Matthew Ragoza
mtr22@pitt.edu

Assignment 11

Problem 1. Investment decision under uncertainty

a. The decision tree for maximizing the expected monetary value of the $10,000 investment is shown in Figure 1. The optimal decision is to invest the money in the stock market. This is the optimal decision because it is the action that maximizes the expected value of the investment, taking into account the two different investment periods and the uncertainty present in the stock movements in each period. The expected value of investing in stocks is $11,280, while the expected value of saving in the bank is $10,500. In other words, in the two-step decision tree, the action that can be taken from the root node with the highest expected value is investing in the stock market instead of the bank.

b. The decision tree for maximizing the expected utility of the $10,000 investment, given the provided utility function, is shown in Figure 2. Again, the optimal decision is to invest the money in the stock market, because the action that can be taken from the root node of the decision tree with the highest expected utility is investing in stocks.

c. Even though the optimal decision is the same in parts a. and b., an investor with the provided utility function is risk averse due to the concave shape of the function. The expected utility of an uncertain lottery is computed as a convex combination of the utilities of the possible outcomes. Since the utility function is concave, the expected utility of the lottery will always be less than the utility of the expected value of the lottery. Therefore, the investor prefers uncertain lotteries less than deterministic outcomes with the same expected value, indicating risk aversion.

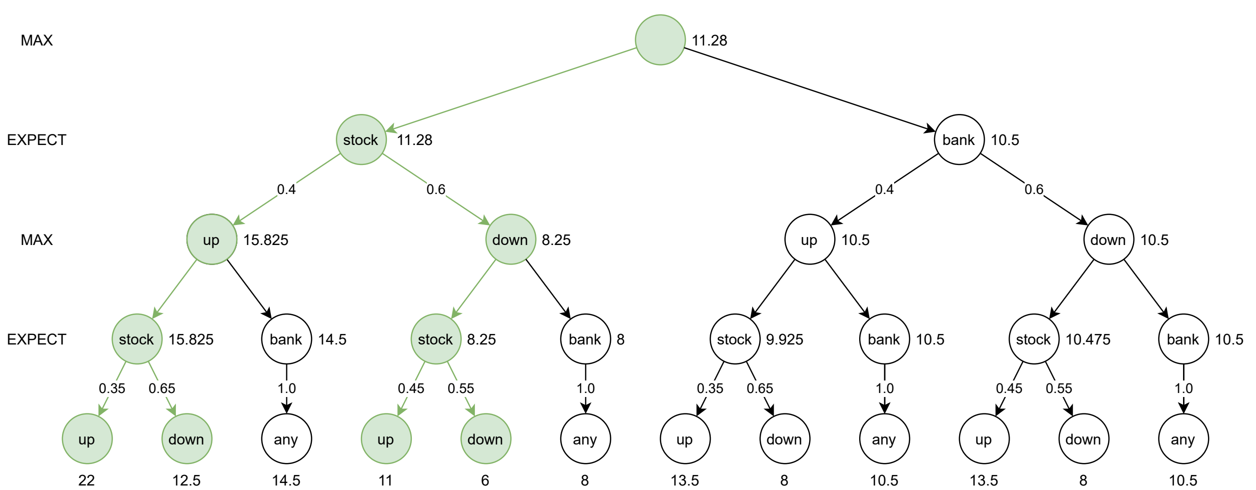

Figure 1: The decision tree for maximizing the expected value of a sum of money through two investment periods. The optimal decision sub-tree is shown in green.

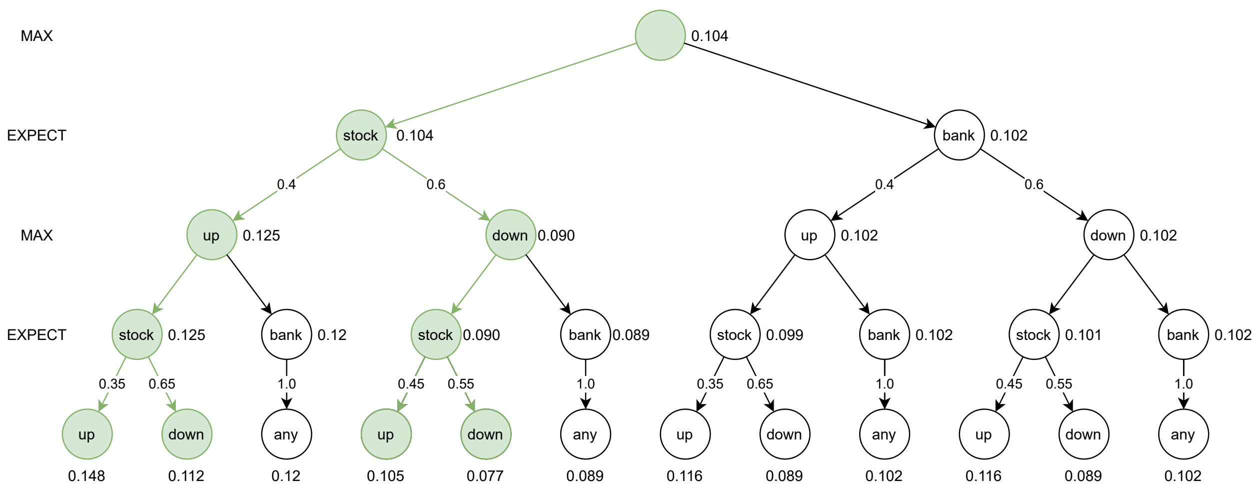

Figure 2: The decision tree for maximizing the expected utility of a sum of money through two investment periods, using the utility function $U(x) = \sqrt{x/1000}$. The optimal decision sub-tree is shown in green.

Problem 2. Buying a car

a. The best car to buy in order to maximize the expected profit is car A, which results in an expected value of \$290 compared to \$220 for car B. This can be seen by the decision tree displayed in Figure 3.

b. We now consider the case where we can perform a test on one of the two cars before making a purchase. The decision tree for this scenario is shown in Figure 4. Constructing this decision tree required calculating additional probabilities, which are shown below. With the option to perform a test first, the optimal decision sequence begins with performing test T1, which is is applied to car A. If the test passes, we should buy car A and obtain an expected profit of \$350. If the test fails, we should buy car B for an expected profit of \$170. The expected value of the overall optimal decision sequence, starting with performing T1, is \$302. This is greater than the \$270 expected by performing T2 and \$290 for not performing either test. The value of information is \$12 for T1 and -\$20 for T2.

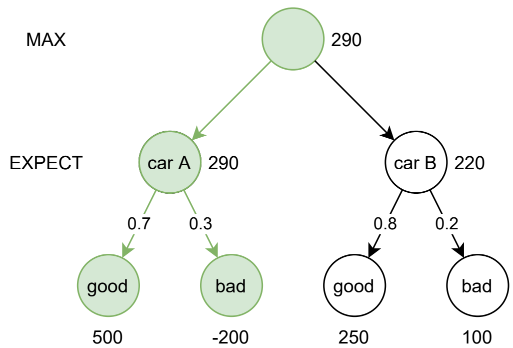

Figure 3: The decision tree for maximizing the expected profit when choosing between two cars to buy. The optimal decision sub-tree is shown in green.

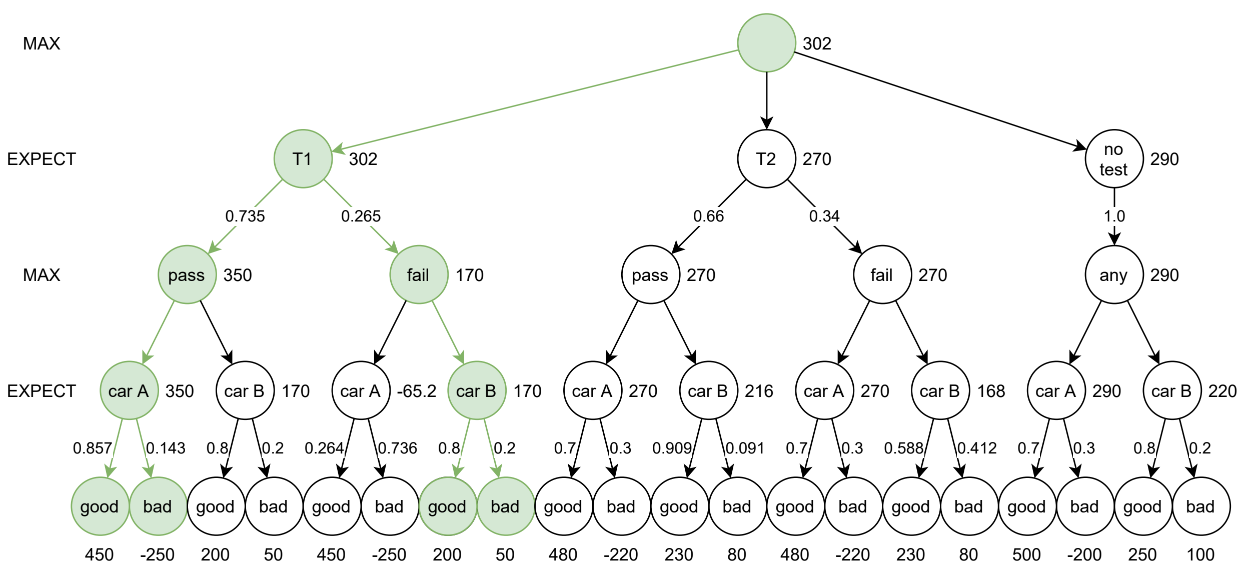

Figure 4: The decision tree for maximizing the expected profit when choosing between two cars to buy with the option to perform a test on one of the cars first. The optimal decision sub-tree is shown in green.

$$\begin{aligned}P(T1 = pass) &= \sum_a P(T1 = pass|A = a)P(A = a)\\&= P(T1 = pass|A = good)P(A = good) + P(T1 = pass|A = bad)P(A = bad)\\&= (0.9)(0.7) + (0.35)(0.3)\\&= 0.735\end{aligned}$$

$$\begin{aligned}P(T1 = fail) &= 1 - P(T1 = pass)\\&= 1 - 0.735\\&= 0.265\end{aligned}$$

$$\begin{aligned}P(A = good|T1 = pass) &= \frac{P(T1 = pass|A = good)P(A = good)}{P(T1 = pass)}\\&= \frac{(0.9)(0.7)}{0.735}\\&\approx 0.857\end{aligned}$$

$$\begin{aligned}P(A = bad|T1 = pass) &= 1 - P(A = good|T1 = pass)\\&= 1 - 0.857\\&= 0.143\end{aligned}$$

$$\begin{aligned}P(A = good|T1 = fail) &= \frac{P(T1 = fail|A = good)P(A = good)}{P(T1 = fail)}\\&= \frac{(0.1)(0.7)}{0.265}\\&\approx 0.264\end{aligned}$$

$$\begin{aligned}P(A = bad|T1 = fail) &= 1 - P(A = good|T1 = fail)\\&= 1 - 0.264\\&= 0.736\end{aligned}$$

$$\begin{aligned}P(T2 = pass) &= \sum_b P(T2 = pass|B = b)P(B = b)\\&= P(T2 = pass|B = good)P(B = good) + P(T2 = pass|B = bad)P(B = bad)\\&= (0.75)(0.8) + (0.3)(0.2)\\&= 0.66\end{aligned}$$

$$\begin{aligned}P(T2 = fail) &= 1 - P(T2 = pass)\\&= 1 - 0.66\\&= 0.34\end{aligned}$$

$$\begin{aligned} P(B = good|T2 = pass) &= \frac{P(T2 = pass|B = good)P(B = good)}{P(T2 = pass)} \\ &= \frac{(0.75)(0.8)}{0.66} \\ &\approx 0.909 \end{aligned}$$

$$\begin{aligned} P(B = bad|T2 = pass) &= 1 - P(B = good|T2 = pass) \\ &= 1 - 0.909 \\ &= 0.091 \end{aligned}$$

$$\begin{aligned} P(B = good|T2 = fail) &= \frac{P(T2 = fail|B = good)P(B = good)}{P(T2 = fail)} \\ &= \frac{(0.25)(0.8)}{0.34} \\ &\approx 0.588 \end{aligned}$$

$$\begin{aligned} P(B = bad|T2 = fail) &= 1 - P(B = good|T2 = fail) \\ &= 1 - 0.909 \\ &= 0.412 \end{aligned}$$