Objectives
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## Objectives

You should be able to ...

## Lambda Calculus Examples

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Here are some examples!

- ► Perform a beta-reduction.
- ▶ Detect  $\alpha$ -capture and use  $\alpha$ -renaming to avoid it.
- ▶ Normalize any given  $\lambda$ -calculus term.

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## Examples

 $(\lambda x.x) a$   $(\lambda x.x x) a$   $(\lambda x.y x) a$   $(\lambda x.\lambda a.x) a$   $(\lambda x.\lambda x.x) a$   $(\lambda x.(\lambda y.y) x) a$ 

## Examples

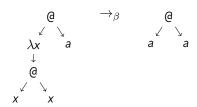
$$\begin{array}{ccc} (\lambda x.x) \, a & \longrightarrow_{\beta} & a \\ (\lambda x.x \, x) \, a & \\ (\lambda x.y \, x) \, a & \\ (\lambda x.\lambda a.x) \, a & \\ (\lambda x.\lambda x.x) \, a & \\ (\lambda x.(\lambda y.y) \, x) \, a & \end{array}$$

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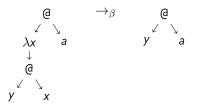
## Examples

$$\begin{array}{ccc} (\lambda x.x) \, a & \to_{\beta} & a \\ (\lambda x.x \, x) \, a & \to_{\beta} & a \, a \\ (\lambda x.y \, x) \, a & \\ (\lambda x.\lambda a.x) \, a & \\ (\lambda x.\lambda x.x) \, a & \\ (\lambda x.(\lambda y.y) \, x) \, a & \end{array}$$



### Examples

$$\begin{array}{cccc} (\lambda x.x) \ a & \rightarrow_{\beta} & a \\ (\lambda x.x \ x) \ a & \rightarrow_{\beta} & a \ a \\ (\lambda x.y \ x) \ a & \rightarrow_{\beta} & y \ a \\ (\lambda x.\lambda a.x) \ a & \\ (\lambda x.\lambda x.x) \ a & \\ (\lambda x.(\lambda y.y) \ x) \ a & \end{array}$$



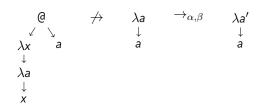
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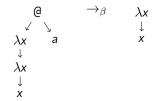
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## Examples

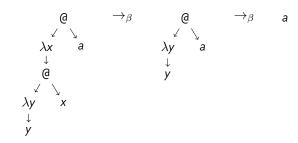


## Examples



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## Examples



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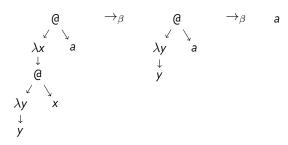
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### Examples

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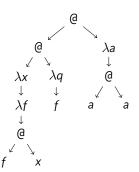
# $\alpha$ capture

$$(\lambda x. \lambda a. x) a \rightarrow_{\alpha} (\lambda x. \lambda a'. x) \rightarrow_{\beta} \lambda a'. a$$

- lacktriangle If a free occurrence of a variable gets placed under a  $\lambda$  that binds it, this is called  $\alpha$ capture.
- ► To resolve this, rename the binder.

## Here's One for You to Try!

- ightharpoonup Convert this tree into an equivalent  $\lambda$  term.
- ► Identify the free variables.
- Simplify it by performing as many  $\beta$  reductions (and necessary  $\alpha$  renamings) as possible.

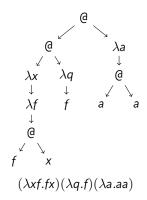


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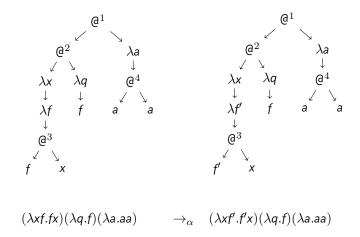
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### Solution



► There is one free variable ....

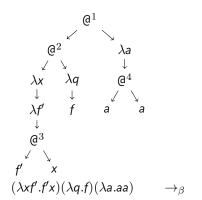
## Solution, Step 1



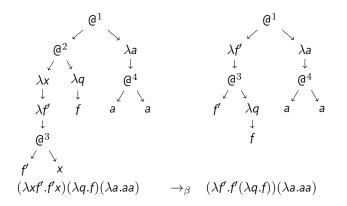
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## Solution, Step 2



## Solution, Step 2



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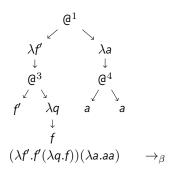
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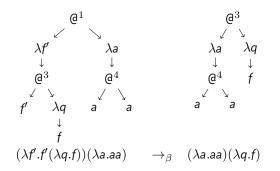
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## Solution, Step 3

# Solution, Step 3





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## Solution, Step 4

# Solution, Step 4

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\downarrow & \downarrow & & \\
\lambda a & \lambda q & & \\
\downarrow & \downarrow & \\
\mathbb{Q}^{4} & f & & \\
\downarrow & & \\
a & a & \\
(\lambda f'.f'(\lambda q.f))(\lambda a.aa) & \rightarrow_{\beta}
\end{array}$$

Solution, Step 5

Solution, Step 5

