[MECH 393]

MACHINE ELEMENT DESIGN

GEARBOX DESIGN FOR THE SOLAR IMPULSE PROJECT

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December 8th, 2017

Section 1

Executive Summary

In this report we present our proposal for the design of a gearbox for the Solar Impulse aircraft. The gearbox's main function is to transmit the power from the input shaft to the propeller, which it achieves via a two-stage compound gear train. The components of the gearbox include: four spur gears, three shafts, and bearings to support these shafts. After designing the gears to meet the required power and rotational speeds, the selected gears had a module of 3mm and $N_2 = 18$ for the input, $N_3 = 45$ $N_4 = 20$ respectively for the intermediate gears, and $N_5 = 64$ for the output to the propeller. In the final design, the gears would fit a size of 295.5mm in the z direction, which is less than the maximum of 300mm. They also fit within the 230mm width. The entire gearbox fits into a length of 291mm, given the maximum length of 350mm. The dimensions of shafts are complicated to describe, but full technical drawings can be found in the annex. For the input shaft (Shaft 1), roller bearing NUP204ECP was selected to last the required amount of cycles. Similarly, for shaft 2, we have 320/32X roller bearings. Finally for shaft 3 we have roller 30205 for the left side and roller 32305 for the right side. The gear specifications can be found in Table 1.2. The critical safety factors can be found in Table 1.1, and the total mass of components can be found in Table 6.4. The final mass of the gearbox was found to be 15.09kg, which is overweight. Given the high safety factors in the shafts, choosing lighter metals and less conservative shaft diameters would reduce the weight.

Table 1.1: Critical Safety Factors in Each Component

Component	Safety Factor
Gear	1.08
Shaft	5.45
Bearing	1.10

Table 1.2: Gear design specifications

Parameter	Gear 2	Gear 3	Gear 4	Gear 5
Module [mm]	3	3	3	3
Facewidth [mm]	24	24	35	35
Teeth	18	45	20	64
Pressure angle [degrees]	20	20	20	20

Table 1.3: Total Mass of Gearbox Design

Component	Total Mass (kg)
Gears	10.84
Shafts	3.12
Bearings	1.13
Total	15.09

Section 2

Contributions

2.1 Matthew Phillips

Design of the gears and MATLAB code debugging and development. Technical drawings and CAD modeling. Various parts of the written report.

2.2 Christopher Roy

Design of the shafts, MATLAB code development. Various parts of the written report.

2.3 Lucas Shtychno

Design of the bearings, sample calculations. Various parts of the written report.

Section 3

Intro to the Design Problem

3.1 Problem Definition

This project tasks us with designing a two stage reduction compound geartrain to be connected to the electric motor at the input and to the propeller at the output. This design serves to reduce weight and cost compared to other gear trains. The cost of the project is not of particular importance but the weight, efficiency, safety and certification are of the utmost importance. To this effect, first of all, 4 gears must be designed such that the torque from the motor is amplified and the rotational speed is decreased in the propeller. Next, three shafts must be designed to support the gears and finally bearings must be designed with particular attention paid to the bearing at the propeller which will transmit an axial force.

3.2 Research of Solutions

A double branch gearbox has the advantage that the reaction forces in the center gear will be eliminated, since the middle gear becomes an idler with gears on either side of it. This means that the output middle shaft will only need to be designed to support the weight of the propeller, the torque from it, and axial forces. The middle shaft at the input will only need to be designed to accept torque from the input.

Other types of gearboxes exist in aerospace for different types of engines. For example, turbofan engines have a very different type of gearing. The newest turbofan engine gearboxes have one or more outer gears that rotate around a sun gear that spins. This allows the shafts at the center of the engine to spin very fast whilst allowing the intake fan to turn at a lower speed[2]. The main advantages of these planetary gear systems in comparison with reduction gear boxes are their high power density, large reductions in small volumes and the fact that coaxial shafting is possible. Some drawbacks however include increased lubrication needs, large bearing stress and difficulty to access for maintenance[3]. Gearboxes in helicopters must also be reduction gearboxes since they have to reduce the high RPM of the engine into a much slower shaft speed for the main rotor, this reduction is often on the order of 100:1. One interesting distinction in helicopters is the failsafe present in the rotor. If the engine fails the rotor continues to rotate freely since the transmission is a free-wheeling unit. This allows the helicopter to safely descend in a controlled matter[4].

Section 4

Methodology

4.1 Assumptions and Justification

Diametral pitch (P_d, imperial units) can effectively be used as a measure of coarseness of a gear. It is the number of teeth per inch (diameter). Thus, a higher diametral pitch results in a finer gear. The inverse of diametral pitch is the module (metric system), which is the number of mm (diameter) per teeth, and a higher module results in a coarser gear. For the purposes of this project, we must design with a diametral pitch smaller than 20 (defined as when a gear becomes fine), this is equivalent to a module of $25.4/20 = 1.27$mm/tooth.
When analyzing the formulas for bending stress, it can be seen that on the numerator is; W_t (tangential force), and on the denominator are; m (module), J (bending geometry factor), and K_v (dynamic factor).Thus, to reduce bending stress for any gear:

- W_t should be reduced, and is a function of the gear's pitch radius.
- m should be increased, and is chosen.
- J should be increased, and is a function of the number of teeth on the gear and pinion.
- K_v should be decreased, and is a function of the pitch radius of the gear.

Now, since size constraints are given for the gearbox, it must be assumed that we must use as much of this space as possible, and the coarseness (and thus module) of the gears and pinions is what will vary. Meaning that increasing m will mean there are less teeth per diameter of the gears. J decreases when the number of teeth on the pinion and gear decrease, so now, if module is kept constant and number of teeth is decreased, the total size of the gearbox decreases. This then allows for the module to increase, which is beneficial. In addition, it is extremely important to recognize that the pinion in the second stage is the most critical point, so we absolutely want to minimize the torque (and thus W_t) on the pinion in the second stage. To do so, we must minimize the reduction of first stage to reduce the torque in the second stage. **Therefore, it will be beneficial to maximize module, and minimize the number of teeth on our gears, maximize face width, and choose a small reduction for the first stage.**

The material chosen to work with for the gears was chosen from tables 12-20 and 12-21 in the textbook. The material's properties can bee seen in Table 4.1.

Table 4.1: Gear Material Properties

Material	Class	Material Designation	S'_{fb}	S'_{fc}
Steel	A1-A5	2.5% Chrome	380 – 450 MPa	1300 – 1500 MPa
.	.	.	55 – 65 kpsi	192 – 216 kpsi

The material for the shaft was chosen from Table A-4 of the textbook, and is as follows in Table 4.2.

Table 4.2: Shaft Material

Material	SAE/AISI Number	Condition	S_y	S_{ut}
Steel	A1-A5	Quench & Temper	807 MPa	1124 MPa
.	.	@ 400F	117 kpsi	163 kpsi

Given that this plane is solar powered, and surely does not have power to spare (due to solar and battery technological limitations), the safety factors must be kept low to ensure weight is reduced, which will help the plane stay airborne. As this is the case, we aim for a 1.1 safety factor. We have not chosen a higher value partly because this is a single pilot aircraft. The life of the pilot is of course extremely important, but compared to a passenger aircraft, something going wrong would only result in the death of one soul, and not hundreds. Furthermore, this is a glider aircraft, and if something were to go wrong, the plane may glide, giving the pilot a fair amount of time to prepare for a mayday. The plane also has other safety features implemented, further reducing the risk of the loss of a soul.

For the purposes of calculations, we must determine the tangential force acting on the pinion at the input and output. The most critical point in the gearbox is the pinion in the second stage. This is because in the second stage, rotational speed is reduced, and power is conserved. From the formula P=T×ω, torque will increase in the second stage. Then, from T=W_t×r, it is obvious that tangential force, W_t will increase, which will dominate in the stress calculations over any changes in corrective factors. **For all stress calculations, we take the worst case scenario, at an input RPM of 4000 and a power of 40HP (takeoff)**. When checking the pinion in the second stage, we again use the worst case, with a power of 40HP, and an RPM equal to 4000 (input) multiplied by the gear reduction of the first stage. It should also be noted that the fatigue bending and surface strengths were found in accordance with the estimated number of cycles undergone by the end of the 2000 hour life of the gearbox, shown in Tables 4.3 and 4.4. Also note that the gear reductions must also be taken in account because lower RPMs result in lower number of cycles.

Journal bearings and balls bearings each have their advantages and disadvantages. Journal bearings may last for (effectively) infinite life, are quiet, and cheap, but require a constant flow of lubricant to replace losses. The disadvantages, of rolling element bearings are that they fail from fatigue, have poor damping ability, higher noise levels, and higher cost. Fortunately, fatigue life can be accounted, for damping ability will not be too important, noise levels are not significant, and cost is not an issue. The most important advantages however are that they can support combined radial and thrust loading, are less sensitive to interruptions in lubrication,and good low temperature starting. The decreased sensitivity to interruptions in lubrication will be extremely important. If a journal bearing were used, a loss of power and/or oil pressure would cause the bearing to lose its ability to a hold load and run smoothly, which could of course be disastrous in an aircraft. For this reason, rolling element bearings are the safe choice.

Throughout all calculations for bending and surface stress, and corrected bending and surface strengths, multiple corrective factors were applied. Firstly, in the bending stress calculations, the corrective factors applied and how they were chosen are as follows;

- J, bending strength geometry factor. Chosen in accordance with pinion and gear teeth, using Table 12-9 of the textbook (full-depth teeth with HPSTC loading). HPSTC, highest point of single-tooth contact, loading was chosen because it assumes that the gears are manufactured with high accuracies, which will be the case as cost is not an issue.

- K_v, dynamic factor. Calculated using equations 12.16si, 12.17a, and 12.17b of the textbook, using a Q_v (gear quality index) of 11 since cost is not an issue.
- K_m, load distribution factor. All face widths, F, we worked with in our calculations were done with a face width smaller than 50mm, thus a K_m of 1.6is used, as described by table 12-16 of the textbook.
- K_a, application factor. We expect a uniform driving and driven machine, thus giving a K_a of 1, as per table 12-17 of the textbook.
- K_S, size factor. We are not using any particularly large gear teeth so we set this corrective factor to 1.
- K_B, rim thickness factor. $K_B = 1$.
- K_I, idler factor. None of the gears in a two stage compound gear train are idlers, so the factor is set to 1.

Next, in the surface stress calculations, the corrective factors applied and how they were chosen are as follows:

- I, surface geometry factor. Calculated using equation 12.22a of the textbook, with equations 12.22b, 12.23, and Table 12-18 of the textbook.
- C_a,application factor. Same as K_a.
- C_m, load distribution factor. Same as K_m.
- C_v, dynamic factor. Same as K_v.
- C_S, size factor. Same as K_S.

- C_f, surface finish factor. AGMA has not yet established standards, set to 1 for gears made of conventional methods. Our gears are made from conventional methods, so C_f = 1.

Third, for bending strength calculations, the corrective factors applied and how they were chosen are as follows:

- K_L, life factor. Chosen from Table 12-24 of the textbook, using the number of load cycles at the end of the desired life of the gearbox, shown in Table 4.3. Note that this estimate is still a conservative one as it assumes the plane will take off for 1 hour, every day, and assumes that its climb is always a steep climb, not a slow one.

- K_T, temperature factor. Calculated in accordance with equation 12.24b of the textbook, using a maximum operating temperature of 40°C, 104 Fahrenheit. K_T = 0.909677.

- K_R, reliability factor. Chosen as 99.0%, giving K_R= 1. This was chosen because again, there are several other safety backups implemented in the plane so choosing an even higher reliability is not necessary.

Table 4.3: Estimation of Load Cycles for 2000 Hour Gearbox Life, for the input pinion.

Operation	Time (hours)	RPM	Cycles/hour	Cycles (cumulative)
Take-off	1	4000	240,000	240,000
Steep Climb	11	3180	190,800	2,338,800
Gliding	4	0	0	2,338,800
Steady Altitude	8	2555	153,300	3,565,200
Total	24		Average Cycles/hour:	148,550
			×2000 hours:	297,100,000

Table 4.4: Summary of Load Cycles for Each Gear

Gear	Total Number of Cycles
2	2.97E8
3	1.19E8
4	1.19E8
5	3.91E8

Lastly, for surface strength calculations, the corrective factors applied and how they were chosen are as follows:

- C_L, surface life factor. Same concept as K_L, but from Figure 12-26 of the textbook.
- C_H, hardness ratio factor. Set to 1since gears are made of the same material.
- C_T, temperature factor. Identical to K_T. $C_T = 0.909677$.
- C_R, reliability factor. Identical to K_R. $C_R = 1$.

The weight of component was determined by first finding the volume of components in SolidWorks, and then applying a density of steel equal to $7.8 Mg/m^3$. For the shafts, made of the same material, a similar process was followed, and we took a density for the shaft steel equal to 7.87g/cc[1].

4.2 MATLAB Gear Code

Almost the entire process for this section was automated in MATLAB. Our first step was to find appropriate gear ratios, with associated tooth numbers that would give us a total reduction of 8, and that fit within the designated size constraints (we used gearratios.m to do this). With this code, we selected a module of 3, and desired reduction of 8. We then input the different numbers of teeth into our graphstress.m script. This script automatically calculates the bending and surface stresses, as well as the bending and surface fatigue strengths for each gear. It then iterates from a module of 1.3, up until the module that means the gear will not fit in the prescribed size. For each module, it then iterates through face-widths that range from 8×m to 16×m. After all this has completed, it graphs the safety factors of each combination of F and m. The countourf plot then places equi-safety-factor lines on the figures. From these figures we can see all the possible combinations of m and F that fall within the acceptable safety factor. We ran graphstress.m for different tooth combinations and selected the tooth combinations that gave us the best safety factors, with the lowest face-widths possible to reduce weight. After all the combinations given by gearration.m were tested, we found the best combinations of number of teeth was 18, 45, 20, and 64, for gears 2, 3, 4, and 5 respectively. The chosen face-widths were 24mm for the first stage and 35mm for the second stage. The F-m plots made in MATLAB for our gear numbers can be

[1] http://www.matweb.com/search/datasheet.aspx?matguid=eeebf996f8684ca5ae67ac7efe147f64&ckck=1

found in Tables 4.1, 4.2, 4.3, and 4.4. The safety factors for these combinations can be found in Table 6.1. Throughout the development of the code to do all this we made a few significant errors and had bugs as a result. This delayed us greatly, but all is fixed now. However, because of this we had to start our shaft design fairly late, and had to make trade-offs and sacrifice some of the design efficiency in order to ensure that a design that met the requirements was provided before the deadline. As a result, we decided not to put spokes in our gears for lack of time. However, it would be quite easy to do so. An easy way to do this would be to drill large holes in a radial pattern around the gear. However, caution must be taken to ensure that the thickness of the rim is at least 1.2 times the gear tooth height to ensure that K_B does not change, and thus the safety factor does not change. What was not required in this project is designing the attachment method to convey toque from the shafts to the gears. There are several ways to do this, including keyways, keys of different styles (parallel, tapered, Woofruff), and collar clamps. The MATLAB code and technical drawings for the gears can both be found at the back of the document in the annex. As cost is not an issue, tolerances were taken to be very tight, and this applies to all components.

4.3 SHAFTS

As mentioned, errors and bugs in the MATLAB code we were forced to start our shaft design slightly late and we had to restart the design as well. To ensure the shaft would have a safety factor greater than one we **overestimated the shaft diameters** to make sure they would be strong enough. The minor steps on the shafts are all 2.5mm and have a 2.5mm radius fillet. Chamfers at the shaft ends were also added to ease assembly. The safety factors for our shafts can be found in Table 6.2. While designing the shafts we also **overestimated how long the bearings would be** and designed the ends of the shafts where the bearings go accordingly. We then calculated the safety factors for the bearings. The actual bearings we chose are shorter than what was designed, which has the effect of **reducing the effective length** of the shafts, and as a result, **reducing the moments** in the shafts. This means that with the new, shorter bearings, the **safety factors will increase**, and we are confident that the shaft would survive. This approach was taken as we did not have the time to recalculate the new safety factors for the shafts. **We thus provide a minimum safety factor for the shafts since the actual safety factor will be larger.**

4.4 BEARINGS

We selected roller bearings for all shafts. Roller bearings were selected because they are more efficient then journal bearings and still last long enough for the purposes of the design. The bearings were chosen from *SKF's*[2] catalog. They are also safer should the plane lose oil pressure. To ensure they would last the corrected life $L_p = k_r \left(\frac{C}{P}\right)^{10/3}$ and made sure it was bigger than the cycles the shaft would undergo in 6000 hours of service. We selected K_r=0.21 for shaft 1 (input) bearings which corresponds to a reliability of 99% to ensure that the bearings do not fail sooner than expected. The larger reaction force was used to get the corrected life for all bearings so that both bearings could be identical on each shaft except 3 due to the axial load. The axially loaded roller bearing's corrected life was computed similarly but P was computed to account for axial and radial forces and K_r was set to 89% because the axial load was very large compared to the available bearings. For the bearings on shaft 2 we had to use a reliability of 90% due to the unusual diameter of 32mm for shaft 2 (intermediate) reducing our choices.

[2] Can be accessed at http://www.skf.com/ca/en/index.html

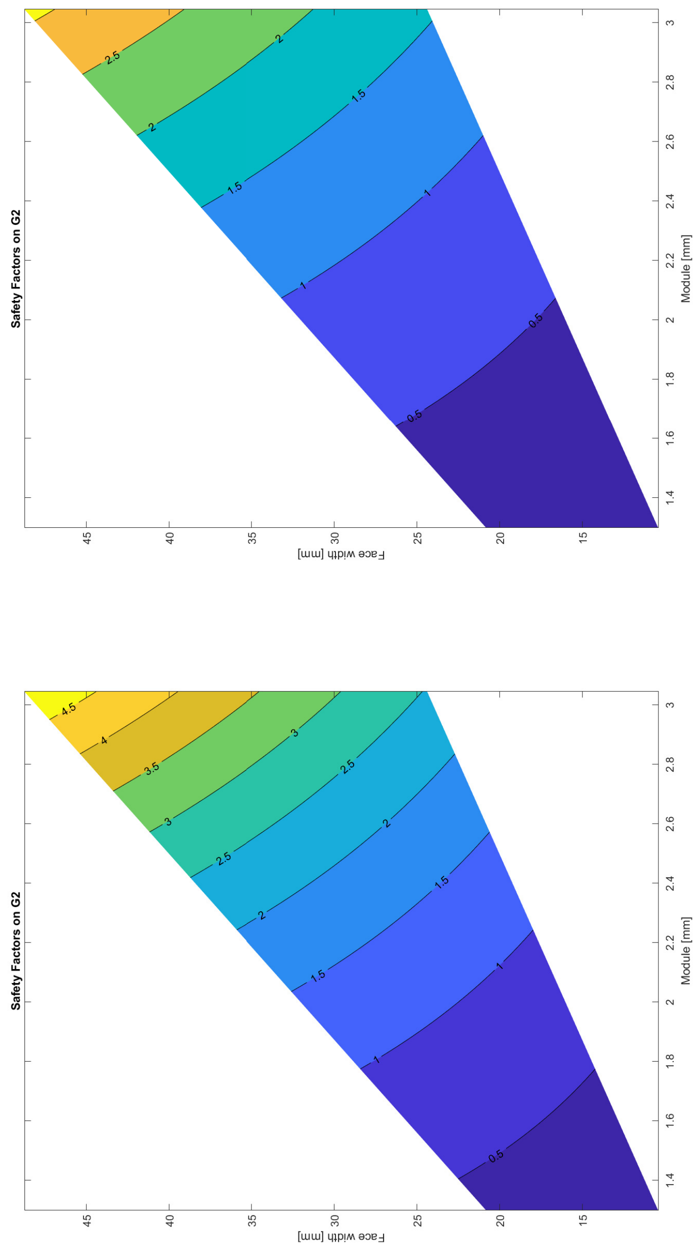
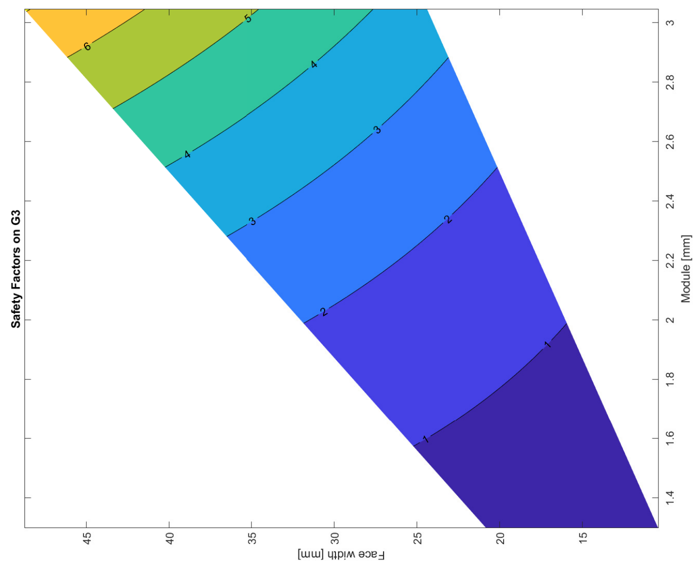


Figure 4.1: MATLAB Plots for Safety Factors on Gear 2



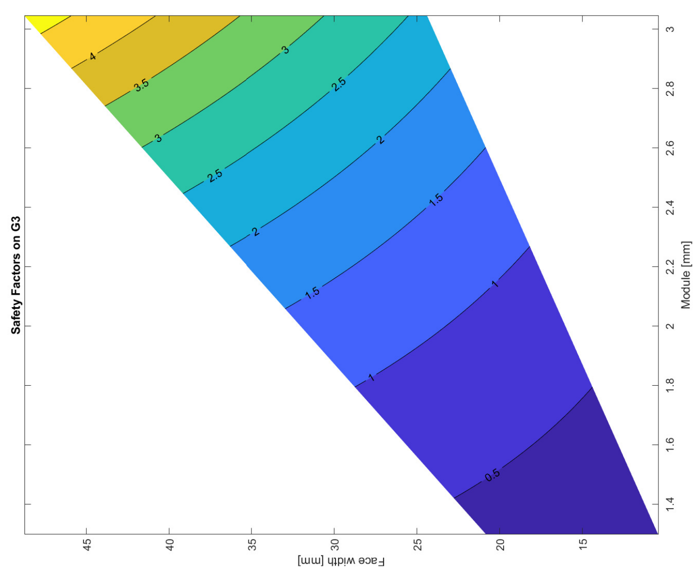


Figure 4.2: MATLAB Plots for Safety Factors on Gear 3

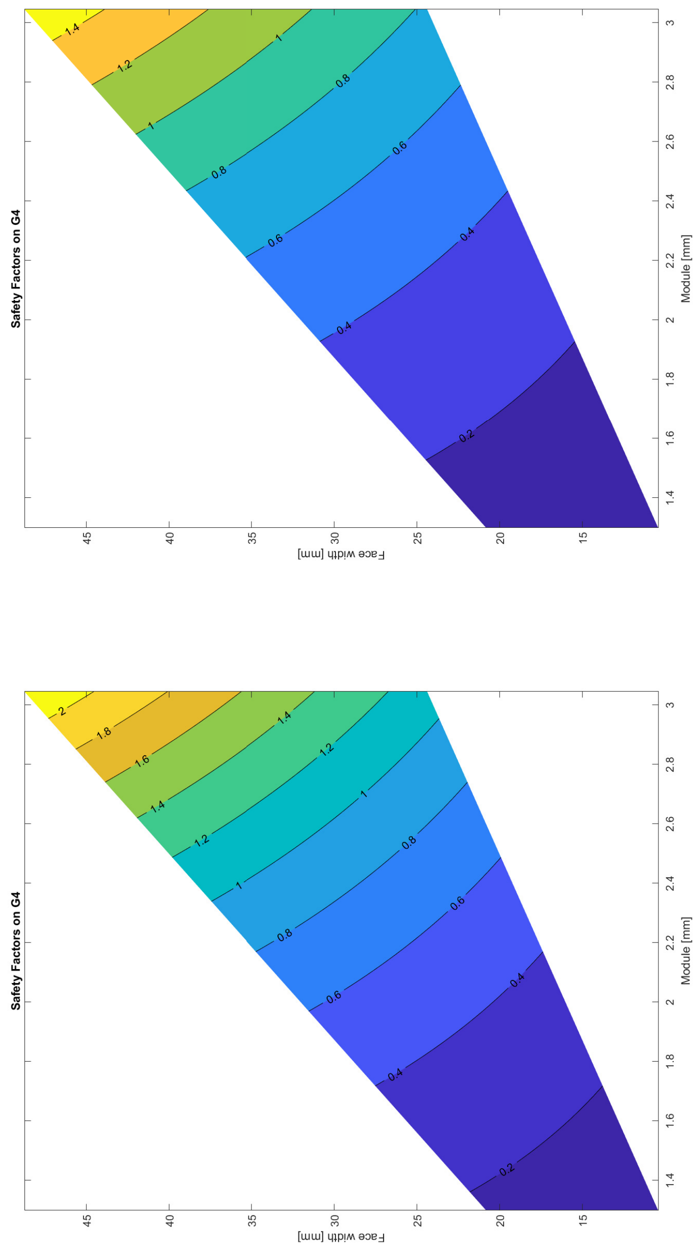


Figure 4.3: MATLAB Plots for Safety Factors on Gear 4

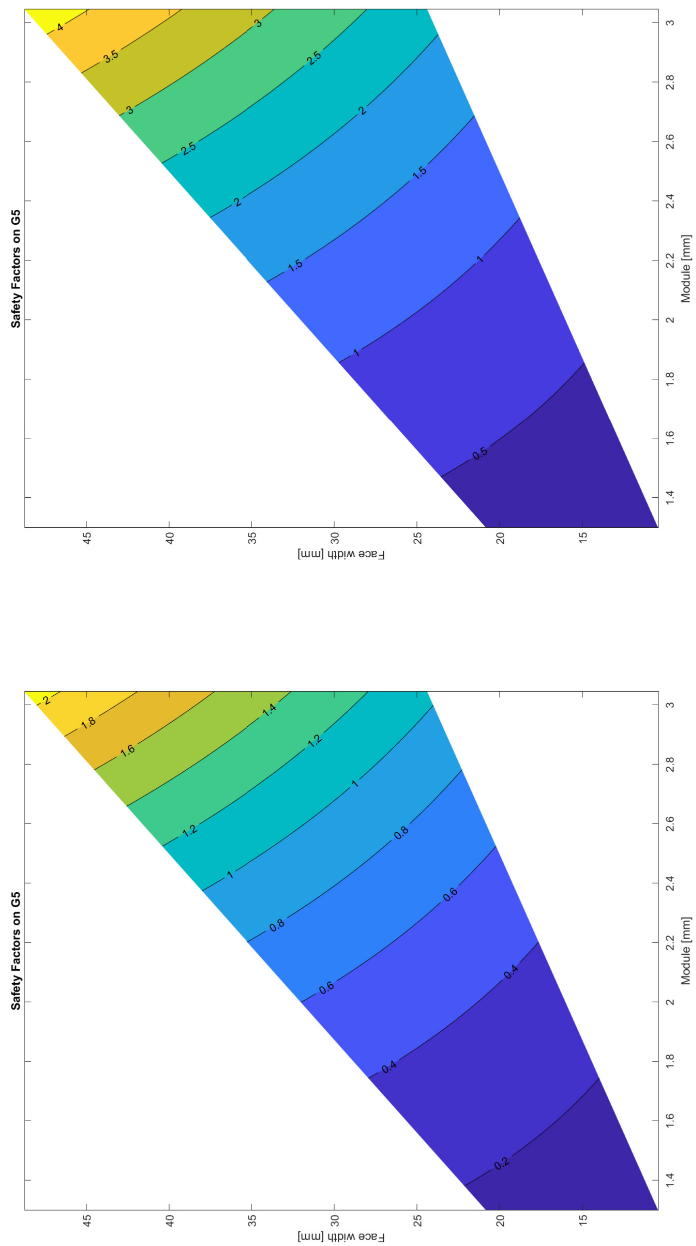


Figure 4.4: MATLAB Plots for Safety Factors on Gear 5

Section 5

Sample Calculations

5.1 Gear Calculations

We include here only the calculation for gear 4 the most critical gear

5.1.1 Basic Parameters

$$\text{Pitch diameter } = d_p = N * m = 20 * 0.003 = 0.06M \tag{5.1}$$

$$\text{Pitch radius } = r_p = 0.5(N * m) = 0.5 * 20 * 0.003 = 0.03M \tag{5.2}$$

$$\text{Tangential Force} = r_p = \frac{T}{r_p} = \frac{178.025Nm}{0.03M} = 5934.166N \tag{5.3}$$

5.1.2 Bending Stress of Pinion

$$\sigma_b = \frac{w_t\ k_a\ k_m\ k_s\ k_i\ k_b}{F\ J\ m\ k_v} \tag{5.4}$$

$$\sigma_b = \frac{5934.166 * 1 * 1.6 * 1 * 1 * 1}{0.035 * 0.33 * 0.003 * 0.9286} = 295.09\text{MPa} \tag{5.5}$$

5.1.3 Surface Stress of Pinion

$$\sigma_c = C_P\sqrt{\frac{W_t\ C_a\ C_m\ C_s\ C_f}{F\ I\ d\ C_v}} \text{ note that } C_a, C_m, C_v, C_s \equiv k_a, k_m, k_v, k_s \tag{5.6}$$

$$I = \frac{\cos(\phi)}{(\frac{1}{\rho_p}+\frac{1}{\rho_g})d_p} \text{ where } \rho_p = \sqrt{\left(r_p+\frac{1+x_p}{p_d}\right)^2-(r_p\cos(\phi))^2}-\frac{\pi\cos(\phi)}{p_d} \text{ and} \rho_g = C\sin(\phi)-\rho_p \tag{5.7}$$

$$I = \frac{\cos(20)}{(\frac{1}{0.3267}+\frac{1}{1.369})(2.36in.)} = 0.1049 \tag{5.8}$$

$$\rho_p = \sqrt{\left(0.5*20*0.003+\frac{1+0}{8.46}\right)^2-(0.5*20*0.003*\cos(20))^2}-\frac{\pi*\cos(20)}{8.46} = 0.3267 \tag{5.9}$$

$$\rho_g = C*\sin(20)-0.3267 = 1.369 \tag{5.10}$$

$$\sigma_c = 1.870\sqrt{\frac{5934.166*1*1.6*1*1*1}{20*0.003*0.035*0.1049*0.9286}} = 1274\text{MPa} \tag{5.11}$$

5.1.4 Fatigue Bending Stress Pinion

$$S_{fb} = \frac{K_L}{K_T\ K_R}S'_{fb} \text{ with } K_T = \frac{460+T}{620} \tag{5.12}$$

$$S_{fb} = \frac{0.97203}{0.909677*\ 1}*450 = 480.8437\text{MPa and } K_T = \frac{460+104}{620} = 0.909677 \tag{5.13}$$

$$N_{sf} = \frac{S_{fb}}{\sigma_b} = \frac{480.37\text{MPa}}{295.09\text{MPa}} = 1.52 \tag{5.14}$$

5.1.5 Fatigue Surface Stress Pinion

$$S_{fc} = \frac{C_L\ C_H}{C_T\ C_R}S'_{fc} \text{ with } C_T, C_R \equiv K_T, K_R \tag{5.15}$$

$$S_{fc} = \frac{0.88660\ *1}{0.909677*\ 1}*1500 = 1461.95\text{MPa} \tag{5.16}$$

$$N_{sfc} = \left(\frac{S_{fc}}{\sigma_c}\right)^2 = \left(\frac{1461.95\text{MPa}}{1274\text{MPa}}\right)^2 = 1.083 \tag{5.17}$$

5.2 Shaft Calculations

5.2.1 Shaft 1 (Input)

A conservative estimate of diameter derived from the Goodman diagram:

$$d = \left(\frac{32 N_f}{\pi} \left[\frac{\sqrt{(k_f M_a)^2 + \frac{3}{4}(k_{fs} T_a)^2}}{S_f} + \frac{\sqrt{(k_{fm} M_m)^2 + \frac{3}{4}(k_{fsm} T_m)^2}}{S_{ut}} \right] \right)^{1/3} \tag{5.18}$$

Determining the alternating and mean components of the Moments and Torques requires solving for the bearing reaction forces. Considering that gear forces will always be in the same plane (due to the 20 degree pressure angle), we work with the magnitudes of the gear forces and solve for the magnitude of the bearing, in this plane. The distances a and b are depicted in Figure 5.1.

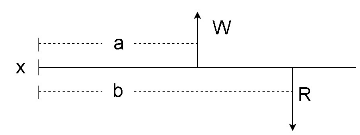


Figure 5.1: Free-body diagram for the input shaft calculations

$$T_a = 0 \quad T_m = 71.21\text{Nm} \tag{5.19}$$
$$W_t = \frac{T_m}{r_p} = \frac{71.21}{0.5 * 54} = 2637.41\text{N} \tag{5.20}$$
$$W_r = W_t \tan\alpha \tag{5.21}$$
$$= 2637.41 \tan(20) = 959.93\text{N} \tag{5.22}$$
$$W = \sqrt{W_t^2 + W_r^2} \tag{5.23}$$
$$= \sqrt{2637.41^2 + 959.93^2} = 2806.63\text{N} \tag{5.24}$$
$$\sum M_{x=0} = 0$$
$$R_{\text{Right}} = \frac{W \times a}{b} = \frac{2806.63 \times 27.5}{70} = 1102.6\text{N} \tag{5.25}$$
$$\sum F = 0 \tag{5.26}$$
$$R_{\text{Left}} = W - R_{\text{Right}} \tag{5.27}$$
$$= 2806.63 - 1102.6 = 1704.03\text{N} \tag{5.28}$$

With the reaction forces, we can find the internal shear, moment and torques at any point. We will compute the safety factor of the input shaft at the first stress concentration (30mm), which is a distance of 15cm away from the center of the first bearing.

$$V = R_{\text{Left}} = 1704\text{N} \tag{5.29}$$

$$M_a \text{ at } x = 15 = M + Vx = 0 + 1704 \times 15 = 25560.42\text{Nm} \tag{5.30}$$

$$\tag{5.31}$$

Knowing the moments and torques at the point of interest, we can calculate the safety factor given a chosen shaft diameter. We choose $d = 20$ as an initial guess. Rearranging Equation 5.18 and simplifying, we get:

$$N_f = \frac{\pi d^3}{32} \times \left[\frac{(k_f M_a)}{S_f} + \frac{0.75 * (k_{fm} T_m)^2}{S_{ut}} \right]^{-1} \tag{5.32}$$

K_f and K_{fs} are found from the values K_t and K_{ts}, which are found from Figures C-3 and C-4 from the textbook. The notch sensitivity q is found from Figure 6-36 in the textbook.

$$D/d = 1.25 \tag{5.33}$$

$$r/d = 0.25 \tag{5.34}$$

$$k_f = 1 + q(K_t - 1) \tag{5.35}$$

$$= 1 + 0.92(1.325 - 1) = 1.299 \tag{5.36}$$

$$k_{fs} = 1 + q(K_{ts} - 1) \tag{5.37}$$

$$= 1 + 0.92(1.15 - 1) = 1.138 \tag{5.38}$$

$$\tag{5.39}$$

$$S_e = C_{load}C_{size}C_{surf}C_{temp}C_{reliab}(0.5S_{UT}) \tag{5.40}$$

$$C_{load} = 1 \tag{5.41}$$

$$C_{size} = 0.8892 \tag{5.42}$$

$$C_{surf} = 0.9224 \tag{5.43}$$

$$C_{temp} = 1 \tag{5.44}$$

$$C_{reliab} = 0.814 \tag{5.45}$$

$$S_e = 375.21\text{MPa} \tag{5.46}$$

$$N_f = \frac{\pi(20)^3}{32} \times \left[\frac{((1.299)(25560)}{375.21} + \frac{(0.75(1.138(71.1)^2}{1124}\right]^{-1} = 8.8699 \tag{5.47}$$

5.3 Bearings

5.3.1 Roller Bearing Corrected life

Target life is 6000 hours. For shaft 1 this is 1782 million cycles. We chose a tapered roller bearing 30204. C = 34.1KN, d= 20mm, S_L =15 000RPM < 4000RPM OK.

$$L_p = K_R\left(\frac{C}{P}\right)^3 = 0.21 * \left(\frac{28.5KN}{1.70KN}\right)^{10/3} = 2512.54\text{Million Cycles} > 1782 \text{ Million Cycles} \tag{5.48}$$

(5.49)

Thus, this bearing will survive the required cycles with a 99% reliability ($K_r = 0.21$).

5.3.2 Tapered Roller Bearing (axial force) Corrected life

S_L=12000RPM < 500RPM → ok.

$$\text{P} = X\ V\ F_r + Y\ F_a = 0.4 * 1 * 3.677 + 2 * 6.67 = 14.81KN \tag{5.50}$$

$$L_p = K_R\left(\frac{C}{P}\right)^3 = 1.07 * \left(\frac{74.1KN}{14.81KN}\right)^{10/3} = 229 \text{ Million Cycles} > 22.6\text{Million Cycles} \tag{5.51}$$

$$\tag{5.52}$$

(Note, we chose a reliability of 89%) And this bearing will survive the amount of cycles required

Section 6

Design Results

6.1 Gears

The chosen teeth numbers are 18, 45, 20, 64 for gears 2, 3, 4, and 5, respectively. These numbers yield safety factors for bending and surface stresses detailed in Table 6.1. The obtained safety factors are within the acceptable range, and the safety factor of 1.08 (1.083) is deemed acceptable. We could have easily gone with a larger safety factor by increasing the face width, but decided to opt for the smaller one to reduce weight.

6.2 Shafts

The shafts we designed are more than strong enough to support the operating conditions, and their safety factors cen be found in Table 6.2. With more time we could have reduced the safety factors, but as a result of delays rooting from the gear calculations, we did not have much time to

Table 6.1: Bending and Surface Stress Safety Factors for all Gears

Component	Bending Safety Factor	Surface Safety Factor
Gear 2	2.36	1.49
Gear 3	2.28	3.37
Gear 4	1.52	**1.08**
Gear 5	**1.45**	3.03

Table 6.2: Nominal Safety Factors at Critical Points for all Shafts

Component	Location	Safety Factor	Note
Shaft 1	Point A_1	**8.86**	Stress Concentration
	Point B_1	12.27	Maximum Bending Moment
	Point C_1	12.71	Stress Concentration
	Point D_1	16.04	Stress Concentration
Shaft 2	Point A_2	9.56	Stress Concentration
	Point B_2	8.55	Stress Concentration
	Point C_2	**5.45**	Stress Concentration
	Point D_2	10.93	Maximum Bending Moment
	Point E_2	6.15	Stress Concentration
Shaft 3	Point A_3	9.93	Stress Concentration
	Point B_3	12.83	Stress Concentration
	Point C_3	7.54	Maximum Bending Moment
	Point D_3	**6.16**	Stress Concentration

iterate, so we opted for making the shafts overly strong.

6.3 Bearings

Roller bearings were selected for each of the bearings. There are two bearings per shaft. The first shaft (input) has two identical NUP204ECP bearings which last about a 1000 million more cycles than required with a reliability of 99%. Shaft 2 has two 320/32X bearings that last about 300 million more cycles than required with a reliability of 90%. Finally the output shaft 3 has a 30205 and a 32305 roller bearing for the left and right sides respectively. The right one was particularly challenging given the fact that the axial load was large compared to the radial load causing us to chose 89% for the reliability in order to obtain the required number of cycles.

6.4 Weight

The total weight of the design is 15.09KG as shown in the figure below. It is obvious that the gears are by far the heaviest component of the design. In the future it would be advantageous to find a design with a lighter steel or other metal and a smaller amount of teeth for the output since the

Table 6.3: Spec'd out Bearing Design

	Bearings Shaft 1	Bearings Shaft 2	Bearing Shaft 3 LHS	Bearing Shaft 3 RHS
Bearing Name	Roller NUP204ECP	Roller 320/32X	Roller 30205	Roller 32305
Max Reaction (KN)	1.704	5.55	2.637	3.677
C (KN)	28.5	45.1	38.1	74.1
C_0 (KN)	22	45.1	33.5	63
S_L (RPM)	19000	11000	13000	12000
F_r (KN)	1.704	5.55	2.637	3.677
F_a (KN)	0	0	0	6.677
X	x	x	x	0.4
V	x	x	x	1
Y	x	x	x	2
P	1.704	5.55	2.637	14.81
K_r	0.21	1	0.21	1.07
L_P Needed	1782.0	714.0	222.6	222.6
L_P Calculated	2512.54	1078.81	1542.65	228.98
Mass(Kg)	0.12	0.19	0.15	0.36

64 tooth gear is very large.

Table 6.4: Total Mass of Gearbox Design

Component	Total Mass (kg)
Gears	10.84
Shafts	3.12
Bearings	1.13
Total	15.09

Section 7

Conclusion

In this design project, we were tasked with designing a two-stage compound gearbox for the Solar Impulse plane. The gearbox needed to have a reduction of 8 (or greater) to reduce an input speed of 4000 RPM to an output of maximum 500 RPM. We designed the gears, shafts and bearings, excluding the attachment methods for the bearings and the attachment methods for the gear-shaft interface to convey torque. Our design achieved the requirements of being safe, having the desired reduction of 8 (our reduction was exactly 8), and fitting within the designated size constraints. However, we did not fall within the recommended maximum weight of 5kg, which is thankfully not critical. the reduction of the first stage of the gearbox was 2.5 and was achieved with numbers of teeth equal to 18 and 45. The reduction of the second stage is equal to 3.2 and was achieved with numbers of teeth equal to 20 and 64. Initial errors in our MATLAB code for the gear design caused significant delays and meant we had to compromise efficiency of the design to ensure we would achieve a working product in the designated time. However, given more time we could have improved the design by making spokes in our gears to reduce their weight. Furthermore, we could have done more iterations on the shafts to decrease their safety factors to be closer to 1.1. It might have also been possible to hollow the shaft to reduce weight.

Since the shafts and bearings were over designed, the weakest link in the design becomes gear 4, the pinion in the second stage. It was designed to last for 2000 hours with a safety factor of 1.083. It should therefore be maintained at or before 2000 hours of use. Although the shafts and bearings are over-engineered, they should be maintained after 2000 and 6000 hours respectively.

Section 8

Annex

8.1 All Gear Calculations (MATLAB Code)

As all our calculations were automated in MATLAB, the MATLAB code that we developed will be included in the following pages. Note that graphstress.m iterates through module and facewidth values, and stores them to matrices. Furthermore, graphstress.m passes an F and m value to stresscalculations.m which then returns eight stress values, bending and surface stress for each of the four gears. In the file gearratios.m we input a modulus and total desired reduction. It then outputs gear ratios that work, with all their associated numbers of teeth that fit into the gearbox dimensions. The way the system functions can be seen in a block diagram presented in Figure 8.1.

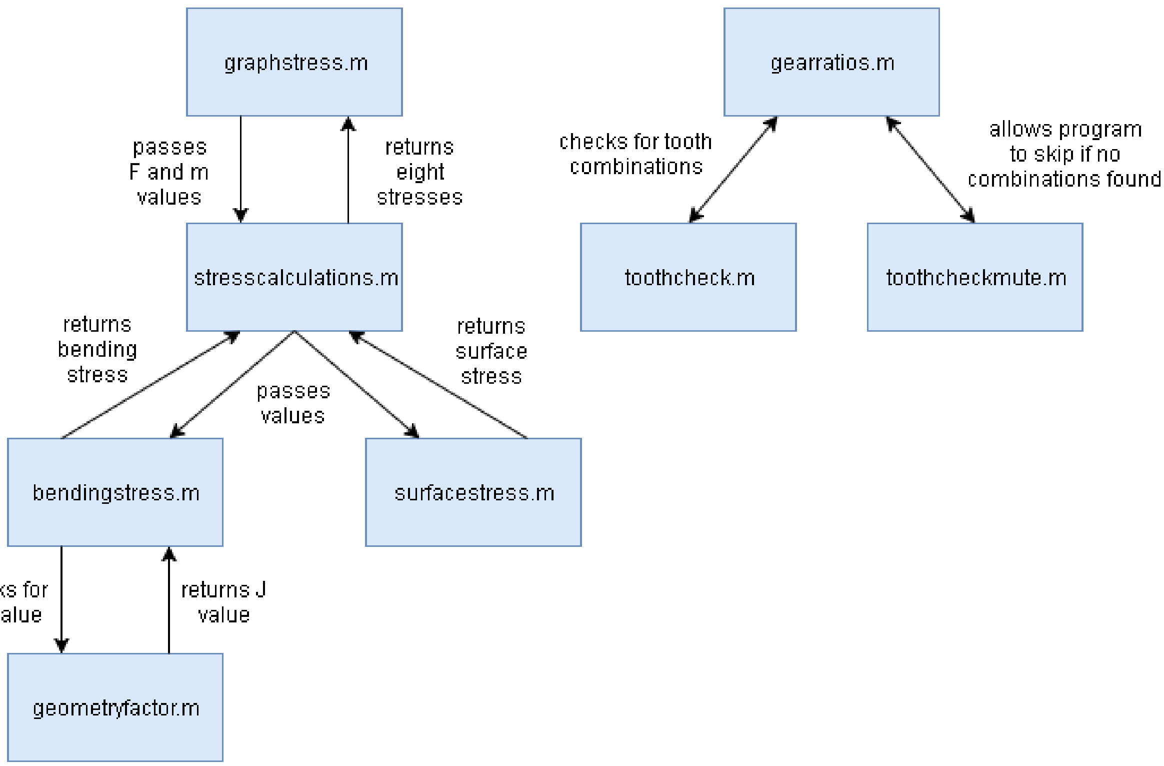


Figure 8.1: Block Diagram of MATLAB gear Code

Gearratios.m

```
clc;
clear all;

%Mechanical Advantage
m_a = 8;

%module
module = 3;

%addendum
ad = module;

%Maximum diameter for ANY gear; based on gearbox geometry
GEAR_MAX_DIAM = 230;

%Standard from books
%       If pinion has 16 teeth, gear can have no more than 101  teeth
%       If pinion has 17 teeth, gear can have no more than 1309 teeth
%We check this manually
min_teeth = 18;

%Maximum tooth (comes from d_max)
max_teeth = floor((GEAR_MAX_DIAM/module)-2);

%variables

%R1 = n3/n2;
%R2 = n5/n4;

%gearbox size
GBOX_MAX_HEIGHT = 300;

fprintf('*********************************************************************
****************\n');
fprintf('*********************************************************************
****************\n');
fprintf('Script was run with the following parameters:\n\n');
fprintf('Mechanical advantage (m_a):\t %d\n', m_a);
fprintf('Minimum teeth on a gear:\t %d\n', min_teeth);
fprintf('Maximum teeth on a gear:\t %d\n', max_teeth);
fprintf('Module (m):\t\t\t %f\n', module);
fprintf('Gearbox size [mm]:\t\t %d\n\n', GBOX_MAX_HEIGHT);
fprintf('*********************************************************************
****************\n');
fprintf('*********************************************************************
****************\n');


for R1=2.0:0.01:4.5
    R2 = 8/R1;
    if(toothcheckmute(R1,min_teeth,max_teeth) ~= 0 &&
toothcheckmute(R2,min_teeth,max_teeth) ~=0)
```

```
fprintf('\n%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%\n');
        fprintf('For R1 = %f, R2 = %f\n', R1, R2);

fprintf('%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%\n');
        [optionsR1, n_valsR1] = toothcheck(R1,min_teeth,max_teeth, module);
        [optionsR2, n_valsR2] = toothcheck(R2,min_teeth,max_teeth, module);

        for j=1:optionsR1
            for k=1:optionsR2
                n2 = n_valsR1(j, 1);
                n3 = n_valsR1(j, 2);
                n4 = n_valsR2(k, 1);
                n5 = n_valsR2(k, 2);

                total_size = module*(2 + n5 + 0.5*n4 + 0.5*n3);

                    if(total_size <= GBOX_MAX_HEIGHT)
                        if(abs(n2 - n4) <=5 && abs(n3 - n5) <=5)
                            fprintf('N2 = %d, N3 = %d, N4 = %d, N5 = %d
yields a size of: %f **********\n', n2, n3, n4, n5, total_size);

                        else
                            fprintf('N2 = %d, N3 = %d, N4 = %d, N5 = %d
yields a size of: %f\n', n2, n3, n4, n5, total_size);
                        end
                    end
            end
        end

    end

end
```

geometryfactor.m

```
function j = geometryfactor(N_p, N_g, targetGear);


if(targetGear == 'g')
     %Pin   21       26       35       55       135      %gear
table = [   0.33,    0.00,    0.00,    0.00,    0.00;    %21
            0.35,    0.35,    0.00,    0.00,    0.00;    %26
            0.37,    0.38,    0.39,    0.00,    0.00;    %35
            0.40,    0.41,    0.42,    0.43,    0.00;    %55
            0.43,    0.44,    0.45,    0.47,    0.49;];   %135

%Pinion column assignment
if(21<= N_p && N_p < 26)
    firstcol = 1;
```

```
    x1 = 21;
    secondcol = 2;
    x2 = 26;
elseif(26<= N_p && N_p < 35)
    firstcol = 2;
    x1 = 26;
    secondcol = 3;
    x2 = 35;
elseif(35<= N_p && N_p < 55)
    firstcol = 3;
    x1 = 35;
    secondcol = 4;
    x2 = 55;
elseif(55<= N_p && N_p <= 135)
    firstcol = 4;
    x1 = 55;
    secondcol = 5;
    x2 = 135;
else
    j=0.33;
    return % For when less than 21 teeth
end

%Gear row assignment
if(21<= N_g && N_g < 26)
    firstrow = 1;
    y1 = 21;
    secondrow = 2;
    y2 = 26;
elseif(26<= N_g && N_g < 35)
    firstrow  = 2;
    y1 = 26;
    secondrow = 3;
    y2 = 35;
elseif(35<= N_g && N_g < 55)
    firstrow  = 3;
    y1 = 35;
    secondrow = 4;
    y2 = 55;
elseif(55<= N_g && N_g <= 135)
    firstrow  = 4;
    y1 = 55;
    secondrow = 5;
    y2 = 135;
else
    j=0.34; return % For when less than 21 teeth
end

x = N_p;
y = N_g;

Q11 = table(firstrow, firstcol);
Q21 = table(firstrow, secondcol);
Q12 = table(secondrow, firstcol);
Q22 = table(secondrow, secondcol);
```

```
j = ((x2-x)*(y2-y)*Q11)/((x2 - x1)*(y2-y1)) + ((x - x1)*(y2-y)*Q21)/((x2 -
x1)*(y2-y1)) + ((x2 - x)*(y - y1)*Q12)/((x2 - x1)*(y2-y1)) + ((x-x1)*(y-
y1)*Q22)/((x2 - x1)*(y2-y1));
end

if(targetGear == 'p')
     %Pin   21      26      35      55      135     %gear
table = [   0.33,   0.00,   0.00,   0.00,   0.00;   %21
            0.33,   0.35,   0.00,   0.00,   0.00;   %26
            0.34,   0.36,   0.39,   0.00,   0.00;   %35
            0.34,   0.37,   0.40,   0.43,   0.00;   %55
            0.35,   0.38,   0.41,   0.45,   0.49;];  %135

%Pinion column assignment
if(21<= N_p && N_p < 26)
    firstcol = 1;
    x1 = 21;
    secondcol = 2;
    x2 = 26;
elseif(26<= N_p && N_p < 35)
    firstcol = 2;
    x1 = 26;
    secondcol = 3;
    x2 = 35;
elseif(35<= N_p && N_p < 55)
    firstcol = 3;
    x1 = 35;
    secondcol = 4;
    x2 = 55;
elseif(55<= N_p && N_p <= 135)
    firstcol = 4;
    x1 = 55;
    secondcol = 5;
    x2 = 135;
else
    j=0.33; return % For when less than 21 teeth
end

%Gear row assignment
if(21<= N_g && N_g < 26)
    firstrow = 1;
    y1 = 21;
    secondrow = 2;
    y2 = 26;
elseif(26<= N_g && N_g < 35)
    firstrow  = 2;
    y1 = 26;
    secondrow = 3;
    y2 = 35;
elseif(35<= N_g && N_g < 55)
    firstrow  = 3;
    y1 = 35;
    secondrow = 4;
    y2 = 55;
elseif(55<= N_g && N_g <= 135)
    firstrow  = 4;
```

```
    y1 = 55;
    secondrow = 5;
    y2 = 135;
else
    j=0.34; return %For when less than 21 teeth
end

x = N_p;
y = N_g;

Q11 = table(firstrow, firstcol);
Q21 = table(firstrow, secondcol);
Q12 = table(secondrow, firstcol);
Q22 = table(secondrow, secondcol);

j = ((x2-x)*(y2-y)*Q11)/((x2 - x1)*(y2-y1)) + ((x - x1)*(y2-y)*Q21)/((x2 -
x1)*(y2-y1)) + ((x2 - x)*(y - y1)*Q12)/((x2 - x1)*(y2-y1)) + ((x-x1)*(y-
y1)*Q22)/((x2 - x1)*(y2-y1));
end
```

bendingstress.m

```
function stress = bendingstress(m, F, N_p, N_g, RPM, targetGear)
%RPM OF PINIONS IS INPUT
% stress = (Wt * Ka * Km * Ks * Kb * Ki)/(F*m*J*Kv)

P=40; %HP

if(targetGear=='g')
%%%%Operating parameters%%%%
    omega = RPM * 2 * pi * (1/60)*(N_p/N_g); %angular velocity [rad/s]
    r_g = 0.5*m*N_g; %pitch radius [mm]
    Vt = r_g*(1/1000) * omega; %Pitch-line velocity [m/s]
    Qv = 11; %Gear quality index
%Application factor (Ka)
    %If uniform driving machine, uniform driven
        Ka = 1;
%Idler Factor (Ki)
    %Our system has no idlers
        Ki = 1;
%Size Factor (Ks)
    %Adjust if massive teeth, otherwise:
        Ks = 1;
%Rim thickness factor (Kb)
    %Applies if gear design has spokes
        Kb = 1;
%Dynamic factor (Kv)
    %From equation 12.17b
        B = 0.25*(12 - Qv)^(2/3);
        A = 50 + 56*(1 - B);
        Kv = (A / (A + sqrt(200*Vt)))^B;
%Load distribution factor (Km)
    %Depends on face width & module/diametral pitch
        %8/pd < F < 16/pd
        if(m <= 3.75)
```

```
            Km = 1.6;
        else
            error("Adjust km value, module exceeds max");
        end
%Bending Geometry Factor (J)
    J = geometryfactor(N_p, N_g, 'g');
%Tangential Load (Wt)
    P = P * 745.699872; %Power conversion to watts
    torque = P/omega;%[N*m]
    Wt = (torque*1000)/r_g; %[N]

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Final calculation
    stress = (Wt * Ka * Km * Ks * Kb * Ki)/(F*m*J*Kv);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
end


if(targetGear=='p')
%%%%Operating parameters%%%%
    omega = RPM * 2 * pi * (1/60); %angular velocity [rad/s]
    r_p = 0.5*m*N_p; %pitch radius [mm]
    Vt = r_p*(1/1000) * omega; %Pitch-line velocity [m/s]
    Qv = 11; %Gear quality index
%Application factor (Ka)
    %If uniform driving machine, uniform driven
        Ka = 1;
%Idler Factor (Ki)
    %Our system has no idlers
        Ki = 1;
%Size Factor (Ks)
    %Adjust if massive teeth, otherwise:
        Ks = 1;
%Rim thickness factor (Kb)
    %Applies if gear design has spokes
        Kb = 1;
%Dynamic factor (Kv)
    %From equation 12.17b
        B = 0.25*(12 - Qv)^(2/3);
        A = 50 + 56*(1 - B);
        Kv = (A / (A + sqrt(200*Vt)))^B;
%Load distribution factor (Km)
    %Depends on face width & module/diametral pitch
        %8/pd < F < 16/pd
        if(m <= 3.75)
            Km = 1.6;
        else
            error("Adjust km value, module exceeds max");
        end
%Bending Geometry Factor (J)
    J = geometryfactor(N_p, N_g, 'p');
%Tangential Load (Wt)
    P = P * 745.699872; %Power conversion to watts
    torque = P/omega;%[N*m]
    Wt = (torque*1000)/r_p; %[N]
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Final calculation
    stress = (Wt * Ka * Km * Ks * Kb * Ki)/(F*m*J*Kv);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
end
```

Surfacestress.m

```
function stress = surfacestress(m, F, N_p, N_g, RPM, targetGear)
%RPM OF PINIONS IS INPUT
% stress = Cp * root((Wt* Ca * Cm * Cs * Cf)/(F * i * d * Cv))

P=40; %HP

if(targetGear == 'g')
%%%%Operating parameters%%%%
        omega = RPM * 2 * pi * (1/60)*(N_p/N_g); %angular velocity [rad/s]
        r_g = 0.5*m*N_g; %pitch radius [mm]
        Vt = r_g*(1/1000) * omega; %Pitch-line velocity [m/s]
        Qv = 11; %Gear quality index
%Application factor (Ca)
        %If uniform driving machine, uniform driven
        Ca = 1;
%Surface finish factor (Cf)
        %No established standards
        Cf = 1;
%Size Factor (Cs)
        %Adjust if massive teeth, otherwise:
        Cs = 1;
%Dynamic factor (Cv)
        %From equation 12.17b
        B = 0.25*(12 - Qv)^(2/3);
        A = 50 + 56*(1 - B);
        Cv = (A / (A + sqrt(200*Vt)))^B;
%Load distribution factor (Cm)
        %Depends on face width & module/diametral pitch
        %   8/pd < F < 16/pd
        if(m <= 3.75)
            Cm = 1.6;
        else
            error("Adjust Cm value, module exceeds max");
        end
%Elastic Coefficient (Cp)
        %All gears of same material
        elastic_mod = 2E5; %[MPa]
        poisson = 0.3;
        Cp = sqrt((1/(pi*(2*(1 - poisson^2)/elastic_mod))));
%Surface Geometry Factor (I)
        %Radius of curvature of teeth of pinion (rho_p)
```

```
            phi = 20; %pressure angle [deg]
            p_d = 25.4/m;%diametral pitch [inch]
            r_p_in = (0.5 * (N_p/p_d)); %pitch radius of pinion [in]
            x_p = 0; %standard full depth teeth, x_p = 0;
        rho_p = sqrt((r_p_in + (1 + x_p)/(p_d))^2 - (r_p_in*cosd(phi))^2) -
(pi*cosd(phi))/p_d; %[in]

        %Radius of curvature of teeth of gear(rho_g)
            pitch_rad_pin = N_p/(2*p_d); %[in]
            pitch_rad_gear = N_g/(2*p_d); %[in]
            C = pitch_rad_pin + pitch_rad_gear; %[in]
        rho_g = C * sind(phi) - rho_p; %[in]

        I = cosd(phi)/(((1/rho_p) + (1/rho_g))* (2*r_p_in));
%Tangential Load (Wt)
    %
    P = P * 745.699872; %Power conversion to watts
    torque = P/omega;%[N*m]
    Wt = (torque*1000)/r_g; %[N]

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Final calculation
    d_p_g = N_g*m;
    stress = Cp * sqrt((Wt* Ca * Cm * Cs * Cf)/(F * I * d_p_g * Cv));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
end



if(targetGear == 'p')
%%%%Operating parameters%%%%
        omega = RPM * 2 * pi * (1/60); %angular velocity [rad/s]
        r_p = 0.5*m*N_p; %pitch radius [mm]
        Vt = r_p*(1/1000) * omega; %Pitch-line velocity [m/s]
        Qv = 11; %Gear quality index
%Application factor (Ca)
        %If uniform driving machine, uniform driven
        Ca = 1;
%Surface finish factor (Cf)
        %No established standards
        Cf = 1;
%Size Factor (Cs)
        %Adjust if massive teeth, otherwise:
        Cs = 1;
%Dynamic factor (Cv)
        %From equation 12.17b
        B = 0.25*(12 - Qv)^(2/3);
        A = 50 + 56*(1 - B);
        Cv = (A / (A + sqrt(200*Vt)))^B;
%Load distribution factor (Cm)
        %Depends on face width & module/diametral pitch
        %   8/pd < F < 16/pd
        if(m <= 3.75)
            Cm = 1.6;
        else
            error("Adjust Cm value, module exceeds max");
```

```
        end
%Elastic Coefficient (Cp)
        %All gears of same material
        elastic_mod = 2E5; %[MPa]
        poisson = 0.3;
        Cp = sqrt((1/(pi*(2*(1 - poisson^2)/elastic_mod))));
%Surface Geometry Factor (I)
        %Radius of curvature of teeth of pinion (rho_p)
            phi = 20; %pressure angle [deg]
            p_d = 25.4/m;%diametral pitch
            r_p_in = (0.5 * (N_p/p_d)); %pitch radius of pinion [in]
            x_p = 0; %standard full depth teeth, x_p = 0;
        rho_p = sqrt((r_p_in + (1 + x_p)/(p_d))^2 - (r_p_in*cosd(phi))^2) -
(pi*cosd(phi))/p_d; %[in]

        %Radius of curvature of teeth of gear(rho_g)
            pitch_rad_pin = N_p/(2*p_d); %[in]
            pitch_rad_gear = N_g/(2*p_d); %[in]
            C = pitch_rad_pin + pitch_rad_gear; %[in]

        rho_g = C * sind(phi) - rho_p; %[in]

        I = cosd(phi)/(((1/rho_p) + (1/rho_g))* 2 * r_p_in);
%Tangential Load (Wt)
    %
    P = P * 745.699872; %Power conversion to watts
    torque = P/omega;%[N*m]
    Wt = (torque*1000)/r_p; %[N]

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Final calculation
    d_p = N_p*m;
    stress = Cp * sqrt((Wt* Ca * Cm * Cs * Cf)/(F * I * d_p * Cv));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
end
```

Stresscalculations.m

```
function [bending2, bending3, bending4, bending5, surf2, surf3, surf4 ,surf5]
= stresscalculations(n2, n3, n4, n5, m, F)



%Calc G2: input pinion
    RPM_n2 = 4000; %PINION RPM
    bending2 = bendingstress(m,F,n2,n3,RPM_n2,'p');
    surf2 = surfacestress(m,F,n2,n3,RPM_n2,'p');

%Calc G3: stage 1 gear
```

```
    RPM_n3 = 4000; %PINION RPM
    bending3 = bendingstress(m,F,n2,n3,RPM_n3,'g');
    surf3 = surfacestress(m,F,n2,n3,RPM_n3,'g');

%Calc G4: stage 2 pinion
    RPM_n4 = 4000*(n2/n3); %PINION RPM
    bending4 = bendingstress(m,F,n4,n5,RPM_n4,'p');
    surf4 = surfacestress(m,F,n4,n5,RPM_n4,'p');

%Calc G5: output gear
    RPM_n5 = 4000*(n2/n3); %PINION RPM
    bending5 = bendingstress(m,F,n4,n5,RPM_n5,'g');
    surf5 = surfacestress(m,F,n4,n5,RPM_n5,'g');
```

graphstress.m

```
function points = graphstress(n2, n3, n4, n5, type)

%Calculating Load Cycles and strengths for each gear
    S_fb_prime = 450; %MPa
    S_fc_prime = 1500; %Mpa

    %0.5 applied because having a reduction >8 and satefy factor above 1.1
practically impossibe
    cycles_n2 = 2.971 * 10^8;
    cycles_n3 = cycles_n2 * (n3/n2);
    cycles_n4 = cycles_n3;
    cycles_n5 = cycles_n3 * (n5/n4);

    K_T = (460+104)/620;
    K_R = 1.00; %reliab 99%

    %LIFE FACTORS, equations found in excel
        K_L_bend_g2 = (-0.034*log(cycles_n2))+1.6042; %log is natural log,
log10 is log base 10
        K_L_bend_g3 = (-0.034*log(cycles_n3))+1.6042;
        K_L_bend_g4 = (-0.034*log(cycles_n4))+1.6042;
        K_L_bend_g5 = (-0.034*log(cycles_n5))+1.6042;

        C_L_surf_g2 = -0.045*log(cycles_n2)+1.7233;
        C_L_surf_g3 = -0.045*log(cycles_n3)+1.7233;
        C_L_surf_g4 = -0.045*log(cycles_n4)+1.7233;
        C_L_surf_g5 = -0.045*log(cycles_n5)+1.7233;

    %corrected_bending_fatigue_strength = 350;  %MPa %This will be deleted
    %corrected_surface_fatigue_strength = 1111; %MPa %This will be deleted

    % STRENGTHS
        S_fb_g2 = (K_L_bend_g2 * S_fb_prime)/(K_T * K_R);
        S_fb_g3 = (K_L_bend_g3 * S_fb_prime)/(K_T * K_R);
        S_fb_g4 = (K_L_bend_g4 * S_fb_prime)/(K_T * K_R);
        S_fb_g5 = (K_L_bend_g5 * S_fb_prime)/(K_T * K_R);

        S_fc_g2 = (C_L_surf_g2 * S_fc_prime)/(K_T * K_R);
        S_fc_g3 = (C_L_surf_g3 * S_fc_prime)/(K_T * K_R);
        S_fc_g4 = (C_L_surf_g4 * S_fc_prime)/(K_T * K_R);
```

```
        S_fc_g5 = (C_L_surf_g5 * S_fc_prime)/(K_T * K_R);

    %Module min
        m_min = 1.3;
        n_biggest = max(n3,n5);
    %Module max
        m_max = min(  (230/(n_biggest+2))   ,  300/(2+n5+0.5*n4+0.5*n3)    );
    %Step Sized
        MASTER_STEP = 500; %Change plot resolution
        m_step = 1/MASTER_STEP;
        F_step = 1/MASTER_STEP;
    i=1;

    points = zeros(MASTER_STEP);
    X = zeros(MASTER_STEP);
    Y = zeros(MASTER_STEP);

    for mod=m_min:((m_max - m_min)*m_step):m_max
        %Face width min
            F_min = 8*mod;
        %Face width max
            F_max = 16*mod;
        j=1;

        for face=F_min:((F_max - F_min)*F_step):F_max
            X(i,j) = mod;
            Y(i,j) = face;
            [bending2,bending3,bending4, bending5, surf2, surf3, surf4 ,
surf5] = stresscalculations(n2,n3,n4,n5,mod,face);

            points2b(i,j) = S_fb_g2/bending2;
            points3b(i,j) = S_fb_g3/bending3;
            points4b(i,j) = S_fb_g4/bending4;
            points5b(i,j) = S_fb_g5/bending5;

            points2c(i,j) = (S_fc_g2/surf2)^2;
            points3c(i,j) = (S_fc_g3/surf3)^2;
            points4c(i,j) = (S_fc_g4/surf4)^2;
            points5c(i,j) = (S_fc_g5/surf5)^2;

            if(i==300 && j == 300)
               fprintf("At a modulus of: %f, a face width of: %f\n", mod,
face);
               fprintf("Bending Stress of Crit (N4) [MPa]: %f\n", bending4);
               fprintf("Surface Stress of Crit (N4) [MPa]: %f\n", surf4);
               reductionratio = (n3/n2)*(n5/n4);
               fprintf("Reduction Ratio: %f, RPM Out:
%f\n",reductionratio,4000*(1/reductionratio));
            end

            j = j +1;
        end

        i = i +1;
    end
```

```
% For contour lines
%zmin= 0.5;
%zmax= 5.0;
%zinc= 0.1;
%zlevs= zmin:zinc:zmax;
%zlevs = 1.1;

%%%%%%%%%%%%%%%%%% Gear 2 %%%%%%%%%%%%%%%%%%
    figure;
    subplot(1,2,1)
        if(type == 'a')
            mesh(X, Y, points2b);
        else
            contourf(X, Y, points2b,'ShowText','on');
        end
        %axis([0 5 0 50 1.1 3]);
            xlabel('Module [mm]');
            ylabel('Face width [mm]');
            zlabel('Safety factor');
            title("Safety Factors on G2")
    subplot(1,2,2)
        if(type == 'a')
            mesh(X, Y, points2c);
        else
            contourf(X, Y, points2c,'ShowText','on');
        end
        %axis([0 5 0 50 1.1 3]);
            xlabel('Module [mm]');
            ylabel('Face width [mm]');
            zlabel('Safety factor');
            title("Safety Factors on G2");



%%%%%%%%%%%%%%%%%% Gear 3 %%%%%%%%%%%%%%%%%%
    figure;
    subplot(1,2,1)
        if(type == 'a')
            mesh(X, Y, points3b);
        else
            contourf(X, Y, points3b,'ShowText','on');
        end
        %axis([0 3 0 3 0 10000]);
            xlabel('Module [mm]');
            ylabel('Face width [mm]');
            zlabel('Safety factor');
            title("Safety Factors on G3")

    subplot(1,2,2)
        if(type == 'a')
            mesh(X, Y, points3c);
        else
            contourf(X, Y, points3c,'ShowText','on');
        end
        %axis([0 3 0 3 0 10000]);
```

```
            xlabel('Module [mm]');
            ylabel('Face width [mm]');
            zlabel('Safety factor');
            title("Safety Factors on G3");



%%%%%%%%%%%%%%%%%% Gear 4 %%%%%%%%%%%%%%%%%%
    figure;
    subplot(1,2,1)
        if(type == 'a')
            mesh(X, Y, points4b);
        else
            contourf(X, Y, points4b,'ShowText','on');
        end
        %axis([0 3 0 3 0 10000]);
            xlabel('Module [mm]');
            ylabel('Face width [mm]');
            zlabel('Safety factor');
            title(sprintf('Safety Factors on G4'));
    subplot(1,2,2)
        if(type == 'a')
            mesh(X, Y, points4c);
        else
            contourf(X, Y, points4c,'ShowText','on');
        end
        %axis([0 3 0 3 0 10000]);
            %For graph titles
                g2=n2;
                g3=n3;
                g4=n4;
                g5=n5;
            xlabel('Module [mm]');
            ylabel('Face width [mm]');
            zlabel('Safety factor');
            %title(sprintf('Surface stresses on G4.\nTeeth: %d, %d, %d,
%d\nReduction: %f',g2,g3,g4,g5,reductionratio));
            title("Safety Factors on G4");



%%%%%%%%%%%%%%%%%% Gear 5 %%%%%%%%%%%%%%%%%%
    figure;
    subplot(1,2,1)
        if(type == 'a')
            mesh(X, Y, points5b);
        else
            contourf(X, Y, points5b,'ShowText','on');
        end
        %axis([0 3 0 3 0 10000]);
            xlabel('Module [mm]');
            ylabel('Face width [mm]');
            zlabel('Safety factor');
            title("Safety Factors on G5");
    subplot(1,2,2)
        if(type == 'a')
```

```
            mesh(X, Y, points5c);
        else
            contourf(X, Y, points5c,'ShowText','on');
        end
        %axis([0 3 0 3 0 10000]);
            xlabel('Module [mm]');
            ylabel('Face width [mm]');
            zlabel('Safety factor');
            title("Safety Factors on G5");

end
```

```
   % mesh(X, Y, points2b);
   % mesh(X, Y, points2c);

    %mesh(X, Y, points3b);
   % mesh(X, Y, points3c);

   % mesh(X, Y, points4b);
   % mesh(X, Y, points4c);

    %colormap(hot);
    %axis([0 3 0 3 0 663]);
```

Toothcheck.m

```
function [options, n_vals]  = toothcheck(R, min_teeth, max_teeth, m)

options = 0;

for tooth1=min_teeth:max_teeth
        tooth2=R*tooth1;
        if(floor(tooth2) > max_teeth)
            break;
        end
        if(floor(tooth2) == tooth2)
          % fprintf("n2 = %d, n3 = %d\n", tooth1, tooth2);
           n_vals(options+1,1) = tooth1;
           n_vals(options+1, 2) = tooth2;
           options = options+1;

        end
end
```

toothcheckmute.m

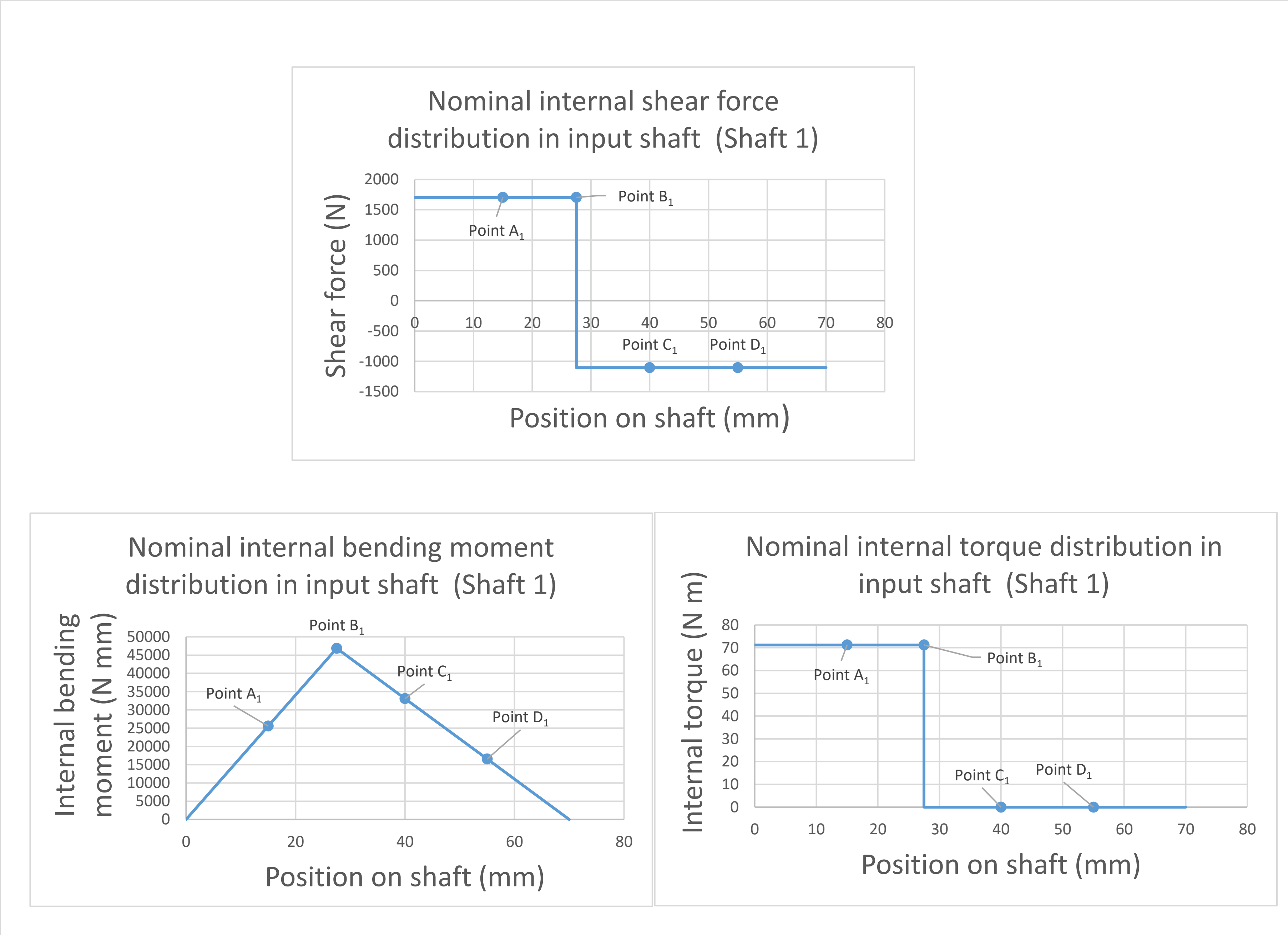
```
function options = toothcheckmute(R, min_teeth, max_teeth)

options = 0;

for tooth1=min_teeth:max_teeth
        tooth2=R*tooth1;
        if(floor(tooth2) > max_teeth)
            break;
        end
        if(floor(tooth2) == tooth2)
           options = options+1;
        end
end
```

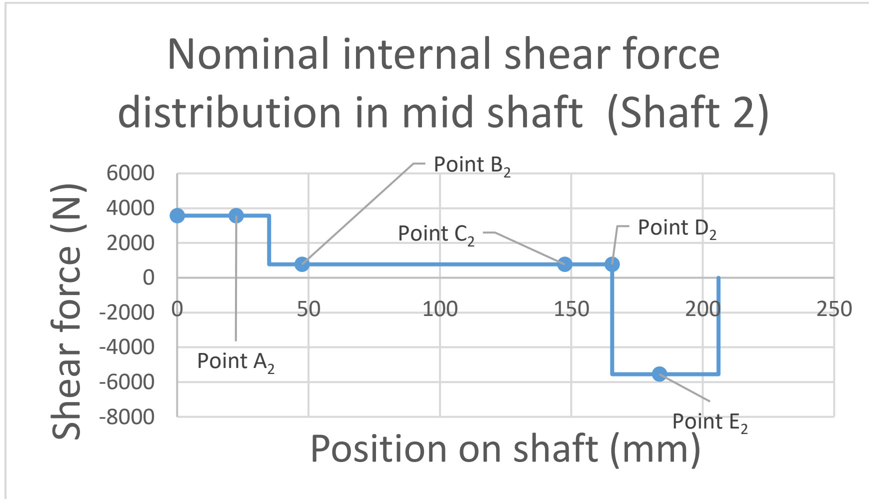
8.2 References to Charts Used

All charts and figures, and equations used were taken from Machine Element Design: An Integrated Approach, 5^{th} edition. Authored by Robert L. Norton, and published by Pearson

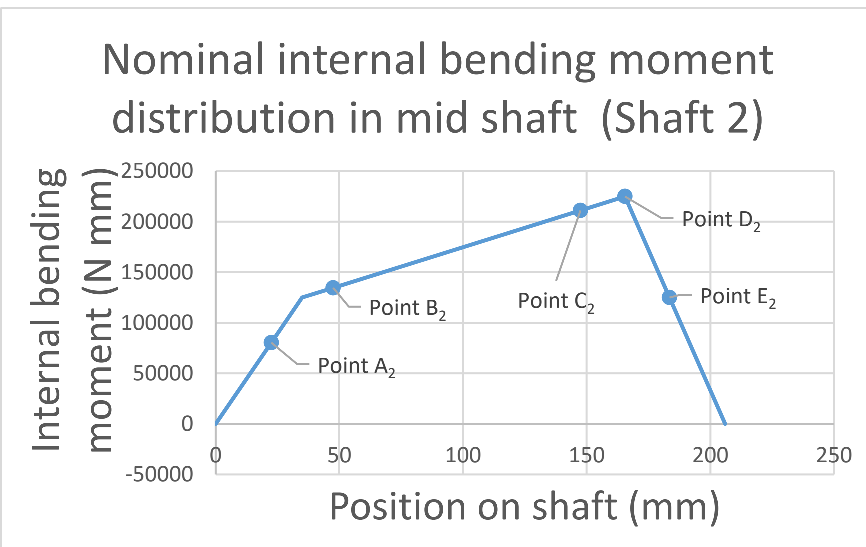
8.3 SHEAR AND MOMENT DIAGRAMS

Seen on the pages to follow.

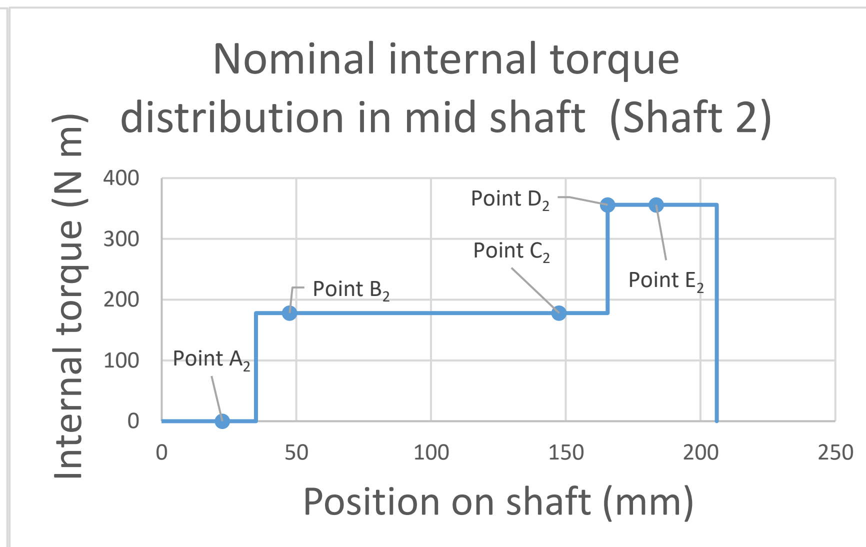

Nominal internal shear force distribution in input shaft (Shaft 1)
Shear force (N)
Position on shaft (mm)
2000
1500
1000
500
0
-500
-1000
-1500
0
10
20
30
40
50
60
70
80
Point A1
Point B1
Point C1
Point D1
Nominal internal bending moment distribution in input shaft (Shaft 1)
Internal bending moment (N mm)
Position on shaft (mm)
50000
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
0
20
40
60
80
Point A1
Point B1
Point C1
Point D1
Nominal internal torque distribution in input shaft (Shaft 1)
Internal torque (N m)
Position on shaft (mm)
80
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
Point A1
Point B1
Point C1
Point D1



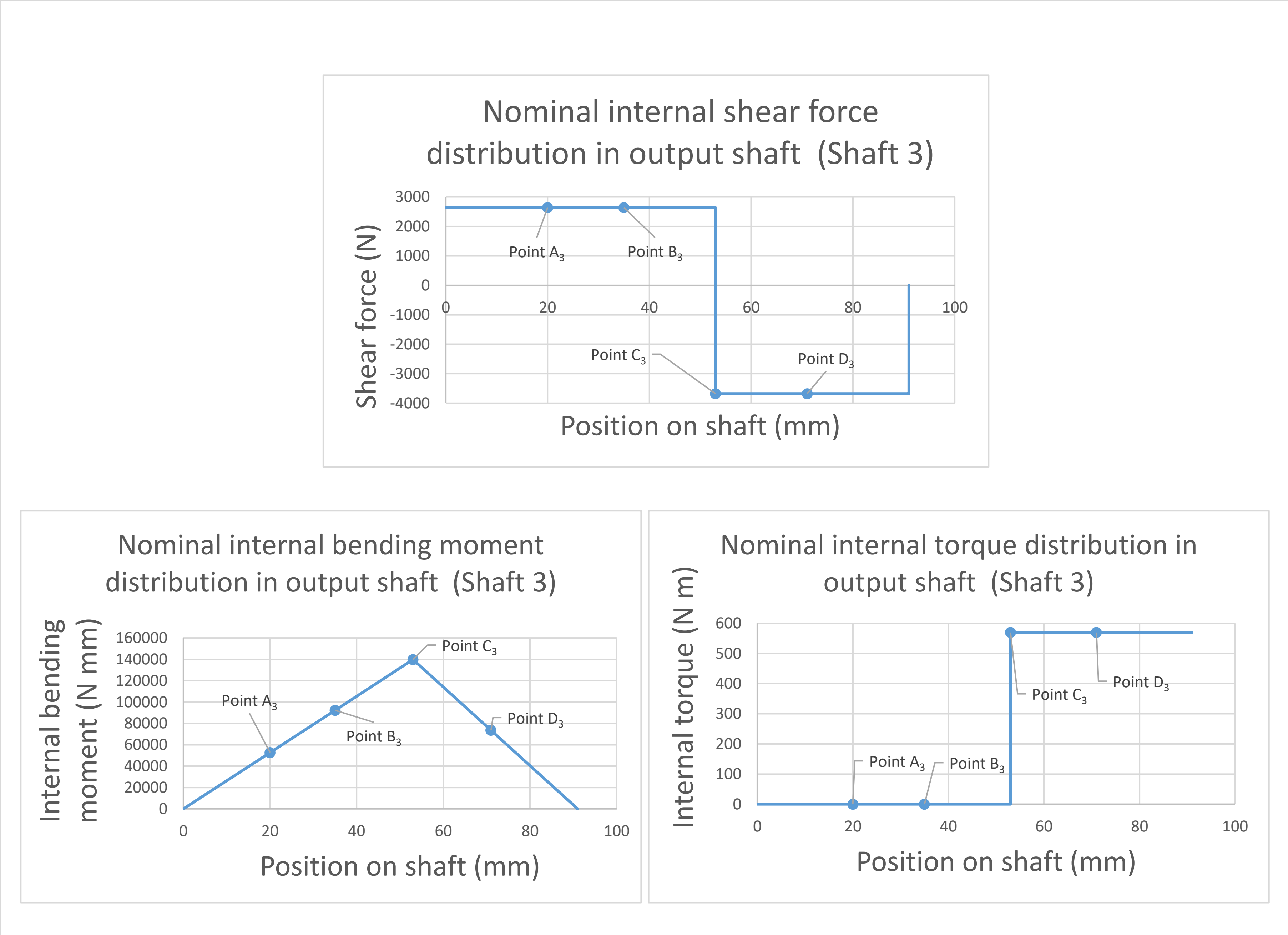
Nominal internal shear force distribution in mid shaft (Shaft 2)
Shear force (N)
Position on shaft (mm)
6000
4000
2000
0
-2000
-4000
-6000
-8000
0
50
100
150
200
250
Point A$_2$
Point B$_2$
Point C$_2$
Point D$_2$
Point E$_2$



Nominal internal bending moment distribution in mid shaft (Shaft 2)
Internal bending moment (N mm)
Position on shaft (mm)
250000
200000
150000
100000
50000
0
-50000
0
50
100
150
200
250
Point A$_2$
Point B$_2$
Point C$_2$
Point D$_2$
Point E$_2$

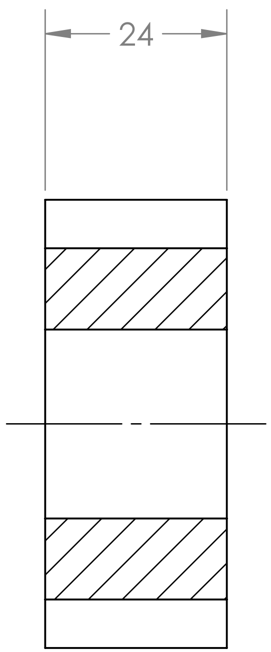


Nominal internal torque distribution in mid shaft (Shaft 2)
Internal torque (N m)
Position on shaft (mm)
400
300
200
100
0
0
50
100
150
200
250
Point A$_2$
Point B$_2$
Point C$_2$
Point D$_2$
Point E$_2$

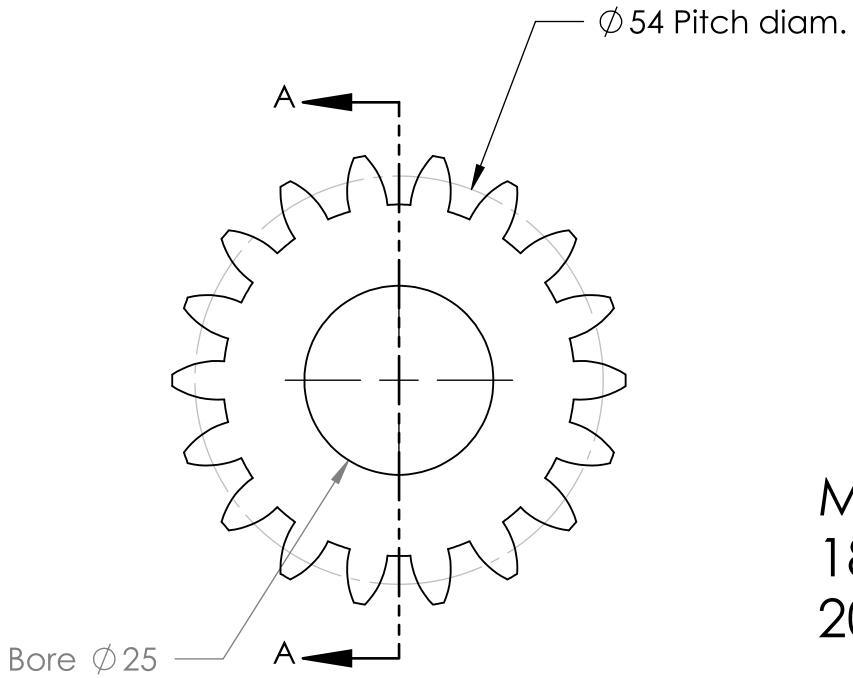

Nominal internal shear force distribution in output shaft (Shaft 3)
Shear force (N)
Position on shaft (mm)
Point A3
Point B3
Point C3
Point D3
3000
2000
1000
0
-1000
-2000
-3000
-4000
0
20
40
60
80
100
Nominal internal bending moment distribution in output shaft (Shaft 3)
Internal bending moment (N mm)
Position on shaft (mm)
Point A3
Point B3
Point C3
Point D3
160000
140000
120000
100000
80000
60000
40000
20000
0
0
20
40
60
80
100
Nominal internal torque distribution in output shaft (Shaft 3)
Internal torque (N m)
Position on shaft (mm)
Point A3
Point B3
Point C3
Point D3
600
500
400
300
200
100
0
0
20
40
60
80
100

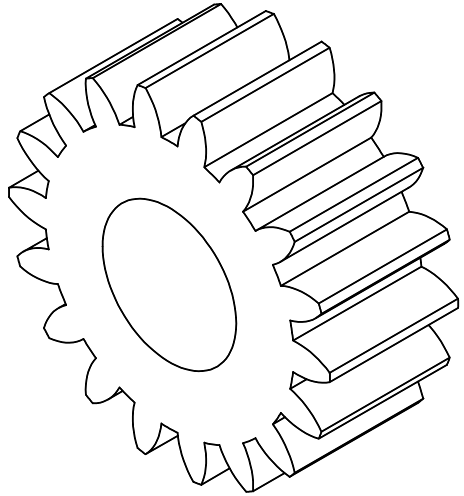
8.4 MECHANICAL DRAWINGS

Seen on the pages to follow.



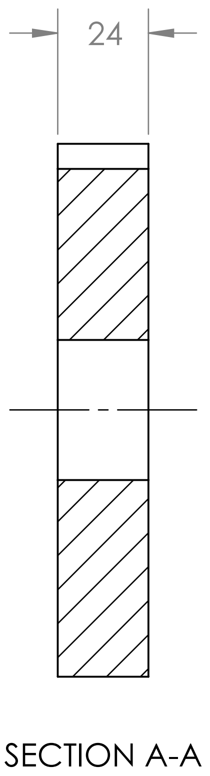
SECTION A-A

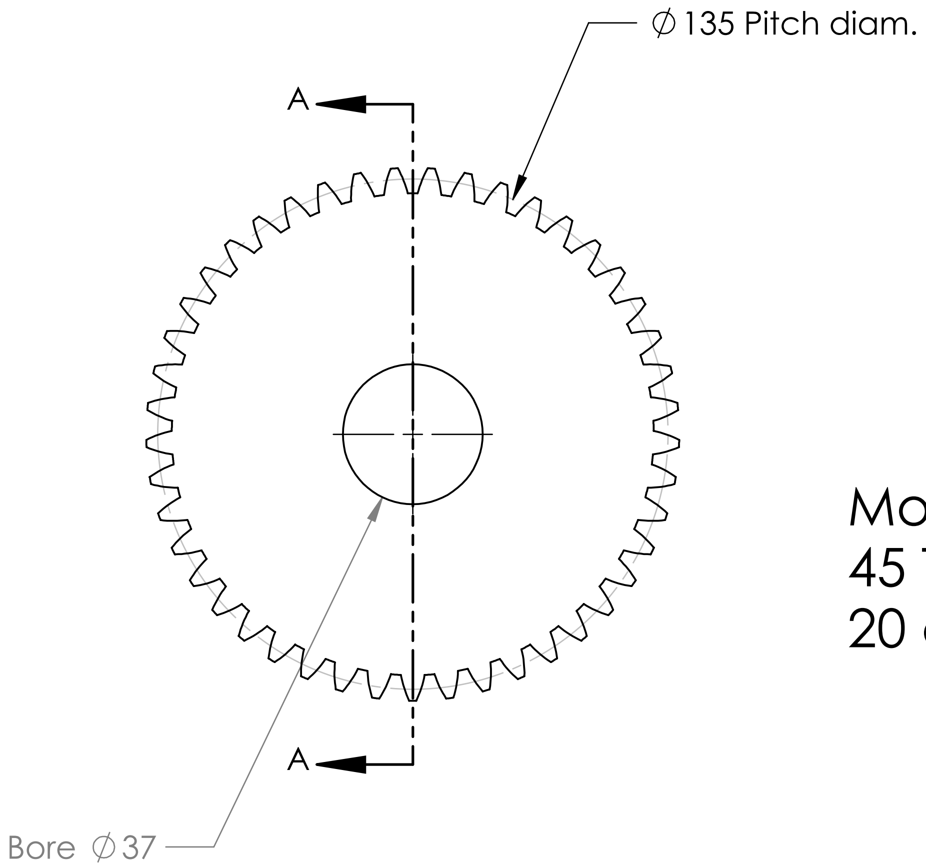


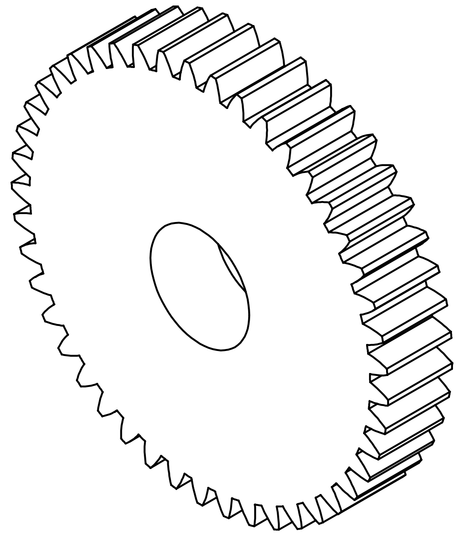


Module: 3mm
18 Teeth
20 deg pressure angle

	NAME	SIGNATURE	DATE	DO NOT SCALE DRAWING — REVISION 1
DRAWN	Matthew Phillips			McGill University
APPV'D				TITLE: Gear 2
UNLESS OTHERWISE SPECIFIED: DIMENSIONS ARE IN MILLIMETERS TOLERANCES: LINEAR: +/- 0.005 ANGULAR: +/- 0.25		QUANTITY: 1		
		MATERIAL: Steel A1-A5 2.5% Chrome Nitrided		DWG NO. 1 — A4
				SCALE:1:1 — SHEET 1 OF 1







Module: 3mm
45 Teeth
20 deg pressure angle

DO NOT SCALE DRAWING
REVISION 1

McGill University

	NAME	SIGNATURE	DATE
DRAWN	Matthew Phillips		
APPV'D			

UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MILLIMETERS

TOLERANCES:
LINEAR: +/- 0.005
ANGULAR: +/- 0.25

QUANTITY: 1

MATERIAL: Steel A1-A5
2.5% Chrome
Nitrided

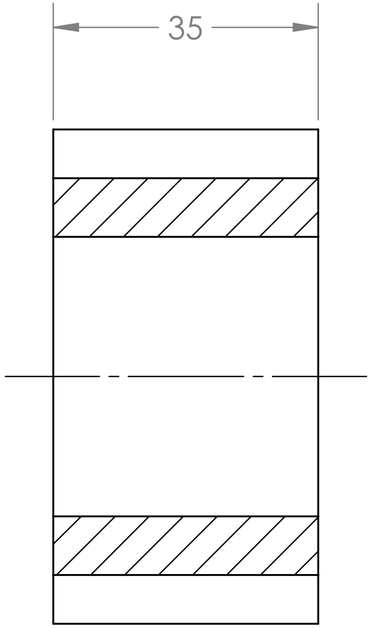
TITLE: Gear 3

DWG NO. 2

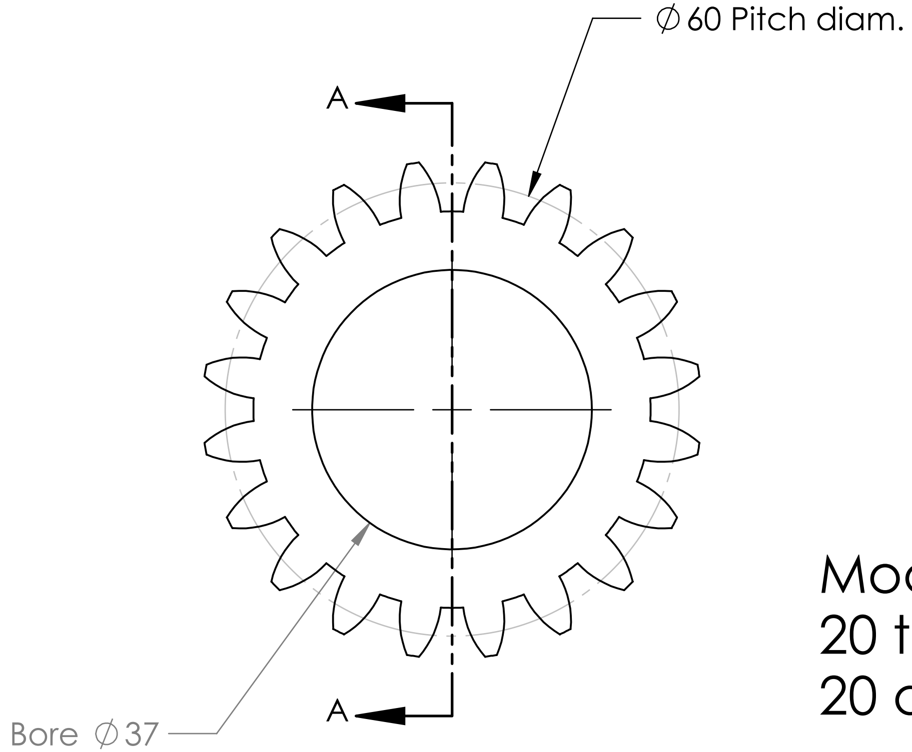
A4

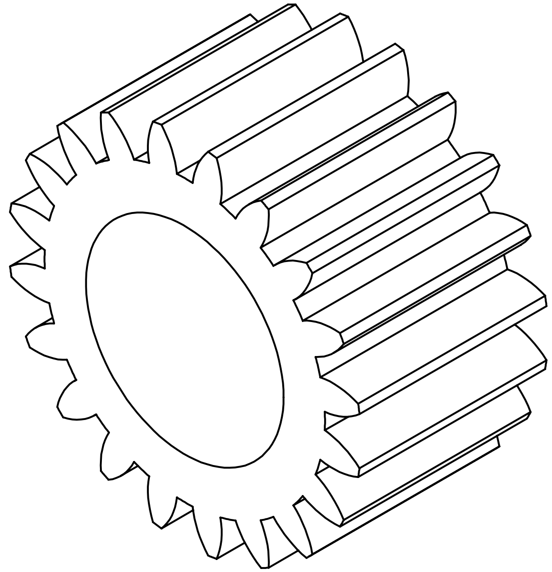
SCALE:1:2

SHEET 1 OF 1



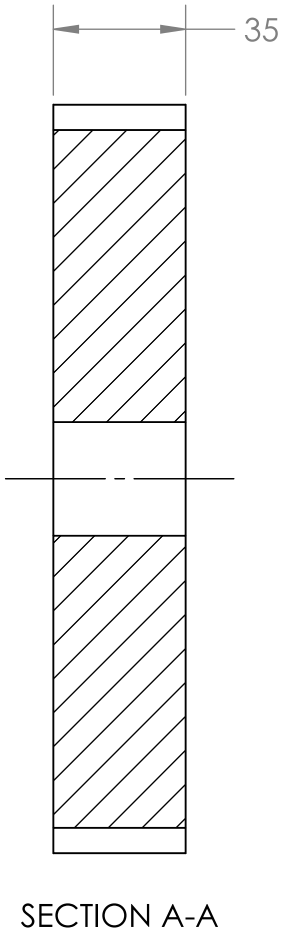
SECTION A-A

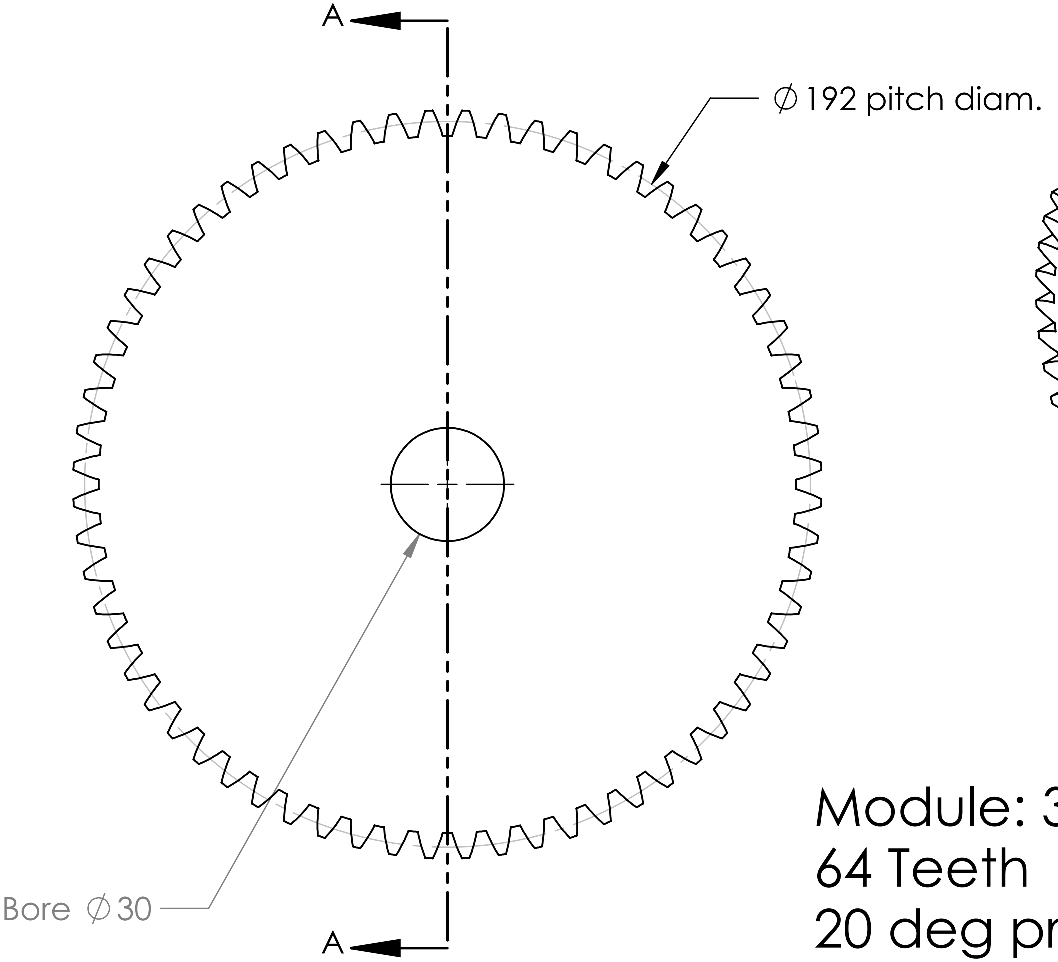


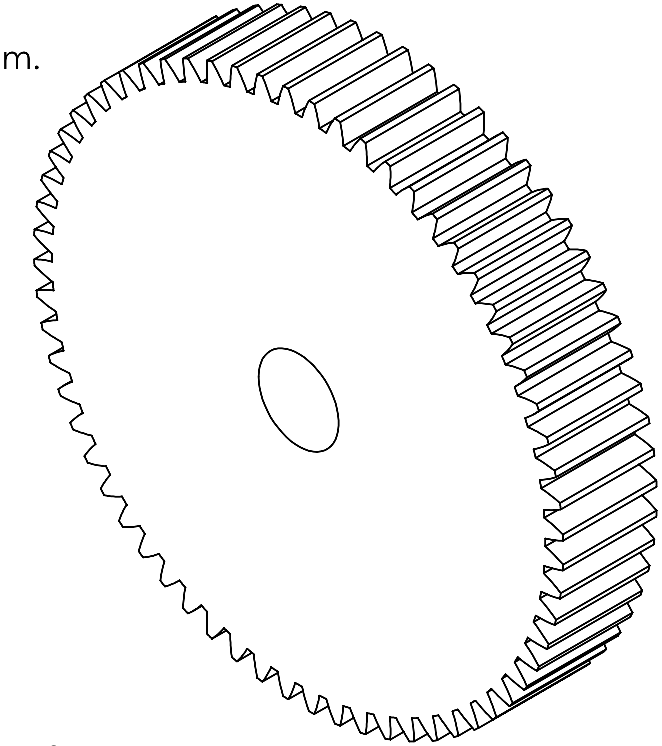


Module: 3mm
20 teeth
20 deg pressure angle

	NAME	SIGNATURE	DATE	DO NOT SCALE DRAWING — REVISION 1
DRAWN	Matthew Phillips			McGill University
APPV'D				TITLE: Gear 4
UNLESS OTHERWISE SPECIFIED: DIMENSIONS ARE IN MILLIMETERS		QUANTITY: 1		
TOLERANCES: LINEAR: +/- 0.005 ANGULAR: +/- 0.25		MATERIAL: Steel A1-A5 2.5% Chrome Nitrided		DWG NO. 3 — A4
				SCALE:1:1 — SHEET 1 OF 1

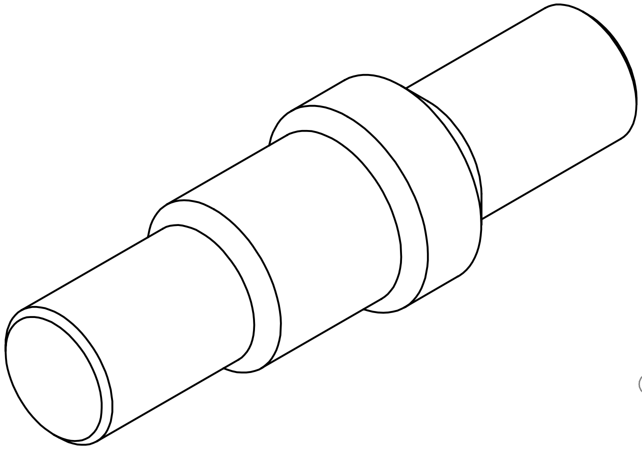


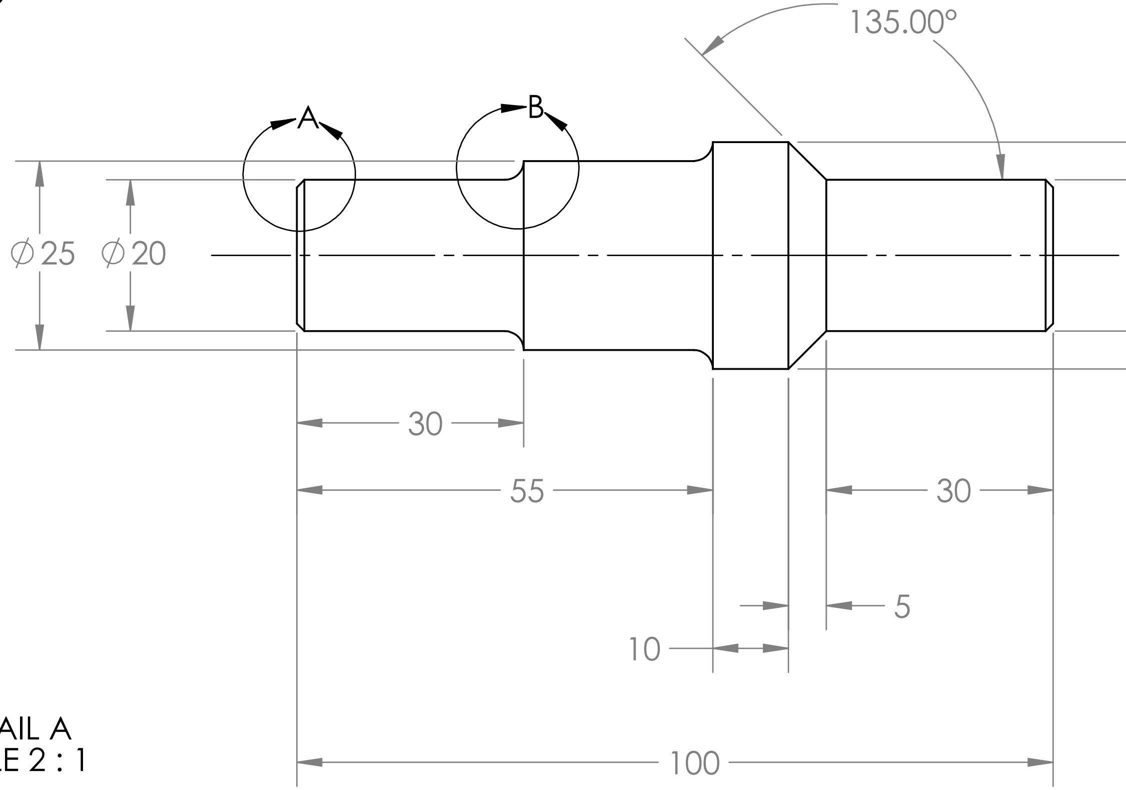




Module: 3mm
64 Teeth
20 deg pressure angle

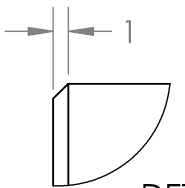
	NAME	SIGNATURE	DATE	DO NOT SCALE DRAWING — REVISION 1
DRAWN	Matthew Phillips			McGill University
APPV'D				TITLE: Gear 5
UNLESS OTHERWISE SPECIFIED: DIMENSIONS ARE IN MILLIMETERS TOLERANCES: LINEAR: +/- 0.005 ANGULAR: +/- 0.25	QUANTITY: 1			DWG NO. 4 — A4
	MATERIAL: Steel A1-A5 2.5% Chrome Nitrided			SCALE:1:2 — SHEET 1 OF 1



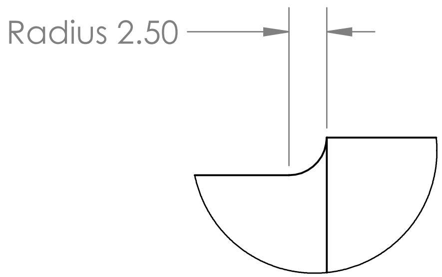


Ends of shaft
have chamfer
1mm, 45 deg.

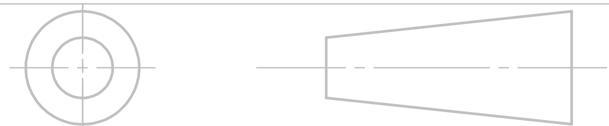
All steps have
2.5mm radius
fillet.



DETAIL A
SCALE 2 : 1



DETAIL B
SCALE 2 : 1



	NAME	SIGNATURE	DATE
DRAWN	Matthew Phillips		
APPV'D			

UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MILLIMETERS

TOLERANCES:
LINEAR: +/- 0.01
ANGULAR: +/- 0.5

QUANTITY: 1

MATERIAL:
AISI 1050 Steel
Quench & Temper @ 400F

DO NOT SCALE DRAWING

REVISION 1

McGill University

TITLE:

Input Shaft

DWG NO. 5

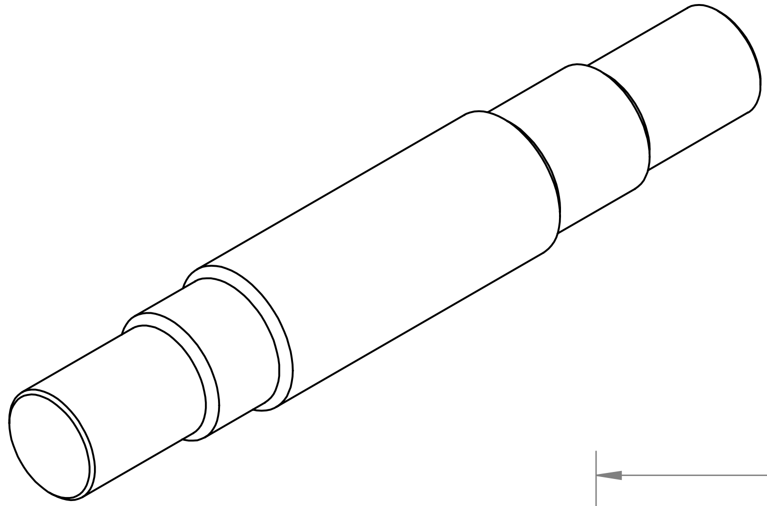
A4

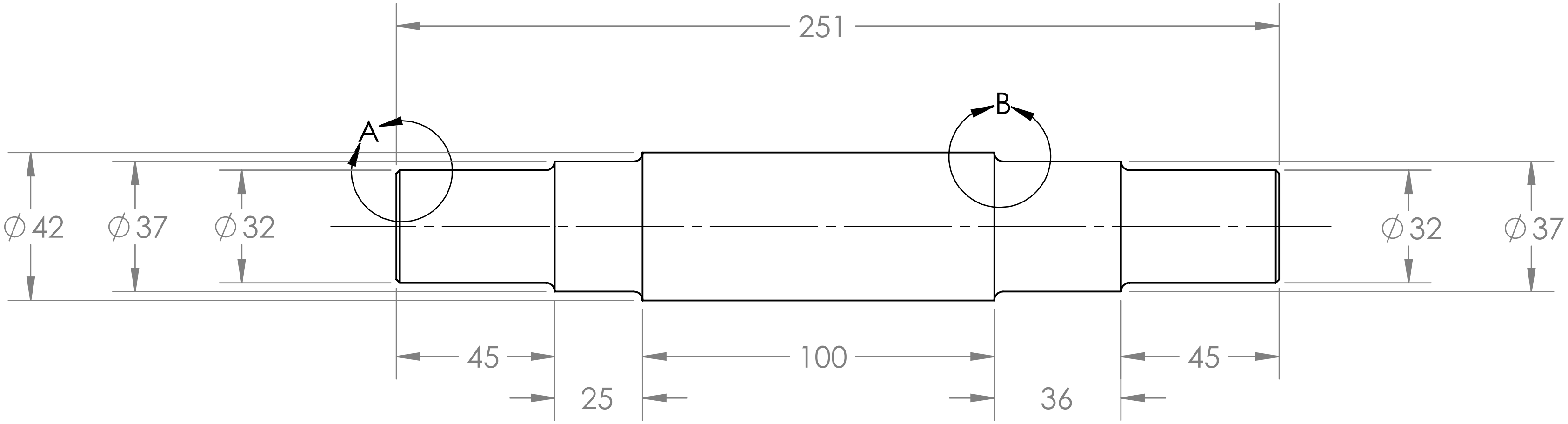
SCALE:1:1

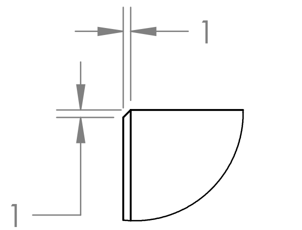
SHEET 1 OF 1

All shaft steps have fillet with radius 2.5mm

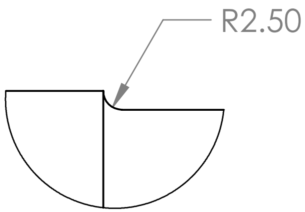
Ends of shaft are chamfered: 1mm, 45 deg







DETAIL A
SCALE 1 : 1



DETAIL B
SCALE 1 : 1

	NAME	SIGNATURE	DATE
DRAWN	Matthew Phillips		
APPV'D			

UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MILLIMETERS

TOLERANCES:
LINEAR: +/- 0.01
ANGULAR: +/- 0.5

QUANTITY: 1

MATERIAL:
AISI 1050 Steel
Quench & Temper @ 400F

DO NOT SCALE DRAWING

REVISION 1

McGill University

TITLE:

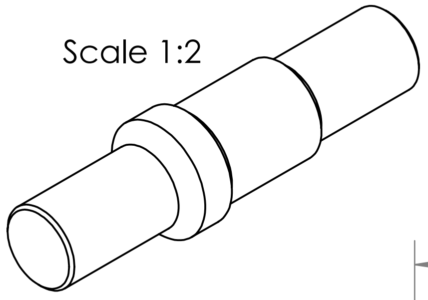
Mid Shaft

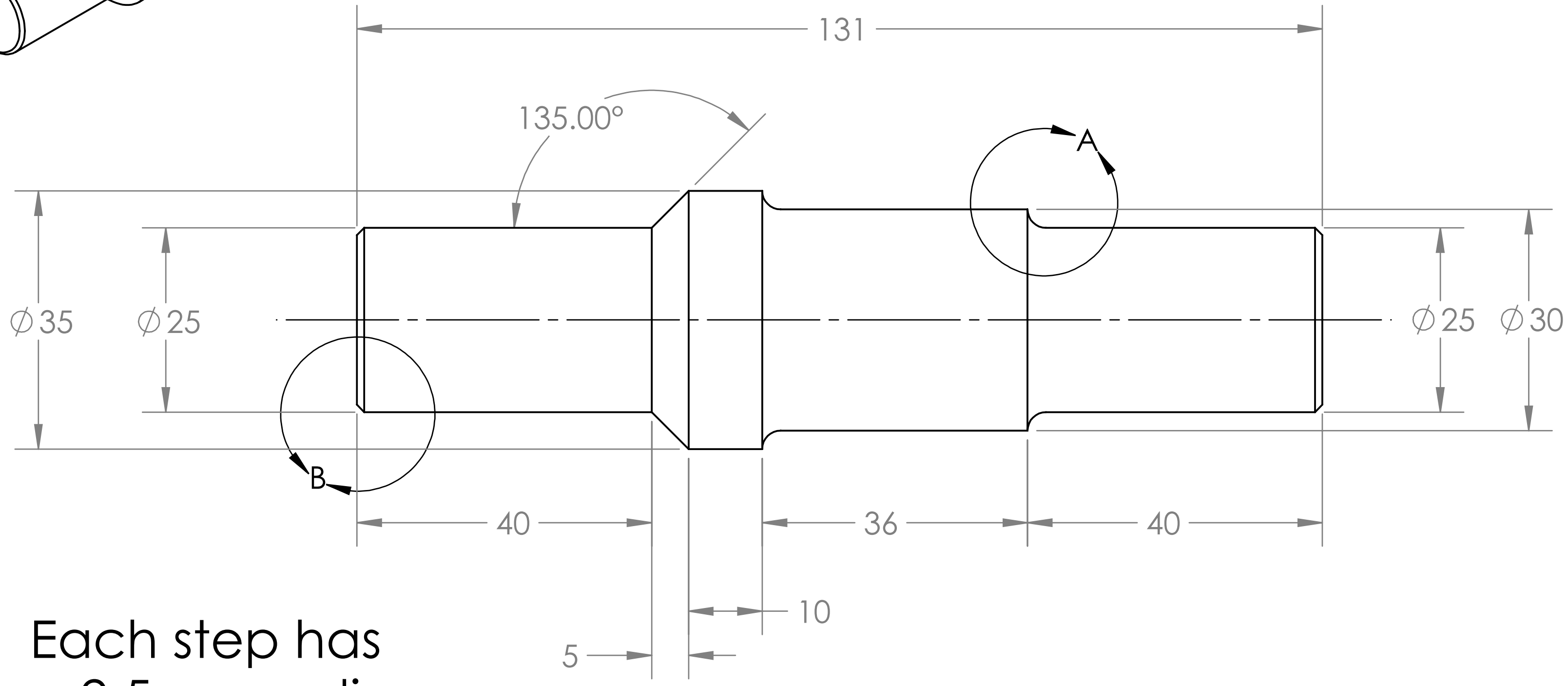
DWG NO. 6

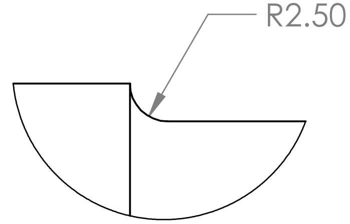
A4

SCALE:1:2

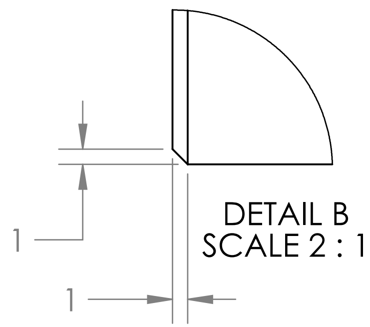
SHEET 1 OF 1







DETAIL A
SCALE 2 : 1



DETAIL B
SCALE 2 : 1

Each step has a 2.5mm radius fillet

Ends of shaft chamfered with 1mm, 45 deg

	NAME	SIGNATURE	DATE
DRAWN	Matthew Phillips		
APPV'D			

UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MILLIMETERS

TOLERANCES:
LINEAR: +/- 0.01
ANGULAR: +/- 0.5

QUANTITY: 1

MATERIAL:
AISI 1050 Steel
Quench & Temper @ 400F

DO NOT SCALE DRAWING

REVISION 1

McGill University

TITLE:

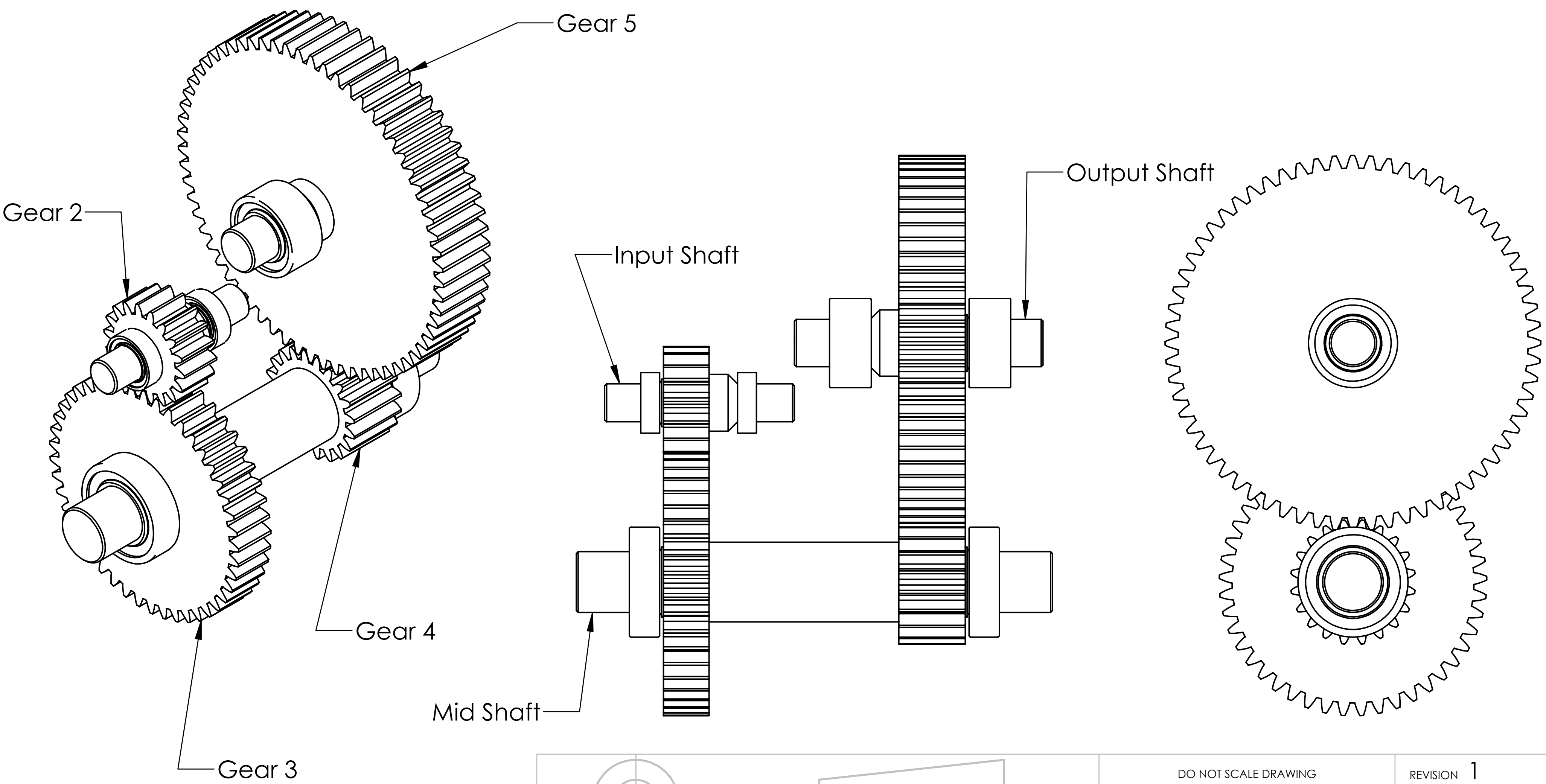
Output Shaft

DWG NO. 7

A4

SCALE:1:1

SHEET 1 OF 1



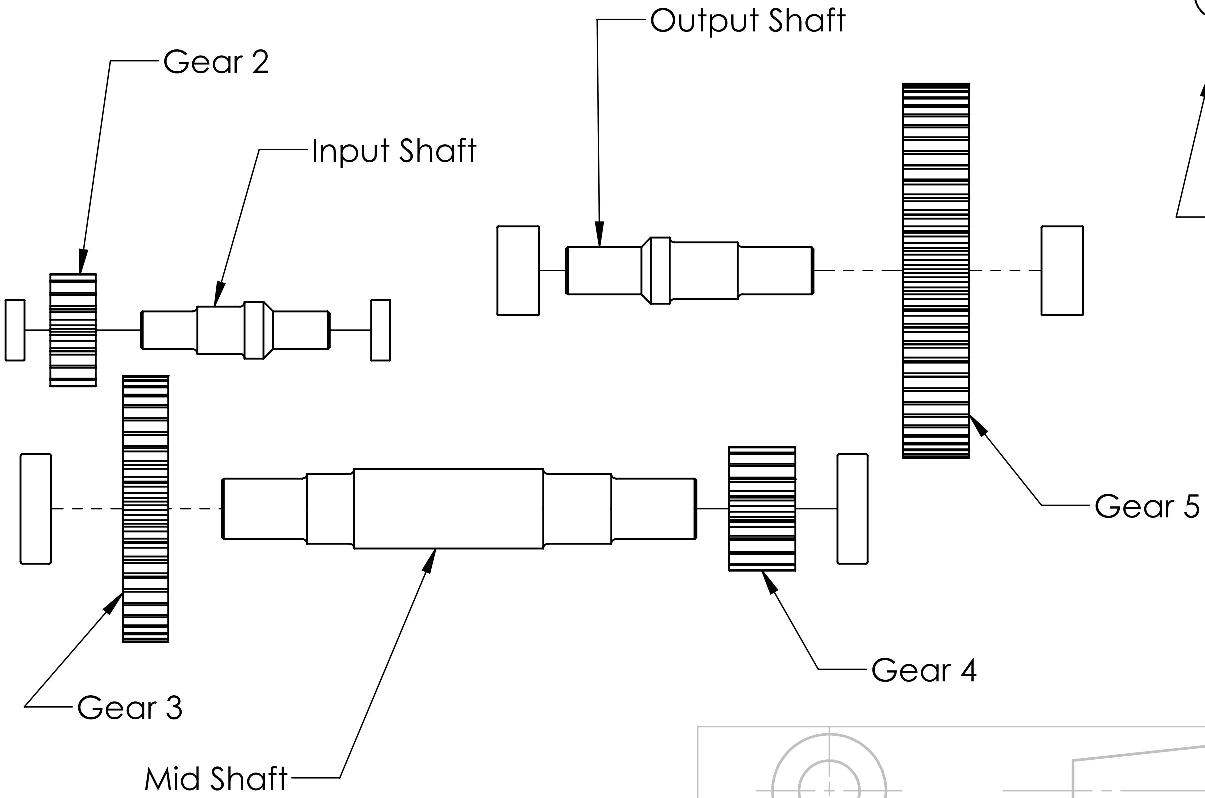
	NAME	SIGNATURE	DATE
DRAWN	Matthew Phillips		
APPV'D			

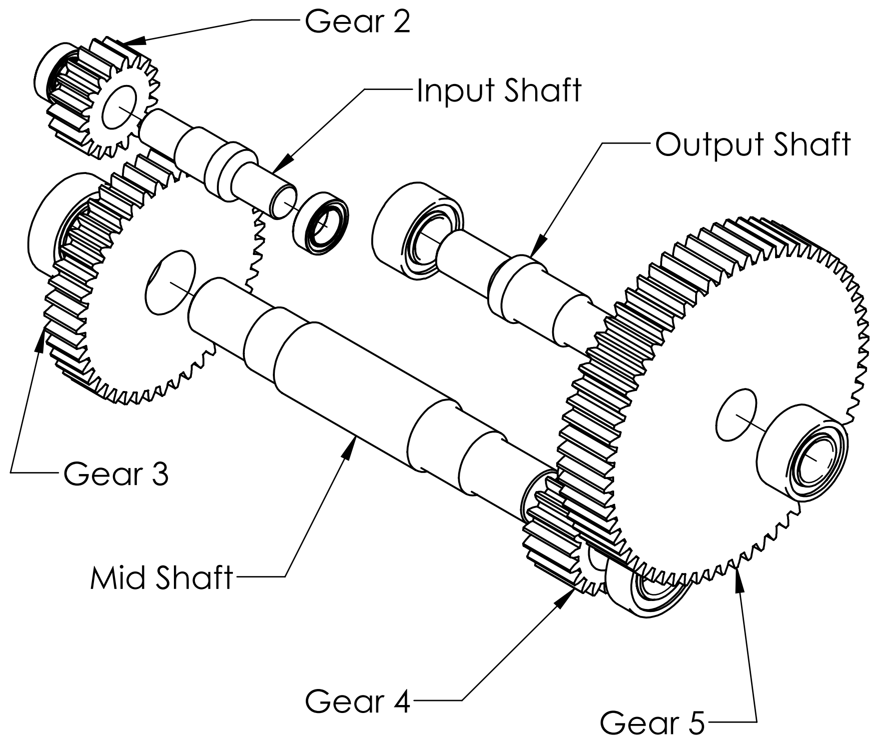
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MILLIMETERS

TOLERANCES:
LINEAR: +/- 0.01
ANGULAR: +/- 0.5

QUANTITY:	1
MATERIAL:	Various

DO NOT SCALE DRAWING	REVISION 1
McGill University	
TITLE:	Gearbox Assembly
DWG NO. 8	A4
SCALE:1:3	SHEET 1 OF 2





			DO NOT SCALE DRAWING	REVISION 1
			McGill University	
	NAME	SIGNATURE	DATE	TITLE: Gearbox Assembly
DRAWN	Matthew Phillips			
APPV'D				
UNLESS OTHERWISE SPECIFIED: DIMENSIONS ARE IN MILLIMETERS TOLERANCES: LINEAR: +/- 0.01 ANGULAR: +/- 0.5		QUANTITY: 1 MATERIAL: Various		DWG NO. 8 — A4
			SCALE:1:4	SHEET 2 OF 2

References

1 Machine Element Design: An Integrated Approach, 5^{th} edition. Robert L Norton. Pearson, 2014.

2 K. Hunecke, *Jet engines: fundamentals of theory, design and operation.* Osceola, WI: Motorbooks International, 2000.

3 D. Jelaska, *Gears and gear drives.* Chichester: John Wiley & Sons, 2012.[3]

4 J. L. Walsh, *Performance optimization of helicopter rotor blades.* Hampton, VA: National Aeronautics and Space Administration, Langley Research Center, 1991.[4]