Fictitious States and Optimizing Measurements

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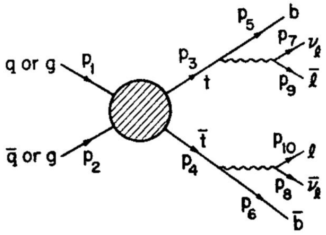
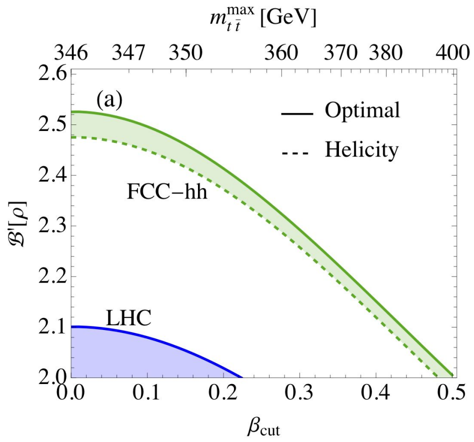
Quantum Tests in Collider Physics, Oxford, UK

Main References:
2311.09166 (Bell inequality)
2407.01672 (Concurrence, Bell inequality, examples, ...)

with Kun Cheng *and* Tao Han

Outline

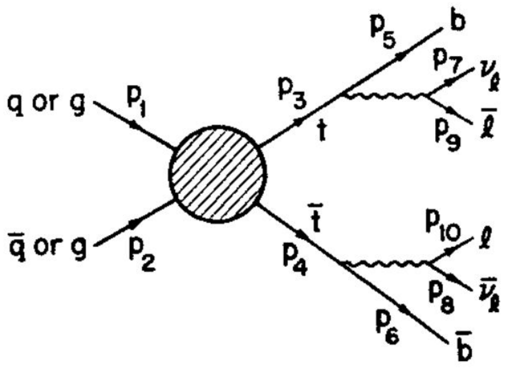




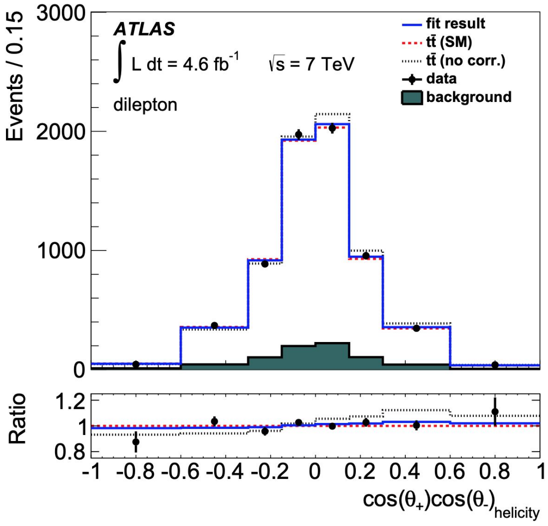
Spin Correlations → Fictitious States → The Optimal Basis

Spin Correlations

- Spin correlations (in the tt system) have been studied for many years
  - At LO tt production has zero *polarization* but non-zero *spin correlations* (*Barger et al 89*)
  - Spin correlations can be detected through the angular decay products *(Mahlon and Parke 95, Stelzer and Willenbrock 95)*
  - Different initial states (qq vs. gg) yield different spin configurations *(Parke et al. 96, 97, ...)*
- Spin correlations have been *measured* in LHC data



ATLAS 1407.4314



Spin Correlations

- Usual method to measure

$$A = \frac{N_{\text{like}} - N_{\text{unlike}}}{N_{\text{like}} + N_{\text{unlike}}} = \frac{N(\uparrow\uparrow) + N(\downarrow\downarrow) - N(\uparrow\downarrow) - N(\downarrow\uparrow)}{N(\uparrow\uparrow) + N(\downarrow\downarrow) + N(\uparrow\downarrow) + N(\downarrow\uparrow)}$$

Need a spin quantization axis

- Extracted from the distribution

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_+ \, d\cos\theta_-} = \frac{1}{4}\left(1 + A\,\alpha_+\alpha_- \cos\theta_+ \cos\theta_-\right) ,$$

θ is angle from the spin quantization axis

- Example from ATLAS ([1407.4314](1407.4314)) uses *helicity* basis and *k-component*

Quantum density matrix

$$\rho = \frac{1}{4}\left(\mathbb{I}_4 + \sum_i B_i^+ \sigma_i \otimes \mathbb{I}_2 + \sum_j B_j^- \mathbb{I}_2 \otimes \sigma_j + \sum_{ij} C_{ij} \sigma_i \otimes \sigma_j\right)$$

Spin correlation matrix in *helicity* basis

"A"

$$C_{ij} = \begin{pmatrix} C_{kk} & C_{kr} & C_{kn} \\ C_{rk} & C_{rr} & C_{rn} \\ C_{nk} & C_{nr} & C_{nn} \end{pmatrix}$$

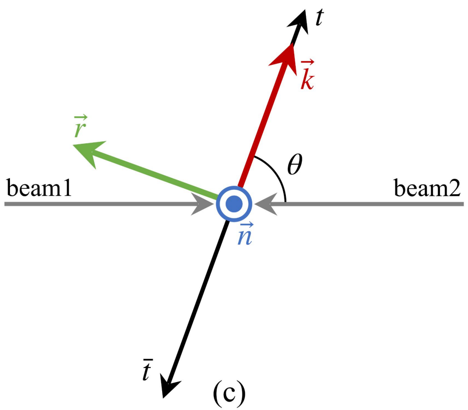
Spin Correlations

- Different quantization bases have different spin correlation matrices

$$\begin{pmatrix} C_{kk} & C_{kr} & C_{kn} \\ C_{rk} & C_{rr} & C_{rn} \\ C_{nk} & C_{nr} & C_{nn} \end{pmatrix} \neq \begin{pmatrix} C_{xx} & C_{xy} & C_{xz} \\ C_{yx} & C_{yy} & C_{yz} \\ C_{zx} & C_{zy} & C_{zz} \end{pmatrix}$$

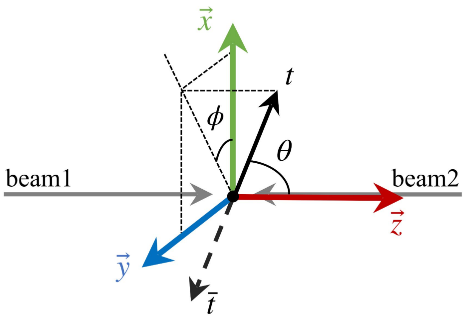
- Bases are related by a rotation $C_{\text{hel}} = R^T C_{\text{beam}} R$

helicity basis



(c)

beam basis



(a)

Spin Correlations

- To estimate one of these entries, we average over many events
  - If each event is using the *same* basis:

$$\Rightarrow C_{kk}$$

  - If each event is using a *different* basis

$$\Rightarrow \langle C \rangle = \frac{1}{N} \sum_{a=1}^{N} C_a$$

- The *averaged* spin correlation is still a form of spin correlation
- In most cases, effectively we are using a different basis for each event
  - We measure *averaged* spin correlations
  - The measured spin correlation matrices are **not** related by rotations any longer

Spin Correlations

- Let C_a^A be the underlying spin correlation matrix in basis A and event a, the measured spin correlation matrix is

$$\langle C \rangle^A = \frac{1}{N} \sum_{a=1}^{N} C_a^A$$

$\leftarrow$ which basis

$\leftarrow$ which event

- The rotation to basis B is *event-dependent* and the **measured** spin correlation matrix is

$$\langle C \rangle^B = \frac{1}{N} \sum_{a=1}^{N} R_a^T C_a^A R_a$$

- In general, no such rotation R exists

$$\langle C \rangle^B = R^T \langle C \rangle^A R$$

- Therefore, due to averaging, spin correlations are **basis-dependent**

Parke, Shadmi [hep-ph/9606419]
Mahlon, Parke [hep-ph/9706304]
Mahlon, Parke [1001.3422]

Spin Correlations

- Example: $q\bar{q} \to t\bar{t}$
  - Helicity Basis

$$C_{\text{hel}} = \begin{pmatrix} 0.66 & 0 & -0.33 \\ 0 & -0.003 & 0 \\ -0.33 & 0 & 0.34 \end{pmatrix}$$

$$\lambda = \{0.87, 0.13, -0.003\}$$

  - Beam Basis

$$C_{\text{beam}} = \begin{pmatrix} 0.003 & 0 & 0.002 \\ 0 & -0.003 & 0 \\ 0.002 & 0 & 0.99 \end{pmatrix}$$

$$\lambda = \{0.99, 0.003, -0.003\}$$

Quantum Information at High Energies

- *Quantum states* do **not** depend on the spin basis

$$\rho = \frac{1}{4}\left(\mathbb{I}_4 + \sum_i B_i^+ \sigma_i \otimes \mathbb{I}_2 + \sum_j B_j^- \mathbb{I}_2 \otimes \sigma_j + \sum_{ij} C_{ij} \sigma_i \otimes \sigma_j\right)$$

- Change of basis is a unitary rotation U

$$\rho \to U^\dagger \rho U$$

- We can directly see quantities of interest are *basis-independent*
  - **Concurrence** $\mathcal{C}(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$ ← Eigenvalues of M

$$M = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$$

$$M \to U^\dagger M U$$

  - **Bell variable** $\mathcal{B}(\rho) = 2\sqrt{\lambda_1 + \lambda_2}$ ← Eigenvalues of $C^T C$

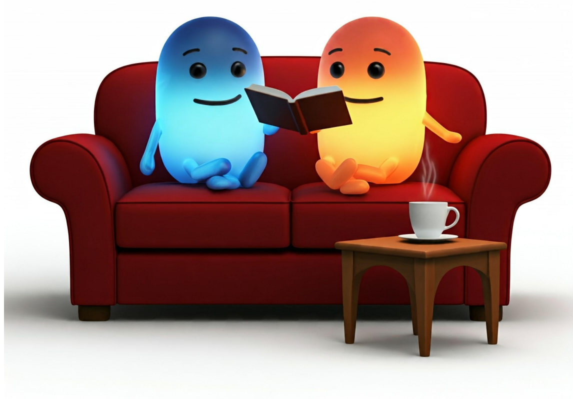
$$C^T C \to R^T C^T C R$$

Quantum Information at High Energies

- Paradox?
  - Quantum states are spin basis-**independent**
  - Spin correlations are spin basis-**dependent**

- We are not using genuine quantum states, we are using **``fictitious states''**

Afik, de Nova 2203.05582
Cheng, Han, ML 2311.09166
Cheng, Han, ML 2407.01672



Quantum Information at High Energies

- ***What*** are fictitious states?
  - *Basis-dependent* state
  - State reconstructed from *averaged* quantities
  - Convex sum of quantum sub-states, **but** with coefficients due to rotations

Quantum state $$\rho_Q = \sum_a \rho_a$$

Fictitious state $$\rho_{\text{fic}} = \sum_a c_a \rho_a$$

$$c_a = \mathrm{tr}(\sigma_{i,a} \otimes \sigma_{j,a} \cdot \sigma_i \otimes \sigma_j)$$

- ***Why*** does it matter?
  - *Breaks* some quantum properties
  - *Preserves* other quantum properties

Note: Physics is **described** by an underlying quantum state, we **reconstruct** the fictitious state

Quantum Information at High Energies

- Fictitious states break: $\langle \mathcal{O} \rangle = \mathrm{tr}(\rho \mathcal{O})$
  - Example: $C_{ij} = \langle \sigma_i \otimes \sigma_j \rangle$

$$C_{ij} = \mathrm{tr}(\rho \sigma_i \otimes \sigma_j)$$
$$C_{ij} \neq \mathrm{tr}(\rho_{\mathrm{fic}} \sigma_i \otimes \sigma_j)$$

→ The numerical value of concurrence *calculated from the fictitious state* is **not** the concurrence of the *underlying quantum state*

- Fictitious states preserve:
  - **Zero** vs. **non-zero** concurrence
  - **Violation** vs. **non-violation** Bell inequality

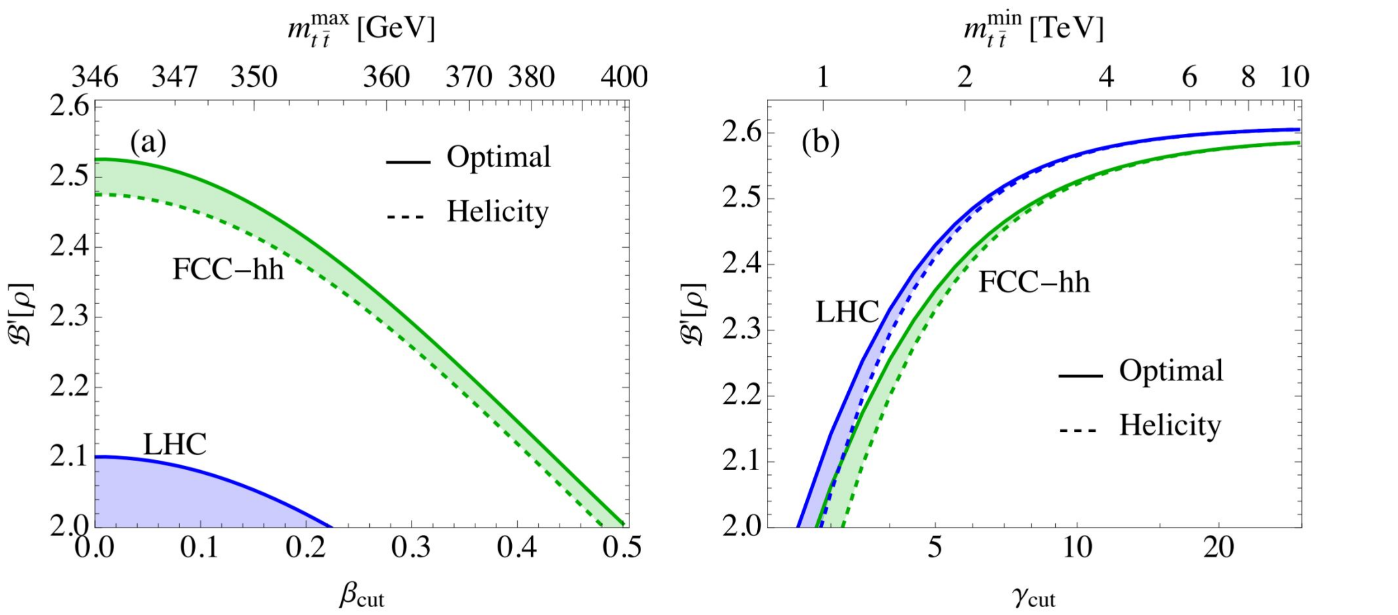
$$\mathcal{C}(\rho_{\mathrm{fic}}) > 0 \quad \Rightarrow \quad \mathcal{C}(\rho_Q) > 0$$
$$\mathcal{B}(\rho_{\mathrm{fic}}) > 2 \quad \Rightarrow \quad \mathcal{B}(\rho_Q) > 2$$

Optimal Basis at Colliders

- Fictitious states are *basis-dependent*
  - There is an **optimal basis** to maximize quantity X (X = concurrence, Bell variable, etc.)
  - Example: $pp \to t\bar{t}$

Threshold region

Boosted region



Optimal Basis at Colliders

- *Optimal basis* is the one that **diagonalizes** the spin correlation matrix
- Same optimal basis for concurrence, Bell variable (eigenvalues), Bell variable (fixed axis)
  - Example: $q\bar{q} \to t\bar{t}$

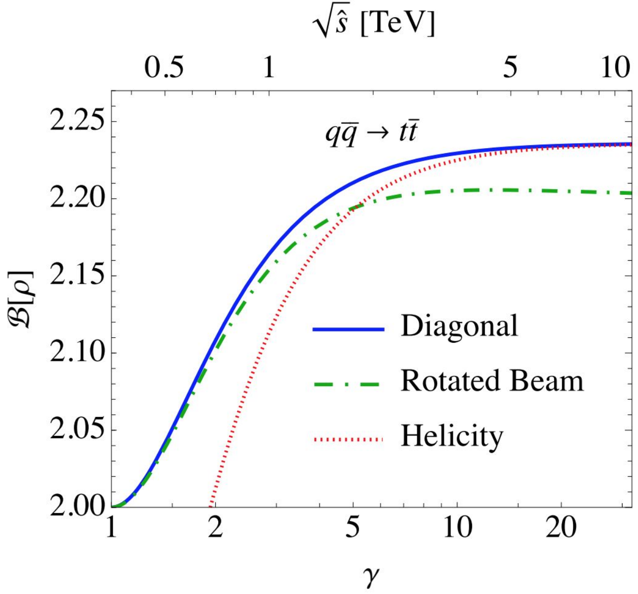
Helicity basis

$$C_{ij} = \begin{pmatrix} \frac{(2-\beta^2)s_\theta^2}{2-\beta^2 s_\theta^2} & 0 & -\frac{2c_\theta s_\theta\sqrt{1-\beta^2}}{2-\beta^2 s_\theta^2} \\ 0 & \frac{-\beta^2 s_\theta^2}{2-\beta^2 s_\theta^2} & 0 \\ -\frac{2c_\theta s_\theta\sqrt{1-\beta^2}}{2-\beta^2 s_\theta^2} & 0 & \frac{2c_\theta^2+\beta^2 s_\theta^2}{2-\beta^2 s_\theta^2} \end{pmatrix}$$

Rotate by angle ξ $\qquad \tan\xi = \frac{1}{\gamma}\tan\theta$

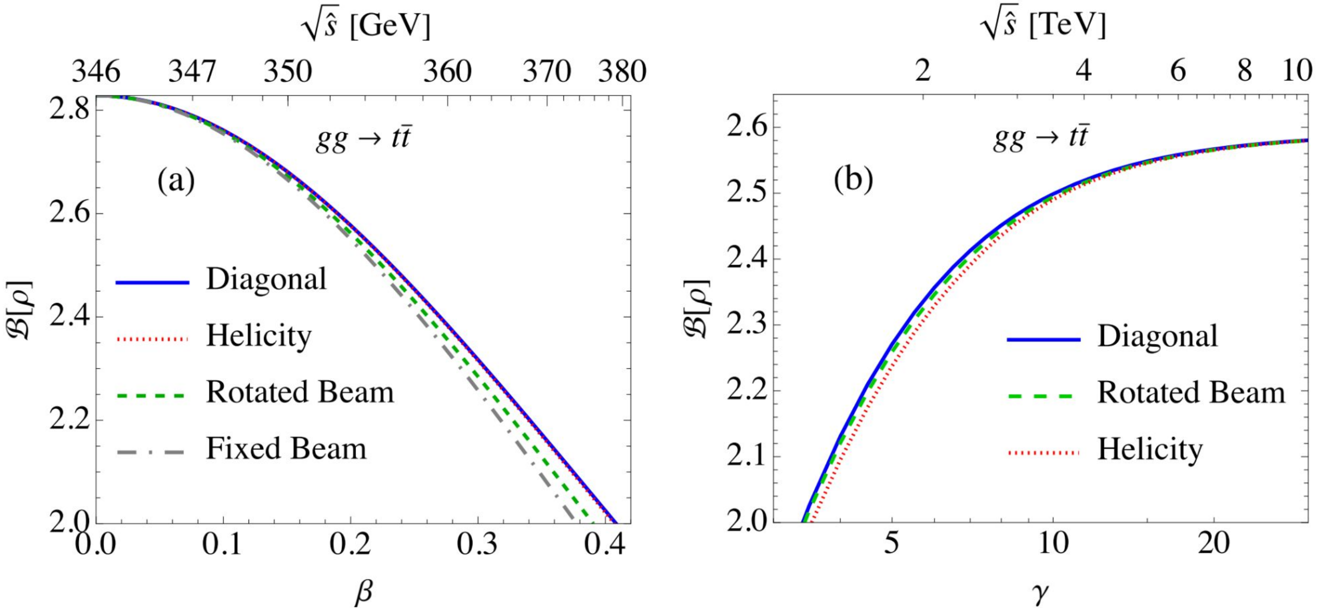
Diagonal/Optimal basis

$$C^{\mathrm{diag}} = \begin{pmatrix} \frac{\beta^2 s_\theta^2}{2-\beta^2 s_\theta^2} & 0 & 0 \\ 0 & -\frac{\beta^2 s_\theta^2}{2-\beta^2 s_\theta^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

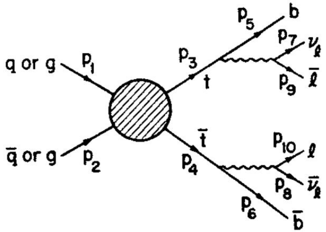


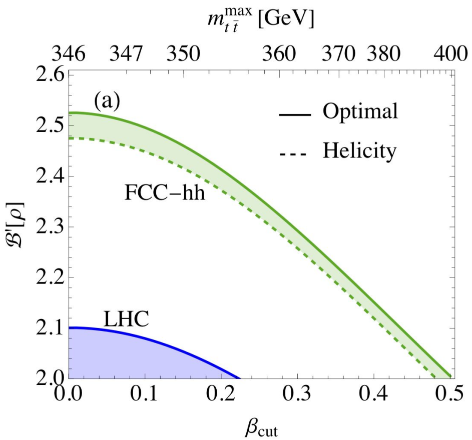
Optimal Basis at Colliders

- *Optimal basis* is the one that **diagonalizes** the spin correlation matrix
- Same optimal basis for concurrence, Bell variable (eigenvalues), Bell variable (fixed axis)
  - Example: $gg \to t\bar{t}$



Conclusions





Spin Correlations → Fictitious States → The Optimal Basis

- Averaging makes spin correlations basis-dependent
- Basis-dependence forces fictitious states rather than quantum states
- Can leverage basis-dependence into optimizing concurrence and Bell violation

Backup: tt Spin Configurations

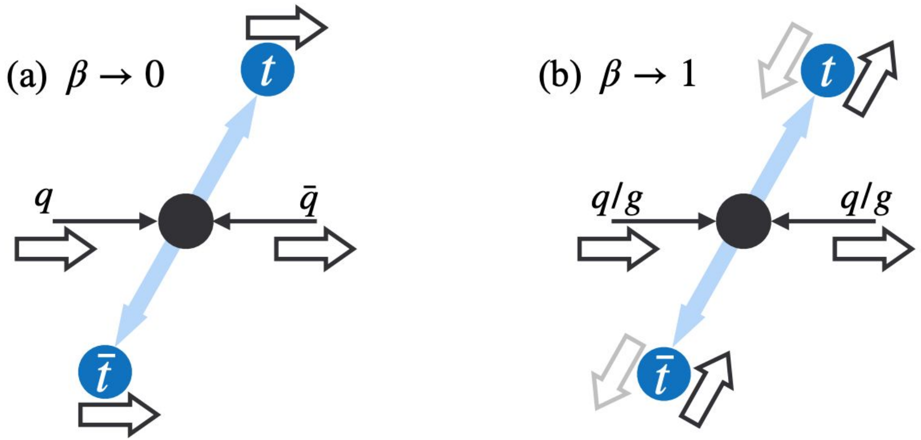


FIG. 3. Spin configurations of $t\bar{t}$ produced from unlike-helicity initial states: (a) for $q\bar{q} \to t\bar{t}$ near threshold, with the cross section proportional to β; and (b) for $q\bar{q}$, $g_L g_R \to t\bar{t}$ in the boosted region. Figure adapted from Ref. [42].

Backup: tt Spin Configurations

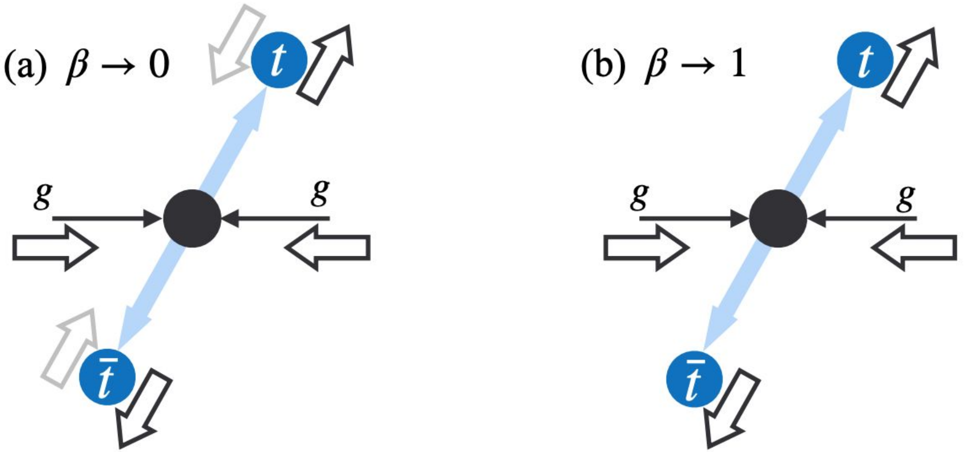


FIG. 4. Spin configurations of $t\bar{t}$ produced from like-helicity gluons near and above threshold. The cross section is proportional to β. Figure adapted from Ref. [42].