Loan Amortization

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A Simple Amortization

- For the simplest loans
 (fixed/constant rate, fully
 amortizing), you only need 3
 pieces of information:
 - Starting Balance
 - Term
 - (Flat) Interest Rate
- Think in terms of a ledger ->

| 1000 | | | | | |
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| 14 | \$467.02 | | | \$41.84 | |
| 15 | \$425.18 | \$43.20 | \$1.24 | \$41.96 | |
| 16 | \$383.21 | \$43.20 | \$1.12 | \$42.09 | |
| 17 | \$341.13 | \$43.20 | \$0.99 | \$42.21 | \$298.92 |
| 18 | \$298.92 | \$43.20 | \$0.87 | \$42.33 | \$256.59 |
| 19 | \$256.59 | \$43.20 | \$0.75 | \$42.45 | \$214.14 |
| 20 | \$214.14 | \$43.20 | \$0.62 | \$42.58 | \$171.56 |
| 21 | \$171.56 | \$43.20 | \$0.50 | \$42.70 | |
| 22 | \$128.86 | \$43.20 | \$0.38 | \$42.83 | \$86.03 |
| 23 | \$86.03 | \$43.20 | \$0.25 | \$42.95 | \$43.08 |
| 24 | \$43.08 | \$43.20 | \$0.13 | \$43.08 | \$0.00 |
| | 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 | 24 3.50% balbop 1 1000 2 \$959.71 3 \$919.31 4 \$878.79 5 \$838.15 6 \$797.39 7 \$756.51 8 \$715.52 9 \$674.40 10 \$633.17 11 \$591.81 12 \$550.33 13 \$508.74 14 \$467.02 15 \$425.18 16 \$383.21 17 \$341.13 18 \$298.92 19 \$256.59 20 \$214.14 21 \$171.56 22 \$128.86 23 \$86.03 | 24 3.50% balbop pmt 1 1000 \$43.20 2 \$959.71 \$43.20 3 \$919.31 \$43.20 4 \$878.79 \$43.20 5 \$838.15 \$43.20 6 \$797.39 \$43.20 7 \$756.51 \$43.20 8 \$715.52 \$43.20 9 \$674.40 \$43.20 10 \$633.17 \$43.20 11 \$591.81 \$43.20 12 \$550.33 \$43.20 13 \$508.74 \$43.20 14 \$467.02 \$43.20 15 \$425.18 \$43.20 16 \$383.21 \$43.20 17 \$341.13 \$43.20 18 \$298.92 \$43.20 19 \$256.59 \$43.20 20 \$214.14 \$43.20 21 \$171.56 \$43.20 22 \$128.86 \$43.20 23 \$86.03 \$43.20 | 24 3.50% balbop pmt ipmt 1 1000 \$43.20 \$2.92 2 \$959.71 \$43.20 \$2.80 3 \$919.31 \$43.20 \$2.68 4 \$878.79 \$43.20 \$2.56 5 \$838.15 \$43.20 \$2.44 6 \$797.39 \$43.20 \$2.33 7 \$756.51 \$43.20 \$2.09 9 \$674.40 \$43.20 \$1.97 10 \$633.17 \$43.20 \$1.85 11 \$591.81 \$43.20 \$1.73 12 \$550.33 \$43.20 \$1.61 13 \$508.74 \$43.20 \$1.48 14 \$467.02 \$43.20 \$1.36 15 \$425.18 \$43.20 \$1.24 16 \$383.21 \$43.20 \$1.12 17 \$341.13 \$43.20 \$0.99 18 \$298.92 \$43.20 \$0.87 20 \$214.14 \$43.20 \$0.62 21 \$171.56 \$43.20 \$0.50 22 \$128.86 \$43.20 \$0.38 23 \$86.03 \$43.20 \$0.25 | 3.50% balbop pmt ipmt ppmt |

Excel Worksheet Functions

- PMT
- IPMT
- PPMT
- FV
- PV

(i) Note

Drag the equations down the Excel spreadsheet to fully amortize the loan

Function Formulas

$$monthly\ payment = pmt = rac{rate}{1-(1+rate)^{-term}}*balance$$
 $monthly\ interest\ payment = ipmt_t = balance_t*rate$
 $monthly\ principal\ payment = ppmt_t = pmt - ipmt_t$
 $ending\ balance = baleop_t = balance_t - ppmt_t = balance_{t+1}$

Floating Rates

Constant Rate:

$$pmt = rac{rate}{1-(1+rate)^{-term_0}}*balance_0$$

Variable Rate:

$$pmt_t = rac{rate_t}{1 - (1 + rate_t)^{-term_t}} *balance_t$$

Day Counting Conventions

Day counting convention impacts the conversion of annual rate to monthly rate 30/360:

$$rate_{monthly} = rate_{annual} * \left(rac{30}{360}
ight) = rac{rate_{annual}}{12}$$

Actual/360:

$$rate_{monthly} = rate_{annual} * \left(rac{days \; in \; month}{360}
ight)$$

Actual/Actual:

$$rate_{monthly} = rate_{annual} * \left(rac{days \ in \ month}{days \ in \ year}
ight)$$