Quantum Discord at the Large Hadron Collider

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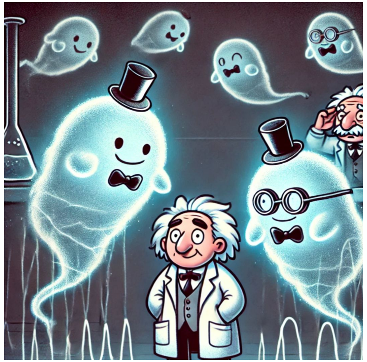
University of Hawaii

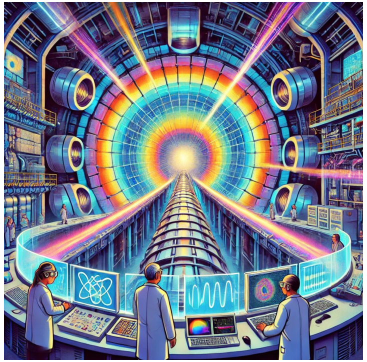
Motivation

- Quantum Mechanics is one of the **cornerstones** of modern physics
- Colliders now enable us to explore QM at the **highest** laboratory energies
- Quantum Information leverages quantum properties of systems for **computation** and **information processing**
- Quantum discord is a weaker correlation than entanglement that distinguishes **classical** and **quantum** systems
    - May be useful for *quantum state merging*, *quantum metrology*, and *identifying systems with quantum advantages*

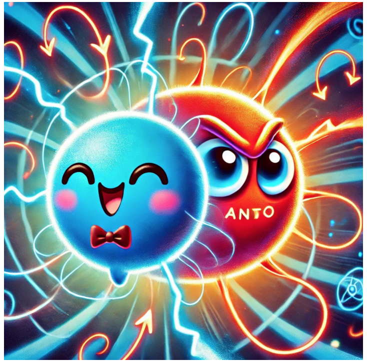
Outline



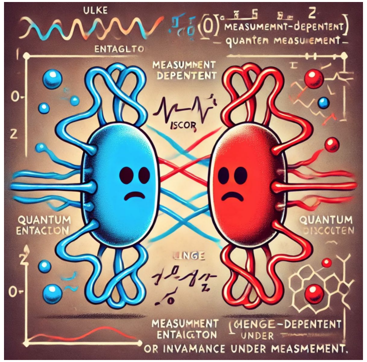
Quantum Mechanics



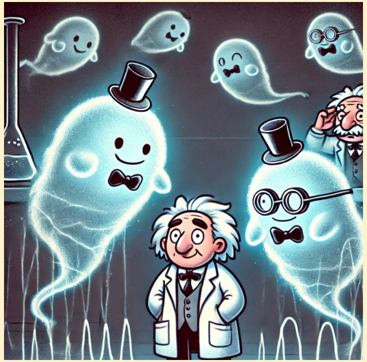
Tomography at Colliders



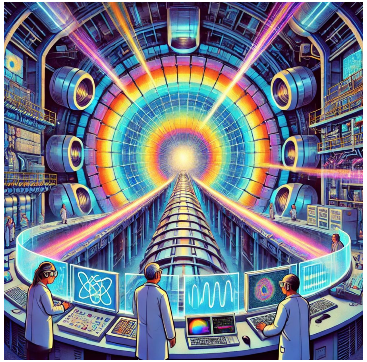
Top-Antitop State



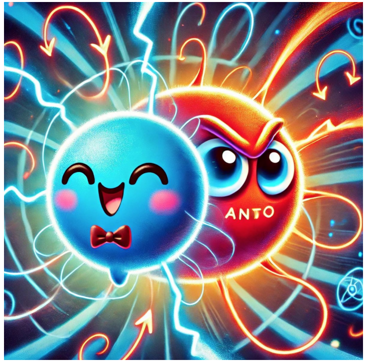
Quantum Discord



Quantum Mechanics



Tomography at Colliders



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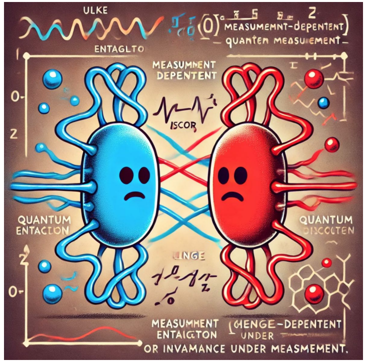


Quantum Discord

Quantum Mechanics

- Single **qubit** $|\psi\rangle$ describes two-level state

$$|\uparrow\rangle, |\downarrow\rangle \qquad \text{or} \qquad |1\rangle, |0\rangle$$

- Generally have a **superposition**

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad \text{or} \qquad |\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

- **Observables** represented by operators A

- **Measurement** of A given by $a = \langle\psi|A|\psi\rangle$

Quantum Mechanics

- *Examples*:

$$|\psi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |\psi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

- Measure spin along the **z-axis** σ_z

$$\langle\psi|\sigma_z|\psi\rangle = 1 \qquad \langle\psi|\sigma_z|\psi\rangle = 0$$

- Measure spin along the **x-axis** σ_x

$$\langle\psi|\sigma_x|\psi\rangle = 0 \qquad \langle\psi|\sigma_x|\psi\rangle = -1$$

Quantum Mechanics

- **Two qubits** occupy the tensor space $\mathcal{H}_1 \otimes \mathcal{H}_2$
- Possible states are

$$|0\rangle \otimes |0\rangle \qquad |0\rangle \otimes |1\rangle \qquad |1\rangle \otimes |0\rangle \qquad |1\rangle \otimes |1\rangle$$

- Here's a **separable** state

$$|\psi\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \qquad |\psi_1\rangle = |\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

- Here's an **entangled** state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Quantum Mechanics

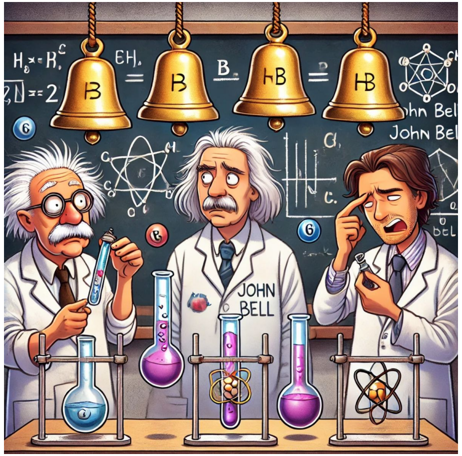
- Consider measuring **both** spins at the same time

$$E(\sigma_x, \sigma_y) = \langle\psi|\sigma_x \otimes \sigma_y|\psi\rangle$$

- Bell (1964) and **Clauser, Horne, Shimony, Holt** (1969) suggested an experiment of 4 configurations

$$|E(a_1, b_1) - E(a_1, b_2) + E(a_2, b_1) + E(a_2, b_2)| \leq 2$$

- All theories of **local realism** obey the inequality
    - *Locality* = at detection, qubits are not interacting
    - *Realism* = qubit observables have definite values before and after measurement



Quantum Mechanics

Nobel Prize in Physics 2022

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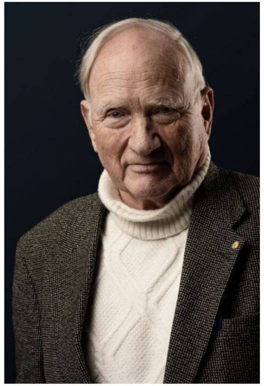
Alain Aspect

Prize share: 1/3



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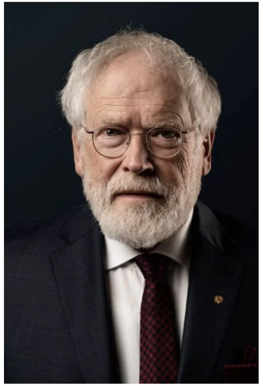
John F. Clauser

Prize share: 1/3



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Anton Zeilinger

Prize share: 1/3

The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

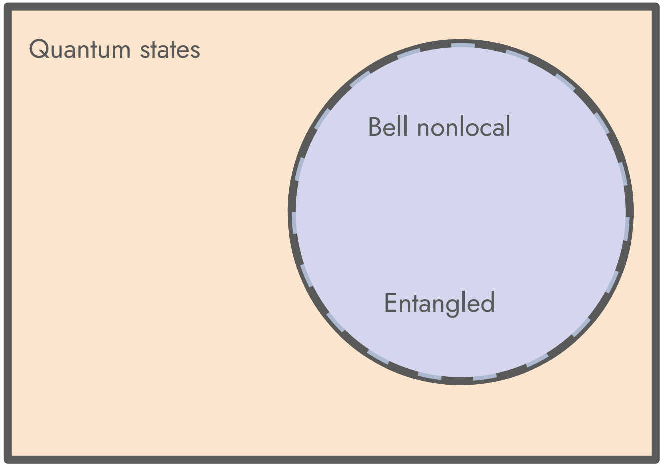
Quantum Mechanics

- For **pure** states, Bell's inequality and entanglement distinguish the same states
- For **mixed** states, there are more entangled states than Bell nonlocal

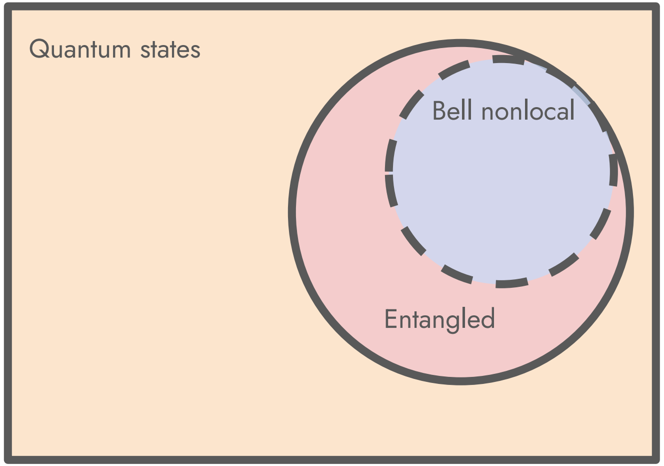
$$\rho = \sum_a p_a |\psi_a\rangle \otimes \langle\psi_a| \qquad \sum_a p_a = 1$$

qubit: **2x2** matrix
two qubits: **4x4** matrix

pure



mixed



Quantum Mechanics

- For one qubit, use **Pauli decomposition**

$$\rho = \frac{1}{2}\left(\mathbb{I}_2 + \sum_a B_a \sigma_a\right)$$

- Qubit described by **3 parameters** B_a

- Clear physical meaning: $B_a = \langle B_a \rangle = \mathrm{tr}(\rho\sigma_a)$
    - *Spin along the a direction*

- Measuring the entire quantum state is called **quantum tomography**

Quantum Mechanics

- For two qubits, use the **Fano-Bloch decomposition**

$$\rho = \frac{1}{4}\left(\mathbb{I}_4 + \sum_i B_i^+ \sigma_i \otimes \mathbb{I}_2 + \sum_j B_j^- \mathbb{I}_2 \otimes \sigma_j + \sum_{ij} C_{ij} \sigma_i \otimes \sigma_j\right)$$

- Two qubits described by:

$$B_i^+ = \mathrm{tr}[\rho\,(\sigma_i \otimes \mathbb{I}_2)]$$

- 3 parameters
- Polarization of qubit 1

$$B_j^- = \mathrm{tr}[\rho\,(\mathbb{I}_2 \otimes \sigma_j)]$$

- 3 parameters
- Polarization of qubit 2

$$C_{ij} = \mathrm{tr}[\rho\,(\sigma_i \otimes \sigma_j)]$$

- 9 parameters
- Spin correlations between qubits

Quantum Mechanics

- For two qubits, use the **Fano-Bloch decomposition**

$$\rho = \frac{1}{4}\left(\mathbb{I}_4 + \sum_i B_i^+ \sigma_i \otimes \mathbb{I}_2 + \sum_j B_j^- \mathbb{I}_2 \otimes \sigma_j + \sum_{ij} C_{ij}\sigma_i \otimes \sigma_j\right)$$

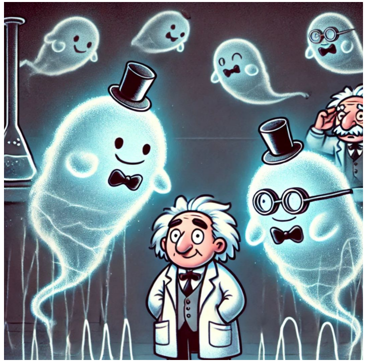
- **Partial Trace** reduces 2 qubit system to 1 qubit system (**Reduced density matrix**)

$$\rho_A = \mathrm{tr}_B \rho_{AB}$$

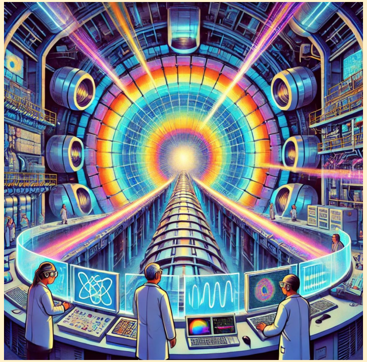
$$\rho_A = \sum_i \rho_a \otimes \langle i|\rho_b|i\rangle$$

$$\rho_B = \mathrm{tr}_A \rho_{AB}$$

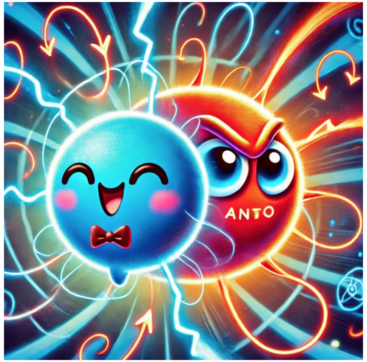
$$\rho_B = \sum_i \langle i|\rho_a|i\rangle \otimes \rho_b$$



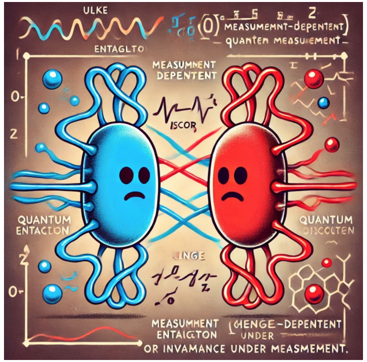
Quantum Mechanics



Tomography at Colliders



Top-Antitop State



Quantum Discord

Tomography at Colliders

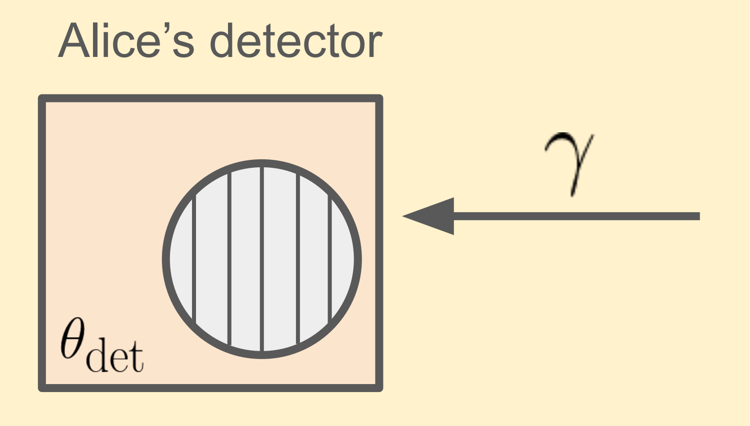
- Low energy photon experiment

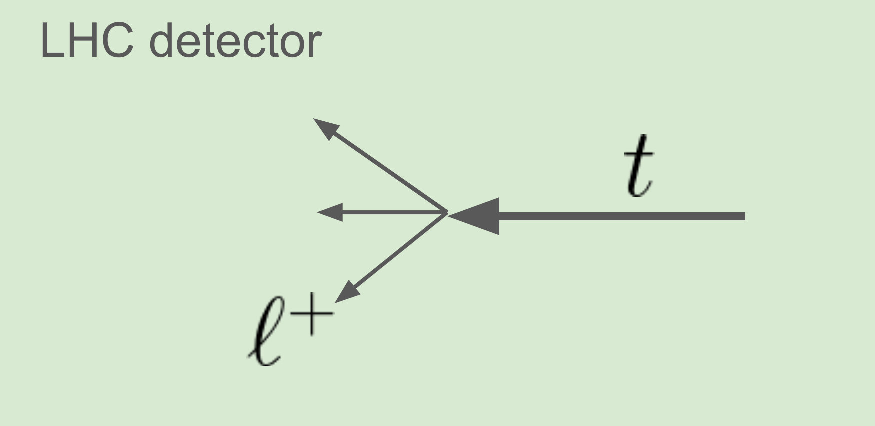
Alice $\xleftarrow{\gamma}$ source $\xrightarrow{\gamma}$ Bob

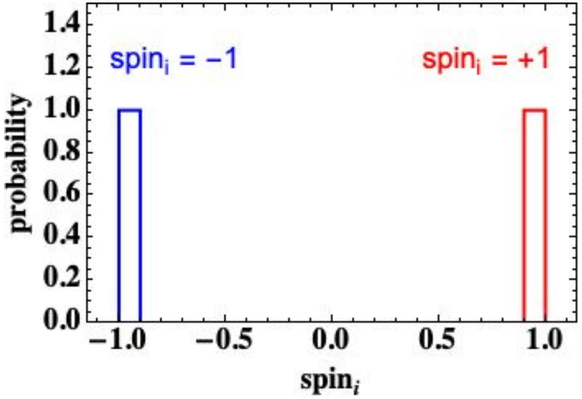
- At LHC, treat the spin of each particle as a qubit

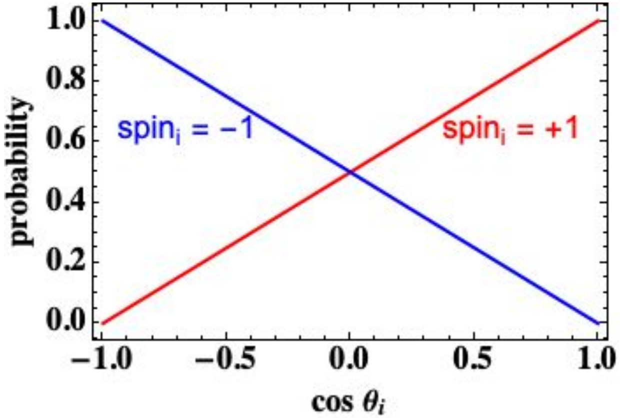
 $\xleftarrow{t}$ collision $\xrightarrow{\bar{t}}$ 

Tomography at Colliders









Tomography at Colliders

Afik, de Nova [2003.02280]

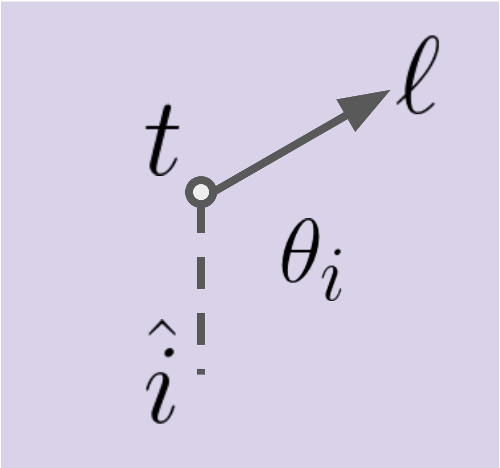
- Full set of spin correlations **reconstructs** the quantum density matrix
- In the decay of a top, consider one particle — **the spin analyzer** — and take the **angle** between its momentum and a reference axis (in the top rest frame)

$$\frac{1}{\Gamma}\frac{\Gamma}{\cos\theta_i} = \frac{1}{2}(1 + \kappa|B|\cos\theta_i)$$

κ spin analyzing power

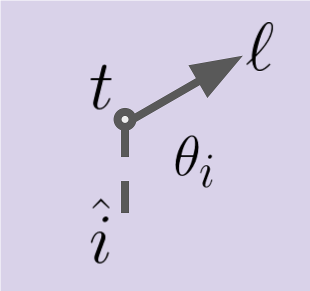
Spin Analyzer	Power
lepton/down-quark	1.00
neutrino/up-quark	-0.34
b-quark or W	$\mp$0.40
soft-quark	0.50
optimal hadronic	0.64

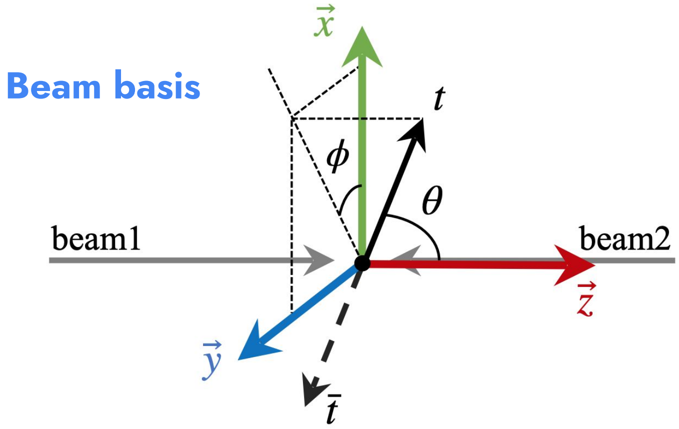
← **most correlated** (lepton/down-quark)
← **least correlated** (neutrino/up-quark)

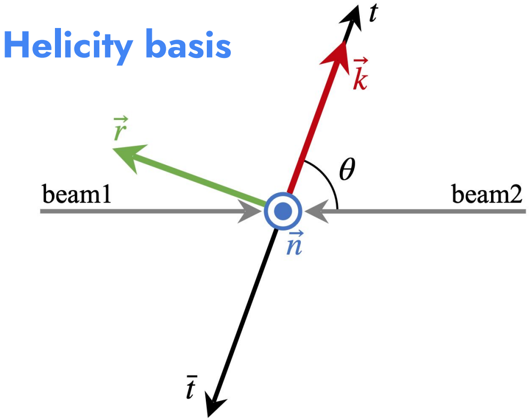


Tomography at Colliders

- Choice of **Basis**
  - Set of 3 **angles** for the momentum measurement
  - Equivalent to **axes** for spin measurement







Tomography at Colliders

- **Decay Method**
  - Parametrize density matrix as $\rho(B_i^+, B_j^-, C_{ij})$

- Universal **angular** distributions

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_{A,i}} = \frac{1}{2}\left(1+\kappa_A B_i^+ \cos\theta_{A,i}\right),$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_{B,j}} = \frac{1}{2}\left(1+\kappa_B B_j^- \cos\theta_{B,j}\right),$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_{A,i}\cos\theta_{B,j}} = -\frac{1}{2}\left(1+\kappa_A\kappa_B C_{ij}\cos\theta_{A,i}\cos\theta_{B,j}\right)\log|\cos\theta_{A,i}\cos\theta_{B,j}|.$$

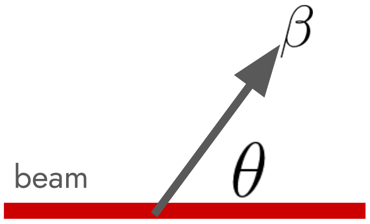
- Extract **components** of the density matrix

$$B_i^+ = \frac{3\langle\cos\theta_{A,i}\rangle}{\kappa_A}, \qquad B_j^- = \frac{3\langle\cos\theta_{B,j}\rangle}{\kappa_B}, \qquad C_{ij} = \frac{9\langle\cos\theta_{A,i}\cos\theta_{B,j}\rangle}{\kappa_A\kappa_B}.$$

Tomography at Colliders

Cheng, Han, ML 2410.08303

- **Kinematic method**
  - Parametrize density matrix as $\rho(\theta, \beta)$



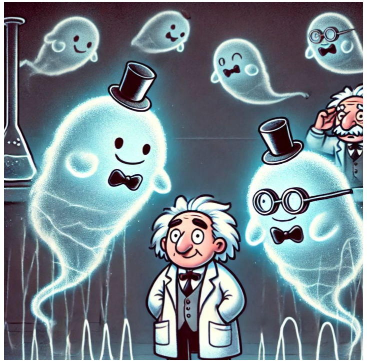
- Process-specific **components** of the density matrix

$$C_{ij} = \frac{1}{2-\beta^2\sin^2\theta}\begin{pmatrix} (2-\beta^2)\sin^2\theta & 0 & \sqrt{1-\beta^2}\sin(2\theta) \\ 0 & -\beta^2\sin^2\theta & 0 \\ \sqrt{1-\beta^2}\sin(2\theta) & 0 & \beta^2+(2-\beta^2)\cos^2\theta \end{pmatrix}_{ij}.$$

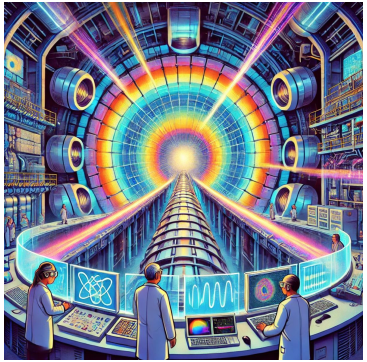
- From θ and β distributions

$$C_{11} = \langle\frac{(2-\beta^2)\sin^2\theta}{2-\beta^2\sin^2\theta}\rangle \qquad C_{12} = 0$$

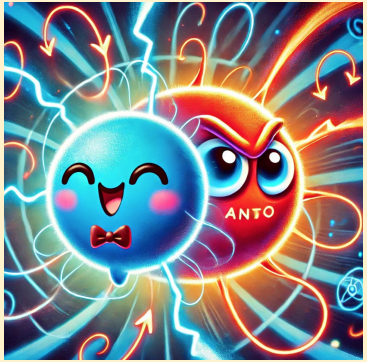
(Entries that are predicted to be zero are often used in practice)



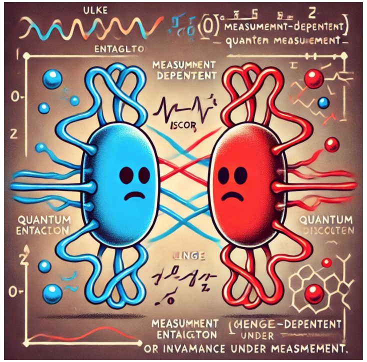
Quantum
Mechanics



Tomography at
Colliders



Top-Antitop
State

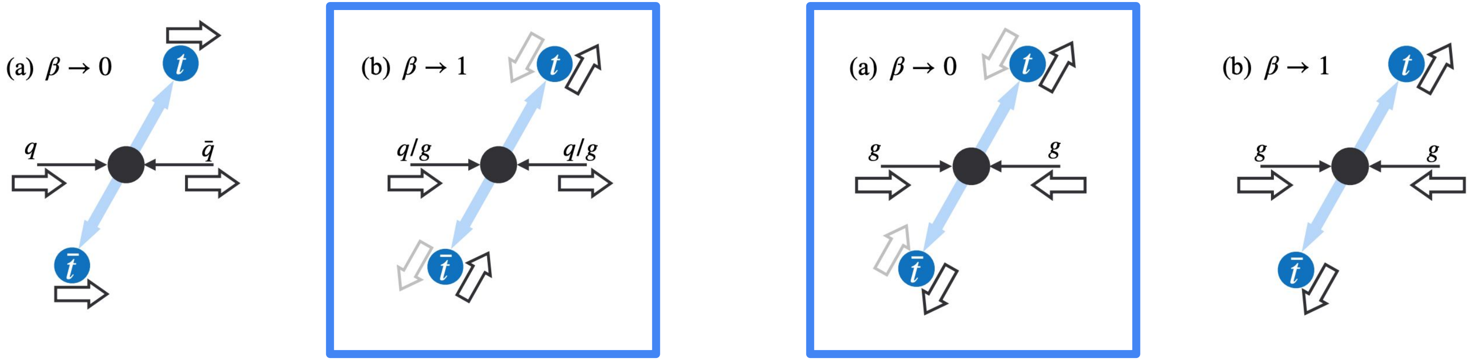


Quantum
Discord

Top-Antitop

- This talk will focus on **$t\bar{t}$ system**
- **Spin correlations** in $t\bar{t}$ have been studied for many years
  - In QCD $t\bar{t}$ is not polarized, but has **spin correlations** — Barger, Ohnemus, Phillips 1989
  - Spin correlations measurable from **angles** of decay products — Mahlon, Parke hep-ph/9512264; Stelzer, Willenbrock hep-ph/9512292

- **qq** and **gg** have different correlations

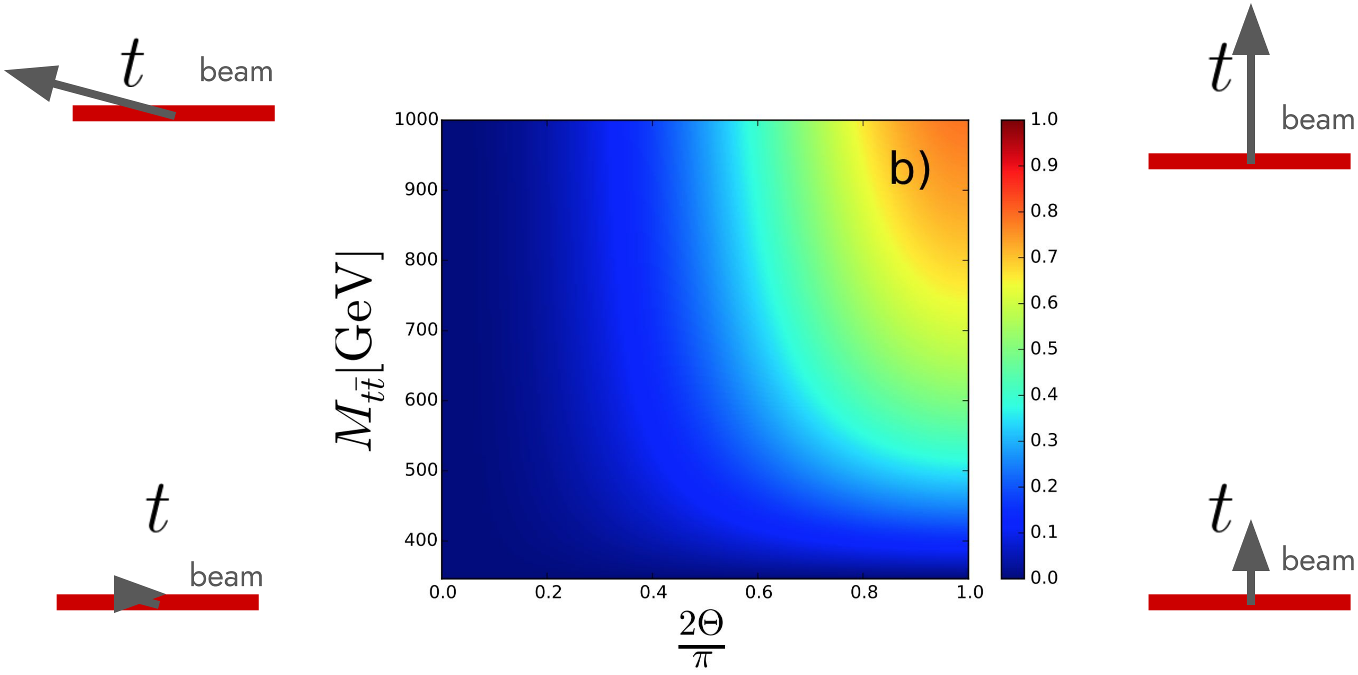


Top-Antitop

- **qq state**

$$\rho_{q\bar{q}} = \underline{a\rho^{(+)}} + \underline{(1-a)\rho^{(X)}_{\text{mix}}}, \qquad a = \frac{\beta^2}{2-\beta^2},$$

Spin-triplet, entangled **Mixed, separable**



Top-Antitop

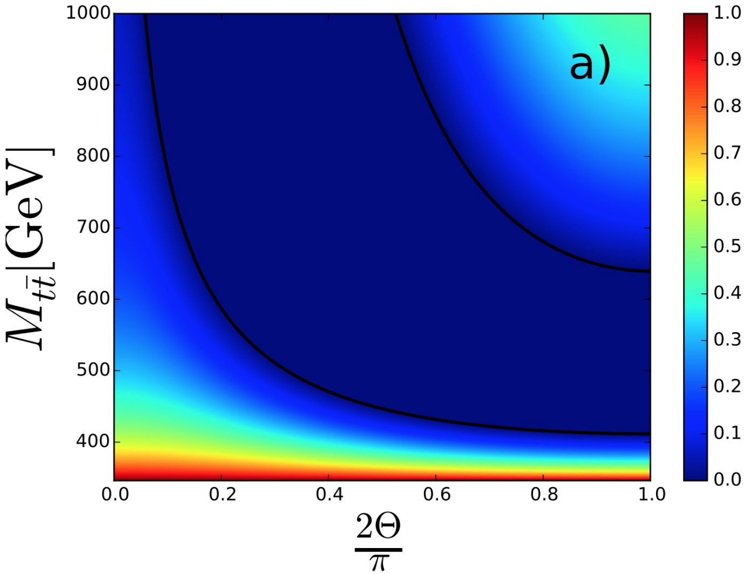
- **gg state**

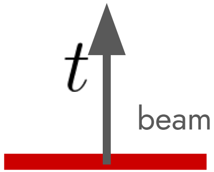
$$\rho_{gg} = a_1\rho^{(+)} + a_2\rho^{(-)} + a_3\rho^{(X)}_{\mathrm{mix}} + a_4\rho^{(Y)}_{\mathrm{mix}},$$

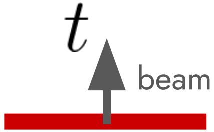
Spin-triplet, entangled (high p_T)

Spin-singlet, entangled (threshold)

Mixed







Top-Antitop

- An entangled quantum state is not separable $\rho \neq \rho_A \otimes \rho_B$
- Given a state ρ, how do we tell if it's entangled or not?
- Compute its **concurrence** $\mathcal{C}$

$\mathcal{C} = 0$	separable
$0 < \mathcal{C} \leq 1$	entangled

- Given in terms of spin correlation **coefficients** (near threshold)

$$\mathcal{C} = -C_{11} - C_{22} - C_{33} - 1$$

- Experimentally used version: the **angle** between the leptons φ

$$\frac{1}{\sigma}\frac{d\sigma}{d\varphi} = \frac{1}{2}(1 - D\cos\varphi) \qquad D = -\frac{1+\mathcal{C}}{3}$$

entangled

$$D < -\frac{1}{3}$$

Top-Antitop

Afik, de Nova 2003.02280

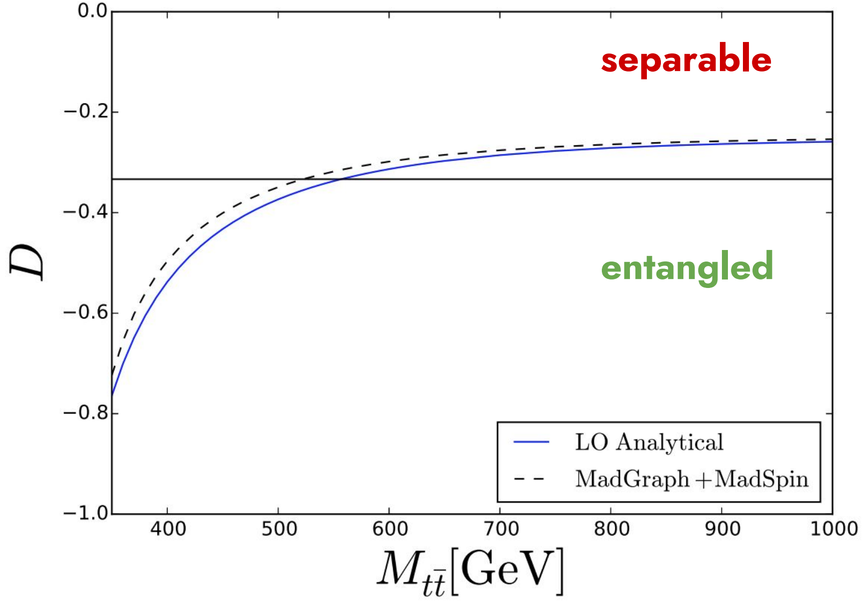
- D was **already measured** (but over the entire phase space)
- Placing an **upper cut** revealed entanglement

Relation to concurrence

$$D = -\frac{1+\mathcal{C}}{3}$$

entangled

$$D < -\frac{1}{3}$$

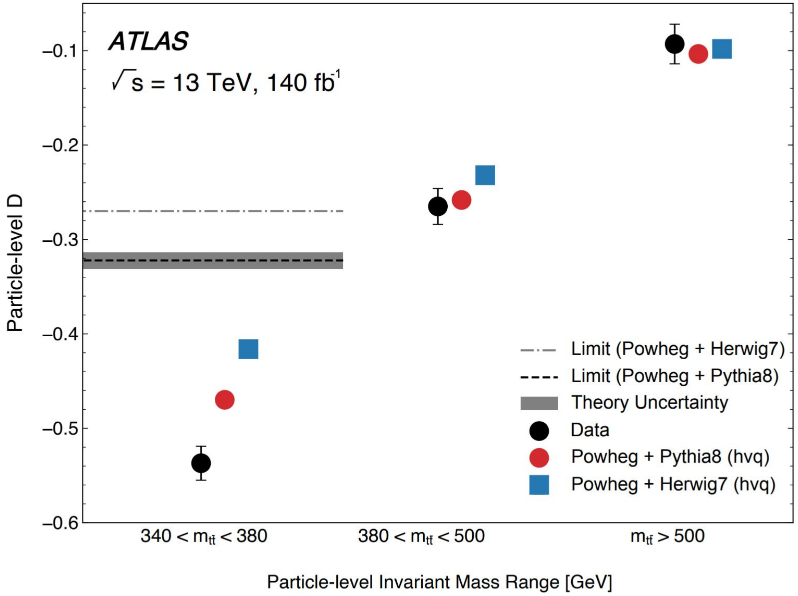


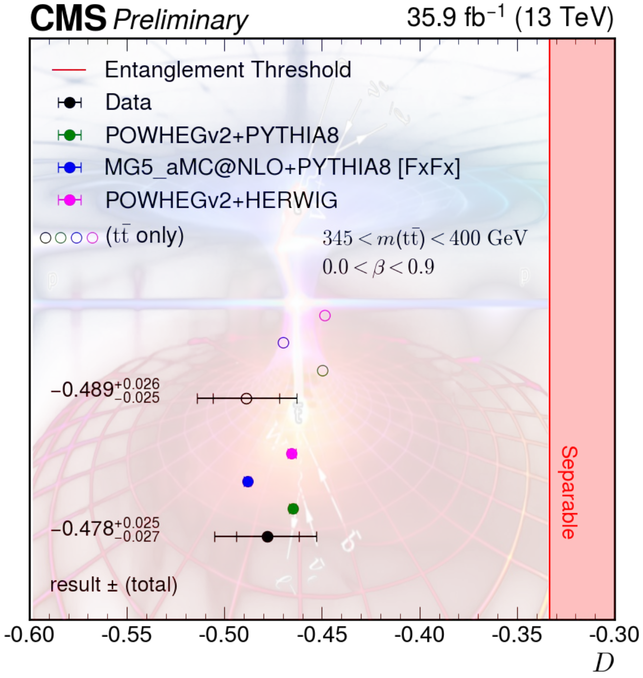
Top-Antitop

ATLAS 2311.07288

CMS 2406.03976

- Already **measured** by both ATLAS and CMS





Top-Antitop

- Bell's inequality given by **CHSH inequality**

$$|E(a_1,b_1)-E(a_1,b_2)+E(a_2,b_1)+E(a_2,b_2)| \leq 2$$

- At a collider

$$E(\vec{a},\vec{b}) = \mathrm{tr}[\rho(\vec{a}\cdot\vec{\sigma}\otimes\vec{b}\cdot\vec{\sigma})] = \langle \vec{a}\cdot\vec{\sigma}\otimes\vec{b}\cdot\vec{\sigma}\rangle$$

- *Example*: Measure qubit 1 along x and qubit 1 along z

$$\vec{a} = (1,0,0) \qquad \vec{b} = (0,0,1) \qquad E(\vec{a},\vec{b}) = C_{xz}$$

- Bell variable is

$$\mathcal{B} = |\langle \vec{a}_1\cdot\vec{\sigma}\otimes\vec{b}_1\cdot\vec{\sigma}\rangle - \langle \vec{a}_1\cdot\vec{\sigma}\otimes\vec{b}_2\cdot\vec{\sigma}\rangle + \langle \vec{a}_2\cdot\vec{\sigma}\otimes\vec{b}_1\cdot\vec{\sigma}\rangle + \langle \vec{a}_2\cdot\vec{\sigma}\otimes\vec{b}_2\cdot\vec{\sigma}\rangle|$$

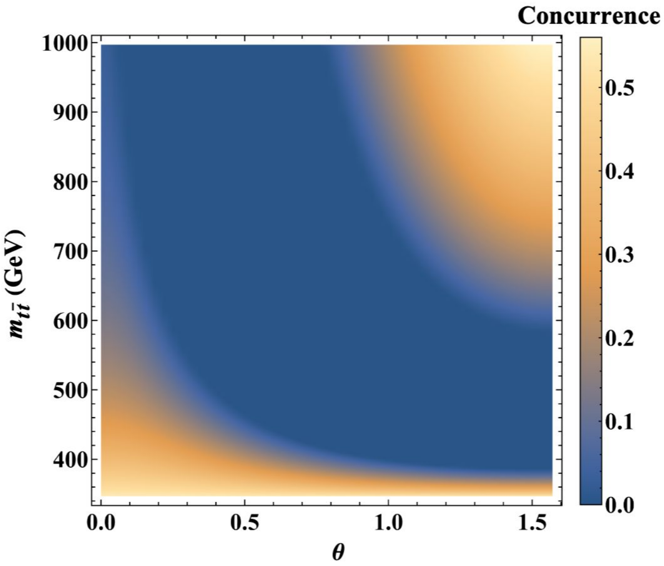
$$\mathcal{B} = \sqrt{2}\,\mathrm{max}_{ij}(C_{ii} \pm C_{jj})$$ (after choosing a_1, a_2, b_1, b_2)

Top-Antitop

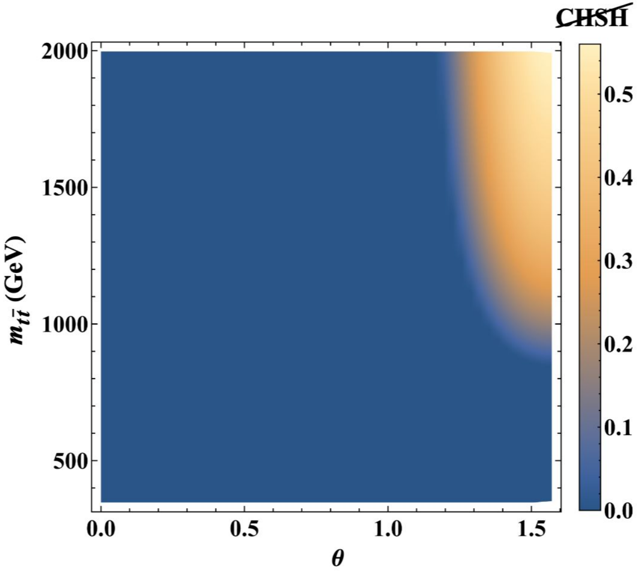
Han, ML, Wu 2310.17696

- In $t\bar{t}$, Bell nonlocality **more difficult** measurement than entanglement

Entanglement



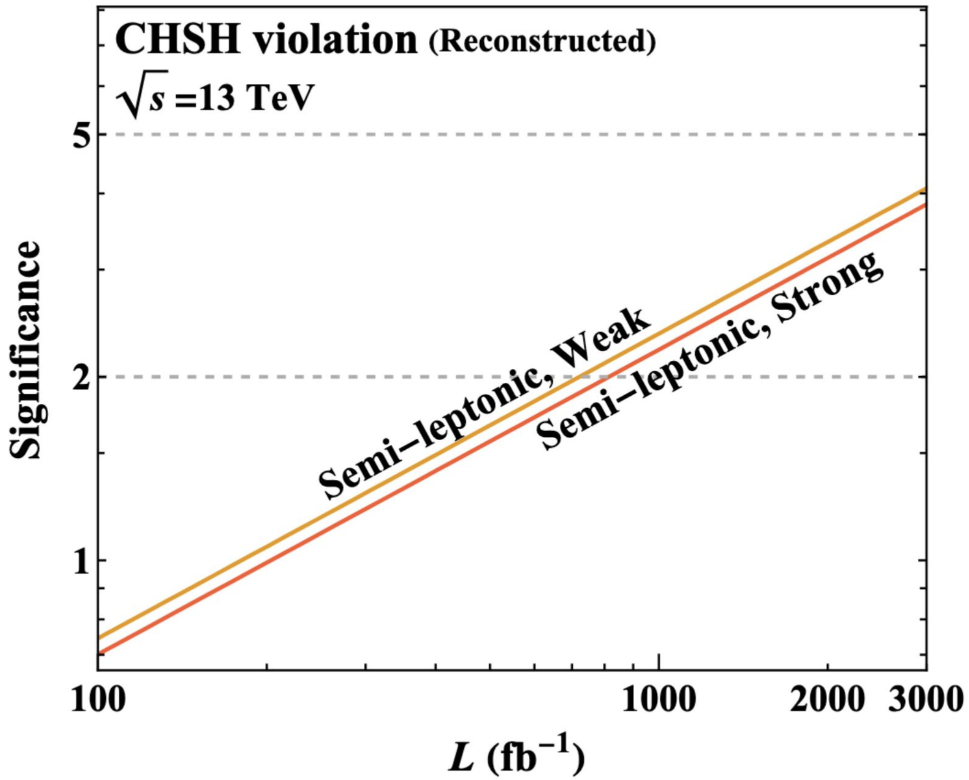
Bell nonlocality

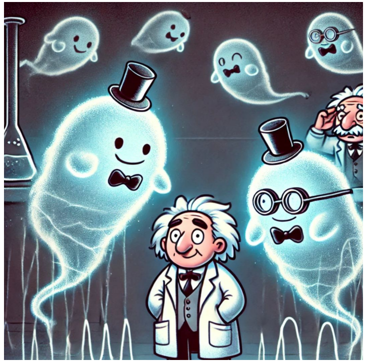


Top-Antitop

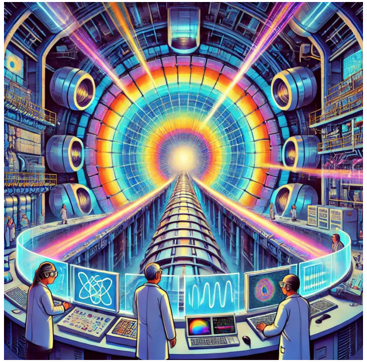
Han, ML, Wu [2310.17696]

- In tt, Bell nonlocality **more difficult** measurement than entanglement

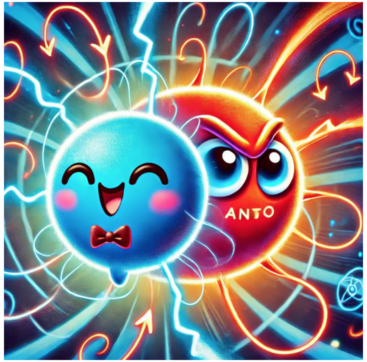




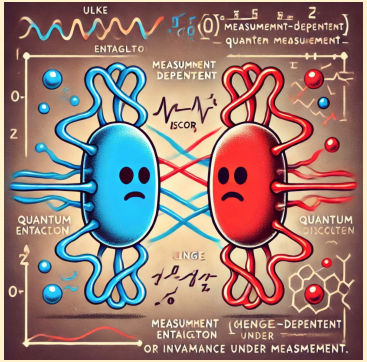
Quantum Mechanics



Tomography at Colliders



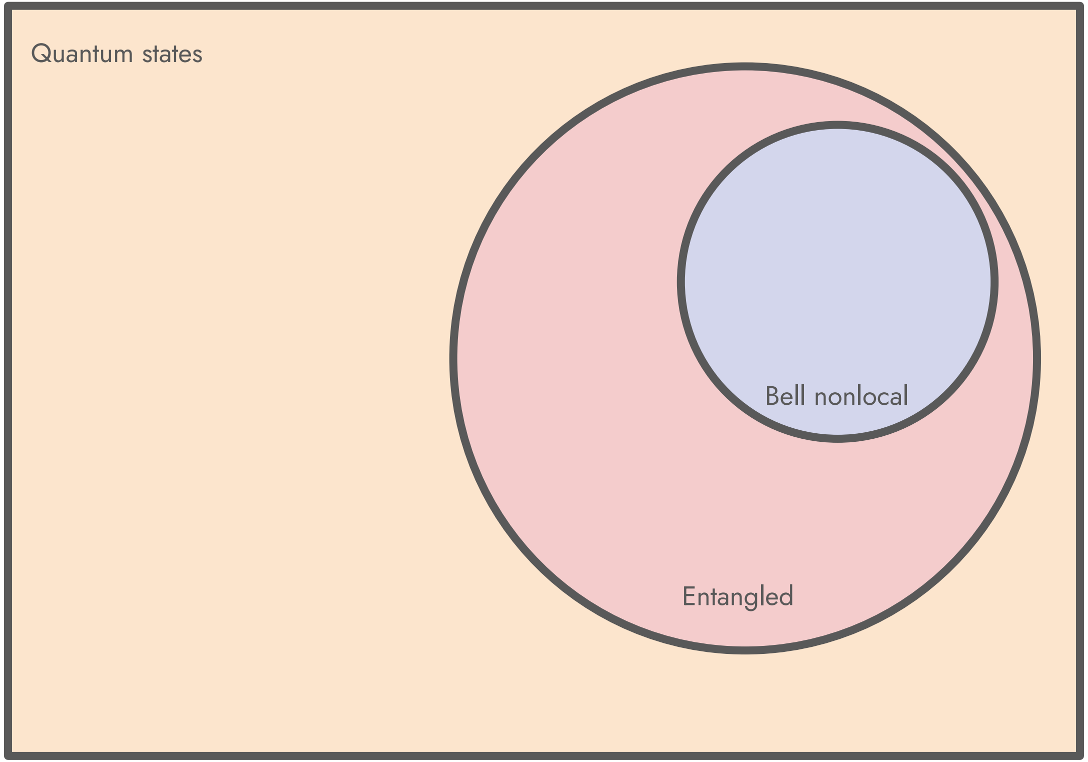
Top-Antitop State



Quantum Discord

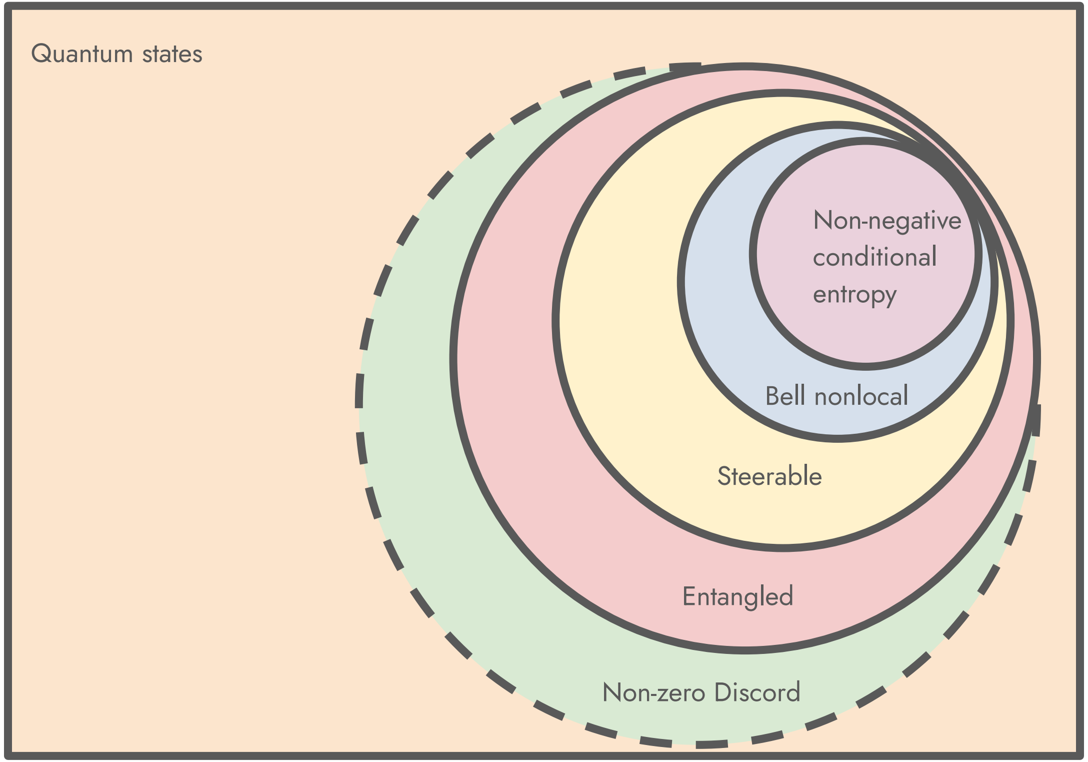
Quantum Discord

- Hierarchy of Correlations



Quantum Discord

- Hierarchy of Correlations



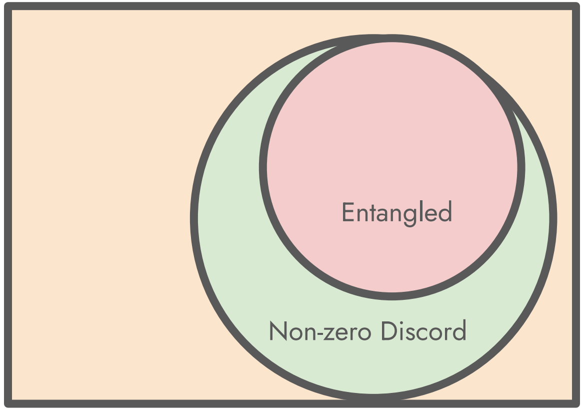
Quantum Discord

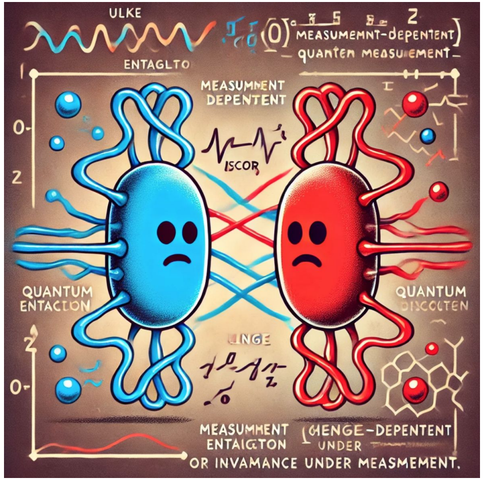
Ollivier, Zurek quant-ph/0105072

- **Quantum Discord**

$$D_A(\rho) = I(\rho) - J_A(\rho)$$

- $I(\rho)$ and $J_A(\rho)$ are equivalent classically
- $J_A(\rho)$ is **measurement-dependent**
- **Non-zero discord** states can be **separable**





Quantum Discord

- Entropy(X) = amount of uncertainty about a variable X
- **Shannon entropy** (classical information theory)

$$H(X) = -\sum_{x \in X} p(x) \log_2(p(x))$$

- *Example*: $X_1=0$

$$H(X_1) = 0$$

(No uncertainty, low entropy)

- *Example*: $X_2=0$ (50% of the time), 1 (50%)

(Large uncertainty, high entropy)

$$H(X_2) = 1$$

- Maximum of Entropy(X_1 ,..., X_N) is N

Quantum Discord

- Entropy(X) = amount of uncertainty about a variable X
- **Von Neumann entropy** (quantum information theory)

$$S(\rho) = -\mathrm{tr}(\rho \log_2(\rho))$$

  - S = 0 for pure states
  - S = N for maximally-mixed states

- _Example_:

$$S(\rho_1) = 2$$

$$\rho_1 = \frac{1}{4}\mathbb{I}_4$$

- _Example_:

$$S(\rho_2) = 0$$

$$\rho_2 = |\psi\rangle\langle\psi|$$

Quantum Discord

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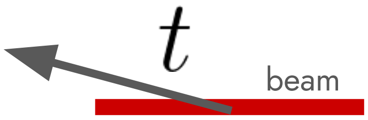
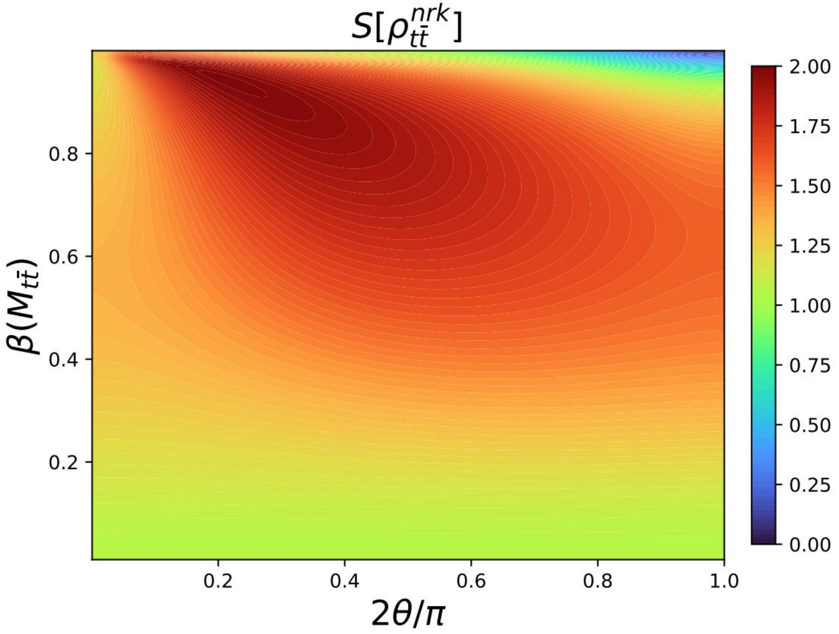
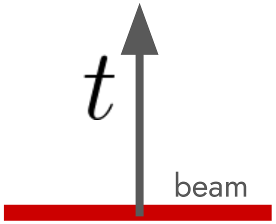
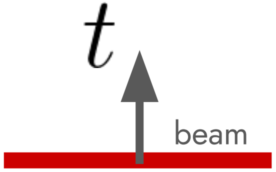
- Entropy(X) = amount of uncertainty about a variable X









Quantum Discord

- **Classical Mutual Information**

$$I(X;Y) = H(X) - H(X|Y)$$
$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

- For two bits X and Y, mutual information is how much **information** you learn about one bit from **observing** the other bit

- *Example*: Alice flips two fair coins c_1 and c_2
  Bob flips two fair coins c_3 and c_4
  The results from Alice reveal **nothing** about Bob

$$I = 0$$

- *Example*: Alice flips two fair coins c_1 and c_2
  Bob flips two fair coins c_2 and c_3
  The results from Alice reveal **1 bit** of information about Bob

$$I = 1$$

Tomography at Colliders

- **Total Mutual Information**

$$I(\rho) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

- The total **information** you learn about one bit from **observing** the other bit, including **both** classical and quantum correlations
- Bounded between 0 and 2 (for two qubits)

$S(\rho_A)$	**Reduced density matrix of A**
$S(\rho_B)$	**Reduced density matrix of B**
$S(\rho_{AB})$	**Total density matrix of A and B**

Quantum Discord

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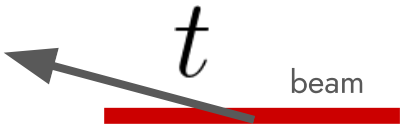
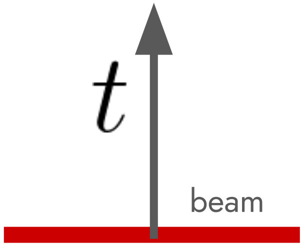
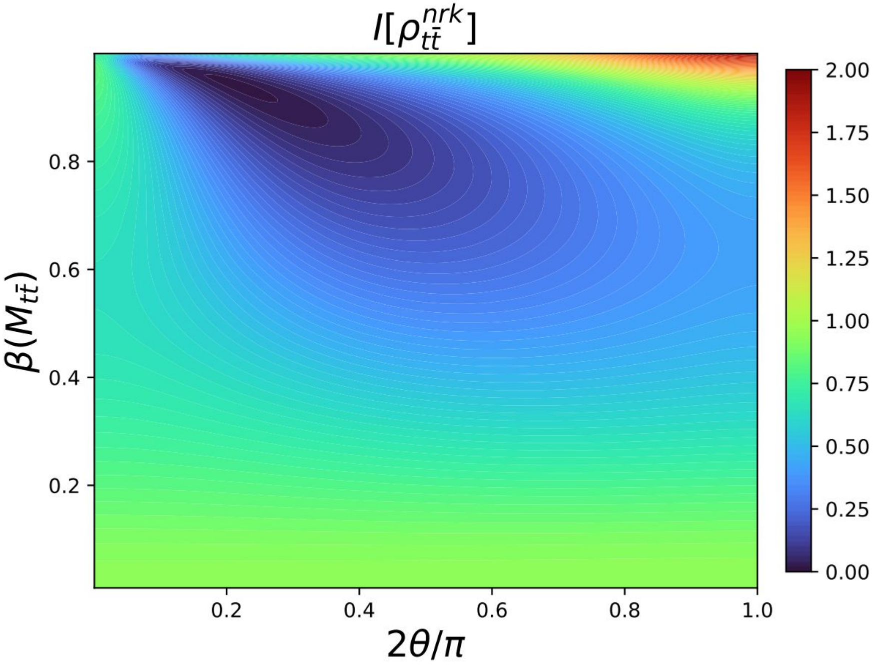
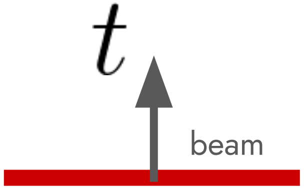
- **Total Mutual Information**









Quantum Discord

- **Classical Mutual Information**

$$J_A(\rho;\hat{n}) = S(\rho_A) - S(\rho_A|\rho_B;\hat{n})$$

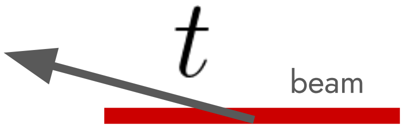
- The mutual information due to **observing B along the axis n**
- **Measurement-dependence** is a quantum effect
- Define classical mutual information as the measurement that **least disturbs** the state

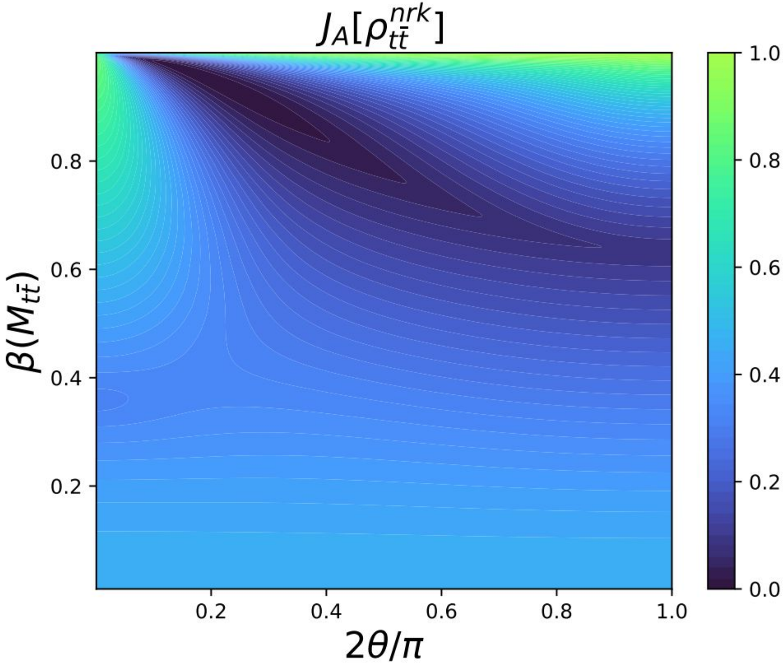
$$J_A(\rho) = \max_{\hat{n}} J_A(\rho;\hat{n})$$

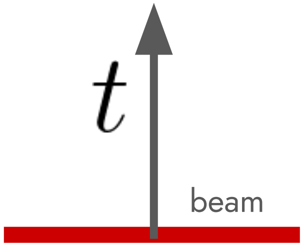
- Maximization makes classical information **hard** to compute and measure
- Bounded between 0 and 1 (for two qubits)

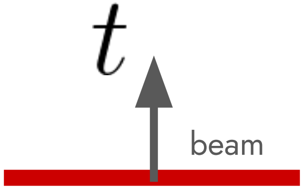
Quantum Discord

- **Classical Mutual Information**









Quantum Discord

- **Quantum Discord**

$$D_A(\rho) = I(\rho) - J_A(\rho)$$

- Difference between **total** mutual information and **classical** mutual information
- Can be **different** for qubit 1 and for qubit 2

$D_A = 0$	Zero discord states (classical-classical, classical-quantum)
$0 < D_A \leq 1$	Non-zero discord states (quantum-classical, quantum-quantum)

- Due to maximization, discord is generally **difficult** to compute
  - Full **analytic solution** for subclass of states: *X*-states
  - Top-antitop state is an ***X*-state**

Luo 2008

Quantum Discord

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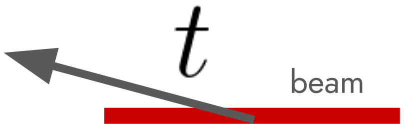
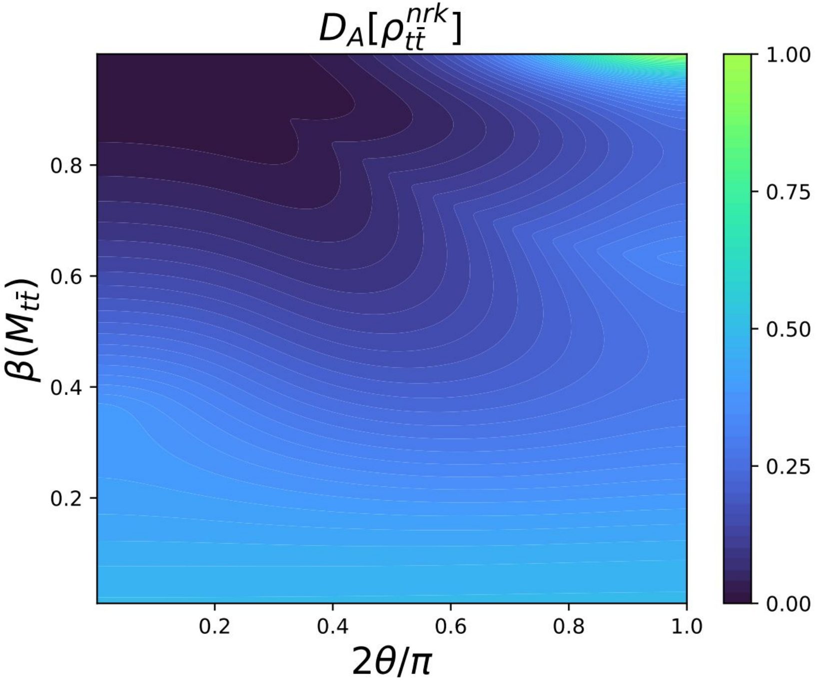
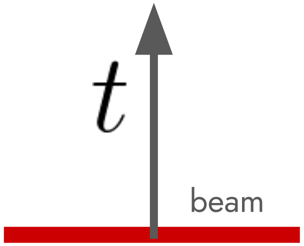
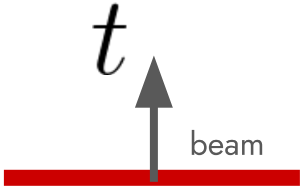
- **Quantum Discord**









Quantum Discord

- **Conditional Entropy**

$$S(\rho_A|\rho_B) = S(\rho) - S(\rho_B)$$

- **Analog** of classical conditional entropy

- **Number of bits** need to be shared with qubit 1 to reconstruct qubit 2
    - S=1 means qubit 1 means 1 bit is needed to reconstruct qubit 2
    - S=0 means qubit 1 doesn't need additional communication to reconstruct qubit 2

- S<0 indicates bits available for **future quantum communication**

Horodecki, Oppenheim, Winter 2005

Quantum Discord

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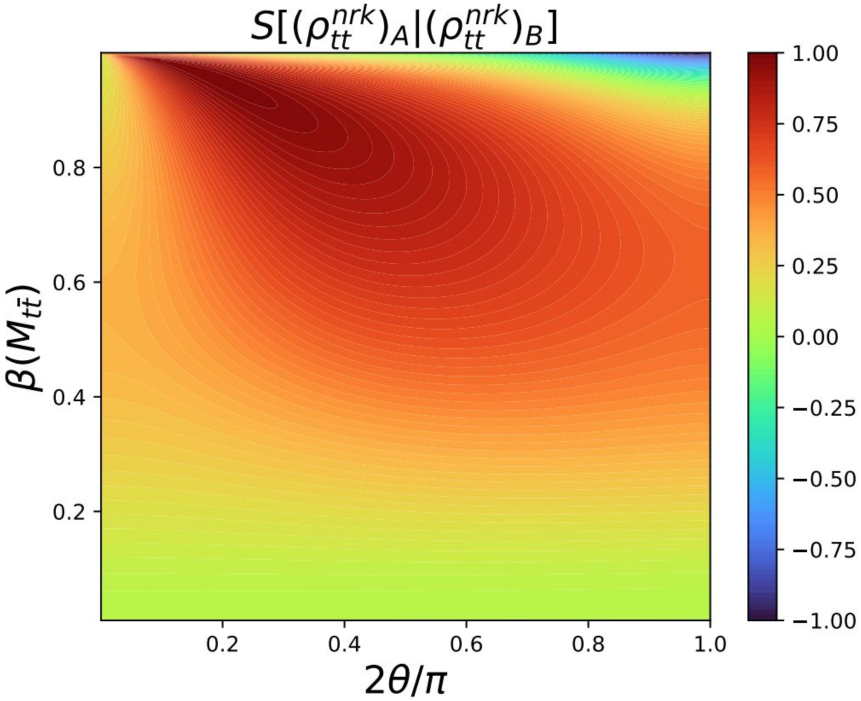
- **Conditional Entropy**



Quantum Discord

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- Simulation of $pp \to t\bar{t} \to (b\ell^+\nu)(\bar{b}\ell^-\bar{\nu})$
  - Event selection

    - At least two jets each of with $p_T > 25$ GeV and $|\eta| < 2.5$.
    - At least one b-tagged jet. If two b-jets are identified, we use these to reconstruct the event. If there is only one b-jet identified, we use the leading non b-tagged jet as the second candidate.
    - Exactly two opposite-sign leptons with $p_T > 25$ GeV and $|\eta| < 2.5$. We consider the ee, $\mu\mu$, and $e\mu$ channels. Leptons must pass an isolation requirement of $I \leq 0.15$.[4]

  - Software

    MadGraph → MadSpin → Pythia → Delphes → RooUnfold

Quantum Discord

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- Signal Regions

  - **Boosted**

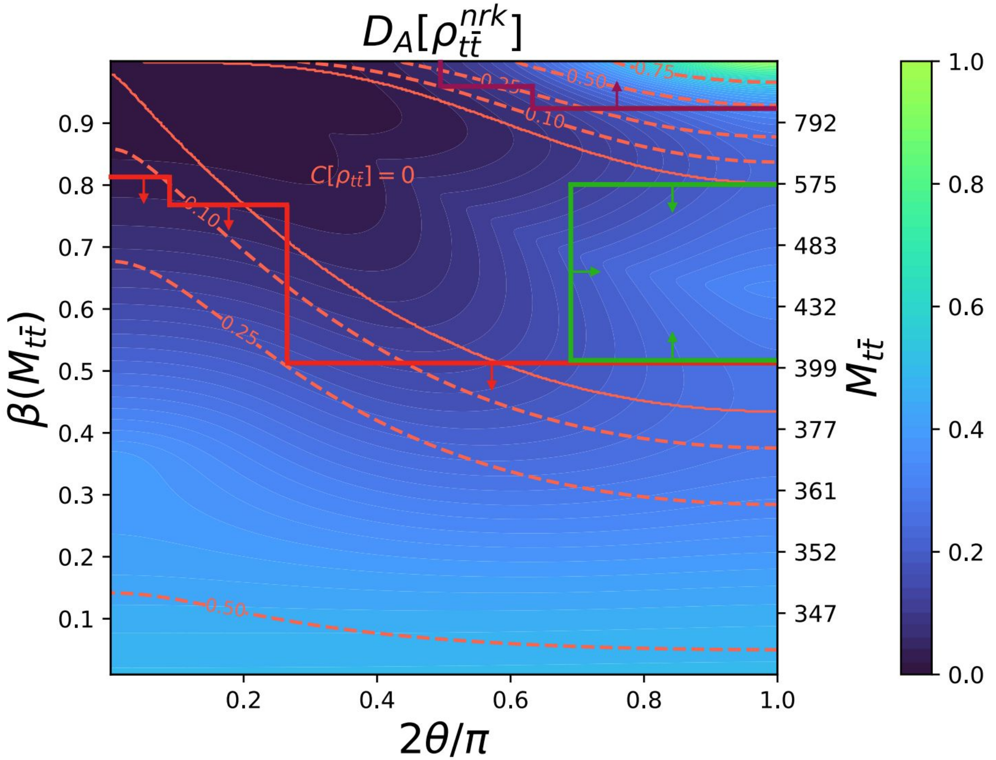
$$800\ \text{GeV} \le M_{t\bar{t}} \quad \text{and} \quad \theta \ge \frac{3\pi}{10},$$
$$1100\ \text{GeV} \le M_{t\bar{t}} \quad \text{and} \quad \theta \ge \frac{\pi}{4}.$$

  - **Separable**

$$400\ \text{GeV} \le M_{t\bar{t}} \le 575\ \text{GeV} \quad \text{and} \quad \theta \ge \frac{3\pi}{8},$$

  - **Threshold**

$$M_{t\bar{t}} \le 400\ \text{GeV},$$
$$M_{t\bar{t}} \le 500\ \text{GeV} \quad \text{and} \quad \theta \le 3\pi/20,$$
$$M_{t\bar{t}} \le 600\ \text{GeV} \quad \text{and} \quad \theta \le \pi/20,$$



Quantum Discord

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- **Results (139 fb^{-1}) – Decay Method**

	Threshold Region		Separable Region		Boosted Region	
	$\langle \epsilon_{rec} \rangle$	$D_A(\rho_{t\bar{t}})$	$\langle \epsilon_{rec} \rangle$	$D_A(\rho_{t\bar{t}})$	$\langle \epsilon_{rec} \rangle$	$D_A(\rho_{t\bar{t}})$
Parton		0.200 ± 0.003		0.255 ± 0.008		0.197 ± 0.003
Reconstructed	0.10	0.23 ± 0.04	0.28	0.18 ± 0.05	0.08	0.20 ± 0.05

5.7σ **3.6σ** **4.2σ**

- *With the kinematic method all results are > 5σ*

Quantum Discord

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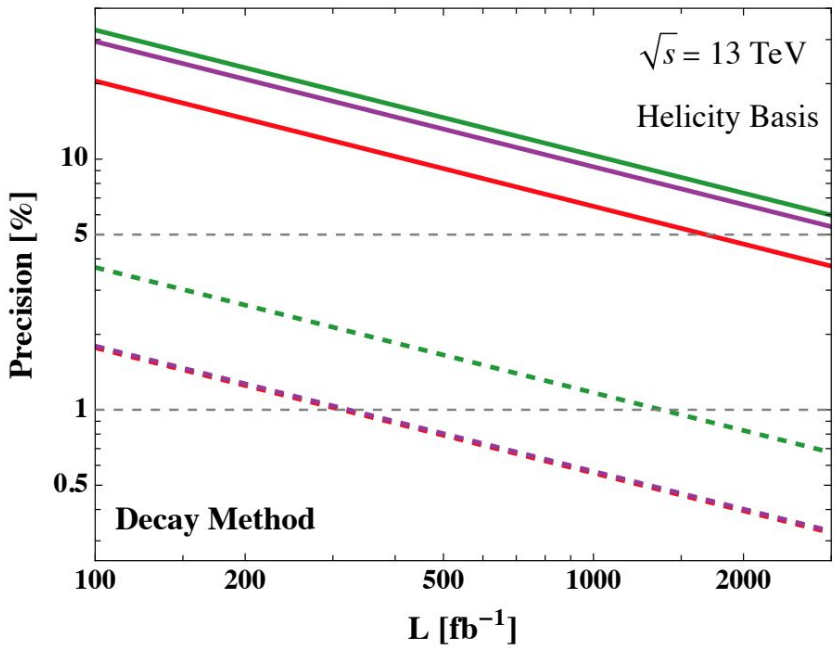
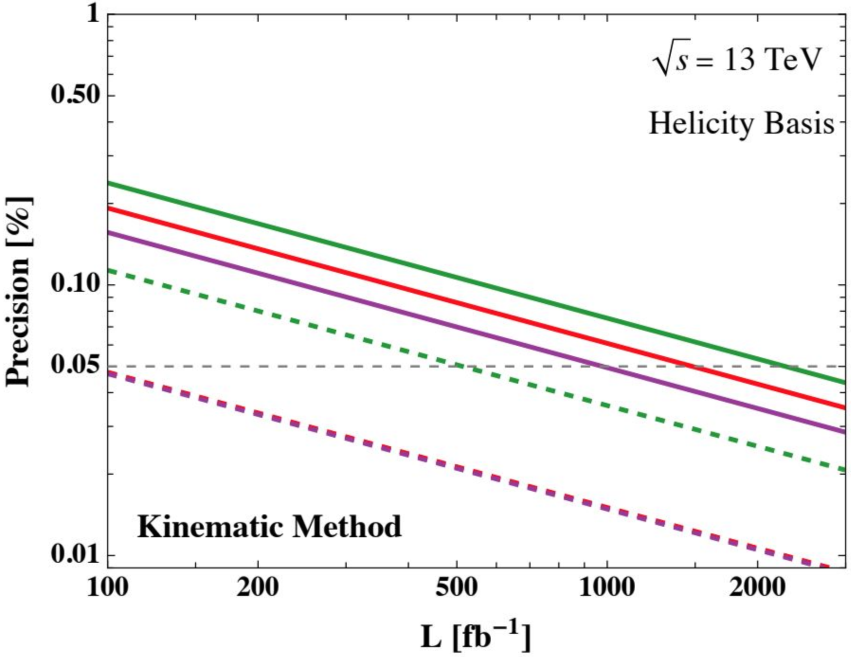
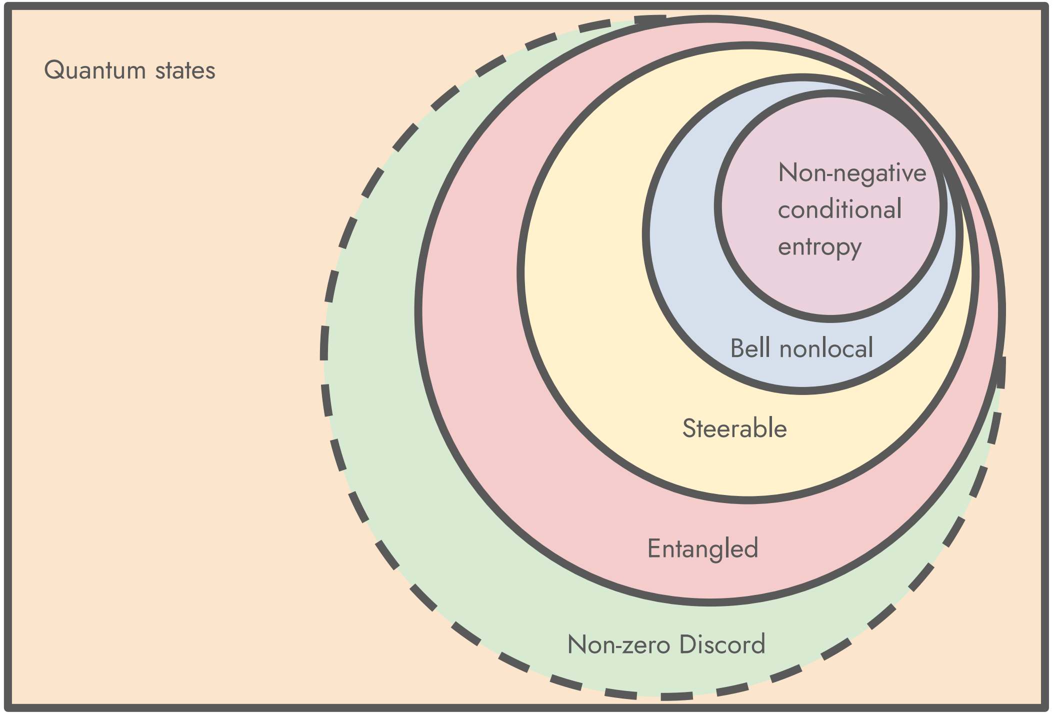
- **Results (139 fb^{-1})**





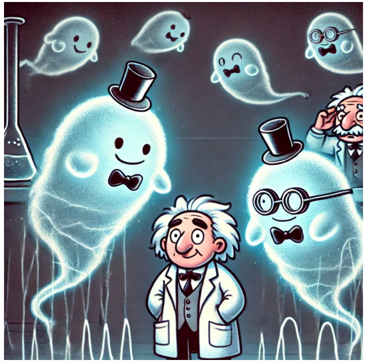
Quantum Discord

- Entanglement, Quantum Discord, Bell nonlocality are measurable in tt
- What about other correlations? What about other final states?



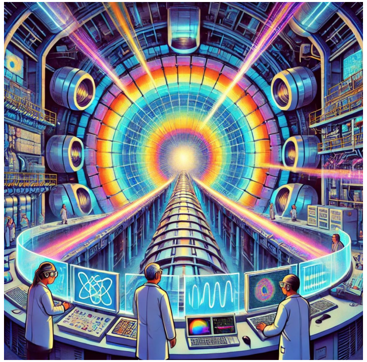
QIT and HEP

- Particle **spins** is one set of quantum systems
  - Choosing spin axes leads to tomography of the density matrix
- Particle **flavor** is another possibility
  - Particle decay reveals flavor
  - Varying **decay times** leads to tomography of the density matrix
- Oscillations of $B^0\bar{B}^0$ at **Belle** is an excellent system
  - Bell nonlocality — Go (Belle) quant-ph/0310192
  - Decoherence models — Hawaii (Belle)
  - Quantum tomography — Cheng, Han, ML, Wu - in preparation



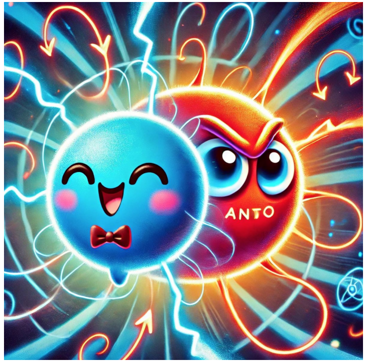
Quantum Mechanics

Non-classical correlations between particles



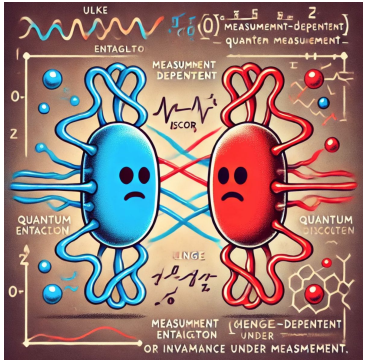
Tomography at Colliders

Can reconstruct quantum states and colliders and compute entanglement, …



Top-Antitop State

Tt system produces many types of quantum states



Quantum Discord

>5σ in current data! Would be the first observation of separable quantum correlations!