CS 421 --- Type Semantics Activity (Polytype Version) --- Solutions

Proofs

Time estimate: 25 minutes.

Create proofs for the following judgements according to the given rules.

Problem 4) `{id:` $\forall\alpha.\alpha \to \alpha$ `, y:Int}` $\vdash (id\ y) :\ Int$

$$\dfrac{\dfrac{\dfrac{}{\{id : \forall\alpha.\alpha \to \alpha, y : Int\} \vdash id :\ \forall\alpha.\alpha \to \alpha}\ \textsc{Var}}{\{id : \forall\alpha.\alpha \to \alpha, y : Int\} \vdash id :\ Int \to Int}\ \textsc{Inst} \quad \dfrac{}{\{id : \forall\alpha.\alpha \to \alpha, y : Int\} \vdash y :\ Int}\ \textsc{Var}}{\{\forall\alpha.\alpha \to \alpha, y : Int\} \vdash (id\ y) :\ Int}\ \textsc{App}$$

Problem 5) `{y:Int,z:String}` $\vdash (\lambda f.(f\ y, f\ z))\ (\lambda x.x) :\ (Int, String)$

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x :\ \alpha}\ \textsc{Var}}{\Gamma \vdash \lambda x.x :\ \alpha \to \alpha}\ \textsc{Abs} \quad \dfrac{\text{No way to use } f \text{ here...}}{\Gamma \cup \{f : \alpha \to \alpha\} \vdash (f\ y, f\ z)) :\ (Int, String)}\ \textsc{Pair}}{\Gamma = \{y : Int, z : String\} \vdash (\lambda f.(f\ y, f\ z))\ (\lambda x.x) :\ (Int, String)}\ \textsc{App}$$

Problem 6) `{x:Int,y:String}` $\vdash$ **`let`** $f = \lambda x.x$ **`in`** $(f\ x, f\ y) :\ (Int, String)$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x :\ \alpha}\ \textsc{Var}}{\Gamma \vdash \lambda x.x :\ \alpha \to \alpha}\ \textsc{Abs}}{\Gamma \vdash \lambda x.x :\ \forall\alpha.\alpha \to \alpha}\ \textsc{Gen} \quad \dfrac{\dfrac{\dfrac{}{\Gamma' \vdash id :\ \forall\alpha.\alpha \to \alpha}\ \textsc{Var}}{\Gamma' \vdash id :\ Int \to Int}\ \textsc{Inst} \quad \dfrac{}{\Gamma' \vdash x :\ Int}\ \textsc{Var}}{\Gamma' \vdash idx :\ Int}\ \textsc{App} \quad \dfrac{\dfrac{\dfrac{}{\Gamma' \vdash id :\ \forall\alpha.\alpha \to \alpha}\ \textsc{Var}}{\Gamma' \vdash id :\ String \to String}\ \textsc{Inst} \quad \dfrac{}{\Gamma' \vdash y :\ String}\ \textsc{Var}}{\Gamma' \vdash idy :\ String}\ \textsc{App}}{\Gamma' = \{\forall\alpha.\alpha \to \alpha\} \cup \Gamma \vdash (idx, id\ y) :\ (Int, String)}\ \textsc{Pair}}{\Gamma = \{x : Int, y : String\} \vdash \textbf{let}\ f = \lambda x.x\ \textbf{in}\ (f\ x, f\ y) :\ (Int, String)}\ \textsc{Let}$$