Quant Interview FAQ — Derivatives (https://bagelquant.com/quant-interview-faq-derivatives/)

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Each question below includes a **Short Answer**, a concrete **Example**, and a **Detailed Explanation** with quant-level depth, including formulas, edge cases, and practical caveats.

> *Full topic regarding derivatives is covered in the Derivatives (https://bagelquant.com/derivatives/) section.*

1) Forward vs. Futures

Short Answer
Forward = OTC, single settlement at T, bilateral credit risk; Futures = exchange-traded, standardized, daily mark-to-market with margin and clearing.

Example
Crude oil June forward struck at K vs. NYMEX CL June futures: same notionally, but futures P&L is realized daily; the forward's P&L is realized once at expiry.

Detailed Explanation

- **Pricing:** With deterministic rates, $F_0^{fut} = F_0^{fwd} = S_0 e^{(r-q)T}$. With stochastic rates and $\mathrm{corr}(S, r) \neq 0$, daily settlement creates a **convexity bias**: $\mathbb{E}[F^{fut}] \gtrless F^{fwd}$ depending on sign of correlation.
- **Credit & Funding:** Forwards embed bilateral CVA/DVA, CSA terms (thresholds, MTA, collateral rate), and potentially different **discounting curves** (OIS vs. legacy IBOR). Futures replace bilateral credit risk with CCP exposure and margin liquidity risk.
- **Operations:** Futures require intraday margin; forwards require CSA collateral management and closeout mechanics (ISDA).
- **Hedging Impact:** Futures P&L realized early changes effective reinvestment rate and can create basis vs a forward hedge.

2) No-Arbitrage Pricing

Short Answer
Portfolios with identical future cash flows must have the same price today.

Example
Equity forward parity: $F_0 = S_0 e^{(r-q)T}$. If market quotes $F_0^\star > S_0 e^{(r-q)T}$, do **cash-and-carry** (borrow cash, buy spot, short forward) to lock risk-free profit.

Detailed Explanation

- **Cash-and-carry:** Profit at T equals $F_0^\star - S_0 e^{(r-q)T}$.
- **Reverse carry:** If $F_0^\star < S_0 e^{(r-q)T}$, short spot, invest proceeds, long forward; profit $= S_0 e^{(r-q)T} - F_0^\star$.
- **Carry components:** q can be dividends (equity), foreign rate (FX: $r_d - r_f$), or convenience yield vs storage (commodities).
- **Real-world frictions:** Bid–ask, short-borrow fees, taxes, discrete dividends, and collateral rates shrink or flip apparent arbitrages.

3) Risk-Neutral Valuation

Short Answer

Under $\mathbb{Q}$, discounted prices are martingales; price = discounted expectation of payoff under $\mathbb{Q}$.

Example

European option: $V_0 = e^{-rT}\mathbb{E}^{\mathbb{Q}}[f(S_T)]$, with $dS_t = (r-q)S_t\,dt + \sigma S_t\,dW_t^{\mathbb{Q}}$.

Detailed Explanation

- **Change of Measure:** Girsanov transforms drift $\mu \to r-q$ to remove risk premia from pricing; risk is handled via replication.
- **Completeness:** If the market is complete (e.g., BSM), replication is unique ⇒ a unique $\mathbb{Q}$. In incomplete markets (jumps, stoch-vol), additional criteria (e.g., minimal martingale measure) pick a $\mathbb{Q}$.
- **Numeraire:** Pricing invariance across numeraires (bank account, T-bond, annuity) leads to forward measures simplifying some products (e.g., caplets).

4) BSM PDE (Derivation)

Short Answer

Delta-hedge $V(S,t)$ to eliminate diffusion; the residual riskless portfolio must earn r ⇒ $V_t + \frac{1}{2}\sigma^2 S^2 V_{SS} + rSV_S - rV = 0$.

Example

For a call with $V(T,S) = \max(S-K,0)$, solving the PDE yields the Black–Scholes formula.

Detailed Explanation

- **Itô's Lemma:** $dV = V_t dt + V_S dS + \frac{1}{2} V_{SS}(dS)^2$. With $dS = \mu S dt + \sigma S dW$.
- **Hedge:** Take $\Delta = V_S$, portfolio $\Pi = V - \Delta S$ eliminates dW.
- **No-arb:** $d\Pi = r\Pi dt$ ⇒ PDE above with appropriate boundary (terminal payoff), and conditions at $S \to 0, \infty$.
- **Dividends:** With continuous yield q, PDE becomes $V_t + \frac{1}{2}\sigma^2 S^2 V_{SS} + (r-q) S V_S - rV = 0$.

5) Meaning of $N(d_1)$ and $N(d_2)$

Short Answer
$N(d_2)$ ≈ risk-neutral probability of finishing ITM; $N(d_1)$ relates to delta/expected exercise under $\mathbb{Q}$.

Example
Call: $C = S_0 e^{-qT} N(d_1) - K e^{-rT} N(d_2)$. If $N(d_2) = 0.60$, there's a 60% $\mathbb{Q}$-chance of $S_T > K$.

Detailed Explanation

- **Term 1:** $S_0 e^{-qT} N(d_1)$ = PV of expected asset delivered at exercise (adjusted for q).
- **Term 2:** $K e^{-rT} N(d_2)$ = PV of exercise price paid, weighted by exercise probability.
- **Delta:** $\Delta_{call} = e^{-qT} N(d_1)$, connecting $N(d_1)$ to hedging ratio.

6) Volatility's Impact

Short Answer
Higher σ raises option value due to convexity (Jensen's inequality).

Example
ATM call $S_0 = K = 100, T = 1, r = q = 0$: $C(\sigma = 10\%) \approx 3.99$ vs $C(\sigma = 30\%) \approx 11.92$ (BSM).

Detailed Explanation

- **Convex Payoff:** Upside unbounded, downside floored at 0 ⇒ dispersion benefits long options.
- **Vega:** $\nu = S_0 e^{-qT} \phi(d_1) \sqrt{T} > 0$. Peaks near ATM and for longer T.
- **Skews:** Market smiles imply state-dependent effective volatility, making sensitivity path-dependent in practice.

7) Delta Hedging

Short Answer
Hold $\Delta = \partial V / \partial S$ shares against an option to neutralize small S moves.

Example
Short 100 calls with $\Delta = 0.55$ ⇒ buy 55 shares to be delta-neutral initially; rebalance as Δ changes.

Detailed Explanation

- **Gamma–Theta:** Frequent rebalancing needed if Γ large; long gamma gains from realized variance but pays theta (time decay).
- **Discrete Hedging Error:** Hedging discretely produces residual P&L $\approx \frac{1}{2}\Gamma(\Delta S)^2 - \Theta \Delta t$ plus transaction costs.
- **Smile Dynamics:** "Sticky-delta" vs "sticky-strike" conventions materially affect hedge slippage.

8) Put–Call Parity

Short Answer
$C - P = S_0 e^{-qT} - K e^{-rT}$ for European options with same (K, T).

Example
$S_0 = 100$, $r = 5\%$, $q = 2\%$, $T = 1$. If $C = 9.0$, parity implies
$P = 9.0 - 100e^{-0.02} + 100e^{-0.05} \approx 6.4.$

Detailed Explanation

- **Replication:** Long call + short put = synthetic forward $S_T - K$.
- **Uses:** Build synthetics (e.g., covered call ↔ short put), detect data inconsistencies, infer missing quotes.
- **Edge Cases:** Early-exercise (American) parity becomes an inequality; discrete dividends must be PV-adjusted.

9) Volatility Smile/Skew

Short Answer
Implied vol varies with strike/maturity due to non-Gaussian returns and supply/demand.

Example
Equities: OTM puts rich (downward skew); FX: more symmetric smiles with risk-reversal asymmetry.

Detailed Explanation

- **Drivers:** Leverage effect ($\rho_{S,\sigma} < 0$), crash risk premia, hedging pressure, jumps/stoch-vol.
- **Modeling:** SVI per maturity, SABR/Heston dynamics, local vol for exact fit vs. dynamics realism trade-off.
- **Arb-free:** Enforce butterfly (convexity in K) and calendar (monotonic in T) constraints.

10) Vega

Short Answer
Sensitivity to volatility: $\nu = S_0 e^{-qT} \phi(d_1)\sqrt{T}$.

Example
$S_0 = K = 100$, $T = 1$, $r = q = 0$, $\sigma = 20\%$ ⇒ $\nu \approx 39.9$ per unit vol (i.e., 0.399 per 1% vol point).

Detailed Explanation

- **Term Structure:** Per-maturity vega buckets; vega not fungible across T.
- **Smile:** Skew vega (dV/d skew) and curvature vega (vomma) matter for surface moves.
- **Hedging:** Use options near ATM and close T to neutralize efficiently.

11) Gamma

Short Answer
Curvature w.r.t. S: $\Gamma = e^{-qT} \phi(d_1)/(S_0 \sigma \sqrt{T})$.

Example
Near-ATM, short-dated options have large Γ (sensitive delta).

Detailed Explanation

- **Risk/Reward:** Long gamma benefits from realized volatility; short gamma earns theta but is exposed to large moves.
- **Inventory:** Market makers run gamma targets and rebalance based on liquidity/vol.

12) Theta

Short Answer
Time decay: typically negative for long options, positive for short.

Example
Short-dated ATM options can lose value rapidly into expiry (theta acceleration).

Detailed Explanation

- **Components:** “Carry” from discounting and from expected drift under $\mathbb{Q}$; discrete dividends can flip signs around ex-dates.
- **Trade Design:** Structures like calendars exploit theta/vega interplay.

13) Cost-of-Carry Forward Pricing

Short Answer
$F_0 = S_0 e^{(r-q+c-\delta)T}$ with storage cost c and convenience yield δ.

Example
Gold with $r = 4\%$, storage $c = 1\%$, $\delta = 0$ ⇒ $F_0 = S_0 e^{0.05T}$.

Detailed Explanation

- **FX:** $F_0 = S_0 e^{(r_d - r_f)T}$.
- **Commodities:** Scarcity $\Rightarrow \delta > 0$ (backwardation).
- **Curve:** Forward curve encodes expectations + risk premia + inventory/flow constraints.

14) Futures Convexity Adjustment

Short Answer
With stochastic rates, futures ≠ forwards due to daily settlement.

Example
Eurodollar futures convexity vs FRA often approximated by $\frac{1}{2}\sigma_r^2 T_1 T_2$ (order-of-magnitude guidance).

Detailed Explanation

- **Mechanism:** Covariance of daily gains with discounting shifts fair futures price.
- **Sign:** If underlying positively co-moves with rates, long futures benefit ⇒ futures > forward.

15) Greeks of Digital Options

Short Answer
Extremely sharp near strike: large gamma/vega, unstable delta.

Example
Cash-or-nothing call price $= e^{-rT}N(d_2)$; $\Delta = e^{-rT}\phi(d_2)/(S_0\sigma\sqrt{T})$.

Detailed Explanation

- **Hedging:** Use tight call spreads to approximate a digital and smooth greeks.
- **Risk:** Jump/announcement risk is acute due to step payoff.

16) Asian Options

Short Answer
Payoff depends on average; lower variance ⇒ cheaper than vanilla.

Example
Arithmetic Asian call: $(\bar{S} - K)^+$ with $\bar{S} = \frac{1}{n}\sum S_{t_i}$.

Detailed Explanation

- **Pricing:** Geometric Asians have closed forms; arithmetic often via MC or analytic approximations (Turnbull–Wakeman).
- **Greeks:** Pathwise estimators preferred; bridge corrections reduce bias.

17) Barrier Options

Short Answer
Activation/extinction based on path crossing; many closed forms via reflection.

Example
Down-and-out call = vanilla call – down-and-in call.

Detailed Explanation

- **Monitoring:** Continuous vs discrete matters (discrete cheaper knock-out); Brownian bridge improves MC accuracy.
- **Greeks:** Discontinuous near barrier (kinks); hedging requires careful sizing and possibly semi-static portfolios.

18) Stochastic Volatility (Heston)

Short Answer
Volatility follows a mean-reverting square-root process; semi-closed forms via characteristic functions.

Example
$dv = \kappa(\theta - v)dt + \eta\sqrt{v}\, dW^v$, $d\langle W^S, W^v\rangle = \rho\, dt.$

Detailed Explanation

- **Calibration:** Fit to smile surface across K, T by minimizing price or IV errors.
- **Dynamics:** Negative ρ creates equity-type skew; mean-reversion sets term structure.
- **Greeks:** Additional vanna, volga exposures; hedging needs both underlyings and volatility instruments.

19) SABR

Short Answer

Rates/FX model producing analytic IV approximations with parameters controlling level (α), skew (ρ), and curvature (ν); β sets log-normal vs normal.

Example

FX often uses $\beta \approx 1$; rates sometimes $\beta < 1$ for low-rate environments.

Detailed Explanation

- **Hagan Formula:** Widely used closed-form IV; care with extreme K, F and very short T.
- **Calibration:** ATM volatility pins α; risk-reversal pins ρ; butterfly pins ν.

20) Local Volatility (Dupire)

Short Answer

Deterministic $\sigma_{loc}(t, S)$ reproducing the entire vanilla surface exactly.

Example

Dupire formula: $\sigma_{loc}^2(t, K) = \frac{\partial_T C + qC - rK\partial_K C}{\frac{1}{2}K^2\partial_{KK}C}\Big|_{T=t}$.

Detailed Explanation

- **Use:** Good for barrier/exotics when exact vanilla fit is mandated.
- **Limit:** Unrealistic dynamics (sticky-strike), may mis-hedge under surface moves; numerically sensitive to noisy $\partial_{KK}C$.

21) Monte Carlo vs PDE

Short Answer
MC handles high-dimensional/path-dependent payoffs; PDE efficient in low dimensions and for early exercise.

Example
American put via finite-difference (PDE with free boundary) vs Bermudan via LSMC.

Detailed Explanation

- **MC:** Error $\mathcal{O}(1/\sqrt{M})$; QMC lowers effective variance; Greeks via pathwise/LR estimators.
- **PDE:** Fast & accurate in 1–2D with well-posed boundaries; tricky beyond 2D or with complex path terms.

22) Gamma–Theta Tradeoff

Short Answer
Long gamma benefits from movement but pays theta; short gamma earns theta but is hurt by movement.

Example
Long straddle: positive gamma/vega, negative theta; P&L thrives on realized vol exceeding implied.

Detailed Explanation

- **P&L Attribution:** $\Delta \mathrm{P\backslash\&L} \approx \Delta\,\Delta S + \frac{1}{2}\Gamma(\Delta S)^2 + \nu\,\Delta\sigma + \Theta\,\Delta t.$
- **Strategy:** Market makers run near-neutral delta and target gamma/theta depending on vol views.

23) Forward-Start Options

Short Answer

Strike set at future date; prices depend on time to maturity after start.

Example

At t_1, strike $K = S_{t_1}$; payoff at T: $(S_T - S_{t_1})^+$.

Detailed Explanation

- **Valuation:** Under BSM, reduces to vanilla with maturity $T - t_1$ and ATM at t_1.
- **Use:** Equity comp, forward vol trades; greeks tied to forward measure over $[t_1, T]$.

24) Variance Swaps

Short Answer

Exchange realized variance for fixed variance strike; priced via strip of OTM options.

Example

Payoff $= N\left(\sigma^2_{\text{real}} - K_{\text{var}}\right)$ with realized variance from high-frequency returns.

Detailed Explanation

- **Replication:** $K_{\text{var}} = \dfrac{2e^{rT}}{T} \int_0^\infty \frac{P(K) - C(K)}{K^2}\, dK$ (OTM strip).
- **Risks:** Vol-of-vol, jumps, discretization. Corridors and gamma swaps extend concept.

25) Portfolio Greeks

Short Answer

Aggregate by summation across positions (linearity).

Example

$\Delta_{book} = \sum_i \Delta_i Q_i$, $\nu_{book} = \sum_i \nu_i Q_i$.

Detailed Explanation

- **Hierarchy:** Position → strategy → book; limits set per Greek and scenario.
- **Surface Risk:** Include $\partial\sigma/\partial K$ and $\partial\sigma/\partial T$ (skew/term risk).
- **Stress:** Nonlinear interactions under jumps/liquidity shocks require scenario P&L beyond first/second order.

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COMMENTS