Intravenous Injection

Midlothian Medicines produce therapeutic drugs for intravenous injection. In particular, they make medicines for patients who suffer an allergic reaction. The injection is not instantaneous, but rather takes around 5 seconds, so that the medicine can be considered as initially uniformly distributed over a length of the vein determined by the mean velocity of blood in the vein. It is believed that the medicine then forms a 'slug', which diffuses whilst also moving through the veins. Midlothian Medicines are interested in the concentration of the medicine in the blood once it reaches the heart approximately 60 seconds later. They are also interested in how much medicine should be injected to obtain a desired concentration in the heart; this target varies by application, so they would like a general approach here. Furthermore, they are interested in the effects of non-constant flow, for example it may be a periodic flow due to the effects of pumping blood around the body.

Midlothian Medicines are unsure about many of the parameters required for modelling, but believe that, in a narrow vein, the diffusion coefficient is approximately $0.1\text{m}^2/\text{s}$, and the blood flow rate is around 0.2m/s.

The main aims of this study are to:

- Identify a suitable model for transport of medicine in the blood stream;
- Does this model show the formation of a 'slug'? If not, why not?
- Determine the concentration as it reaches the heart, in terms of the initial amount of medicine injected. How much medicine needs to be injected if an effective concentration to treat the allergic reaction is $1 \times 10^{-3}\text{g/m}$?
- Investigate the effects of non-constant flow of blood. How does this affect how much medicine should be administered?
- If possible, determine relevant parameters, such as blood flow rates and chemical diffusivity, from real-world data. How do these compare to the model parameters given, and do any changes affect your conclusions? In particular, how are the results affected by different velocities in different parts of the circulatory system?

Hints:

- You likely want to start with a 1D advection-diffusion equation and consider possible boundary conditions. You may want to assume that the veins are infinitely long.
- You may want to investigate what happens under a change of variables where you follow the flow (e.g., $\eta = x - vt$, where x is the position, t the time, and v the constant velocity).
- You may find that solution to a diffusion equation from a point source to be useful, especially when combined with the method of superpositions (you could start by assuming that the injection is instantaneous).

- Try to find an analytic solution to your problem, even if you have to make some reasonable approximations.
- For numerics, you could use something simple such as forward Euler combined with finite difference, but be sure to validate your choice of step size and other parameters.
- It is likely that you will need to use your numerical simulations for the later aims of the project, especially when non-constant velocities are included.