First Order ODEs Continued 1/31/23

Ex: $\frac{dw}{dt} + t^2 w = 0$, $a(t) = t^2$. Find $A(t)$ s.t. $A'(t) = a(t) = t^2$

$A(t) = \frac{1}{3}t^3 + 6$ → educated guess → $w(t) = e^{-\frac{1}{3}t^3 - 6} = e^{-\frac{1}{3}t^3} e^{-6}$

(guess integral of it) ← Not all solutions

Other solutions: $w(t) = 0$, $w(t) = -5e^{-\frac{1}{3}t^3}$

↳ more general solution: $w(t) = \boxed{e^{-\frac{1}{3}t^3} e^{-6} C}$ or $e^{-\frac{1}{3}t^3} C$ ← include e^{-6} in constant

- This method works for linear homogeneous DEs

Nonhomogeneous DEs

$\frac{dy}{dt} + a(t) y = f(t)$

$\frac{d}{dt}\left(e^{A(t)} y\right) = e^{A(t)} A'(t) y + e^{A(t)} \frac{dy}{dt} = e^{A(t)} \left(A'(t) y + \frac{dy}{dt}\right)$

If $A(t)$ satisfies $A'(t) = a(t)$...

$\underbrace{\frac{d}{dt}\left(e^{A(t)} y\right)}_{B(t)} = e^{A(t)}\left(\frac{dy}{dt} + a(t) y\right)$

then solve:

$\frac{dB}{dt} = e^{A(t)} f(t)$

Note: IVP → Initial Value Problem

Example: $\frac{dx}{dt} = -3x + e^{2t}$ ⇒ $\frac{dx}{dt} + 3x = e^{2t}$, initial value: $x(0) = 2$

$a(t) = 3$, $f(t) = e^{2t}$

⇒ $A(t) = \int a(t)\,dt = \int 3\,dt = 3t$

⇒ $B(t) = \int e^{A(t)} f(t)\,dt = \int e^{3t} e^{2t}\,dt = \int e^{5t}\,dt = \frac{1}{5}e^{5t} + C$

$x(t) = e^{-A(t)} B(t)$

$= e^{-3t}\left(\frac{1}{5}e^{5t} + C\right)$

$x(t) = \frac{1}{5}e^{2t} + Ce^{-3t}$ $x(0) = \frac{1}{5} + C = 2$ $C = 9/5$

- Thm: If $a(t)$ and $f(t)$ are defined and continuous near t_I, then there is a unique solution to the IVP on the biggest interval around t_I on which $a(t)$ and $f(t)$ are defined/continuous.

Ex: $dy/dt + (1/1-t)y = \sin(t)$, $y(0) = 0$

① 1 fn is undefined ∴ I.D $= (-\infty, 1)$ (undefined ~~bad~~, ok for discontinuous)

↳ There is a solution (bc initial time is not undefined)

1 SQUARE =