, the demand we will receive for high fare class. i.e., $F_1(x)$ is the probability that we receive x or less requests for the high fare class.
- Let F_2 be the distribution of D_2, the demand we will receive for low fare class, i.e., $F_2(x)$ is the probability that we receive x or less requests for the low fare class.
- Suppose we increase the booking limit from b to b +1. What is the effect on the revenue?



Optimal booking limits with two fare classes

- In summary, if we increase the booking limit from b to b +1, the expected change in revenue is:

$(1 - F_2(b)) \times F_1(C - b) \times p_2$ → Potential revenue increase from booking one more discount passenger

$- (1 - F_2(b)) \times (1 - F_1(C - b)) \times (p_1 - p_2)$ → Potential revenue decrease from displacing a full fare passenger

At the optimal b:

$$(1 - F_2(b)) \times F_1(C - b) \times p_2 = (1 - F_2(b)) \times (1 - F_1(C - b)) \times (p_1 - p_2)$$

$$\Downarrow$$

$$\boxed{F_1(C - b) = (p_1 - p_2) / p_1} \rightarrow \text{Littlewood's Rule}$$

If we let y denote the protection level for the high fare class, y=C - b

$$\Downarrow$$

$$\boxed{F_1(y) = (p_1 - p_2) / p_1}$$

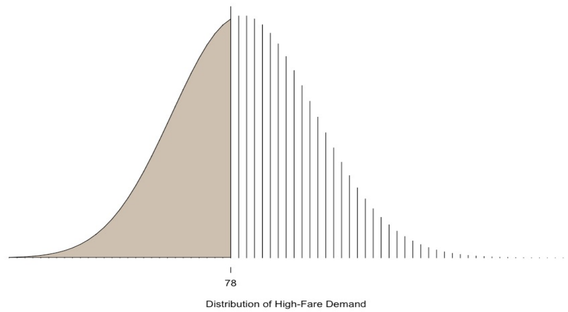
Optimal booking limits with two fare classes

- If $p_2 = 0$, set the protection level for high fare as the capacity C, and the booking limit for the low fare to 0.
- As p_2 increases, the optimal number of seats to protect for full fare decreases until when $p_2 = p_1$.
- If $p_2 = p_1$, then it is optimal to set the protection level for full fare to 0 and the booking limit to the capacity, C.
- Interesting fact: The optimal booking limit for low fare does not depend in any way on the forecast of discounted demand!

Optimal booking limits with two fare classes

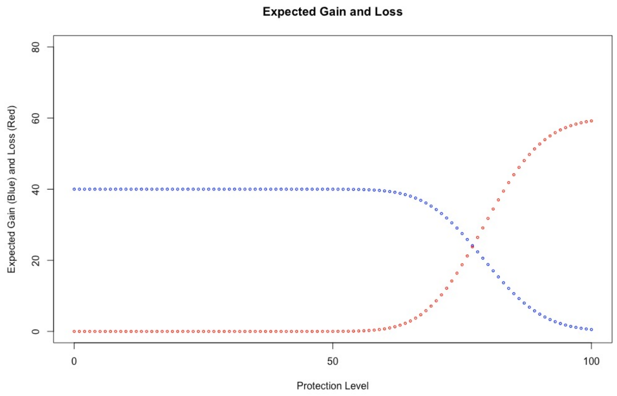
Analytical Solution:

- Compute a Critical Fractile: CF = $(p_1 - p_2)/p_1$ = (100-60)/100=0.4.
- Protect as many seats to guarantee that the the probability that the high fare demand realization falling below this many seats is CF



Optimal booking limits with two fare classes

- Marginal Gain vs Marginal Loss:



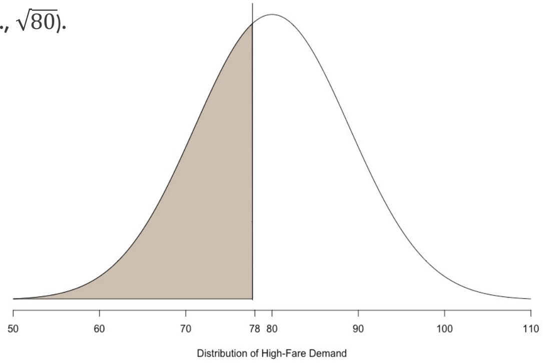
Optimal booking limits with two fare classes

- Marginal Gain vs Marginal Loss:

```
ExpGain=rep(0,capacity+1)
ExpLoss=rep(0,capacity+1)
for (i in 0:(capacity+1)){
    protect=i-1
    ExpGain[i]=(1-ppois(protect,mH))*(pH-pL)
    ExpLoss[i]=ppois(protect,mH)*pL
}
xaxis=0:capacity
plot(xaxis,ExpGain,type="p",cex=0.5,col="blue",ylim=c(0,80),xaxt="n",
     xlab = "Protection Level", ylab = "Marginal Gain (Blue) and Loss (Red)")
lines(xaxis,ExpLoss,type="p",cex=0.5,col="red")
xtick<-seq(0, 100, by=50)
axis(side=1, at=xtick)
```

Optimal booking limits with two fare classes

- Continuous Demand Distributions
  - Suppose high-fare demand is normal with mean 80 and standard deviation 8.94 (i.e., $\sqrt{80}$).



Optimal booking limits with two fare classes

- A Lower Bound for Expected Revenue (FCFS)

  Suppose we admit demand on a first come first serve (FCFS) basis, i.e., set protection level to zero.

```
mL=100          # Mean Demand for Low-Fare, Poisson
mH=80           # Mean Demand for High-Fare, Poisson
pL=60           # Price for Low-Fare
pH=100          # Price for Low-Fare
capacity=100    # Capacity
ExpRevenue=rep(0,capacity+1)
for (i in 1:1){
    protect=i-1
    availforLowFare=capacity-protect;
    ExpRevenue[i]=0;
    for(dL in 0:200){
        soldLowFare=min(availforLowFare,dL)
        remainforHighFare=capacity-soldLowFare
        for(dH in 0:200){
            soldHighFare=min(remainforHighFare,dH)
            RevenueThisIter=pL*soldLowFare+pH*soldHighFare
            ExpRevenue[i]=ExpRevenue[i]+RevenueThisIter*dpois(dL,mL)*dpois(dH,mH)
        }
    }
}
RevenueFCFS=ExpRevenue[1]
print(paste("Lower Bound for Expected Revenue (FCFS):", round(RevenueFCFS,1)))
```

Optimal booking limits with two fare classes

- An Upper Bound for Expected Revenue (Perfect Foresight)
  Suppose we have perfect foresight, i.e., can see future demand and apply best allocation.

```
mL=100          # Mean Demand for Low-Fare, Poisson
mH=80           # Mean Demand for High-Fare, Poisson
pL=60           # Price for Low-Fare
pH=100          # Price for Low-Fare
capacity=100    # Capacity
ExpRevenueUB=0
for(dL in 0:200){
for(dH in 0:200){
    soldHighFare=min(dH,capacity)
    remainforLowFare=capacity-soldHighFare
    soldLowFare=min(dL,remainforLowFare)
    RevenueThisIter=pL*soldLowFare+pH*soldHighFare
    ExpRevenueUB=ExpRevenueUB+RevenueThisIter*dpois(dL,mL)*dpois(dH,mH)
}
}
print(paste("Upper Bound for Expected Revenue (Perfect Foresight):", round(ExpRevenueUB,1)))
```

Optimal booking limits with two fare classes

- Revenue Potential of the Optimal Protection Level.
  - Optimal protection achieves: 89% of revenue potential of perfect foresight.