Funding cycle simulations

Brad Duthie

12/10/2021

```r
f <- function(t, w, cs, cf, cd) {
  ft <- dc.component + sum( cs * sin(cf*w*t + cd));
  return(ft);
}
```

The goal here is to generate some function of budget over time. What we want is some way to have the budget go through high and low periods, but not in a way that would be recognised as predictable. That is, we do not want the budget to simply oscillate over time as would be the case if we just made budget a function of a sine wave. To give some degree of irregularity to the budget, we can use a Fourier series approach to creating irregular curves by adding sine waves of different frequency (f), delay (φ), and strength (A). There is a distinction mate between 'fundamental frequency' and 'wave frequency', but I do not understand what is meant by this or where it appears in the code. I will just use 'frequency'. Individual sine waves are generated from these parameter values as below,

$$b(t) = A \sin\left(\frac{2\pi}{T} t f + \varphi\right).$$

In the above, T is the total number of time steps (t). What we have done is sampled random values for four different sign wave frequencies (f_i), delay (φ_i), and strength (A_i). Frequency values are sampled are sampled from a uniform distribution for frequency,

$$f_i \sim U(0.01, 0.08).$$

We can express the probability density function, if desired, like this,

$$f_i(x) = \begin{cases} \frac{1}{0.08-0.01} & \text{for } 0.01 \leq x \leq 0.08, \\ 0 & \text{for } x < 0.01 \text{ or } 0.08 > b \end{cases}.$$

Delay values are similarly sampled from a uniform distribution (unless this was a discrete distribution?),

$$\varphi_i \sim U(0, 180).$$

The probability density function is below,

$$\varphi_i(x) = \begin{cases} \frac{1}{180} & \text{for } 0 \leq x \leq 180, \\ 0 & \text{for } x < 0 \text{ or } 180 > b \end{cases}.$$

Lastly, strength (amplitude) values are sampled from a uniform distribution between 1 and 150 (again, assuming that this was not discrete),

$$A_i \sim U(1, 150).$$

The probability density function is below,

$$A_i(x) = \begin{cases} \frac{1}{150-1} & \text{for } 1 \leq x \leq 150, \\ 0 & \text{for } x < 1 \text{ or } 150 > b \end{cases}.$$

The sampling of each f_i, φ_i, and A_i was then done four times to make four unique sine waves ($b_i(t)$),

$$b_i(t) = A_i \sin\left(\frac{2\pi}{T} t f_i + \varphi_i\right).$$

These sign waves were then summed and then added to a constant $C = 500$ (assuming that this constant value does not change?) to determine the manager budget in a time step $B(t)$,

$$B(t) = C + \sum_{i=1}^{4} b_i(t)$$

The consequence of this is an irregular oscillation of budget over time. The figure below shows the budget B (black line), determined by the sum of the four $b_i(t)$ (coloured lines).