Mean-Reverting and Jump-Diffusion Models for Electricity Prices

Ornstein–Uhlenbeck, Schwartz Type I and II Models, and Cross-Commodity Integration

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Abstract

Electricity prices display unique dynamics distinct from other commodities: high volatility, sharp spikes, strong mean reversion, and multi-scale seasonality. This note develops the main stochastic models used in quantitative energy trading: the Ornstein–Uhlenbeck (OU) and Schwartz Type I mean-reverting diffusions, their jump-diffusion extensions, and the two-factor forward-curve model (Schwartz Type II / Gibson–Schwartz). We conclude with joint electricity–gas modeling for spark spread pricing and cross-commodity hedging.

1 Stylized Facts of Electricity Spot Prices

- **Mean reversion:** Prices revert rapidly to equilibrium levels due to dispatch, generation costs, and balancing mechanisms.
- **High volatility:** Weather shocks, outages, and demand spikes cause strong deviations.
- **Seasonality:** Daily, weekly, and annual load and generation cycles.
- **Spikes:** Short-lived upward jumps reflecting scarcity pricing.
- **Non-storability:** No physical arbitrage between spot and forward markets.

These empirical facts make power prices incompatible with geometric Brownian motion (GBM). Instead, mean-reverting and jump-diffusion processes describe the economics of power markets more realistically.

2 Ornstein–Uhlenbeck (OU) Model

Theoretical Model

The Ornstein–Uhlenbeck (OU) process is the canonical continuous-time mean-reverting process:

$$dX_t = \kappa(\theta - X_t)\,dt + \sigma\,dW_t,$$

where κ controls the speed of reversion, θ the long-term mean, and σ the instantaneous volatility.

Economic intuition. - X_t is the *deseasonalized log spot price*. - When $X_t > \theta$, prices are high, and the drift term pulls them back down. - When $X_t < \theta$, prices are low, and drift is positive (pulling upward). - The process continuously "snaps back" to the equilibrium mean θ, mimicking market rebalancing.

Analytical solution and moments.

$$X_t = X_0 e^{-\kappa t} + \theta(1 - e^{-\kappa t}) + \sigma \int_0^t e^{-\kappa(t-s)} dW_s.$$

$$\mathbb{E}[X_t | X_0] = X_0 e^{-\kappa t} + \theta(1 - e^{-\kappa t}), \quad \mathrm{Var}[X_t | X_0] = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa t}).$$

Stationarity. As $t \to \infty$, the distribution converges to

$$X_t \sim \mathcal{N}\left(\theta, \frac{\sigma^2}{2\kappa}\right),$$

and the autocorrelation function decays exponentially as $\rho(\tau) = e^{-\kappa\tau}$.

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Interpretation: The OU process models the core balancing mechanism in electricity markets — prices revert quickly after shocks (e.g., generator outage, demand surge).

3 Schwartz Type I Model (Mean-Reverting GBM)

Theoretical Model

Mean reversion applied to the price level:

$$dS_t = \kappa(\theta - S_t)\, dt + \sigma S_t\, dW_t.$$

Explanation. Here, the spot price S_t itself, not its logarithm, reverts to θ. This ensures $S_t > 0$ and keeps proportional volatility. In low-volatility regimes, this behaves similarly to OU on log prices.

Analytical expectation.

$$F(t, T) = \mathbb{E}_t[S_T] = S_t e^{-\kappa(T-t)} + \theta(1 - e^{-\kappa(T-t)}).$$

Hence, the forward price approaches θ as $T \to \infty$ — consistent with the long-term equilibrium.

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Interpretation: This model is appropriate when physical mean-reversion acts on price levels (e.g., marginal generation cost anchoring). It provides tractable closed forms for expected prices and is used in simple risk dashboards.

4 Jump-Diffusion Extensions

Theoretical Model

To capture price spikes, augment OU with jumps:

$$dX_t = \kappa(\theta - X_t)\,dt + \sigma\,dW_t + J_t\,dN_t,$$

where N_t is a Poisson process with intensity λ and J_t is the jump size.

Why jumps? Electricity's non-storability and tight capacity cause abrupt, short-lived price spikes. These cannot be reproduced by continuous Brownian motion.

Conditional moments.

$$\mathbb{E}[X_t] = X_0 e^{-\kappa t} + \theta(1 - e^{-\kappa t}) + \frac{\lambda \mu_J}{\kappa}(1 - e^{-\kappa t}),$$

$$\mathrm{Var}(X_t) = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa t}) + \frac{\lambda(\mu_J^2 + \sigma_J^2)}{2\kappa}(1 - e^{-2\kappa t}).$$

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Interpretation: The diffusion term captures base volatility; the jump term models rare, large price spikes. Mean reversion ensures spikes decay rapidly once system balance is restored.

5 Forward Curve Dynamics: Schwartz Type II / Gibson–Schwartz

Spot-only models lack the term-structure effects seen in forward prices. The **Schwartz Type II** (or **Gibson–Schwartz**) model adds a second factor.

Theoretical Model

$$\ln S_t = \chi_t + \xi_t, \quad d\chi_t = -\kappa_\chi \chi_t\,dt + \sigma_\chi\,dW_t^{(\chi)}, \quad d\xi_t = \mu_\xi\,dt + \sigma_\xi\,dW_t^{(\xi)}, \quad \mathrm{corr}(dW_t^{(\chi)}, dW_t^{(\xi)}) = \rho_{\chi\xi}.$$

Interpretation.

- χ_t: short-term deviation — fast mean reversion, captures transient volatility and spikes.
- ξ_t: long-term equilibrium level — slow-moving trend driven by fuel costs or macro variables.

Forward price under risk-neutral measure.

$$F(t,T) = f(T)\,\exp[A(t,T) + B_\chi(t,T)\chi_t + B_\xi(t,T)\xi_t],$$

with $B_\chi(t,T) = e^{-\kappa_\chi(T-t)}$, $B_\xi(t,T) = 1$.

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Economic meaning: Short-term factor drives nearby volatility; long-term factor shapes the overall level of the forward curve. As maturity increases, $B_\chi(t,T) \to 0$ — forward prices reflect mainly long-term expectations.

6 Cross-Commodity Integration: Electricity–Gas and Spark Spreads

Power generation margins link electricity and fuel markets. The **spark spread** reflects the gross margin of gas-fired generation:

$$\text{Spark}_t = P_{,t} - \ P_{,t}.$$

Joint dynamics.

$$\begin{aligned} dX_t^{(e)} &= -\kappa_e(X_t^{(e)} - \theta_e)\,dt + \sigma_e\,dW_t^{(e)}, \\ dX_t^{(g)} &= -\kappa_g(X_t^{(g)} - \theta_g)\,dt + \sigma_g\,dW_t^{(g)}, \\ dW_t^{(e)}dW_t^{(g)} &= \rho_{eg}\,dt. \end{aligned}$$

Instantaneous spread variance.

$$\sigma_S^2 = \sigma_e^2 +^2 \sigma_g^2 - 2\rho_{eg}\sigma_e\sigma_g.$$

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Interpretation: The correlation ρ_{eg} captures the linkage between fuel costs and power prices — high during normal markets, lower during system stress. The spread variance determines the risk in spark spread options or generation portfolios.

Spark Spread Option Pricing (Margrabe Formula). Assuming lognormal forward dynamics:

$$V_0 = e^{-rT}[F_e(t,T)\Phi(d_1) - F_g(t,T)\Phi(d_2)],$$

$$d_{1,2} = \frac{\ln\frac{F_e(t,T)}{F_g(t,T)} \pm \frac{1}{2}\sigma_S^2 T}{\sigma_S\sqrt{T}}.$$

This gives a closed-form spark spread option price under correlated lognormal forwards.

7 Calibration and Simulation

Calibration workflow.

1. Remove deterministic seasonality $f(t)$ from spot or forward series.
2. Estimate κ, θ, and σ from deseasonalized data (OLS or MLE on AR(1) form).
3. Fit jump parameters $(\lambda, \mu_J, \sigma_J)$ using excess kurtosis or EM algorithm.
4. Calibrate correlation ρ_{eg} using joint forward returns.
5. For two-factor models, fit $(\kappa_\chi, \sigma_\chi, \sigma_\xi, \rho_{\chi\xi})$ to forward vol term structures.

Simulation scheme (OU with jumps).

$$X_{t+\Delta t} = X_t e^{-\kappa \Delta t} + \theta(1 - e^{-\kappa \Delta t}) + \sigma \sqrt{\tfrac{1-e^{-2\kappa \Delta t}}{2\kappa}} \varepsilon_t + \sum_{i=1}^{N_t} J_i,$$

with $N_t \sim \text{Poisson}(\lambda \Delta t)$.

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Use case: Simulated price paths support risk metrics (VaR, CVaR), structured product pricing, and scenario testing for portfolio hedging.

8 Summary Table

Model	Equation	Feature	Application
OU	$dX_t = \kappa(\theta - X_t)dt + \sigma dW_t$	Mean reversion (log)	Spot price dynamics
Schwartz I	$dS_t = \kappa(\theta - S_t)dt + \sigma S_t dW_t$	Level reversion, positivity	Long-term mean anchor
Jump-OU	$+ J_t dN_t$	Spikes, heavy tails	Short-term volatility
Schwartz II	2-factor (χ_t, ξ_t)	Term-structure of vol	Forward curves
Joint OU (Elec–Gas)	Correlated diffusions	Cross-commodity link	Spark spread pricing

9 Conclusion

Electricity prices are driven by strong mean reversion, seasonal cycles, and occasional jumps — reflecting the physical balance of generation and demand. The Ornstein–Uhlenbeck and Schwartz Type I models provide the foundation for spot and short-term pricing. Extending to the two-factor (Schwartz Type II) specification captures forward-curve term structures, while integrating correlated gas dynamics links generation costs directly to electricity pricing.

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Takeaway: Mean-reverting and jump-diffusion frameworks bridge the gap between market data and physical reality. They underpin risk models, option pricing, and inter-commodity hedging strategies in the power and gas complex.

Key References

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