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Assignment 6

Problem 1. Logical inference with propositional rules

Table 1 summarizes the results of applying propositional logic rules of inference on the provided animal identification knowledge base. Out of the fives theorems we were interested in, three were entailed by the knowledge base, and two were not. Both the forward and backward chaining algorithms were able to correctly determine whether the knowledge base entailed each of the five theorems, which was expected since resolution with Horn normal form is a sound and complete inference procedure.

Theorem		Size of fact base		Num. rules checked	
#	$KB \models \alpha$	Forward	Backward	Forward	Backward
1	T	10	9	12	5
2	F	10	6	26	3
3	T	7	7	2	2
4	T	6	6	0	0
5	F	10	6	26	2

Table 1: Comparison of forward and backward chaining for logical inference.

We can compare the two algorithms based on the size of the fact base produced while trying to prove each theorem. For Theorems 2 and 5, which were not entailed by the knowledge base, forward chaining produced a fact base of size 10. This is the maximum number of facts that could be inferred, due to the completeness of resolution. Forward chaining also exhausted the entire knowledge base when proving Theorem 1. Backward chaining produced a smaller fact base on Theorems 1, 2 and 5. Both methods generated the same number of facts for Theorems 3 and 4. We were given six initial facts, so only one new fact was needed to prove Theorem 3, and Theorem 4 was provided in the initial set.

Another measure we can use to compare the algorithms is the number of rules that were checked during inference. Theorems 3 and 4 were trivial to prove, so there is not much insight to be gained from them. But for the other three theorems, forward chaining applied far more rules than backward chaining. Forward chaining checked 12 total rules to prove Theorem 1, compared to only 5 rules that were checked by backward chaining. The differences were even more drastic on Theorems 2 and 5. For these, forward chaining was able to prove lack of entailment using only 2 or 3 rules, while forward chaining needed to check 26.

It is clear from these results that backward chaining is more efficient at proving entailment of a specific theorem from a knowledge base. It checked fewer rules than forward chaining, and generated fewer unnecessary facts. This is because forward chaining must generate every possible fact from the knowledge base in order to prove lack of entailment. In contrast, backward chaining only proceeds until it exhausts the rules that have the theorem as an consequent (recursively). If our goal were to explore all facts that are entailed by a knowledge base, the forward chaining algorithm would be the better option. Each algorithm has advantages and disadvantages that depend on the goal of the inference procedure.

Problem 2. Translation of first-order logic

Let $Student(x)$ represent "x is a student."
Let $History(x)$ represent "x is history class."
Let $Biology(x)$ represent "x is biology class."
Let $Took(x, y)$ represent "x took y."
Let $Spring(x)$ represent "x occurred in Spring 2020."
Let $Failed(x, y)$ represent "x failed y."
Let $Person(x)$ represent "x is a person."
Let $Buys(x, y)$ represent "x buys y."
Let $Insurance(x)$ represent "x is an insurance policy."
Let $Smart(x)$ represent "x is smart."
Let $Expensive(x)$ represent "x is expensive."
Let $Woman(x)$ represent "x is a woman."
Let $Likes(x, y)$ represent "x likes y."
Let $Vegetarian(x)$ represent "x is a vegetarian."
Let $Barber(x)$ represent "x is a barber."
Let $Shaves(x, y)$ represent "x shaves y."
Let $Man(x)$ represent "x is a man."
Let $Town(x)$ represent "x is in town."
Let $Professor(x)$ represent "x is a professor."

a. $\exists x\, \exists y\, \exists z\, (Student(x) \land Took(x, y) \land History(y) \land Took(x, z) \land Biology(z) \land Spring(y) \land Spring(z))$

b. $\exists x\, \exists y\, (Student(x) \land Failed(x, y) \land History(y))$

c. $\exists x\, \exists y\, \exists z\, (Student(x) \land Failed(x, y) \land History(y) \land Failed(x, z) \land Biology(z))$

d. $\forall x\, (Student(x) \land \exists y[Took(x, y) \land History(y)] \implies \exists z\, [Took(x, z) \land Biology(z)])$

e. $\forall x\, (Person(x) \land \exists y[Buys(x, y) \land Insurance(y)] \implies Smart(x))$

f. $\neg \exists x\, \exists y\, (Person(x) \land Buys(x, y) \land Expensive(y) \land Insurance(y))$

g. $\exists x\, (Woman(x) \land \forall y\, [Man(y) \land \neg Vegetarian(y) \implies Likes(x, y)])$

h. $\exists x\, (Barber(x) \land \forall y\, [Man(y) \land Town(y) \land \neg Shaves(y, y) \implies Shaves(x, y)])$

i. $\forall x\, (Professor(x) \land \neg Smart(x) \implies \neg \exists y\, Likes(y, x))$

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Problem 3. Proof in first-order logic

Let $L(x, y) = Loves(x, y)$ and let $H(x) = Happy(x)$. We can prove the following equivalence:

$$\forall x\, (\exists y\, L(x, y) \implies H(x)) \iff \forall x\, \forall y\, (L(x, y) \implies H(x))$$

Proof.

1. $\forall x\, \forall y\, (L(x, y) \implies H(x))$ (Assumption)
2. $\forall y\, (L(a, y) \implies H(a))$ (Universal elimination, from 1)
3. $L(a, b) \implies H(a)$ (Universal elimination, from 2)
    4. $\exists y\, L(a, y)$ (Assumption)
    5. $L(a, b)$ (Existential elimination, from 4)
    6. $H(a)$ (Conditional elimination, from 3, 5)
7. $\exists y\, L(a, y) \implies H(a)$ (Conditional introduction, from 4-6)
8. $\forall x\, (\exists y\, L(x, y) \implies H(x))$ (Universal introduction, from 7)

9. $\forall x\, \forall y\, (L(x, y) \implies H(x)) \implies \forall x\, (\exists y\, L(x, y) \implies H(x))$ (Conditional introduction, from 1-8)
    10. $\forall x\, (\exists y\, L(x, y) \implies H(x))$ (Assumption)
    11. $\exists y\, L(a, y) \implies H(a)$ (Universal elimination, from 10)
        12. $L(a, b)$ (Assumption)
        13. $\exists y\, L(a, y)$ (Existential introduction, from 12)
        14. $H(a)$ (Conditional elimination, from 11, 13)
    15. $L(a, b) \implies H(a)$ (Conditional introduction, from 12-14)
    16. $\forall y\, (L(a, y) \implies H(a))$ (Universal introduction, from 15)
    17. $\forall x\, \forall y\, (L(x, y) \implies H(x))$ (Universal introduction, from 16)
18. $\forall x\, (\exists y\, L(x, y) \implies H(x)) \implies \forall x\, \forall y\, (L(x, y) \implies H(x))$ (Conditional introduction, from 10-17)
19. $\forall x\, (\exists y\, L(x, y) \implies H(x)) \iff \forall x\, \forall y\, (L(x, y) \implies H(x))$ (Biconditional introduction, from 9, 18)

$\square$