TRANSFORM	TIME-DOMAIN	FREQUENCY-DOMAIN
Fourier series	cont._p	Discrete
CFT	cont	cont
DTFT	Discrete	cont.
DFT	Discrete	Discrete

NEXT TIME
PG. 251 (DTFT, DFT)

FFT = DFT w/
cooley-Tukey implementation
EQ. 7, 1.20, 21

SEPT. 10, 2013

EE310 TOPICS

1) continuous systems
2) Discrete systems
3) Laplace transform
4) z-Transform
5) continuous Fourier transform
6) DTFT
7) Intro to sampling

HW #1 POINTERS

① P2.30

$y_H = c_1 n 4^n + c_2 (-1)^n$

$y_p = K n 4^n u(n)$

$y_T(n) = [\frac{6}{5} n 4^n + c_1 4^n + c_2 (-1)^n] u(n)$

$c_1 = \frac{26}{25}$

$c_2 = \frac{-1}{25}$

initial Guess

$K n 4^n u(n) - 3K(n-1) 4^{n-1} u(n-1) - 4K(n-2) 4^{n-2} u(n-2) = 4^n u(n) + 2(4)^{n-1} u(n-1)$

For n=2

$K(32-12) = 4^2 + 8 = 24$

$K = 6/5$

② P3.49 see e-mail for notation

③ P2.22(a)

④ M3.3 see e-mail for correction.
(EQ. 2.6 and EQ. 2.7 for Geometric series – Pg #36)

$X(\omega) = 2 \sum_{-\infty}^{\infty} 0.5^n u(n+2) e^{-jn\omega}$ ($\sum_{0}^{\infty}$)

Get closed-form solution.

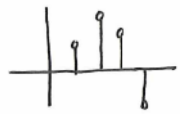
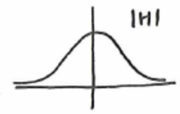
⑤ 1.10

$F_s = \frac{10{,}000 \text{ bits/sec}}{10 \text{ bits/sample}} = 1000$ samples/sec.

(For Part D) → Add amplitude of cosine.

Res. $= \Delta = \frac{x_{max} - x_{min}}{m-1} = \frac{5-(-5)}{1023} = \frac{10}{1023}$

TRANSFORMS (CON'T)

TRANSFORMS	TIME	FREQUENCY
Discrete Time Fourier Transform (DTFT)		

* assume frequency domain is continuous.

Pg. 251, EQ. 4.2.28, 4.2.29 Notation

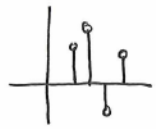
$$f(n) = \frac{1}{2\pi}\int_0^{2\pi} F(\omega)\, e^{jn\omega}\, d\omega$$

$$F(\omega) = \sum_{n=-\infty}^{\infty} f(n)\, e^{-jn\omega}$$

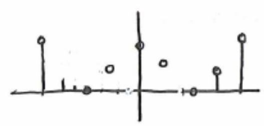
Discrete Fourier Transform (DFT)

same as the FFT w/ cooley-Tuckey implementation for faster computation (chpt. 8)

TIME (Discrete)



FREQUENCY (Discrete)



Pg. 456 EQ. 7.1.20, 7.1.21

$$f(n) = \frac{1}{N}\sum_{k=0}^{N-1} F(k)\, e^{j2\pi kn/N}$$

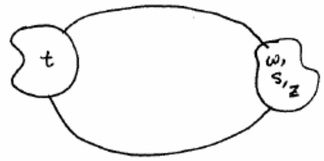
$$F(k) = \sum_{n=0}^{N-1} f(n)\, e^{-j2\pi kn/N}$$

N^2 calculations add complexity w/ cooley-Tuckey is faster b/c $N\log(\frac{N}{2})$ } for N values

---

SUMMARY

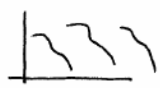
TRANSFORM	TIME	FREQ.
Fourier series	cont.	disc.
CFT	cont.	cont.
DTFT	disc.	cont.
DFT, FFT	disc.	disc.

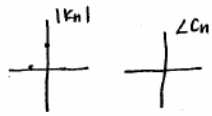


- Phasors
- Laplace
- Z
- Fourier

TIME DOMAIN & FREQUENCY DOMAIN

TRANSFORMS	TIME-DOMAIN	FREQ. DOMAIN
FOURIER SERIES	• continuous	• Discrete
	• f(t)	• a_n, b_n, c_n
$\omega_0 = \frac{2\pi}{T} = 2\pi f_0$	• periodic	





$$f(t) = \sum_{-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$= a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

PG. 229
EQ'n. 4.18, 4.19 K, f, t

$$a_0 = \frac{1}{T} \int f(t)\, dt$$

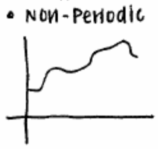
$$a_n = \frac{2}{T} \int f(t) \cos(n\omega_0 t)\, dt$$

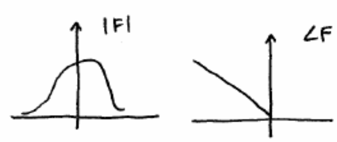
$$b_n = \frac{2}{T} \int f(t) \sin(n\omega_0 t)\, dt$$

$$c_n = \frac{1}{T} \int f(t)\, e^{-jn\omega_0 t}\, dt$$

$$c_n = \frac{1}{2} (a_n + jb_n)$$

CONTINUOUS FOURIER TRANSFORM (CFT)	• continuous	• cont.
	• Non-Periodic	





$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t}\, d\omega$$

PG. 236, 4.1, 29, 30

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt$$

SECOND ORDER DIFF. EQ.

$y'' + 5y' + 6y = 12$

$y'(0) = 0$

$y(0) = 0$

characteristic EQ.

$\lambda^2 + 5\lambda + 6 = 0$

$\lambda_1 = -2, \quad \lambda_2 = 3$

$y_h = Ae^{-2t} + Be^{-3t}$

$y_p = K$ Guess

$y' = 0$

$y'' = 0$

substitute into D.E.

$0 + 0 + 6K = 12$

$6K = 12 \Rightarrow K = 2$

$y_T = y_H + y_p = 2 + Ae^{-2t} + Be^{-3t}$

initial conditions $\underbrace{A = -3 \quad B = 1}$

$y(0) = 0 = 2 + A + B$

$y'(0) = -7A - 3B = 0$

$y_T(t) = 2 - 3e^{-2t} + 1e^{-3t}$

NOW, Discrete Difference EQ'n.

$$y' = \frac{dy}{dt} = \frac{y(n+1) - y(n)}{\Delta t}$$

$$y'' = \frac{dy'}{dt} = \frac{y'(n+1) - y'(n)}{\Delta t}$$

$$= \frac{1}{\Delta t}\left[\frac{y(n+2) - y(n+1)}{\Delta t} - \frac{y(n+1) - y(n)}{\Delta t}\right]$$

$$= \frac{1}{\Delta t^2}\left[y(n+2) - 2y(n+1) + y(n-1)\right]$$

$$\frac{1}{\Delta t^2}(y(n+2)) - 2y(n+1) + y(n) + \frac{5}{\Delta t}y(n+1) - \frac{5}{\Delta t}y(n) + 6y(n) = 12$$

simplify,

$$y(n+2) + (-2 + 5\Delta t)y(n+1) + (1 - 5\Delta t + 6\Delta t^2)y(n) = 12\Delta t^2$$

could

$\lambda^2 + \alpha\lambda + \beta = 0$

OR Direct Application of Diff. EQ.

$$y(n+2) = -(-2 + 5\Delta t)y(n+1) - (1 - 5\Delta t + 6\Delta t^2)y(n) + 12\Delta t^2$$

show why the signal repeats

Expand $p(t)$ using complex Fourier series

$$= \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} = \sum \tfrac{1}{T} e^{jn\omega_0 t}$$

$2\pi\delta(\omega)$ continuous Fourier transform table pg.303 T.C. Chen

$$P(\omega) = \sum_{-\infty}^{\infty} \frac{2\pi}{T}\,\delta(\omega - n\omega_0)$$

$$X_s(f) = \frac{1}{T}\sum_{-\infty}^{\infty} X(f - nf_s)$$

$$X_s(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(\omega-\beta)\,P(\beta)\,d\beta$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(\omega-\beta)\left[\sum_{-\infty}^{\infty}\frac{2\pi}{T}\,\delta(\beta - n\omega_s)\,d\beta\right]$$

use sifting property of delta fx.

$$= \frac{1}{T}\sum_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_1(\omega-\beta)\,\delta(\beta-n\omega_s)\,d\beta$$

$$X_s(\omega) = \frac{1}{T}\sum_{-\infty}^{\infty} F_1(\omega - n\omega_s)$$

SEPT. 24, '13 EXAMPLE FROM MB (ALIASING)

$x_a(t) = 4 + 2\cos(150\pi t + \pi/3) + 4\sin(350\pi t)$

$F_s = 200$

$F_{max} = 175$

The Nyquist rate is $> 2F_{max}$ → 350 Hz

$T_s = \frac{1}{F_s} = 0.005$

$x(n) = x_a(nT_s) = x_a(0.005n) = 4 + 2\cos(150\pi(0.005n) + \pi/3) + 4\sin(350\pi(0.005n))$

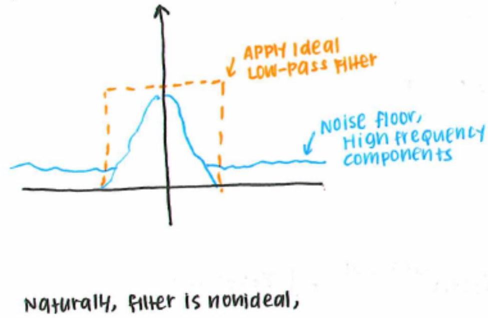
$x(n) = 4 + 2\cos(\underbrace{0.75\pi n}_{\text{in the range of } -\pi \text{ to } \pi} + \pi/3) + 4\sin(\underbrace{1.75\pi n}_{\text{out of range of } -\pi \text{ to } \pi})$

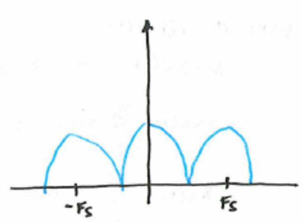
so then $\sin(1.75\pi n \pm 2\pi n)$

shift until it is in the interval

$\therefore\ + \sin(-0.25\pi n)$

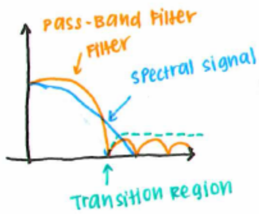
If we have spectrum





$F_s > 2F_{max}$

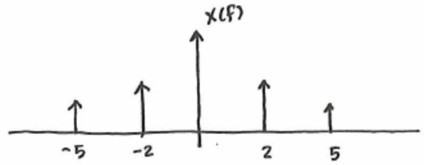
Naturally, filter is nonideal,



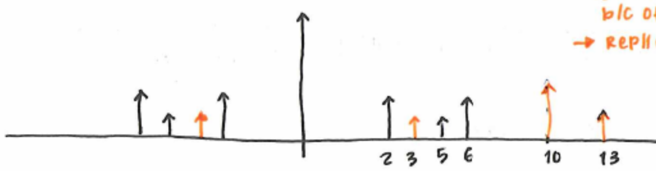
For aliased case,

$x(t) = 5\cos(2\pi \cdot 2000t) + 1\cos(2\pi \cdot 5000t)$

$f_s = 8000$

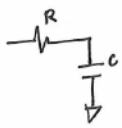


→ Replicas are corrupting original signal b/c of overlap.
→ Replicas in orange



RC FILTERS

LPF



$Z_C = \frac{1}{j\omega C}$

As $\omega\uparrow$, $Z_C\downarrow$, $V_{out}\downarrow$

using voltage divider,

$$H(s) = \frac{1}{sC(R+\frac{1}{sC})} = \frac{1}{1+sRC}$$

$$H(\omega) = \frac{1}{1+j\omega RC} = \frac{1}{1+j\omega/\omega_c}$$

$\omega = 2\pi f$ $\qquad \omega_c = \frac{1}{RC} = 2\pi f_c$

$$f_c = \frac{1}{2\pi RC}$$

we have seen the signal repeat in the frequency domain,

$$X_s(f) = \frac{1}{T}\sum_{n=-\infty}^{\infty} X(f-nf_s)$$ ← show

continuous Fourier transform
$x_1(t) + x_2(t)$
the summation is linearity / superposition

$$P(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

$$x(t)p(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-nt)$$

Expand $x(t)p(t)$

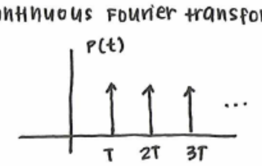
$$x(t)\delta(t-T) + x(t)\,\delta_2(t-2T) + \ldots +$$

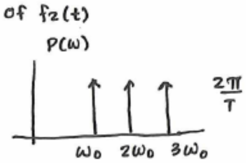
$$x_s(t) = \sum_{n=-\infty}^{\infty} x(t)\,\delta(t-nT)$$

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)$$

$$\mathcal{F}[f_1(t) f_2(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f_1(t)e^{-j(\omega-\beta)t}\,dt\right]F_2(\beta)\,d\beta$$

continuous Fourier transform of $f_2(t)$





Z - TRANSFORMS

$X(z) = \mathcal{Z}[x(n)]$

Take transform

discrete

$$= \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$= x(0) z^{-0} + x(1) z^{-1} + x(2) z^{-2} + \ldots + x(n) z^{-n}$$

Pg. 170, Table 3

If we have $x(n) = a^n u(n)$, where $u(n)$ is a unit-step discrete function.

$$X(z) = \sum_{n=0}^{\infty} a^n u(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} (az^{-1})^n$$

$$\sum_{n=0}^{\infty} (az^{-1})^n = 1 + az^{-1} + (az^{-1})^2 + \ldots + (az^{-1})^n$$

$$az^{-1} \sum_{n=0}^{\infty} (az^{-1})^n = az^{-1} + (az^{-1})^2 + \ldots + (1 - az^{-1}) S = 1$$

$$\sum_{n=0}^{\infty} (az^{-1})^n = \frac{1}{1 - az^{-1}}$$

#3 Entry, Table 3.3

LINEARITY

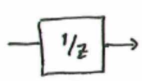
- Additivity
- Homogeneity
- Superposition

$$\mathcal{Z}(ax_1(n) + bx_2(n)) = a\mathcal{Z}(x_1(n)) + b\mathcal{Z}(x_2(n))$$

SHIFT THEOREM

$$\boxed{\mathcal{Z}(x(n-m)) = z^{-m}(X(z))}$$

Graphically, this is



shift 1

CONVOLUTION

$$\boxed{x(n) = x_1(n) * x_2(n)}$$

EX. For 2 probability distributions, convolve to find the combined distribution.

$$x(n) = \sum_{k=0}^{\infty} x_1(n-k) x_2(k)$$

$X(z) = X_1(z) X_2(z) \Rightarrow$ convolution is multiplication in z-domain

$x(n) = \mathcal{Z}^{-1}(X(z))$

1) Take $\mathcal{Z}$ transform of x_1, x_2
2) Multiply
3) Take $\mathcal{Z}^{-1}$

EX.

$x_1(n) = 3\,\underbrace{\delta(n)}_{\text{Kronnecker Delta function}} + 2\,\delta(n-1)$

$x_2(n) = 2\,\delta(n) - \delta(n-1)$

using DIRECT CONVOLUTION

$x(n) = 3.2\,\delta(n) + (2\cdot 2 + (-1)(3)\,\delta(n-1)) + (-1)(2)\,\delta(n-2)$

$\therefore\ x(n) = 6\,\delta(n) + \delta(n-1) - 2\,\delta(n-2)$

$X_1(z) = 3z^{-0} + 2z^{-1}$

$X_2(z) = 2z^{-0} - 1z^{-1}$

$X_1(z)X_2(z) = 6 + 1z^{-1} - 2z^{-2}$

Take z^{-1},

$\therefore\ x(n) = 6\,\delta(n) + \delta(n-1) - 2\,\delta(n-2)$

ANOTHER FUNCTION...

if $Y(z) = \dfrac{z^2(z+1)}{(z-1)(z^2-z+0.5)}$

$\dfrac{Y(z)}{z} = \dfrac{z(z+1)}{(z-1)(z^2-z+0.5)}$

$= \dfrac{B}{(z-1)} + \dfrac{A}{(z-0.5-j0.5)} + \dfrac{A^*}{(z-0.5+j0.5)}$ } get this from "P"

use residue function in MATLAB

num = [1 1 0] Numerator Elements

den = [1 -2 1.5 -0.5] Denominator Elements

Then [R P K] = residue(num, den)

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SEPT. 12, 2013

Solving the Difference Equation

1) Apply z-transform to ODE
2) use initial conditions
3) solve for $X^+(z)$ in P.F.E form.
4) Take inverse of z-transform

$$Z(y(n)) = Y(z)$$
$$Z(y(n-1)) = y(-1) + z^{-1}Y^+(z)$$
$$Z(y(n-2)) = z^{-2}Y^+(z) + z^{-1}y(-1) + y(-2)$$
$$Z(y(n-m)) = z^{-m}Y^+(z) + y(-1)z^{-1(m-1)} + \ldots + y(-1)z^{-1} + y(-m)$$

EXAMPLE $y(n) - 0.5y(n-1) = 5(0.2)^n u(n)$ $\qquad y(-1) = 1$

Forcing function

$$Y^+(z) - 0.5[z^{-1}Y^+(z) + y(-1)] = \frac{5}{1-0.2z^{-1}}$$

$$Y^+(z)[1-0.5z^{-1}] = \frac{5}{1-0.2z^{-1}} + 0.5$$

$$= \frac{5 + 0.5 - 0.1z^{-1}}{1-0.2z^{-1}}$$

$$Y^+(z) = \frac{5.5 - 0.1z^{-1}}{(1-0.2z^{-1})(1-0.5z^{-1})} = \frac{A}{1-0.2z^{-1}} + \frac{B}{1-0.5z^{-1}}$$

$$A = \left.\frac{5.5-0.1z^{-1}}{(1-0.5z^{-1})}\right|_{z^{-1}=\frac{1}{0.5}} = \frac{53}{6}$$

$$B = \left.\frac{5.5-0.1z^{-1}}{1-0.2z^{-1}}\right|_{z^{-1}=}$$

$$Y^+(z) = \frac{8.33}{(1-0.5z^{-1})} - \frac{3.33}{(1-0.2z^{-1})}$$

$$y(n) = [8.33(0.5)^n - 3.33(0.2)^n]u(n)$$

EXAMPLE Repeating Root

$$y(n) - 0.8y(n-1) + 0.16y(n-2) = 2u(n)$$

Relaxed initial conditions $y(-1) = 0$
$y(-2) = 0$

$$Y^+(z) - 0.8[z^{-1}Y^+(z) + y(-1)] + 0.16[z^{-2}Y^+(z) + z^{-1}y(-1) + y(-2)] = 2\frac{1}{1-z^{-1}}$$

$$Y^+(z) - 0.8Y^+(z)z^{-1} + 0.16Y^+(z)z^{-2} = \frac{2}{1-z^{-1}}$$

$$Y(z)[1 - 0.8z^{-1} + 0.16z^{-2}] = \frac{2}{1-z^{-1}}$$

$$Y(z) = \frac{z}{(1-z^{-1})(1-0.4z^{-1})^2} \left[\frac{z}{z}\right]$$

$$\therefore \frac{Y(z)}{z} = \frac{z}{(z-1)(1-0.4z^{-1})} \left[\frac{z^2}{z^2}\right]$$

$$\therefore \frac{Y(z)}{z} = \frac{2z^2}{(z-1)\underbrace{(z-0.4)^2}_{(z^2-0.8z+0.16)}}$$

$$\frac{Y(z)}{z} = \frac{A}{z-1} + \frac{B}{z-0.4} + \frac{C}{(z-0.4)^2}$$

$$A = \left.\frac{2z^2}{(z-0.4)^2}\right|_{z=1} = \frac{2}{0.6^2} = 5.55$$

$$C = \left.\frac{2z^2}{(z-1)}\right|_{z=0.4} = -0.53$$

$2 = A+B$

$B = 3.55$

$$\frac{Y(z)}{z} = \frac{5.55}{(z-1)} + \frac{3.55}{(z-0.4)} - \frac{0.53}{(z-0.4)^2}$$

$$Y(z) = \frac{5.55}{1-z^{-1}} + \frac{3.55}{1-0.4z^{-1}} - \frac{\overbrace{0.53}^{(1.325)(0.4)}z^{-1}}{(1-0.4z^{-1})^2}$$

$$y(n) = [5.55(1)^n + 3.55(0.4)^n - 1.325n\, 0.4^n]u(n) //$$

EXAMPLE complex roots

$$Y(z) = \frac{z^2(z+1)}{(z-1)(z^2-z+0.5)}$$

$$\frac{Y(z)}{z} = \frac{z(z+1)}{(z-1)(z^2-z+0.5)}$$

fXML: roots([1 -1 0.5])

$$= \frac{B}{z-1} + \frac{A}{(z-0.5-j0.5)} + \frac{A^*}{(z-0.5+j0.5)}$$

fx ML: residue(numerator, denominator)

$$B = \left.\frac{z(z+1)}{z^2-z+0.5}\right|_{z=1} = 4$$

$$A = \left.\frac{z(z+1)}{(z-1)(z-0.5+j0.5)}\right|_{z=0.5+j0.5} = 1.58 \angle -161.57^\circ$$

$$\therefore A^* = 1.58 \angle 161.57^\circ$$

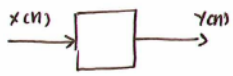
$$H(z) = \frac{Y(z)}{X(z)}$$

PULSE TRANSFER FUNCTION

$h(n)$ = impulse response

$= z^{-1}\{H(z)\}$

$Y(z) = H(z)X(z)$



t-Domain

PFE > residue

Difference EQ. $\leftrightarrow$ $H(z)$

EXAMPLE

$$H(z) = \frac{z^2+1}{z^2+1.3z+0.36}$$

$$\frac{H(z)}{z} = \frac{z^2+1}{z(z+0.9)(z+0.4)}$$

$$= \frac{A}{z} + \frac{B}{z+0.9} + \frac{C}{z+0.4}$$

Find A, B, C, z^{-1}

For $h(n)$,

$Y(z)_{step} = H(z)X(z)$

step $X(z) = \frac{z}{z-1}$

$$\frac{Y(z)}{z} = \frac{z^2+1}{(z-1)(z+0.9)(z+0.4)}$$

$$= \frac{A}{z-1} + \frac{B}{z+0.9} + \frac{C}{z+0.4}$$

Then, find $y(n)_{step}$

$y(n) = 0.5y(n-1) + x(n)$

Relaxed

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-0.5z^{-1}}$$

$Y(z) - 0.5Y(z)z^{-1} = X(z)$

$Y(z)(1-0.5z^{-1}) = X(z)$

$Y(z) = H(z)X(z)$

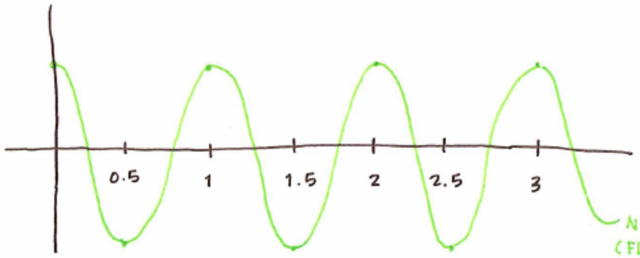
$x(n) = 10\cos(\frac{\pi}{4}n)\,u(n)$

$$X(z) = \frac{10\,[1-(1/\sqrt{2})]\,z^{-1}}{1-\sqrt{2}z^{-1}+z^{-2}}$$

Use lookup table on pg #170, #7,

$\cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$

EXAMPLE



$x(n+N) = x(n)$

$N \triangleq$ Fundamental Period

Let $f = \frac{5}{10} = \frac{1}{2}$

Pg. 15

$f_1 = \frac{31}{60} \Rightarrow N = 60$

$f_2 = \frac{30}{60} \Rightarrow N = 2$

NOW, $f = \frac{6}{10} = \frac{3.2}{5.2} = \frac{3}{5}$ $\quad N = 5$

SEPT. 17, '13 $\quad$ ω, f RANGE FOR DISCRETE SYSTEMS & SIGNALS

$\cos[(\omega_0 + 2\pi)n + \theta] = \cos(\omega_0 n + 2\pi + \theta) = \cos(\omega_0 n + \theta)$

$x_k(n) = A \cos(\omega_k n + \theta)$

$k = 0, 1, 2 \ldots$

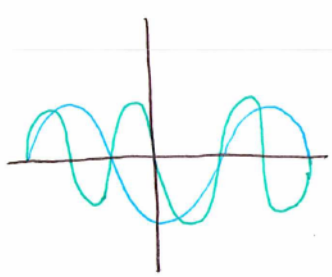
$\omega_k = \omega_0 + 2\pi k$

are identical

$|\omega| > \pi$

$= |\omega| < \pi$

$|f| > \frac{1}{2} = |f| < \frac{1}{2}$



$-\pi < \omega < \pi$ $\qquad$ $-\frac{1}{2} < f \leq \frac{1}{2}$

EXAMPLE

$$y(n) + 1.3y(n-1) + 0.36\,y(n-2) = x(n) - x(n-2)$$

Performing z-transform,

$$Y(z) + 1.3Y(z)z^{-1} + 0.36\,Y(z)z^{-2} = X(z) - X(z)z^{-2}$$

$$\therefore\ Y(z)\,[1 + 1.3z^{-1} + 0.36z^{-2}] = X(z)\,[1 - z^{-2}]$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{1 - z^{-2}}{1 + 1.3z^{-1} + 0.36z^{-2}}$$

$$[1 - z^{-2}]\,X(z) = 1$$

$$X(z) = \frac{1}{1 - z^{-2}}\left[\frac{z^2}{z^2}\right] = \frac{z}{1-z}$$

$$\frac{H(z)}{z} = \frac{z^2+1}{z(z+0.9)(z+0.4)} = \frac{A}{z} + \frac{B}{z+0.9} + \frac{C}{z+0.4}$$

Then,

Find A, B, C, $h(n)$

Take the inverse z of $h(n)$

The step response is

$$X(z) = \frac{z}{z-1}$$

$$Y(z) = H(z)X(z) = \frac{z(z^2+1)}{(z-1)(z+0.9)(z+0.4)}$$

$$\frac{Y(z)}{z} = \frac{z^2+1}{(\ )(\ )(\ )} = \frac{A}{z-1} + \frac{B}{z+0.9} + \frac{C}{z+0.4}$$

NOTATION

Typically,

continuous domain: $x(t) = A\cos(\omega t + \theta)$

Phase [radians]

Amplitude

Angular frequency $= 2\pi f$

$f = \frac{1}{T}$ cycles/sec.

sequential domain: $x_a(t) = A\cos(\Omega t + \theta)$

$$\Omega = 2\pi F$$

$$F = \frac{1}{T}$$

Rewriting using Euler's identity,

$$x_a(t) = \frac{A}{2}e^{j(\Omega t + \theta)} + \frac{A}{2}e^{-j(\Omega t + \theta)}$$

$+F$: CCW rotation

$-F$: CW rotation

Frequency Range : $-\infty < F < \infty$

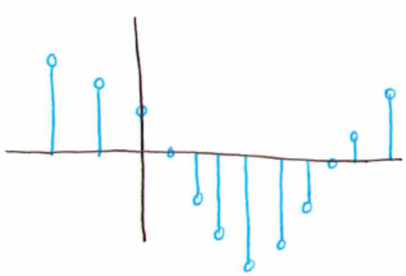
Amplitude

discrete: $x(n) = A\cos(\omega n + \theta)$

$\omega = 2\pi f$ [radians/sample]

$f \triangleq$ cycles/sample.

EXAMPLE



Here,

$\omega = \pi/6$

$f = \frac{1}{12}$ cycles/sample

$\theta = \pi/3$

Z-INVERSE for exception w/ complex numbers
Table, EQ. 3.4.34, pg. 190

$\therefore\ y(n) = 4u(n) + 3.16\,(0.707)^n \cos(45°n - 161.57°)\,u(n)$

SOLVING WITH MATLAB

$$F(Z) = \frac{z-1}{(z+2)(z^2+2z+4)}$$

$$\therefore\ \frac{F(Z)}{Z} = \frac{z-1}{z(\quad)(\quad)} = \frac{B}{Z} + \frac{C}{z+2} + \frac{A}{z-P} + \frac{A^*}{z+P}$$

```
num = [1 -1];
den = [1 4 8 8 0];
[R P K] = residue (num, den)
```

R = 0.375
- 0.125 − j0.144
- 0.125 + j0.144
- 0.125

P = −2
−1 + 1.73
−1 − 1.73
0

B = −0.125

C = 0.325

A = $0.19 \angle -131°$

A* = $0.19 \angle 131°$

P = $2 \angle 120$

$f(n) = -0.125\,\delta(n) + 0.375\,(-2)^n + 0.38\,(2)^n \cos(120°n - 131°)$ //

PULSE TRANSFER FUNCTION

similar to Transfer Function in $\mathcal{L}$. [H(s)]
"pulse" is reminder we are in discrete domain.

$$\boxed{H(Z) = \frac{Y(Z)}{X(Z)}}$$ output, input

For single in, single out systems (SISO)

$h(n) = Z^{-1}[H(Z)]$
impulse response

$$\boxed{y(n) = Z^{-1}[Y(Z)] = Z^{-1}[X(Z)H(Z)]}$$

$$Y(Z) = H(Z)X(Z) = \frac{10\left(1-\left(\frac{1}{\sqrt{2}}\right)\right)z^{-1}}{(1-0.5z^{-1})(1-e^{j\pi/4}z^{-1})(1-e^{-j\pi/4}z^{-1})}$$

using P.F.E,

$$Y(Z) = \frac{6.3}{1-0.5z^{-1}} + \frac{6.78e^{-j28.7°}}{1-e^{+j\pi/4}z^{-1}} + \frac{6.78e^{j28.7°}}{1-e^{-j\pi/4}z^{-1}}$$

Pg # 190, EQ. 3.4.34

$$y(n) = \underbrace{6.3(0.5)^n u(n)}_{\text{Transient Response}} + \underbrace{13.56\cos\left(\frac{\pi}{4}n - 28.7°\right)u(n)}_{\substack{\text{steady-state Response} \\ \text{(Frequency Response)}}}$$

ω : same

A : May scale

φ : shifts

CONVOLUTION

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\,x(n-k)$$ CONVOLUTION SUMMATION

Let's excite the signal w/ complex exponential (sinusoid)

$$x(n) = Ae^{j\omega n}$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\,Ae^{j\omega(n-k)}$$

$$= A\sum_{k=-\infty}^{\infty} h(k)e^{-j\omega k}\,e^{j\omega n}$$

$$H(\omega) = \sum_{k=-\infty}^{\infty} h(k)e^{-j\omega k}$$

compare the DTFT

$$F(\omega) = \sum_{n=-\infty}^{\infty} f(n)e^{-j\omega n}$$

$H(\omega)$ DTFT of $h(k)$

$$y(n) = A\,H(\omega)e^{j\omega n}$$

consider $h(n) = \left(\frac{1}{2}\right)^n u(n)$

$$H(\omega) = \sum_{n=-\infty}^{\infty} h(n)e^{-j\omega n} = \sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^n u(n)\,e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{2}e^{-j\omega}\right)^n$$

using Geometric series,

MB 2.6 EQ

$$= \frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

Let input $x(n) = Ae^{j\pi n/2}$

$\omega = \pi/2$

$$H\left(\frac{\pi}{2}\right) = \frac{1}{1+\frac{1}{2}e^{-j\pi/2}} = \frac{1}{1+j\frac{1}{2}} \cdot \frac{(1-j\,1/2)}{(1-j\,1/2)}$$

$$= \frac{2e^{-j26.6}}{\sqrt{5}}$$

$$y(n) = A\left(\frac{2}{\sqrt{5}}e^{-j26.6}\right)e^{jn\pi/2}$$

$$= \underbrace{\frac{2}{\sqrt{5}}}_{\text{scaled}} A e^{j(\frac{\pi n}{2} \underbrace{- 26.6^\circ}_{\text{shifted}})}$$

let $x(n) = Ae^{j\pi n}$

$$H(\omega = \pi) = \frac{1}{1-\frac{1}{2}e^{-j\pi}} = \frac{1}{3/2} = \frac{2}{3}$$

$$y(n) = \underbrace{\frac{2}{3}}_{\text{scaled}} A e^{j\pi n}$$ No shift

$H(\omega)$ is the DTFT of $h(k)$. It is also periodic with period 2π.

- Also continuous in the freq. domain
- Time-domain is discrete.

Time → continuous
Freq. → Discrete } Fourier series

$$h(k) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(\omega)e^{j\omega t}\,dt$$

MOVING AVERAGE SYSTEM

Ex. $y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1)]$

Plot the magnitude/phase $H(\omega)$

$$h(n) = \frac{1}{3}\delta(n+1) + \frac{1}{3}\delta(n) + \frac{1}{3}\delta(n-1)$$

$$H(\omega) = \sum_{k=-\infty}^{\infty} h(k)e^{-j\omega k}$$

$$= \sum_{k=-1}^{1} h(k)e^{-j\omega k}$$

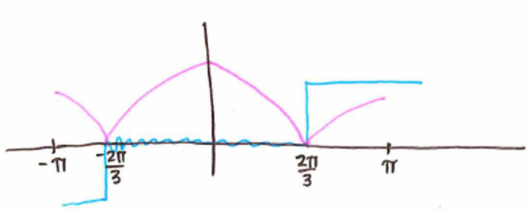
$$H(\omega) = \frac{1}{3}(e^{j\omega} + 1 + e^{-j\omega})$$

using trig. relations,

$$= \frac{1}{3}(1 + 2\cos(\omega))$$

$$|H(\omega)| = \frac{1}{3}|1 + 2\cos\omega|$$

$$\theta(\omega) = \begin{cases} 0 & 0 \le \omega \le 2\pi/3 \\ \pi & 2\pi/3 \le \omega \le \pi \end{cases}$$



(MOVING AV. SYSTEM)

EX. For $y(n) = 0.9y(n-1) + 0.1x(n)$

Determine magnitude/phase of the freq. response.

$h(n) = 0.1(0.9)^n u(n)$

writing out the terms,

$h(n) = 0.9h(n-1) + 0.1\delta(n)$

$h(0) = 0$

$h(1) = 0.9(0.1)$

$h(2) = \underbrace{0.9(0.9)(0.1)}_{\text{From previous iteration}}$

$h(3) = (0.9)^3(0.1)$

$h(n) = (0.9)^n(0.1)u(n)$

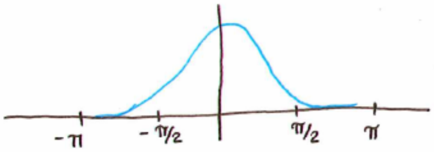
OR...

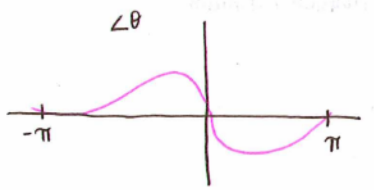
$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.1}{1-0.9z^{-1}}$$

$h(n) = 0.1(0.9)^n u(n)$
Table 3.3, Pg # 170

$\cos(60) = \frac{1}{2}$

For $|H|$





$$H(\omega) = \sum_{-\infty}^{\infty} h(n)e^{-j\omega n}$$

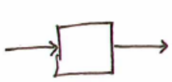
$$= \sum_{n=0}^{\infty} (0.1)(0.9)^n e^{-j\omega n}$$

$$= 0.1\sum_{n=0}^{\infty} (0.9e^{-j\omega})^n$$

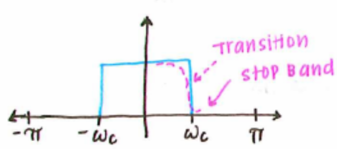
$$H(\omega) = \frac{0.1}{1-0.9e^{-j\omega}}$$

$$|H(\omega)| = \frac{0.1}{\sqrt{1-0.9^2-2(0.9)\cos(\omega)}}$$

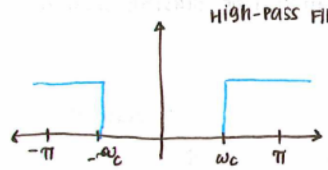
$$\angle\theta = -\tan^{-1}\left(\frac{0.9\sin\omega}{1-0.9\cos\omega}\right)$$



$|H(\omega)|$ Low-Pass Filter

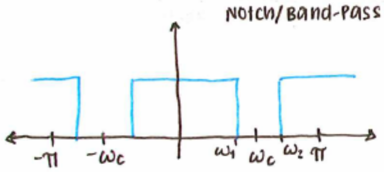


High-pass Filter

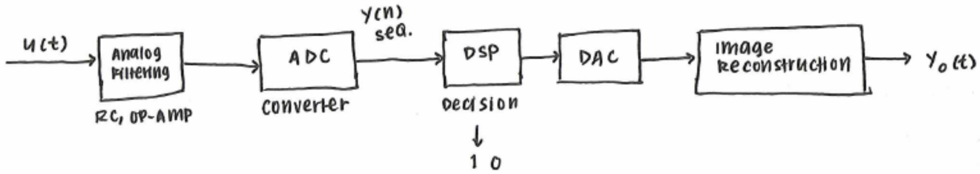


* NOTE: In real life, noise floor exists

Notch/Band-pass Filter



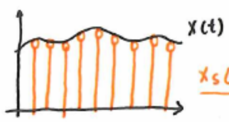
SAMPLING



consider an impulse train

$$P(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

we want to sample a function

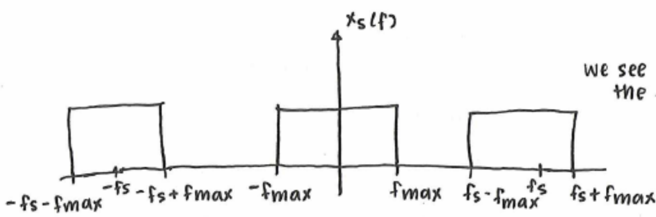


$x_s(t) = x(t) \cdot \sum \delta(t-nT)$

Multiply in time-domain
convolution in frequency domain

$$X_s(f) = X(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s)$$

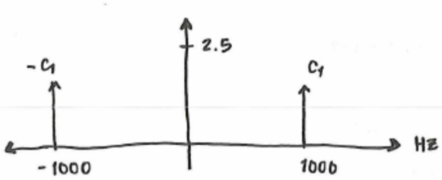
$$X_s(f) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(f-nf_s)$$



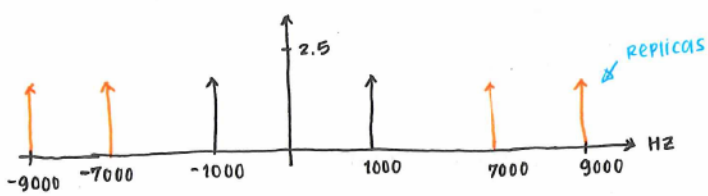
we see replicas in the frequency domain

EX. $X(f) = 5\cos(2\pi \cdot 1000t)$

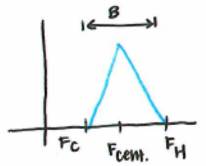
$f_s = 8000$ HZ



For replicas, sampling fast enough that no aliasing occurs.



Bandpass signal



$F_s \geq 2F_H$

$F_s \geq 2B$

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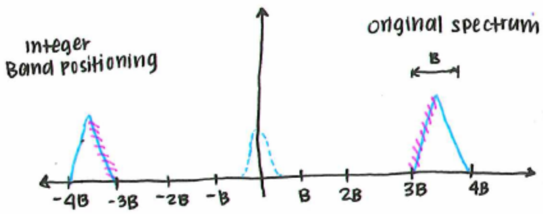
Integer Band Positioning

SEPT. 26, '13 BANDPASS SIGNAL

B: Bandwidth



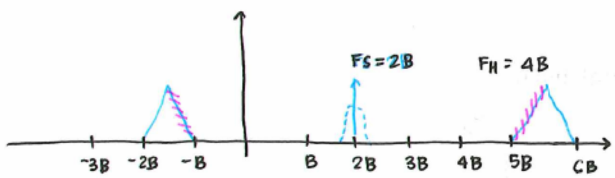
Filter everywhere but bandpass

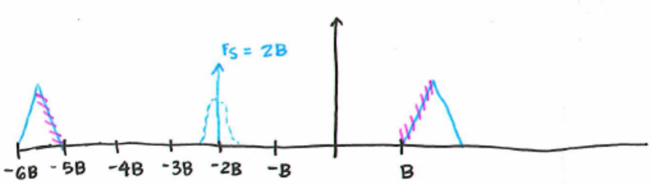
$2F_H \leq F_s$ Not optimal

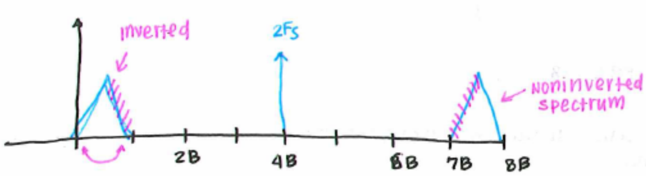
$2B \leq F_s$

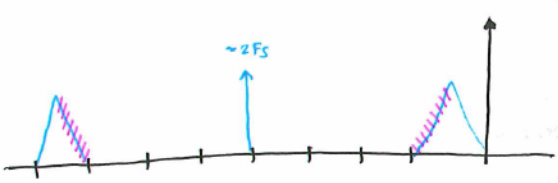
choose at least twice the bandwidth to avoid aliasing.

we can shift w/ the impulse train









$F_s \geq 2B$

Number of bands $K_{max} = \frac{F_H}{B}$

DISCRETE SIGNALS, DECIMATION IN SEQUENCE DOMAIN

we have some sampled analog signal,

$$x(n) = x_a(nT)$$

$$X(F) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_a(F - kF_s)$$

↑ Replicas

$$x_d(n) = x(nD)$$

D = Decimation Factor

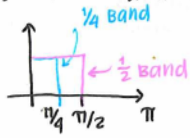
$$X_d(F) = \frac{1}{DT} \sum_{k=-\infty}^{\infty} X_a\left(F - k\frac{F_s}{D}\right)$$

$$\frac{F_s}{D} \geq 2B$$

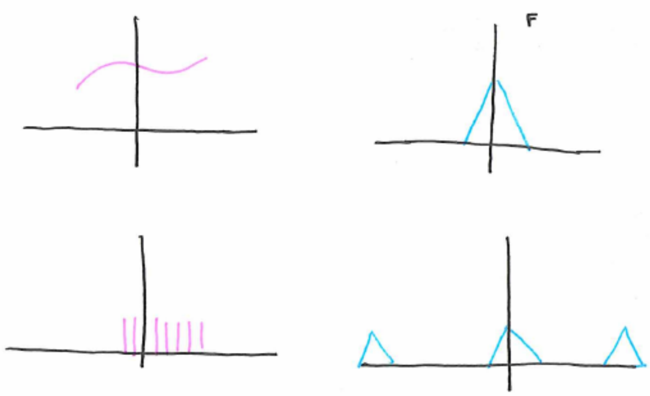
Decimation by 2 using half band digital filter.

D = 2

use Digital LPF to smooth data and avoid aliasing in the digital domain.



For D=4, use a Quarter Band digital filter.



SEE Plot 6.5 in DSP Textbook → Pg #428

By changing B, there is limit w/ B being so narrow that information in the narrow B is not discernible.

Thus, filter digitally before downsampling (decimation).

NEXT WEEK: Chpt. 6 Quantization Reading

Look at SQNR= 6.02б

* introducing something nonreversible.

From pg 388 (sampling)

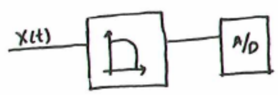
GROUP QUIZ on aliasing : show aliasing, demonstrate, hand in one sheet

EX. using 24 fps wagon wheel, fan, etc.

DUE Tuesday, sept. 30th

RELATIONSHIPS of time & freq. domain ⇒ pg # 391 *

ALIASING DISTORTION Fig 6.1.8, pg # 394



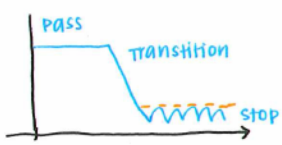
BUTTERWORTH FILTER ANALOG DOMAIN

$$|H(f)| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^{2n}}}$$

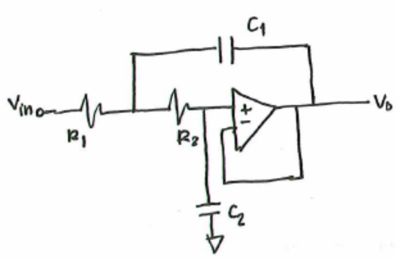
n - order of the filter

The higher the order, the better the performance

Performance cost when realization



n=2, op-amp Realization circuit



Circuit Transfer Function

$$\frac{\frac{1}{R_1 R_2 C_1 C_2}}{s^2 + \left(\frac{1}{R_1 C_2} + \frac{1}{R_2 C_2}\right) s + \frac{1}{R_1 R_2 C_1 C_2}}$$

compare Butterworth filter n=2

$$\frac{(2\pi f_c)^2}{s^2 + 1.41(2\pi f_c)s + (2\pi f_c)^2}$$

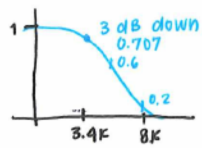
$n=2$ $\quad f_s = 8K$ $\quad f_c = 3.4K$

$C_2 = 0.01\,\mu F$

$R_1 = 6.62\,k\Omega$

$C_1 = 5\,nF$

$R_2 = 6.62\,K\Omega$



$$\text{Aliasing } \% = \frac{|H(f)|_{f=f_s-f_a}}{|H(f)|_{f=f_a}}$$

$$= \frac{\sqrt{1+\left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1+\left(\frac{(f_s-f_a)}{f_c}\right)^{2n}}}$$

$$a\% = \frac{\sqrt{1+\left(\frac{3.4}{3.4}\right)^4}}{\sqrt{1+\left(\frac{(8-3.4)}{3.4}\right)^4}} = 67.8\%$$

at 1000 Hz,

$$a\% = \frac{\sqrt{1+\left(1/3.4\right)^4}}{\sqrt{1+\left(\frac{(8-1)}{3.4}\right)^4}} = 23.05\%$$

$f_s = 16{,}000$

keep $f_c = 3{,}400$ $\quad n=2$

$$a\% = \frac{\sqrt{1+\left(\frac{3.4}{3.4}\right)^4}}{\sqrt{1+\left(\frac{(16-4)}{3.4}\right)^4}} = 10.26\%$$

$f_s = 40{,}000$, $\quad f_c = 8K$

$f_a = 8K$

$n=1$, $a\% = 34.3\%$
$n=2$, $a\% = 8.82\%$
$n=3$, $a\% = 2.21\%$
$n=4$, $a\% = 0.55\%$

The higher the order,
the sharper the transition

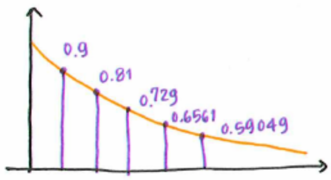
$X_q(n) = \text{quantize}[x(n)]$

* This is a lossy operation.

Define Error

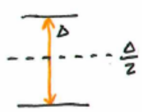
$$e_q(n) = X_q(n) - x(n)$$

Ex. when $x(n) = 0.9$



$\Delta = 0.01$ Round

$x(0) = 1$	$X_q(0) = 1$
$x(1) = 0.9$	$X_q(1) = 0.9$
$x(2) = 0.81$	$X_q(2) = 0.8$
$x(3) = 0.729$	$X_q(3) = 0.7$
$x(4) = 0.6561$	$X_q(4) = 0.7$
$x(5) = 0.59049$	$X_q(5) = 0.6$



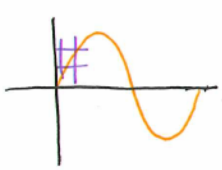
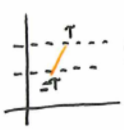
$$-\frac{\Delta}{2} \le e_q(n) \le \frac{\Delta}{2}$$

Test A/D sine wave

$$x_a(t) = A\cos(\Omega_0 t)$$

Signal-to-Noise Ratio Quantization

$$\text{SQNR} = \frac{P_x}{P_q} \begin{array}{l} \leftarrow \text{Signal Power} \\ \leftarrow \text{Noise Power} \end{array}$$

 $\Rightarrow$ 

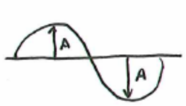
$$P_{q_\Delta} = \frac{1}{2T}\int_{-T}^{T} e_q^2(t)\,dt$$

Then Mean Square Error:

$$P_{q_\Delta} = \frac{1}{T}\int_0^T e_q^2(t)\,dt$$

$$= \frac{1}{T}\int_0^T \left(\frac{\Delta}{2T}\right)^2 t^2\,dt$$

$$= \frac{\Delta^2}{4T^3}\,\frac{t^3}{3}\Big|_0^T = \frac{\Delta^2}{12}$$



Range = 2A

$$\Delta = \frac{\text{Range}}{2^b} = \frac{2A}{2^b}$$

OCT. 3, '13

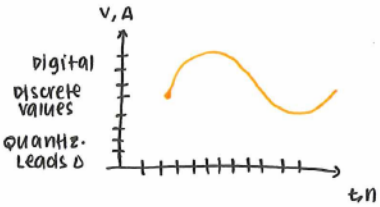
1) Finish chpt. 6

2) Read chpt. 7 - DTFT

3) Exam 1 in 2 weeks → 10/17

   1. H(t), Diff-EQ
   2. DTFT, $H(\omega)$
   3. sampling
   4. Quantization
   5. Reconstruction

QUANTIZATION



Common Range

- 0 - 5 Volts
- -5 - 5 Volts
- 0 - 10 V
- 4 - 20 mA

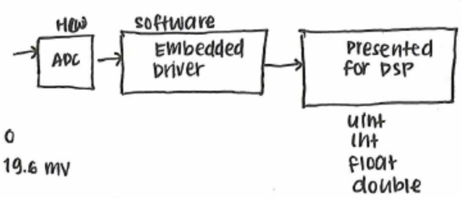
EX. 8 bits → 255
0 - 255 int.
0 - 5 int.

Quantization

unipolar

$$\Delta = \frac{R}{2^b - 1} = \frac{5-0}{2^8 - 1} = \frac{5}{255} = 19.6 \text{ mV}$$

32 bit A/D



b7	b0	
0000	0000	0
	0001	19.6 mV
0000	000010	2 × 19.6 mV

$255 \times \Delta = 5$

Bipolar,

$$\Delta = \frac{+5-(-5)}{256} = 39.06 \text{ mV}$$

Two complement form

we have 127 bits : (127)(39.06 mV) = 4.96

$$\Delta = \frac{5-(-5)}{255} = 39.215 \text{ mV}$$

39.215 * 127 = 4.98

$$P_q = \frac{4A^2}{2^{2b}(12)} = \frac{A^2}{3 \cdot 2^{2b}}$$

$$SQNR = \frac{P_x}{P_q}$$

signal

$$P_x = \frac{1}{T_p}\int_0^{T_p} (A\cos(\Omega_0 t)^2)\,dt$$

$$= \frac{A^2}{T_p}\int_0^{T_p} \frac{1}{2}(1+\cos(2\Omega_0 t))\,dt$$

$$= \frac{A^2}{T_p}\,\frac{1}{2}t\Big|_0^{T_p} = \frac{A^2}{2}$$

$$SQNR = \frac{A^2}{2}\cdot\frac{3\cdot 2^{b}}{A^2}$$

$$= \frac{3}{2}2^{2b}$$

$$SQNR\,[dB] = 10\log_{10}(SQNR)$$

$$= 10\log_{10}\left(\frac{3}{2}\right) + 10\log_{10}(2^{2b})$$

$$= 1.76 + 20\log_{10}(2^{2b}) = 1.76 + 20\log_{10}(2)$$

$$= 1.76 + 6.02b \quad [dB]$$

where $b \triangleq$ bits

statistical signals

$$SQNR = 6.02b + 16.81 - 20\log\left(\frac{R_x}{\sigma_x}\right)$$

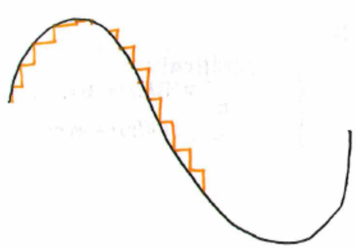
sampled, simulation Data:

$$SQNR = \frac{\cancel{\frac{1}{N}}\sum_{n=0}^{N-1} x^2(n)}{\cancel{\frac{1}{N}}\sum_{n=0}^{N-1} e^2(n)}$$

know your $x(n)$

Experimental

$$SQNR = 4.77 + 20\log_{10}\left(\frac{X_{RMS}}{|x|_{max}}\right) + 6.02b$$



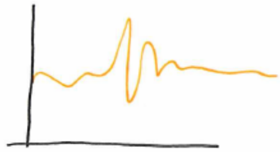
$X(t) = 4.5\sin(2\pi 100t)$

$f_s = 8000$

Range $= -5, 5$

$SQNR = 1.76 + 6.02b = 25.84$ dB

EXAMPLE : speech



f_s = 8KHz

-5-5 range

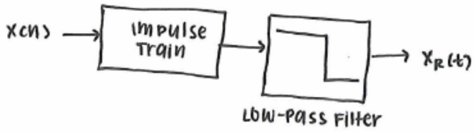
$$SQNR = 4.77 + 6.02(4) + 20\log_{10}\left(\frac{x_{RMS}}{|x|_{max}}\right) = 15 \text{ dB}$$

$$\frac{x_{RMS}}{|x|_{max}} = 0.263$$

using MATLAB → 15.01 dB

RECONSTRUCTION

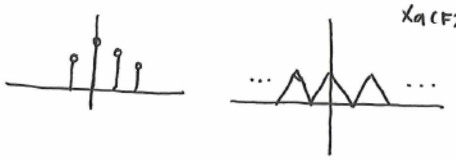
$\sum x(n)\,\delta(t - nt_s)$



$$x_R(t) = \sum_{n=-\infty}^{\infty} x(n)\, \text{sinc}(F_s(t - nT_s))$$

6.1.17 – 6.1.20

Given $x(n)$



$$X_a(F) = \frac{1}{F_s} X(F)$$

OH! OH! NOOO!

using DTFT,

$$X(F) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi F n/F_s}$$

using continuous Fourier transform,

$$x_a(t) = \int_{-F_s/2}^{F_s/2} X_a(F)\, e^{j2\pi F t}\, dF$$

$$x_a(t) = \frac{1}{F_s} \sum x(n) \left. \frac{e^{j2\pi F(t - n/F_s)}}{j2\pi(t - n/F_s)} \right|_{-F_s/2}^{F_s/2}$$

$$F_s = \frac{1}{T}$$

simplification

$e^{j2\pi(F_s/\cancel{2})(t-nT)}$ $\quad F_s = 1/T$

$\therefore = e^{j(\pi/T)(t-nT)}$

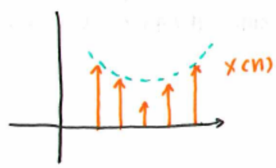
$$= \sum_{n=-\infty}^{\infty} x_a(nT) \underbrace{\frac{\sin((\pi/T)(t-nT))}{(\pi/t)(t-nT)}}_{\text{sinc function}}$$

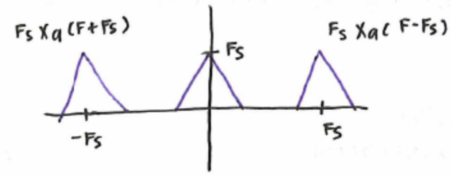
NEXT TIME : DFT $\quad H(\omega)$

OCT. 8, 2013

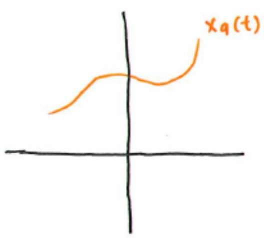
1) Reconstruction
2) DFT (chpt. 7) → FFT (chpt. 8)
3) Exam 1 – 10/17
   1. H(Z), P.F.E } signals & systems
   2. DTFT, CFT
   3. sampling
   4. quantization
   5. reconstruction

RECONSTRUCTION – Given $x(n) = x_a(nT)$

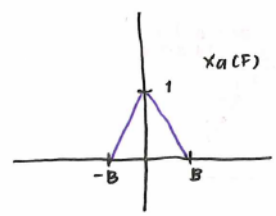




6.1.17 – 6.1.20
$$X(F) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi F n/F_s}$$
continuous



CFT



$$X_a(F) = \frac{1}{F_s} X(F) \qquad |F| \leq \frac{F_s}{2}$$
$$T = 1/F_s$$

inverse CFT $$x_a(t) = \int_{-F_s/2}^{F_s/2} X_a(F) e^{j2\pi F t} \, dF$$

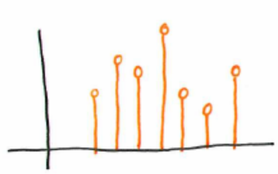
SHANNON'S RECONSTRUCTION THEOREM (ideal recon.)

$$x_a(t) = \sum_{n=-\infty}^{\infty} x(nT) \operatorname{sinc}[(\pi/T)(t-nT)]$$

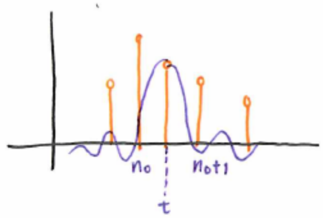
consider $t = n_0 T$

$$x_a(n_0 T) = \sum_{n=-\infty}^{\infty} x(n_0 T)(1)$$ sinc goes to 1 when arg. goes to 0.

$= x(n_0)$ value at the point



Now t located between $n_0 T$ and $(n_0+1)T$



$$x_a(t=(n_0T+t_\Delta)) = \sum_{-\infty}^{\infty} x(nT)\,\text{sinc}[\;]$$

$n = 3, \dots, 6$

$n_0 = 4 \quad T = 1$

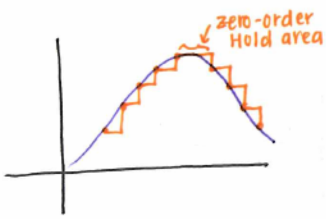
$$x_a(t) = x(3)\,\text{sinc}[(\pi)(4+t_\Delta-3)] + x(4)\,\text{sinc}[\pi(4+t_\Delta-4)] + x(5)\,\text{sinc}[\pi(4+t_\Delta-5)] + x(6)\,\text{sinc}[\pi(4+t_\Delta-6)]$$

Let $t_\Delta = 0$.

$$x_a(t) = \sum_{n=-\infty}^{\text{truncate}(t/T)} x(nT)\,\underbrace{h(t-nT)}_{\text{Reconstruction impulse Response}}$$

Zero-order Hold

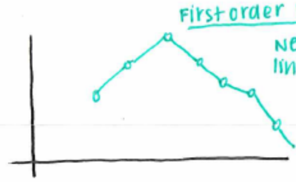
$$h_{ZOH}(t) = \begin{cases} 1 & 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases}$$



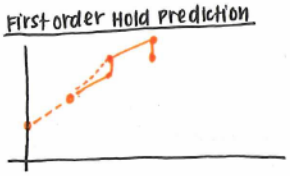
$x_a(t) = x(n) \qquad nT \le t \le nT+T$
For all n

First order Hold

Next sample linearly increases by 1.



First order Hold prediction



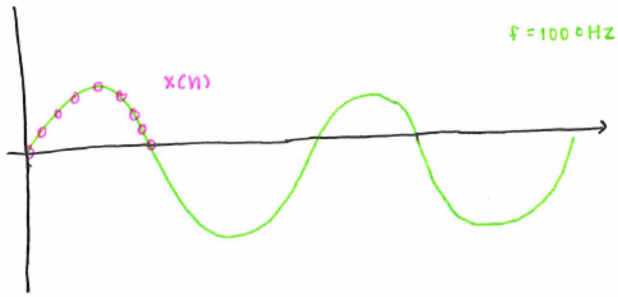
$$x_a(t) = \sum_{-\infty}^{\infty} x(n)\,\underbrace{h(t-nT)}_{h_{FOHP}}$$

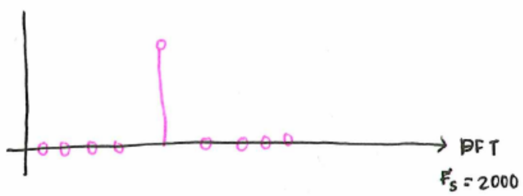
Given $X(k)$, find $x(n)$ IDFT

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j2\pi kn/N}$$

For $n = 0, 1, 2.., N-1$ so we have N^2 operations

OCT. 10, '13 DFT





IDFT

$$X(k) = \sum_{n=0}^{N-1} x(n)\, \underbrace{e^{-j2\pi kn/N}}_{\text{Twiddle Factor}}$$

$$x(n) \; F \quad \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j2\pi kn/N}$$

$\left.\begin{array}{l} T = \text{sampling period} \\ T_0 = \text{period} \end{array}\right\} \; T_0 = NT$

$$f_0 = \frac{1}{T_0}$$

when $k = 0$, $N = 4$,

$$X(k=0) = \sum_{n=0}^{3} x(n)\, e^{-j(0)}$$

$$= x(0) + x(1) + x(2) + x(3) = 10$$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$$

$$X(k=1) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi}{2}n} = x(0)\, e^{-j0} + x(1)\, e^{-j\frac{\pi}{2}} + x(2)\, e^{-j\pi} + x(3)\, e^{-j3\pi/2} = -2 + j2$$

$$X(2) = \sum_{n=0}^{3} x(n) e^{-jn\pi} = x(0) e^{-j(0)} + x(1)\, e^{-j\pi} + x(2) e^{-j2\pi} + x(3) e^{-j3\pi}$$

$$= x(0) - x(1) + x(2) - x(3) = -2$$

$$X(3) = \sum_{n=0}^{3} x(n) e^{-j\frac{3\pi n}{2}}$$

DISCRETE FOURIER TRANSFORM

Discrete in ① Freq. $X(k)$

② Time $x(n)$

so we are in the sequence domain

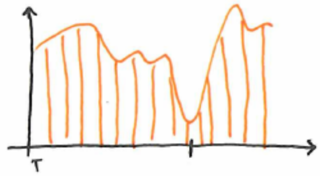
EXPONENTIAL FOURIER SERIES

$$c_k = \frac{1}{T_0}\int_{T_0} \underbrace{x(t)}_{\text{periodic}} e^{-jk\omega_0 t}\, dt$$

similar to a_n, b_n in EE234

$$f_0 = \frac{1}{T_0}$$

$$\omega_0 = \frac{2\pi}{T_0}$$



periodic

$T_0 = NT$

$x(0), \ldots, x(N-1)$ for N samples

$\int \rightarrow \Sigma$

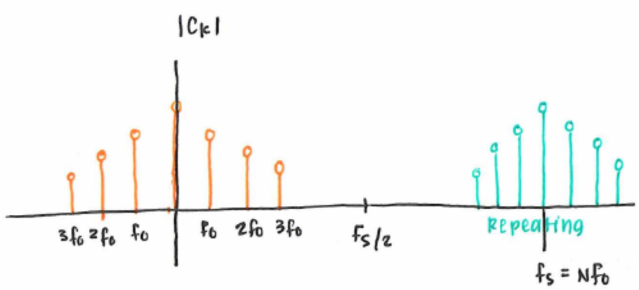
$$c_k = \frac{1}{NT}\sum_{n=0}^{N-1} x(n) e^{-j\frac{k 2\pi n T}{NT}} T$$

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

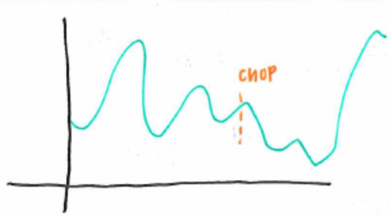
$-\infty < k < \infty$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N} \underbrace{e^{-j2\pi n}}_{\text{Always 1}}$$

$c_{k+N} = c_k$ periodic Frequency Domain



Assume periodic



$$X(k) = Nc_k = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

Given $x(n)$ – Find the k^{th} value be DFT

$$X(k) = \sum_{n=0}^{N-1} x(n) \underbrace{e^{-j2\pi kn/N}}_{\text{Twiddle Factor} = W^{+k}}$$

for $k = 0, 1, 2, \ldots, N-1$

$N \cdot N = N^2$ operations

$f_s = 10$ Hz $N = 10$ points

$T = \frac{1}{f_0} = \frac{1}{10} = 0.1$ s

$\Delta f = \frac{f_s}{N} = \frac{10}{10} = 1$ Hz

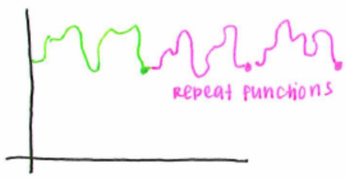
Bink corresponds to 1 Hz units,

$f_s = 10$KHz, $N = 1024$

$\Delta f = \frac{f_s}{N} = 9.776$ Hz

$f_{max} = \frac{N\Delta f}{2} = \frac{f_s}{2} = 500$ Hz

WINDOW FUNCTION



we choose an $\underline{N}$ when doing a DFT.

windowing

$x_w(n) = x(n)w(n)$ (Element-wise)

If window big enough, end at zero to avoid discontinuous jump.

(Triangle window)

$$W_T = 1 - \frac{|2n - N + 1|}{N-1}$$

Hamming window

$$W_{HM} = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)$$

Hanning window

$$W_N = 0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right)$$

FFT

1) Decimation in Time

2) Decimation in frequency

FFT - Fast   DFT - N^2 operations

$N\log(N)$

Decimation in Freq:

$X(k)$ is split into two parts

$$X(k) = \sum_{n=0}^{(N/2)-1} x(n) W_N^{kn} + \sum_{n=N/2}^{N-1} -x(n) W_N^{kn} + W_N^{(N/2)} \sum_{n=0}^{(N/2)-1} x(n+\tfrac{N}{2}) W_N^{kn}$$

$$W_N^{kn} = e^{-j2\pi kn/N} \qquad e^{-j\pi} = -1$$

$$W_N^{(N/2)} = e^{-j2\pi kn/N} = e^{-j\pi} = -1$$

$$X(k) = [\underbrace{\sum_{n=0}^{(N/2)-1} (x(n) + (-1)^k \cdot x(n+\tfrac{N}{2}))}_{\text{Reduced by } \frac{N}{2}}] W_N^{kn}$$

OCT. 22, '13 FAST FOURIER TRANSFORM

computation cost: N= Number of samples

N^2 DFT

$\frac{N}{2}\log(n)$ FFT

COOLEY-TUKEY IMPLEMENTATION

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

for $k = 0, 1, \ldots, N-1$

$W_N = e^{-j2\pi kn/N}$

split into two:

$$X(k) = \sum_{n=0}^{(N/2)-1} x(n)\, W_N^{kn} + \sum_{n=N/2}^{N-1} x(n) W_N^{kn} + W_N^{k(N/2)} \sum_{n=0}^{(N/2)-1} x(n+\tfrac{N}{2})\, W_N^{kn}$$

$W_N^{(N/2)} = e^{-j2\pi(N/2)/N} = e^{-j\pi} = -1$

$$X(k) = \sum_{n=0}^{(N/2)-1} \left(x(n) + (-1)^k\, x(n+\tfrac{N}{2})\right) W_n^{kn}$$

Even

$$X(2m) = \sum_{n=0}^{(N/2)-1} \left(x(n) + x(n+\tfrac{N}{2})\right) W_N^{2mn}$$

odd

$$X(2m+1) = \sum_{n=0}^{(N/2)-1} \left(x(n) - x(n+\tfrac{N}{2})\right) W_N^{n}\, W_N^{2mn}$$

DFT Even N/2 points $\left(\frac{N}{2}\right)^2$ complex Multiplications

DFT odd W_N^n $\frac{N}{2}$ points $\frac{N}{2} + \left(\frac{N}{2}\right)^2$ complex Mults.

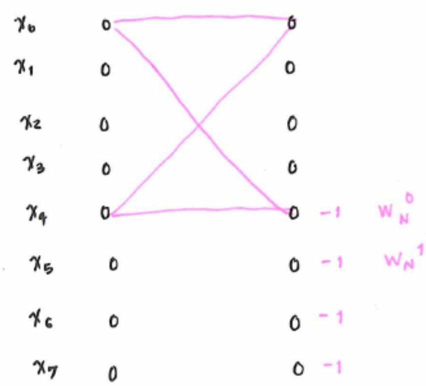
$N = 8$ $N^2 = 64$

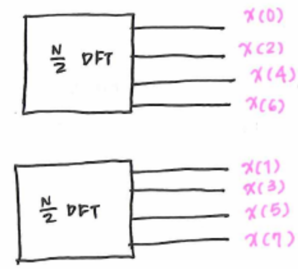
$$\left(\tfrac{N}{2}\right)^2 = \left(\tfrac{N}{2}\right)^2 + \tfrac{N}{2} + \left(\tfrac{N}{2}\right)^2$$

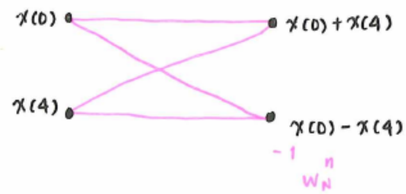
$$= 16 + 4 + 16 = 36$$

Butterfly operation

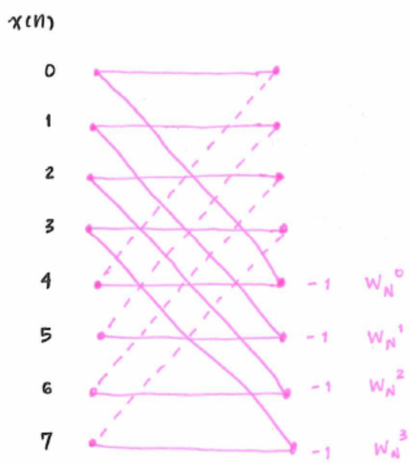
N = 8 → 12 complex Mults.



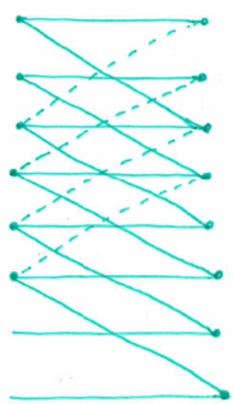




EXAMPLE



pg # 528
skip every other one



clusterfuck!

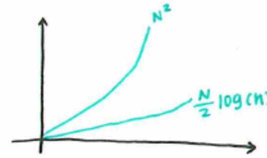
DECIMATION IN TIME

$x(n)$ Even $x(2m)$

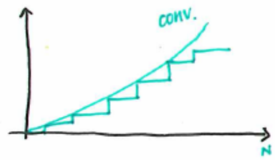
odd $x(2m+1)$

$$X(k) = \underbrace{\sum_{m=0}^{(N/2)-1} x(2m)\, W_{\frac{N}{2}}^{mk}}_{\text{DFT: } \frac{N}{2} \text{ points}} + \underbrace{W_N^k}_{\frac{N}{2} \text{ mults}} \underbrace{\sum_{m=0}^{(N/2)-1} x(2m+1)\, W_{N/2}^{mk}}_{\text{DFT: } \frac{N}{2} \text{ points}}$$

see Matlab Book for plot 5.21



plot 5.2 compatibility



$$X_1(k) = \sum_{n=0}^{N-1} x_1(n)\, e^{-j2\pi nk/N}$$

$$X_2(k) = \sum_{n=0}^{N-1} x_2(n)\, e^{-j2\pi nk/N}$$

$$X_3(k) = X_1(k)\, X_2(k)$$

$$x_3(m) = \frac{1}{N}\sum_{k=0}^{N-1} X_3(k)\, e^{j2\pi km/N}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X_1(k) X_2(k)\, e^{j2\pi km/N}$$

$$x_3(m) = \frac{1}{N}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{N-1} x_1(n)\, e^{-j2\pi kn/N}\right]\left[\sum_{l=0}^{N-1} x_2(l)\, e^{-j2\pi kl/N}\right] e^{j2\pi km/N}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(l) \sum_{k=0}^{N-1} e^{j2\pi k(m-n-l)/N}$$

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N, & l = m-n+pN \\ 0, & \text{Elsewhere} \end{cases} \qquad p = \text{integer}$$

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n)\, x_2(m-n+pN)$$

circular convolution

$x_1(n) = [2, 1, 2, 1]$

$x_2(n) = [1, 2, 3, 4]$

$$x_3(0) = \text{conv} = \sum_{n=0}^{3} x_1(n)\, x_2(-n)$$

$$x_3(0) = (2)(1) + (2)(1) + (3)(2) + (1)(4) = 14$$

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2(1-n)$$

$$= (2)(2) + (1)(1) + (2)(4) + (3)(1) = 16$$

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2(2-n) = 14$$

$$x_3(3) = \sum_{n=0}^{3} x_1(n)\, x_2(3-n) = 16$$

$$x_3(n) = [14, 16, 14, 16]$$

DFT

$$X_1(k) = \sum_{n=0}^{3} x_1(n)\, e^{-j2\pi nk/4}$$

$$= 2 + e^{-j\pi k/2} + 2e^{-j\pi k} + e^{-j2\pi k/2}$$

$$= [6, 0, 2, 0]$$

$$X_2(k) = [10, -2+j2, -2, -2-j2]$$

$$= 1 + 2e^{-j\pi k/2} + 3e^{-j\pi k} + 4e^{-j3\pi k/2}$$

$$X_3(k) = [60, 0, -4, 0]$$

IDFT

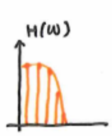
$$x_3(n) = \frac{1}{4}\sum_{k=0}^{3} X_3(k)\, e^{j2\pi nk/4} = \frac{1}{4}(60 - 4e^{-j\pi n})$$

$$= [14, 16, 14, 16]$$

circular convolution nice

Linear Filtering

$x(n)$ length L



FIR Filter $h(n)$ length M

$$y(n) = \sum_{k=0}^{M-1} h(k)\, x(n-k)$$

$y(n)$ length $L+M-1$

$Y(k) = X(k)H(k)$

N-point DFTs

Pad $x(n)$, $h(n)$ w/ zeros to ↑ N.

$h(n) = [1,2,3]$ $\quad m=3$

$x(n) = [1,2,2,1]$ $\quad L=4$

$N \geq L+m-1 = 4+3-1 = 6$

choose $N=8$ (Factor of 2)

$$X(k) = \sum_{n=0}^{7} x(n) e^{-j2\pi kn/8}$$

$$= [\,6,\ \frac{2+\sqrt{2}}{2} - j\frac{4+3\sqrt{2}}{2},\ -1-j,\ \frac{2-\sqrt{2}}{2} + j\frac{4-3\sqrt{2}}{2},\ 0,\ \frac{2-\sqrt{2}}{2} - j\frac{4-3\sqrt{2}}{2},\ -1+j,\ \frac{2+\sqrt{2}}{2} + j\frac{4+3\sqrt{2}}{2}\,]$$

$H(k) = [\,6,\ 1+\sqrt{2}-j(3+\sqrt{2}),\ -2-j2,\ 1-\sqrt{2}+j(3-\sqrt{2}),\ 2,\ 1-\sqrt{2}-j(3-\sqrt{2}),\ -2+j2,\ 1+\sqrt{2}+j(3+\sqrt{2})\,]$

$Y(k) = [\,36,\ -14.07-j17.48,\ j4,\ 0.07+j0.515,\ 0,\ 0.07\ \ j0.515,\ -j4,\ -14.07+j17.48\,]$

$$y(n) = \frac{1}{8}\sum_{n=0}^{7} Y(k) e^{j2\pi kn/8}$$

$$= [\,1,4,9,11,8,3,0,0\,]$$

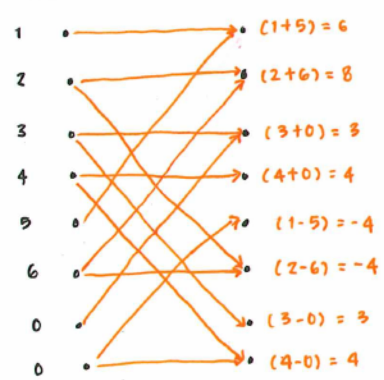
OCT. 24, '13 $\quad$ For butterfly, we have $\frac{N}{2}$ stages.

No class 10/31

8-POINT FFT $\quad x(n) = [1,2,3,4,5,6,0,0]$

* EXAM TYPE

stage 1



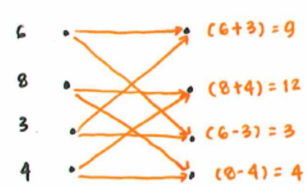
$\cdot W_8^0 = -4$

$W_8^1 = -4e^{-j2\pi(1/8)} = -2.828+j2.828$

$W_8^2 = 3e^{-j2\pi(2/8)} = -j3$

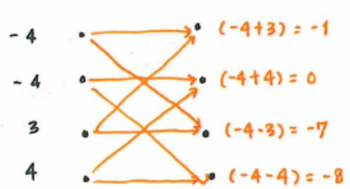
$W_8^3 = 4e^{-j2\pi(4/8)} = -2.828-j2.828$

stage 2



$\cdot W_8^0 = 3$

$W_8^2 = 4e^{-j2\pi(2/8)} = -j4$



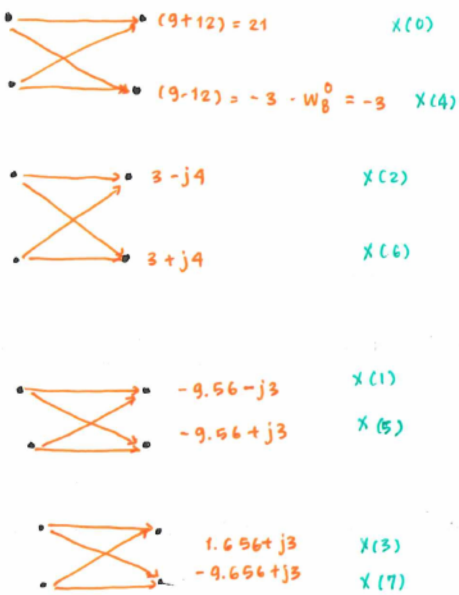
$(-4-j3)$

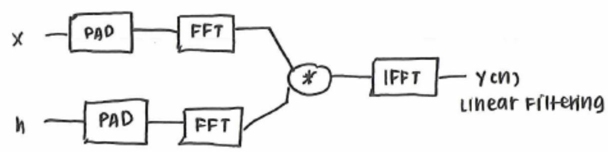
(-5.656)

$W_8^0 = -4+j3 \quad (-4+j3)$

$W_8^2 = 5.656$

stage 3





IF N > 60, use freq. Domain approach; otherwise, just convolve.

$h(n) = [1, 2, 3]$

$x(n) = [1, 2, 2, 1]$

circular convolution ≠ linear convolution

$H(k) = [6, -2-j2, 2, -2+j2]$

$X(k) = [6, -1-j, 0, -1+j]$

$Y(k) = [36, j4, 0, -j4]$

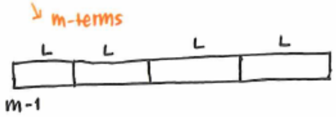
$y(n) = [9, 7, 9, 11]$

1 8 4 3

x(n) = Real time long sequence

h(n) finite

m-terms



Block processing

DFT length N = L + m - 1

Block size

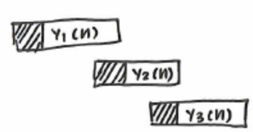
Filter size

h(n) padded with L-1 zeros

$x_1(n) = [0, 0, 0, 0, x(0), x(1), \ldots x(L-1)]$

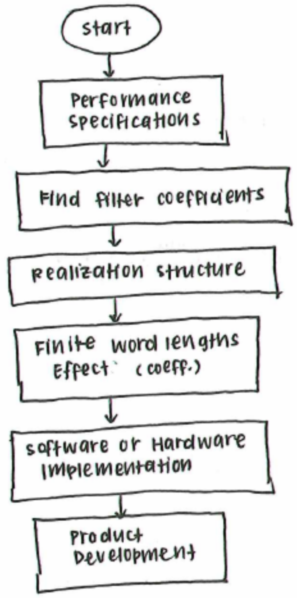
$x_2(n) = [x_1(L-m+1), \ldots, x_1(L-1), x(L), \ldots]$

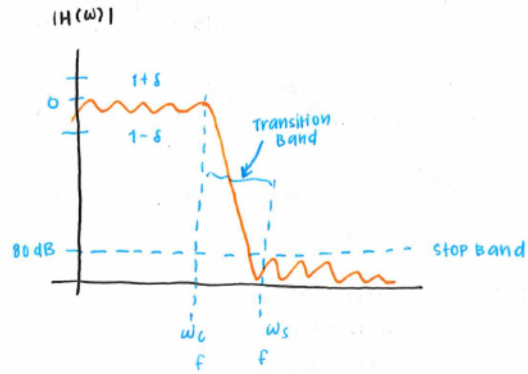
$x_3(n) = [$



OCT. 29, '13 **FILTERS**

chpt. 9, 10





Determined by Hardware, software, how fast sampling should be

sptool, fvtool, fdatool

Choice b/w FIR, IIR filtering

* IIR is recursive

writing out the difference eq'n

$$y(n) = \sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

Pulse Transfer Function

* FIR:

$$y(n) = \sum_{k=0}^{M-1} b_k (n-k)$$

Phase Distortion

$$H(\omega) = |H| e^{j\phi}$$

$$\tau_p = \frac{-\phi(\omega)}{\omega} \triangleq \text{Phase Delay}$$

Passband constant or ϕ

$$\tau_g = \frac{-d\phi(\omega)}{d\omega} \triangleq \text{Group Delay}$$

Frequency components treated equally or not

Linear Phase : FIR, Symmetry odd, Even

IIR, Forward/Reverse phase cancellation not common.

FIR TYPES

consider $N=7$

$h(0) = h(6)$

$h(1) = h(5)$

$h(2) = h(4)$

$h(3)$

DTFT

$$H(\omega) = \sum_{n=0}^{6} h(n) e^{-j\omega n} = h(0) + h(1) e^{-j\omega n} + h(2) e^{-2j\omega} + h(3) e^{-3j\omega} + h(4) e^{-4j\omega} + h(5) e^{-5j\omega} + h(6) e^{-6j\omega}$$

$$H(\omega) = e^{-j3\omega} [h(0)(e^{j3\omega} + e^{-j3\omega}) + h(1) (e^{j2\omega} + e^{-j2\omega}) + h(2)(e^{j\omega} + e^{-j\omega}) + h(3)]$$

$$= e^{-j3\omega} [2h(0) \cos(3\omega) + 2h(1) \cos(2\omega) + 2h(2) \cos(\omega) + h(3)]$$

Linear phase

For $N=8$,

$h(0) = h(7)$

$h(1) = h(6)$

$h(2) = h(5)$

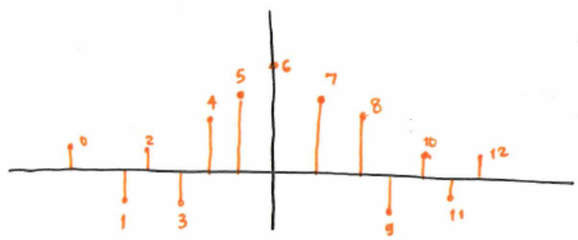
$h(3) = h(4)$

$H(\omega)$ DTFT

4 TYPES

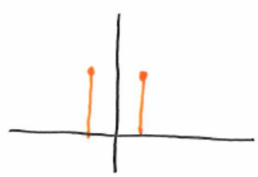
count from 0-12, total 13 terms that is order: Even b/c M-1

①



N = 13
order: Even
High
Bandpass
Band stop

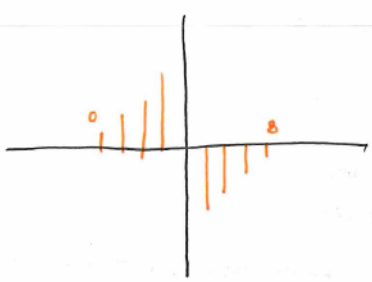
②



constant Phase Delay

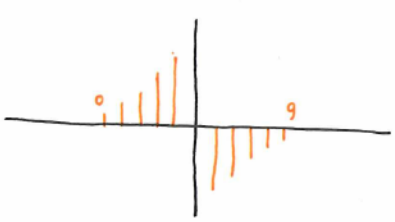
N = 12
order: odd
LP, BP, constant Phase Delay

③



N = 9
order: Even
Antisymmetric
Bandpass
constant group delay

④



N = 10
order: odd
constant Group Delay

LINEAR PHASE FIRs USING WINDOWS

$H_{desired}(\omega)$ spec.

Find $h_{desired}(n)$

$$H_d(\omega) = \sum h_j(n) e^{-j\omega n}$$

$$h_j(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} \, d\omega$$

DTFT

$$h(n) = h_j(n) \underset{\text{windowed}}{w(n)}$$

window Rect $\quad W(\omega) = e^{-j\omega(M-1)/2} \dfrac{\sin(\frac{\omega M}{2})}{\sin(\frac{\omega}{2})}$

Hanning =

Bartlett =

Blackman =

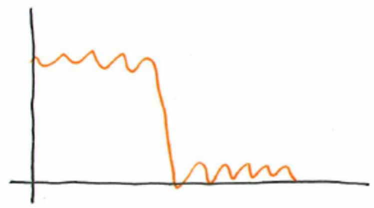
Hamming =

$$H_d(\omega) = \begin{cases} 1 e^{-j\omega(M-1)/2} & 0 < \omega < \omega_c \\ 0 & \text{Elsewhere} \end{cases}$$

$$h_j(n) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega(n - \frac{M-1}{2})} \, d\omega$$

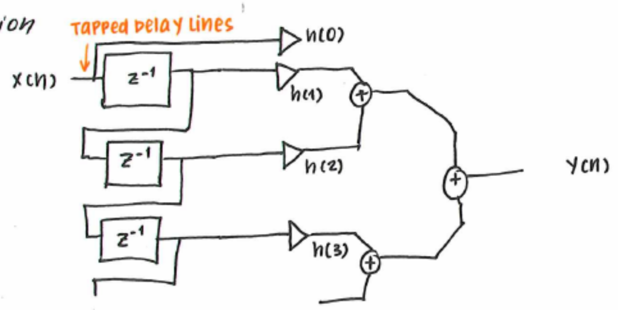
$$= \text{sinc}\left[\omega_c \left(n - \frac{M-1}{2}\right) \right]$$

$$h(n) = w(n) h_d(n)$$

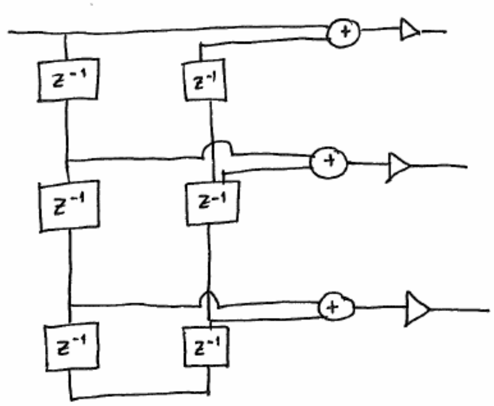


window	Peak side lobe (in stop Band)
Rectangular	-13 dB
Bartlett	-27 dB
Hamming	-32 dB
Hanning	-43 dB
Blackman	-58 dB

Realization



since multiplication is expensive, do a pre-add.



Might not have a value on the centre tap.

NOV. 5, 2013 EXAM II - Tuesday before Thanksgiving (NOV. 26)

Chpt. 7, 8, 9, 10

JCI : 10 8601

GENERAL FILTER

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

$\text{IIR} \to$ Recursive FIR

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

FIR PART:

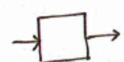
$$y(n) = \sum_{k=0}^{M-1} b_k x(n-k)$$

<u>convolution</u>

$$y(n) = \sum_{k=0}^{M-1} h(k) x(n-k)$$

Q : is it a filter or a system?

A : yes both.
Depends on complexity of expression and recursive element.



$\boxed{z^{-1}}$

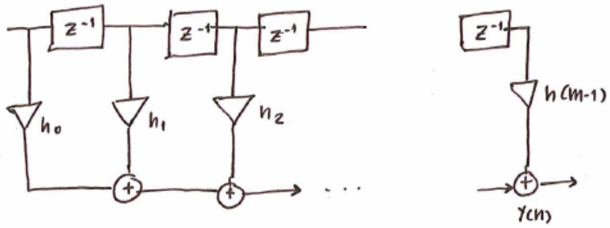
$\uparrow$
in sequence domain w/ time delay (D flip-flop)

h(1)
Multiplier (Represented as gain block)

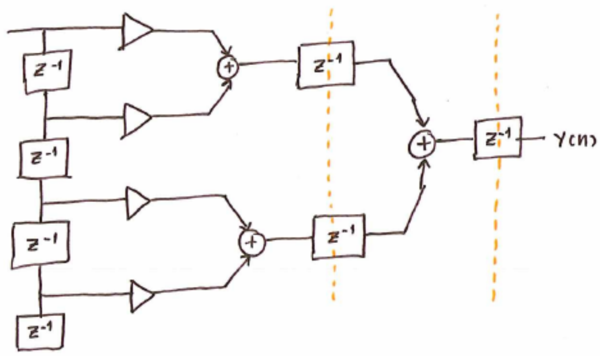
$\oplus$
summation

FIR STRUCTURES:
- Direct
- cascaded
- Transposed
- Lattice

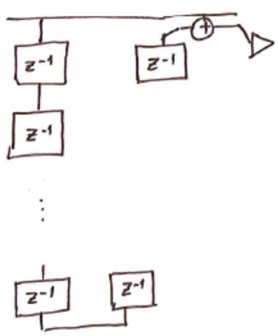
Direct Delay Line



Think of S.W. and H.W. implementation



Multiplier is expensive so when we have FIR w/ symmetry we can pre-add before multiplying. (use FIR for linear phase req., otherwise use IIR).



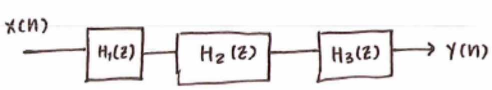
cycles are taking up silicon real estate.

cascaded structure

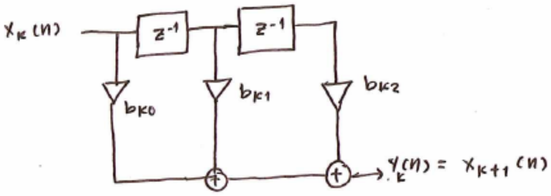
$$H_k(z) = b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2}$$

sections

$$H(z) = \prod_{k=1}^{K} H_k(z)$$

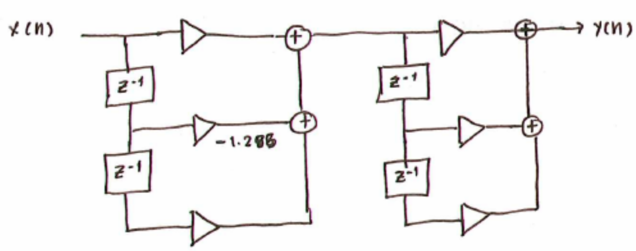


Each section 2nd order (IIR, second order bi quad)



$$H(z) = 1 - 2.818z^{-1} + 3.97z^{-2} - 2.18z^{-3} + z^{-4}$$

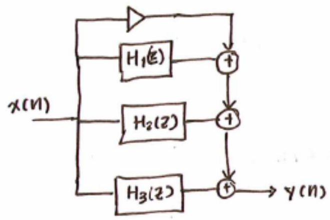
$$H(z) = H_1(z) \cdot H_2(z) = (1 - 1.285z^{-1} + z^{-2})(1 - 1.5337z^{-1} + z^{-2})$$



Parallel structures

$$H(z) = C + \sum_{K=1}^{N} \frac{A_K}{1 + P_K z^{-1}}$$

$$C = \frac{b_N}{a_N}$$



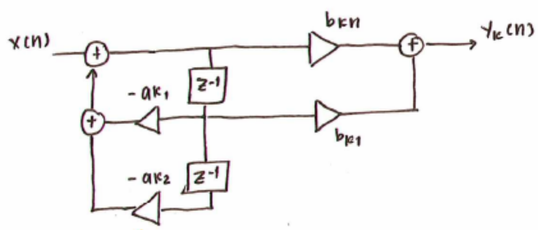
Partial Fraction Expansion

we get complex poles,

$$H_K(z) = \frac{b_{K0} + b_{K1} z^{-1}}{1 + a_{K1} z^{-1} + a_{K2} z^{-2}}$$

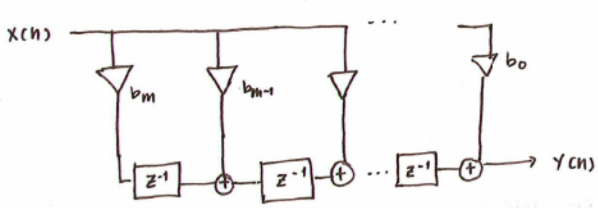
2nd order section

For second order section,



Recursive

FIR Transpose structure



D-flip flops also expensive in HW implementation.

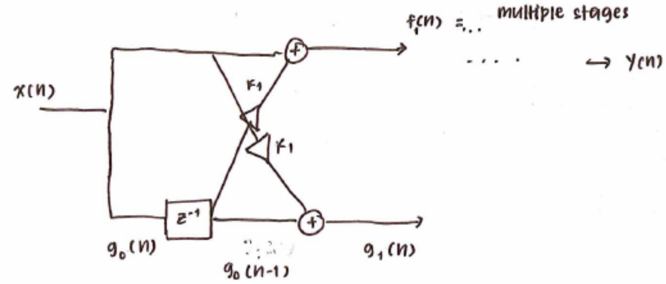
Let $W_m(n) = b_m x(n)$ at each stage

$W_K(n) = W_{K+1}(n-1) + b_K x(n)$

then

$y(n) = W_1(n-1) + b_0 x(n)$

FIR Lattice Structures



$$y(n) = x(n) + \sum_{k=1}^{m} \alpha_n(k)\, x(n-k)$$

M = 1

$$y(n) = x(n) + \alpha_\psi(1)\, x(n-1)$$

m = 2

$$y(n) = x(n) + \alpha_2(\text{-})\, x(n-\text{.}) + \alpha_1(1)\, x(n-2)$$

$$f_1(n) = x(n) + k_1 x(n-1)$$
$$y_1(n) = k_1 x(n) + x(n-1)$$

cascade

$$f_2(n) = f_1(n) + k_2\, g_1(n-1)$$
$$g_2(n) = k_2 f_1(n) + g_2(n-1)$$

$f_2(n)$ by substitution,

$$y(n) = f_2(2)$$
$$= x(n) + k_1 x(n-1) + k_2 (x(n-2))$$
$$= x(n) + k_1(1 + k_2)\, x(n-1) + k_2 x(n-2)$$

compare: $k_2 = \alpha_2(2)$

$$k_1 = \frac{\alpha_2(1)}{1 + \alpha_2(2)}$$

MATLAB Book, Pg. 290

Recursive algorithm computes k_m from α_m
$m-1 \rightarrow 1$

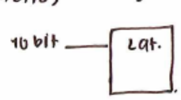
IIR Systems

- Direct
- Transpose
- Cascade
- Parallel
- Lattice / Ladder

$$y(n) = -\sum_{k=1}^{m} a_k y(n-k) + \ldots \text{FIR}$$

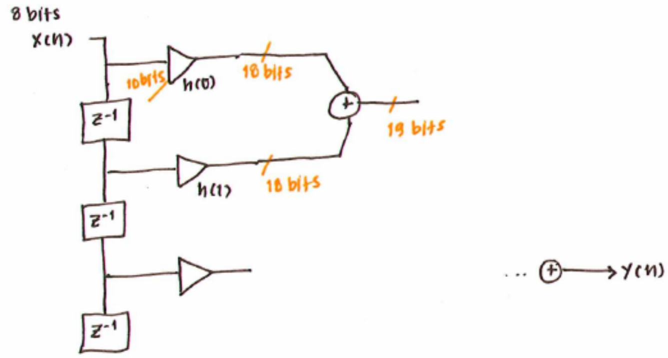
Recursive so coefficients are important to avoid next calculation to blow up :)

DA-FIR is "multiplier-less" (Distributed Arithmetic) } Developed by Les Mintzer.



For Tapped delay line...

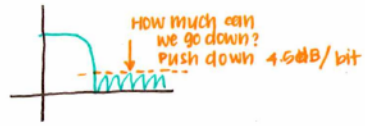
we're counting bits:



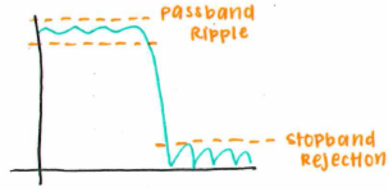
The dominating factor is the amount we need to go down

$\boxed{\text{4.5 dB per bit}}$

(ie) can't get below 45 dB for 10 bits

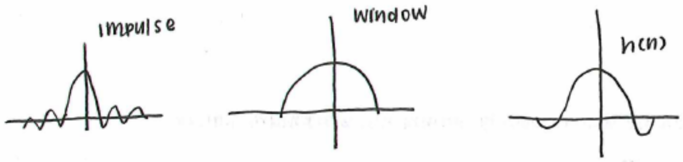


Linear Phase



FIR IMPLEMENTATION

1) window
2) Frequency sampling
3) Remez Exchange Algorithm



Equiripple allows you to specify both edges.

4.5 dB/bit Resolution : industry standard

can have IIR lattice w/ poles and/or zeros → this is a ladder-lattice.

FIR LATTICE MBP9#240

$$H(z) = \sum_{m=0}^{m-1} b_m z^{-m} = b_0\left(1 + \sum_{m=1}^{m-1} \frac{b_m}{b_0} z^{-m}\right)$$

Let $$A_{m-1}(z) = \left(1 + \sum_{m=1}^{m-1} \alpha_{m-1}(m) z^{-m}\right)$$

$$\alpha_{m-1} = \frac{b_m}{b_0} \qquad m = 1, 2, \ldots, m-1$$

Direct Form : b_n to K_m in lattice

Recursive algorithm for K_m

$$K_0 = b_0$$
$$K_{m-1} = \alpha_{m-1}(m-1)$$
$$J_m = z^{-m} A_m(z^{-1})$$
$$A_{m-1}(z) = \frac{A_m(z) - K_m J_m(z)}{1 - K_m^2}$$
$$K_m = \alpha_m(m)$$

EX. 6.8 FIR → No recursion

$$y(n) = 2x(n) + \frac{13}{12}x(n-1) + \frac{5}{4}x(n-2) + \frac{2}{3}x(n-3)$$

FIR

$$b = [2, \tfrac{13}{12}, \tfrac{5}{4}, \tfrac{2}{3}] \qquad A = \frac{b}{b_0} = [\tfrac{13}{24}, \tfrac{5}{8}, \tfrac{1}{3}]$$

$K_0 = 2$

$K_3 = A_3 = 1/3$

$J = [\frac{1}{3}, \frac{5}{8}, \frac{13}{24}]$ J is the flip-flop of A

$$A = \frac{1}{1-\frac{1}{9}}[A - \frac{1}{3}J]$$

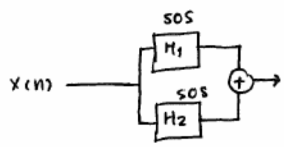
$$= \frac{9}{8}\left[\left(\frac{13}{24}, \frac{5}{8}, \frac{1}{3}\right) - \frac{1}{3}\left(\frac{1}{3}, \frac{5}{8}, \frac{13}{24}\right)\right]$$

$A = A(1:2)$

$K_2 = A(2) = \frac{1}{2}$

$K_1 = \frac{1}{4}$

IIR cascade, parallel Realizations TB. 592, Ex.9.31



FIR 1) window → smooth & truncate using Hanning, Hamming, Blackman, Kaiser

2) Freq. sampling Method → IDFT → h(n)

3) Park-m, Remez

* optimization theory

IIR 1) impulse invariant Method

2) Bilinear transforms → z

H(s) Analog continuous → z-domain

NOV. 14, '13

EXAM 2

1) DFT

2) FFT

3) Filter Realizations

4) FIR filter design

FINAL EXAM

1) IIR Filter Design

2) Multi-rate Filters

3) Adaptive Filters

4) Linear prediction

5) Exam 1

6) Exam 2

TIMELINE

R (Today) — working, HW #9

T - presentation : FIR, IIR

R - HW Review

T - Exam (26^{th})

FIR FILTER DESIGN

1) Window Method, w_c

2) Freq. sampling, sampled.

3) Remez Exchange Algorithm, all parameters.

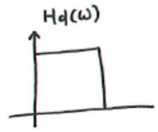
1.1 impulse
1.2 windows
1.3 MATLAB compare h(n)
1.4 fvtool

2.1 Freq. sampling
2.2 DFT
2.3 fvtool
2.4 Examples

3.1 Algorithm (min(max()))
3.2 MATLAB functions
3.3 Examples

DESIGN OF FILTER USING WINDOW

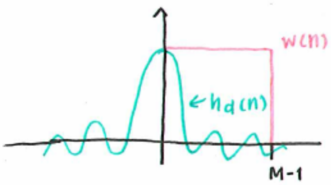
1) convert $H_d(\omega) \rightarrow h_d(n)$



desired LPF

2) Multiply window w/ impulse function

$h(n) = h_d(n)\, w(n)$

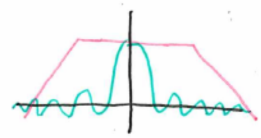


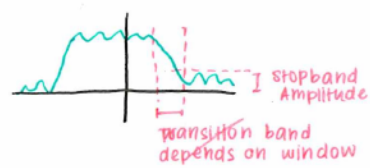
M = taps

M-1 = order

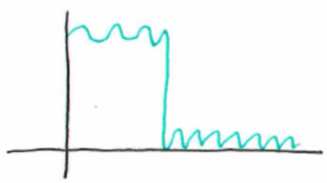
Depending on window type,
we sacrifice transition band or stopband Amplitude.

(ie)

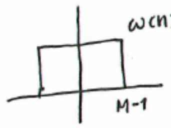




3) Freq. Resp: $H(\omega)$



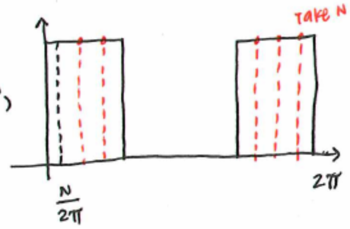
NOV. 19, '13



- Take FFT

$$W(\omega) = \sum_{k=0}^{M-1} e^{-j\omega n} = \frac{\sin(\omega M/2)}{\sin(\omega M/2)}$$

FREQUENCY SAMPLING FILTER



Linear phase

$$h(n) = \frac{1}{N} \sum H(k) e^{j2\pi kn/N}$$

$$\angle H(k) = \begin{cases} -\left(\frac{M-1}{2}\right)\left(\frac{2\pi k}{M}\right) & k=0, \frac{M-1}{2} \\ +\left(\frac{M-1}{2}\right)\left(\frac{2\pi}{M}(m-k)\right) & k=0, \frac{M-1}{2} \end{cases}$$

IFFT: Expression TB

$h(n)$ · 673 = $G(k)$

$G(k) = H_m(k)$

$m = 20, 40, 60$

NO T	1 T	2 T

Optimum Equiripple FIR Filter Design

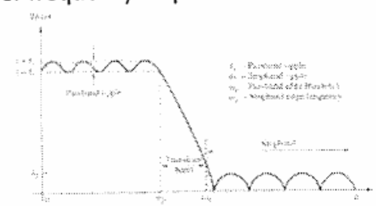
Step by Step Process

By: The Graduates

EE410 HW#9

Why choose this more complicated method?

- Precise control over critical frequencies
- Weighted approximation error spread evenly over frequency response of filter



Definitions

- $H_{dr}(\omega)$: desired filter
- Assuming Low Pass Type 1 Filter, $Q(\omega) = 1$
- Normalized weighting function

  $W(\omega) = \delta 2/\delta 1$, in the passband

  $W(\omega) = 1$, in the stopband
- Best Weighted Approximation to Hdr(w):

$$P(\omega) = \sum \alpha(k) x^k, \qquad x = \cos(\omega)$$

Remez Algorithm

- Compute δ analytically (ω_n in the range: $0 \le \pi/(M+1) < \pi$):

$$\delta = \frac{\gamma_0 H_{dr}(\omega_0) + \gamma_1 H_{dr}(\omega_1) + \cdots + \gamma_{L+1} H_{dr}(\omega_{L+1})}{\frac{\gamma_0}{W(\omega_0)} - \frac{\gamma_1}{W(\omega_1)} + \cdots + \frac{(-1)^{L+1}\gamma_{L+1}}{W(\omega_{L+1})}} \qquad \gamma_k = \prod_{\substack{n=0 \\ n \ne k}}^{L+1} \frac{1}{\cos\omega_k - \cos\omega_n}$$

- Use δ to find $P(\omega_n)$:

$$P(\omega_n) = H_{dr}(\omega_n) - \frac{(-1)^n \delta}{W(\omega_n)}, n = 0,1,\ldots,L+1$$

Algorithm (Contd.)

- Use Lagrange Interpolation to find P(ω):

$$P(\omega) = \frac{\sum_{k=0}^{L} P(\omega_k)[\beta_k/(x - x_k)]}{\sum_{k=0}^{L} [\beta_k/(x - x_k)]}$$

$$\beta_k = \prod_{\substack{n=0 \\ n\neq k}}^{L} \frac{1}{x_k - x_n}$$

Algorithm (Contd.)

- Compute the Error Function:

$$E(\omega) = W(\omega)[H_{dr}(\omega) - P(\omega)]$$

- If $|E(\omega)| \geq \delta$

  then, select a new set of frequencies corresponding to L +2 largest peaks and repeat computations beginning with δ.
- Repeat process, until $|E(\omega)| \leq \delta$

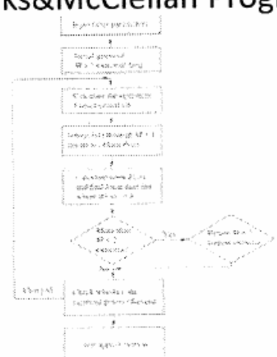
Finally,

- Optimal solution is reached:

$$H_d(\omega) = P(\omega)$$

- Use Table 10.5 to find h(n)

Parks&McClellan Program



Matlab Algorithm

- Design parameters
  1)
  - Critical frequencies: ω_p and ω_s
  2)
  - Passband Ripple (R_p) and Stopband Attenuation (A_s)
  -or-
  - Passband Tolerance (δ_1) and Stopband Tolerance (δ_2)

$$R_p = -20\log_{10}\frac{1-\delta_1}{1+\delta_1} \qquad A_s = -20\log_{10}\frac{\delta_2}{1+\delta_1}$$

Side Note: Calculating filter order

- Can we use formula given in class 4.5dB/bit and our desired stopband attenuation to calculate the order number instead of using firpmord() matlab function

[n,fo,ao,w] = FIRPMORD(f,a,dev)

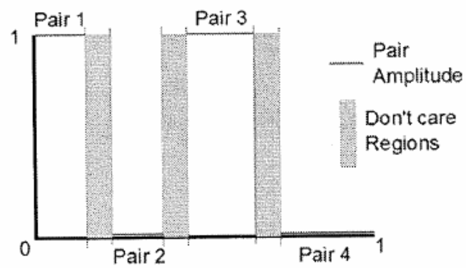
- We use this to calculate our initial order number value for filter, and all inputs for firpm.
- f = vector of cutoff frequencies
  - f = [ω_{p1} ω_{s1} ω_{s2} ω_{p2} ω_{p3} ω_{s3}];
  - Length of F must be even
- a = vector specifying the desired function's amplitude
  - a = [1 0 1 0];
- dev = [δ_1 δ_2 δ_1 δ_2]; %maximum allowable ripple size

b = FIRPM(n,f,a,w)

- n = order of filter
- f = PAIRS of normalized frequency points between 0 and 1, where 1 corresponds to the Nyquist frequency. Must be even.
  - f = [0 ω_{p1} ω_{s1} ω_{s2} ω_{p2} ω_{p3} ω_{s3} 1];
- a = desired amplitude pairs which correspond to the frequency pairs. Must be same size as f
  - a = [1 1 0 0 1 1 0 0];
- w = weights assign an importance factor for the accuracy to each f pair relative to other pairs
  - w = [1 2 1 2]; % half the length of f

Figure – Plotting f and a (firpm)

f = [0 ω_{p1} ω_{s1} ω_{s2} ω_{p2} ω_{p3} ω_{s3} 1];
a = [1 1 0 0 1 1 0 0];

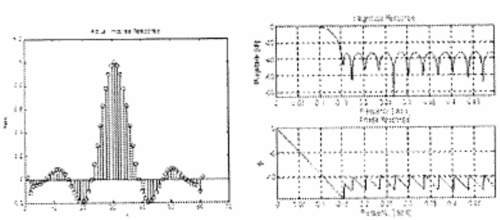


Check if correct attenuation has been reached.

- Convert discrete domain impulse response (from firpm function) into frequency domain impulse response.
- Compare:
  - IF: Maximum stopband attenuation of our optimized impulse response is more than our desired stopband attenuation, THEN increment the order and re-run firpm UNTIL less than

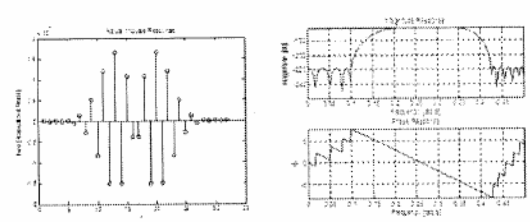
Lowpass Filter

As in example TB 10.2.3
- fpass = 0.1
- fstop = 0.15



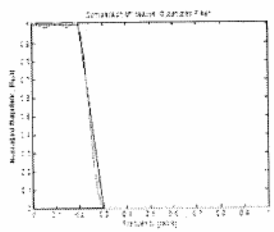
Bandpass Filter

As in example TB 10.2.4
- fpass = [0.2 0.35]
- fstop = [0.1 0.425]

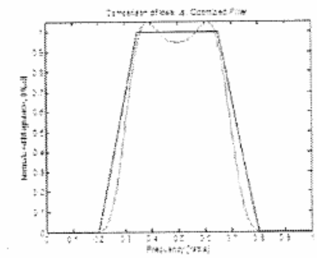


Optimized Lowpass Filter

As in MB Ex. 7.23
- fp = 0.2;
- fs = 0.3;



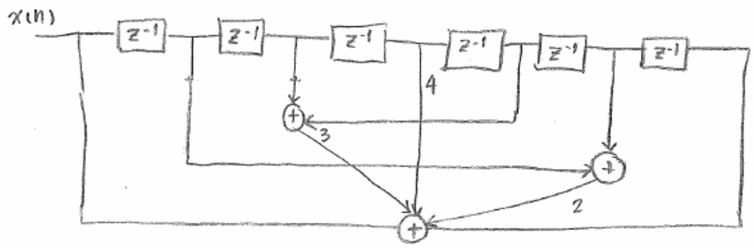
Bandpass Filter



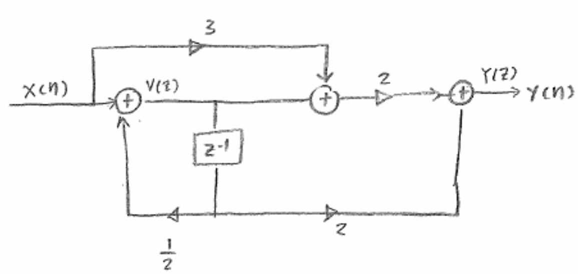
9.1) Determine a direct-form realization for the linear phase filters:

(A) $h(n) = \{ \underset{\uparrow}{1}, 2, 3, 4, 3, 2, 1 \}$

$$H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 3z^{-4} + 2z^{-5} + z^{-6}$$



9.3) Determine the system function and impulse response



$$V(z) = X(z) + \tfrac{1}{2} z^{-1} V(z)$$

$$v(n) = x(n) + \tfrac{1}{2} v(n-1)$$

$$Y(z) = 2[\, 3X(z) + V(z) \,] + 2z^{-1} V(z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$X(z) = V(z) - \frac{1}{2}z^{-1}V(z)$$

$$Y(z) = 2\,[\,3X(z) + V(z)\,] + 2z^{-1}V(z)$$

$$= 6V(z) - 3z^{-1}V(z) + 2V(z) + 2z^{-1}V(z)$$

$$= 8V(z) - z^{-1}V(z)$$

$$H(z) = \frac{V(z)\,[\,8 - z^{-1}\,]}{V(z)\,[\,1 - \frac{1}{2}z^{-1}\,]} = \frac{8 - z^{-1}}{1 - \frac{1}{2}z^{-1}}$$

$$h(n) = 8(0.5)^n u(n) - (0.5)^{n-1} u(n-1) \;//$$

9.2) $H(z) = 1 + 2.88z^{-1} + 3.4048z^{-2} + 1.74z^{-3} + 0.4z^{-4}$

$A_4 = H(z)$

$\alpha_4(4) = 0.4 = K_4$

$B_4 = 0.4 + 1.74z^{-1} + 3.4048z^{-2} + 2.88z^{-3} + z^{-4}$

$$A_3 = \frac{A_4 - KB_4}{1 - K_4^2} = 1 + 2.6z^{-1} + 2.432z^{-2} + 0.7z^{-3}$$

$B_3 = 0.7 + 2.432z^{-1} + 2.6z^{-2} + z^{-3}$

$\alpha_3(3) = K_3 = 0.7$

$$A_2 = \frac{A_3 - KB_3}{1 - K_3^2} = 1 + 1.76z^{-1} + 1.2z^{-2}$$

$B_2 = 1.2 + 1.76z^{-1} + z^{-2}$

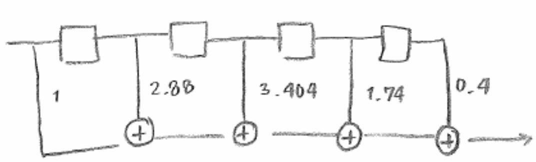
$K_2 = 1.2$

$A_1 = 1 + 0.8z^{-1}$

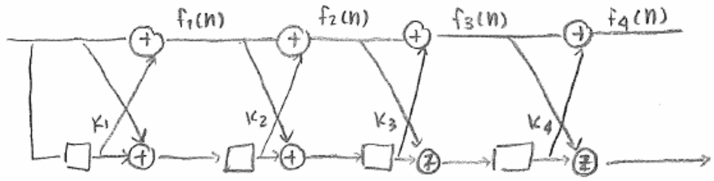
$K_1 = 0.8$

Not min. phase since $K_2 > 1$.

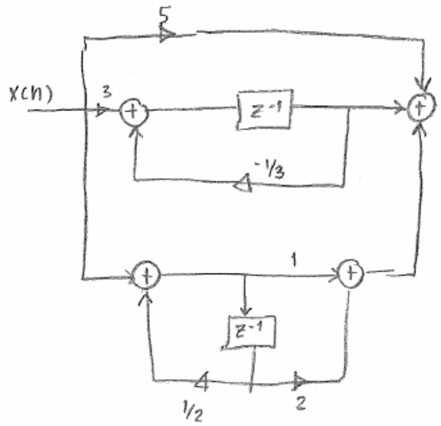
Direct



Lattice

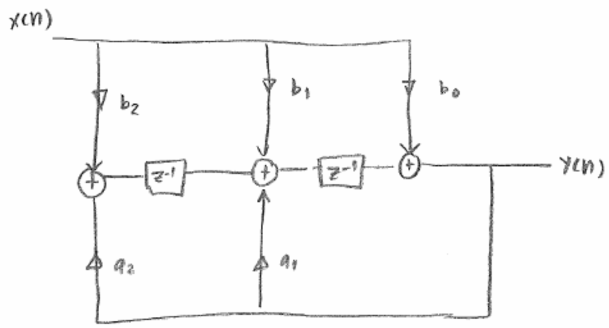


9.4) Determine the system function and impulse resp.



$$H(z) = 5 + \frac{3z}{1+\frac{1}{3}z^{-1}} + \frac{1+2z}{1-\frac{1}{2}z^{-1}}$$

9.7)



$$y(n) = a_1 y(n-1) + a_2 y(n-2) + b_0 x(n) + b_1 x(n-1) + b_2 x(n-2)$$

$$Y(z) = a_1 z^{-1} Y(z) + a_2 z^{-2} Y(z) + b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

Poles should be in unit circle to be stable.

9.9)

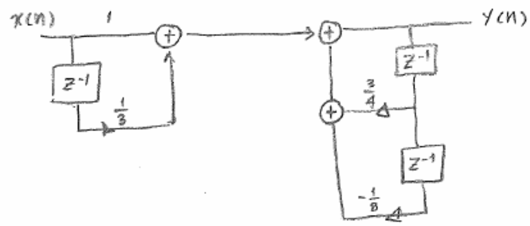
(A) $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$ $\qquad H(z) = \frac{Y(z)}{X(z)}$

$$Y(z) = \frac{3}{4}z^{-1}Y(z) - \frac{1}{8}z^{-2}Y(z) + X(z) + \frac{1}{3}z^{-1}X(z)$$

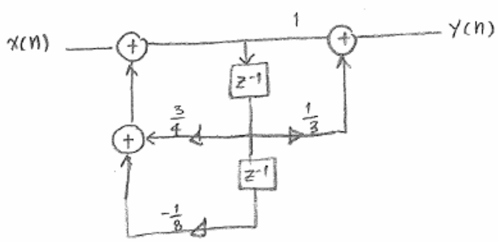
$$\frac{Y(z)\left[1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right]}{X(z)\left[1 + \frac{1}{3}z^{-1}\right]} = 1$$

$$H(z) = \frac{1 + \frac{1}{3}z^{-1}}{\underbrace{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}_{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}} = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1-\frac{1}{4}z^{-1}}$$

Direct Form I



Direct Form II



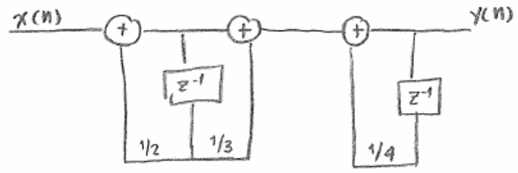
cascade

$$A = \left.\frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{1}{4}z^{-1}}\right|_{z^{-1}=2} = \frac{10}{3}$$

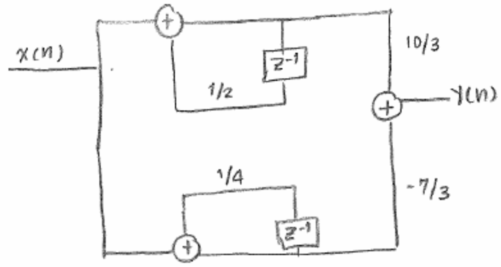
$$B = \left.\frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{1}{2}z^{-1}}\right|_{z^{-1}=4} = -\frac{7}{3}$$

$$H(z) = \frac{(10/3)}{1-\frac{1}{2}z^{-1}} + \frac{-(7/3)}{1-\frac{1}{4}z^{-1}}$$

cascade



parallel



ALANA M. SOEHARTONO
NOVEMBER 21, 2013
EE410
HW#8

TB9.9 D)

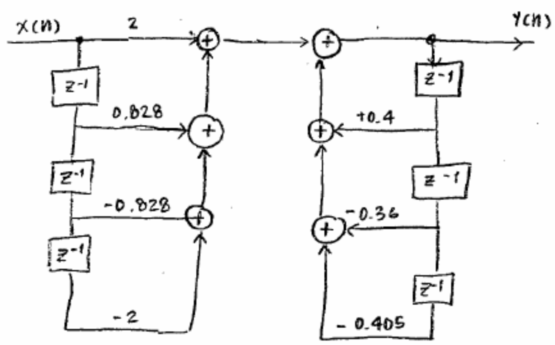
Direct Form I
Direct Form II
Cascade
Parallel

$$H(z) = \frac{2(1-z^{-1})(1+\sqrt{2}z^{-1}+z^{-2})}{(1+0.5z^{-1})(1-0.9z^{-1}+0.81z^{-2})}$$

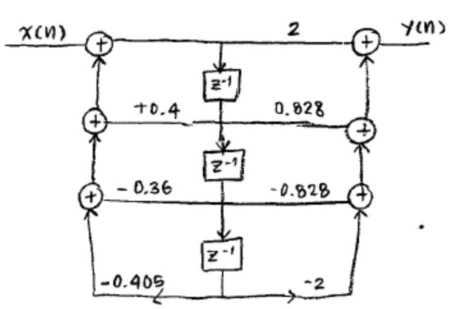
$$= \frac{2\sqrt{2}z^{-1} - 2z^{-1} - 2\sqrt{2}z^{-2} + 2z^{-2} - 2z^{-3} + 2}{(-0.4z^{-1} + 0.36z^{-2} + 0.405z^{-3} + 1)}$$

$$= \frac{0.828z^{-1} - 0.828z^{-2} - 2z^{-3} + 2}{(1 - 0.4z^{-1} + 0.36z^{-2} + 0.405z^{-3})}$$

Direct Form I

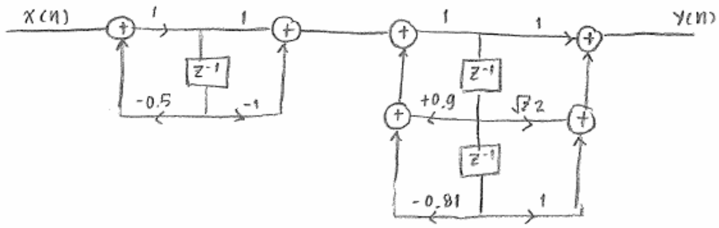


Direct Form II



cascade

$$H(z) = 2\left[\frac{(1-z^{-1})}{(1+0.5z^{-1})} \cdot \frac{(1+\sqrt{2}z^{-1}+z^{-2})}{(1-0.9z^{-1}+0.81z^{-2})}\right]$$



parallel

$$H(z) = \frac{A}{1+0.5z^{-1}} + \frac{B+Cz^{-1}}{(1-0.9z^{-1}+0.81z^{-2})}$$

$$A = \left.\frac{2(1-z^{-1})(1+\sqrt{2}z^{-1}+z^{-2})}{(1-0.9z^{-1}+0.81z^{-2})}\right|_{z^{-1}=\frac{-1}{0.5}} = \frac{13.0294}{6.04} \approx 2.157$$

$$2\left[\frac{(1-z^{-1})}{(1+0.5z^{-1})} \cdot \frac{(1+\sqrt{2}z^{-1}+z^{-2})}{(1-0.9z^{-1}+0.81z^{-2})}\right] = \frac{A}{1+0.5z^{-1}} + \frac{B+Cz^{-1}}{(1-0.9z^{-1}+0.81z^{-2})}$$

$$2(1-z^{-1})(1+\sqrt{2}z^{-1}+z^{-2}) = A(1-0.9z^{-1}+0.81z^{-2}) + (B+Cz^{-1})(1+0.5z^{-1})$$

$$2+0.828z^{-1}-0.828z^{-2}-2z^{-3} = A-0.9Az^{-1}+0.81Az^{-2}+B+0.5Bz^{-1}+Cz^{-1}+0.5Cz^{-2}$$

Matching terms,

$$z^{-3}[-2] = z^{-3}[0]$$

$$z^{-2}[-0.82] = z^{-2}[0.81A+0.5C]$$

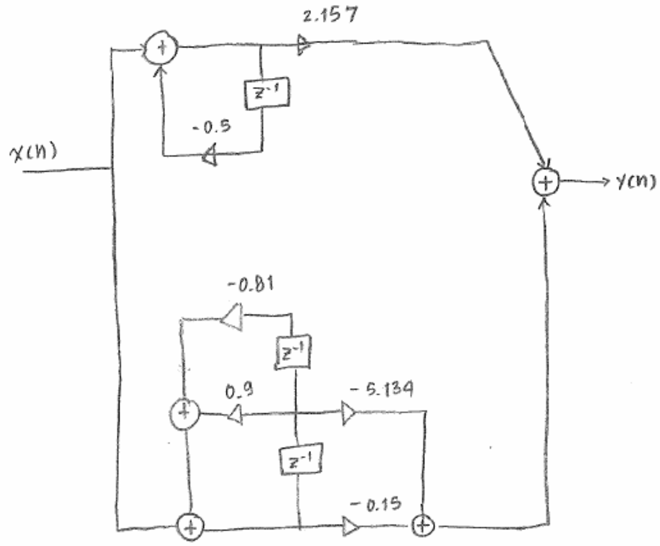
$$C = \frac{-0.82-0.81(A)}{0.5} = -5.134$$

$$z^{-1}[0.828] = z^{-1}[-0.9A+0.5B+C]$$

$$B = \frac{0.828+0.9A-C}{0.5} \qquad ??$$

$$z^{0}[2] = z^{0}[A+B]$$

$$B = 2-A = -0.157$$

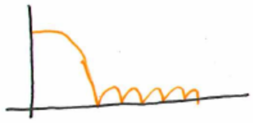

2.157
z^{-1}
-0.5
x(n)
y(n)
-0.81
z^{-1}
0.9
-5.134
z^{-1}
-0.15

DEC. 3, '13

IIR FILTER DESIGN

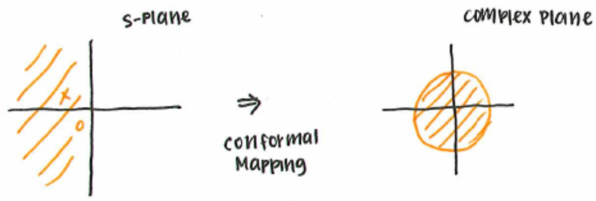
Less taps, simpler

FIR - Linear phase



APPROACH

Design H(s)
Analog filter proto
Map to H(z)
complex plane



Analog Filters (Ω - Analog, ω - Digital)

1. Butterworth
2. Chebyshev 1
3. Chebyshev 2
4. Elliptical

① $$|H_a(s)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$

$\Omega_c \triangleq$ cutoff
$N \triangleq$ order
$\varepsilon \triangleq$ Ripple param.
$= \sqrt{10^{R_p/10}-1}$

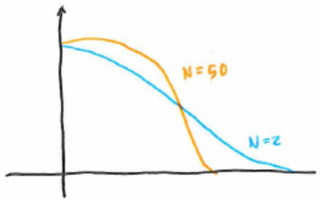
② $$|H_a(s)|^2 = \frac{1}{1+\left[\varepsilon^2 T_N^2\left(\frac{\Omega}{\Omega_c}\right)\right]}$$

$T_N \triangleq$ N^{th} order chebyshev polynomial
U_N = N^{th} order Elliptical Jacobian Func.

③ $$|H_a(s)|^2 = \frac{1}{1+\left[\varepsilon^2 T_N^2\left(\frac{\Omega}{\Omega_c}\right)\right]^{-1}}$$

④ $$|H_a(s)|^2 = \frac{1}{1-\varepsilon^2 U_N^2\left(\frac{\Omega}{\Omega_c}\right)}$$

$|H_a(s)|$



Given: Ω_p = passband freq.

Ω_s = stopband freq.

R_p = passband ripple

A_s = stopband Attenuation

pg# 413 MB

Butter, find N-order

Ω_c

Design eq'n. to calculate

chebyshev 1, 2

Elliptical

Find ε, Ω_c, N, H(s)

EX. 8.54

a

b

$$H_a(s) = \frac{K}{\prod(s - p_K)}$$

$$\varepsilon = \sqrt{10^{R_p/10} - 1}$$

$$A = 10^{A_s/20}$$

$$\Omega_c = \Omega_p$$

$$\Omega_{ratio} = \frac{\Omega_s}{\Omega_p}$$

$$g = \sqrt{(A^2-1)/\varepsilon^2}$$

$$N = \left\lceil \frac{\log_{10}(g + \sqrt{g^2-1})}{\log_{10}(\Omega_{ratio} + \sqrt{\Omega_R^2 - 1})} \right\rceil$$

Mappings

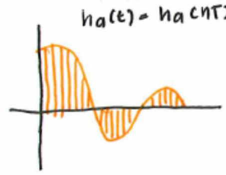
1. Impulse invariance method
2. Bi-linear transform

$z = e^{st}$

T = sample time

$H_a(s)$

$h_a(t) \cdot h_a(nT)$



1. IMPULSE INVARIANCE METHOD

Now, given w_p, w_s, R_p, A_s, find $H(z)$

1. choose sampling T

$$\Omega_p = w_p/T$$

$$\Omega_s = w_s/T$$

2. Design $H_a(s)$ given $\Omega_p, \Omega_s, R_p, A_s$

3. Partial fraction expansion

$$H_a(s) = \sum \frac{K_k}{s - P_k}$$

4. Map the poles

$$s - P_k \rightarrow 1 - e^{P_k T} z^{-1}$$

$$H(z) = \sum_{k=1}^{N} \frac{R_k}{1 e^{P_k T} z^{-1}}$$

5. Multiply out if necessary

EX. $H_a(s) = \dfrac{s+1}{s^2+5s+6} = \dfrac{2}{s+3} - \dfrac{1}{s+2}$

$P_1 = -3$

$P_2 = -2$

Let $T = 0.1$

$$H(z) = \frac{2}{1 - e^{-3T} z^{-1}} - \frac{1}{1 - e^{-2T} z^{-1}}$$

$$= \frac{1 - 0.896 z^{-1}}{1 - 1.56 z^{-1} + 0.600 z^{-2}}$$

Pg. # 427-429 MB

$w_p = 0.2\pi$

$w_s = 0.3\pi$

$R_p = 1$ dB

$A_s = 15$ dB

N = 6 Butterworth

$H(z) = (\quad) + (\quad) + (\quad)$

cheby-1

$$H(z) = \frac{-0.083 - 0.0242z^{-1}}{1 - 1.5z^{-1} + 0.84z^{-2}} + \frac{-0.083 + 0.024z^{-1}}{1 - 1.566z^{-1} + 0.65z^{-2}}$$

cheby-2 }
Elliptical } These Methods failed (!!)
use Bi-linear trans.

2. BI-LINEAR TRANSFORM

$$s = \frac{2}{T}\,\frac{1 - z^{-1}}{1 + z^{-1}}$$

$$z = \frac{1 + sT/2}{1 - sT/2}$$

$$z = re^{jw}$$

$$s = \sigma + j\Omega$$

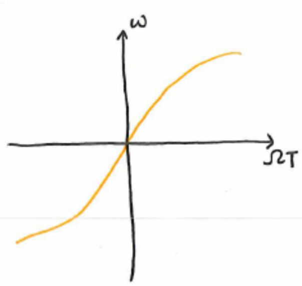
$$s = 2\,\frac{re^{jw} - 1}{re^{jw} + 1}$$

$$\sigma + jw = \frac{2}{T}\left(\frac{r^2 - 1}{1 + r^2 + 2r\cos w} + j\,\frac{2r\cdot\sin w}{1 + r^2 + 2r\cos w}\right)$$

$r = 1, \quad \sigma = 0$

$$\Omega = \frac{2}{T}\tan\left(\frac{w}{2}\right)$$

$$w = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)$$



2. BI-LINEAR $\qquad w_p, w_s, R_p, A_s$

1. choose T
2. prewarp :

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right)$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{w_s}{2}\right)$$

3. H(s) design
4. For every $s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$
5. simplify

EX. $\quad H(s) = \dfrac{s+1}{s^2+5s+6}$

Let T = 1

$$H(z) = \frac{2\frac{1-z^{-1}}{1+z^{-1}} + 1}{\left(2\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 5\left(2\frac{1-z^{-1}}{1+z^{-1}}\right) + 6}$$

$$= \frac{3+2z^{-1}-z^{-2}}{20+4z^{-1}} = \frac{0.15+0.1z^{-1}-0.05z^{-2}}{1+0.2z^{-1}}$$

Benefits: stability, good results

MB. Pg. 436 - 441

Given w_p, w_s, R_p, A_s

4 proto analog filters

compare Results

Ω_p = prewarp EQ'n.

Ω_s =

DEC. 10, '13 *FINAL EXAM - 5 Q'S.*

12/17 - 5-7PM

1. DFT, DTFT, FFT
2. Sampling/Quantiz./Recons.
3. FIR Filter Design,
4. IIR Filter Design
5. (1) Multirate
   (2) Linear Prediction
   (3) Adaptive Filtering

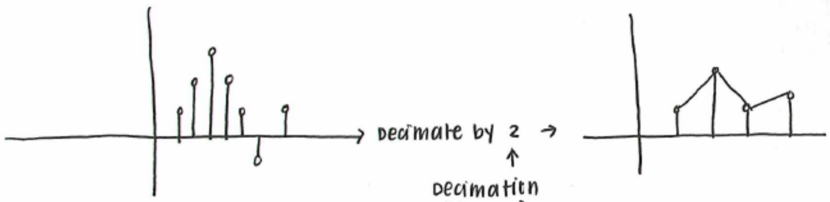
Convolution: Circular, Linear

* Thurs. - Teach Evaluations

MULTI-RATE SAMPLING

- Vandanythan

· a signal processing application



Decimation Factor, D

symbol: $\boxed{\downarrow D}$ so here we have $\boxed{\downarrow 2}$

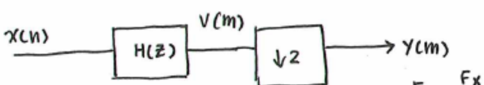
$y(m) = x(mD)$

$y(1) = x(1(2)) = x(2)$

$y(2) = x(2(2)) = x(4)$

$\vdots$

so here we see mathematically we are skipping samples by 2.



Aliasing Filter $h(n)$

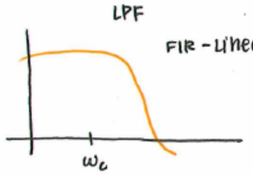
$F_x = \frac{1}{T_s}$

$\omega_c = \frac{\pi}{2} = \frac{\pi}{D}$

$F_y = \frac{F_x}{D}$

$\therefore F_y = \frac{1}{2} F_x$

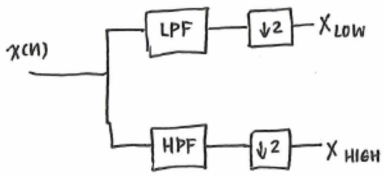
LPF



FIR - Linear Phase

$$v(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k)$$

$$y(m) = v(mD) = \sum_{k=0}^{\infty} h(k)\, x(mD-k)$$

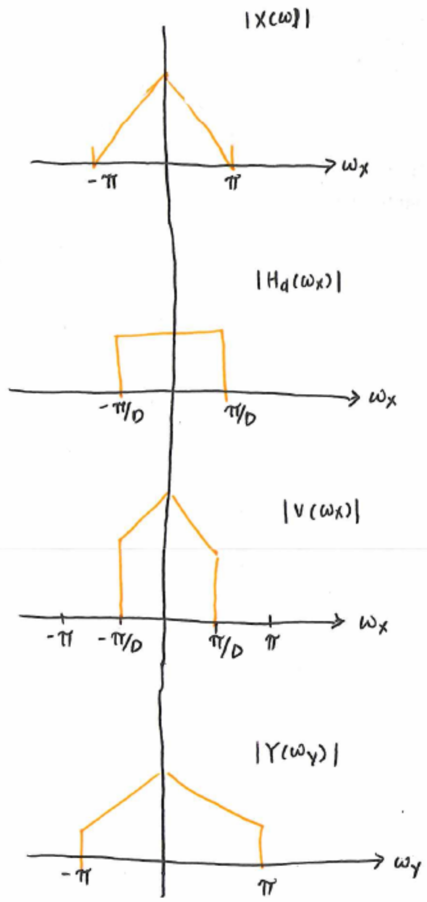


wavelet Transform – A decimation-type compression method

(EX) JPG-2000 compression

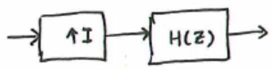
LL	HL
LH	HH

when we have



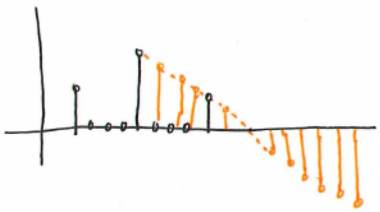
$M = 30$
0.1 dB ripple
30 dB stopband rejection

Interpolation I-factor



FIR

insert zeros



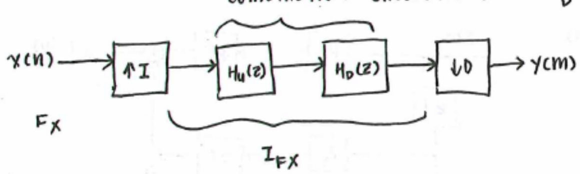
$$y(m) = \sum h(m-kI)\,x(k)$$

$$\omega_c = \frac{\pi}{I}$$

For an image with an $\frac{I}{D}$ ratio, we can use re-sampling, decimation and interpolation for drag & drop resizing.

drag

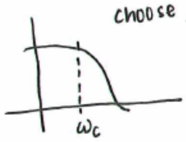
combine H(z) choose worst case: $\frac{\pi}{D}$ or $\frac{\pi}{I}$ for filter



$\frac{I}{D} F_x = F_y$

EX. $D = 2$, $I = 5$

choose $\frac{\pi}{I} = \frac{\pi}{5}$



Time-varying periodic coefficients by change of variable, pg # 764, 765, in TB.

$g(n,m) = h(nI + (mD)I)$

$g(0,m) = h(0) \quad h(2) \quad h(4) \quad h(1) \quad h(3)$

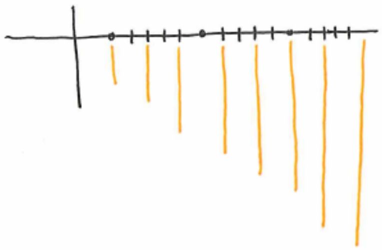
$g(1,m) = h(5) \quad h(7) \quad h(9) \quad h(6) \quad h(8)$

$g(2,m) =$ $\vdots$

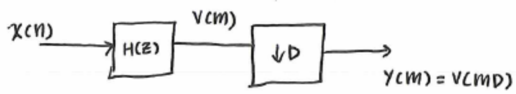
$\vdots$ $h(29) \quad h(26) \quad h(28)$ we start seeing banks of coefficients.

Remez
30 coeffs.
0.1 ripple
30 dB stopband
$\omega_c = \frac{\pi}{5}$

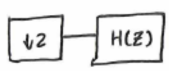
we find coefficients to keep w/ periodicity that looks like



POLY PHASE FILTER REALIZATIONS



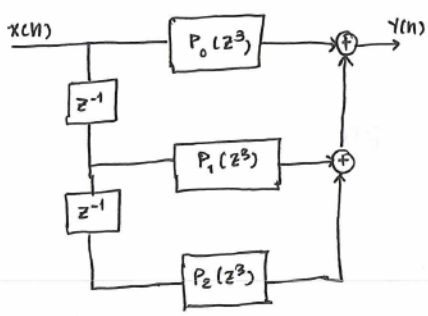
Run H(z) at high rate!
can I do that at a lower rate?

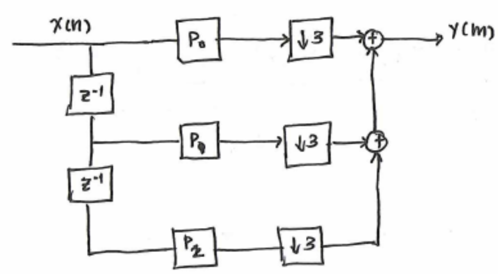


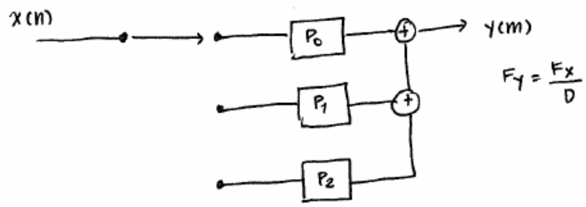
$$H(z) = \sum_{i=0}^{M-1} z^{-1} P_i(z^{n})$$

Noble identities

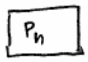
$$P_i(z) = \sum_{n=-\infty}^{\infty} h(nm+i)\, z^{-n}$$





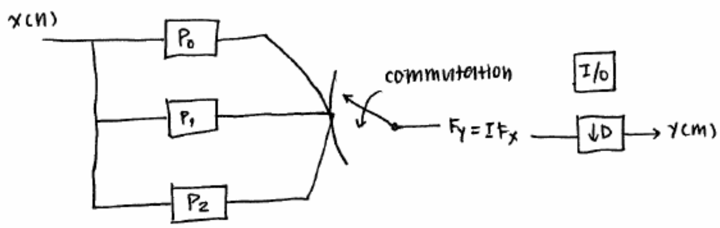


commutative Rate F_x



Filter coeff. sequence

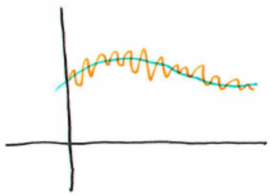
Interpolation Realization



DEC. 12, '13 LINEAR PREDICTION & ADAPTIVE FILTERS

DSP APPS

Kalman - to moon and back



"Estimation"
- Random variable:
- stationary Random Process

Digital controls, modern

$$x = \underset{\sim}{A} x + \underset{\sim}{B} u + \underline{\underline{w}}$$

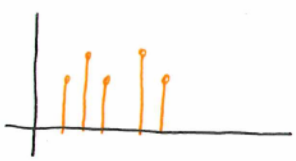
$$Y = \underset{\sim}{C} x + \underset{\sim}{D} u + \underline{\underline{v}}$$

$w \triangleq$ Process Noise

$v \triangleq$ Measurement Noise

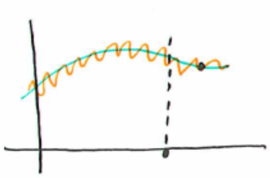
* white, Gaussian, σ, μ

Stochastic system (statistical)



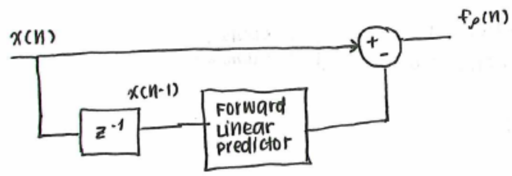
Estimate $\hat{x}(n) = -\sum_{k=1}^{p} a_p(k)\, x(n-k)$

By convention: $-a_p(k)$



$$f_p(n) = x(n) - \hat{x}(n)$$
$$= x(n) + \sum_{k=1}^{p} a_p(k)\, x(n-k)$$

implementations typically lattice: criteria $\rightarrow h(k)$



Minimize mean, square error.

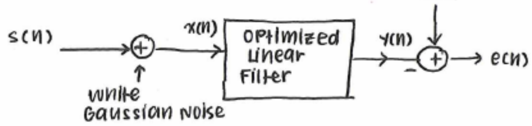
$$\gamma_{xx}(\ell) = -\sum_{k=1}^{p} a_p(k)\, \gamma_{xx}(\ell-k)$$ ← coefficients

$\ell = 0, 2, 3, \ldots, p$
set linear eq'ns.

$\gamma \triangleq$ Auto-correlation sequence of $x(n)$

$\underbrace{\text{Wiener}}_{\text{whole set}}$ Filter, $\underbrace{\text{Kalman}}_{\text{Recursively solved}}$ Filter

Wiener Filter Model



* Minimizes mean square error

* zero μ, wide sence stationary

FIR WIENER FILTER

$$y(n) = \sum_{k=0}^{M-1} h(k)\, x(n-k)$$

MSE, $d(n)$, $y(n)$

Expected $|e(n)|^2$

$$E\left| d(n) - y(n) \right|^2$$

$$E\left| d(n) - \sum h(k)\, x(n-k) \right|^2$$

quadratic : $h(k)$

Derivative

$$\sum_{k=1}^{M-1} h(k)\, \gamma_{xx}(\ell-k) = \gamma_{dx}(\ell)$$

↑ Autocorrelation ↑ cross-correlation

$x(n) = s(n) + w(n)$

$s(n) = 0.6\, s(n-1) + v(n)$

$v(n) \Rightarrow \sigma_v^2 = 0.64$

$w(n) \Rightarrow \sigma_w^2 = 1$

Use M=2 filter

$$\left.\begin{aligned} 2h(0) + 0.6h(1) &= 1 \\ 0.6h(0) + 2h(1) &= 0.6 \end{aligned}\right\} \text{2 eq'ns., 2 unknowns}$$

$h(0) = 0.451$
$h(1) = 0.165$

ADAPTIVE FILTERS – "Echo cancellation"

LMS : Least Mean square ← still recursive, but simple.

RLS : Recursive Least square

LMS – solving minimization-type problem

$$\sum_{k=0}^{m-1} h(k)\, r_{xx}(l-k) = r_{dx}(l+D)$$

D.

set linear solve

For non-stationary signals:

$r_{xx} \to \gamma$ Actual autocorrelation

$r_{dx} \to \gamma$ Actual cross-correlation

These aren't sample estimates of auto-correlation

LMS : steepest Descent } Numerical Analysis
conjugate Gradient

$$\vec{h}_m(n+1) = \vec{h}_m(n) + \frac{1}{2}\Delta_n \vec{S}(n)$$

At each step in time. speed of convergence

RLS – using Kalman Estimation (algorithm on pg 920)

Algorithm:

1. $\vec{a} = \underset{\sim}{X}_m \vec{h}_m(n-1)$
2. $e_m(n) = d(n) - \hat{d}(n)$
3. K_m : Kalman gain
4. $\underset{\sim}{P}$: correlation Matrix
5. $\vec{h}_m(n) = \vec{h}_m(n-1) + \underset{\sim}{K}_m(n)\, e_m(n)$

EE410-001 FINAL EXAM R: 12/17 5.30-7.30PM

1. H(Z), Y(n), Diff. EQ.
2. DFT, FFT, DTFT
3. sampling, quant., recon.
4. FIR, IIR filter design
5. convolution: circular, Linear

ALANA M. SOEHARTONO
DECEMBER 12, 2013
EE410 - DSP
HW #10

TB10.10) use the Bilinear Transformation to convert the analog filter with the system function

$$H(s) = \frac{s+0.1}{(s+0.1)^2 + 9}$$

into a digital IIR filter. select $T = 0.1$ and compare the location of the zeros in $H(z)$ with the locations of zeros obtained by applying the impulse variance method of the conversion of $H(s)$.

$$H(z) = \frac{b}{s+a}$$

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) = \alpha + j\Omega$$

zeros: $s = -0.1$

Poles: $p = \alpha \pm j\omega = -0.1 + j3$
$= -0.1 - j3$

Mapping: $z = e^{sT}$
$= e^{-0.1T} = e^{-0.1(0.1)} = 0.99$

$$z_k = e^{p_k T}$$

$$z_1 = e^{(-0.1+j3)T} = e^{-0.01+j0.3} = 0.99e^{j0.3} = 0.99[\cos(0.3) + j\sin(0.3)]$$

$$z_2 = e^{(-0.1-j3)T} = 0.99e^{-j0.3} = 0.99[\cos(0.3) - j\sin(0.3)]$$

From 10.3.31,

$$H(z) = \sum_{k=1}^{N} \frac{C_k}{1 - e^{p_k T} z^{-1}}$$

From EX. 10.3.3, $C_k = \frac{1}{2}, \frac{1}{2}$

$$= \frac{\frac{1}{2}}{1 - 0.99e^{j0.3}z^{-1}} + \frac{\frac{1}{2}}{1 - 0.99e^{-j0.3}z^{-1}}$$

$$= \frac{1}{2}\left[\frac{(1-0.99e^{-j0.3}z^{-1}) + (1-0.99e^{j0.3}z^{-1})}{1 - 1.89z^{-1} + 0.981z^{-2}}\right]$$

$$= \frac{1 - 0.946z^{-1}}{1 - 1.89z^{-1} + 0.981z^{-2}}$$

circular convolution

$h(n) = \{1, 2, 3, 0, 0, 0\}$

$x(n) = \{1, 2, 2, 1, 0, 0\}$

1	2	3	0	0	0	
0	0	1	2	2	1	+

$0 + 0 + 3 + 0 + 0 + 0 = 3$ ⑥

1	2	3	0	0	0	
0	1	2	2	1	0	+

$0 + 2 + 6 + 0 + 0 + 0 = 8$ ⑤

1	2	3	0	0	0	
1	2	2	1	0	0	+

$1 + 4 + 6 + 0 + 0 + 0 = 11$ ④

1	2	3	0	0	0	
2	2	1	0	0	1	+

$2 + 4 + 3 + 0 + 0 + 0 = 9$ ③

1	2	3	0	0	0	
2	1	0	0	1	2	+

$2 + 2 = 4$ ②

1	2	3	0	0	0	
1	0	0	1	2	2	+

$1 = 1$ ①

$y(n) = \{1, 4, 9, 11, 8, 3\}$

Linear convolution

1	2	3			
1	2	2	1		
1	2	3			
	2	4	6		
		2	4	6	
			1	2	3

$y(n) = \{1, 4, 9, 11, 8, 3\}$

P2.33 (B) Determine the impulse resp. and unit-step response

$$y(n) = 0.7y(n-1) - 0.1y(n-2) + 2x(n) - x(n-2)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$Y(z) = 0.7\,[z^{-1}Y(z)] - 0.1\,[z^{-2}Y(z)] + 2X(z) - z^{-2}X(z)$$

$$Y(z)\,[1 - 0.7z^{-1} + 0.1z^{-2}] = X(z)\,[2 - z^{-2}]$$

$$H(z) = \frac{2 - z^{-2}}{1 - 0.7z^{-1} + 0.1z^{-2}} = \frac{A}{1 - 0.5z^{-1}} + \frac{B}{1 - 0.2z^{-1}}$$

NOTE TI-89 = $\frac{(z-.5)(z-0.2)}{z^2}$ so just replace z w/ 1 and multiply numbers w/ z^{-1}

$$A = \left.\frac{2 - z^{-2}}{1 - 0.2z^{-1}}\right|_{z^{-1} = 2} = \frac{2-4}{0.6} = \frac{-2}{\frac{6}{10}} = -\frac{10}{3}$$

$$z^{-1} = \frac{1}{\frac{1}{5}}$$

$$B = \left.\frac{2 - z^{-2}}{1 - 0.5z^{-1}}\right|_{z^{-1} = 5} = \frac{-23}{-1.5} = \frac{46}{3}$$

$$H(z) = \frac{(-10/3)}{1 - 0.5z^{-1}} + \frac{(46/3)}{1 - 0.2z^{-1}}$$

$$h(n) = z^{-1}\{H(z)\} \qquad a^n u(n) = \frac{1}{1 - az^{-1}}$$

$$h(n) = -\frac{10}{3}(0.5)^n u(n) + \frac{46}{3}(0.2)^n u(n)$$

the step response is when $X(z) =$ $\left[z\{u(n)\} = \frac{1}{1 - z^{-1}}\right]$

$$Y(z) = \frac{1}{(1 - z^{-1})(1 - 0.2z^{-1})(1 - 0.5z^{-1})}$$

$$z^{-1}\{Y(z)\} = y(n)$$

$$A = \left.\frac{1}{1 - 0.7z^{-1} + 0.1z^{-2}}\right|_{z^{-1} = 1} = \frac{1}{0.4} = \frac{5}{2}$$

$$B = \left.\frac{1}{(1 - z^{-1})(1 - 0.5z^{-1})}\right|_{z^{-1} = 5} = \frac{1}{(-4)(-1.5)} = \frac{1}{6}$$

$$C = \left.\frac{1}{(1 - z^{-1})(1 - 0.2z^{-1})}\right|_{z^{-1} = 2} = \frac{1}{(-1)(0.6)} = -\frac{10}{6} = -\frac{5}{3}$$

$$Y(z) = \frac{(5/2)}{1-z^{-1}} + \frac{1}{6}\frac{1}{1-0.2z^{-1}} + \frac{(-5/3)}{1-0.5z^{-1}}$$

$$y(n) = \left[\frac{5}{2} + \frac{1}{6}(0.2)^n - \frac{5}{3}(0.5)^n\right] u(n)$$

using Bi-linear Transform

$$z = e^{(-0.1+j3)T} = \underbrace{e^{-0.1T}}_{r} e^{j3T}$$

$T = 0.1$

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) = 20\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$$H(z) = \frac{20\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.1}{\left[20\frac{(1-z^{-1})}{(1+z^{-1})} + 0.1\right]^2 + 3^2}$$

$$\left(\frac{1-z^{-1}}{1+z^{-1}}\right) = 1 - \frac{2}{z+1} = \frac{z+1-2}{z+1} = \frac{z-1}{z+1}$$

EX. 10.3.3 convert the analog filter w/ system function

$$H_a(s) = \frac{s+0.1}{(s+0.1)^2 + 3^2}$$

into a digital filter (IIR) using the impulse invariance Method.

zero: $s = -0.1$

Poles: $P_1 = -0.1 + j3$

$P_2 = -0.1 - j3$

For impulse invariance, no need to find $h_a(t)$.

Find $H(z)$, first P.F.E of $H_a(s)$

$$H_a(s) = \frac{A}{s+0.1-j3} + \frac{A^*}{s+0.1+j3}$$

$$A = \left.\frac{s+0.1}{s+0.1+j3}\right|_{s=-0.1+j3} = \frac{j3}{2j3} = \frac{1}{2}$$

$$= \frac{\frac{1}{2}}{s+0.1-j3} + \frac{\frac{1}{2}}{s+0.1+j3}$$

For $H(z) = \sum_{K=1}^{N} \frac{c_K}{1-e^{P_K T} z^{-1}}$

$$= \frac{1}{2}\left[\frac{1}{1-e^{(-0.1+j3)T} z^{-1}} + \frac{1}{1-e^{(-0.1-j3)T} z^{-1}}\right]$$

$$= \frac{1}{2}\left[\frac{1}{1-e^{-0.1T} e^{j3T} z^{-1}} + \frac{1}{1-e^{-0.1T} e^{-j3T} z^{-1}}\right].$$

$$D(z) = 1 - \underbrace{e^{-0.1T} e^{-j3T} z^{-1} - e^{-0.1T} e^{j3T}}z^{-1} + e^{-0.2T} z^{-2}$$

$$- e^{-0.1T}[\cos(3T) - j\sin(3T) + \cos(3T) + j\sin(3T)]$$

$$= -2e^{-0.1T}\cos(3T)$$

$$N(s) = 1 - e^{-0.1T} e^{-j3T} z^{-1} + 1 - e^{-0.1T} e^{j3T} z^{-1}$$

$$= 2 - e^{-0.1T} z^{-1}[\cos(3T) - j\sin(3T) + \cos(3T) + j\sin(3T)]$$

$$= 2 - 2e^{-0.1T}\cos(3T) z^{-1}$$

$$H(z) = \frac{1 - e^{-0.1T}\cos(3T) z^{-1}}{1 - 2e^{-0.1T}\cos(3T) z^{-1} + e^{-0.2T} z^{-2}}$$

In general,

$$H(z) = \frac{1 - r\cos(\omega T) z^{-1}}{1 - 2r\cos(\omega T) z^{-1} + r^2 z^{-2}}$$

$$H(s) = \frac{s+1}{s^2+5s+6}$$

$T=1 \qquad s = 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$

let $\left(\frac{1-z^{-1}}{1+z^{-1}}\right) = a = 1 - \frac{2}{z+1}$

$$\frac{1}{z+1}\left(\frac{z^{-1}}{z^{-1}}\right) = \frac{z^{-1}}{1+z^{-1}}$$

$$= \frac{2a+1}{(2a)^2 + 10a + 6}$$

$$a^2 = \left(1-\frac{2}{z+1}\right)\left(1-\frac{2}{z+1}\right) = 2 - \frac{4}{z+1} + \frac{4}{(z+1)^2}$$

$$= 2\left[1 - \frac{2}{z+1} + \frac{2}{(z+1)^2}\right]$$

$$= \frac{3 - \frac{4}{z+1}}{\left[4 - \frac{8}{z+1} + \frac{8}{(z+1)^2}\right] + 10 - \frac{20}{z+1} + 6}$$

$$D(s) = 20 - \frac{28}{z+1} + \frac{8}{z^2+2z+1}$$

$$H(z) = \frac{0.5(3z^2+2z-1)}{z^2+6z-0.8}\left[\frac{z^{-2}}{z^{-2}}\right] = \frac{0.5(3+2z^{-1}-z^{-2})}{1-6z^{-1}-0.8z^{-2}}$$

$$= \frac{-(z^{-2}-2z^{-1}-3)}{2-12z^{-1}-0.16z^{-2}}$$

TB10.2

$$H_d(\omega) = \begin{cases} 1 & |\omega| \le \pi/6 \\ 0 & \pi/6 < |\omega| < \pi/3 \\ 1 & \pi/3 \le |\omega| \le \pi \end{cases}$$

Tap = 25

$$\alpha = \frac{25-1}{2} = \frac{24}{2} = 12$$

$$H_d(\omega) = 1e^{-j12\omega} \qquad |\omega| \le \pi/6, \quad \pi/3 \le |\omega| \le \pi$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega$$

$$= \delta(n) - \frac{\sin\frac{\pi}{3}(n-12)}{\pi(n-12)} + \frac{\sin\frac{\pi}{6}(n-12)}{\pi(n-12)}$$

TB9.8 For $y(n) = \frac{1}{4}y(n-2) + x(n)$

(A) Find the impulse resp. $h(n)$

$$Y(z) = \frac{1}{4} Y(z) z^{-2} + X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{1}{4} z^{-2}} = \frac{A}{1 - \frac{1}{2} z^{-1}} + \frac{B}{1 + \frac{1}{2} z^{-1}}$$

$$A = \left. \frac{1}{1 + \frac{1}{2} z^{-1}} \right|_{z^{-1} = 2} = \frac{1}{2}$$

$$B = \left. \frac{1}{1 - \frac{1}{2} z^{-1}} \right|_{z^{-1} = -2} = \frac{1}{2}$$

$$h(n) = \left[\frac{1}{2}\left(\frac{1}{2}\right)^n + \frac{1}{2}\left(\frac{1}{2}\right)^n \right] u(n)$$

(B) when $x(n) = \left[\left(\frac{1}{2}\right)^n + \left(-\frac{1}{2}\right)^n \right] u(n)$

$$X(z) = \frac{1}{1 - \frac{1}{2} z^{-1}} + \frac{1}{1 + \frac{1}{2} z^{-1}} = \frac{2}{1 - \frac{1}{4} z^{-2}}$$

$$Y(z) = H(z) \cdot X(z)$$

$$= \frac{2}{(1 - \frac{1}{4} z^{-2})^2}$$

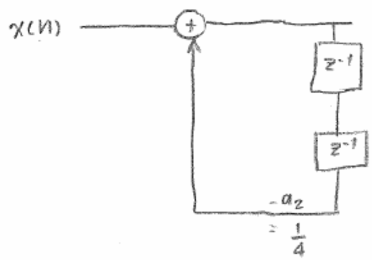
$$= \frac{A_2}{(1 - \frac{1}{2} z^{-1})^2} + \frac{A_1}{1 - \frac{1}{2} z^{-1}} + \frac{B_2}{(1 + \frac{1}{2} z^{-1})^2} + \frac{B_1}{1 + \frac{1}{2} z^{-1}}$$

$$A_2 = \left. \frac{2}{(1 + \frac{1}{2} z^{-1})^2} \right|_{z^{-1} = 2} = \frac{2}{4} = \frac{1}{2}$$

$$B_2 = \left. \frac{2}{(1 - \frac{1}{2} z^{-1})^2} \right|_{z^{-1} = -2} = \frac{1}{2}$$

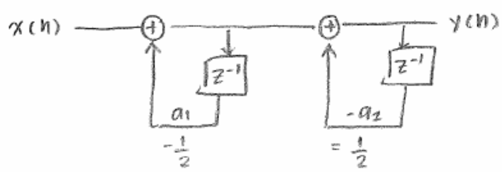
$\frac{1}{2}$

$$H(z) = \frac{1}{1 - \frac{1}{4}z^{-2}}$$

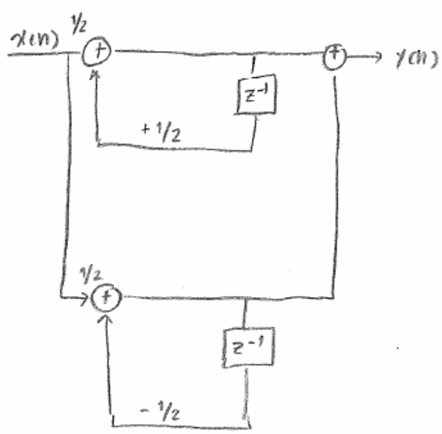


c <u>Direct</u>

<u>cascade</u> using $H(z) = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1+\frac{1}{2}z^{-1}}$



<u>Parallel</u>



Q I

$e^{j\pi/6} = \cos(\frac{\pi}{6}) + j\sin(\frac{\pi}{6}) = \frac{\sqrt{3}}{2} + j\frac{1}{2}$

$e^{j\pi/4} = \cos(\frac{\pi}{4}) + j\sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$

$e^{j\pi/3} = \cos(\frac{\pi}{3}) + j\sin(\frac{\pi}{3}) = \frac{1}{2} + j\frac{\sqrt{3}}{2}$

$e^{j\pi/2} = \cos(\frac{\pi}{2}) + j\sin(\frac{\pi}{2}) = 0 + j = j$

Q II

$e^{j2\pi/3} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$

$e^{j3\pi/4} = -\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$

$e^{j5\pi/4} = -\frac{\sqrt{3}}{2} + j\frac{1}{2}$

$e^{j\pi} = -1 + j(0) = -1$

Q III

$e^{j7\pi/6} = \frac{-\sqrt{3}}{2} - j\frac{1}{2}$

$e^{j5\pi/4} = \frac{-\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$

$e^{j3\pi/4} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$

$e^{j3\pi/2} = 0 - j = -j$

Table 6-2. Common Fourier Transform Pairs

$x[n]$	$X(\Omega)$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\Omega n_0}$
$x[n]=1$	$2\pi\delta(\Omega), \lvert\Omega\rvert \le \pi$
$e^{j\Omega_0 n}$	$2\pi\delta(\Omega-\Omega_0), \lvert\Omega\rvert, \lvert\Omega_0\rvert \le \pi$
$\cos \Omega_0 n$	$\pi[\delta(\Omega-\Omega_0)+\delta(\Omega+\Omega_0)], \lvert\Omega\rvert, \lvert\Omega_0\rvert \le \pi$
$\sin \Omega_0 n$	$-j\pi[\delta(\Omega-\Omega_0)-\delta(\Omega+\Omega_0)], \lvert\Omega\rvert, \lvert\Omega_0\rvert \le \pi$
$u[n]$	$\pi\delta(\Omega)+\dfrac{1}{1-e^{-j\Omega}}, \lvert\Omega\rvert \le \pi$
$-u[-n-1]$	$-\pi\delta(\Omega)+\dfrac{1}{1-e^{-j\Omega}}, \lvert\Omega\rvert \le \pi$
$a^n u[n], \lvert a\rvert < 1$	$\dfrac{1}{1-ae^{-j\Omega}}$
$-a^n u[-n-1], \lvert a\rvert > 1$	$\dfrac{1}{1-ae^{-j\Omega}}$
$(n+1)a^n u[n], \lvert a\rvert < 1$	$\dfrac{1}{(1-ae^{-j\Omega})^2}$
$a^{\lvert n\rvert}, \lvert a\rvert < 1$	$\dfrac{1-a^2}{1-2a\cos\Omega+a^2}$
$x[n]=\begin{cases} 1 & \lvert n\rvert \le N_1 \\ 0 & \lvert n\rvert > N_1 \end{cases}$	$\dfrac{\sin\left[\Omega\left(N_1+\frac{1}{2}\right)\right]}{\sin(\Omega/2)}$
$\dfrac{\sin Wn}{\pi n}, 0 < W < \pi$	$X(\Omega)=\begin{cases} 1 & 0 \le \lvert\Omega\rvert \le W \\ 0 & W < \lvert\Omega\rvert \le \pi \end{cases}$
$\displaystyle\sum_{k=-\infty}^{\infty} \delta[n-kN_0]$	$\displaystyle\Omega_0 \sum_{k=-\infty}^{\infty} \delta(\Omega-k\Omega_0), \Omega_0=\frac{2\pi}{N_0}$

6.5 THE FREQUENCY RESPONSE OF DISCRETE-TIME LTI SYSTEMS

A. Frequency Response:

In Sec. 2.6 we showed that the output $y[n]$ of a discrete-time LTI system equals the convolution of the input $x[n]$ with the impulse response $h[n]$; that is,

$$y[n]=x[n]*h[n] \tag{6.67}$$

Applying the convolution property (*6.58*), we obtain

$$Y(\Omega)=X(\Omega)H(\Omega) \tag{6.68}$$

Table 6-1. Properties of the Fourier Transform

Property	Sequence	Fourier transform
	$x[n]$	$X(\Omega)$
	$x_1[n]$	$X_1(\Omega)$
	$x_2[n]$	$X_2(\Omega)$
Periodicity	$x[n]$	$X(\Omega + 2\pi) = X(\Omega)$
Linearity	$a_1 x_1[n] + a_2 x_2[n]$	$a_1 X_1(\Omega) + a_2 X_2(\Omega)$
Time shifting	$x[n - n_0]$	$e^{-j\Omega n_0} X(\Omega)$
Frequency shifting	$e^{j\Omega_0 n} x[n]$	$X(\Omega - \Omega_0)$
Conjugation	$x^*[n]$	$X^*(-\Omega)$
Time reversal	$x[-n]$	$X(-\Omega)$
Time scaling	$x_{(m)}[n] = \begin{cases} x[n/m] & \text{if } n = km \\ 0 & \text{if } n \neq km \end{cases}$	$X(m\Omega)$
Frequency differentiation	$nx[n]$	$j\dfrac{dX(\Omega)}{d\Omega}$
First difference	$x[n] - x[n-1]$	$(1 - e^{-j\Omega})X(\Omega)$
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\pi X(0)\delta(\Omega) + \dfrac{1}{1 - e^{-j\Omega}} X(\Omega)$ $\|\Omega\| \le \pi$
Convolution	$x_1[n] * x_2[n]$	$X_1(\Omega) X_2(\Omega)$
Multiplication	$x_1[n] x_2[n]$	$\dfrac{1}{2\pi} X_1(\Omega) \otimes X_2(\Omega)$
Real sequence	$x[n] = x_e[n] + x_o[n]$	$X(\Omega) = A(\Omega) + jB(\Omega)$ $X(-\Omega) = X^*(\Omega)$
Even component	$x_e[n]$	$\mathrm{Re}\{X(\Omega)\} = A(\Omega)$
Odd component	$x_o[n]$	$j\,\mathrm{Im}\{X(\Omega)\} = jB(\Omega)$
Parseval's relations		

$$\sum_{n=-\infty}^{\infty} x_1[n] x_2[n] = \frac{1}{2\pi} \int_{2\pi} X_1(\Omega) X_2(-\Omega)\, d\Omega$$

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{2\pi} |X(\Omega)|^2\, d\Omega$$

T 2

EE410-001 **FALL 2013**

Digital Signal Processing

Homework 1

Problems from course books:

TB = Text Book = Digital Signal Processing, 4th Ed.
John Proakis & Dimitris Manolakis, Pearson Prentice Hall 2007.

MB = Matlab Book = Digital Signal Processing using MATLAB, 3rd Ed.
Vinay Ingle & John Proakis, CENGAGE Learning, 2012.

TB 2.30

TB 3.49 — need ';' between expressions. x(n)'; x(n)

TB 2.22a by hand and Matlab — pad 2 0's in matlab

MB P3.3 a, b, plots in Matlab

TB 1.10

1/2

Due: 9-12-2013

P3.3 1.

$$x(n) = 2(0.5)^n u(n+2)$$

$$X(e^{j\omega}) = 2 \sum_{-\infty}^{\infty} 0.5^n u(n+2) e^{-jn\omega}$$

$$= 2 \sum_{-2}^{\infty} 0.5^n e^{-jn\omega}$$

$$= 2(0.5)^{-2} e^{j2\omega} \sum_{n=0}^{\infty} 0.5^n e^{-jn\omega}$$

using MB (2,6) $\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$

$$X(e^{j\omega}) = \frac{2}{(1/2)^2} e^{j2\omega} \sum_{n=0}^{\infty} (0.5e^{-j\omega})^n$$

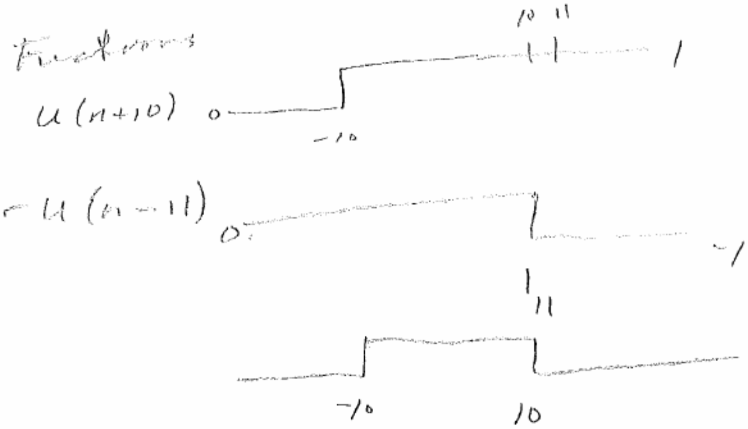
$$X(e^{j\omega}) = \frac{8e^{j2\omega}}{1-0.5e^{-j\omega}}$$

plot with matlab

P3.3 2.

$$X(\omega) = \sum_{-\infty}^{\infty} (0.6)^{|n|}\left[u(n+10) - u(n-11)\right] e^{-jn\omega}$$

Remove Step Functions



$$= \sum_{-10}^{10} 0.6^{|n|} e^{jn\omega}$$

$$= \sum_{-10}^{0} 0.6^{-n} e^{-jn\omega} + \sum_{0}^{10} 0.6^{n} e^{-jn\omega} \underline{-1}$$

Extra 1 because of Counted Twice

M.B. Eq 2.7: $\displaystyle\sum_{n=0}^{N-1} \alpha^{n} = \frac{1-\alpha^{N}}{1-\alpha}$

$X(w) = \sum_0^{10} 0.6^n e^{jnw} + \sum_0^{10} 0.6^n e^{-jnw} - 1$ ✓ (3)

$$\frac{1-0.6^{11} e^{j11w}}{1-0.6e^{jw}} + \frac{1-0.6^{11} e^{-j11w}}{1-0.6e^{-jw}} - 1$$

$$\sum_0^{10} 0.6^n (e^{jnw} + e^{-jnw})$$

$$2 \sum 0.6^n (\cos(nw))$$

$N=11$

$$\sum_{n=0}^{N-1} (0.6 e^{jw})^n \qquad \sum_{n=0}^{N-1} (0.6 e^{-jw})^n \qquad -1$$

$$= \frac{1-0.6^{11} e^{jw11}(\;)}{1-0.6e^{jw}(1-0.6e^{-jw})} + \frac{1-0.6^{11} e^{-jw11}(\;)}{1-0.6e^{-jw}(1-0.6e^{jw})} - 1\frac{(\;)}{(\;)}$$

Denom $\rightarrow 1 - 0.6e^{jw} - 0.6e^{-jw} + 0.36$

$1.36 - 1.2\cos(w)$ ✓ denom ok

$-0.6(e^{jw} + e^{-jw})$

$2\cos(w) = e^{jw} + e^{-jw}$

Now numerator:

$$(1-0.6^{11} e^{jw11})(1-0.6e^{-jw}) + (1-0.6^{11} e^{-jw11})(1-0.6e^{jw})$$

$$-(1.00 - 0.6e^{jw} - 0.6e^{-jw} + 0.36)$$

(4)

numerator Expanded:

$$1 - 0.6e^{-j\omega} - 0.6^{11}e^{j\omega 11} + 0.6^{12}e^{j\omega(11-1)}$$

$$+ 1 - 0.6e^{j\omega} - 0.6^{11}e^{-j\omega 11} + 0.6^{12}e^{j\omega(1-11)}$$

$$- 1 + 0.6e^{j\omega} + 0.6e^{-j\omega} - 0.36$$

$$= 0.64 - 2(0.6)^{11}\cos(11\omega) + 2(0.6)^{12}\cos(10\omega)$$

$$X(e^{j\omega}) = \frac{0.64 - 2(0.6)^{11}\cos(11\omega) + 2(0.6)^{12}\cos(10\omega)}{1.36 - 1.2\cos(\omega)}$$

plot with matlab.

ALANA M. SOEHARTONO
SEPTEMBER 12, 2013
EE410 - DSP
HW # 1

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100

TB 2.30) For $y(n)$, $n \geq 0$

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

where $x(n) = 4^n u(n)$

Determine the response.

① We assume the solution to the homogeneous eq.

$$y_h(n) = \lambda^n$$

Thus the characteristic eq.

$$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 3\lambda - 4) = 0$$

$$(\lambda - 1)(\lambda + 4) = 0$$

$$\lambda_1 = -1, \ \lambda_2 = 4$$

$$\therefore \ y_h = C_1 \lambda_1^n + C_2 \lambda_2^n = C_1(-1)^n + C_2(4)^n$$

② The particular solution is of the form,

$$y_p(n) = Kn4^n u(n)$$

substituting $y_p(n)$ and $x(n)$ into the original expression,

$$Kn4^n u(n) - 3K(n-1)4^{n-1}u(n-1) - 4K(n-2)4^{n-2}u(n-2) = 4^n u(n) + 2[4^{n-1}u(n-1)]$$

Evaluate for K by selecting $n=2$, to maintain unit step terms.

$$32Ku(n) - 12Ku(n-1) = 16u(n) + 8u(n-1)$$

$$32K - 12K = 16 + 8 \Rightarrow K(20) = 24 \Rightarrow K = \frac{6}{5}$$

$$\therefore \ y_p(n) = \frac{6}{5} n 4^n u(n)$$

③ $y_T = y_h + y_p = C_1(-1)^n + C_2(4)^n + \frac{6}{5} n 4^n u(n)$

④ Apply initial conditions, where $n=0$, $n=1$

For original equation:

$$y(0) = 3y(-1) + 4y(-2) + x(0) + 2x(-1)$$

$$y(1) = 3y(0) + 4y(-1) + x(1) + 2x(0)$$

$$x(0) = 4^n u(n) = 4^0 u(0) = 1$$

$$x(-1) = 4^{-1}u(-1) = 4^{-1} \cdot 0 = 0$$

$$x(1) = 4^1 u(1) = 4$$

$$\therefore \ y(0) = 3y(-1) + 4y(-2) + 1$$

$$y(1) = 3y(0) + 4y(-1) + 4 + 2 = 3y(0) + 4y(-1) + 6$$

$$= 3[3y(-1) + 4y(-2) + 1] + 4y(-1) + 6$$

$$= 13y(-1) + 12y(-2) + 9$$

$$y(0) = C_1(-1)^0 + C_2(4)^0 + \frac{6}{5} 0 4^0 = C_1 + C_2$$

$$y(1) = -C_1 + 4C_2 + \frac{24}{5}$$

Equating on both sides,

$$3y(-1)+4y(-2)+1 = C_1 + C_2$$
$$13y(-1)+12y(-2)+9 = -C_1 + 4C_2 + \frac{24}{5}$$

setting $y(-1) = y(-2) = 0$,

$$C_1 + C_2 = 1 \quad \Rightarrow \quad C_1 = 1 - C_2$$
$$-C_1 + 4C_2 + \frac{24}{5} = 9$$
$$-1 + C_2 + 4C_2 = 9 - \frac{24}{5}$$
$$5C_2 = 10 - \frac{24}{5} = \frac{50-24}{5} = \frac{26}{5}$$
$$C_2 = \frac{26}{25}$$
$$C_1 = \frac{25}{25} - \frac{26}{25} = -\frac{1}{25}$$

(5) Then, $y_T = -\frac{1}{25}(-1)^n + \frac{26}{25}(4)^n + \frac{6}{5}n4^n \qquad n \geq 0$ //

TB 3.49) use the one-sided z-transform to determine $y(n)$, $n \geq 0$

(A) $y(n) + \frac{1}{2}y(n-1) - \frac{1}{4}y(n-2) = 0 \qquad y(-1) = y(-2) = 1$

$$Z^{+}\{y(n) + \tfrac{1}{2}y(n-1) - \tfrac{1}{4}y(n-2)\}$$
$$= Y^{+}(z) + \frac{1}{2}[z^{-1}Y^{+}(z) + y(-1)] - \frac{1}{4}[z^{-2}Y^{+}(z) + z^{-1}y(-1) + y(-2)] = 0$$

Based on properties

$$x(n) \xleftrightarrow{z^+} X^{+}(z) \qquad [\text{EQ. } 3.6.1]$$
$$z^{+}\{x(n-k)\} = z^{-k}\left[\sum_{l=-1}^{-k} x(l)z^{-l} + X^{+}(z)\right] \qquad \text{shifting property Time Delay } [3.6.3]$$

Replacing $y(-1)$ and $y(-2)$,

$$\Rightarrow Y^{+}(z) + \frac{1}{2}[z^{-1}Y^{+}(z) + 1] - \frac{1}{4}[z^{-2}Y^{+}(z) + z^{-1} + 1] = 0$$
$$Y^{+}(z) + \frac{1}{2}z^{-1}Y^{+}(z) + \frac{1}{2} - \frac{1}{4}z^{-2}Y^{+}(z) - \frac{1}{4}z^{-1} - \frac{1}{4} = 0$$
$$Y^{+}(z)\left[\frac{1}{2}z^{-1} - \frac{1}{4}z^{-2} + 1\right] = -\frac{1}{2} + \frac{1}{4}z^{-1} + \frac{1}{4} = -\frac{1}{4} + \frac{1}{4}z^{-1}$$
$$Y^{+}(z) = \frac{-\frac{1}{4} + \frac{1}{4}z^{-1}}{\underbrace{\frac{1}{2}z^{-1} - \frac{1}{4}z^{-2} + 1}_{(1-0.309z^{-1})(1+0.809z^{-1})}} = \frac{A}{(1-0.309z^{-1})} + \frac{B}{(1+0.809z^{-1})}$$

$$A = \left.\frac{-\frac{1}{4} + \frac{1}{4}z^{-1}}{(1+0.809z^{-1})}\right|_{z^{-1} = \frac{1}{0.309}} = \frac{0.559}{3.618} = 0.155$$

$$B = \left.\frac{-\frac{1}{4} + \frac{1}{4}z^{-1}}{1-0.309z^{-1}}\right|_{z^{-1} = -\frac{1}{0.809}} = -0.405$$

$$Y^{+}(z) = \frac{0.155}{1-0.309z^{-1}} - \frac{0.405}{1+0.809z^{-1}}$$

$$y(n) = [0.155(0.309)^n - 0.405(-0.809)^n]\,u(n) //$$

(B) $y(n) - 1.5y(n-1) + 0.5y(n-2) = 0 \qquad y(-1) = 1 \qquad y(-2) = 0$

$$Z^+\{ y(n) - 1.5y(n-1) + 0.5y(n-2) = 0 \}$$

$$= Y^+(z) - 1.5[z^{-1}Y^+(z) + y(-1)] + 0.5[z^{-2}Y^+(z) + z^{-1}y(-1) + y(-2)] = 0$$

$$= Y^+(z) - 1.5z^{-1}Y^+(z) - 1.5 + 0.5z^{-2}Y^+(z) + 0.5z^{-1} = 0$$

$$Y^+(z)[1 - 1.5z^{-1} + 0.5z^{-2}] = 1.5 - 0.5z^{-1}$$

$$Y^+(z) = \underbrace{\frac{1.5 - 0.5z^{-1}}{1 - 1.5z^{-1} + 0.5z^{-2}}} = \frac{A}{(1-z^{-1})} + \frac{B}{(1-0.5z^{-1})}$$

$$\frac{(z-1)(z-0.5)}{z^2} = (1-z^{-1})(1-0.5z^{-1})$$

$$A = \left.\frac{1.5 - 0.5z^{-1}}{(1-0.5z^{-1})}\right|_{z^{-1}=1} = \frac{1}{0.5} = 2$$

$$B = \left.\frac{1.5 - 0.5z^{-1}}{(1-z^{-1})}\right|_{z^{-1}=2} = \frac{0.5}{-1} = -\frac{1}{2}$$

$$Y^+(z) = \frac{2}{(1-z^{-1})} - \frac{(1/2)}{(1-0.5z^{-1})}$$

From Table 3.3

$$y(n) = [2(1)^n - \tfrac{1}{2}(\tfrac{1}{2})^n] u(n)$$

(c) $y(n) - \frac{1}{2}y(n-1) - x(n) = 0 \qquad x(n) = (\frac{1}{3})^n u(n) \quad y(-1) = 1$

$$Y^+(z) - \frac{1}{2}[z^{-1}Y^+(z) + y(-1)] - \underbrace{X^+(n)}_{\frac{1}{1-\frac{1}{3}z^{-1}}} = 0$$

$$\underbrace{Y^+(z) - \frac{1}{2}z^{-1}Y^+(z)}_{Y^+(z)[1 - \frac{1}{2}z^{-1}]} = \frac{1}{1-\frac{1}{3}z^{-1}} + \frac{1}{2} = \frac{1 + 1/2 - \frac{1}{6}z^{-1}}{1-\frac{1}{3}z^{-1}} = \frac{\frac{3}{2} - \frac{1}{6}z^{-1}}{1-\frac{1}{3}z^{-1}}$$

$$Y^+(z) = \frac{\frac{3}{2} - \frac{1}{6}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{2}z^{-1})} = \frac{A}{1-\frac{1}{3}z^{-1}} + \frac{B}{1-\frac{1}{2}z^{-1}}$$

$$A = \left.\frac{\frac{3}{2} - \frac{1}{6}z^{-1}}{1-\frac{1}{2}z^{-1}}\right|_{z^{-1}=3} = \frac{1}{\frac{-1}{2}} = -2$$

$$B = \left.\frac{\frac{3}{2} - \frac{1}{6}z^{-1}}{1-\frac{1}{3}z^{-1}}\right|_{z=2} = \frac{(\frac{3}{3})\frac{3}{2} - \frac{1}{3}(\frac{2}{2})}{(\frac{3}{3})1 - \frac{2}{3}} = \frac{\frac{7}{6}}{\frac{1}{3}} = \frac{7}{2}$$

$$Y^+(z) = \frac{-2}{1-\frac{1}{3}z^{-1}} + \frac{7/2}{1-\frac{1}{2}z^{-1}}$$

$$y(n) = [-2(\tfrac{1}{3})^n + \tfrac{7}{2}(\tfrac{1}{2})^n] u(n) \,//$$

(D) $y(n) - \frac{1}{4}y(n-2) - x(n) = 0$ $\quad x(n) = u(n) \quad y(-1) = 0 \quad y(-2) = 1$

$$Y^+(z) - \frac{1}{4}\left[z^{-2}Y^+(z) + z^{-1}y(-1) + y(-2) \right] - \underbrace{X^+(n)}_{\frac{1}{1-z^{-1}}} = 0$$

$$Y^+(z)\left[1 - \frac{1}{4}z^{-2}\right] = +\frac{1}{4} + \frac{1}{1-z^{-1}}$$

$$= \frac{\frac{1}{4} - \frac{1}{4}z^{-1} + \frac{4}{4}}{1-z^{-1}} = \frac{\frac{5}{4} - \frac{1}{4}z^{-1}}{1-z^{-1}}$$

$$Y^+(z) = \frac{\frac{5}{4} - \frac{1}{4}z^{-1}}{(1-z^{-1})\underbrace{(1-\frac{1}{4}z^{-2})}_{(1+\frac{1}{2}z^{-1})(1-\frac{1}{2}z^{-1})}} = \frac{A}{(1-z^{-1})} + \frac{B}{(1+\frac{1}{2}z^{-1})} + \frac{C}{(1-\frac{1}{2}z^{-1})}$$

$$A = \left.\frac{\frac{5}{4} - \frac{1}{4}z^{-1}}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1})}\right|_{z^{-1}=1} = \frac{1}{(\frac{1}{2})(\frac{3}{2})} = \frac{4}{3}$$

$$B = \left.\frac{\frac{5}{4} - \frac{1}{4}z^{-1}}{(1-z^{-1})(1-\frac{1}{2}z^{-1})}\right|_{z^{-1}=-2} = \frac{\frac{7}{4}}{\underbrace{(1+2)(1+1)}_{6}} = \frac{7}{24}$$

$$C = \left.\frac{\frac{5}{4} - \frac{1}{4}z^{-1}}{(1-z^{-1})(1+\frac{1}{2}z^{-1})}\right|_{z^{-1}=2} = \frac{\frac{3}{4}}{(1-2)(1+1)} = \frac{3}{-8}$$

$$Y^+(z) = \frac{\frac{4}{3}}{1-z^{-1}} + \frac{7/24}{1+\frac{1}{2}z^{-1}} - \frac{3/8}{1-\frac{1}{2}z^{-1}}$$

$$y(n) = \left[\frac{4}{3}(1)^n + \frac{7}{24}\left(-\frac{1}{2}\right)^n - \frac{3}{8}\left(\frac{1}{2}\right)^n \right] u(n) //$$

TB1.10) $x_a(t) = 3\cos 600\pi t + 2\cos 1800\pi t$

Rate $= 10{,}000$ bits/s $\quad$ Levels $= 1024$ [Binary numbers]

(A) sampling Freq.

From 1.4.5, we need to know number of bits from coder.

Number of bits $= \log_2 1024 = 10$ bits

$$F_s = \frac{\text{Rate}}{\text{Number of bits}} = \frac{10{,}000 \text{ bits/s}}{10 \text{ bits}} = 1{,}000\,\frac{1}{s} = 1\text{ KHz} //$$

Folding frequency:

$$F_f = \frac{F_s}{2} = 500\text{ Hz} //$$

(B) $F_1 = \frac{f_1}{2} = 300$ Hz $\qquad F_2 = 900$ Hz

$F_{max} = F_2$

$F_N = 2F_2 = 1800$ Hz //

Nyquist frequency

(c) $F_s = 1000$ Hz $\qquad F_s > 2F_{max}$ $\qquad$ where $2F_{max} = 2000$ Hz

folding frequency $= \frac{F_s}{2} = 500$ Hz

$x(n) = x_a(nT) = x_a\left(\frac{n}{F_s}\right)$ $\qquad x_a = 3\cos 600\pi t + 2\cos 1800\pi t$

$\frac{N}{F_s}$

if $F_n > \frac{F_s}{2}$, aliasing will be observed.

$x(n) = 3\cos 2\pi\left(\frac{300}{1000}\right)n + 2\cos 2\pi\left(\frac{900}{1000}\right)n$

we need to make $F_2 < \frac{F_s}{2}$ (which is 500 Hz)

for F_2, $\left(1 - \frac{9}{10}\right) = \frac{1}{10}$

$x(n) = 3\cos 2\pi\left(\frac{3}{10}\right)n + 2\cos 2\pi\left(\frac{1}{10}\right)n$

in other words, $f_1 = \frac{3}{10}$ and $f_2 = \frac{1}{10}$ //

(D) $\Delta = \frac{x_{max} - x_{min}}{1023} = \frac{5-(-5)}{1023} = \frac{10}{1023}$

Because $\cos_{max} = 1$ and $\cos_{min} = -1$ //

MB3.3) Determine analytically the DTFT.

(1) $x(n) = 2(0.5)^n u(n+2)$

$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x(n) e^{-j\omega n}$$

when $n < -2$, $x(n) = 0$

$$\therefore X(e^{j\omega}) = 2\left[0 + (0.5)^{-2} e^{-j2\omega} + (0.5)^{-1} e^{-j\omega} + \sum_{n=0}^{\infty} (0.5)^n e^{-j\omega n}\right]$$

For an infinite geometric series,

$$\sum_{n=0}^{\infty} (0.5)^n e^{-j\omega n} = \frac{(0.5e^{-j\omega})^{\infty \to 0} - 1}{(0.5e^{-j\omega}) - 1} = \frac{1}{1 - (0.5e^{-j\omega})}$$

$$\therefore X(e^{j\omega}) = 2\left[(0.5)^{-2} e^{-j2\omega} + (0.5)^{-1} e^{-j\omega} + \frac{1}{1 - 0.5e^{-j\omega}}\right] //.$$

(2) $x(n) = (0.6)^{|n|}[u(n+10) - u(n-11)]$

For $n < -10$ and $n > 11$, $x(n) = 0$

since we use $|n|$,

$$X(e^{j\omega}) = \sum_{n=1}^{10} (0.6)^n e^{j\omega n} + \sum_{n=0}^{11} (0.6)^n e^{-j\omega n}$$

Please see next pages for magnitude and phase plots.

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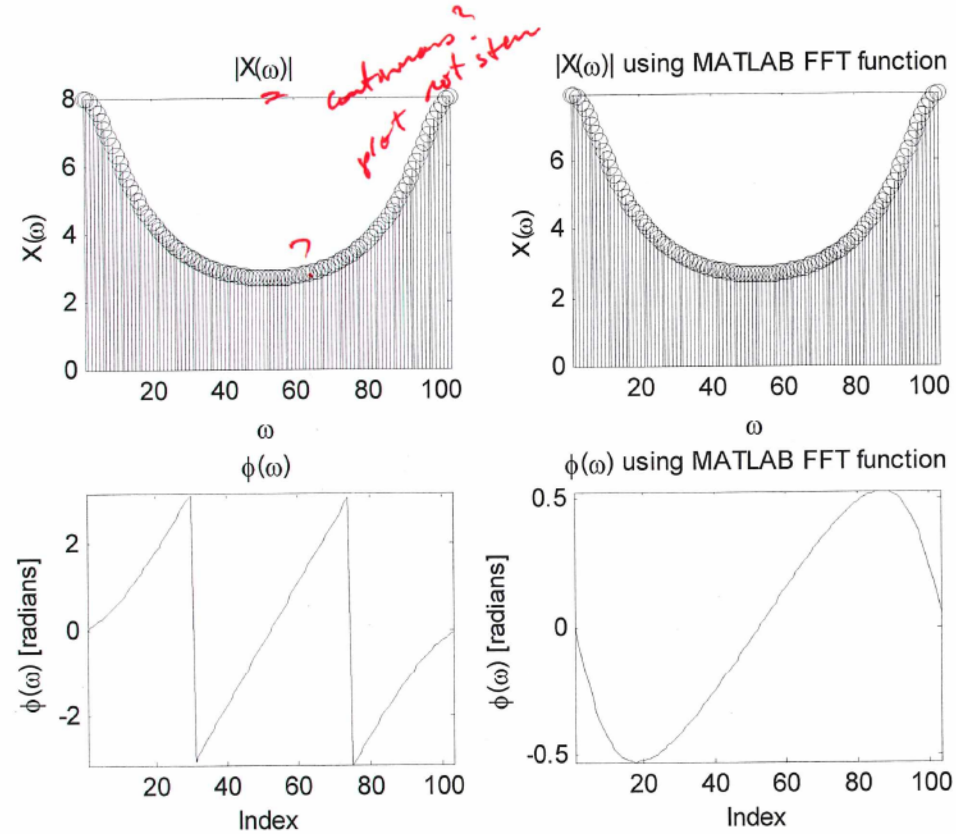
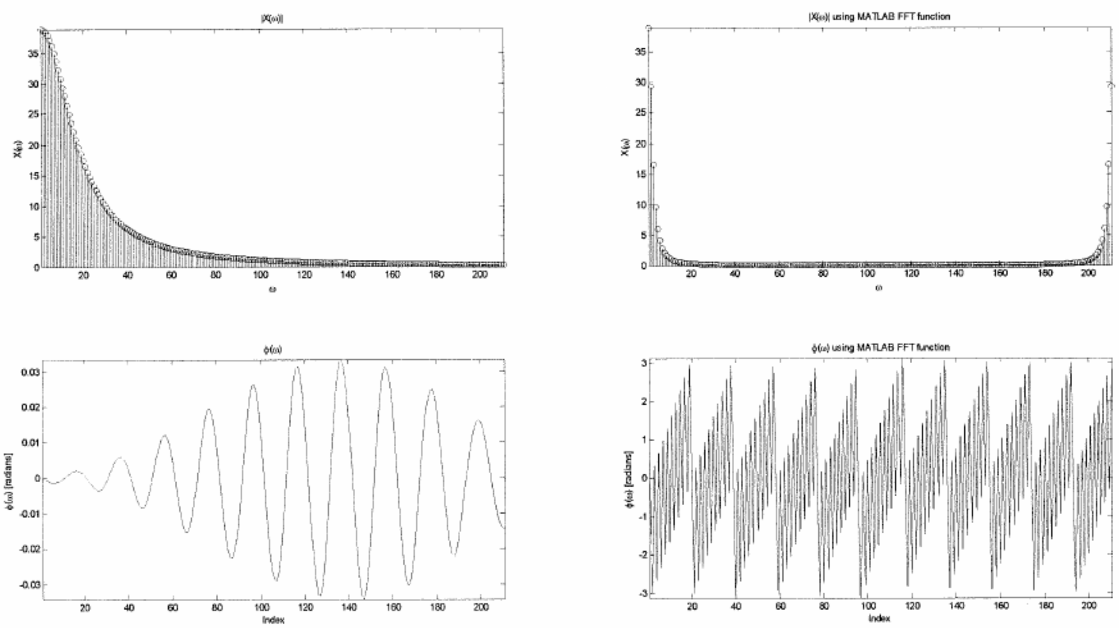
Shown is the plot comparing the discrete computation of x[n], shown on the top and bottom left of the figure, along with a comparison of the MATLAB FFT function, shown on the top and bottom right of the figure for MB3.3 (1). The results are in agreement for the magnitude of the expressions. The difference lies in the phase of the two signals. The first phase, resembling a saw tooth wave, takes the exponent from the expression starting at n = -2. In MATLAB, the FFT begins when n = 0, thus a difference is observed.




```
%%%%%%%% EE410 Digital Signal Processing %%%%%%
% Submitted by: Alana M. Soehartono
% HW1 - MB3.3
%
%
% The purpose of this program is to compute the discrete-time Fourier
% transform (DTFT) of a given input x(n) = (0.5)^n
%
% The computation is compared to MATLAB's FFT algorithm and both are
% plotted.

clear; clc;

% Performing n iterations of x(n)
n = [-2:1:100];
counter = 1;
for w = 0:2*pi/length(n):2*pi-2*pi/length(n)
    w(counter) = w;
```

```
    x(counter) = sum((0.5).^n.*exp(-j*w(counter).*n));
    counter = counter + 1;
end

% For comparison, the Matlab FFT
X = fft((0.5.^n));
figure, plot(abs(X));

figure, subplot(2,2,1); stem(abs(x)); title('|X(\omega)|'); xlabel('\omega');
ylabel('X(\omega)'); axis tight;
        subplot(2,2,2); stem(abs(X)); title('|X(\omega)| using MATLAB FFT
function'); xlabel('\omega'); ylabel('X(\omega)'); axis tight;
        subplot(2,2,3); plot(angle(x)); title('\phi(\omega)');
xlabel('Index'); ylabel('\phi(\omega) [radians]'); axis tight;
        subplot(2,2,4); plot(angle(X)); title('\phi(\omega) using MATLAB FFT
function'); xlabel('Index');ylabel('\phi(\omega) [radians]'); axis tight;
```

Shown is the plot comparing the discrete computation of x[n], shown on the top and bottom left of the figure, along with a comparison of the MATLAB FFT function, shown on the top and bottom right of the figure for Problem MB3.3 (2). The MATLAB magnitude plot shows symmetry around the centrepoint, while this is not observed in the other plot.



Alana M. Soehartono
EE410 – Digital Signal Processing, HW#1

```
%%%%%%%% EE410 Digital Signal Processing %%%%%%
% Submitted by: Alana M. Soehartono
% HW1 - MB3.3
%
%
% The purpose of this program is to compute the discrete-time Fourier
% transform (DTFT) of a given input x(n) = (0.6)^abs(n)
%
% The computation is compared to MATLAB's FFT algorithm and both are
% plotted.

clear;clc;

% Performing n iterations of x(n)
n = [-10:0.1:11];
counter = 1;
for w = 0:2*pi/length(n):2*pi-2*pi/length(n)
    w(counter) = w;
    x(counter) = sum((0.6).^abs(n).*exp(-j*w(counter).*n));
    counter = counter + 1;
end

% For comparison, the Matlab FFT
X = fft((0.6.^abs(n)));
figure, plot(abs(X));

figure, subplot(2,2,1); stem(abs(x)); title('|X(\omega)|'); xlabel('\omega');
ylabel('X(\omega)'); axis tight;
        subplot(2,2,2); stem(abs(X)); title('|X(\omega)| using MATLAB FFT
function'); xlabel('\omega'); ylabel('X(\omega)'); axis tight;
        subplot(2,2,3); plot(angle(x)); title('\phi(\omega)');
xlabel('Index'); ylabel('\phi(\omega) [radians]'); axis tight;
        subplot(2,2,4); plot(angle(X)); title('\phi(\omega) using MATLAB FFT
function'); xlabel('Index');ylabel('\phi(\omega) [radians]'); axis tight;
```

TB 2.22 (A) Let $x(n)$ = input signal
$h_i(n)$ = impulse response
$y_i(n)$ = corresponding output

$x(n) = \{1, 4, 2, 3, 5, 3, 3, 4, 5, 7, 6, 9\}$

$y(n) = x(n) * h(n)$

$h_1(n) = \{1, 1\}$

$y_1(n) = x(n) + x(n-1)$

$= \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 13, 15, 9\}$

$h_2(n) = \{1, 2, 1\}$

$y_2(n) = x(n) + 2x(n-1) + x(n-2)$

$= \{1, 6, 11, 11, 13, 16, 14, 13, 16, 21, 25, 28, 24, 9\}$

$h_3(n) = \{\frac{1}{2}, \frac{1}{2}\}$

$y_3(n) = \frac{1}{2}x(n) + \frac{1}{2}x(n-1)$

$= \{\frac{1}{2}, \frac{5}{2}, 3, \frac{5}{2}, 4, 4, 3, \frac{7}{2}, \frac{9}{2}, 6, \frac{13}{2}, \frac{15}{2}, \frac{9}{2}\}$

$h_4(n) = \{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$

$y_4(n) = \frac{1}{4}x(n) + \frac{1}{2}x(n-1) + \frac{1}{4}x(n-2)$

$= \{\frac{1}{4}, \frac{3}{2}, \frac{11}{4}, \frac{11}{4}, \frac{13}{4}, 4, \frac{7}{2}, \frac{13}{4}, 4, \frac{21}{4}, \frac{25}{4}, 7, 6, \frac{9}{4}\}$

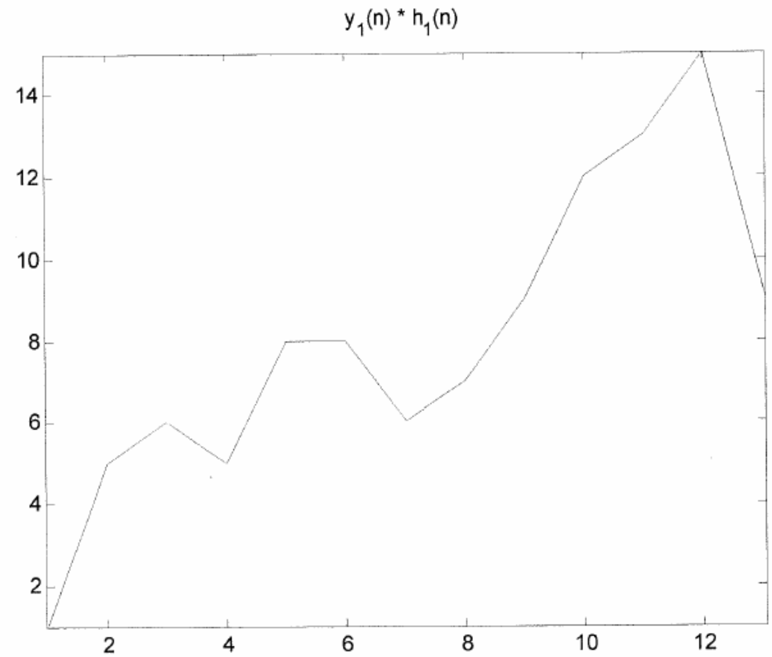
$h_5(n) = \{\frac{1}{4}, -\frac{1}{2}, \frac{1}{4}\}$

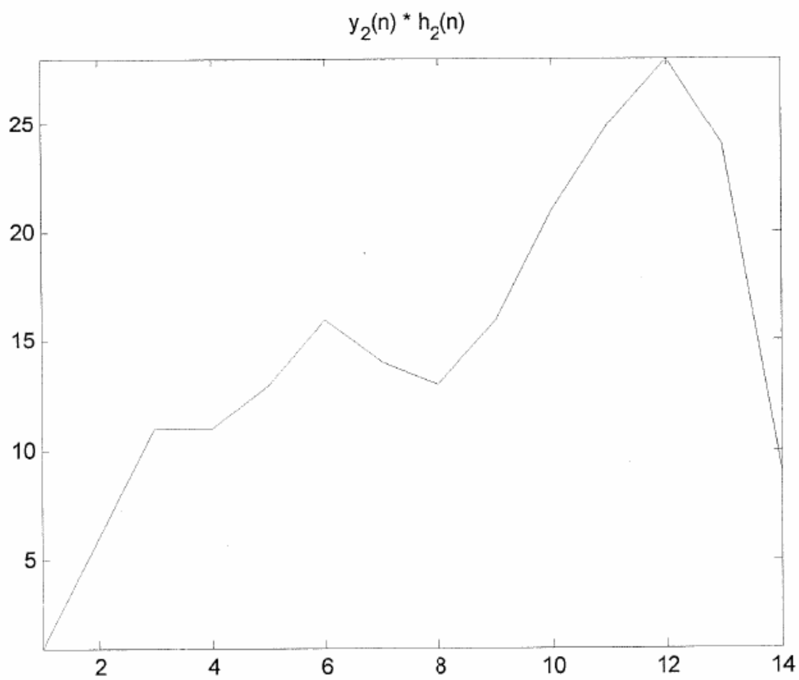
$y_5(n) = \frac{1}{4}x(n) - \frac{1}{2}x(n-1) + \frac{1}{4}x(n-2)$

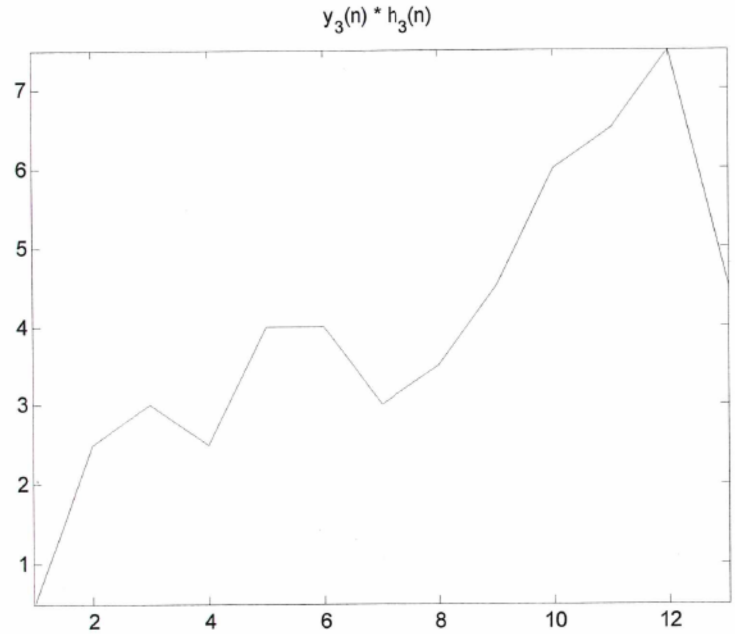
$= \{\frac{1}{4}, \frac{1}{2}, -\frac{5}{4}, \frac{3}{4}, \frac{1}{4}, -1, \frac{1}{2}, \frac{1}{4}, 0, \frac{1}{4}, -\frac{3}{4}, 1, -3, \frac{9}{4}\}$ //

Alana M. Soehartono
EE410 – Digital Signal Processing, HW#1

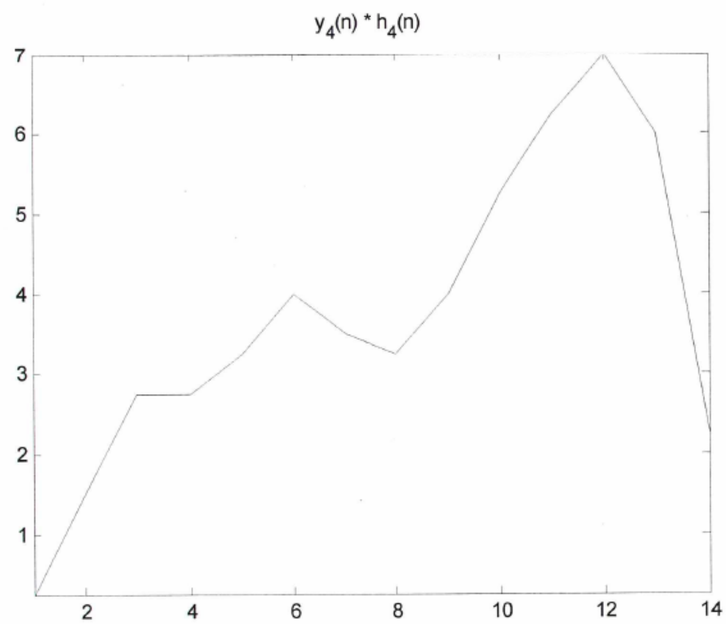
TB2.2







stem



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```
clear;clc;

x = []; h = [];
y = [1 4 2 3 5 3 3 4 5 7 6 9];
h = [1 1];
x = conv(y,h);
figure, plot(x); title('y_1(n) * h_1(n)'); axis tight;

x = []; h = [];
y = [1 4 2 3 5 3 3 4 5 7 6 9];
h = [1 2 1];
x = conv(y,h);
figure, plot(x); title('y_2(n) * h_2(n)'); axis tight;

x = []; h = [];
y = [1 4 2 3 5 3 3 4 5 7 6 9];
h = [.5 .5];
x = conv(y,h);
figure, plot(x); title('y_3(n) * h_3(n)'); axis tight;

x = []; h = [];
y = [1 4 2 3 5 3 3 4 5 7 6 9];
h = [.25 .5 .25];
x = conv(y,h);
figure, plot(x); title('y_4(n) * h_4(n)'); axis tight;

x = []; h = [];
y = [1 4 2 3 5 3 3 4 5 7 6 9];
h = [.25 -.5 .25];
x = conv(y,h);
figure, plot(x); title('y_5(n) * h_5(n)'); axis tight;
```

EE410-001 **FALL 2013**

Digital Signal Processing

Homework 2

Problems from course books:

TB = Text Book = Digital Signal Processing, 4th Ed.
John Proakis & Dimitris Manolakis, Pearson Prentice Hall 2007.

MB = Matlab Book = Digital Signal Processing using MATLAB, 3rd Ed.
Vinay Ingle & John Proakis, CENGAGE Learning, 2012.

TB 3.42 a

TB 3.42 b

TB 3.47 b

TB 3.47 d

MB P3.14

Due: 9-19-2013

TB 3.42 (B) Find the zero-state unit step response of $H(z)$

$$H(z) = \frac{Y(z)}{X(z)} \Rightarrow Y(z) = H(z)X(z)$$

$$u(n) \xleftrightarrow{z} \frac{1}{1-z^{-1}}$$

$$Y(z) = \frac{z^{-1} + \frac{1}{2}z^{-2}}{(1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2})}\left[\frac{1}{1-z^{-1}}\right] = \frac{z^{-1}+\frac{1}{2}z^{-2}}{(1-z^{-1})(1-\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2})}\left[\frac{z^2}{z^2}\right]$$

$$= \frac{z+\frac{1}{2}}{\underbrace{(z^2-z)}_{z(z-1)}(z-\frac{2}{5})(z-\frac{1}{5})} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-\frac{2}{5}} + \frac{D}{z-\frac{1}{5}}$$

* is pole-zero cancellation applicable here?

$$A = \left.\frac{z+\frac{1}{2}}{(z-1)(z-\frac{2}{5})(z-\frac{1}{5})}\right|_{z=0} = \frac{\frac{1}{2}}{(-1)(-\frac{2}{5})(-\frac{1}{5})} = \frac{1}{2}\left(\frac{-25}{2}\right) = -\frac{25}{4}$$

$$B = \left.\frac{z+1/2}{z(z-\frac{2}{5})(z-\frac{1}{5})}\right|_{z=1} = \frac{3/2}{(\frac{3}{5})(\frac{4}{5})} = \frac{3}{2}\left(\frac{25}{12}\right) = \frac{25}{8}$$

$$C = \left.\frac{z+1/2}{z(z-1)(z-1/5)}\right|_{z=\frac{2}{5}} = \frac{(\frac{2}{2})\frac{2}{5}+\frac{1}{2}(\frac{5}{5})}{(\frac{2}{5})(-\frac{3}{5})(\frac{1}{5})} = \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{10}}}\left(\frac{-\overset{25}{\cancel{125}}}{\underset{2}{\cancel{6}}}\right) = \frac{75}{4}$$

$$D = \left.\frac{z+1/2}{z(z-1)(z-\frac{2}{5})}\right|_{z=\frac{1}{5}} = \frac{\frac{7}{10}}{\frac{1}{5}(-\frac{4}{5})(-\frac{1}{5})} = \frac{7}{\underset{2}{\cancel{10}}}\left(-\frac{\overset{5}{\cancel{125}}}{4}\right) = \frac{35}{8}$$

$$Y(z) = z^{-1}\left[A + \frac{B}{1-z^{-1}} + \frac{C}{1-\frac{2}{5}z^{-1}} + \frac{D}{1-\frac{1}{5}z^{-1}}\right]$$

$$y(n) = \left[-\frac{25}{4}\delta(n) + \frac{25}{8}(1)^n + \frac{75}{4}\left(\frac{2}{5}\right)^n + \frac{35}{8}\left(\frac{1}{5}\right)^n\right]u(n) \,//$$

TB 3.47 (B) compute the convolution using time-domain and one sided z-transformed.

$x_1(n) = \left(\frac{1}{2}\right)^n u(n) \qquad x_2(n) = \left(\frac{1}{3}\right)^n u(n)$

$X_1(z) = \dfrac{1}{1-\frac{1}{2}z^{-1}} \qquad X_2(z) = \dfrac{1}{1-\frac{1}{3}z^{-1}}$

$Y(z) = X_1(z) \cdot X_2(z)$

$= \dfrac{1}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})} = \dfrac{A}{1-\frac{1}{2}z^{-1}} + \dfrac{B}{1-\frac{1}{3}z^{-1}}$

$A = \left.\dfrac{1}{1-\frac{1}{3}z^{-1}}\right|_{z^{-1}=2} = \dfrac{1}{1-\frac{2}{3}} = \dfrac{1}{\frac{1}{3}} = 3$

$B = \left.\dfrac{1}{1-\frac{1}{2}z^{-1}}\right|_{z^{-1}=3} = \dfrac{1}{1-\frac{3}{2}} = \dfrac{1}{-\frac{1}{2}} = -2$

$Y(z) = \dfrac{3}{1-\frac{1}{2}z^{-1}} - \dfrac{2}{1-\frac{1}{3}z^{-1}}$

$y(n) = \left[3\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n\right] u(n)$ //

TB3.47 (D) compute the convolution of the signal pair in the time domain and using the one-sided z-transform.

$x_1(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$ $\qquad$ $x_2(n) = \{\underset{\uparrow}{1}, 1, 1\}$

$X_1(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}$

$X_2(z) = 1 + z^{-1} + z^{-2}$

$Y(z) = X_1(z) \cdot X_2(z) = (1 + z^{-1} + z^{-2} + z^{-3} + z^{-4})(1 + z^{-1} + z^{-2})$

$= 1 + z^{-1} + z^{-2} + z^{-2} + z^{-1} + z^{-3} + z^{-2} + z^{-3} + z^{-4} + z^{-3} + z^{-4} + z^{-5}$
$+ z^{-4} + z^{-5} + z^{-6}$

$= 1 + 2z^{-1} + 3z^{-2} + 3z^{-3} + 3z^{-4} + 2z^{-5} + z^{-6}$

$y(n) = \{1, 2, 3, 3, 3, 2, 1\} = x_1(n) * x_2(n)$

$y(n) = x_1(n) * x_2(n)$

$= x_1(n) + x_1(n-1) + x_1(n-2)$

$= \{1, 2, 3, 3, 3, 2, 1\}$ //

SAMPLE CALCULATION – similar arithmetic for $y(n)$

For $y(2) = x_1(2) + x_1(1) + x_1(0) = 1 + 1 = 2$

$y(3) = x_1(3) + x_1(2) + x_1(1) = 1 + 1 + 1 = 3$

$y(1) = x_1(1) + x_1(0) + x_1(-1) = 1 + 0 + 0 = 1$

MB 3.14

$$H_d(e^{j\omega}) = \begin{cases} 1e^{-j\alpha\omega} & |\omega| \le \omega_c \\ 0 & \omega_c < |\omega| \le \pi \end{cases}$$

(1) Determine the ideal impulse response $h_d(n)$ using [3.2]

$$x(n) \triangleq \mathcal{F}^{-1}[X(e^{j\omega})] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$h_d(n) = \frac{1}{2\pi}\int_{\omega_c}^{\pi} 0\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\alpha\omega}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega(n-\alpha)}\, d\omega = -\frac{j}{2(n-\alpha)}\, e^{j\omega(n-\alpha)}\Big|_{-\omega_c}^{\omega_c} + C$$

$$= \frac{1}{2j(n-\alpha)}\, e^{j\omega(n-\alpha)}\Big|_{-\omega_c}^{\omega_c} + C$$

$$= \frac{1}{2j(n-\alpha)}\left[e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}\right] + C$$

NOTE: $\frac{1}{nx}\left[\frac{e^{inx} - e^{-inx}}{2i}\right] = \frac{\sin(nx)}{nx} = \operatorname{sinc}(nx)$

$$= \frac{1}{\pi(n-\alpha)}\left[\frac{e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}}{2j}\right] + C$$

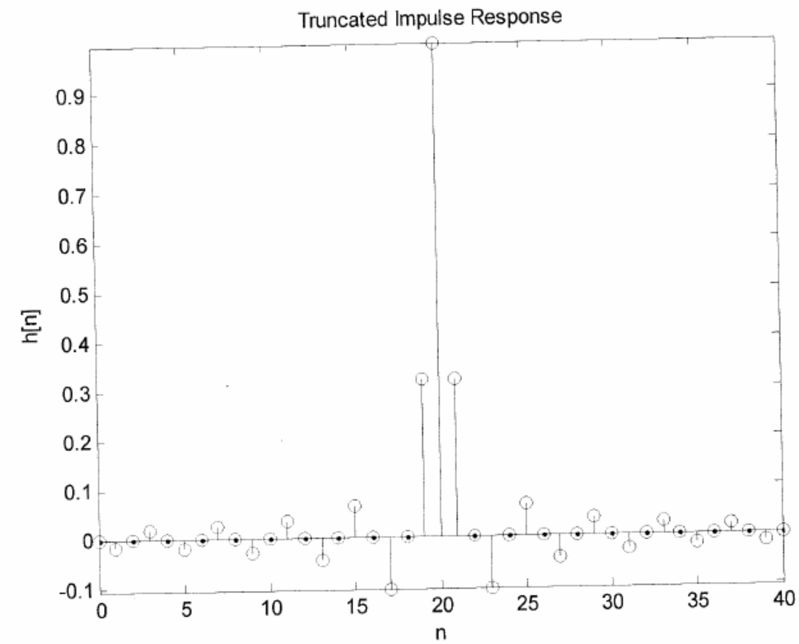
$$= \frac{\sin(\omega_c(n-\alpha))}{\pi(n-\alpha)}\left[\frac{\omega_c}{\omega_c}\right] = \frac{\omega_c}{\pi}\,\frac{\sin(\omega_c(n-\alpha))}{\omega_c(n-\alpha)}$$

$$= \frac{\omega_c}{\pi}\operatorname{sinc}[\omega_c(n-\alpha)]$$

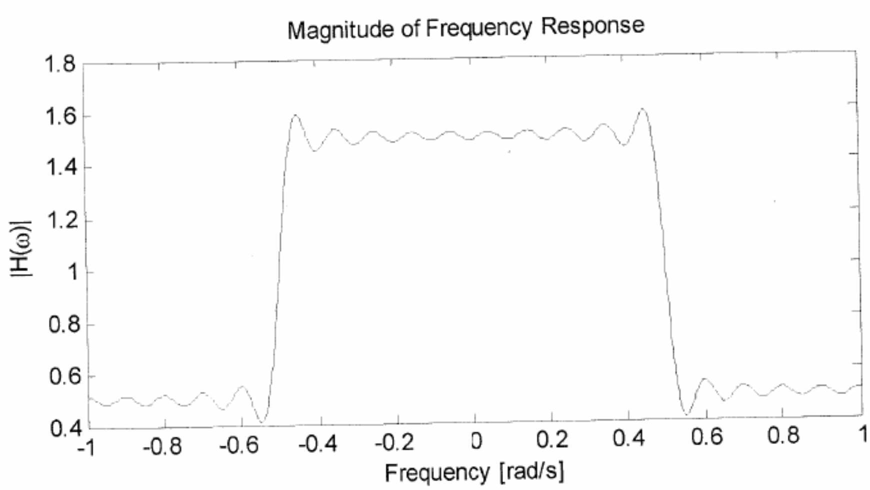
This is a low-pass filter. Ideally the filter would not have ringing in the edges (observed in the plots due to convolution with a sinc function.

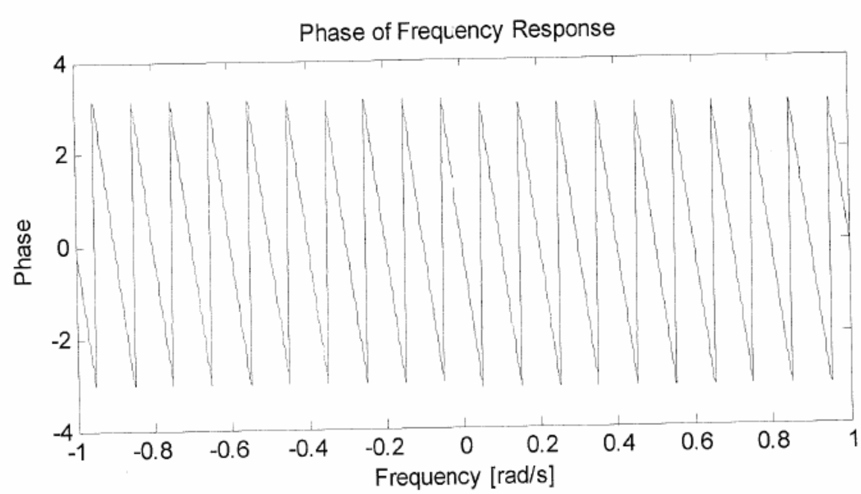
Please see next page for plots.

MB3.14.2



MB3.14.3





```
% EE410 - HW #2
%
% MB3.14 - Plot the truncated impulse response of h[n]
% and frequency response.
%
%
% Written By: Alana M. Soehartono
%             September 19, 2013
%             EE410 - Digital Signal Processing
%             University of Wisconsin-Milwaukee
%
% % % % % % % % % % % % % % % % % % % % % % % % %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear;clc;

%% PROBLEM 3.14.2 %%

% Define parameters
N = 41;
n = [0:N-1];
alpha = 20;
wc = 0.5*pi;

% Compute truncated impulse response
h = sin(wc*(n-alpha))./(pi*(n-alpha));

% For 'NaN' values, replace with '1'
check = isnan(h);
index = find(check == 1);
h(index) = 1;

% Plot figure
figure, stem(n,h);
title('Truncated Impulse Response'); xlabel('n'); ylabel('h[n]'); axis tight;

%% PROBLEM 3.14.3 %%

% Define parameters
M = 500;
k = -M:M;
w = (pi/M)*k;

% Compute frequency response
H = dtft(h,n,M);

% Plot figure
figure, subplot(2,1,1); plot(w/pi,abs(H)); title('Magnitude of Frequency
Response'); xlabel('Frequency [rad/s]'); ylabel('|H(\omega)|');
        subplot(2,1,2); plot(w/pi,angle(H)); title('Phase of Frequency
Response'); xlabel('Frequency [rad/s]'); ylabel('Phase');
```

```
function [X] = dtft(x,n,M)
%
% Compute the Discrete-Time Fourier Transform
%
% INPUTS:
%   x - Finite duration sequence over n
%   n - Sample position vector
%   M - Number of samples
%
% OUTPUTS:
%   X - DTFT values computed at w frequencies
%
% Written By: Alana M. Soehartono
%             September 19, 2013
%             EE410 - Digital Signal Processing
%             University of Wisconsin-Milwaukee
%
% % % % % % % % % % % % % % % % % % % % % % % % %

k = -M:M;
X = x*(exp(-i*pi./M)).^(n'*k);
end
```

EE410-001 **FALL 2013**

Digital Signal Processing

Homework 3

Problems from course books:

TB = Text Book = Digital Signal Processing, 4th Ed.
John Proakis & Dimitris Manolakis, Pearson Prentice Hall 2007.

MB = Matlab Book = Digital Signal Processing using MATLAB, 3rd Ed.
Vinay Ingle & John Proakis, CENGAGE Learning, 2012.

All plots in Matlab.

TB 5.4 j

TB 5.7 a

MB 3.11 1

MB 3.11 3

MB P3.11 5

Due: 9-26-2013

ALANA M. SOEHARTONO
SEPT. 24, 2013
EE410 – HW #3

DEPT. OF ELECTRICAL ENGINEERING
SUBMITTED AT 9:30 PM, SEPT. 26, '13

100

TB 5.4(j) Determine and sketch the magnitude and phase response.

$$y(n) = \frac{1}{4}[x(n) + x(n-1) + x(n-2) + x(n-3)]$$

$$Y(\omega) = H(\omega)X(\omega)$$

$$h(n) = \left\{\underset{\uparrow}{\frac{1}{4}}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$$

$$H(\omega) = \frac{1}{4}(1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega})$$

$$= \frac{1}{4}[(1 + e^{-j2\omega}) + e^{-j\omega} + e^{-j3\omega}]$$

$$= \frac{1}{4}[e^{-j\omega}(e^{j\omega} + e^{-j\omega}) + e^{-j2\omega}(e^{j\omega} + e^{-j\omega})]$$

$$= \frac{1}{4}[e^{-j\omega}(2\cos\omega) + e^{-j2\omega}(2\cos\omega)]$$

$$= \frac{1}{2}\cos\omega[e^{-j\omega} + e^{-j2\omega}]$$

Please see attachment for plots. (Next page)

TB 5.7(a) For the FIR filter, plot the magnitude & phase response.

$$y(n) = x(n) + x(n-4)$$

$$h(n) = \{1, 1\}$$

$$H(\omega) = 1 + e^{-j4\omega}$$

$$= e^{-j\omega}[e^{j\omega} + e^{-j3\omega}] \Rightarrow \text{check: } [1 + e^{-j4\omega}]$$

Please see attachment for plots. (Next page)

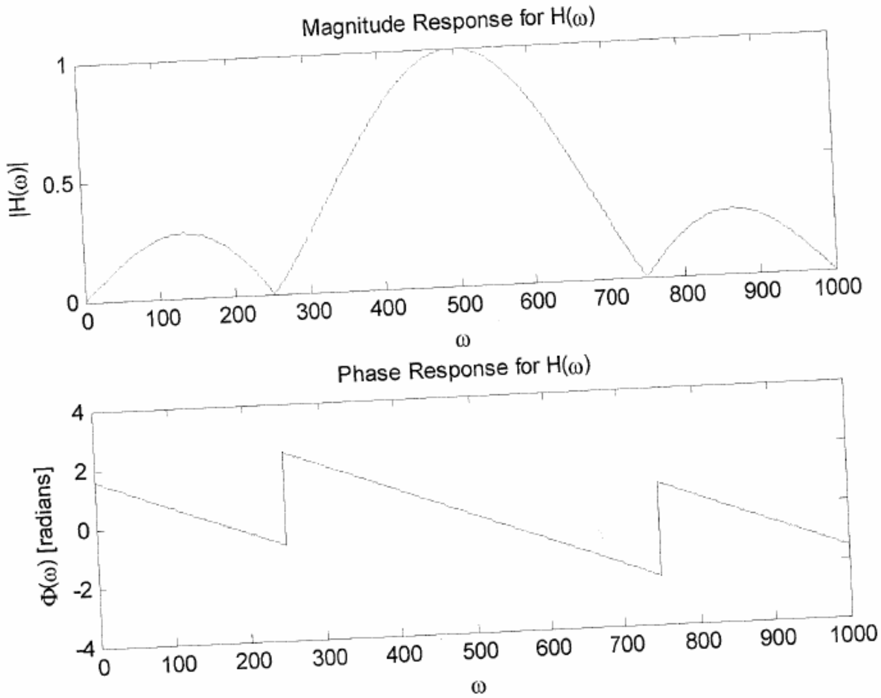
MB 3.11 (1) Determine the freq. response $H(e^{j\omega})$ for the LSI system described by the impulse response. Plot the response over $[-\pi, \pi]$

$$h(n) = (0.9)^{|n|}$$

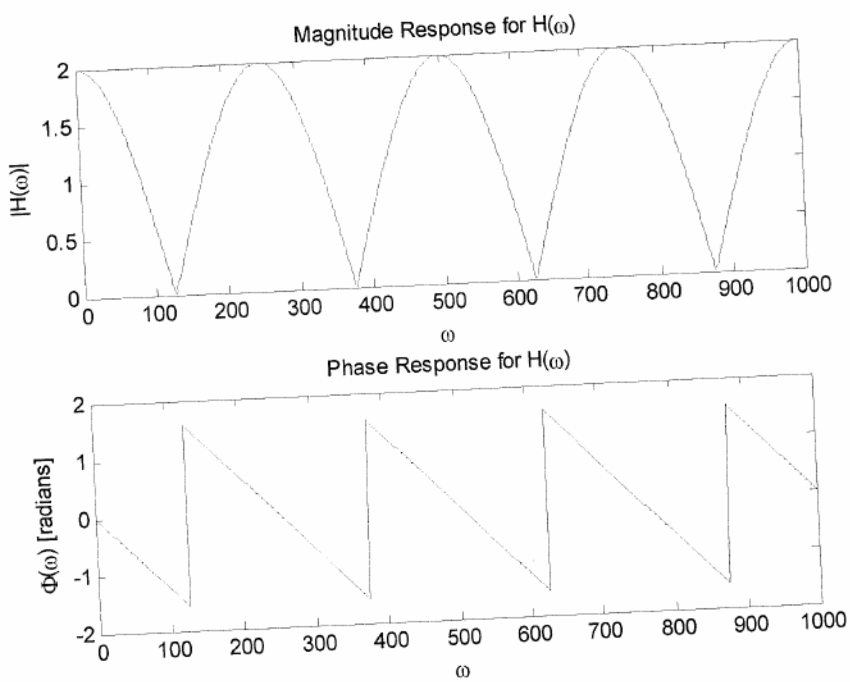
$$H(e^{j\omega}) = \sum_{-\infty}^{\infty} h(n)e^{-j\omega n} = \sum_{-\infty}^{\infty} (0.9)^{|n|} e^{-j\omega n}$$

Alana M. Soehartono
EE410 – Digital Signal Processing, HW#3

TB5.4(j)



TB5.7(a)



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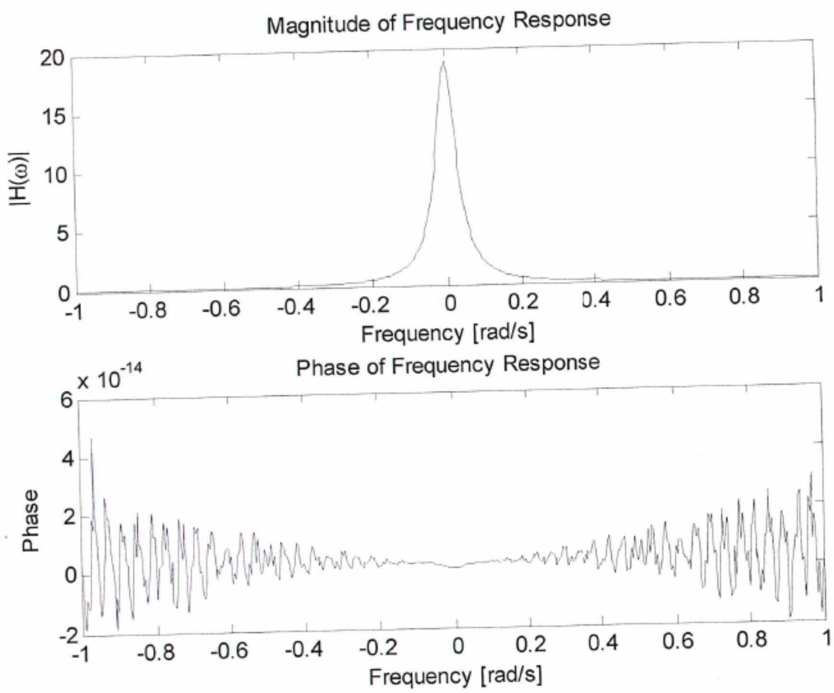
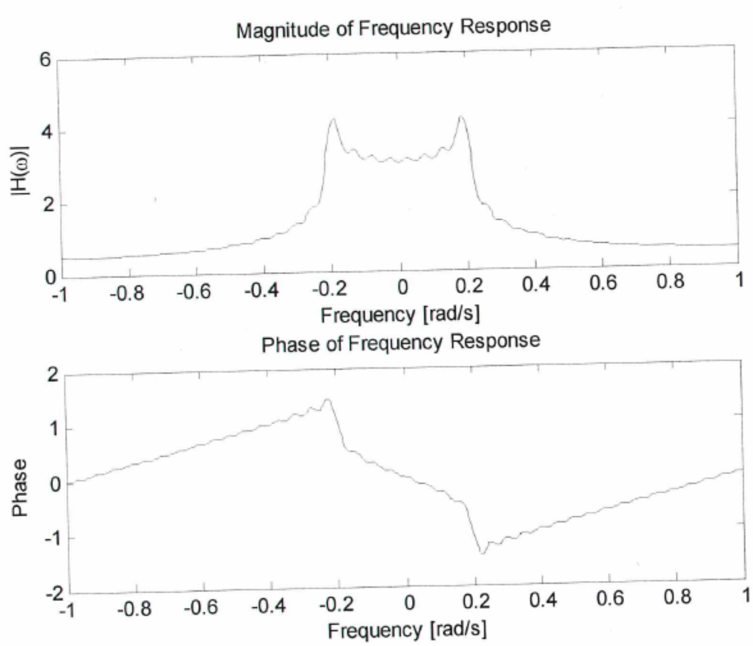
EE410 – Digital Signal Processing, HW#3

MB3.11(1)



MB3.11(3)



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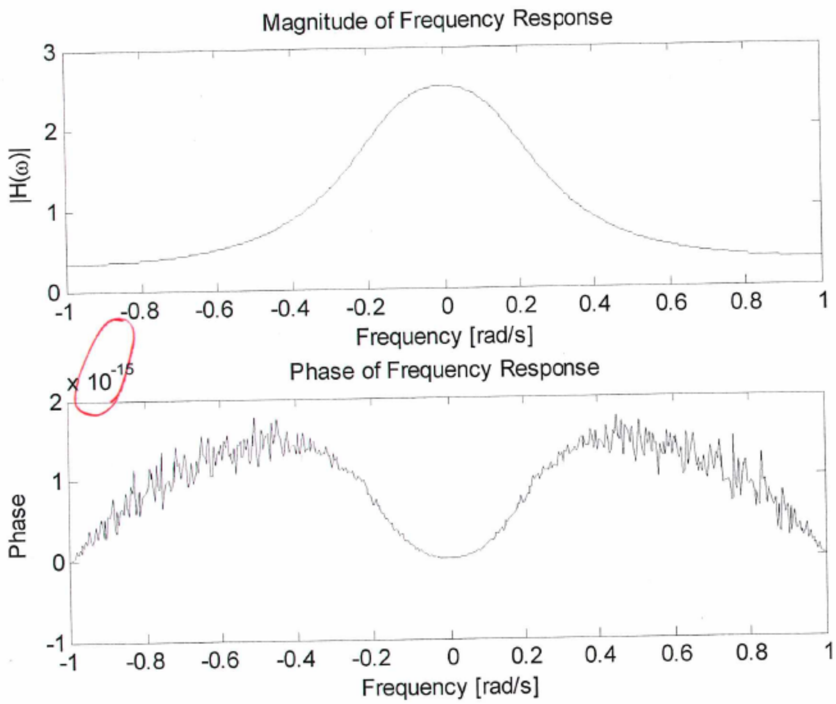
EE410 – Digital Signal Processing, HW#3

MB3.11(5)



Alana M. Soehartono
EE410 – Digital Signal Processing, HW#3

```
% EE410 - HW #3
%
% Plots of the Frequency Response for TB5.4(j), TB5.7(a), MB3.11(1),(3),(5)
%
% Written By: Alana M. Soehartono
%             September 25, 2013
%             EE410 - Digital Signal Processing
%             University of Wisconsin-Milwaukee
%
% % % % % % % % % % % % % % % % % % % % % % % % %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear;clc;

%% TB 5.4(j) Plots for frequency response of H(omega)

w = linspace(-pi,pi,1000);
H = 0.5*cos(w).*(exp(-i*w) + exp(-i*2.*w));

figure, subplot(2,1,1); plot(abs(H)); title('Magnitude Response for
H(\omega)');
xlabel('\omega'); ylabel('|H(\omega)|');

subplot(2,1,2); plot(angle(H)); title('Phase Response for H(\omega)');
xlabel('\omega'); ylabel('\Phi(\omega) [radians]');

%% TB 5.7(a) Plots for frequency response of H(omega)

H = [];
H = exp(-i.*w).*(exp(i.*w) + exp(-i*3.*w));

figure, subplot(2,1,1); plot(abs(H)); title('Magnitude Response for
H(\omega)');
xlabel('\omega'); ylabel('|H(\omega)|');

subplot(2,1,2); plot(angle(H)); title('Phase Response for H(\omega)');
xlabel('\omega'); ylabel('\Phi(\omega) [radians]');

%% MB 3.11(1) Plot of frequency response of H(omega)

M = 250; k = -M:M;
w = (pi/M)*k;

n = -50:50;
h = (0.9).^abs(n);

% Compute frequency response
H = dtft(h,n,M);

% Plot figure
figure, subplot(2,1,1); plot(w/pi,abs(H)); title('Magnitude of Frequency
Response');
xlabel('Frequency [rad/s]'); ylabel('|H(\omega)|');
```

Alana M. Soehartono


```
subplot(2,1,2); plot(w/pi,angle(H)); title('Phase of Frequency Response');
xlabel('Frequency [rad/s]'); ylabel('Phase');

%% MB 3.11(3) Plot of frequency response of H(omega)

M = 250; k = -M:M;
w = (pi/M)*k;

% Note: For plotting sequence, we use [(n - n0) >= 0]
n1 = 0; n2 = 40;

n = -50:50;
h = sinc(0.2.*n).*([(n-n1) >= 0]-[(n-n2) >= 0]);

% Compute frequency response
H = dtft(h,n,M);

% Plot figure
figure, subplot(2,1,1); plot(w/pi,abs(H)); title('Magnitude of Frequency
Response');
xlabel('Frequency [rad/s]'); ylabel('|H(\omega)|');

subplot(2,1,2); plot(w/pi,angle(H)); title('Phase of Frequency Response');
xlabel('Frequency [rad/s]'); ylabel('Phase');

%% MB 3.11(5) Plot of frequency response of H(omega)

M = 250; k = -M:M;
w = (pi/M)*k;

n = -50:50;
h = (0.5).^abs(n).*cos(0.1*pi.*n);

% Compute frequency response
H = dtft(h,n,M);

% Plot figure
figure, subplot(2,1,1); plot(w/pi,abs(H)); title('Magnitude of Frequency
Response');
xlabel('Frequency [rad/s]'); ylabel('|H(\omega)|');

subplot(2,1,2); plot(w/pi,angle(H)); title('Phase of Frequency Response');
xlabel('Frequency [rad/s]'); ylabel('Phase');
```

Alana M. Soehartono
EE410 – Digital Signal Processing, HW#3

```
function [X] = dtft(x,n,M)
%
% Compute the Discrete-Time Fourier Transform
%
% INPUTS:
%   x - Finite duration sequence over n
%   n - Sample position vector
%   M - Number of samples
%
% OUTPUTS:
%   X - DTFT values computed at w frequencies
%
% Written By: Alana M. Soehartono
%             September 19, 2013
%             EE410 - Digital Signal Processing
%             University of Wisconsin-Milwaukee
%
% % % % % % % % % % % % % % % % % % % % % % % % %

k = -M:M;
X = x*(exp(-i*pi./M)).^(n'*k);
end
```

EE410-001 FALL 2013

Digital Signal Processing

Homework 4

Problems from course books:

TB = Text Book = Digital Signal Processing, 4th Ed.
John Proakis & Dimitris Manolakis, Pearson Prentice Hall 2007.

MB = Matlab Book = Digital Signal Processing using MATLAB, 3rd Ed.
Vinay Ingle & John Proakis, CENGAGE Learning, 2012.

TB 6.9

TB 6.10

TB 6.11

MB 3.19

Due: 10-8-2013

ALANA M. SOEHARTONO
SEPT. 30, 2013
EE410 - HW # 4

100

TB.6.9 consider the continuous-time signal

$$x_a(t) = \begin{cases} e^{-j2\pi F_0 t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Freq. shifting

step fx $e^{-j2\pi F_0 t} u(t)$

$\frac{1}{j2\pi F} + \delta(F)$

using sifting fx:

$f(t) e^{j2\pi F_0 t} = F(F-F_0)$

(A) compute analytically the spectrum $X_a(F)$ of $x_a(t)$.

$$X_a(F) = \int_{-\infty}^{\infty} x_a(t) e^{-j2\pi F t} dt$$

$$= \int_0^{\infty} e^{-j2\pi F_0 t} e^{-j2\pi F t} dt = \int_0^{\infty} e^{-j2\pi t(F_0+F)} dt$$

$$= \frac{1}{-j2\pi(F_0+F)} e^{-j2\pi t(F_0+F)} \Big|_0^{\infty} = \frac{1}{-j2\pi(F_0+F)} [\cancel{e^{-\infty}}^{0} - \cancel{e^{0}}^{1}]$$

$\Delta = \frac{R}{2^{b+1}}$

$$= \frac{1}{j2\pi(F_0+F)} \;//$$

$e^{j0} = [\cos(0) + j\sin(0)]$

$e^{j\infty} = [\cos(\infty) + j\sin(\infty)]$

(B) compute analytically the spectrum of the signal $x(n) = x_a(nT)$

$T = \frac{1}{F_s}$

苏、安娜 那是我的书。
我不是中国人。
他们没有朋友。

$$x(n) = \begin{cases} e^{-j2\pi F_0 n/F_s} & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$X(F) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi f n} \qquad f = F/F_s$$

$$= \sum_{n=0}^{\infty} e^{-j2\pi F_0 n/F_s} e^{-j2\pi \frac{F}{F_s} n} = \sum_{n=0}^{\infty} e^{-j2\pi n/F_s (F_0+F)}$$

what's a better method?

$e^{j\infty} = ?? 0$

For an infinite geometric series,

$$\sum_{n=0}^{\infty} e^{-j2\pi n \frac{1}{F_s}(F_0+F)} = \frac{\cancel{e^{-j2\pi(\infty)\frac{1}{F_s}(F_0+F)}}^{0} - \cancel{e^{-j2\pi(0)\frac{1}{F_s}(F_0+F)}}^{1}}{e^{-j2\pi \frac{1}{F_s}(F_0+F)} - 1} = \frac{1}{1 - e^{-j2\pi(F_0+F)/F_s}} \;//$$

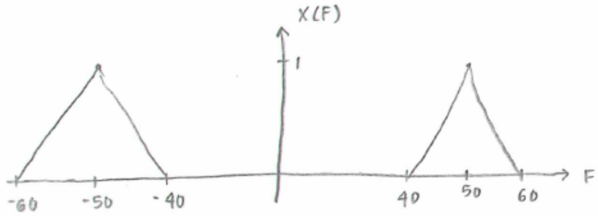
(C) Plot the magnitude spectrum $|X_a(F)|$ for $F_0 = 10$ Hz

(D) Plot the magnitude spectrum $|X(F)|$ for $F_s = 10, 20, 40$ and 100 Hz. (next page)

(E) Based on our plots, we observe aliasing when the frequency range is limited to $F = F_0 = F_s$. Here we are unable to see the entire signal in comparison with other plots.

-- ling ling.

TB 6.10 consider the sampling of the bandpass signal whose spectrum is shown. Determine the minimum sampling rate F_s to avoid aliasing.



$F_c = 50$

$$r = \frac{F_c + B/2}{B} = \frac{60}{20} = 3$$

$$b = \frac{F_c + B/2}{r} = \frac{60}{3} = 20 \text{ Hz}$$

$F_s = 2b = 40 \text{ Hz}$

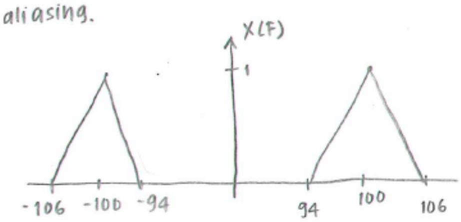
For aliasing to be avoided, sampling should be at least $F_s > 2B$.

$B = 60 - 40 = 20 \text{ Hz}$

$F_s = 2(20 \text{ Hz}) = 40 \text{ Hz} = 2b$

The sampling frequency should at least be 40 Hz to avoid aliasing.

TB 6.11 consider the sampling of the bandpass signal shown. Determine the minimum F_s to avoid aliasing.



Hello!

$B = 106 - 94 = 12$

$F_c = 100$

$$r = \frac{F_c + \frac{B}{2}}{B} = \frac{106}{12} = 8.83 = 8$$

$$b = \frac{F_c + B/2}{r} = \frac{106}{8} = \frac{53}{4}$$

$K_{max} \leq \frac{F_H}{B}$ $K_{max} = 8$

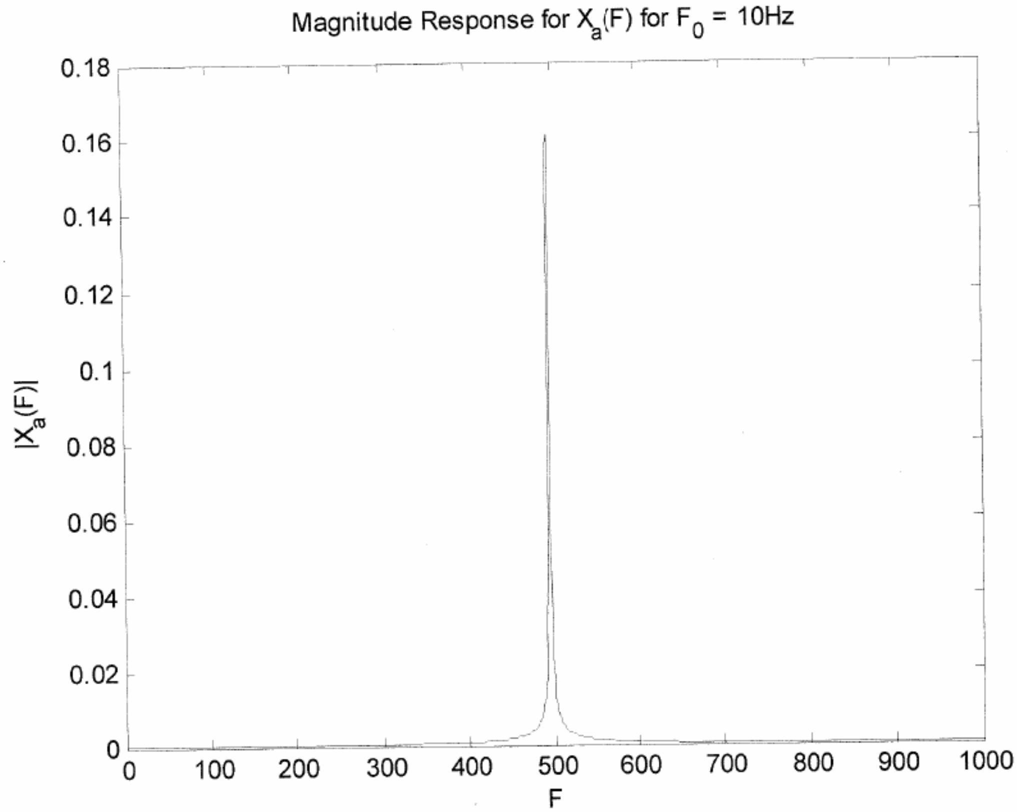
$$F_{s\,max} = \frac{2F_H}{K_{max}} = \frac{2(106)}{8} = 26.5 \text{ Hz}.$$

$F_s = 2b = \frac{53}{2}$ Hz. α.

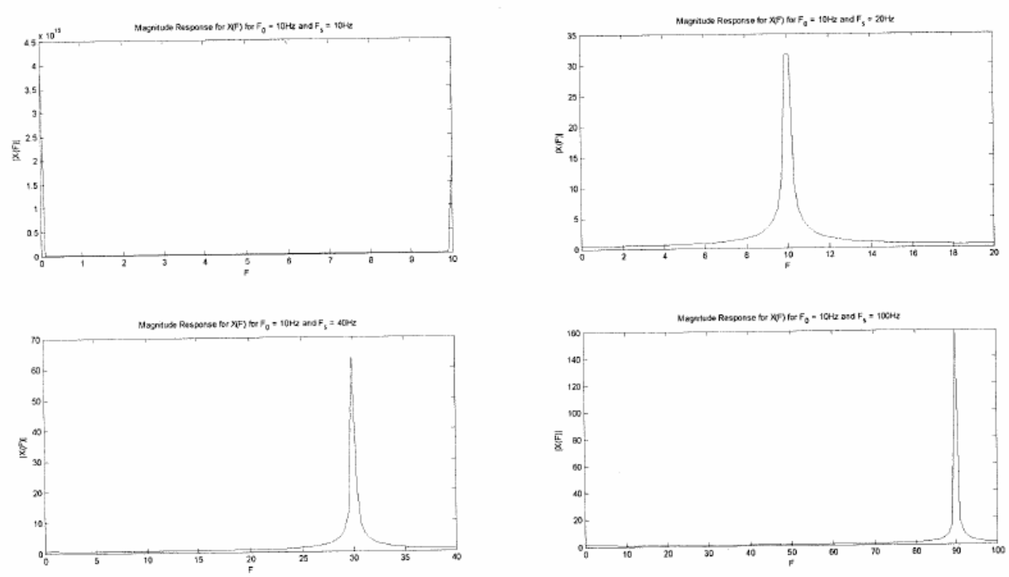
widens the BW.

Alana M. Soehartono
EE410 – Digital Signal Processing, HW#4

TB6.9(c)

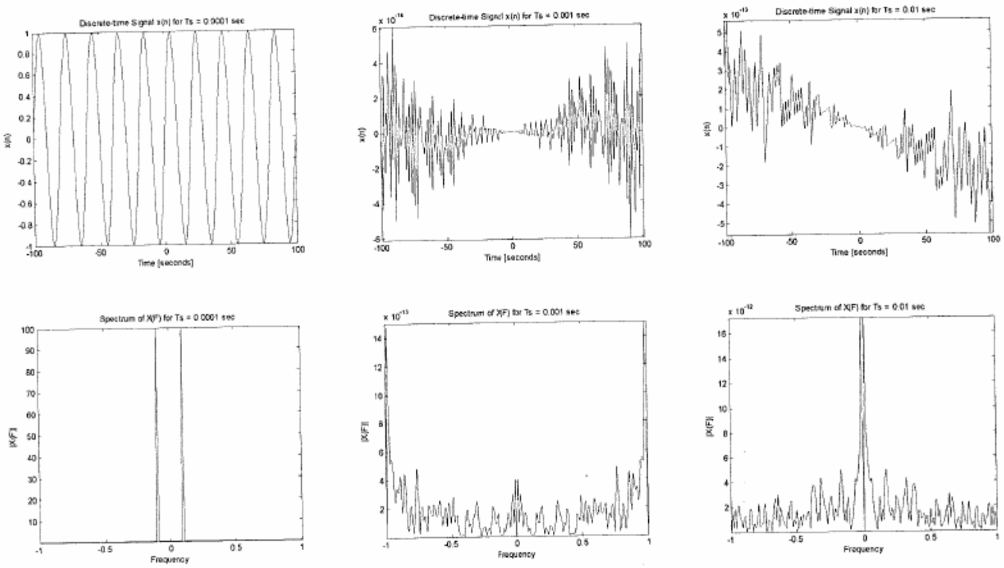


TB6.9(d)



Alana M. Soehartono
EE410 – Digital Signal Processing, HW#4

MB3.19



Alana M. Soehartono
EE410 – Digital Signal Processing, HW#4

```
% EE410 - HW #4
%
% Plots of the Frequency Response for TB6.9 and MB3.19
%
% Written By: Alana M. Soehartono
%             October 8, 2013
%             EE410 - Digital Signal Processing
%             University of Wisconsin-Milwaukee
%
% % % % % % % % % % % % % % % % % % % % % % % % %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%% TB 6.9(c) Plots for frequency response of H(omega)

clear;clc;
F = linspace(-1E3,1E3,1E3);
Fo = 10;
Xa = 1./(j*2*pi*(Fo+F));

figure, plot(abs(Xa)); title('Magnitude Response for X_a(F) for F_0 = 10Hz');
xlabel('F'); ylabel('|X_a(F)|');

%% TB 6.9(d) Plots for frequency response of H(omega)

clear;clc;
fs = [10 20 40 100];
Fo = 10;
figure,
for iter = 1:length(fs)
    Fs = fs(iter);
    F = linspace(0,Fs,100);
    X(iter,:) = 1./(1 - exp(-j*2*pi*(F + Fo)/Fs));
    subplot(2,2,iter); plot(F,abs(X(iter,:)));
    title(['Magnitude Response for X(F) for F_0 = 10Hz and F_s =
',num2str(Fs), 'Hz']);
    xlabel('F'); ylabel('|X(F)|');
end

%% MB 3.19 Spectrum plots of discrete-time signal of x_a(t)

clear;clc;

% Plotting the Discrete-Time Signal x(n)
ts = [0.0001 0.001 0.01];
figure,
for iter = 1:length(ts)
    Ts = ts(iter);
    n = -100:100;
    x(iter,:) = sin(1000*pi*(n*Ts));
    subplot(2,3,iter); plot(n,x(iter,:));
    title(['Discrete-time Signal x(n) for Ts = ',num2str(Ts), ' sec']);
    xlabel('Time [seconds]'); ylabel('x(n)'); axis tight;
end
```

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```
% Plotting the DTFT of x(n)
M = 99; k = -M:M;
w = (pi/M)*k;
for iter = 1:length(ts)
    X(iter,:) = dtft(x(iter,:),n,M);
    X2(iter,:) = fft(x(iter,:));
    subplot(2,3,3+iter); plot(w/pi,abs(X(iter,:)));
    title(['Spectrum of X(F) for Ts = ',num2str(ts(iter)), ' sec']);
    xlabel('Frequency'); ylabel('|X(F)|'); axis tight;
end
```

```
function [X] = dtft(x,n,M)
%
% Compute the Discrete-Time Fourier Transform
%
% INPUTS:
%   x - Finite duration sequence over n
%   n - Sample position vector
%   M - Number of samples
%
% OUTPUTS:
%   X - DTFT values computed at w frequencies
%
% Written By: Alana M. Soehartono
%             September 19, 2013
%             EE410 - Digital Signal Processing
%             University of Wisconsin-Milwaukee
%
% % % % % % % % % % % % % % % % % % % % % % % % %

k = -M:M;
X = x*(exp(-i*pi./M)).^(n'*k);
end
```

ALANA SOEHARTONO
EE410
OCT. 24, 2013
QUIZ #2

100

DFT TIME N^2

FFT Time $N \log(\frac{N}{2})$

$\frac{N}{2} \log_2(N)$

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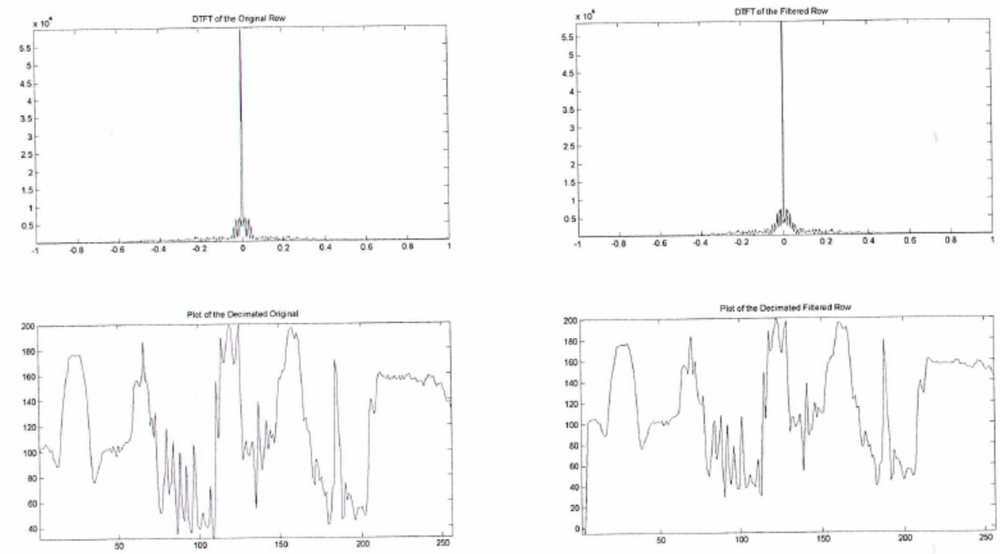
EE410 – HW#5

90/100

P1)

Shown below is the DTFT of the original and filtererd rew vector 256, along with its corresponding



discrete time signals. Both the original and filtered images are decimated by a factor of 2.

Shown below is the plot of the differences ebtween the decimated original row and filtered row.

2)

(1) $L = 2^b = 2^8 = 256$

(2) $\Delta = \dfrac{x_{max} - x_{min}}{L - 1} = \dfrac{2.5 - (-2.5)}{255} = \dfrac{5}{255} = 0.0196\ V = 19.6\ mV$

(3) $\Delta l = 1.5$

$l = \dfrac{1.5}{\Delta} = 76.5$

(4) 0100 1100

(5) ?

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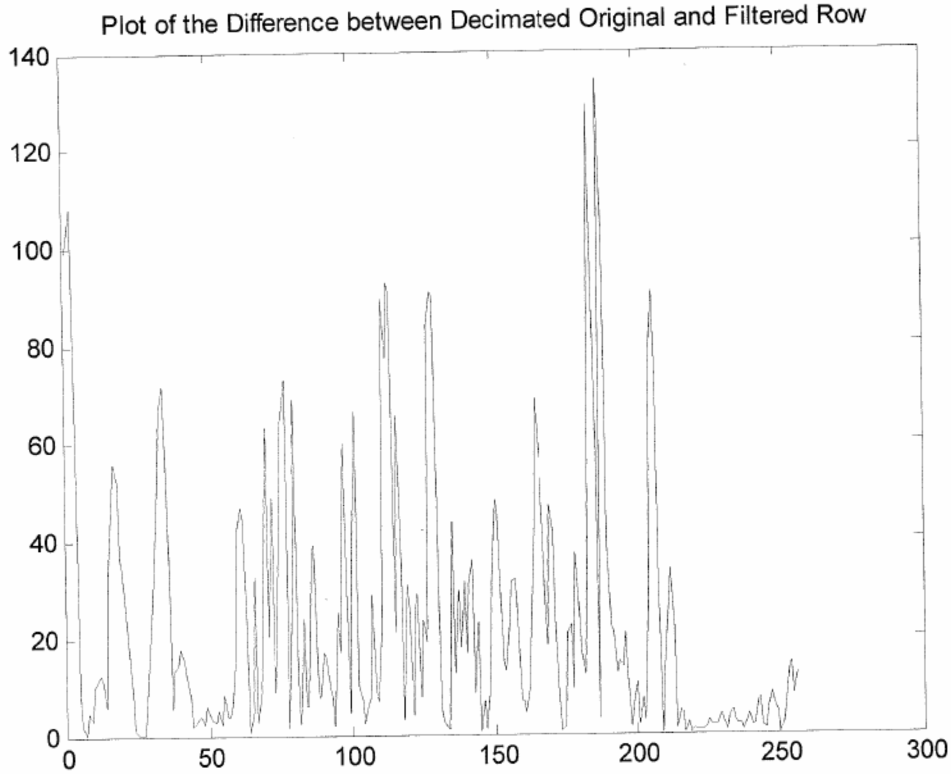
EE410 – HW#5



P3.3)

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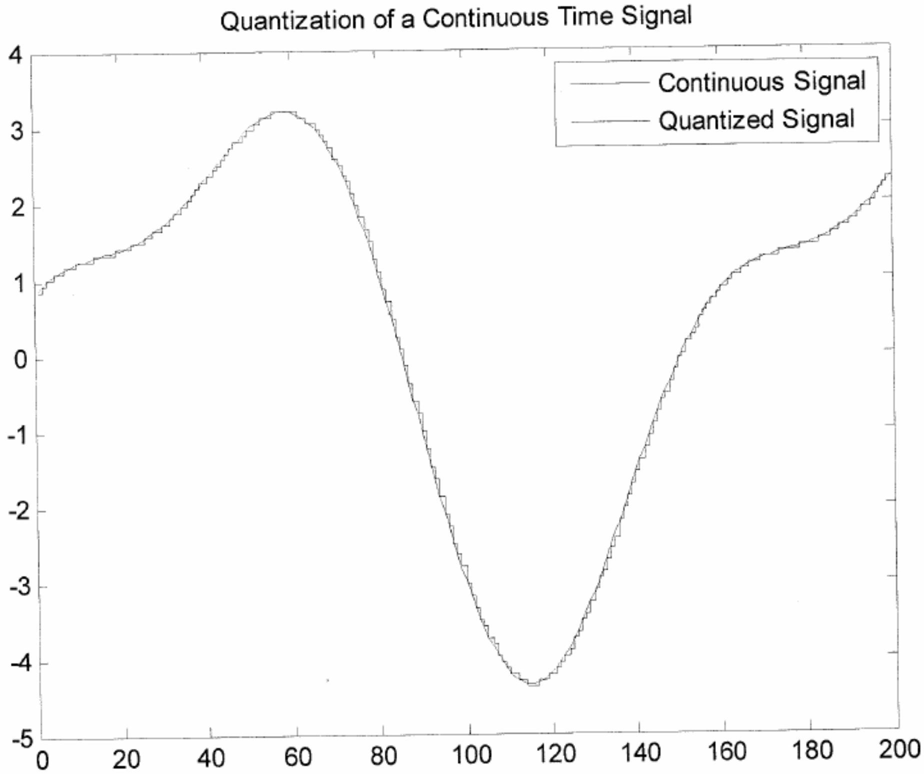

P3.4)

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Original Image

Quantized Image with b =4

Quantized Image with b =5

Quantized Image with b =6

Quantized Image with b =7

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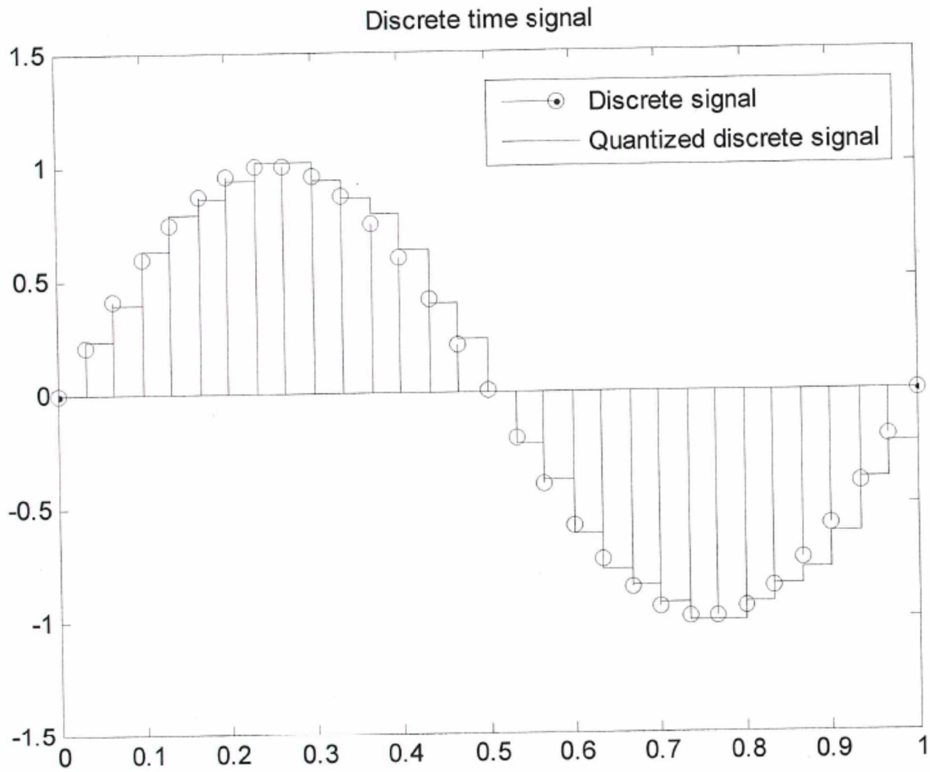
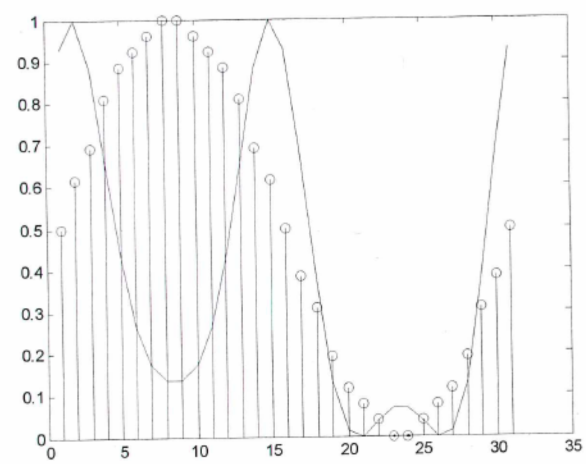
P3.5)





?

-10

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October 17, 2013
EE410 – HW#5

```
% EE410 - HW #5
%
%
%
% Written By: Alana M. Soehartono
%             October 17, 2013
%             EE410 - Digital Signal Processing
%             University of Wisconsin-Milwaukee
%
% % % % % % % % % % % % % % % % % % % % % % % % %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%% Problem 1.1

clear;clc;
% Load image
lena512 = double(imread('lena512.bmp'));

% Filtering row vector 256
x = lena512(256,:);
b = fir1(15,0.5);
g = filter(b,1,x);

% DTFT of the filtered row
n = -length(g)/2:length(g)/2-1;
M = n(end); k = -M:M;
w = (pi/M)*k;
G = dtft(g,n,M);

% DTFT of the original row
n = -length(g)/2:length(g)/2-1;
M = n(end); k = -M:M;
w = (pi/M)*k;
X = dtft(x,n,M);

figure, plot(w/pi,abs(X)); title('Half-Band Filtered of Row 256');

% Decimation (written as separate function)
F = 2;
y1 = Decim(x,2);
y2 = Decim(g,2);

% Compare with MATLAB function decimate
% y1 = decimate(x,2);
% y2 = decimate(g,2);

figure, subplot(2,1,1); plot((y1));
        subplot(2,1,2); plot((y2));

% Finding the difference between decimated and original
y3 = y2 - y1;
```

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EE410 – HW#5

```
figure, plot(abs(y3)); title('Plot of the Difference between Decimated
Original and Filtered Row');

% Optional plot
figure, subplot(2,2,1); plot(w/pi,abs(X)); title('DTFT of the Original Row');
axis tight
        subplot(2,2,2); plot(w/pi,abs(G)); title('DTFT of the Filtered Row');
axis tight
        subplot(2,2,3); plot((y1)); title('Plot of the Decimated Original');
axis tight
        subplot(2,2,4); plot((y2)); title('Plot of the Decimated Filtered
Row'); axis tight


%% Problem 3.3

clear;clc;
% Continuous Time Signal
t = 0:1/8000:(1-1/8000); f1 = 50; f2 = 100; phase = pi/4;
x = 3.25.*sin(2*pi*f1.*t) + 1.25.*cos(2*pi*f2.*t + phase);

% Quantizer Parameter
b = 6; % Number of bits
L = 2^b; % Quantization Level
k = 0:L;
range = [2.5 -2.5]; % Range max to min
delta = (range(1) - range(2))/L; % Resolution

%%% Initial Quantizer I used
%%% Q = sign(x).*delta.*((abs(x)./delta) + 0.5);

% Quantizer
[I, Q]= biquant(b,range(2),range(1),x);
figure, stairs(Q(1:200)); hold on; plot(x(1:200),'r'); title('Quantization of
a Continuous Time Signal');


%% Problem 3.4

clear;clc;
% Original Image display
[aa,map] = imread('lena512.bmp');
figure, image(aa)
colormap(map)
axis off
axis image
title('Original Image');

figure,
for iter = 1:4
    bits = iter + 3;
    [I, Q]= biquant(bits,min(aa(:)),max(aa(:)),aa);
    subplot(2,2,iter);
    image(Q); colormap(map)
```

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EE410 – HW#5

```
    axis off
    axis image
    title(['Quantized Image with b =',num2str(bits)]);
end

%% Section 3.5

clear;clc;
% Discrete time signal
f = 30; T = 1/f; dt = 1/15;
t = 0:T:1;
x = sin(2*pi*t);

% Quantized discrete time signal
b = 8;
[I, Q]= biquant(b,-10,10,x);

createfigure(t,x,Q);

% Reconstruction
To = 1/f;
r = sinc((2*pi+b + To + x));
r = (r - min(r))/(max(r) - min(r));

q = biqtdec(b,-10,10,I);
q = (q - min(q))/(max(q) - min(q));

figure, stem(q); hold on; plot(r,'r');



function out = Decim(x,F)
% The purpose of this function is to decimate a signal by an integer value.
%
% INPUTS: x - The input vector
%         F - Integer decimation factor
%
%
% Written By: Alana M. Soehartono
%             October 11, 2013
%             EE410 - Digital Signal Processing
%             University of Wisconsin-Milwaukee
%
% % % % % % % % % % % % % % % % % % % % % % % % %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for M = 1:size(x,2)/F;
    ind = (M - 1)*F + 1;
    out(1,M) = (x(1,ind) + x(1,ind+1))*(1/F);
end
```

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October 17, 2013
EE410 – HW#5

```
function createfigure(X1, Y1, Y2)
%CREATEFIGURE(X1,Y1,Y2)
%  X1:  stem x
%  Y1:  stem y
%  Y2:  stairs y

%  Auto-generated by MATLAB on 17-Oct-2013 09:06:08

% Create figure
figure1 = figure;

% Create axes
axes1 = axes('Parent',figure1);
box(axes1,'on');
hold(axes1,'all');

% Create stem
stem(X1,Y1,'Parent',axes1,'DisplayName','Discrete signal');

% Create stairs
stairs(X1,Y2,'Color',[1 0 0],'Parent',axes1,...
    'DisplayName','Quantized discrete signal');

% Title
title('Discrete time signal');

% Create legend
legend1 = legend(axes1,'show');
set(legend1,...
    'Position',[0.544850316478987 0.786027167277167 0.316846986089645
0.100961538461538]);


function [X] = dtft(x,n,M)
%
% Compute the Discrete-Time Fourier Transform
%
% INPUTS:
%   x - Finite duration sequence over n
%   n - Sample position vector
%   M - Number of samples
%
% OUTPUTS:
%   X - DTFT values computed at w frequencies
%
% Written By: Alana M. Soehartono
%             September 19, 2013
%             EE410 - Digital Signal Processing
%             University of Wisconsin-Milwaukee
%
% % % % % % % % % % % % % % % % % % % % % % % % %

k = -M:M;
X = x*(exp(-i*pi./M)).^(n'*k);
```

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EE410 – HW#5

```
end
```

EE410-001 **FALL 2013**

Digital Signal Processing

Homework 6

Problems from course books:

TB = Text Book = Digital Signal Processing, 4th Ed.
John Proakis & Dimitris Manolakis, Pearson Prentice Hall 2007.

MB = Matlab Book = Digital Signal Processing using MATLAB, 3rd Ed.
Vinay Ingle & John Proakis, CENGAGE Learning, 2012.

1. TB 7.8
2. TB 7.9
3. Use the data set from TB 7.8 and perform linear filtering.
4. Use the data set from TB7.8 and perform linear filtering with DFT, IDFT
   4.1. With zero padding to N=8.
   4.2. With zero padding to N=16.
5. MB P5.10.1 Do both the DFT (N=200) and DTFT and plot.

Due: 11-5-2013

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EE410
NOVEMBER 5, 2010
HW#6

SUBMITTED AT 8PM

100

TB7.8) Determine the circular convolution of the sequences.

$$x_1(n) = \{ \underset{\uparrow}{1}, 2, 3, 1 \}$$

$$x_2(n) = \{ \underset{\uparrow}{4}, 3, 2, 2 \}$$

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n)\, x_2((m-n))_N \qquad m = 0, 1, \ldots, N-1$$

$x_3(0) = (4)(1) + (2)(2) + (2)(3) + (3)(1) = 4 + 4 + 6 + 3 = 17$

$x_3(1) = (3)(1) + (4)(2) + (2)(3) + (2)(1) = 3 + 8 + 6 + 2 = 19$

$x_3(2) = (2)(1) + (3)(2) + (4)(3) + (2)(1) = 2 + 6 + 12 + 2 = 22$

$x_3(3) = (2)(1) + (2)(2) + (3)(3) + (4)(1) = 2 + 4 + 9 + 4 = 19$

$x_3 = \{ 17, 19, 22, 19 \}$

1	2	3	1		
2	2	3	4	+	4
2+4+9+4 = 19					
1	2	3	1		
2	3	4	2	+	3
2+6+12+2 = 22					
1	2	3	1		
3	4	2	2	+	2
3+8+6+2 = 19					
1	2	3	1		
4	2	2	3	+	1
4+4+6+3 = 17					

TB7.9) $x_3(n) = x_1(n)$ Ⓝ $x_2(n)$

$$X_1(k) = \sum_{n=0}^{3} x_1(n)\, e^{-j2\pi nk/4} \qquad k = 0, 1, 2, 3$$

$$= 1 + 2e^{-j\pi k/2} + 3e^{-j\pi k} + e^{-j3\pi k/2}$$

$X_1(0) = 7$

$X_1(1) = 1 + 2e^{-j\pi/2} + 3e^{-j\pi} + e^{-j3\pi/2}$

$= 1 + 2[\cancel{\cos(\frac{\pi}{2})}^{0} - j\,\cancel{\sin(\frac{\pi}{2})}^{1}] + 3[\cancel{\cos\pi}^{-1} - j\,\cancel{\sin\pi}^{0}] + [\cancel{\cos(\frac{3\pi}{2})}^{0} - j\,\cancel{\sin(\frac{3\pi}{2})}^{-1}]$

$= 1 - j2 - 3 + j = -2 - j$

$X_1(2) = 1 + 2e^{-j\pi} + 3e^{-j2\pi} + e^{-j3\pi}$

$= 1 + 2[\cancel{\cos\pi}^{-1} - j\,\cancel{\sin\pi}^{0}] + 3[\cancel{\cos 2\pi}^{1} - j\,\cancel{\sin 2\pi}^{0}] + \cancel{\cos 3\pi}^{-1} - j\,\cancel{\sin 3\pi}^{0}$

$= 1 - 2 + 3 - 1 = 1$

$X_1(3) = 1 + 2e^{-j3\pi/2} + 3e^{-j3\pi} + e^{-j9\pi/2}$

$= 1 + j2 - 3 - j = -2 + j$

$X_1 = \{ 7, -2-j, 1, -2+j \}$ // ✓

$$X_2(k) = \sum_{n=0}^{3} x_2(n)\, e^{-j2\pi nk/4} \qquad k = 0, 1, 2, 3$$

$$= 4 + 3e^{-j\pi k/2} + 2e^{-j\pi k} + 2e^{-j3\pi/2}$$

$X_2(0) = 4 + 3 + 2 + 2 = 11$

$X_2(1) = 4 + 3e^{-j\pi/2} + 2e^{-j\pi} + 2e^{-j3\pi/2} = 4 - j3 - 2 + j2 = 2 - j$

$X_2(2) = 4 + 3e^{-j\pi} + 2e^{-j2\pi} + 2e^{-j3\pi} = 4 - 3 + 2 - 1 = 1$

$X_2(3) = 4 + 3e^{-j3\pi/2} + 2e^{-j3\pi} + 2e^{-j9\pi/2} = 4 + j3 - 2 - j2 = 2 + j$ ✓

$X_2 = \{ 11, 2-j, 1, 2+j \}$ //

$X_3(K) = X_1(K) \cdot X_2(K)$

$X_3(0) = (7)(11) = 77$

$X_3(1) = (-2-j)(2-j) = -4 + 2j - 2j + j^2$

$= -5$

$X_3(2) = (1)(1) = 1$

$X_3(3) = (-2+j)(2+j) = -4 - 2j + 2j + j^2 = -5$

$x_3(n) = \sum_{K=0}^{3} X_3(K) e^{j2\pi nK/4} \qquad n = 0, 1, 2, 3$

$= \frac{1}{4}(77 - 5e^{j\pi n/2} + e^{j\pi n} - 5e^{j3\pi n/2})$

$x_3(0) = (77 - 5 + 1 - 5)\frac{1}{4} = \frac{68}{4} = 17$

$x_3(1) = \frac{1}{4}(77 - 5e^{j\pi/2} + e^{j\pi} - 5e^{j3\pi/2})$

$= \frac{1}{4}(77 - j5 - 1 + j5) = 19$

$x_3(2) = \frac{1}{4}(77 - 5e^{j\pi} + e^{j2\pi} - 5e^{j3\pi})$

$= \frac{1}{4}(77 + 5 + 1 + 5) = 22$

$x_3(3) = \frac{1}{4}(77 - 5e^{j3\pi/2} + e^{j3\pi} - 5e^{j9\pi/2})$

$= \frac{1}{4}(77 + j5 - 1 - j5) = 19$

$x_3 = \{17, 19, 22, 19\}$ //

$x_2 = 4 \ 3 \ 2 \ 2$

3) $x_1 = [1 \ 2 \ 3 \ 1]$

$x_2(-K) = [2 \ 2 \ 3 \ \underset{\uparrow}{4}]$

$x_3(0) = (4)(1) = 4$

$x_3(1) = (3)(1) + (4)(2) = 3 + 8 = 11$

$x_3(2) = (2)(1) + (3)(2) + (3)(4) = 2 + 6 + 12 = 20$

$x_3(3) = (2)(1) + (2)(2) + (3)(3) + (4)(1) = 2 + 4 + 9 + 4 = 19$

$x_3(4) = (0)(1) + (2)(2) + (2)(3) + (3)(1) = 4 + 6 + 3 = 13$

$x_3(5) = (0)(1) + (0)(2) + (2)(3) + (2)(1) = 8$

$x_3(6) = (0)(1) + (0)(2) + (0)(3) + (2)(1) = 2$

$x_3(7) = (0)(1) + (0)(2) + (0)(3) + (0)(1) = 0$

HW6.4.1)

x3 = {4, 11, 20, 19, 13, 8, 2, 5.3E-15}

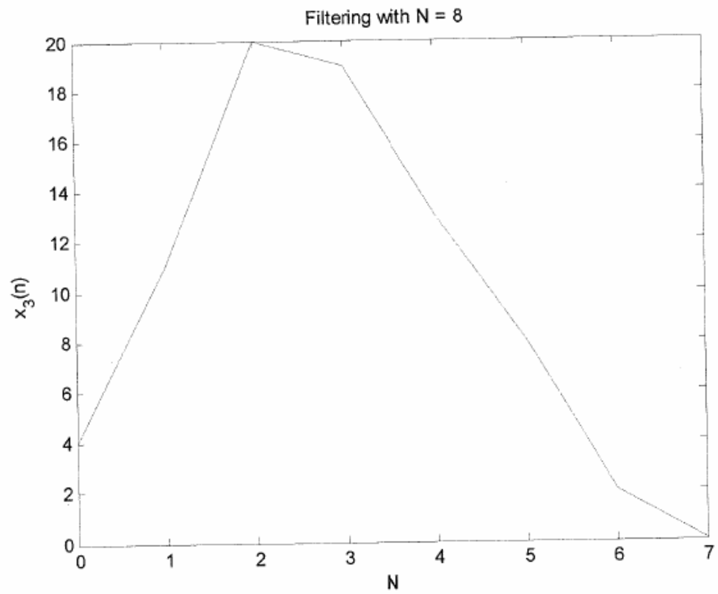


Figure 1 - Linear filtering with DFT and IDFT, N = 8.

HW6.4.2)

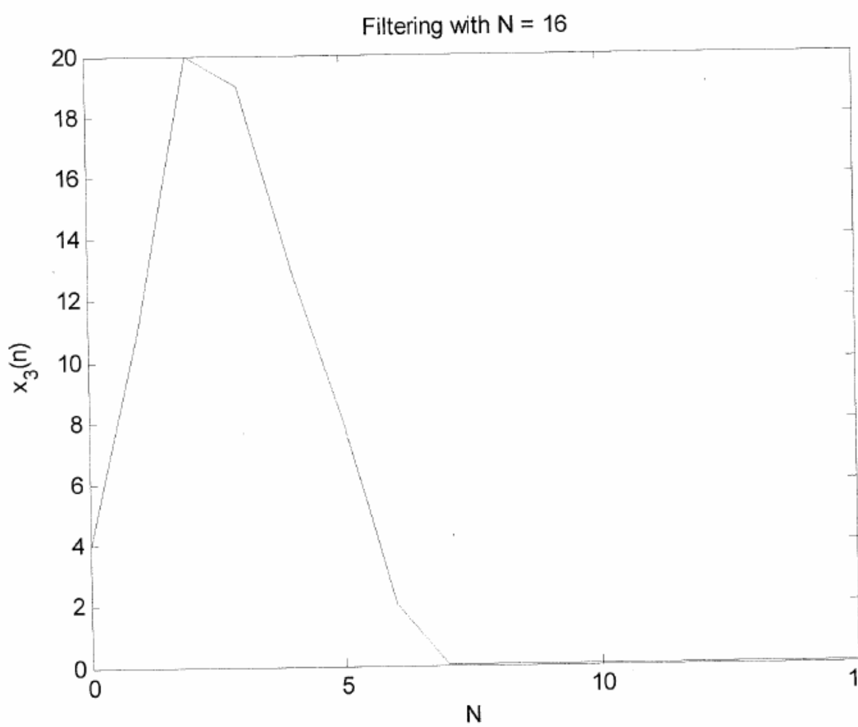


Figure 2 - Linear filtering with DFT and IDFT, N = 16.

HW6.5)

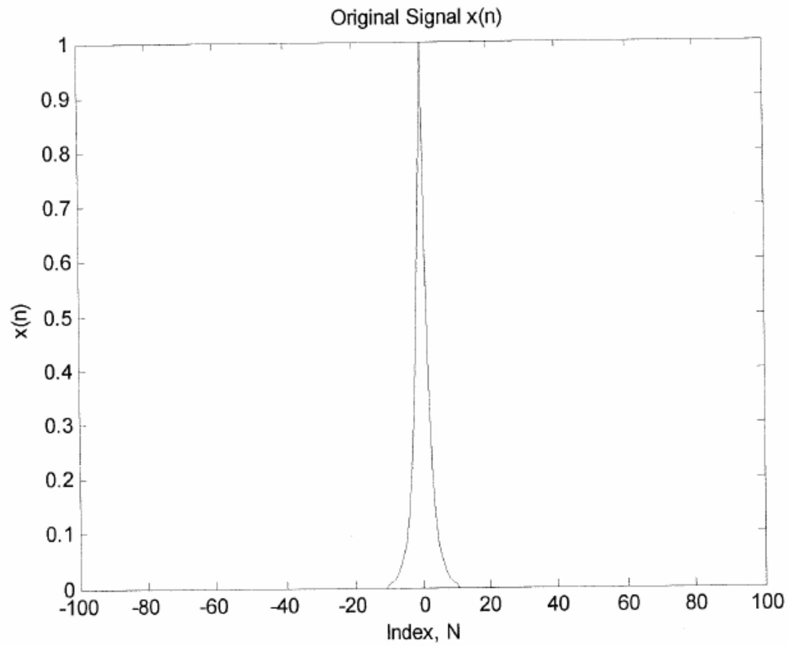


Figure 3 - Original signal x(n).

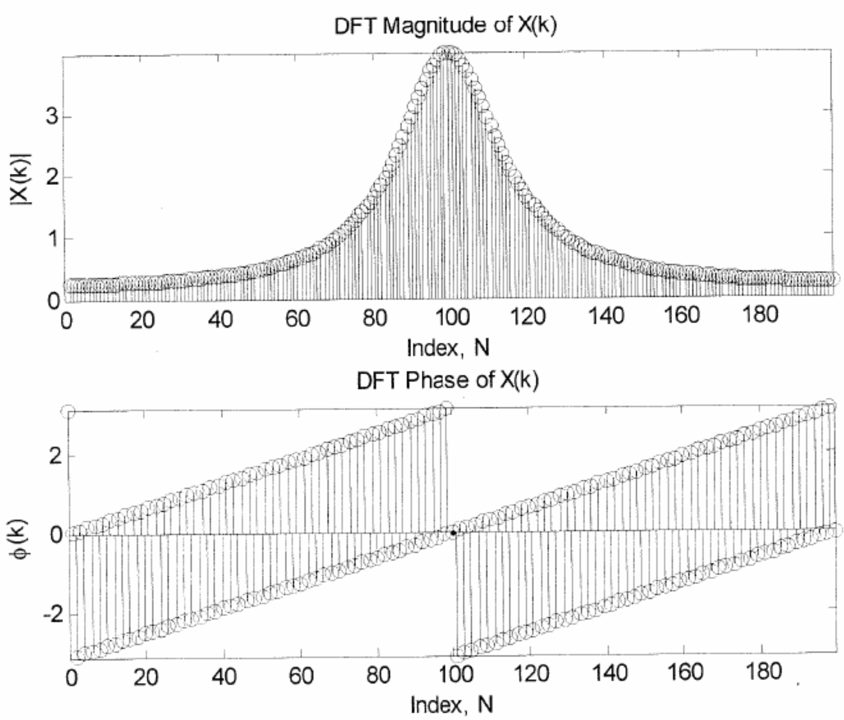


Figure 4 - Magnitude and phase plot of the original signal x(n) with DTFT, N = 200.

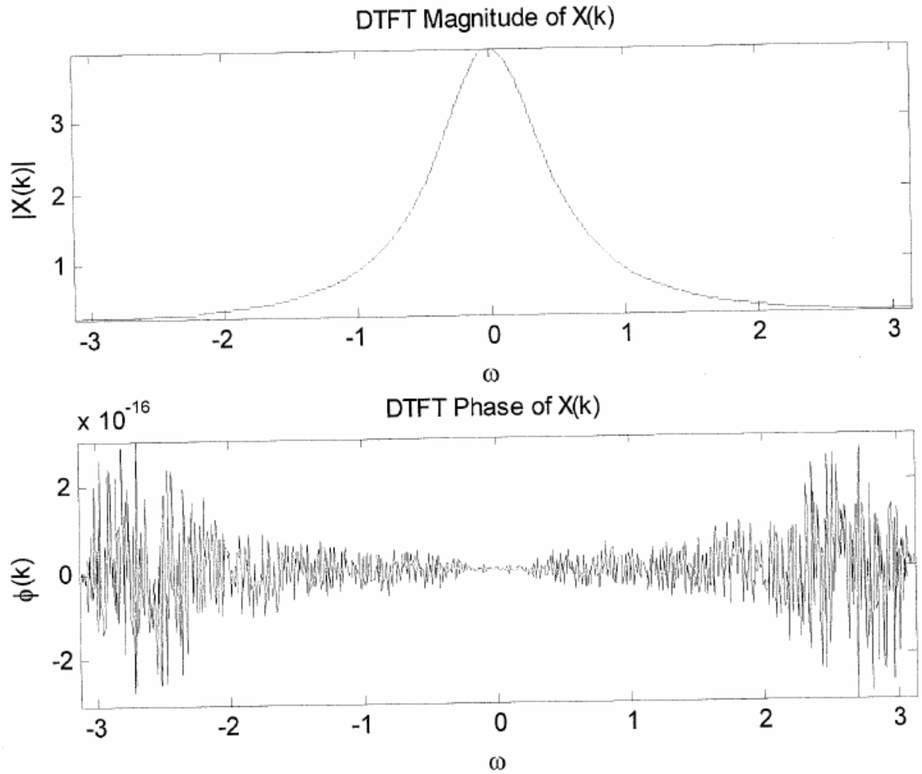


Figure 5 - Magnitude and phase plot of the original signal x(n) with DFT, N = 200.

APPENDIX)

```
% EE410 - HW #6
%
%
%
% Written By: Alana M. Soehartono
%             November 5, 2013
%              EE410 - Digital Signal Processing
%              University of Wisconsin-Milwaukee
%
% % % % % % % % % % % % % % % % % % % % % % % % %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear;clc

%% Problem 4.1

% Zero-padding
N = 8;
x1(:,N) = 0;
x2(:,N) = 0;

% DFT
X1 = dft(x1);
X2 = dft(x2);

% Filtering
X3 = X1.*X2;

% IDFT
x3 = (1/N)*idft(X3);
Index = 0:N-1;
figure, plot(Index,abs(x3)); title('Filtering with N = 8');
        xlabel('Index, N'); ylabel('x_3(n)'); axis tight;

%% Problem 4.2

% Zero-padding
N = 16;
x1(:,N) = 0;
x2(:,N) = 0;

% DFT
X1 = dft(x1);
X2 = dft(x2);

% Filtering
X3 = X1.*X2;

% IDFT
x3 = (1/N)*idft(X3);
Index = 0:N-1;
figure, plot(Index,abs(x3)); title('Filtering with N = 16');
```

```
        xlabel('Index, N'); ylabel('x_3(n)'); axis tight;

%% Problem 5 - MB5.10.1

% Generate the Signal
n1 = -10; n2 = 11;
n = -99:100;
x = 0.6.^abs(n).*([(n-n1) >= 0]-[(n-n2) >= 0]);
figure, plot(n,x); title('Original Signal x(n)');
                   xlabel('Index, N'); ylabel('x(n)');

% Zero-padding
N = 200;
x(:,N) = 0;

% DFT and plot
xx = CentreShift(x);
X = dft(xx);

Index = 0:N-1;

figure, subplot(2,1,1); stem(Index,abs(X)); title('DFT Magnitude of X(k)');
        xlabel('Index, N'); ylabel('|X(k)|'); axis tight;

subplot(2,1,2); stem(Index,angle(X)); title('DFT Phase of X(k)');
        xlabel('Index, N'); ylabel('\phi(k)'); axis tight;

% DTFT and plot
w = [-500:500]*pi/500;

% Compute frequency response
XX = dtft(x,n,w);

figure, subplot(2,1,1); plot(w,abs(XX)); title('DTFT Magnitude of X(k)');
        xlabel('\omega'); ylabel('|X(k)|'); axis tight;

subplot(2,1,2); plot(w,angle(XX)); title('DTFT Phase of X(k)');
        xlabel('\omega'); ylabel('\phi(k)'); axis tight;
```

```
function [P] = dft(p)
% The purpose of this function is to compute the one-dimensional Discrete
% Fourier transform of an input row/vector x.
%
% INPUTS: p - Input vector
% OUTPUT: p - Fourier transformed input vector x
%
% Written By: Alana M. Soehartono
%             November 5, 2013
%             EE410 - Digital Signal Processing
%             University of Wisconsin-Milwaukee
%
% % % % % % % % % % % % % % % % % % % % % % % % %

% Find size of vector
[M N] = size(p);

% Pre-allocate
P = zeros(M,N);

% DFT Loop
for k = 0:N-1
    for n = 0:N-1
        P(k+1) = P(k+1) + p(n+1)*exp(-j*2*pi*k*n/N);
    end
end
end


function [P] = idft(p)
% The purpose of this function is to compute the one-dimensional Discrete
% Fourier transform of an input row/vector x.
%
% INPUTS: p - Input vector
% OUTPUT: p - Fourier transformed input vector x
%
% Written By: Alana M. Soehartono
%             November 5, 2013
%             EE410 - Digital Signal Processing
%             University of Wisconsin-Milwaukee
%
% % % % % % % % % % % % % % % % % % % % % % % % %

% Find size of vector
[M N] = size(p);

% Pre-allocate
P = zeros(M,N);

% DFT Loop
for k = 0:N-1
    for n = 0:N-1
        P(k+1) = P(k+1) + p(n+1)*exp(j*2*pi*k*n/N);
    end
end
```

```
end



function fs = CentreShift(f)
% The purpose of this function is to a centre shift on a 2D image.
%
% INPUTS: f - Input image
% OUTPUT: fs - Centre shifted image
%
% Written By: Alana M. Soehartono
%             November 5, 2013
%             EE410 - Digital Signal Processing
%             University of Wisconsin-Milwaukee
%
% % % % % % % % % % % % % % % % % % % % % % % % %

[P Q] = size(f);

for y = 0:P-1
    for x = 0:Q-1
        fs(y+1,x+1) = f(y+1,x+1)*(-1)^(x + y);
    end
end
end


% function for Discrete Time Fourier Transform
%
function [X] = dtft(x,n,w)
%
% X = DTFT values computed at w frequencies
% x = finite duration sequence over n (row vector)
% n = sample position row vector
% w = frequency row vector
%
X = x*exp(-j*n'*w);
```

4.1) $x_1(n) = \{1, 2, 3, 1, 0, 0, 0, 0\}$

$x_2(n) = \{4, 3, 2, 2, 0, 0, 0, 0\}$

For long filters (> 60), use DFT approach.

DFT:

$$X_1(K) = \sum_{n=0}^{7} x_1(n) e^{-j2\pi \frac{nK}{8}}$$

$$= 1 + 2e^{-j\pi K/4} + 3e^{-j\pi K/2} + e^{-j\pi 3K/4}$$

$X_1(0) = 1 + 2 + 3 + 1 = 7$

$X_1(1) = 1 + 2\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} j\right) + 3(-j) + 1\left(-\frac{\sqrt{2}}{2}\right.$

8.8) Compute 8 point FFT using decimation in frequency algorithm

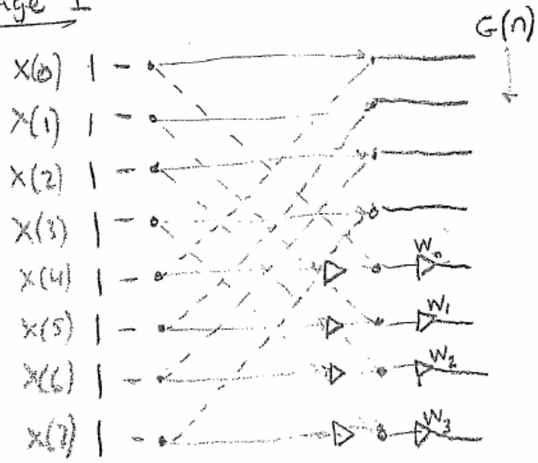
$W_8^n = e^{-j2\pi n/8}$

$\triangleright^n = *(W_8^n)$

$\triangleright = *(-1)$

$$x(n) = \begin{cases} 1, & 0 \le n \le 7 \\ 0, & \text{Otherwise} \end{cases}$$

Stage 1



$G(0) = x(0) + x(4) = 2$

$G(1) = x(1) + x(5) = 2$

$G(2) = x(2) + x(6) = 2$

$G(3) = x(3) + x(7) = 2$

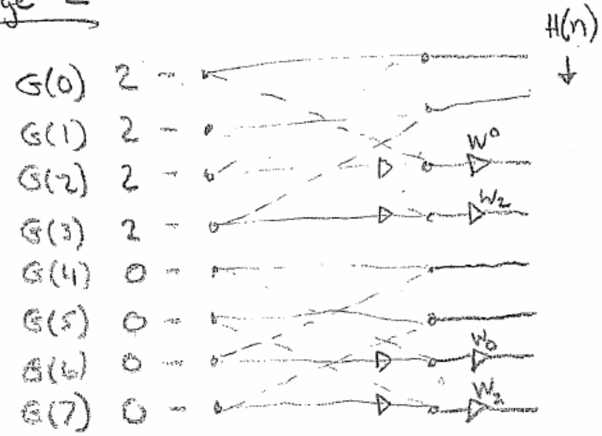
$G(4) = [x(0) - x(4)]W_8^0 = [0]W_8^0 = 0$

$G(5) = [x(1) - x(5)]W_8^1 = [0]W_8^1 = 0$

$G(6) = [x(2) - x(6)]W_8^2 = [0]W_8^2 = 0$

$G(7) = [x(3) - x(7)]W_8^3 = [0]W_8^3 = 0$

Stage 2



$H(0) = [G(0) + G(2)] = 4$

$H(1) = [G(1) + G(3)] = 4$

$H(2) = [G(0) - G(2)]W_8^0 = [0]W_8^0 = 0$

$H(3) = [G(1) - G(3)]W_8^2 = [0]W_8^2 = 0$

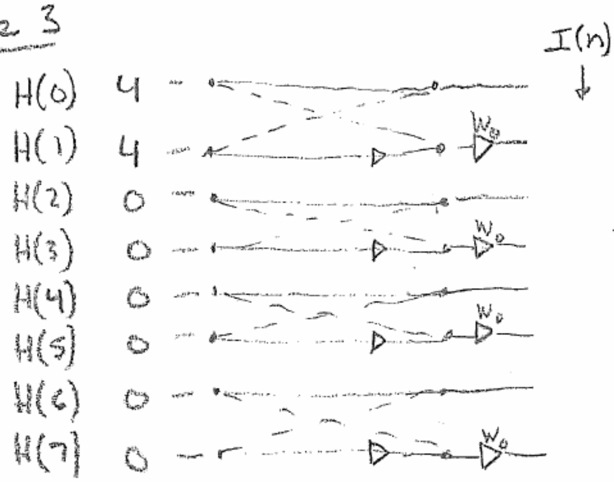
$H(4) = [G(4) + G(6)] = 0$

$H(5) = [G(5) + G(7)] = 0$

$H(6) = [G(4) - G(6)]W_8^0 = 0$

$H(7) = [G(5) + G(7)]W_8^2 = 0$

Stage 3



$I(0) = [H(0) + H(1)] = 8 = X(0)$

$I(1) = [H(0) - H(1)]W_8^0 = [0]W_8^0 = 0 = X(4)$

$I(2) = [H(2) + H(3)] = 0 = X(2)$

$I(3) = [H(2) - H(3)]W_8^0 = [0]W_8^0 = 0 = X(6)$

$I(4) = [H(4) + H(5)] = 0 = X(1)$

$I(5) = [H(4) - H(5)]W_8^0 = [0]W_8^0 = 0 = X(3)$

$I(6) = [H(6) + H(7)] = 0 = X(5)$

$I(7) = [H(6) - H(7)]W_8^0 = [0]W_8^0 = 0 = X(7)$

$$\boxed{X = [8, 0, 0, 0, 0, 0, 0, 0]}$$

EE410-001 **FALL 2013**

Digital Signal Processing

Homework 7

Problems from course books:

TB = Text Book = Digital Signal Processing, 4th Ed.
John Proakis & Dimitris.Manolakis, Pearson Prentice Hall 2007.

MB = Matlab Book = Digital Signal Processing using MATLAB, 3rd Ed.
Vinay Ingle & John Proakis, CENGAGE Learning, 2012.

1. Download the Lena512 grayscale image from the Rice website. Use row 256 as a 1 dimensional data set.
   1.1. Perform linear filtering with an 8 tap filter with normalized cutoff frequency of 0.25.
   1.2. Perform linear filtering using the Overlap-Save method and block size of 128.
   1.3. Compare your results to confirm they are the same.

2. Download the Lena512 grayscale image from the Rice website. Use row 256 as a 1 dimensional data set.
   2.1. Perform linear filtering with a 16 tap filter with normalized cutoff frequency of 0.1.
   2.2. Perform linear filtering using the Overlap-Add method and block size of 64.
   2.3. Compare your results to confirm they are the same.

3. TB 8.8

4. TB 8.11

5. Use Matlab's Run & Time feature to make 2 curves for computational time vs N for N = 4, 8, 16 … 8192.
   5.1. DFT compute time.
   5.2. FFT compute time.

Due: 11-7-2013

$(1-\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1})$

8.8) Compute 8 point FFT using decimation in frequency algorithm

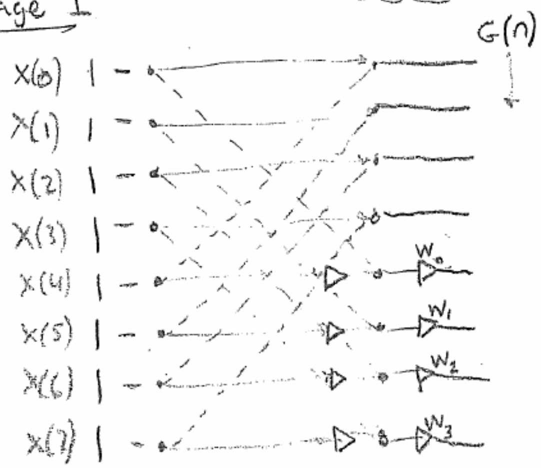
$W_8^n = e^{-j2\pi n/8}$

$\triangleright^n = *(W_8^n)$

$\triangleright = *(-1)$

$x(n) = \begin{cases} 1, & 0 \le n \le 7 \\ 0, & \text{Otherwise} \end{cases}$

Stage 1



$G(0) = x(0) + x(4) = 2$

$G(1) = x(1) + x(5) = 2$

$G(2) = x(2) + x(6) = 2$

$G(3) = x(3) + x(7) = 2$

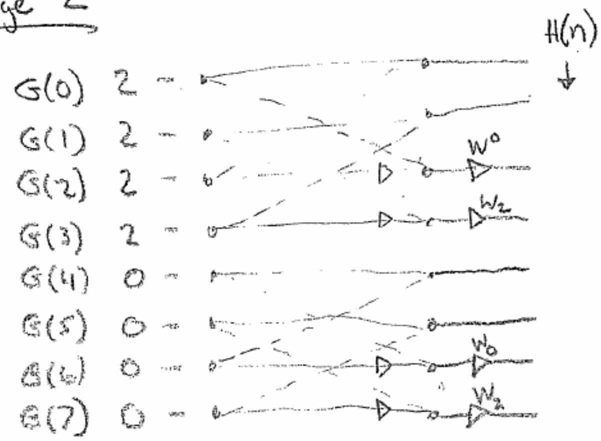
$G(4) = [x(0) - x(4)] W_8^0 = [0] W_8^0 = 0$

$G(5) = [x(1) - x(5)] W_8^1 = [0] W_8^1 = 0$

$G(6) = [x(2) - x(6)] W_8^2 = [0] W_8^2 = 0$

$G(7) = [x(3) - x(7)] W_8^3 = [0] W_8^3 = 0$

Stage 2



$H(0) = [G(0) + G(2)] = 4$

$H(1) = [G(1) + G(3)] = 4$

$H(2) = [G(0) - G(2)] W_8^0 = [0] W_8^0 = 0$

$H(3) = [G(1) - G(3)] W_8^2 = [0] W_8^2 = 0$

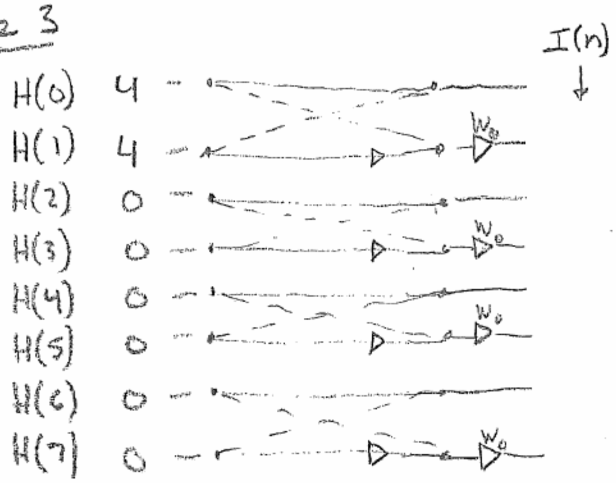
$H(4) = [G(4) + G(6)] = 0$

$H(5) = [G(5) + G(7)] = 0$

$H(6) = [G(4) - G(6)] W_8^0 = 0$

$H(7) = [G(5) + G(7)] W_8^2 = 0$

Stage 3



$I(0) = [H(0) + H(1)] = 8 = X(0)$

$I(1) = [H(0) - H(1)] W_8^0 = [0] W_8^0 = 0 = X(4)$

$I(2) = [H(2) + H(3)] = 0 = X(2)$

$I(3) = [H(2) - H(3)] W_8^0 = [0] W_8^0 = 0 = X(6)$

$I(4) = [H(4) + H(5)] = 0 = X(1)$

$I(5) = [H(4) - H(5)] W_8^0 = [0] W_8^0 = 0 = X(3)$

$I(6) = [H(6) + H(7)] = 0 = X(5)$

$I(7) = [H(6) - H(7)] W_8^0 = [0] W_8^0 = 0 = X(7)$

$\boxed{X = [8, 0, 0, 0, 0, 0, 0, 0]}$

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EE410 – HW#7

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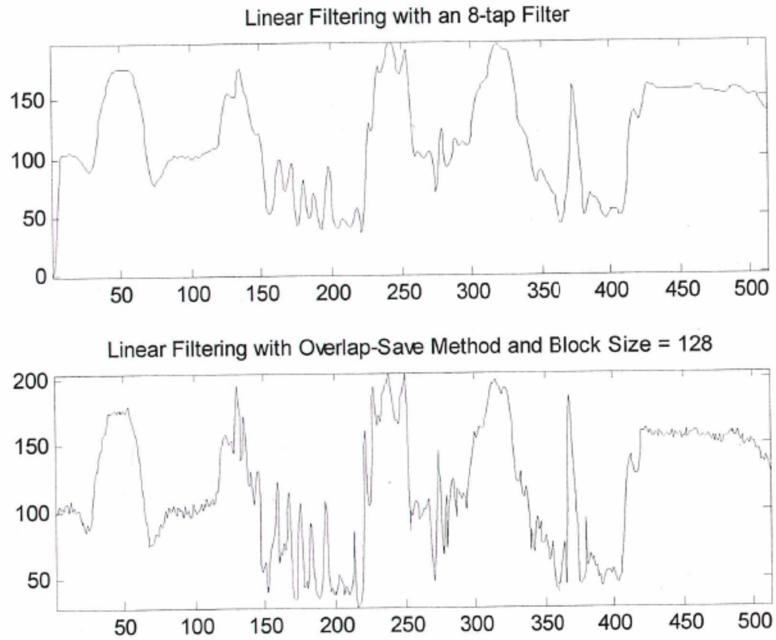
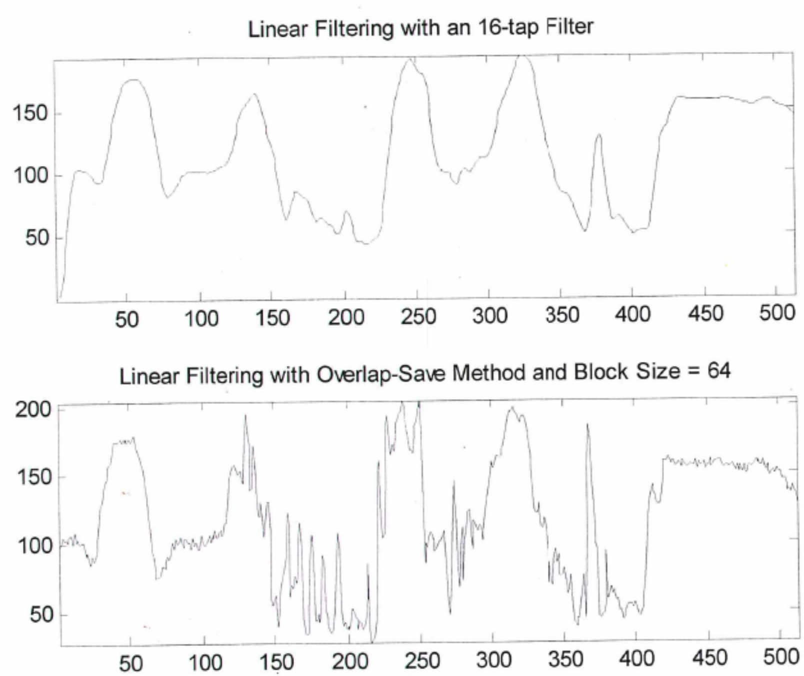
P1) Based on the plots using an 8-tap linear filter and the overlap-save method, the plots indicate similarity in the results. The second plot however, displays for fringes in the signal.



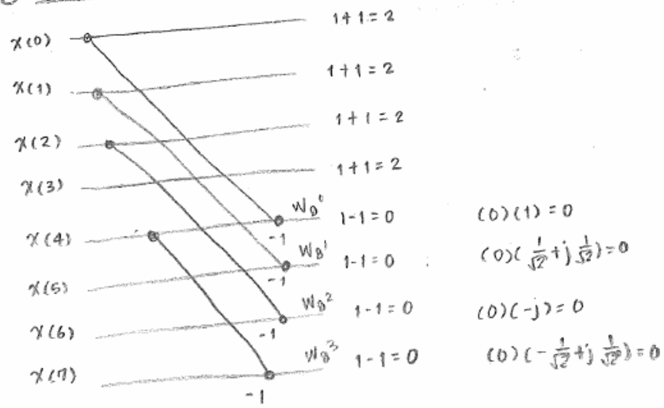
P2)



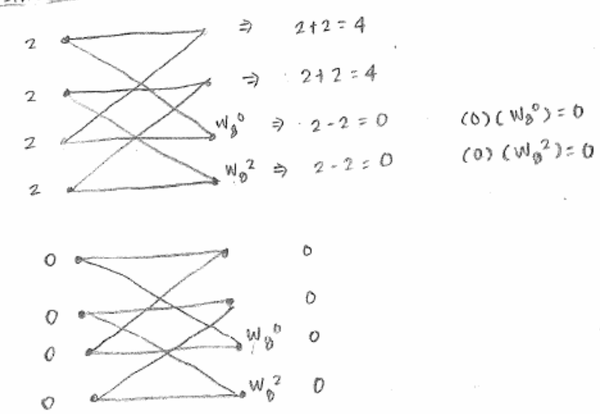
TB8.8) compute the 8-point DFT of the sequence using decimation in freq. algorithm

$$x(n) = \begin{cases} 1, & 0 \le n \le 7 \\ 0, & \text{otherwise} \end{cases}$$

① STAGE 1

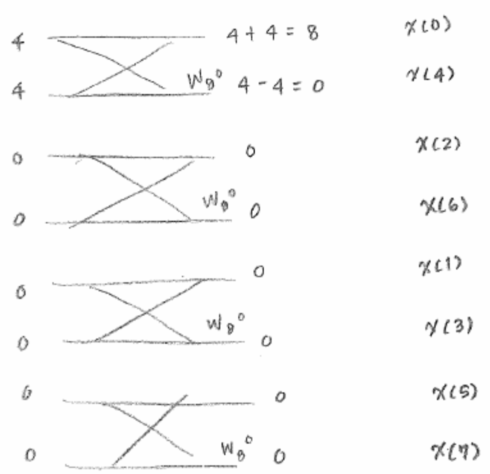


② STAGE 2



1

③ STAGE 3

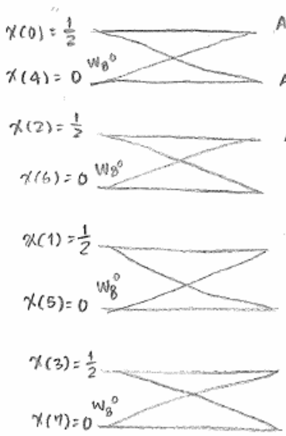


TB8.11) compute the 8-point DFT using in-place radix-2 decimation-in-time and radix-2 decimation-in-frequency.

$$x(n) = \left\{ \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0, 0, 0 \right\}$$

① STAGE 1

$$W_8^0 = e^{-j2\pi(0/8)} = 1$$



$A_0 = a_0 + W_8^0 b_0 = \frac{1}{2} + (1)(0) = \frac{1}{2}$

$A_4 = a_0 - W_8^0 b_4 = \frac{1}{2} - (1)(0) = \frac{1}{2}$

$A_2 = a_2 + W_8^0 b_2 = \frac{1}{2} + (1)(0) = \frac{1}{2}$

$A_6 = a_2 - W_8^0 b_6 = \frac{1}{2} - (1)(0) = \frac{1}{2}$

$A_1 = a_1 + W_8^0 b_1 = \frac{1}{2} + (1)(0) = \frac{1}{2}$

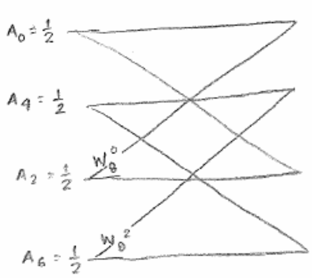
$A_5 = a_1 - W_8^0 b_5 = \frac{1}{2} - (1)(0) = \frac{1}{2}$

$A_3 = a_3 + W_8^0 b_3 = \frac{1}{2} + (1)(0) = \frac{1}{2}$

$A_7 = a_3 - W_8^0 b_7 = \frac{1}{2} - (1)(0) = \frac{1}{2}$

② STAGE 2

$$W_8^2 = e^{-j2\pi(2/8)} = e^{-j\pi/2} = -j$$

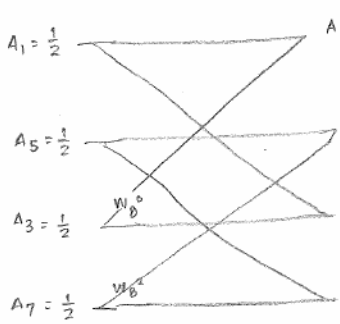


$A_{0,2} = A_0 + W_8^0 A_2 = \frac{1}{2} + (1)(\frac{1}{2}) = 1$

$A_{4,2} = A_4 + W_8^2 A_6 = \frac{1}{2} + (-j)(\frac{1}{2}) = \frac{1}{2} - j\frac{1}{2}$

$A_{2,2} = A_0 - W_8^0 A_2 = \frac{1}{2} - (1)(\frac{1}{2}) = 0$

$A_{6,2} = A_4 - W_8^2 A_6 = \frac{1}{2} - (-j)(\frac{1}{2}) = \frac{1}{2} + j\frac{1}{2}$



$A_{1,2} = A_1 + W_8^0 A_3 = \frac{1}{2} + (1)(\frac{1}{2}) = 1$

$A_{5,2} = A_5 + W_8^2 A_7 = \frac{1}{2} + (-j)(\frac{1}{2}) = \frac{1}{2} - j\frac{1}{2}$

$A_{3,2} = A_1 - W_8^0 A_3 = \frac{1}{2} - (1 \cdot (\frac{1}{2}) = 0$

$A_{5,2} = A_5 - W_8^2 A_7 = \frac{1}{2} - (-j)(\frac{1}{2}) = \frac{1}{2} + j\frac{1}{2}$

③ STAGE 3 Next page

③ STAGE 3

$A_{0,2} = 1$

$A_{4,2} = \frac{1}{2}(1-j)$

$A_{2,2} = 0$

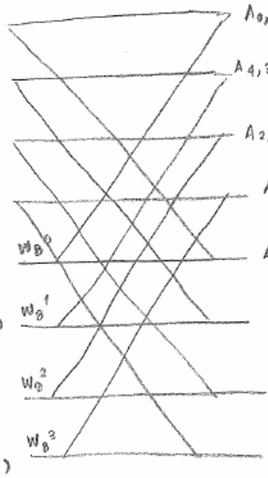
$A_{6,2} = \frac{1}{2}(1+j)$

$A_{1,2} = 1$

$A_{5,2} = \frac{1}{2}(1-j)$

$A_{3,2} = 0$

$A_{7,2} = \frac{1}{2}(1+j)$



$A_{0,3} = A_{0,2} + W_8^0 A_{1,2} = 1 + (1)(1) = 2$

$A_{4,3} = A_{4,2} + W_8^1 A_{5,2} = \frac{1}{2}(1-j) + \left[\frac{1}{\sqrt{2}}(1-j) \ \frac{1}{2}(1-j) \right]$

$A_{2,3} = A_{2,2} + W_8^2 A_{3,2} = 0 + (-j)(0) = 0$

$A_{6,3} = A_{6,2} + W_8^3 A_{7,2} = \frac{1}{2}(1+j) + \left[\frac{1}{\sqrt{2}}(-1+j) \ \frac{1}{2}(1+j) \right]$

$A_{1,3} = A_{0,2} - W_8^0 A_{1,2} = 1 - (1)(1) = 0$

$A_{5,3} = A_{4,2} - W_8^1 A_{5,2} = \frac{1}{2} - j\frac{1}{2} - \left[\frac{1}{2}(1-j) \ \frac{1}{\sqrt{2}}(1-j) \right]$

$A_{3,3} = A_{2,2} - W_8^2 A_{3,2} = 0 - (-j)(0) = 0$

$A_{7,3} = A_{6,2} - W_8^3 A_{7,2} = \frac{1}{2} + j\frac{1}{2} - \left[+\frac{1}{\sqrt{2}}(-1+j) \ \frac{1}{2}(1+j) \right]$

$W_8^0 = 1$

$W_8^1 = e^{-j\pi/4} = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

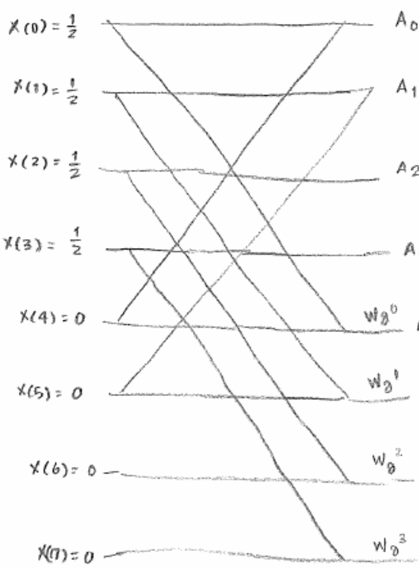
$W_8^2 = -j$

$W_8^3 = e^{-j3\pi/4} = -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$ corrected Twiddle

For Decimation in Freq.

see pg #523

① STAGE 1



$A_0 = A_0 + W_8^0 A_4 = \frac{1}{2}$

$A_1 = A_1 + W_8^1 A_5 = \frac{1}{2}$

$A_2 = \frac{1}{2}$

$A_3 = \frac{1}{2}$

$A_4 : x(0) - W_8^0\, x(4) = 0 - 0 = \frac{1}{2}$

$A_5 = x(1) - W_8^1\, x(5) = \frac{1}{2} - 0$

$A_6 = x(2) - W_8^2\, x(6) = \frac{1}{2}$

$A_7 = x(3) - W_8^3\, x(7) = \frac{1}{2}$

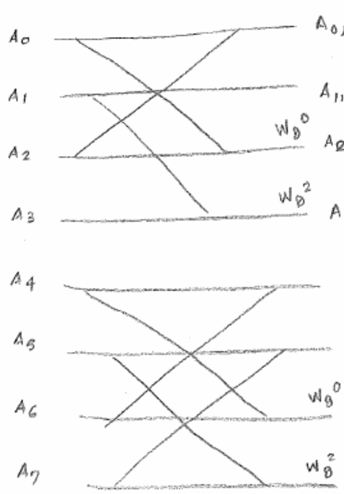
$W_8^0 = 1$

$W_8^1 = \frac{1}{\sqrt{2}}(1-j)$

$W_8^2 = -j$

$W_8^3 = \frac{1}{\sqrt{2}}(-1+j)$

② STAGE 2



$A_{0,2} = A_0 + W_8^0 A_2 = \frac{1}{2} + \frac{1}{2} = 1$

$A_{1,2} = A_1 + W_8^2 A_3 = \frac{1}{2} - j\frac{1}{2}$

$A_{2,2} = A_0 - W_8^0 A_2 = \frac{1}{2} - \frac{1}{2} = 0$

$A_{3,2} = A_1 - W_8^2 A_3 = \frac{1}{2} + j\frac{1}{2}$

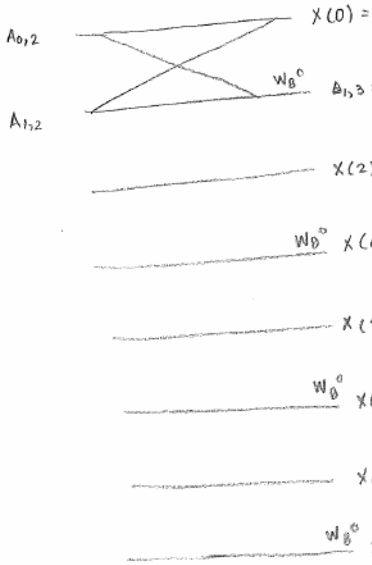
$A_{4,2} = A_6 + W_8^0 A_4 = 1$

$A_{5,2} = A_5 + W_8^2 A_7 = \frac{1}{2} - j\frac{1}{2}$

$A_{6,2} = A_4 - W_8^0 A_6 = \frac{1}{2} - \frac{1}{2} = 0$

$A_{7,2} = A_5 - W_8^2 A_7 = \frac{1}{2} + j\frac{1}{2}$

③ STAGE 3



$X(0) = 1 + \frac{1}{2} - j\frac{1}{2} = \frac{3}{2} - j\frac{1}{2}$

$A_{1,3} = X(4) = 1 - \frac{1}{2} + j\frac{1}{2} = \frac{1}{2} + j\frac{1}{2}$

$X(2) = 0 + \frac{1}{2} + j\frac{1}{2}$

$X(6) = 0 - \frac{1}{2} - j\frac{1}{2}$

$X(1) = 1 + \frac{1}{2} - j\frac{1}{2} = \frac{3}{2} - j\frac{1}{2}$

$X(5) = 1 - \frac{1}{2} + j\frac{1}{2} = \frac{1}{2} + j\frac{1}{2}$

$X(3) = 0 + \frac{1}{2} + j\frac{1}{2}$

$X(7) = -\frac{1}{2} - j\frac{1}{2}$

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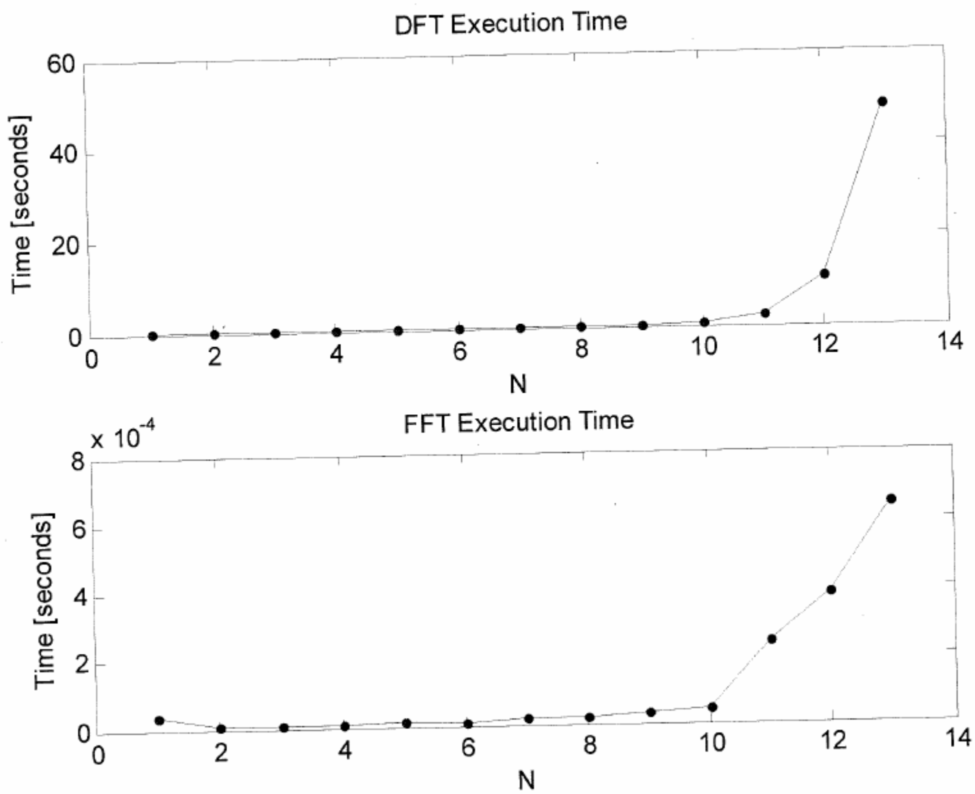
P5) The plot shows the time advantadge of using the Fast Fourier transform instead of Discrete Fourier transform. For data with a length of 2^{13}, FFT execution time is on the order of 10^{-4} seconds while using the DFT is almost one minute to compute.



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November 7, 2013
EE410 – HW#7

```
% EE410 - HW #7
%
%
% Written By: Alana M. Soehartono
%             November 7, 2013
%             EE410 - Digital Signal Processing
%             University of Wisconsin-Milwaukee
%
% % % % % % % % % % % % % % % % % % % % % % % % % % %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear;clc

%% HW7.1
load('lena512.mat');

% Row 256 of Lena image
X = lena512(256,:);

%%% Linear Filtering %%%
% Create filter with 8 taps and Wn = 0.25
tap = 8; Wn = 0.25;
b = fir1(tap+1,Wn);

% Perform filtering
Y = filter(b,1,X);

%%% Overlap and Save Method %%%
% Initialize parameters
n = 0:9;
h = 1;    % Impulse response
N = 128; % Block length

% Perform overlap and save
y = ovrlpsav(X,h,N);

figure, subplot(2,1,1); plot(Y); axis tight;
        title('Linear Filtering with an 8-tap Filter');
        subplot(2,1,2); plot(y); axis tight;
        title('Linear Filtering with Overlap-Save Method and Block Size =
128');


%% HW7.2

%%% Linear Filtering %%%
% Create filter with 16 taps and Wn = 0.1
tap = 16; Wn = 0.1;
b = fir1(tap+1,Wn);

% Perform filtering
Y = filter(b,1,X);
```

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EE410 – HW#7

```
%%% Overlap-Add Method %%%
% Initialize parameters
h = 1;  % Impulse response
N = 64; % Block length

% Perform overlap and add
y = ovrlpadd(X,h,N);

figure, subplot(2,1,1); plot(Y); axis tight;
        title('Linear Filtering with an 16-tap Filter');
        subplot(2,1,2); plot(y); axis tight;
        title('Linear Filtering with Overlap-Save Method and Block Size =
64');

%% HW7.5
clear;clc;

% Computing DFT
for n = 1:13
    x = rand(1,2^n);
    tic
    dft(x,length(x));
    dft_time(n) = toc;
end

% Computing FFT
for n = 1:13
    x = rand(1,2^n);
    tic
    fft(x);
    fft_time(n) = toc;
end

n = 1:13;
figure, subplot(2,1,1); plot(n,dft_time,'.-'); xlabel('N'); ylabel('Time
[seconds]');
        title('DFT Execution Time');

subplot(2,1,2), plot(n,fft_time,'.-'); xlabel('N'); ylabel('Time [seconds]');
        title('FFT Execution Time');
```

Alana M. Soehartono
November 7, 2013
EE410 – HW#7

```
function y = ovrlpsav(x,h,N)
% Overlap-save method of block convolution
%
% [y] = ovrlpsav(x,h,N)
% y = output sequence
% x = input sequence
% h = impulse response
% N = block length

Lenx = length(x);
M = length(h);

M1 = M - 1;
L = N - M1;

h = [h zeros(1,N-M)];

x = [zeros(1,M1),x,zeros(1,N-1)];       % Pre-append (M-1) zeros
K = floor((Lenx+M1-1)/(L));             % # of blocks
Y = zeros(K+1,N);

% Convolution with successive blocks
for k = 0:K
    xk = x(k*L+1:k*L+N);
    Y(k+1,:) = circonvt(xk,h,N);
end

Y = Y(:,M:N)';                          % Discard the first (M-1) samples
y = (Y(:))';                            % Assemble output
```

```
function y = ovrlpadd(x,h,N)
% Overlap-save method of block convolution
%
% [y] = ovrlpsav(x,h,N)
% y = output sequence
% x = input sequence
% h = impulse response
% N = block length


Lenx = length(x);
M = length(h);

M1 = M - 1;
L = N - M1;

h = [h zeros(1,N-M)];

y = fftfilt(h,x,N);
```

EE410-001 **FALL 2013**

Digital Signal Processing

Homework 8

Problems from course books:

TB = Text Book = Digital Signal Processing, 4^{th} Ed.
John Proakis & Dimitris Manolakis, Pearson Prentice Hall 2007.

MB = Matlab Book = Digital Signal Processing using MATLAB, 3^{rd} Ed.
Vinay Ingle & John Proakis, CENGAGE Learning, 2012.

1. TB 9.9 d

2. Design a 4^{th} order FIR filter with cutoff frequency of 0.1 with sptool.
   - 2.1. Using matlab filter command obtain the impulse response.
   - 2.2. Using Simulink design a direct form realization and obtain the impulse response.
   - 2.3. Using Simulink design a transpose realization and obtain the impulse response.

3. Design an IIR filter with cutoff frequency of 0.1 with sptool.
   - 3.1. Using matlab filter command obtain the impulse response.
   - 3.2. Using Simulink design a direct form realization and obtain the impulse response.
   - 3.3. Using Simulink design a transpose realization and obtain the impulse response.

4. Design an IIR filter with 2 cascaded biquads with sptool.
   - 4.1. Using matlab filter command obtain the impulse response.
   - 4.2. Using Simulink design a cascaded realization and obtain the impulse response.

5. Design a FIR filter with cutoff frequency of 0.1 and 80 dB rejection in the stop band with sptool.
   - 5.1. Using matlab quantize the coefficients to 8 bits.
   - 5.2. Using the fvtool compare the magnitude response for both filters noting the change in stop band rejection.

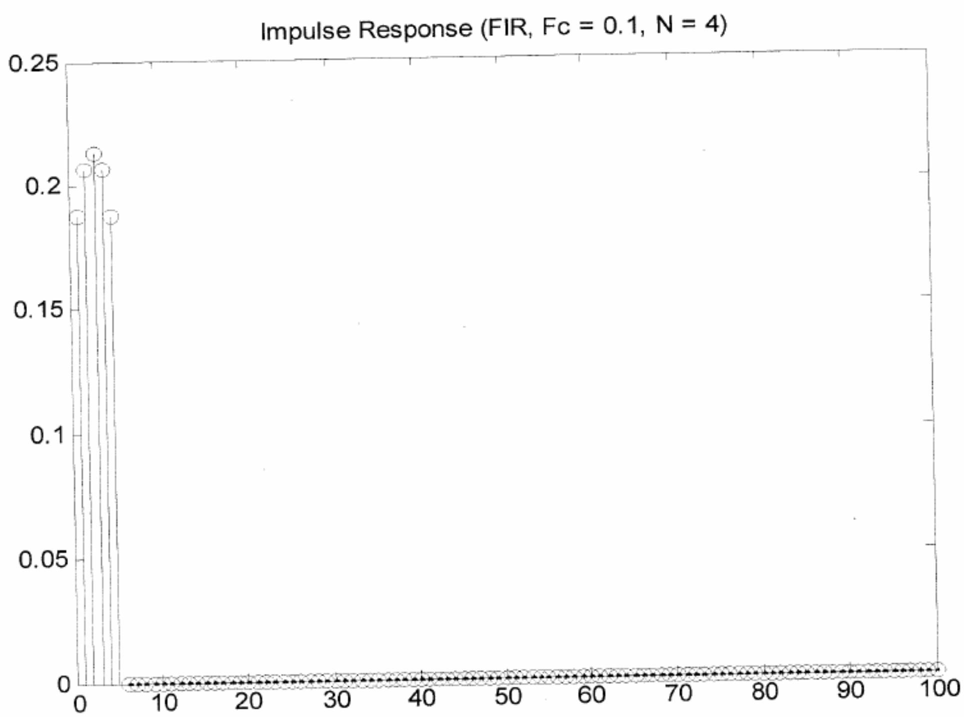
Due: 11-19-2013

Ryan Baumgartner
Homework #8

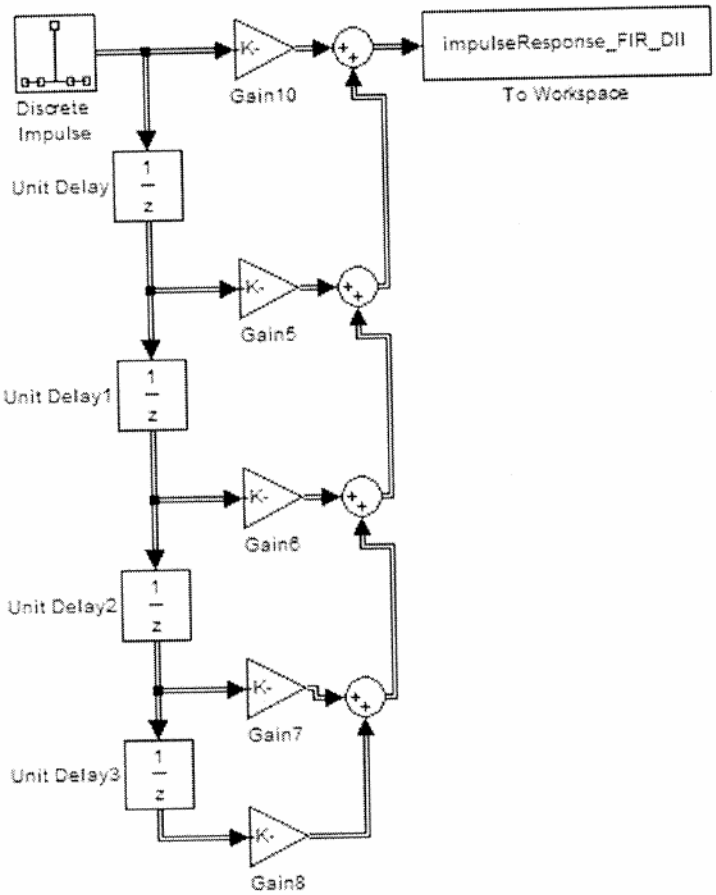
Problem #2

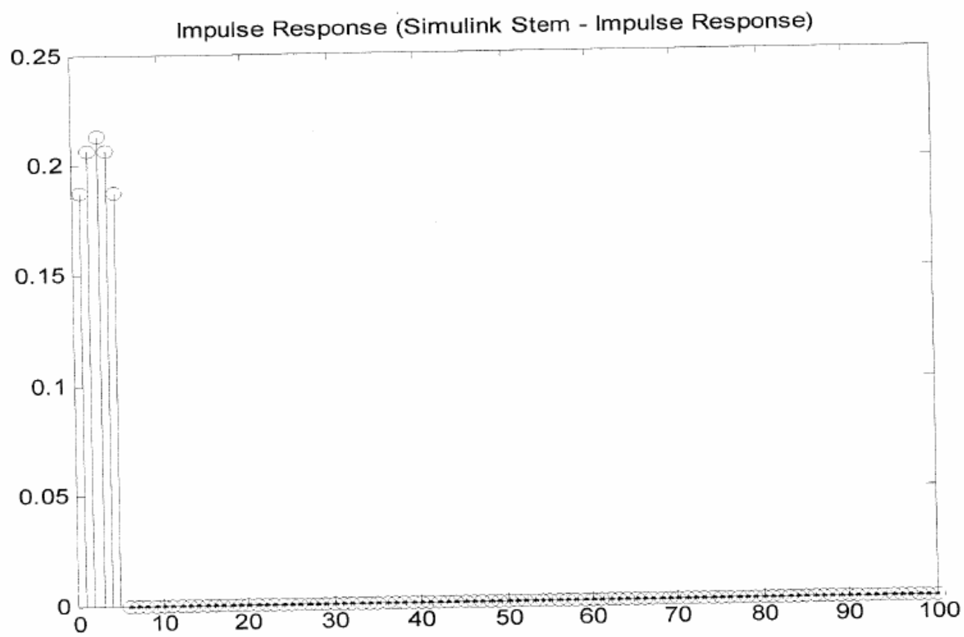
Matlab Code

```
%% Problem 2
clear
clc
N    = 4;          % Order
Fc   = 0.1;        % Cutoff Frequency
flag = 'scale';    % Sampling Flag
Beta = 0.5;        % Window Parameter

% Create the window vector for the design algorithm.
win = kaiser(N+1, Beta);

% Calculate the coefficients using the FIR1 function.
bFIR  = fir1(N, Fc, 'low', win, flag);

% Filter an impulse signal using the coeff to obtain the impulse response
a = 1;
X = zeros(100,1);
X(1,1)  = 1;
impulseResponse = filter(bFIR,a,X);
figure
stem(impulseResponse);
title('Impulse Response (FIR, Fc = 0.1, N = 4)')
```

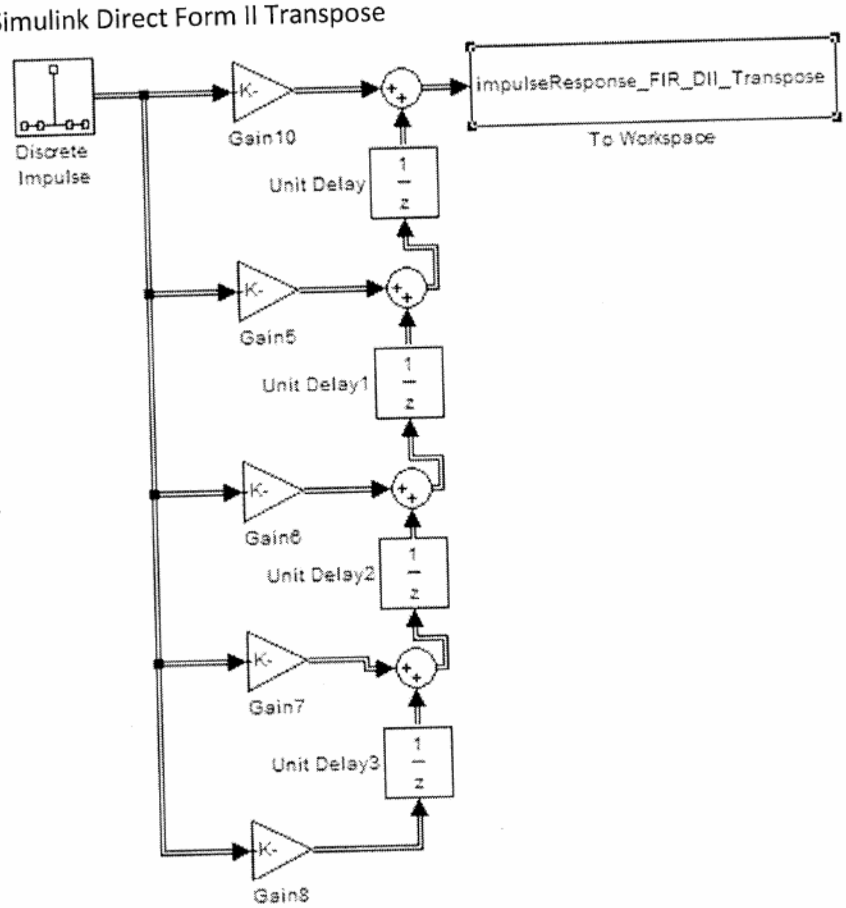


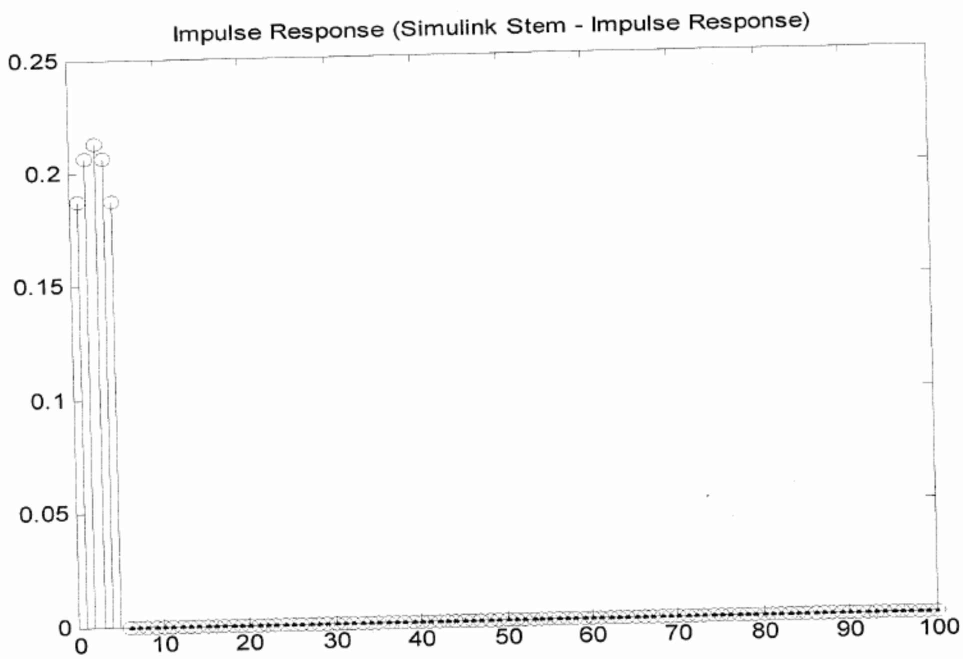
Simulink Direct Form II





Simulink Direct Form II Transpose

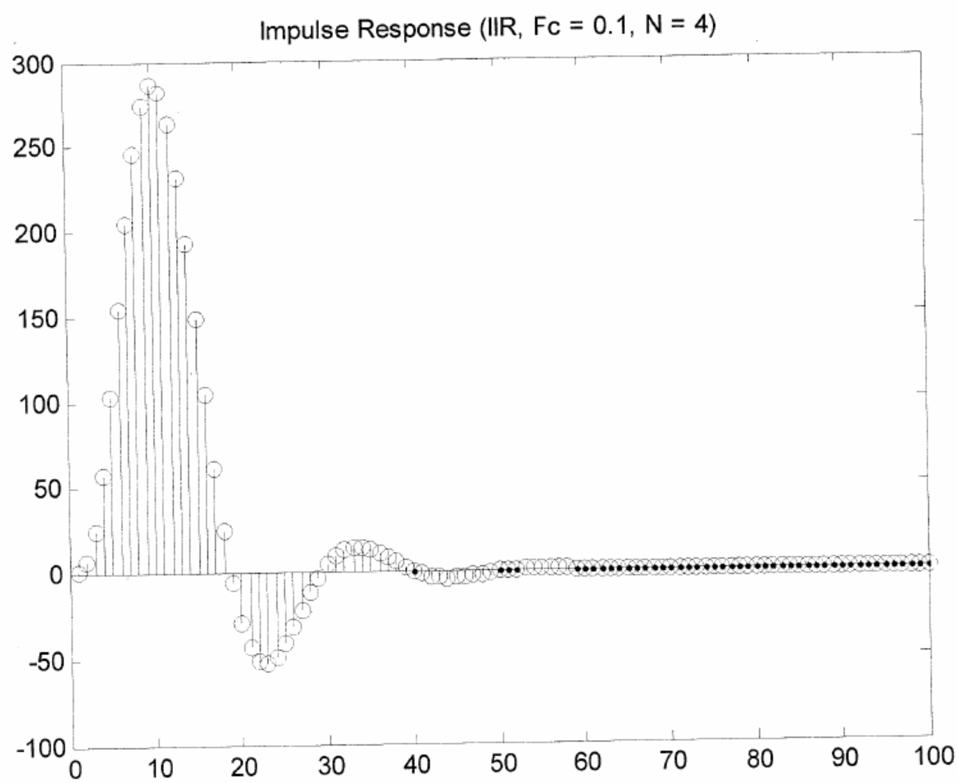




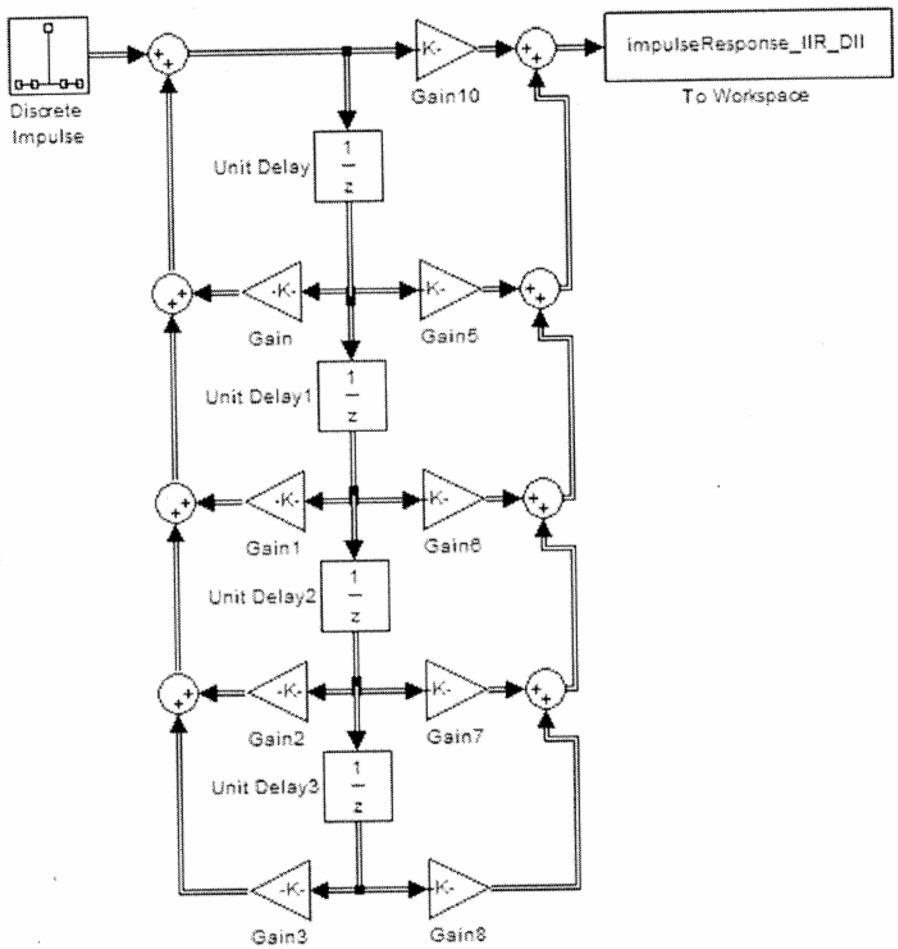
Problem #3

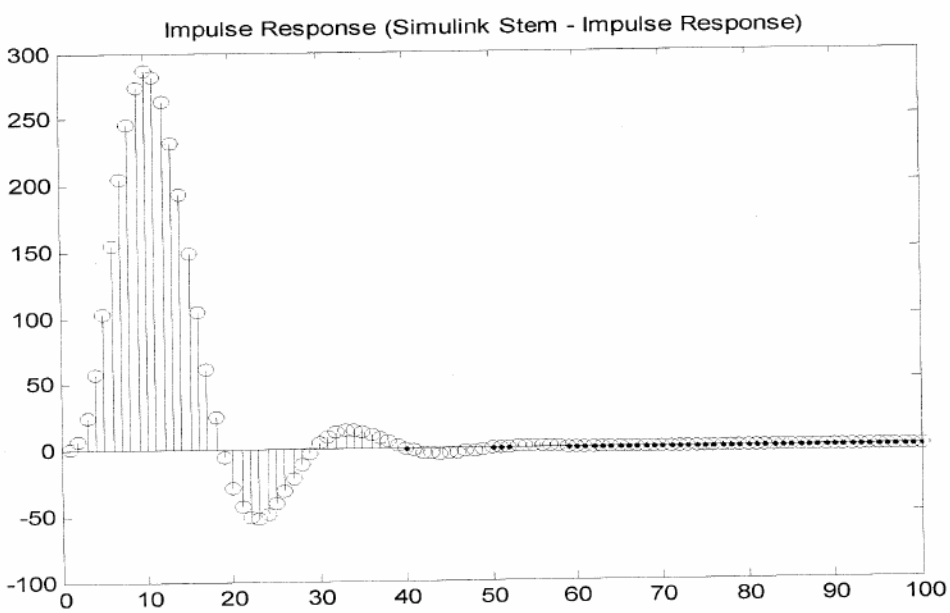
Matlab Code

```
%% Problem 3
% clear
% clc
N  = 4;      % Order
Fc = 0.1;    % Cutoff Frequency

% Construct an FDESIGN object and call its BUTTER method.
h  = fdesign.lowpass('N,F3dB', N, Fc);
Hd = design(h, 'butter');
sosIIR = Hd.sosMatrix; % Create BiQuad coefficients
[bIIR,aIIR]=sos2tf(sosIIR)
% Filter an impulse signal using the coeff to obtain the impulse response
X = zeros(100,1);
X(1,1) = 1;

impulseResponse = sosfilt(sosIIR,X);
figure
stem(impulseResponse);
title('Impulse Response (IIR, Fc = 0.1, N = 4)')
sumIIR = sum(impulseResponse);
```

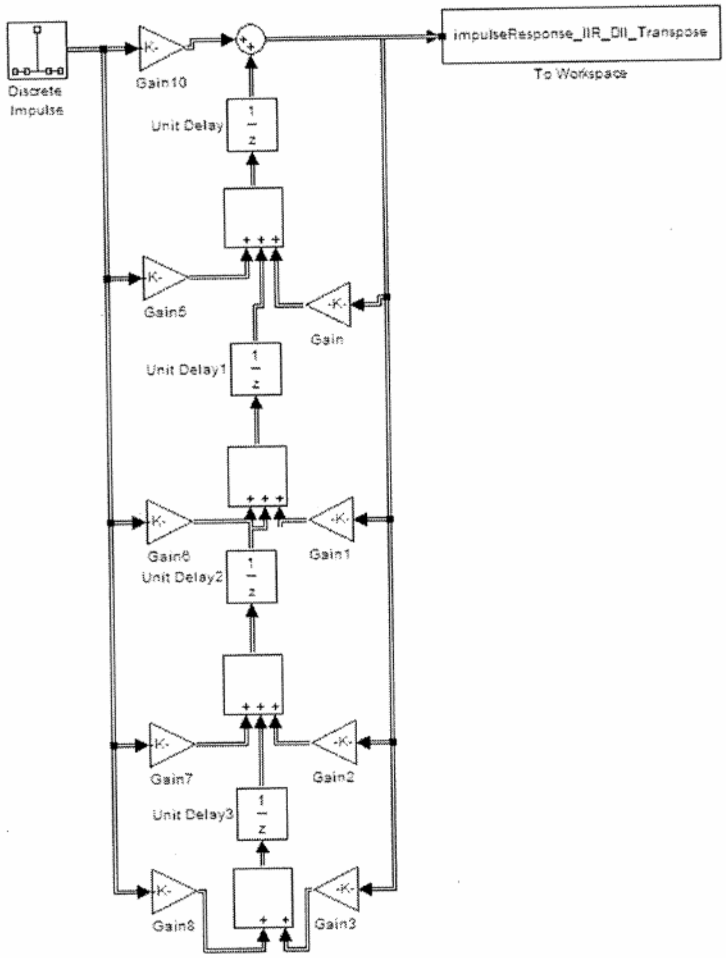


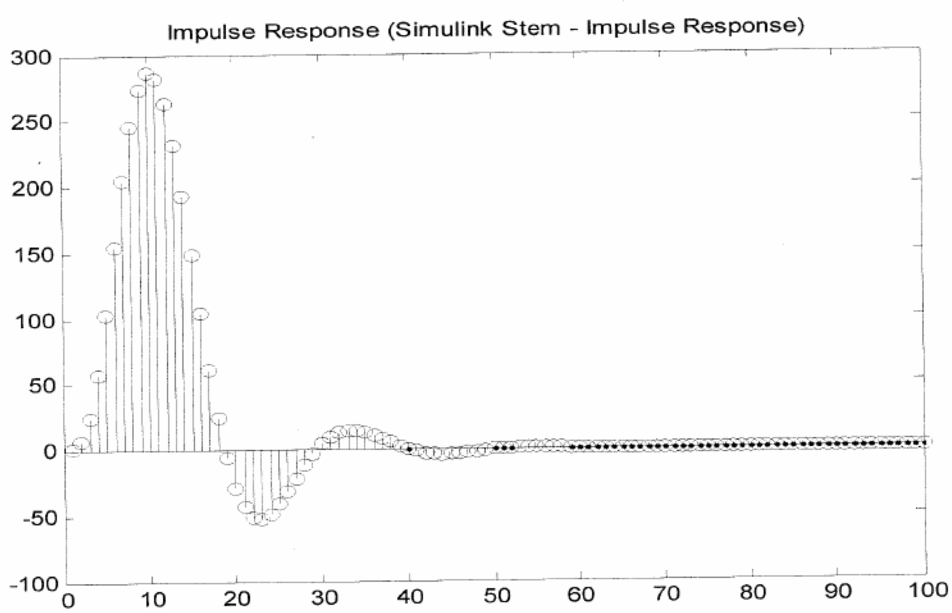
Simulink Direct Form II





Simulink Direct Form II Transpose

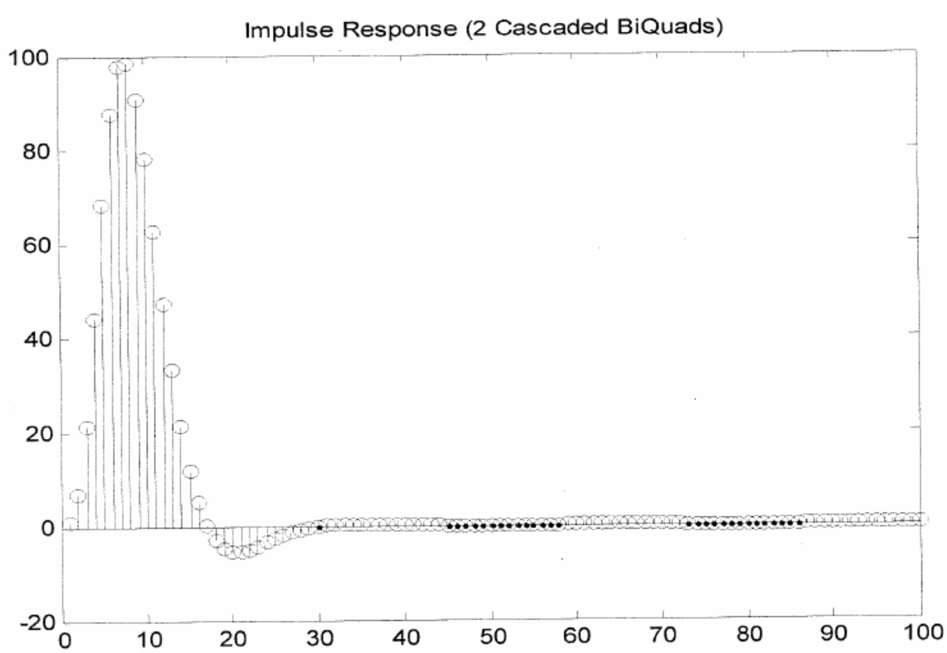




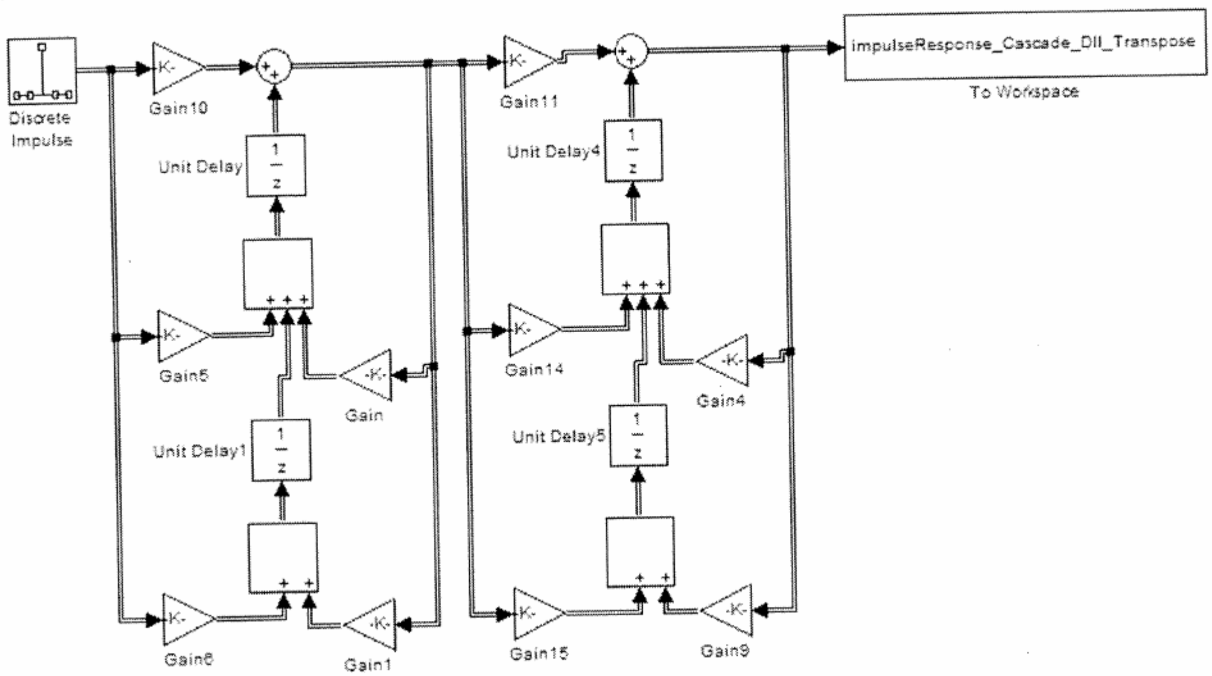
Problem #4

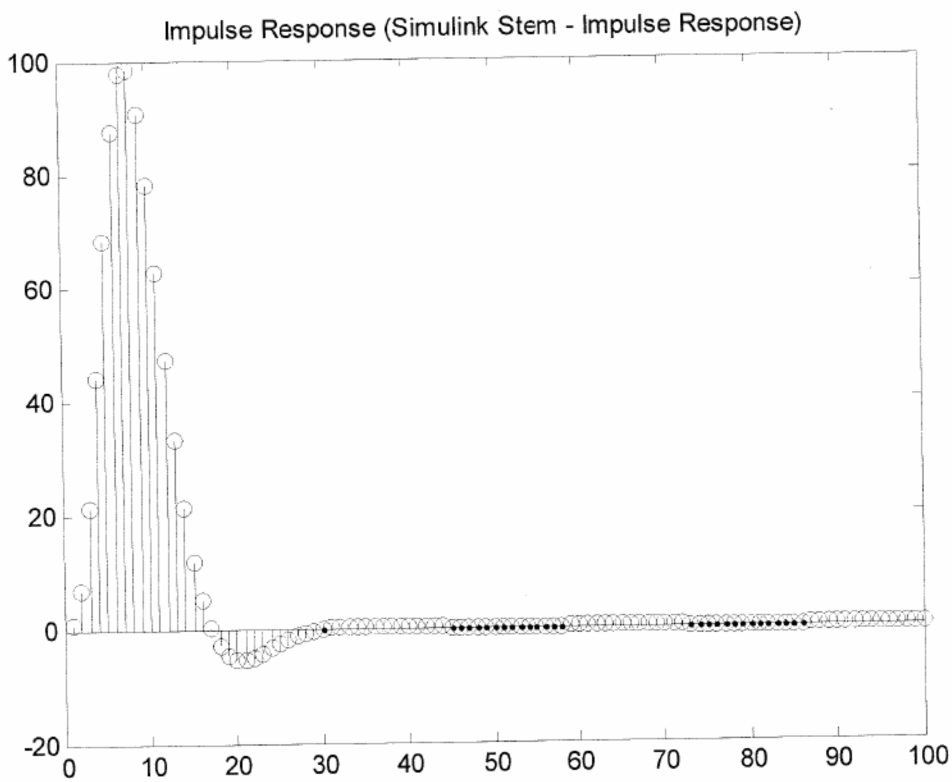
Matlab Code

```
%% Problem 4
% Section 1
N1 = 2;     % Order
Fc1 = 0.1;  % Cutoff Frequency
% Construct an FDESIGN object and call its BUTTER method.
h1 = fdesign.lowpass('N,F3dB', N1, Fc1);
Hd1 = design(h1, 'butter');
sos1 = Hd1.sosMatrix; % Create BiQuad coefficients
% Section 2
N2 = 2;     % Order
Fc2 = 0.2;  % Cutoff Frequency
% Construct an FDESIGN object and call its BUTTER method.
h2  = fdesign.lowpass('N,F3dB', N2, Fc2);
Hd2 = design(h2, 'butter');
sos2 = Hd2.sosMatrix; % Create BiQuad coefficients
% Filter an impulse signal using the coeff to obtain the impulse response
X = zeros(100,1);
X(1,1) = 1;
halfFilt = sosfilt(sos1,X);
impulseResponse = sosfilt(sos2,halfFilt);
figure
stem(impulseResponse);
title('Impulse Response (2 Cascaded BiQuads)')
sumIIR = sum(impulseResponse)
% Direct II Transpose IIR Model
simOut = sim('Cascade_DII_Transpose');
figure
stem(impulseResponse_Cascade_DII_Transpose);
title('Impulse Response (Simulink Stem - Impulse Response)')
```

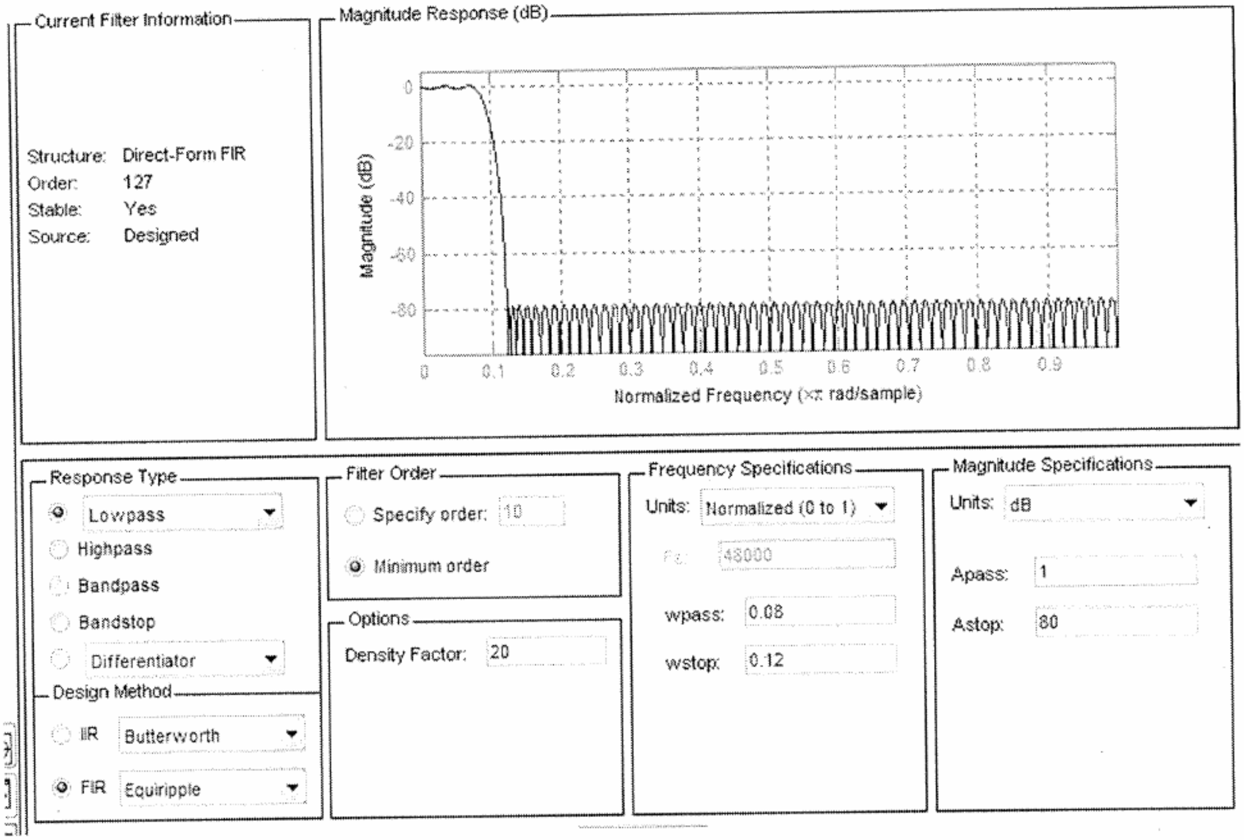


Simulink Direct Form II Transpose Cascade System





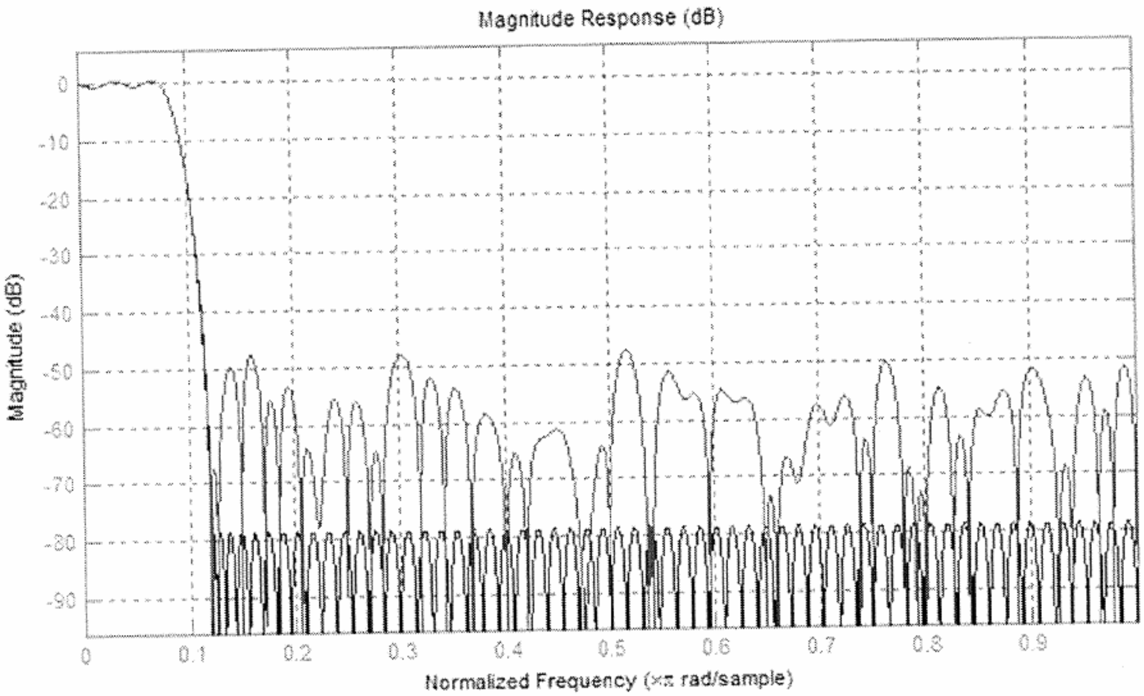
Problem #5



Matlab Code

```
%% Problem 5

Fpass = 0.08;               % Passband Frequency
Fstop = 0.12;               % Stopband Frequency
Dpass = 0.057501127785;     % Passband Ripple
Dstop = 0.0001;             % Stopband Attenuation
dens  = 20;                 % Density Factor
% Calculate the order from the parameters using FIRPMORD.
[N, Fo, Ao, W] = firpmord([Fpass, Fstop], [1 0], [Dpass, Dstop]);
% Calculate the coefficients using the FIRPM function.
b  = firpm(N, Fo, Ao, W, {dens});
Hd = dfilt.dffir(b);
maxVal = max(b);
minVal = min(b);
Range = maxVal - minVal;
bits = 8;
% quantizedCoeff = quant(b,8,Range);
res = Range/(2^bits - 1);
val = b./res;
for l = 1:length(b)
    if sign(round(val(l))) ~= 0
        quant(l) = (sign(round(val(l))) .* abs(round(val(l))))*res;
        errorQ(l) = val(l) - round(val(l));
    else
```

```
        quant(l) = (round(val(l)))*res;
        errorQ(l) = val(l) - round(val(l));
    end
end
expectedDB = 4.5*bits;
fvtool(b,1,quant,1);
```



EE410-001 **FALL 2013**

Digital Signal Processing

Homework 9

Problems from course books:

TB = Text Book = Digital Signal Processing, 4th Ed.
John Proakis & Dimitris Manolakis, Pearson Prentice Hall 2007.

MB = Matlab Book = Digital Signal Processing using MATLAB, 3rd Ed.
Vinay Ingle & John Proakis, CENGAGE Learning, 2012.

1. Group 1 Review the Window Method of designing FIR filters and present a short lecture on the topic.
   1.1. Impulse calculation.
   1.2. Window calculations.
   1.3. Example in Matlab.
   1.4. FVTOOL to show response.

2. Group 2 Review the Frequency Sampling Method of designing FIR filters and present a short lecture on the topic.
   2.1. What is frequency sampling?
   2.2. From the IDFT to a form that can be used for calculation of coefs.
   2.3. Example in Matlab.
   2.4. FVTOOL to show response.

3. Group 3 (Grad) Review the Remez Exchange Algorithm method of designing FIR filters and present a short lecture on the topic.
   3.1. Optimal FIR filter design.
   3.2. Algorithm overview to setup the min(max(E)) problem.
   3.3. Matlab functions explained.
   3.4. Example in Matlab.

Due: 11-19-2013 Lecture in class

Name ALANA M. SOEHARTONO

EE410-001

FALL 2013

Digital Signal Processing

Exam 1

Problem 1 10

Problem 2 17

Problem 3 ~~20~~ 19

Problem 4 17

Problem 5 20

Total 83

1. Given the following difference equation

$$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$$

Find

a) The Pulse Transfer function H(z).

b) The impulse response.

c) The step response.

(A) $Y(z) = 0.6[z^{-1}Y(z)] - 0.08z^{-2}Y(z) + X(z)$

using Time-shifting property

$\therefore\ x(n-k) \longleftrightarrow z^{-k}X(z)$

$H(z) = \frac{Y(z)}{X(z)}$

$Y(z)[1 - 0.6z^{-1} + 0.08z^{-2}] = X(z)$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{\underbrace{0.08z^{-2} - 0.6z^{-1} + 1}_{(z^{-1}-0.4)(z^{-1}-0.2)}} = \frac{A}{(z^{-1}-0.4)} + \frac{B}{(z^{-1}-0.2)}$

$= z^{-2} - 0.2z^{-1} - 0.4z^{-1} + 0.08z^{-2}$ ✓

-2

$A = \left.\frac{1}{(z^{-1}-0.2)}\right|_{z^{-1}=0.4} = \frac{1}{0.2} = 5$

$H(z) = \frac{1}{(1-\frac{1}{5}z^{-1})(1-\frac{2}{5}z^{-1})}$

$B = \left.\frac{1}{(z^{-1}-0.4)}\right|_{z^{-1}=0.2} = \frac{1}{-0.2} = -5$

$H(z) = \frac{5}{z^{-1}-0.4} - \frac{5}{z^{-1}-0.2} = \frac{5}{-0.4}\,\frac{1}{1-2.5z^{-1}} + \frac{5}{0.2}\,\frac{1}{1-5z^{-1}}$ //

CHECK: $\frac{5}{0.2-z^{-1}} = -\frac{5}{z^{-1}-0.2}$ ✓

(B) $h(n) = Z^{-1}\{H(z)\}$

using Table 3.3 $\quad a^n u(n) \longleftrightarrow \frac{1}{1-az^{-1}} \quad |z| > |a|$

$= \frac{-5}{0.4}(2.5)^n u(n) + \frac{5}{0.2}(5)^n u(n)$ // $\quad$ ROC: $|z| > |a|$

$h(n) = [2(\frac{2}{5})^n - (\frac{1}{5})^n]u(n)$

(C) The step response is $X(z) = 1$ //

$X(z) = \frac{1}{1-z^{-1}}$

-8

$Y(z) = \frac{1}{(1-z^{-1})(1-\frac{1}{5}z^{-1})(1-\frac{2}{5}z^{-1})}$

use P.F.E.

$y(n) = [\frac{25}{12} + \frac{1}{4}(\frac{1}{5})^n - \frac{4}{3}(\frac{2}{5})^n]u(n)$

2. a) Show that the following sequence

$$\sum_{n=0}^{\infty} n\alpha^n$$

can be represented in closed form as, for $|\alpha < 1|$

$$\frac{\alpha}{(1-\alpha)^2}$$

b) Determine the Discrete-Time Fourier Transform in closed form for the following sequence.

$$x(n) = n(0.9)^n u(n)$$

(A) $S_1 = \sum_{k=0}^{\infty} kx^k = 0 + x + 2x^2 + 3x^3 + \dots$

$-x S_1 = x \sum kx^k = 0 + x^2 + 2x^3 + 3x^4 + \dots$

$= \underbrace{x(1 + x^2 + x^3 + \dots + \dots)}_{\frac{1}{1-x}}$

$S_1(1-x) = \frac{x(1)}{1-x}$

$S_1 = \frac{x}{(1-x)^2}$

(A) $\sum_{n=0}^{\infty} n\alpha^n$ using sum property, where $r = \alpha$ and $k = n$

for $\sum_{k=0}^{n} kr^k = \frac{r + [n(r-1)-1]r^{n+1}}{(r-1)^2}, r \neq 1 = \frac{\alpha + [n(\alpha-1)-1]\alpha^{n+1}}{(\alpha-1)^2}$

$= \frac{\alpha + [\infty(\cancel{\alpha-1})^0 - 1]\cancel{\alpha^{\infty+1}}}{(\alpha-1)^2}$ · where $[\infty\alpha - \infty] = 0$ $= \frac{\alpha + [\cancel{\infty\alpha}^0 - \cancel{\infty}^0 - 1]\alpha^{\infty+1}}{(\alpha-1)^2}$

$= \frac{\alpha}{(1-\alpha)^2}$

Here I meant to include the fact that $|\alpha < 1|$, which in that case when, say $\alpha = -1$ we get $\alpha^{\infty+1} = -1$, $(-1)^{\infty+1} = -1$

(B) $x(n) = n(0.9)^n u(n)$

$n a^n u(n) \longleftrightarrow \frac{az^{-1}}{(1-az^{-1})^2}$

$X(z) = \frac{0.9z^{-1}}{(1-0.9z^{-1})^2}$

$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x(n) e^{-j\omega n}$

$= \sum_{-\infty}^{\infty} n(0.9)^n e^{-j\omega n}$

-3

$\sum_{k=m}^{n} kr^k = \frac{r + [n(r-1)-1]r^{n+1}}{(r-1)^2}$

similarly using the property above, For terms, I meant to multiply with z^{-1} ← DTFT

$= \frac{0.9 + [n(0.9-1)-1]0.9^{n+1}}{(0.9-1)^2} = \frac{0.9 + [n(0.9) - n - 1]0.9^{n+1}}{(0.9-1)^2}$

$= \frac{0.9 + n0.9^{n+2} - n0.9^{n+1} - 0.9^{n+1}}{(0.9-1)^2} = \frac{0.9 + \infty\cancel{0.9^{\infty+2}}^0 - \infty\cancel{0.9^{\infty+1}}^0 - \cancel{0.9^{\infty+1}}^0}{(0.9-1)^2}$

Because $0.9^{\infty} = 0$

(B) $X(\omega) = \sum_{n=0}^{\infty} n(0.9)^n e^{-jn\omega} = \frac{0.9e^{-j\omega}}{(1-0.9e^{-j\omega})^2}$

3. An analog signal

$$x_a(t) = \sin(480\pi t) + 3\sin(720\pi t)$$

is sampled at 600 times per second. Determine

a) the Nyquist sampling rate for $x_a(t)$.

b) the folding frequency.

c) the resulting discrete time signal, x(n).

d) Is there aliasing ?

$F_s = 600\frac{1}{s} = 600$ Hz

(A) $F_N = 2F_{max}$ $F_1 = \frac{480}{2} = 240$ Hz

$F_{max} = F_2$ $F_2 = \frac{720}{2} = 360$ Hz

$F_N = 2(360)$ Hz $= 720$ Hz //

(B) $F_f = \frac{F_s}{2} = \frac{600}{2} = 300$ Hz //

(C) $x(n) = x_a(nT) = x_a(\frac{n}{F_s})$

60 Above Folding go 60 below and − sign!

$= \sin 2\pi(\frac{240}{600})n + 3\sin 2\pi(\frac{360}{600})n$

$-\pi < \omega < \pi$

$= \sin 2\pi(\frac{8}{10})n + 3\sin 2\pi(\frac{12}{10})n$

we want $F_n < \frac{F_s}{2}$, so for F_2, $(1-\frac{3}{5}) = \frac{2}{5}$

$x(n) = \sin 2\pi(\frac{2}{5})n - 3\sin 2\pi(\frac{2}{5})n$ //

−1

(D) Aliasing will occur for the second term where $f_2 = \frac{3}{5}$ since this is greater than Fs. we can avoid aliasing by setting $F_n < \frac{F_s}{2}$, which is reflected in the final result of (C). //

4. Using a 4-bit ADC quantize an analog input ranging from 0 to 5 volts.
   a. How many quantization levels.
   b. What is the resolution of the ADC.
   c. At an input of 3.2 volts what is the quantized value.
   d. At an input of 3.2 volts what binary code is produced.
   e. At an input of 3.2 volts what is the quantization error.

(A) $L = 2^b = 2^4 = 64$ -3

(B) $\Delta = \frac{\text{Range}}{L-1} = \frac{V_{max} - V_{min}}{L-1} = \frac{5-0}{63} = 0.07936$ V = 79.36 mV/step

(C) $l\Delta = 3.2V$

$l = \frac{3.2V}{\Delta} = 40.32 \Rightarrow \text{floor}(l) = 40$

(D) 128 64 32 8 4 2 1 $40 = 32 + 8$

0011 0000

(E) $e_q(n) = x_q(n) - x(n)$

At 3.2 V, $l = 3.2$,

$l(\Delta) = (40)(0.07936) = 3.1744$

$e_q(n) = 3.2 - 3.1744 = 0.0256$

max error $= \frac{\Delta}{2} = 0.0396$ so, $e_q(3.2) = 0.0256$ OK.

5. Using the Reconstruction formula 6.1.20

   a. find $x_a(t = 5.3)$

      for T=1, x(n)=n, for the range of n=3 to n=8.

   b. What value did you expect in part a ?

   c. How would you make your answer in part (a) more accurate?

$$x_a(t) = \sum_{n=3}^{8} x(n) \frac{\sin(\pi/T)(t-nT)}{(\pi/T)(t-nT)} \qquad x(n) = x_a(nT)$$

(A) $x_a(5.3) = ?$ $\quad T=1 \quad x(n)=n \quad n=[3,8]$

nT but T=1

$$x_a(t) = \sum_{n=3}^{8} x(n) \frac{\sin(\pi)(t-n)}{(\pi)(t-n)} = \sum_{n=3}^{8} nT \frac{\sin(\pi)(t-n)}{\pi(t-n)}$$

$x(n) = n$

$$= (3)\frac{\sin(\pi)(5.3-3)}{\pi(5.3-3)} + (4)\frac{\sin\pi(5.3-4)}{\pi(5.3-4)} + (5)\frac{\sin(\pi)(5.3-5)}{\pi(5.3-5)}$$

$$+ (6)\frac{\sin\pi(5.3-6)}{\pi(5.3-6)} + (7)\frac{\sin\pi(5.3-7)}{\pi(5.3-7)} + (8)\frac{\sin\pi(5.3-8)}{\pi(5.3-8)}$$

in other words,

$$3\,\text{sinc}[\pi(5.3-3)] + 4\,\text{sinc}[\pi(5.3-4)] + 5\,\text{sinc}[\pi(5.3-5)]$$

$$+ 6\,\text{sinc}[\pi(5.3-6)] + 7\,\text{sinc}[\pi(5.3-7)] + 8\,\text{sinc}[\pi(5.3-8)]$$

$$= 3(0.111916) + 4(-0.1980) + 5(0.8584) + 6(0.3679) + 7(-0.1515) + 8(0.09538)$$

$$= 5.74582 \checkmark$$

(B) For $x(n)=n$, we could have expected to have a value of 5.3 by direct application of $x_a(t=5.3) = x(n) = x_a(nT)$. ✓

(C) To make the answer in (A) more accurate, ie. be closer to the expected 5.3, we can try to interpolate at more points. ✓

Name ALANA M. SOEHARTONO

EE410-001

FALL 2013

Digital Signal Processing

Exam 2

Problem 1 23

Problem 2 25

Problem 3 25

Problem 4 17

Total 90

1. Given the following sequence

$$x(n) = [1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

a) Compute by hand the DFT showing all terms. No credit given without all work shown explicitly.

b) Compute and sketch the Magnitude of the DFT.

c) Compute and sketch the Phase of the DFT.

(A) $X(k) = \sum_{n=0}^{7} x(n) e^{-j2\pi nk/8} \qquad k = 0, 1, 2, \ldots, 7$

$= (1)e^{-j(0/8)} + (1)e^{-j\pi n/4} + 0e^{-j\pi n/2} + 0e^{-j6\pi n/8} + 0e^{-j\pi n} + 0e^{-j10\pi n/8}$

$+ 0e^{-j12\pi n/8} + 0e^{-j14n\pi/8}$

$= 1 + e^{-j\pi n/4}$ Also: $e^{-j\theta} \equiv \cos\theta - j\sin\theta$

$X(0) = 1 + e^{-j\pi(0)/4} = 2$

$X(1) = 1 + e^{-j\pi/4} = (\frac{\sqrt{2}}{2} + 1) - j\frac{\sqrt{2}}{2} \simeq 1.701 - j.707$

$X(2) = 1 + e^{-j\pi/2} = 1 - j$

$X(3) = 1 + e^{-j3\pi/4} = (1 - \frac{\sqrt{2}}{2}) - j\frac{\sqrt{2}}{2} \simeq 0.293 - j0.707$

$X(4) = 1 + e^{-j\pi} = 0$

$X(5) = 1 + e^{-j5\pi/4} = (1 - \frac{\sqrt{2}}{2}) + j\frac{\sqrt{2}}{2} \simeq 0.293 + j0.707$

$X(6) = 1 + e^{-j3\pi/2} = 1 + i$

$X(7) = 1 + e^{-j7\pi/4} = (\frac{\sqrt{2}}{2} + 1) + j\frac{\sqrt{2}}{2} \simeq 1.707 + j0.707$

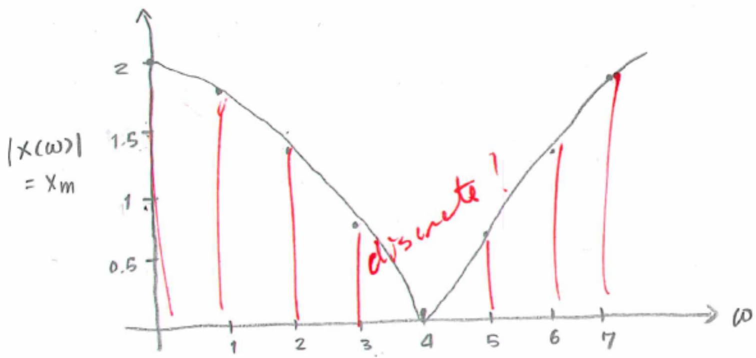
(B) Magnitude for $z = x + iy$

$|z| = \sqrt{x^2 + y^2}$

$X_m(0) = 2$ $\qquad X_m(2) = \sqrt{2}$ $\qquad X_m(4) = 0$ $\qquad X_m(6) = \sqrt{2}$

$X_m(1) = 1.842$ $\qquad X_m(3) = 0.765$ $\qquad X_m(5) = 0.765$ $\qquad X_m(7) = 1.842$

$X_m = [2, 1.842, \sqrt{2}, 0.765, 0, 0.765, \sqrt{2}, 1.842]$



(c) $\theta = \tan^{-1}\left(\frac{Y}{X}\right)$ -2

$X_\theta(0) = \tan^{-1}\left(\frac{0}{2}\right) = \frac{\pi}{2} \simeq 1.57$

$X_\theta(1) = \tan^{-1}\left(\frac{-0.707}{1.707}\right) = -0.393$

$X_\theta(2) = \tan^{-1}\left(\frac{-1}{1}\right) = -\frac{\pi}{4} \simeq -0.786$

$X_\theta(3) = \tan^{-1}\left(\frac{-0.707}{0.293}\right) = -1.178$

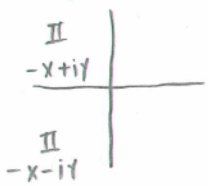
$X_\theta(4) = \tan^{-1}(0) = 0$

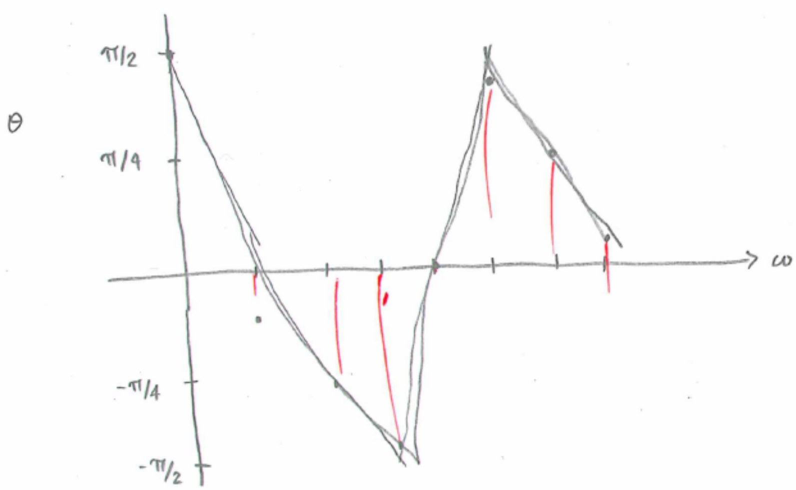
$X_\theta(5) = \tan^{-1}\left(\frac{0.707}{0.293}\right) = 1.178$

$X_\theta(6) = \tan^{-1}(1) = \frac{\pi}{4} \simeq 0.786$

$X_\theta(7) = \tan^{-1}\left(\frac{0.707}{1.707}\right) = 0.393$

For II, III, $\theta + \pi$



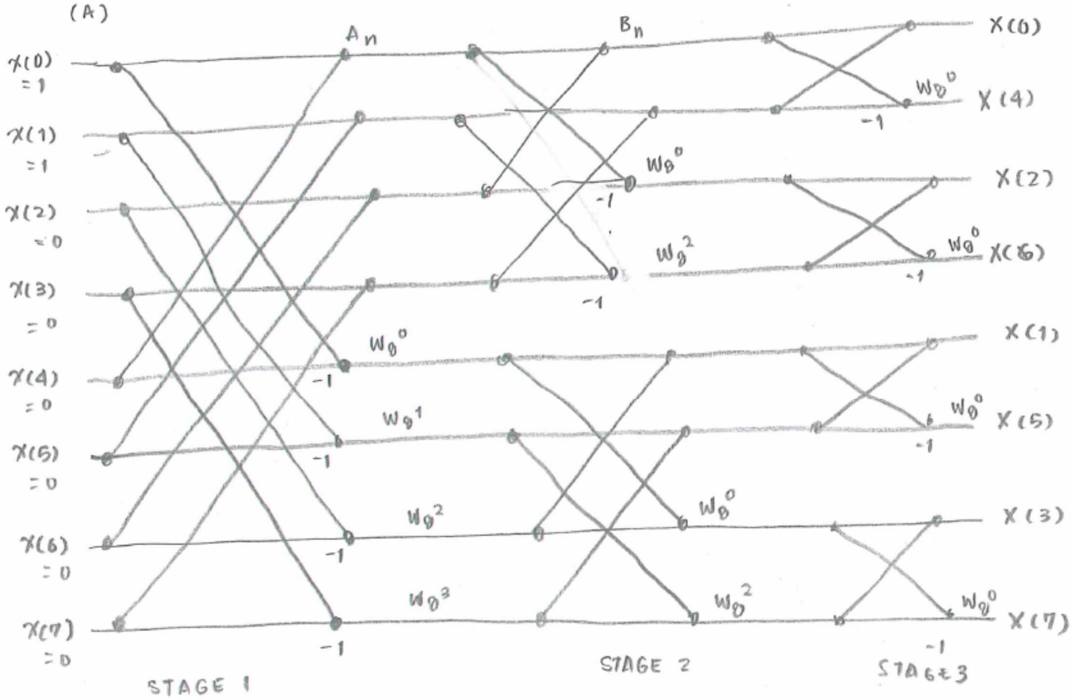


2. Compute the 8 point radix-2 decimation in frequency FFT for the given sequence. Show all work. No credit for just the answer.

$$x(n) = [1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

a) Show the results of each stage of the FFT calculation.
b) What is the main advantage of using the FFT over DFT ?

(A)



STAGE 1 $\quad W_8^n = e^{-j2\pi n/8}$

$W_8^0 = e^{-j0} = 1$ $\qquad W_8^2 = e^{-j\pi/2} = -j$

$W_8^1 = e^{-j\pi/4} = \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$ $\qquad W_8^3 = e^{-j3\pi/4} = -\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$

$A(0) = x(0) + x(4) = 1 + 0 = 1$

$A(1) = x(1) + x(5) = 1 + 0 = 1$

$A(2) = x(2) + x(6) = 0$

$A(3) = x(3) + x(7) = 0$

$A(4) = [x(0) - x(4)]\, W_8^0 = 1(1) = 1$

$A(5) = [x(1) - x(5)]\, W_8^1 = 1\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)$

$A(6) = [x(2) - x(6)]\, W_8^2 = (0)(-j) = 0$

$A(7) = [x(3) - x(7)]\, W_8^3 = (0)\, W_8^3 = 0$

STAGE 2

$B(2) = [A(0) - A(2)] W_8^0 = [1-0](1) = 1$

$B(3) = [A(1) - A(3)] W_8^2 = [1-0][-j] = -j$

$B(0) = [A(0) + A(2)] = 1$

$B(1) = A(1) + A(3) = 1$

$B(4) = A(4) + A(6) = 1 + 0$

$B(5) = A(5) + A(7) = \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2} + 0$

$B(6) = [A(4) - A(6)] W_8^0 = (1)(1)$

$B(7) = [A(5) - A(7)] W_8^2 = \left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)(-j) = -\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$

STAGE 3

$X(0) = B(0) + B(1) = 1 + 1 = 2$

$X(4) = [B(0) - B(1)] W_8^0 = [1-1][1] = 0$

$X(2) = [B(2) + B(3)] = 1 - j$

$X(6) = [B(2) - B(3)] W_8^0 = (1+j)(1) = 1 + j$

$X(1) = B(4) + B(5) = \left(1 + \frac{\sqrt{2}}{2}\right) - j\frac{\sqrt{2}}{2}$

$X(5) = [B(4) - B(5)] W_8^0 = \left[1 - \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right][1] = \left(1 - \frac{\sqrt{2}}{2}\right) + j\frac{\sqrt{2}}{2}$

$X(3) = B(6) + B(7) = \left(1 - \frac{\sqrt{2}}{2}\right) - j\frac{\sqrt{2}}{2}$

$X(7) = [B(6) - B(7)] W_8^0 = \left(1 + \frac{\sqrt{2}}{2}\right) + j\frac{\sqrt{2}}{2}$

(B) The main advantage of FFT vs. DFT is the amount of time to compute. complexity for FFT is $\frac{N}{2}\log_2 N$ vs. N^2 for DFT.

3. Given the following difference equation

$$y(n) = y(n-1) - \frac{1}{2}y(n-2) + x(n) - x(n-1) + x(n-2)$$

a) Find the Pulse Transfer function H(z). Show all work.
b) Draw the Direct Form 1 filter realization of the system labeling all of the relevant coefficients.
c) Draw the Direct Form 2 filter realization of the system labeling all of the relevant coefficients.

(A)

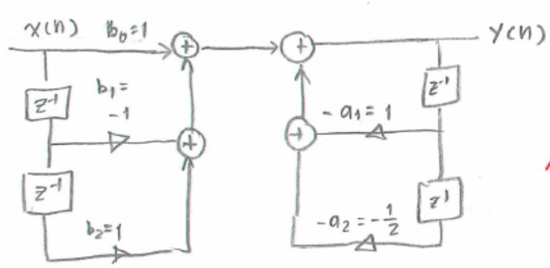
$$Y(z) = z^{-1}Y(z) - \frac{1}{2}z^{-2}Y(z) + X(z) - z^{-1}X(z) + z^{-2}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

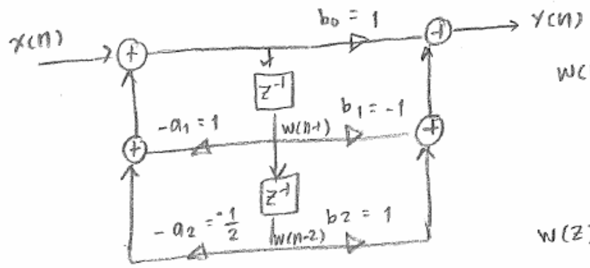
$$Y(z)[1 - z^{-1} + \tfrac{1}{2}z^{-2}] = X(z)[1 - z^{-1} + z^{-2}]$$

$$\therefore\ H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-1} + z^{-2}}{1 - z^{-1} + \frac{1}{2}z^{-2}} =$$

(B) Direct I



(c)



$$w(n) = -\sum_{k=1}^{2} a_k w(n-k) + x(n)$$

$$= a_1 w(n-1) - a_2 w(n-2) + x(n)$$

$$W(z) = -1\, z^{-1} W(z) - \frac{1}{2} z^{-2} W(z) + X(z)$$

4. Design an M=7 Hamming window FIR linear-phase digital filter with frequency response:

$$H_d(\omega) = \begin{cases} 1, & for\ |\omega| \le \dfrac{\pi}{6} \\ 0, & for\ \dfrac{\pi}{6} < |\omega| \le \pi \end{cases}$$

a) Show the calculated values for $h_d(n)$

b) Show the calculated values for $w_{\text{Hamming}}(n)$

c) Show the final values for $h(n)$

All values calculated and shown by hand or hand calculator. Show all work. No credit for results only.

$\omega(n) = [\ 0, \frac{\pi}{6}, \frac{2\pi}{6}, \frac{3\pi}{6}, \frac{4\pi}{6}, \frac{5\pi}{6}, \pi\ ]$

(A) $h_d(n) = \sum_{K=0}^{6} H_d(\omega) e^{+j2\pi nK/7}$ $\quad n = 0, 1, 2, 3 \ldots, 7$

$h_d(n) = 1e^{j2(0)n} + 1e^{j2\pi n/7} + 0e^{j4\pi n/7} + 0e^{j6\pi n/7} + 0e^{j8\pi n/7} + 0e^{j10\pi n/7} + 0e^{j12\pi n/7}$

X

8

$h_d(0) = 1 + e^{j(0)} = 2$

$h_d(1) = 1 + e^{j2\pi/7} = 1.62 + j0.781$

$h_d(2) = 1 + e^{j4\pi/7} = 0.77 + j0.97$

$h_d(3) = 1 + e^{j6\pi/7} = 0.099 + j0.434$

$h_d(4) = 1 + e^{j8\pi/7} = 0.099 - j0.434$

$h_d(5) = 1 + e^{j10\pi/7} = 0.77 - j0.99$

$h_d(6) = 1 + e^{j12\pi/7} = 1.62 - j0.781$

(B) $w(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{\underbrace{M-1}_{6}}\right)$

$w(0) = 0.54 - 0.46\cos[0] = 0.08$

$w(1) = 0.31$

$w(2) = 0.77$

$w(3) = 1$

$w(4) = 0.77$

$w(5) = 0.31$

$w(6) = 0.08$

(C) $h(n) = h_d(n)\,w(n)$

ok

$h(0) = (2)(0.08) = 0.16$

$h(1) = (0.31)(1.62 + j0.781) = 0.5022 + j0.242$

$h(2) = (0.77 + j0.97)(0.77) = 0.5929 + j0.745$

$h(3) = (0.99 + j0.434)(1) = 0.99 + j0.434$

$h(4) = (0.99 - j0.434)(0.77) = -0.7623 - j0.33$

$h(5) = (0.77 - j0.99)(0.31) = 0.239 - j0.31$

$h(6) = (1.62 - j0.781)(0.08) = 0.1296 - j0.06248$

Name T J

EE410-001

FALL 2013

Digital Signal Processing

Exam 2

Problem 1 25

Problem 2 25

Problem 3 25

Problem 4 25

Total 100

1. Given the following sequence

$$x(n) = [1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

a) Compute by hand the DFT showing all terms. No credit given without all work shown explicitly.

b) Compute and sketch the Magnitude of the DFT.

c) Compute and sketch the Phase of the DFT.

a) $X(k) \triangleq \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$ Defn.

$x(n) = [1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$

$N = 8$

$X(k=0) = \sum_{n=0}^{N-1} x(n) e^{-j0}$

$= 1 + 1 + 0 + 0 + 0 + 0 + 0 + 0 = 2$

$X(k=1) = 1e^{0} + 1e^{-j\pi/4} + \underbrace{0e^{-j2\pi n/N} + \ldots}_{\text{Rest Terms zero}} = 1.707 - j0.707$

$X(k=2) = 1e^{0} + 1e^{-j\pi/2} + 0e^{-j4\pi n/N} + \ldots$

$= 1 - j$

$X(k=3) = 1e^{0} + 1e^{-j3\pi/4} = 0.293 - j0.707$

$X(k=4) = 1e^{0} + 1e^{-j\pi 4/4} = 0$

$X(k=5) = 1e^{0} + 1e^{-j5\pi/4} = 0.293 + j0.707$

$X(k=6) = 1e^{0} + 1e^{-j6\pi/4} = 1 + j$

$X(k=7) = 1e^{0} + 1e^{-j7\pi/4} = 1.707 + j0.707$

b) Magnitude of each Term $X(k)$ by

$$|X(k)| = \sqrt{Re^2 + Im^2}$$

$|X(0)| = 2$

$|X(1)| = 1.85$

$|X(2)| = 1.41$

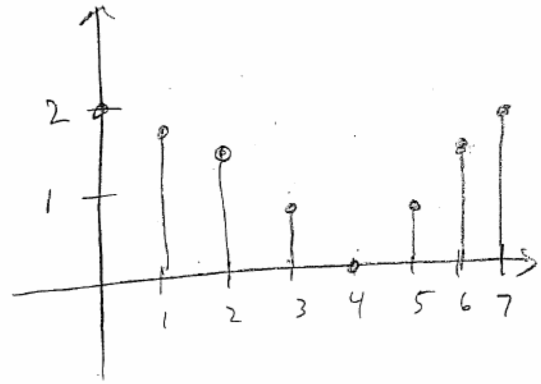
$|X(3)| = 0.765$

$|X(4)| = 0$

$|X(5)| = 0.765$

$|X(6)| = 1.41$

$|X(7)| = 1.85$



c) Phase of the DFT

$$\angle X(k) = \tan^{-1}\left(\frac{Im}{Re}\right)$$

In Radians

$\angle X(0) = 0$

$\angle X(1) = -0.3927$

$\angle X(2) = -0.7854$

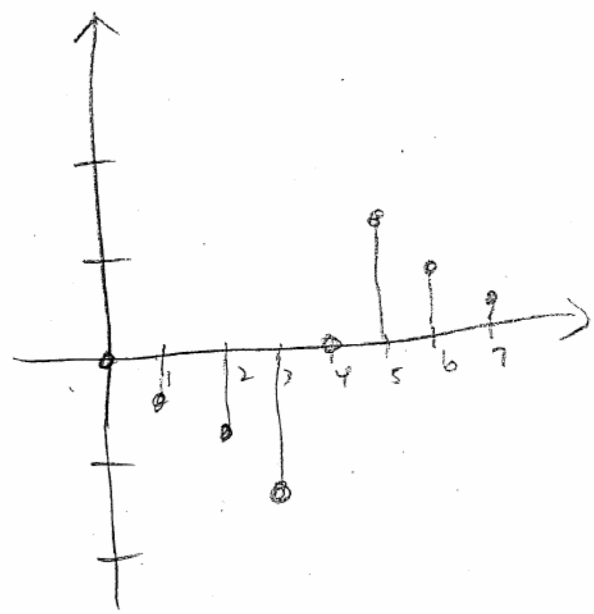
$\angle X(3) = -1.1781$

$\angle X(4) = 0$

$\angle X(5) = 1.1781$

$\angle X(6) = 0.7854$

$\angle X(7) = 0.3927$



2. Compute the 8 point radix-2 decimation in frequency FFT for the given sequence. Show all work. No credit for just the answer.

$$x(n) = [1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

a) Show the results of each stage of the FFT calculation.
b) What is the main advantage of using the FFT over DFT ?

N^2 complex mults - DFT

$\frac{N}{2}\log_2(N)$ complex mults - FFT

a) Given Figure 8.1.11 p.527 of Text Book.

Stage 1

$x(0) = 1$	a+b	1
$x(1) = 1$	a-b	1
$x(2) = 0$		0
$x(3) = 0$		0
$x(4) = 0$		1
$x(5) = 0$		$1e^{-2\pi/8} = 0.707 - j0.707$
$x(6) = 0$		0
$x(7) = 0$		0

Feed these Terms into stage 2.

Stage 2

1	1
1	1
0	1
0	$1 e^{-j4\pi/8} = -j$
1	1
$0.707 - j0.707$	$0.707 - j0.707$
0	1
0	$(0.707 - j0.707)\,-j = -0.707 - j0.707$

use these as inputs to stage 3.

Stage 3

		Assign values
1	2	X(0)
1	0	X(4)
1	$1-j$	X(2)
$-j$	$1+j$	X(6)
1	$1.707 - j0.707$	X(1)
$0.707 - j0.707$	$0.293 + j0.707$	X(5)
1	$0.293 - j0.707$	X(3)
$-0.707 - j0.707$	$1.707 + j0.707$	X(7)

3. Given the following difference equation

$$y(n) = y(n-1) - \frac{1}{2}y(n-2) + x(n) - x(n-1) + x(n-2)$$

a) Find the Pulse Transfer function H(z). Show all work.
b) Draw the Direct Form 1 filter realization of the system labeling all of the relevant coefficients.
c) Draw the Direct Form 2 filter realization of the system labeling all of the relevant coefficients.

a) $H(z) = \frac{Y(z)}{X(z)}$

$$y(n) - y(n-1) - \tfrac{1}{2}y(n-2) = x(n) - x(n-1) + x(n-2)$$

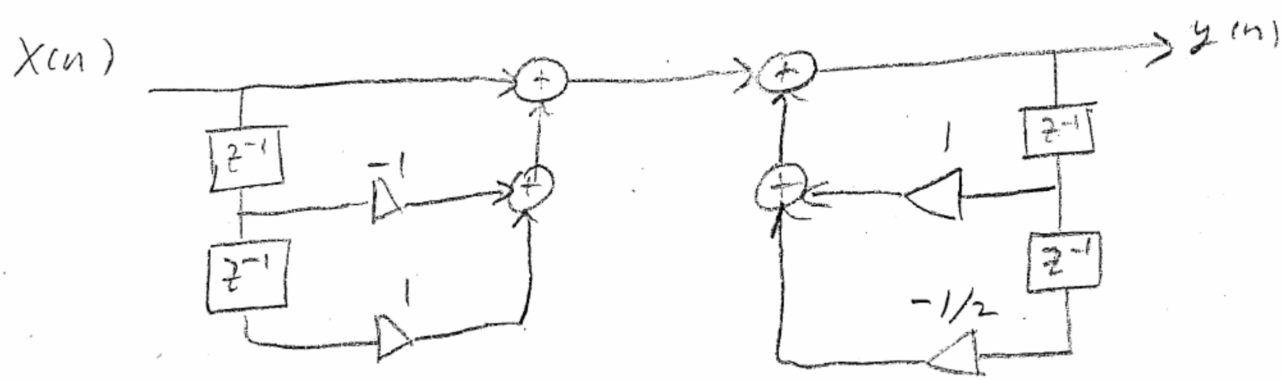
z-transform

$$Y(z) - z^{-1}Y(z) - \tfrac{1}{2}z^{-2}Y(z) = X(z) - z^{-1}X(z) + z^{-2}X(z)$$

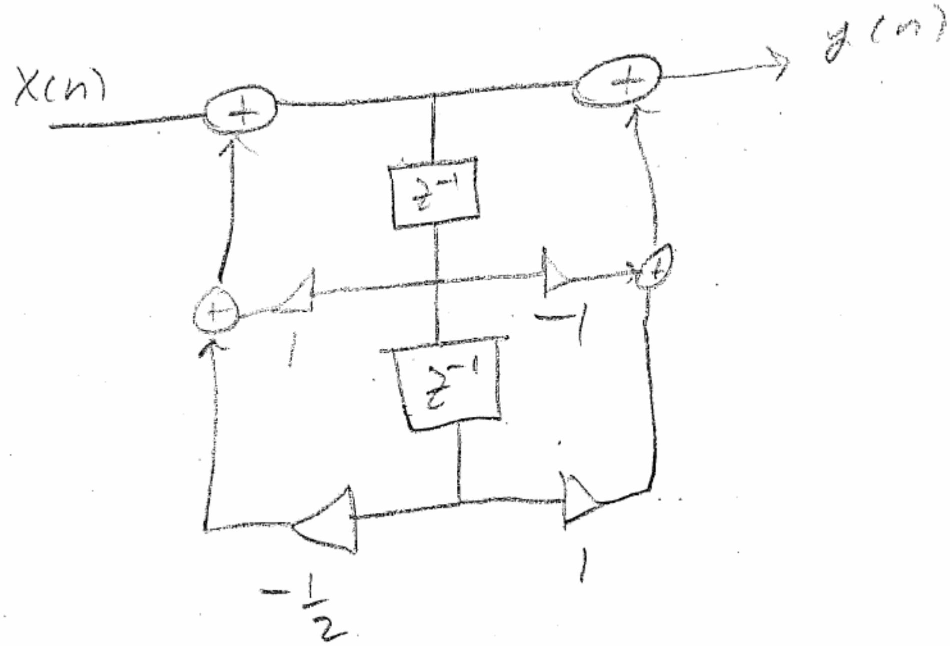
$$Y(z)\left(1 - z^{-1} - \tfrac{1}{2}z^{-2}\right) = X(z)\left(1 - z^{-1} + z^{-2}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-1} + z^{-2}}{1 - z^{-1} + \frac{1}{2}z^{-2}}$$

b) Direct Form 1



c) Direct Form 2



4. Design an M=7 Hamming window FIR linear-phase digital filter with frequency response:

$$H_d(\omega) = \begin{cases} 1, & for\,|\omega| \le \dfrac{\pi}{6} \\ 0, & for\,\dfrac{\pi}{6} < |\omega| \le \pi \end{cases}$$

a) Show the calculated values for $h_d(n)$

b) Show the calculated values for $w_{\text{Hamming}}(n)$

c) Show the final values for $h(n)$

All values calculated and shown by hand or hand calculator. Show all work. No credit for results only.

a) using 10.2.27

$$h_d(n) = \frac{\sin\left[w_c\left(n - \frac{M-1}{2}\right)\right]}{\pi\left(n - \frac{M-1}{2}\right)}$$

10.2.29

$$h_d\left(\frac{M-1}{2}\right) = h_d\left(\frac{7-1}{2}\right) = h(3) = \frac{w_c}{\pi}$$

$$w_c = \frac{\pi}{6}$$

$$h(3) = 1/6 = 0.167$$

$$h_d(n) = \frac{\sin\left(\frac{\pi}{6}(n-3)\right)}{\pi(n-3)}$$

$$h_d(0) = \sin\left(\frac{\pi}{6}(-3)\right) = \frac{-1}{\pi(-3)} = 0.1061$$

$$h_d(1) = \frac{\sin\left(\frac{\pi}{6}(-2)\right)}{\pi(-2)} = \frac{0.866}{\pi(2)} = 0.1378$$

$$h_d(2) = \frac{\sin\left(\frac{\pi}{6}(-1)\right)}{\pi(-1)} = 0.1592$$

$h(4) = h(2)$; $h(5) = h(1)$, $h(6) = h(0)$

$$W_{Hamming} = 0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right)$$ p. 666

$M = 7$

$W_H(0) = 0.08$

$W_H(1) = 0.31$

$W_H(2) = 0.77$

$W_H(3) = 1$

$W_H(4) = W(2) = 0.77$

$W_H(5) = W(1) = 0.31$

$W_H(6) = W(0) = 0.08$

$h(n) = h_d(n)\, W_H(n)$ for $n = 0, 1 \ldots 6$.

$h(0) = (0.1061)(0.08) = 0.0085$

$h(1) = (0.1378)(0.31) = 0.0427$

$h(2) = (0.1592)(0.77) = 0.1226$

$h(3) = (0.167)(1.0) = 0.167$

$h(4) = h(2) = 0.1226$

$h(5) = h(1) = 0.0427$

$h(6) = h(0) = 0.0085$

TB7.24 For the finite duration signal $x(n) = \{1, 2, 3, 1\}$

(A) compute the 4-point DFT by solving explicitly the system of linear eq'ns, using IDFT

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi nk}{N}}$$

$$N x(n) = \sum_{k=0}^{N-1} X(k)\, e^{j2\pi nk/N}$$

For $n=0$,

$$X(0) + X(1) + X(2) + X(3) = 4x(0) = 4$$

For $n=1$,

$$X(0)e^{j2\pi(0)/4} + X(1)e^{j\pi/2} + X(2)e^{j\pi} + X(3)e^{j3\pi/2} = 4(2) = 8$$

For $n=2$

$$X(0) + X(1)e^{j\pi} + X(2)e^{j2\pi} + X(3)e^{j3\pi} = 4(3) = 12$$

For $n=3$,

$$X(0) + X(1)e^{j3\pi/2} + X(2)e^{j3\pi} + X(3)e^{j9\pi/2} = 4$$

1 1 1 1 $\qquad e^{j\pi/2} = \cos(\frac{\pi}{2}) + j\sin(\frac{\pi}{2}) =$

1