

Name: \_\_\_\_\_

## CS 421 — Type Semantics Activity (Polytype Version)

Mattox Beckman

### The Rules

#### Constants

$$\frac{}{\Gamma \vdash n : \mathbf{int}} \text{CONST, when } n \text{ is an integer.}$$

Similarly for True and False.

#### Variables

$$\frac{}{\Gamma \vdash x : \sigma} \text{VAR, when } x : \sigma \in \Gamma$$

#### Binary Arithmetic

$$\frac{\Gamma \vdash e_1 : \mathbf{int} \quad \Gamma \vdash e_2 : \mathbf{int}}{\Gamma \vdash e_1 \oplus e_2 : \mathbf{int}} \text{BINOP}$$

#### Integer Relations

$$\frac{\Gamma \vdash e_1 : \mathbf{int} \quad \Gamma \vdash e_2 : \mathbf{int}}{\Gamma \vdash e_1 \sim e_2 : \mathbf{bool}} \text{RELOP}$$

#### If

$$\frac{\Gamma \vdash e_1 : \mathbf{bool} \quad \Gamma \vdash e_2 : \tau \quad \Gamma \vdash e_3 : \tau}{\Gamma \vdash \mathbf{if } e_1 \mathbf{ then } e_2 \mathbf{ else } e_3 : \tau} \text{IF}$$

#### Application

$$\frac{\Gamma \vdash e_1 : \tau \rightarrow \tau' \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1 e_2 : \tau'} \text{APP}$$

#### Abstraction

$$\frac{\Gamma \cup \{x : \tau\} \vdash e : \tau'}{\Gamma \vdash \lambda x. e : \tau \rightarrow \tau'} \text{ABS}$$

#### Let

$$\frac{\Gamma \vdash e_1 : \sigma \quad \Gamma \cup [x : \sigma] \vdash e_2 : \tau}{\Gamma \vdash \mathbf{let } x = e_1 \mathbf{ in } e_2 : \tau} \text{LET}$$

#### Gen

$$\frac{\Gamma \vdash e : \sigma}{\Gamma \vdash e : \forall \alpha. \sigma} \text{GEN, where } \alpha \text{ is not free in } \Gamma$$

#### Inst

$$\frac{\Gamma \vdash e : \sigma'}{\Gamma \vdash e : \sigma} \text{INST, where } \sigma' \geq \sigma$$

# Proofs

Create proofs for the following judgements according to the given rules.

## Problem 1)

$\{\text{id} : \forall \alpha. \alpha \rightarrow \alpha, \ y : \text{Int}\} \vdash (\text{id } y) : \text{Int}$

## Problem 2)

$\{y : \text{Int}, z : \text{String}\} \vdash (\lambda f. (f \ y, f \ z)) (\lambda x. x) : (\text{Int}, \text{String})$

## Problem 3)

$\{x : \text{Int}, y : \text{String}\} \vdash \text{let } f = \lambda x. x \text{ in } (f \ x, f \ y) : (\text{Int}, \text{String})$