Derivatives and Fixed Income — Lecture Notes

Program: PGE M1 — Quantitative Finance Track
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Lecturer: Alexandre Landi

The Greeks

In previous lectures, we studied how to *price* options using Black–Scholes and Monte Carlo. Now we turn to the next question: how sensitive are these prices to changes in the underlying variables?

This is where the **Greeks** come in. Each Greek measures the sensitivity of the option price to one of the inputs of the Black–Scholes model.

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1 Introduction to the Greeks

The Greeks are partial derivatives of the option price with respect to model parameters:

$$\Delta = \frac{\partial V}{\partial S}, \quad \Gamma = \frac{\partial^2 V}{\partial S^2}, \quad \Theta = \frac{\partial V}{\partial T}, \quad \rho = \frac{\partial V}{\partial r}, \quad \nu \text{ (Vega)} = \frac{\partial V}{\partial \sigma}.$$

Each symbol corresponds to:

- Δ: sensitivity to underlying price S.
- Γ: curvature in S, second-order sensitivity.
- Θ: sensitivity to time (time decay).
- ρ: sensitivity to interest rate r.
- ν: sensitivity to volatility σ.

Intuitively: the Greeks are the "risk gauges" traders watch daily.

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2 Delta

General definition:

$$\Delta = \frac{\partial V}{\partial S}$$

For a European call and put under Black–Scholes:

$$\Delta_{\text{call}} = N(d_1), \quad \Delta_{\text{put}} = N(d_1) - 1$$

Interpretation: Delta tells us how much the option price moves when the underlying price changes by one unit. A call option behaves like a fraction of the underlying asset: if $\Delta = 0.6$, then a \$1 increase in the stock raises the option value by about \$0.60.

At-the-money calls typically have $\Delta \approx 0.5$. Deep in-the-money calls approach $\Delta \to 1$, while deep out-of-the-money calls have $\Delta \to 0$.

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3 Vega

General definition:

$$\nu = \frac{\partial V}{\partial \sigma}$$

Interpretation: Vega measures how much option value changes for a 1% change in volatility. Long options (both calls and puts) have positive Vega: higher volatility increases value. Short options have negative Vega: they lose when volatility spikes.

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4 Theta

General definition:

$$\Theta = \frac{\partial V}{\partial T}$$

Interpretation: Theta captures time decay. All else equal, as maturity decreases, option value shrinks. This effect is especially strong for at-the-money options. Long option positions generally have negative Theta (they lose value as time passes). Short option positions have positive Theta (they earn the decay).

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5 Rho

General definition:

$$\rho = \frac{\partial V}{\partial r}$$

Interpretation: Rho is probably the least important Greek in practice, since interest rates move slowly. Call values increase with higher rates; put values decrease. This is because higher rates reduce the present value of the strike to be paid at maturity.

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6 Gamma

General definition:

$$\Gamma = \frac{\partial^2 V}{\partial S^2}$$

Interpretation: Gamma measures how much Delta changes when the underlying moves. High Gamma means Delta reacts strongly to small moves. At-the-money short-maturity options have the highest Gamma — this is why market makers fear being short Gamma: it makes hedging unstable.

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7 Signs of the Greeks

To summarize, the Greeks for basic positions can be organized as follows:

	Δ	ν	Θ	ρ	Γ
Call Buyer	$+$	$+$	$-$	$+$	$+$
Call Seller	$-$	$-$	$+$	$-$	$-$
Put Buyer	$-$	$+$	$-$	$-$	$+$
Put Seller	$+$	$-$	$+$	$+$	$-$

The signs reflect intuition: long options gain from volatility (positive Vega) but lose from time passing (negative Theta). Short positions are the mirror image.