

# Lab Report: Simulation of Differential Drive Mobile Robot Trajectory Planning

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## Objective

The aim of this lab was to simulate a differential drive mobile robot using matlab. The robot had a goal position and our goal was to generate a smooth trajectory to the goal. The robot was modeled using a set of ordinary differential equations (ODEs) and the control law was based on three error components.

## Methodology

### Differential Equations

We model the robot's dynamics using the following set of ordinary differential equations (ODEs):

$$\begin{aligned}\dot{x} &= v \cos(\theta) \\ \dot{y} &= v \sin(\theta) \\ \dot{\theta} &= \omega\end{aligned}$$

Where  $v$  and  $\omega$  are the linear and angular velocities, respectively, and  $\theta$  is the robot's heading.

### Control

The control law for the robot is based on three error components:

1.  $\rho$  - Distance to the goal
2.  $\alpha$  - Angle to the goal from the current heading
3.  $\beta$  - Angle to the goal

The control law is formulated as:

$$\begin{aligned}v &= k_p * \rho \\ \omega &= k_\alpha * \alpha + k_\beta * \beta\end{aligned}$$

Where  $k_p$ ,  $k_\alpha$ , and  $k_\beta$  are the proportional gains for the three error components.

### Parameters Used

- $r = 50$ : Initial radius of the circle
- $\text{angle} = -\frac{3\pi}{4}$ : Initial angle in radians
- $k_p = 3$ : Proportional Gain for positional error
- $k_\alpha = 8$ : Proportional Gain for angular error  $\alpha$
- $k_\beta = -1.5$ : Proportional Gain for angular error  $\beta$
- $t_{\text{span}} = [0, 10]$ : Time span of the simulation

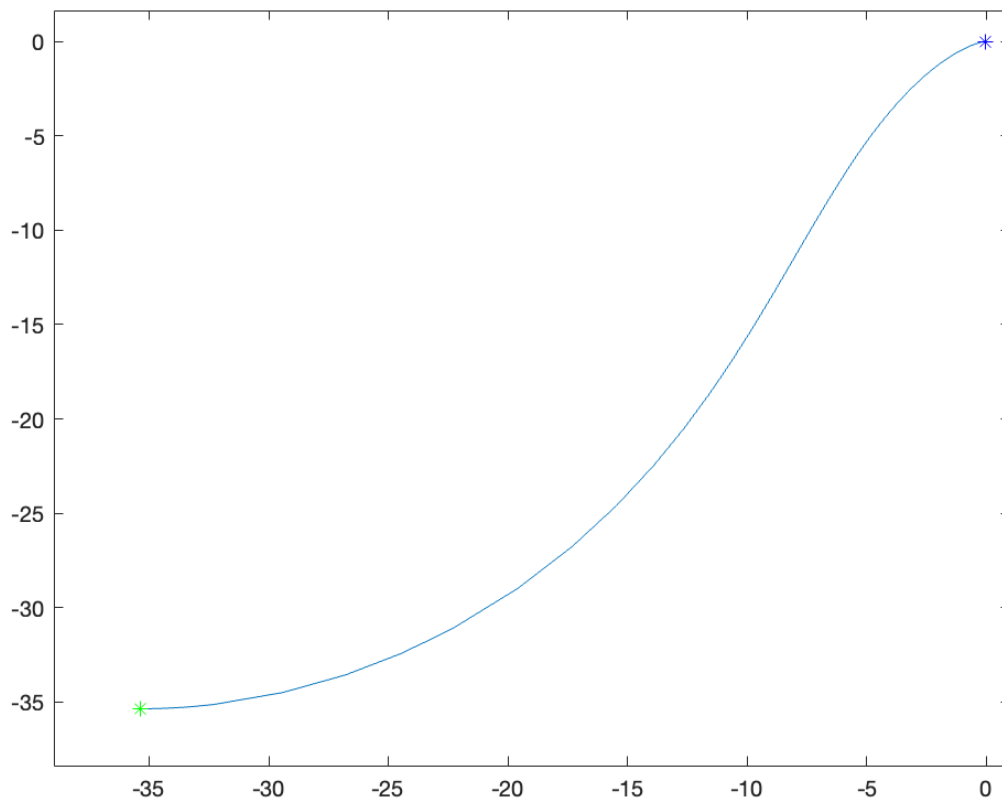
## Implementation

The Matlab code starts by setting the initial position of the robot based on  $r$  and angle. The code uses Matlab's ode45 function to solve the system of ODEs over the time span specified.

The code then plots the trajectory.

## Simulation Results

### Trajectory Plot



### Observations

- The robot starts from the initial point and moves toward the center.
- The trajectory is smooth, which indicates that the control strategy is effective.

## Conclusion

The experiment successfully simulates the path of a differential drive robot moving from an initial position on a circle to a goal position at the center of the circle. The control law effectively guides the robot along a smooth path to the goal.

## Full Code

```
% For a differential drive mobile robot as shown in the picture,  
program in Matlab to simulate the paths shown in Figure 3.20 of the  
Textbook, where the robot is initially on a circle in the xy plane.  
All movements should have smooth trajectories toward the goal in the  
center.
```

```
% Initial Position  
r = 50  
angle = -3*pi/4  
x_0 = r*cos(angle)  
y_0 = r*sin(angle)
```

```
theta_0 = 0
X0 = [x_0; y_0; theta_0]

% Solve ODE
tspan = [0, 10]
[t, X] = ode45(@f, tspan, X0)

% Plot
plot(X(:,1), X(:,2))

hold on
% Put a thing at the initial point
plot(x_0, y_0, 'g*')
hold on
% Put a thing at the last point
plot(X(end,1), X(end,2), 'b*')
hold off

% ODE Function for xdot
function xdot = f(t, X) % t is time, X is state vector
x = X(1)
y = X(2)
theta = X(3)

% Parameters
k_p = 3 % Proportional Gain for positional error
k_alpha = 8 % Proportional Gain for angular error
k_beta = -1.5 % Proportional Gain for angular error

% Goal
x_f = 0
y_f = 0
theta_f = 0

rho = sqrt((x_f - x)^2 + (y_f - y)^2) % Distance to goal

alpha = atan2(y_f - y, x_f - x) - theta % Angle to goal from current heading
beta = -alpha - theta % Angle to goal

% Control Law
v = k_p*rho
omega = k_alpha*alpha + k_beta*beta

% Define xdot
xdot = [v*cos(theta); v*sin(theta); omega]
end
```