Assignment 1

October 25, 2023

To be submitted at the beginning of class on Monday November 6. This assignment will be graded. You may either type or handwrite your answers, but please write legibly. Your Dynare simulations should be included as pictures in your document and you must include your Dynare codes (the .mod file). Please do not include the matlab numerical output - only the figures. Any plagairism (see SFU guidelines) will result in an immediate grade of zero and reported to FASS.

Part I

Consumption-Saving Model (For Grading)

Consider Hitoshi who lives for two periods. In period 1, he is young and in period 2 he is old. He enjoys consuming, c_t, and experiences distutility from labour, n_t, according to the following preference function:

$$U(c_1, c_2, n_1, n_2) = 2c_1^{0.5} + 2\beta c_2^{0.5} - 0.5n_1^2 - 0.5\beta n_2^2$$

He earns income from supplying labour in period 1 of $w_1 n_1$ and in period 2 of $w_2 n_2$. Any saving he earns in period 1 can earn a return of $(1 + r_1)$ that is paid out to him at the beginning of period 2. He also knows when he is young that he will receive an inheritence of a_2 in period 2.

a) Write out Hitoshi's one and two-period budget constraints. Combine these to form an intertemporal budget constraint.

b) What is Hitoshi's choice problem? What variables does he have control over? What variables does he take as given?

c) Solve Hitoshi's two-period optimization problem by taking FOCs and deriving his intertemporal and intra-temporal tradeoff conditions.

d) Does Hitoshi's inheritence affect his intra-temporal tradeoffs? What about his inter-temporal tradeoffs? Explain why or why not.

e) Does the amount that Hitoshi expects to inherit in period 2 change how much Hitoshi would spend in Period 1? Why or why not? Explain and show your math.

Part II

Real Business Cycle Model

For Practice (Done in class together) - Do not submit

Consider the following real business cycle model. A representative household seeks to maximize its expected discounted utility given by

$$E_t \sum_{t=0}^{\infty} \beta^t \left(lnc_t + \theta ln(1 - n_t) \right)$$

where c_t is period t consumption and n_t is period t (fraction of) time spent working. The household faces a per-period budget constraint given by

$$w_t n_t + r_t k_t + (1 - \delta)k_t + R_t b_t = c_t + k_{t+1} + b_{t+1}$$

where b_{t+1} is bond purchases in period t, k is the capital stock available in period $t+1$, and $\delta \in (0,1)$ is the depreciation rate, $\gamma > 0$, $\eta > 0$. A representative firm produces output using the following constant returns to scale production function given by

$$Y_t = e^{Z_t} K_t^{\alpha} N_t^{1-\alpha}$$

where it rents capital and labour from the household. Assume that produdctivity follows an AR(1) process:

$$Z_t = \rho Z_{t-1} + \epsilon_t$$

where ρ<1 and ϵ is a mean zero, white noise process. Assume agents form rational expectations and make optimal decisions.

a) What are the state variables in this problem? (State variables are known to the household when it makes its optimization decisions). What are the endogenous variables?

b) What is the household's choice problem? What is the firm's choice problem?

c) Derive the first order conditions of this problem for both the household and firm. Use them to formulate an intratemporal tradeoff condition and intertemporal tradeoff conditions for consumption and labour.

d) With the equilibrium conditions you obtained, program up this model in Dynare. You may use the parameters provided in the class tutorial.

e) Suppose the economy faces a one-time unanticipated negative productivity shock: $\epsilon_t < 0$. Thereafter, the shocks all remain 0. Describe - in careful detail - the transition of the the economy in response to the productivity shock. **Make sure to explain in detail the tradeoffs the household and firm face in response to the shock.**

f) Compare the variability/variance of output, consumption, and investment. Are these relative volatilies consistent with the business cycle facts?

Preference shock (For Grading)

Instead of a productivity shock[1], now consider the effects of a one-time unanticipated shock to preferences. Now suppose that consumption becomes suddenly more enjoyable (possibly because of a near death experience, like the Pandemic). The utility function becomes:

$$E_t \sum_{t=0}^{\infty} \beta^t \left((1+\phi_t) ln c_t + \theta ln(1-n_t) \right)$$

where $\phi_t = \rho\phi_{t-1} + \epsilon_t$, and $\rho < 1$, $\epsilon_t \sim iidN(0, \sigma_\epsilon^2)$ is a white noise process. Assume agents form rational expectations and make optimal decisions.

The representative household has the same budget constraint as in the previous problem, while the representative firm faces the sames production function (albeit, with no productivity shocks).

a) Derive the household's first order conditions based on this new preference function. State the household's inter-temporal and intra-temporal tradeoff conditions.

b) Derive the firm's first order conditions.

c) Derive the equilibrium conditions for the economy. What equilibrium conditions change because of changes to ϕ_t?

d) Simulate with Dynare the effects of a positive, temporary but persistent increase in utility from consumption (a one-time positive ϵ_t). Be clear what values you assign to your parameters and what the steady state values are. Include the simulated inpulse response function graphs.

e) Explain how the household's labour supply, consumption, and investment decisions are altered relative to their steady state level, both on impact of the shock and in the subsequent periods. What happens to the wage and rental rate on impact of the shock, and in the periods that follow? More importantly, explain intuitively through the equilibrium equations why these variables are changing in response to a temporary increase in utility from consnumption.

f) Find four different business cycle facts from various sources. Pplease cite your sources.

[1] Assume $Z_t = 0$ for all t

g) In what ways does the model do a good job at fitting these business cycle facts? In what ways could it be improved? **Hint:** Use the Moments, Correlation, and Autocorrelation of Simulated Variables tables from Dynare to evaluate your model.

h) How can you change the parameters of your simulation to improve the fit of your model to these business cycle facts?

Make sure to include your Dynare codes in your assignment. Please do not include the entire matlab output.

Part I

Consumption-Saving Model (For Grading)

Consider Hitoshi who lives for two periods. In period 1, he is young and in period 2 he is old. He enjoys consuming, c_t, and experiences distutility from labour, n_t, according to the following preference function:

$$U(c_1, c_2, n_1, n_2) = 2c_1^{0.5} + 2\beta c_2^{0.5} - 0.5n_1^2 - 0.5\beta n_2^2$$

He earns income from supplying labour in period 1 of $w_1 n_1$ and in period 2 of $w_2 n_2$. Any saving he earns in period 1 can earn a return of $(1 + r_1)$ that is paid out to him at the beginning of period 2. He also knows when he is young that he will receive an inheritence of a_2 in period 2.

a) Write out Hitoshi's one and two-period budget constraints. Combine these to form an intertemporal budget constraint.

one period budget constraint:

$$c_1 + s_1 = w_1 n_1$$

c_1 = consumption in period 1

s_1 = saving in period 1

w_1 = wage rate in period 1

n_1 = level of labour in period 1

Two period budget constraint:

$$c_2 = w_2 n_2 + (1 + r_1) s_1 + a_2$$

c_2 = consumption in period 2

w_2 = wage rate in period 2

n_2 = level of labour in period 2

a_2 = inheritance in period 2

Intertemporal budget constraint:

$$c_1 + \frac{c_2}{1+r_1} = w_1 n_1 + \frac{w_2 n_2}{1+r_1} + \frac{a_2}{1+r_1}$$

b) What is Hitoshi's choice problem? What variables does he have control over? What variables does he take as given?

Hitoshi's choice problem:

$$L = U(c_1, c_2, n_1, n_2) + \lambda_1 (w_1 n_1 - c_1 - s_1) + \lambda_2 (w_2 n_2 + (1+r_1) s_1 + a_2 - c_2)$$

$$L = 2c^{0.5} + 2\beta c_2^{0.5} - 0.5 n_1^2 - 0.5\beta n_2^2 + \lambda_1 (w_1 n_1 - c_1 - s_1) + \lambda_2 (w_2 n_2 + (1+r_1) s_1 + a_2 - c_2)$$

Variables he has control over:	Variables he takes:
C_1 = consumption in period 1	w_1 = wage rate in period 1
C_2 = consumption in period 2	w_2 = wage rate in period 2
n_1 = labour supply period 1	a_2 = inheritance in period 2
n_2 = labour supply period 2	r_1 = return on savings period 1
S_1 = saving in period 1	β = preferen

c) Solve Hitoshi's two-period optimization problem by taking FOCs and deriving his intertemporal and intratemporal tradeoff conditions.

$$L = u(c_1, c_2, n_1, n_2) + \lambda_1 (w_1 n_1 - c_1 - s_1) + \lambda_2 (w_2 n_2 + (1+r_1) s_1 + a_2 - c_2)$$

$$L = 2C_1^{0.5} + 2\beta C_2^{0.5} - 0.5 n_1^2 - 0.5 \beta n_2^2 + \lambda [w_2 n_2 + w_1 n_1 (1+r) + a_2 - c_2 - c_1 (1+r_1)]$$

FOC:

Eqn 1 $[c_1]$: $C_1^{-0.5} - \lambda (1+r_1) = 0 \rightarrow \lambda = \dfrac{C_1^{-0.5}}{1+r_1}$

Eqn 2 $[c_2]$: $\beta C_2^{-0.5} - \lambda = 0 \rightarrow \lambda = \beta C_2^{-0.5}$

Eqn 3 $[n_1]$: $-n_1 + w\lambda (1+r_1) = 0 \rightarrow \beta C_2^{-0.5} = \dfrac{C_1^{-0.5}}{1+r_1}$

Eqn 4 $[n_2]$: $-\beta n_2 + w_2 \lambda = 0 \rightarrow C_1^{-0.5} = \beta C_2^{-0.5} (1+r_1)$

Combine $[c_1]$ $[n_1]$ for intratemporal trade off condition

$$\lambda = \frac{c_1^{-0.5}}{1+r_1}$$

$$n_1 = w_1 \lambda (1+r) \rightarrow \lambda = \frac{n_1}{w_1(1+r_1)}$$

$$\frac{c_1^{-0.5}}{1+r_1} = \frac{n_1}{w_1(1+r_1)} \qquad \frac{n_1}{c_1^{-0.5}} = \frac{w_1(1+r_1)}{1+r_1} = w_1$$

$$\frac{n_1}{c_1^{-0.5}} = w_1$$

Combine $[n_1]$ and $[n_2]$ for inter temporal trade off condition

$$[n_1] \quad \lambda = \frac{n_1}{w(1+r_1)}$$

$$[n_2] \quad \lambda = \frac{\beta n_2}{w_2}$$

$$\frac{n_1}{w_1(1+r_1)} = \frac{\beta n_2}{w_2}$$

$$\frac{n_1}{\beta n_2} = \frac{w_1(1+r_1)}{w_2}$$

$$\frac{n_1}{n_2} = \frac{\beta w_1(1+r_1)}{w_2}$$

d) Does Hitoshi's inheritence affect his intra-temporal tradeoffs? What about his inter-temporal tradeoffs? Explain why or why not.

Hitoshi's inheritence does effect his intra-temporal trade-offs since he will be receiving the inheritance in period 2. Intra temporal trade off is linked with trade offs in that single given period.

His inter-temporal trade offs would be affected, impacting his decision on consumption and saving in period 2. The inheritence will influence how he chooses to allocate his consumption, saving, labour supply, and investment. He may work less knowing he will be getting inheritenle

e) Does the amount that Hitoshi expects to inherit in period 2 change how much Hitoshi would spend in Period 1? Why or why not? Explain and show your math.

Yes, if Hitoshi expects a large amount of inheritance in period 2, he might choose to spend more and save less in period 1 since he is expecting a large inherItence.

$$c_1 + \frac{c_2}{1+r_1} = w_1 n_1 + \frac{a_2}{1+r_1} + \frac{w_2 n_2}{1+r_1}$$

But if he expects for a smaller inheritence, he will change the way he consumes and spends in period 1, knowing he will get a small amount for inheritence.

Preference shock (For Grading)

Instead of a productivity shock[1], now consider the effects of a one-time unanticipated shock to preferences. Now suppose that consumption becomes suddenly more enjoyable (possibly because of a near death experience, like the Pandemic). The utility function becomes:

$$E_t \sum_{t=0}^{\infty} \beta^t \left((1+\phi_t) lnc_t + \theta ln(1-n_t)\right)$$

where $\phi_t = \rho\phi_{t-1} + \epsilon_t$, and $\rho < 1$, $\epsilon_t \sim iidN(0, \sigma_\epsilon^2)$ is a white noise process. Assume agents form rational expectations and make optimal decisions.
The representative household has the same budget constraint as in the previous problem, while the representative firm faces the sames production function (albeit, with no productivity shocks).

a) Derive the household's first order conditions based on this new preference function. State the household's inter-temporal and intra-temporal tradeoff conditions.

Household:

$$L = E_t \sum_{t=0}^{\infty} \beta^t \left((1+\phi_t)\ln c_t + \theta \ln(1-n_t)\right) + E_t \sum_{t=0}^{\infty} \lambda_t \left[w_t n_t + r_t k_t + (1-\delta)k_t + R_t b_t - c_t - k_{t+1} - b_{t+1}\right]$$

$$\frac{\partial L}{\partial c_t} = \beta^t \left((1+\phi_t)\frac{1}{c_t}\right) - \lambda_t = 0$$

$$\frac{\partial L}{\partial n_t} = \theta E_t (\beta^t) \lambda_t (-1) + \lambda_t w_t = 0$$

$$\frac{\partial L}{\partial k_{t+1}} = E_t (\beta^t) \lambda_t (-1) + \lambda_t R_t = 0$$

$$\frac{\partial L}{\partial b_{t+1}} = E_t (\beta^t) \lambda_t (-1) + \lambda_t (1) = 0$$

Inter temporal trade-off condition: $\beta^t \left(\frac{1+\phi_t}{c_t}\right) = E_t \beta^t \left(\frac{1}{1+R_t}\right) \beta^{t+1} \left(\frac{1+\phi_{t+1}}{c_{t+1}}\right)$

Intra temporal trade off condition: $\theta E^t (\beta^t) \left(\frac{1}{1-n_t}\right) = -\lambda_t w_t$

b) Derive the firm's first order conditions.

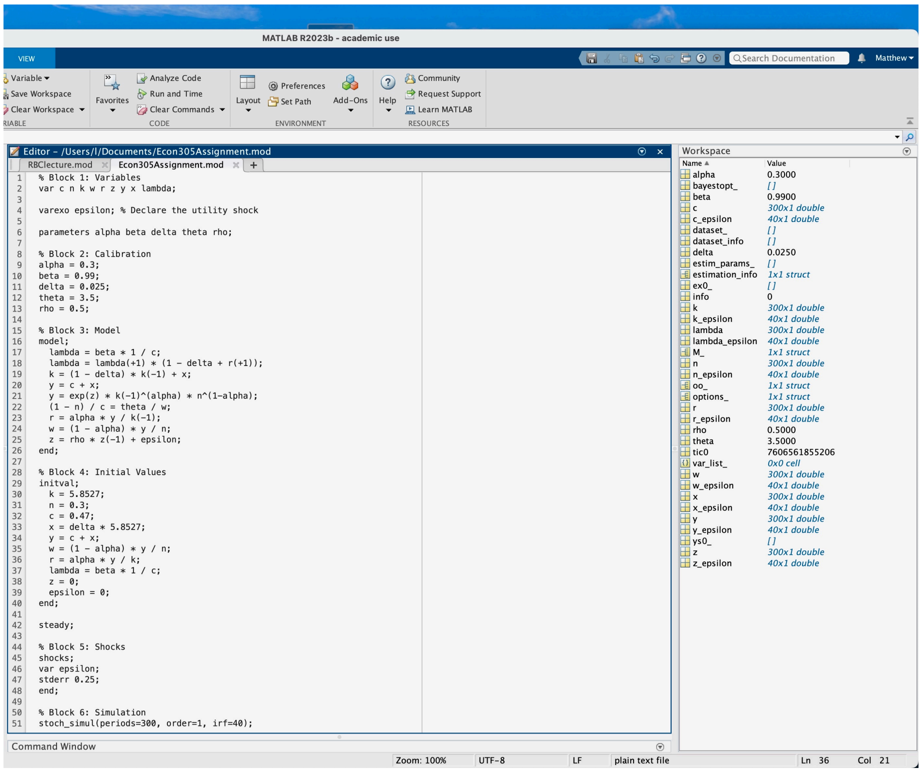
Firm: $\quad L = Z_t \, (K_t^{\alpha}) \, (N_t^{1-\alpha}) - r_t K_t - w_t N_t$

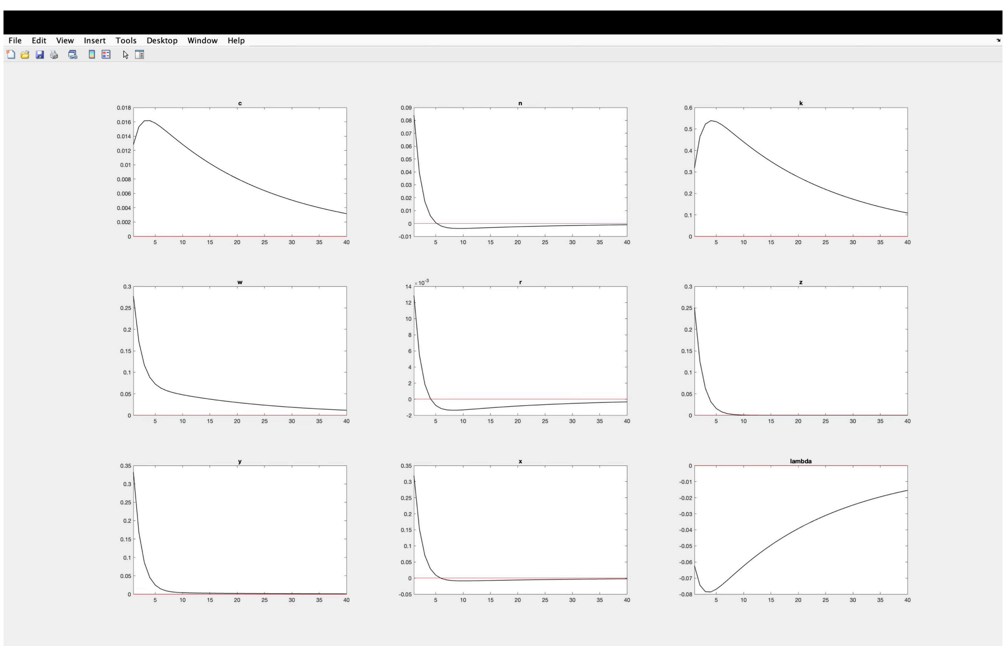
$$[K_t] = \alpha (Z_t)(K_t^{\alpha-1})(N_t^{1-\alpha}) - r_t = 0$$

$$[N_t] = (1-\alpha)(Z_t)(K_t^{\alpha})(N_t^{(-\alpha)}) - w_t = 0$$

c) Derive the equilibrium conditions for the economy. What equilibrium conditions change because of changes to ϕ_t?

d) Simulate with Dynare the effects of a positive, temporary but persistent increase in utility from consumption (a one-time positive ϵ_t). Be clear what values you assign to your parameters and what the steady state values are. Include the simulated inpulse response function graphs.





e) Explain how the household's labour supply, consumption, and investment decisions are altered relative to their steady state level, both on impact of the shock and in the subsequent periods. What happens to the wage and rental rate on impact of the shock, and in the periods that follow? More importantly, explain intuitively through the equilibrium equations why these variables are changing in response to a temporary increase in utility from consnumption.

The immediate impact of the shock is a reduction in the household's labour supply, driven by the increase of consumption. The following behavior of labour supply may vary, with the possibility of it fully or partially returning to pre-shock levels. Due to its higher utility, the shock initially causes consumption to surge. However, in the periods after the shock, consumption is expected to gradually regress to its pre-shock levels. Investment decisions are dependent on the specific characteristics of the model, so it is uncertain whether investment decisions will be altered. The change in wage and rental rates relies on the impact of the shock on labour supply. Wages are higher when the labour supply decreases, and lower when labour supply increases. The rental rate responds by increasing with greater demand for capital and decreasing with reduced demand. As the shock's effects are not expected to last for long, wage and rental rates are expected to return to steady-state levels.

f) Find four different business cycle facts from various sources. Pplease cite your sources.

A business cycle is like the ups and downs of the economy. It goes through four stages: expansion, peak, contraction, and trough. We figure out which stage we're in by looking at stuff like how much money we're making, how many jobs there are, and how much people are spending.

1. Unemployment Rate Behavior: "The seasonally adjusted unemployment rate for October 2023 is 3.9%. This percentage represents the proportion of the labour force that is unemployed and actively seeking employment. In this context, it suggests that 3.9% of the labour force in October 2023 was without a job and looking for work." (BLS, 2023)

- Source: U.S. Bureau of Labor Statistics (https://www.bls.gov/cps/)

2. Interest Rate Policy: "The key goal of monetary policy is to support maximum employment. This means that the Federal Reserve aims to create conditions in which as many people who want to work can find employment. It does so by influencing economic factors that impact job creation, such as interest rates and the overall level of economic activity." (Federal Reserve, 2023)

- Source: Federal Reserve, [Monetary Policy](https://www.federalreserve.gov/monetarypolicy.htm).

3. Inflationary Pressures: "Over the past 12 months, the index for all items less food and energy increased by 4.1%. This indicates that when compared to the same month in the previous year, the prices for these core goods and services have risen by 4.1%." (BLS, 2023)

- Source: U.S. Bureau of Labor Statistics, [Consumer Price Index](https://www.bls.gov/cpi/).

4. Consumer and Business Confidence: "Conversely, 29% of CEOs anticipated that conditions in their industries would worsen, which is down from the 30% who held this view in the previous quarter. Consumer and business confidence indices tend to fluctuate over the business cycle, with increased confidence during expansions and decreased confidence during contractions." (Conference Board, 2023)

- Source: The Conference Board, [Consumer Confidence Index](https://www.conference-board.org/data/consumerconfidence.cfm) and [CEO Confidence](https://www.conference-board.org/data/ceoconfidence.cfm).

g) In what ways does the model do a good job at fitting these business cycle facts? In what ways could it be improved? **Hint:** Use the Moments, Correlation, and Autocorrelation of Simulated Variables tables from Dynare to evaluate your model.

The effectiveness of a model in fitting business cycle facts depends on several factors. For a model to be well-fitted, it should mimic key characteristics of real-world business cycles, including matching moments (means, variances, covariances) of economic variables with empirical data. To accurately reflect the real-life dynamics of business cycles, it must capture correlation patterns between variables and display autocorrelations that are consistent with them. Models should be capable of representing the four phases of the business cycle-expansion, peak, contraction, and trough-as well as transitions between them. Finally, a well-fitting model should exhibit robustness, maintaining its accuracy under various scenarios and sensitivity tests, thus providing reliable insights into the complex dynamics of business cycles. A way it could be improved is by incorporating shocks. By adjusting the parameters of exogenous shocks in the model can better capture unexpected events that influence the business cycle. Using dynare and playing around with the size, persistence, or frequency of shocks can lead to a more accurate model fit.

h) How can you change the parameters of your simulation to improve the fit of your model to these business cycle facts?

By using the real-world data from the business cycle facts, I could adjust the model's parameters to better match how the real economy behaves to improve fit to these business cycle facts. Then I would see how changes in these settings make my model behave differently. For example, I could change the shock, Compare your model with other models and use any technique to find the best settings. By doing this, my model will do a better job of explaining how the economy works as it would be able to replicate the real economy based on the business cycle facts.

Alpha (α): Alpha can affect the output and labor dynamics in the model.

Beta (β): Beta can affect intertemporal decision-making and the discounting of future cash flows.

Delta (δ): Delta can influence the rate at which the capital stock in the economy deteriorates over time.

Theta (θ): Theta can affect the behavior of prices, consumption, or substitution patterns.

Rho (ρ): Rho affects the persistence of shocks and the speed of convergence to equilibrium.