CS 421 — Hoare Triples and Loop Partial Correctness

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The First Four Rules

Axiom 1: Skip

 $\{p\} \mathtt{skip} \; \{p\}$

Axiom 2: Assignment

 $\{p[u:=t]\}u:=t\{p\}$

Rule 3: Composition

 $\frac{\{p\}S_1\{r\},\{r\}S_2\{q\}}{\{p\}S_1;S_2\{q\}}$

Rule 4: Conditional

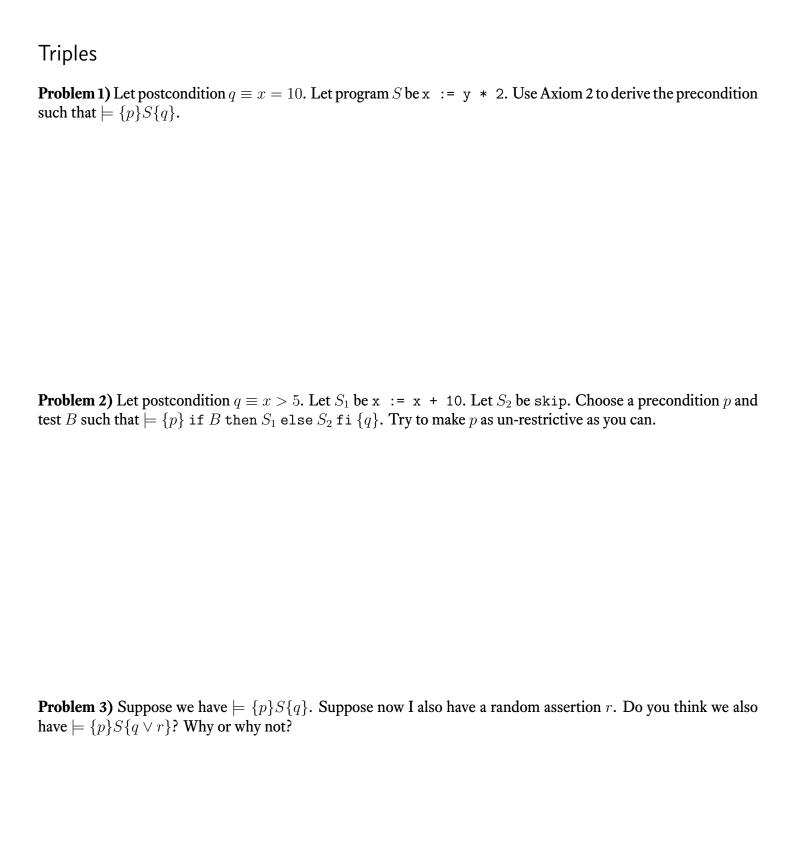
 $\frac{\{p \wedge B\}S_1\{q\}, \{p \wedge \neg B\}S_2\{q\}}{\{p\} \text{if } B \text{ then } S_1 \text{ else } S_2 \text{ fi } \{q\}}$

Rule 5: Loop

 $\frac{\{p \wedge B\}S\{p\}}{\{p\} \text{while } B \text{ do } S \text{ od } \{p \wedge \neg B\}}$

Rule 6: Consequence

$$\frac{p \to p_1, \{p_1\} S\{q_1\}, q_1 \to q}{\{p\} S\{q\}}$$



Weakness

Problem 4) Rank the following logical assertions from strongest to weakest. Note that the ranking is not necessarily linear.

- $a \equiv \text{false}$
- $b \equiv \mathbf{true}$
- $c \equiv x > 10 \lor y < 10$
- $d \equiv x > 10$
- $e \equiv x > 5 \lor y < 5$
- $f \equiv x > 5 \land y < 5$
- $g \equiv x > 5$

Problem 5) What can you say about x + y = 10 in regards to the ordering of the previous question?

Problem 6) Suppose $\{x > 0\}S\{y < 0\}$. Which of the following are also true?

- 1. $\{x > 0\}S\{y < 0 \lor x > 0\}$.
- 2. $\{x > 0 \land y < 0\}S\{y < 0\}$.
- 3. $\{y < 0\}S\{x > 0\}$.
- **4.** $\{x > 0\}S\{y < 0 \land x > 0\}$.
- 5. $\{x > 0 \lor y < 0\}S\{y < 0\}$.
- **6.** $\{x > 0\}S\{y < 10\}$.
- 7. $\{x > -10\}S\{y < 0\}$.

Loop Invariants

We want to take the product of the elements of an array.

- The postcondition is $r\equiv x=\Pi_{j=0}^{|A|-1}a[j]$.
- The loop invariant is $p \equiv x = \prod_{j=0}^{i} a[j]$.
- The loop bound is i < |A|.

Problem 7) Write the code to establish the loop invariant, and give a proof outline for it.

Problem 8) Write the loop, and show that the loop body preserves the loop invariant.

Problem 9) Show that the loop achieves the postcondition on termination.