### **Objectives**

You should be able to ...

### The CPS Transform

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After today's lecture, you will

- ► Convert a direct-style function into CPS:
  - ▶ Both simple and complex, involving nested continuations.



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### The CPS Transform, Simple Expressions

Top Level Declaration To convert a declaration, add a continuation argument to it and then convert the body.

$$C[[f arg = e)] \Rightarrow f arg k = C[[e]]_k$$

Simple Expressions A simple expression in tail position should be passed to a continuation instead of returned.

$$C[a]_k \Rightarrow k a$$

- "Simple" = "No available function calls."
- f a is available in 3 + f a, but not in  $\lambda x.x + f$  a.

Try converting these functions ...

```
1f x = x
2pi1 a b = a
3const x = 10
```

# Simple Expression Examples

### Before:

```
1 f x = x
2 pi1 a b = a
3 const x = 10
```

### After:

```
1 f x k = k x
2 pi1 a b k = k a
3 const x k = k 10
```



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### The CPS Transform, Function Calls

Function Call on Simple Argument To a function call in tail position (where arg is simple), pass the current continuation.

$$C[f arg]_k \Rightarrow f arg k$$

Function Call on Non-simple Argument If arg is not simple, we need to convert it first.

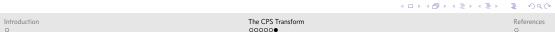
$$C[f arg]_k \Rightarrow C[arg]_{(\lambda v.f \ v \ k)}$$
, where  $v$  is fresh.

Try converting these functions.

```
1 foo 0 = 0
2 foo n | n < 0
                    = foo n
```

### Example

```
1 foo 0 = 0
2 foo n | n < 0
                   = foo n
       | otherwise = inc (foo n)
1 foo 0 k = k 0
2 foo n k | n < 0
                     = foo n k
         | otherwise = foo n (\v -> inc v k)
```



# | otherwise = inc (foo n)

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# The CPS Transform, Operators

Introduction

Operator with Two Simple Arguments If both arguments are simple, then the whole thing is simple.

$$C[e_1 + e_2]_k \Rightarrow k(e_1 + e_2)$$

Operator with One Simple Argument If  $e_2$  is simple, we transform  $e_1$ .

$$C[e_1 + e_2]_k \Rightarrow C[e_1]_{(\lambda v - > k(v + e^2))}$$
 where v is fresh.

Operator with No Simple Arguments If both need to be transformed ...

$$C[\![e_1+e_2]\!]_k \Rightarrow C[\![e_1]\!]_{(\lambda v_1->C[\![e_2]\!]_{\lambda v_2->k(v_1+v_2)})}$$
 where  $v_1$  and  $v_2$  are fresh.

Notice that we need to nest the continuations!

## Examples

```
_1 foo a b = a + b
_2 bar a b = inc a + b
3baz a b = a + inc b
quux a b = inc a + inc b
```



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### Examples

```
1 foo a b = a + b
2 bar a b = inc a + b
3 baz a b = a + inc b
4 quux a b = inc a + inc b
1 foo a b k = k (a + b)
```

```
Examples
```

```
1 foo a b = a + b
2 bar a b = inc a + b
3 baz a b = a + inc b
4 quux a b = inc a + inc b

1 foo a b k = k (a + b)
2 bar a b k = inc a (\v -> k (v + b))
```





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### Examples

```
1 foo a b = a + b
2 bar a b = inc a + b
3 baz a b = a + inc b
4 quux a b = inc a + inc b

1 foo a b k = k (a + b)
2 bar a b k = inc a (\v -> k (v + b))
3 baz a b k = inc b (\v -> k (a + v))
```

### Examples

```
1 foo a b = a + b
2 bar a b = inc a + b
3 baz a b = a + inc b
4 quux a b = inc a + inc b

1 foo a b k = k (a + b)
2 bar a b k = inc a (\v -> k (v + b))
3 baz a b k = inc b (\v -> k (a + v))
4 quux a b k = inc a (\v1 -> inc b (\v2 -> k (v1 + v2)))
```

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### References

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