CS 421 --- Type Semantics Activity (Polytype Version)

Manager	Keeps team on track	
Recorder	Records decisions	
Reporter	Reports to class	
Reflector	Assesses team performance	

Objectives

When you are done with this activity, you will have

- Described the $\geq$ relation on types.
- Described the relation between the GEN and INST rules with $\forall$.
- Used a proof tree to show the difference between monomorphic application and polymorphic **`let`** .

Note that the list of rules can be found at the end of the problems.

The $\geq$ Operation for Types

Time estimate: 5 -- 10 minutes

Instructions Consider the following table of $\geq$ relations. Some are marked ``valid'' and some are marked ``invalid.''

$$\begin{array}{lcrl}
\forall\alpha.\alpha \rightarrow \alpha & \geq & \mathtt{Int} \rightarrow \mathtt{Int} & \text{valid} \\
\forall\alpha.\forall\beta.\alpha \rightarrow \beta & \geq & \forall\beta.\mathtt{Int} \rightarrow \beta & \text{valid} \\
\forall\alpha.\alpha \rightarrow \alpha & \geq & \forall\beta.\beta \rightarrow \beta & \text{valid} \\
\forall\alpha.\alpha \rightarrow \alpha & \geq & \mathtt{Int} \rightarrow \mathtt{String} & \text{invalid} \\
\forall\alpha.\alpha \rightarrow \alpha & \geq & \forall\alpha.\forall\beta.\beta \rightarrow \alpha & \text{invalid}
\end{array}$$

Problem 1) Review the relations above and try to come up with a rule that describes when the $\geq$ relation is valid.

Problem 2) Is the relation $\forall\alpha.\forall\beta.\alpha \rightarrow \beta \quad \geq \quad \mathtt{Int} \rightarrow \mathtt{Int}$ a valid one?

1 Gen and Inst

Time estimate 10--15 minutes.

Problem 3) Consider the following two proof trees.

$$\dfrac{\dfrac{}{\Gamma \vdash id : \mathtt{Int} \to \mathtt{Int}}\,\textsc{Var} \qquad \dfrac{}{\Gamma \vdash x : \mathtt{Int}}\,\textsc{Var}}{\Gamma \vdash id\, x : \mathtt{Int}}\,\textsc{App} \qquad\qquad \dfrac{\dfrac{}{\Gamma \vdash id : \forall \alpha.\alpha \to \alpha}\,\textsc{Var} \qquad \dfrac{}{\Gamma \vdash x : \mathtt{Int}}\,\textsc{Var}}{\Gamma \vdash id\, x : \mathtt{Int}}\,\textsc{App}$$

One of these trees is not correct. Which one? Show how to fix it using the Inst rule.

Problem 4) Consider this proof tree:

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\{x : \alpha\} \vdash x : \alpha}\,\textsc{Var}}{\{\} \vdash \lambda x.x : \alpha \to \alpha}\,\textsc{Abs}}{\{\} \vdash \lambda x.x : \forall \alpha.\alpha \to \alpha}\,\textsc{Gen} \qquad \dfrac{}{\{f : \forall \alpha.\alpha \to \alpha\} \vdash f : \forall \alpha.\alpha \to \alpha}\,\textsc{Var}}{\Gamma \equiv \{\} \vdash \texttt{let}\ f\ =\ \lambda\, x.x\ \texttt{in}\ f : \forall \alpha.\alpha \to \alpha}\,\textsc{Let}$$

Yeah, we stole if from the slides because you're likely to pull them up anyway. So now we are going to ask about the Gen step. How do you know when you need to use it?

Proofs

Time estimate: 25 minutes.

Create proofs for the following judgements according to the given rules.

Problem 5) `{id:` $\forall\alpha.\alpha \rightarrow \alpha$ `, y:Int}` $\vdash (id\, y) : \; Int$

Problem 6) `{y:Int,z:String}` $\vdash (\lambda f.(f\ y, f\ z))\,(\lambda x.x) : \; (Int, String)$

Problem 7) `{x:Int,y:String}` $\vdash$ **`let`** $f = \lambda x.x$ **`in`** $(f\ x, f\ y) : \; (Int, String)$

The Rules

Constants

$$\frac{}{\Gamma \vdash n : \texttt{int}} \textsc{Const}, \text{ when } n \text{ is an integer.}$$

Similarly for `True` and `False`.

Variables

$$\frac{}{\Gamma \vdash x : \sigma} \textsc{Var}, \text{ when } x : \sigma \in \Gamma$$

Binary Arithmetic

$$\frac{\Gamma \vdash e_1 : \texttt{int} \qquad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash e_1 \oplus e_2 : \texttt{int}} \textsc{BinOp}$$

Integer Relations

$$\frac{\Gamma \vdash e_1 : \texttt{int} \qquad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash e_1 \sim e_2 : \texttt{bool}} \textsc{RelOp}$$

If

$$\frac{\Gamma \vdash e_1 : \texttt{bool} \qquad \Gamma \vdash e_2 : \tau \qquad \Gamma \vdash e_3 : \tau}{\Gamma \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 : \tau} \textsc{If}$$

Application

$$\frac{\Gamma \vdash e_1 : \tau \to \tau' \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1\ e_2\ : \tau'} \textsc{App}$$

Abstraction

$$\frac{\Gamma \cup \{x : \tau\} \vdash e : \tau'}{\Gamma \vdash \lambda x.e : \tau \to \tau'} \textsc{Abs}$$

Let

$$\frac{\Gamma \vdash e_1 : \sigma \qquad \Gamma \cup [x : \sigma] \vdash e_2 : \tau}{\Gamma \vdash \texttt{let } \mathsf{x} = e_1 \texttt{ in } e_2 : \tau} \textsc{Let}$$

Gen

$$\frac{\Gamma \vdash e : \sigma}{\Gamma \vdash e : \forall \alpha.\sigma} \textsc{Gen}, \text{ where } \alpha \text{ is not free in } \Gamma$$

Inst

$$\frac{\Gamma \vdash e : \sigma'}{\Gamma \vdash e : \sigma} \textsc{Inst}, \text{ where } \sigma' \geq \sigma$$

Type Semantics Activity (Polytype Version) --- Team's Assessment (SII)

Manager or Reflector: Consider the objectives of this activity and your team's experience with it, and then answer the following questions after consulting with your team.

1. What was a **strength** of this activity? List one aspect that helped it achieve its purpose.

2. What is one things we could do to **improve** this activity to make it more effective?

3. What **insights** did you have about the activity, either the content or at the meta level?

Type Semantics Activity (Polytype Version)--- Reflector's Report

Manager	Keeps team on track	
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1. What was a strength of your team's performance for this activity?

2. What could you do next time to increase your team's performance?

3. What insights did you have about the activity or your team's interaction today?