CS 421 — Small Step Semantics Activity
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The Rules

$< \mathtt{skip}, \sigma > \rightarrow < E, \sigma >$
$< u := t, \sigma > \rightarrow < E, \sigma[u := \sigma(t)] >$

$$\frac{< S_1, \sigma > \rightarrow < S_2, \tau >}{< S_1; S, \sigma > \rightarrow < S_2; S, \tau >}$$
$E; S \equiv S$

$< \mathtt{if}\ B\ \mathtt{then}\ S_1\ \mathtt{else}\ S_2\ \mathtt{fi}, \sigma > \rightarrow < S_1, \sigma >$ where $\sigma \models B$
$< \mathtt{if}\ B\ \mathtt{then}\ S_1\ \mathtt{else}\ S_2\ \mathtt{fi}, \sigma > \rightarrow < S_2, \sigma >$ where $\sigma \models \neg B$

$< \mathtt{while}\ B\ \mathtt{do}\ S_1\ \mathtt{od}, \sigma > \rightarrow < S_1; \mathtt{while}\ B\ \mathtt{do}\ S_1\ \mathtt{od}, \sigma >$ where $\sigma \models B$
$< \mathtt{while}\ B\ \mathtt{do}\ S_1\ \mathtt{od}, \sigma > \rightarrow < E, \sigma >$ where $\sigma \models \neg B$

Reductions

Reduce the following programs according to the semantic rules given.
Problem 1)

```
< if x>y then m:=x else skip fi; if x<y then m:=y else skip fi;, {x:=10,y:=30}>
```

Problem 2)

```
<n := 0; while x > 1 do x:=x/2; n:=n+1 od, {x:=8}>
```

Problem 3)
(Don't spend too much time on this one.)
```
<p:=1; n:=3; while n>1 do p:=p*x od, {x:=3}>
```

Make your own rules!

Problem 4)
Write a rule to explain the `when B S` statement. It executes S only if B is true.

Problem 5)
Write a rule for `do S while B od`. It is like while, but executes S at least one time.

Church Rosser

Problem 6) Consider this semantic rule:

$$x_1 \circ x_2 \circ \cdots \circ x_i \circ x_{i+1} \circ \cdots x_n \rightarrow x_1 \circ x_2 \circ \cdots \circ (x_i * x_{i+1}) \circ \cdots x_n$$

Does it have the Church-Rosser property? Try to prove it.

Problem 7) Consider this semantic rule:

$$x_1 \circ x_2 \circ \cdots \circ x_i \circ x_{i+1} \circ \cdots x_n \rightarrow x_1 \circ x_2 \circ \cdots \circ (x_i - x_{i+1}) \circ \cdots x_n$$

Does it have the Church-Rosser property? Try to prove it.