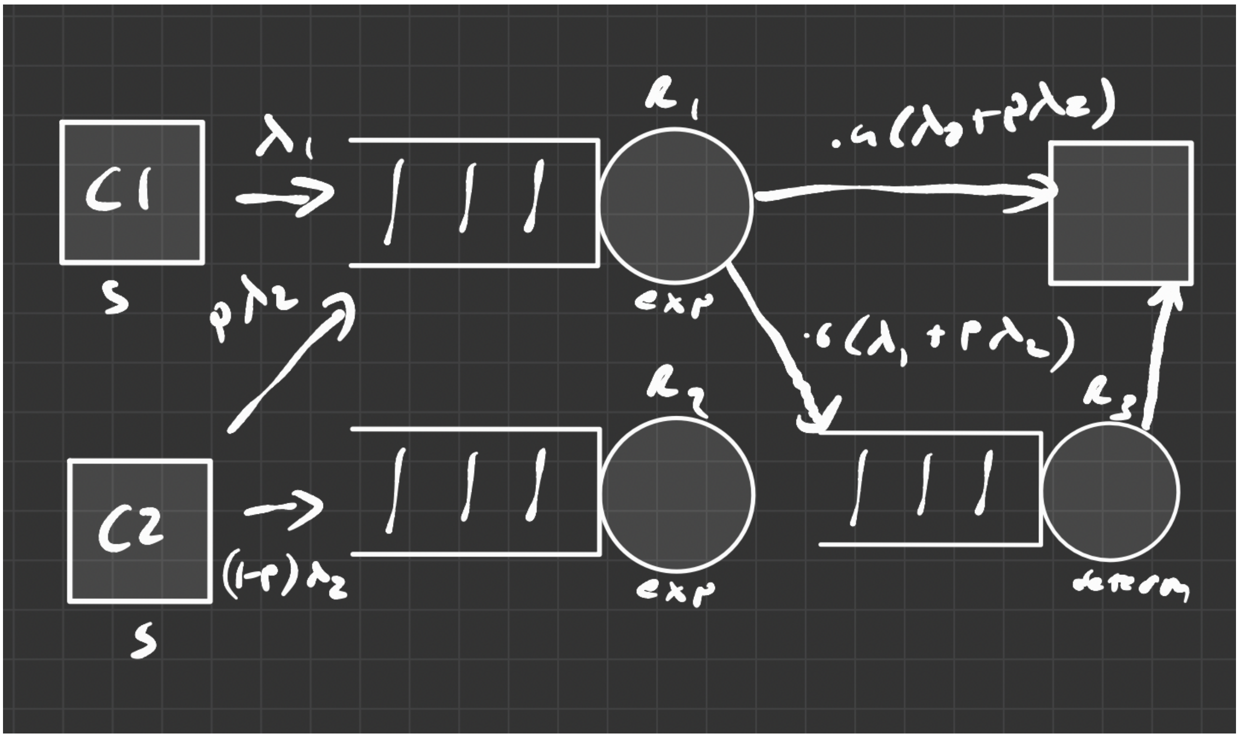
Network of Queues

Block Diagram



Types of Queues

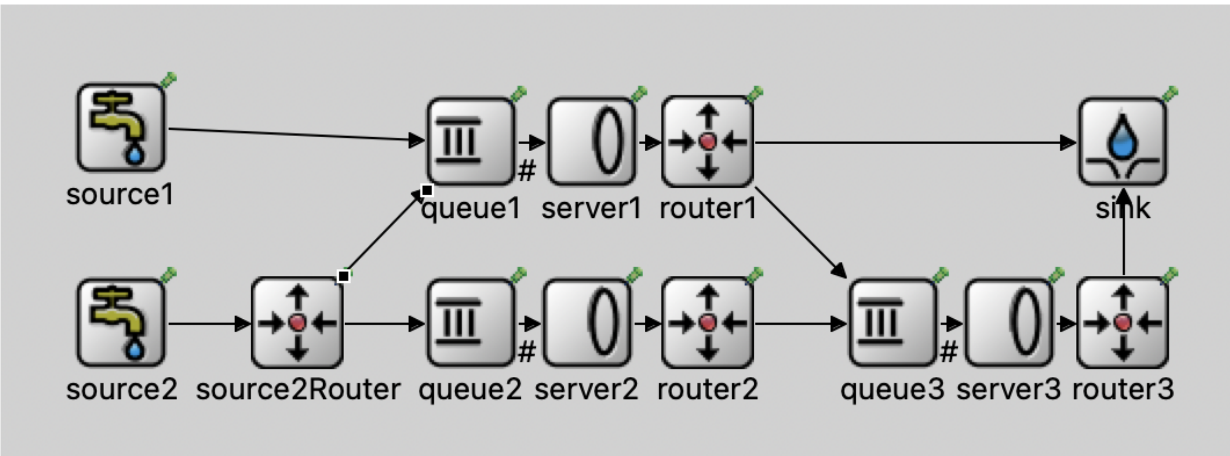
- 2x MM1 Queues
- 1x MD1 Queue

Stability

Yes the queue is stable because for all routers, $\rho < 1$, or $\lambda < \mu$.

The calculations are shown below.

Node Structure



Long-Run Average Time Spent in System per Customer

Router 1

For a M/G/1 Queue we can use the following formula to calculate the long-run average time spent in system per customer

$$w \approx \frac{1}{\mu} + \frac{\lambda\left(\frac{1}{\mu^2} + \sigma^2\right)}{2\,(1-\rho)}$$

Where:

- μ is the service rate
- λ is the arrival rate
- ρ is the utilization
- σ^2 is the variance of the service time

We know:

$$\mu = 4$$

$$\rho = \frac{\lambda}{\mu}$$

From the diagram, we can see router 1 is effected by both source 1 and 2. So the arrival rate is

$$\lambda = \lambda_1 + p * \lambda_2 = 1 + 0.2 * 2 = 1.4$$

Because it is an exponential distribution, the variance is equal to one over the square of the mean.

$$\sigma^2 = \frac{1}{\mu^2} = 1/16$$

So the formula becomes:

$$w \approx \frac{1}{4} + \frac{1.4\left(\frac{1}{16} + \frac{1}{16}\right)}{2\,(1 - \frac{1.4}{4})} \approx 0.385$$

This is close to the actual value of **0.363145** that was outputted from the **cHistogram** module.

```
(cHistogram) count=1199988 mean=0.363145 stddev=0.364883 min=6.345e-09 max=5.87011
```

Router 2

The same concepts can be applied to the other routers. I won't waste time explaining the same thing again.

- $\lambda = 0.8 * 2$
- $\mu = 4$
- $\rho = \frac{\lambda}{\mu} = \frac{2}{4} = 0.5$
- $\sigma^2 = \frac{1}{\mu^2} = \frac{1}{16}$

$$w \approx \frac{1}{4} + \frac{1.6\left(\frac{1}{16} + \frac{1}{16}\right)}{2\,(1 - \frac{1.6}{4})} \approx 0.417$$

Actual Value = 0.41683

```
(cHistogram) count=800012 mean=0.41683 stddev=0.416391 min=4.55766e-07 max=6.37963
```

Router 3

Router 3 recieves input from two source. 60% of Router 1's output and all of Router 2's output.

- $\lambda = 0.6 * (\lambda_1 + p * \lambda_2) + (1 - p) * \lambda_2 = 0.6 * (1 + 0.2 * 2) + (1 - 0.2) * 2 = 2.44$
- $\mu = 4$
- $\rho = \frac{\lambda}{\mu} = \frac{2.44}{4}$
- $\sigma^2 = 0$ (because it's a deterministic service time)

$$w_3 \approx \frac{1}{4} + \frac{2.44\,(\frac{1}{16})}{2\,(1 - \frac{2.44}{4})} \approx 0.446$$

This value of w does not take in account the time spent in the queue from the prior routers. So we need to add it to the weighted average of the w values from the prior routers.

$$w = (0.6 * (1 + 0.2 * 2)/2.44) * 0.385 + ((1 - 0.2) * 2/2.44) * .417 + w_3 \approx 0.823$$

Actual Value = 0.802755

```
(cHistogram) count=1519950 mean=0.802755 stddev=0.470071 min=0.25 max=6.62963
```

Entire System

The sink is used to collect the average delay for the entire system.

It recieve input from Router's 1 and 3. So the total delay is a weighted sum of the delays from each router.

$$\lambda_{sink} = 0.4\lambda_{R1} + \lambda_{R3}$$

$$w_{sink} = \frac{0.4\lambda_{R1} w_{R1} + \lambda_{R3} w_{R3}}{\lambda_{sink}}$$

$$\lambda_{R1} = \lambda_1 + p * \lambda_2 = 1 + 0.2 * 2 = 1.4$$

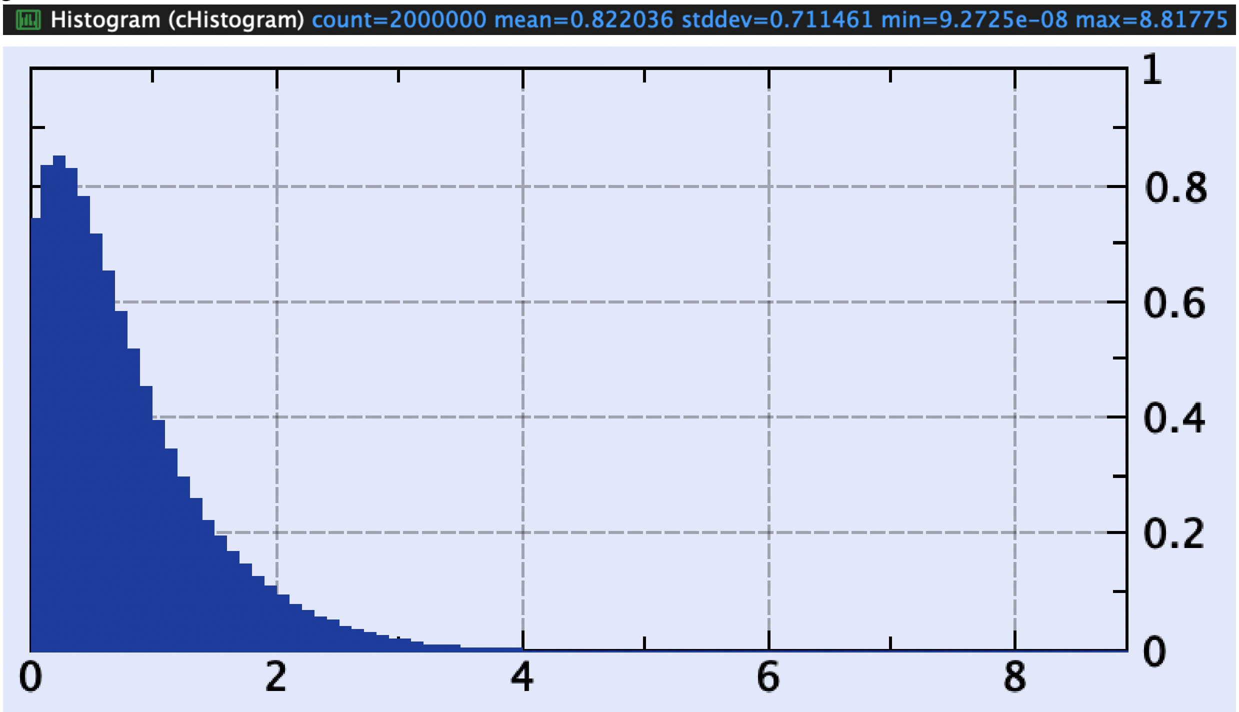
$$\lambda_{R3} = 0.6 * (\lambda_1 + p * \lambda_2) + (1 - p) * \lambda_2 = 0.6 * (1 + 0.2 * 2) + (1 - 0.2) * 2 = 2.44$$

- $w_{R1} = 0.385$
- $w_{R3} = 0.823$

This calculates to:

$$w_{sink} = \frac{0.4 * 1.4 * 0.385 + 2.44 * 0.823}{1.4 * 0.4 + 2.44} \approx 0.741$$

Actual Value = 0.822



Code

The code can be found at my GitHub repo.