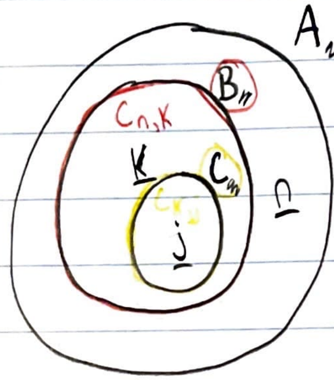
HW8

1O.) $\binom{n}{k}\binom{k}{j}=\binom{n}{j}\binom{n-j}{k-j}$

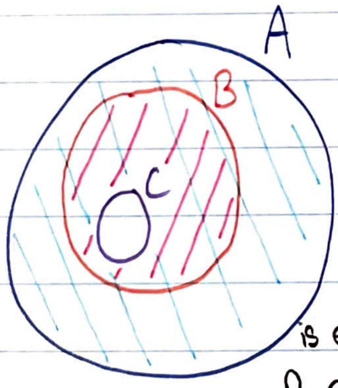
a) $\binom{n}{k}\binom{k}{j} = \frac{n!}{(n-k)!\cancel{k!}}\cdot\frac{\cancel{k!}}{(k-j)!j!} = \boxed{\frac{n!}{(n-k)!(k-j)!j!}}$

$\binom{n}{j}\binom{n-j}{k-j} = \frac{n!}{\cancel{(n-j)!}j!}\cdot\frac{\cancel{(n-j)!}}{(n-j-(k-j))!(k-j)!} = \boxed{\frac{n!}{j!(n-k)!(k-j)!}}$

b) A, $|A|=n$; $|B|=k$; $|C|=j$



$C_{n,k}\cdot C_{k,j} = C_{n,j}\cdot C_{n-j,k-j}$



$C_{n,k}\cdot C_{k,j} = C_{n,j}\cdot C_{n-j,k-j}$

times B fills A times # times C fills B is equal to # times C fills A times # times B-C fills A-C.

Consider sets A B and C such that $C \subseteq B \subseteq A$ and the sizes are n, k, j respectively. The # of subsets of size k out of the set A times the # of subsets of size j out of the set B is equal to the number of subsets of size j out of the set A multiplied with the number of subsets of size k-j out of the set A-C because the proportions of subsets.