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Assignment 5

ISSP 2160
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Problem 1.

Part a. See Figure 1 for truth table.

Part b. Proof by rules of inference:

Proof.

1. $U \Rightarrow (\neg T \Rightarrow (\neg S \wedge P))$ (Given)
2. $\neg U \vee (\neg T \Rightarrow (\neg S \wedge P))$ (Conditional equivalence, from 1)
3. $\neg U \vee \neg\neg T \vee (\neg S \wedge P)$ (Conditional equivalence, from 2)
4. $\neg U \vee T \vee (\neg S \wedge P)$ (Double negation, from 3)
5. $\neg U \vee (\neg S \wedge P) \vee T$ (Commutative property, from 4)
6. $\neg(T \vee Q)$ (Given)
7. $\neg T \wedge \neg Q$ (De Morgan's law, from 6)
8. $\neg T$ (And-elimination, from 7)
9. $\neg Q$ (And-elimination, from 7)
10. $\neg U \vee (\neg S \wedge P)$ (Unit resolution, from 5, 8)
11. $\neg(P \wedge \neg Q) \vee \neg(\neg S \wedge \neg T)$ (Given)
12. $\neg(P \wedge \neg Q) \vee \neg\neg S \vee \neg\neg T$ (De Morgan's law, from 11)
13. $\neg(P \wedge \neg Q) \vee \neg\neg S \vee T$ (Double negation, from 12)
14. $\neg(P \wedge \neg Q) \vee S \vee T$ (Double negation, from 13)
15. $\neg P \vee \neg\neg Q \vee S \vee T$ (De Morgan's law, from 14)
16. $\neg P \vee Q \vee S \vee T$ (Double negation, from 15)
17. $\neg P \vee Q \vee S$ (Unit resolution, from 8, 16)
18. $\neg P \vee S \vee Q$ (Commutative property, from 17)
19. $\neg P \vee S$ (Unit resolution, from 9, 18)
20. $S \vee \neg P$ (Commutative property, from 19)
21. $\neg\neg S \vee \neg P$ (Double negation, from 20)
22. $\neg(\neg S \wedge P)$ (De Morgan's law, from 21)
23. $\neg U$ (Unit resolution, from 10, 22)

□

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						KB_1	KB_2	KB_3	α
P	Q	R	S	T	U	$\neg(P \wedge \neg Q) \vee \neg(\neg S \wedge T)$	$\neg(T \vee Q)$	$U \Rightarrow (\neg T \rightarrow (\neg S \wedge P))$	$\neg U$
						true	***true***	***true***	***true***
					true	*true*	*true*		
				true		*true*		*true*	*true*
				true	*true*	*true*		*true*	
			true			***true***	***true***	***true***	***true***
			true		*true*	*true*	*true*		
			true	*true*		*true*		*true*	*true*
			true	*true*	*true*	*true*		*true*	
		true				***true***	***true***	***true***	***true***
		true			*true*	*true*	*true*		
		true		*true*		*true*		*true*	*true*
		true		*true*	*true*	*true*		*true*	
		true	***true***			***true***	***true***	***true***	***true***
		true	*true*		*true*	*true*	*true*		
		true	*true*	*true*		*true*		*true*	*true*
		true	*true*	*true*	*true*	*true*		*true*	
	true					*true*		*true*	*true*
	true				*true*	*true*			
	true			*true*		*true*		*true*	*true*
	true			*true*	*true*	*true*		*true*	
	true		*true*			*true*		*true*	*true*
	true		*true*		*true*	*true*			
	true		*true*	*true*		*true*		*true*	*true*
	true		*true*	*true*	*true*	*true*		*true*	
	true	*true*				*true*		*true*	*true*
	true	*true*			*true*	*true*			
	true	*true*		*true*		*true*		*true*	*true*
	true	*true*		*true*	*true*	*true*		*true*	
	true	*true*	*true*			*true*		*true*	*true*
	true	*true*	*true*		*true*	*true*			
	true	*true*	*true*	*true*		*true*		*true*	*true*
	true	*true*	*true*	*true*	*true*	*true*		*true*	
true							*true*	*true*	*true*
true					*true*		*true*	*true*	
true				*true*		*true*		*true*	*true*
true				*true*	*true*	*true*		*true*	
true			***true***			***true***	***true***	***true***	***true***
true			*true*		*true*	*true*	*true*		
true			*true*	*true*		*true*		*true*	*true*
true			*true*	*true*	*true*	*true*		*true*	
true		*true*					*true*	*true*	*true*
true		*true*			*true*		*true*	*true*	
true		*true*		*true*		*true*		*true*	*true*
true		*true*		*true*	*true*	*true*		*true*	
true		***true***	***true***			***true***	***true***	***true***	***true***
true		*true*	*true*		*true*	*true*	*true*		
true		*true*	*true*	*true*		*true*		*true*	*true*
true		*true*	*true*	*true*	*true*	*true*		*true*	
true	*true*					*true*		*true*	*true*
true	*true*				*true*	*true*		*true*	
true	*true*			*true*		*true*		*true*	*true*
true	*true*			*true*	*true*	*true*		*true*	
true	*true*		*true*			*true*		*true*	*true*
true	*true*		*true*		*true*	*true*			
true	*true*		*true*	*true*		*true*		*true*	*true*
true	*true*		*true*	*true*	*true*	*true*		*true*	
true	*true*	*true*				*true*		*true*	*true*
true	*true*	*true*			*true*	*true*		*true*	
true	*true*	*true*		*true*		*true*		*true*	*true*
true	*true*	*true*		*true*	*true*	*true*		*true*	
true	*true*	*true*	*true*			*true*		*true*	*true*
true	*true*	*true*	*true*		*true*	*true*			
true	*true*	*true*	*true*	*true*		*true*		*true*	*true*
true	*true*	*true*	*true*	*true*	*true*	*true*		*true*	

Figure 1: Proof by truth table. Empty cells represent false values. The interpretations that are models of the knowledge base are highlighted in bold. Since the theorem is true for every model of the knowledge base, the knowledge base entails the theorem.

Part c. Proof by resolution with refutation:

Proof.

1. $U \Rightarrow (\neg T \Rightarrow (\neg S \wedge P))$ (Given)
2. $\neg U \vee (\neg T \Rightarrow (\neg S \wedge P))$ (Conditional equivalence, from 1)
3. $\neg U \vee \neg\neg T \vee (\neg S \wedge P)$ (Conditional equivalence, from 2)
4. $\neg U \vee T \vee (\neg S \wedge P)$ (Double negation, from 3)
5. $(\neg U \vee T \vee \neg S) \wedge (\neg U \vee T \vee P)$ (Distributive property, from 4)
6. $\neg(T \vee Q)$ (Given)
7. $\neg T \wedge \neg Q$ (De Morgan's law, from 6)
8. $\neg(P \wedge \neg Q) \vee \neg(\neg S \wedge \neg T)$ (Given)
9. $\neg(P \wedge \neg Q) \vee \neg\neg S \vee \neg\neg T$ (De Morgan's law, from 8)
10. $\neg(P \wedge \neg Q) \vee \neg\neg S \vee T$ (Double negation, from 9)
11. $\neg(P \wedge \neg Q) \vee S \vee T$ (Double negation, from 10)
12. $\neg P \vee \neg\neg Q \vee S \vee T$ (De Morgan's law, from 11)
13. $\neg P \vee Q \vee S \vee T$ (Double negation, from 12)
14. $(\neg U \vee T \vee \neg S) \wedge (\neg U \vee T \vee P) \wedge \neg T \wedge \neg Q \wedge (\neg P \vee Q \vee S \vee T)$ (And-introduction, from 5, 7, 13)
15. U (Assumption)
16. $(\neg U \vee T \vee \neg S) \wedge (T \vee P) \wedge \neg T \wedge \neg Q \wedge (\neg P \vee Q \vee S \vee T)$ (Unit resolution, from 14, 15)
17. $(T \vee \neg S) \wedge (T \vee P) \wedge \neg T \wedge \neg Q \wedge (\neg P \vee Q \vee S \vee T)$ (Unit resolution, from 15, 16
18. $\neg T$ (And-elimination, from 17)
19. $(T \vee \neg S) \wedge (T \vee P) \wedge \neg T \wedge \neg Q \wedge (\neg P \vee Q \vee S)$ (Unit resolution, from 17, 18)
20. $(T \vee \neg S) \wedge P \wedge \neg T \wedge \neg Q \wedge (\neg P \vee Q \vee S)$ (Unit resolution, from 18, 19)
21. $\neg S \wedge P \wedge \neg T \wedge \neg Q \wedge (\neg P \vee Q \vee S)$ (Unit resolution, from 18, 20)
22. $\neg S$ (And-elimination, from 21)
23. $\neg S \wedge P \wedge \neg T \wedge \neg Q \wedge (\neg P \vee Q)$ (Unit resolution, from 21, 22)
24. $\neg Q$ (And-elimination, from 23)
25. $\neg S \wedge P \wedge \neg T \wedge \neg Q \wedge \neg P$ (Unit resolution, from 23, 24)
26. $P \wedge \neg P$ (And-elimination, from 25)
27. $\neg U$ (Contradiction, from 15, 26)

$\square$

Problem 2.

Part a. Let Y represent the proposition, "The unicorn is mythical."
Let O represent the proposition, "The unicorn is mortal."
Let M represent the proposition, "The unicorn is a mammal."
Let H represent the proposition, "The unicorn is horned."
Let G represent the proposition, "The unicorn is magical."

The provided knowledge base can then be represented in propositional logic as:

$$
\begin{aligned}
Y &\Rightarrow \neg O \\
\neg Y &\Rightarrow (O \land M) \\
(\neg O \lor M) &\Rightarrow H \\
H &\Rightarrow G
\end{aligned}
$$

Part b. Proof by resolution that the knowledge base does not entail that the unicorn is mythical, i.e. that no contradictions result from assuming the opposite:

Proof.

1. $Y \Rightarrow \neg O$ (Given)
2. $\neg Y \lor \neg O$ (Conditional equivalence, from 1)
3. $\neg Y \Rightarrow (O \land M)$ (Given)
4. $\neg\neg Y \lor (O \land M)$ (Conditional equivalence, from 3)
5. $Y \lor (O \land M)$ (Double negation, from 4)
6. $(Y \lor O) \land (Y \lor M)$ Distributive property, from 5)
7. $(\neg O \lor M) \Rightarrow H$ (Given)
8. $\neg(\neg O \lor M) \lor H$ (Conditional equivalence, from 7)
9. $(\neg\neg O \land \neg M) \lor H$ (De Morgan's law, from 8)
10. $(O \land \neg M) \lor H$ (Double negation, from 9)
11. $(O \lor H) \land (\neg M \lor H)$ (Distributive property, from 10)
12. $H \Rightarrow G$ (Given)
13. $\neg H \lor G$ (Conditional equivalence, from 12)
14. $(\neg Y \lor \neg O) \land (Y \lor O) \land (Y \lor M) \land$
$(O \lor H) \land (\neg M \lor H) \land (\neg H \lor G)$ (And-introduction, from 2, 6, 11, 13)
15. $\neg Y \lor \neg O$ (And-elimination, from 14)
16. $Y \lor O$ (And-elimination, from 14)

17. $Y \lor M$ (And-elimination, from 14)
18. $O \lor H$ (And-elimination, from 14)
19. $\neg M \lor H$ (And-elimination, from 14)
20. $\neg H \lor G$ (And-elimination, from 14)
21. $\neg Y \lor Y$ (Resolution, from 15, 16)
22. $\neg O \lor O$ (Resolution, from 15, 16)
23. $\neg O \lor M$ (Resolution, from 15, 17)
24. $\neg Y \lor H$ (Resolution, from 15, 18)
25. $Y \lor H$ (Resolution, from 17, 19)
26. $M \lor H$ (Resolution, from 17, 24)
27. $O \lor G$ (Resolution, from 18, 20)
28. $\neg M \lor G$ (Resolution, from 19, 20)
29. $H \lor \neg O$ (Resolution, from 19, 23)
30. $H \lor H$ (Resolution, from 19, 26)
31. $G \lor \neg Y$ (Resolution, from 20, 24)
32. $G \lor Y$ (Resolution, from 20, 25)
33. $G \lor M$ (Resolution, from 20, 26)
34. $G \lor \neg O$ (Resolution, from 20, 29)
35. $G \lor H$ (Resolution, from 20, 30)
36. $G \lor G$ (Resolution, from 27, 34)
37. H (Idempotence, from 30)
38. G (Idempotence, from 36)
39. $\neg Y$ (Assumption)
40. O (Unit resolution, from 16, 39)
41. M (Unit resolution, from 17, 39)
42. $KB \not\models Y$ (Resolution without contradiction, from 14, 39)

$\square$

Part c. Proof that the unicorn is magical:

Proof.

1. $Y \Rightarrow \neg O$ (Given)
2. $\neg Y \vee \neg O$ (Conditional equivalence, from 1)
3. $\neg Y \Rightarrow (O \wedge M)$ (Given)
4. $\neg\neg Y \vee (O \wedge M)$ (Conditional equivalence, from 3)
5. $Y \vee (O \wedge M)$ (Double negation, from 4)
6. $\neg O \vee (O \wedge M)$ (Resolution, from 2, 5)
7. $(\neg O \vee O) \wedge (\neg O \vee M)$ (Distributive property, from 6)
8. $\neg O \vee M$ (And-elimination, from 7)
9. $(\neg O \vee M) \Rightarrow H$ (Given)
10. H (Modus ponens, from 8, 9)
11. $H \Rightarrow G$ (Given)
12. G (Modus ponens, from 10, 11)

□

Part d. Proof that the unicorn is horned:

Proof.

1. $Y \Rightarrow \neg O$ (Given)
2. $\neg Y \vee \neg O$ (Conditional equivalence, from 1)
3. $\neg Y \Rightarrow (O \wedge M)$ (Given)
4. $\neg\neg Y \vee (O \wedge M)$ (Conditional equivalence, from 3)
5. $Y \vee (O \wedge M)$ (Double negation, from 4)
6. $\neg O \vee (O \wedge M)$ (Resolution, from 2, 5)
7. $(\neg O \vee O) \wedge (\neg O \vee M)$ (Distributive property, from 6)
8. $\neg O \vee M$ (And-elimination, from 7)
9. $(\neg O \vee M) \Rightarrow H$ (Given)
10. H (Modus ponens, from 8, 9)

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Problem 3.

Proof.

1. The animal gives milk. (Given, fact)
2. If the animal gives milk then it is a mammal. (Given, rule 2)
3. The animal is a mammal. (Modus ponens, from 1, 2)
4. The animals chews cud. (Given, fact)
5. The animal is a mammal and it chews cud. (And-introduction, from 3, 4)
6. If the animal is a mammal and it chews cud then it is an ungulate. (Given, rule 8)
7. The animal is an ungulate. (Modus ponens, from 5, 6)
8. The animal has long legs. (Given, fact)
9. The animal has a long neck. (Given, fact)
10. The animal has a tawny color. (Given, fact)
11. The animal has dark spots. (Given, fact)
12. The animal is an ungulate and it has long legs and it has a long neck and it has a tawny color and it has dark spots. (And-introduction, from 7-11)
13. If the animal is an ungulate and it has long legs and it has a long neck and it has a tawny color and it has dark spots then it is a giraffe. (Given, rule 12)
14. The animal is a giraffe. (Modus ponens, from 12, 13)
15. The animal is not a penguin. (Assumption)
16. The knowledge base does not entail that the animal is a penguin. (Resolution without contradiction, from 15)
17. The animal is a penguin. (Assumption)
18. The knowledge base does not entail that the animal is not a penguin. (Resolution without contradiction, from 17)

□

As can be seen from the above proof, both Theorem1 ("the animal is a giraffe") and Theorem3 ("the animal is a mammal") can be derived from the provided facts using the sequence of rules 2, 8, and 12. While it might seem like we should be able to disprove Theorem3 ("the animal is a penguin") based on these facts as well, that is not the case without additional facts and rules. For instance, there is no rule in the knowledge base explicitly stating that an animal cannot be both a mammal and a bird, or that an animal can only have a tawny color or be black and white. There isn't even a rule stating than an animal cannot be both a giraffe and a penguin. These rely on common sense not present in the knowledge base. To check whether the knowledge base entails whether or not the animal is a penguin, I added assumptions to the end of the proof, and found that there were no resolution rules that could be applied to generate additional facts. Since no contradictions were reached by assuming that the animal was or was not a penguin, we can only conclude that the knowledge base does not entail whether the animal is a penguin or not.