Intro to Differential Equations

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Definition: A D.E. is an equation with a derivative

- DE Classifications:
  - Order → The highest derivative in the eq
    Ex: $y'' + (y')^3 = e^t$ → Order 2
  - Linear → if you can write in these forms: Note: no exponents
    $y'(t) + a(t)y(t) = b(t)$
    $y''(t) + a(t)y'(t) + b(t)y(t) = c(t)$
    ← $a(t)$, $b(t)$ are fixed, known fns

- How to Solve first order DE
  - Special cases: Don't Forget!!!
    - $a(t) = 0 \rightarrow y'(t) = b(t)$
      Ex: $y'(t) = t^3 \rightarrow y = t^4/4 + C$
    - $b(t) = 0 \rightarrow y'(t) = g(t)y(t)$ (separable)
      Ex: $y' = 3t \cdot y$, $y(0) = 7$
      $\int y'/y \, dx = \int 3t \, dt \rightarrow \ln(y) = 3t^2/2 + C$
      $y = e^{3t^2/2 + C}$
      ↳ can be rewritten as $y(t) = \tilde{C} e^{3t^2/2}$ where $\tilde{C} = e^C$

  - When $a(t) \neq 0$, $b(t) \neq 0$ (First order Linear)
    ↳ set it up into one of the special cases
    Ex: $y' + 3t^2 y = 2te^{-t^3}$
    $(y' + 3t^2 y)e^{t^3} = 2t$ — 1. Multiply both sides by integrating factor
    $\frac{d}{dt}(y \cdot e^{t^3}) = 2t$ — 2. put in form $\frac{d}{dt}(\ldots) = g(t)$
    $ye^{t^3} = t^2 + C$ — 3. Integrate
    $y = t^2 e^{-t^3} + Ce^{-t^3}$ — 4. Solve

- Integrating Factor $= e^{\int a(t)\,dt}$

- Equations with lots of variables or fns are harder to solve
  dependent variable
- Conventions: x, y, z are unknown fn, t is independent variable (default)
- Ordinary Differential equations only involve one independent variable

1 SQUARE = _____