Put-Call Parity

Saturday, October 25, 2025 4:41 AM

PUT - CALL PARITY

Protective put

Underlying stock = Apple Inc. = AAPL

Current price = S_0 = 200 \$

Strike price = K = 200 \$

} ATM = at the money

Maturity = T = 3 months = 0.25 years

Risk-free rate = r = 5%

Volatility = σ = 20%

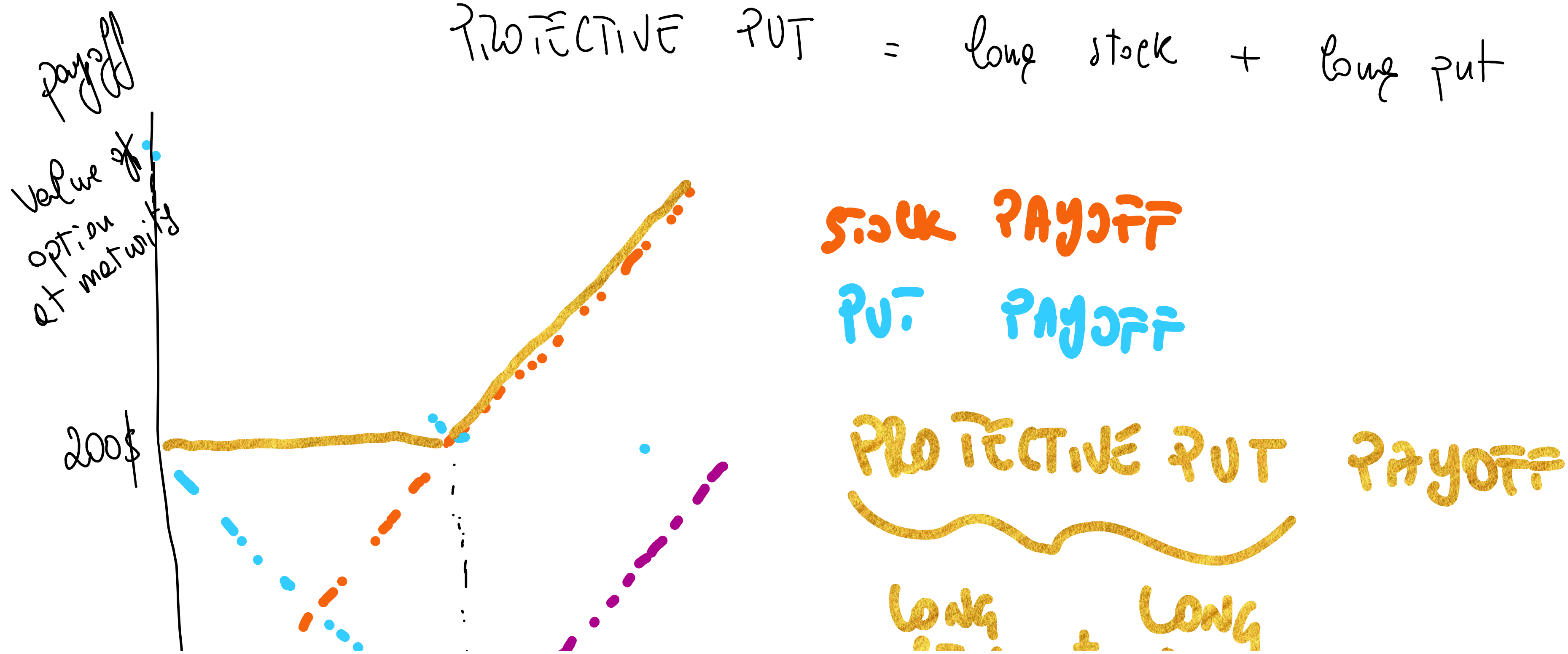
Put premium = p_0 = 10 \$

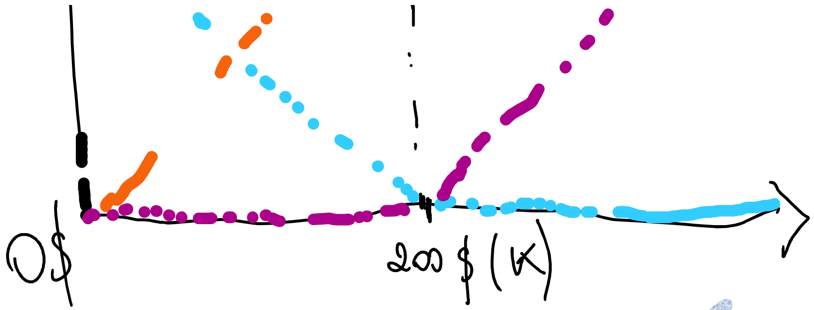
Suppose a trader:

1) Buy 1 share of AAPL

2) Buy 1 put with strike (K) 200 \$ for 10 \$

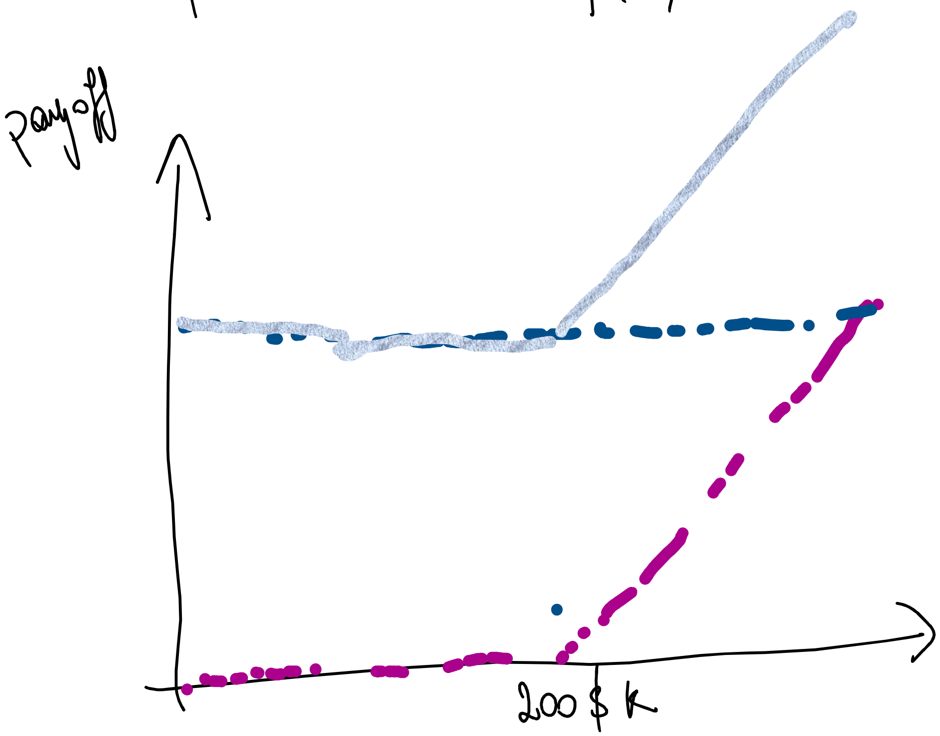
PROTECTIVE PUT = long stock + long put





LONG STOCK + LONG PUT

LONG STOCK + LONG PUT ≠ LONG CALL



ZERO-COUPON BOND PAYOFF

LONG CALL PAYOFF

SAME PAYOFF AS PROTECTIVE PUT

$$\text{Stock} + \text{put} = \text{bond} + \text{call}$$

$$S_T + p_T = K + c_T$$

$$S_0 e^{rT} + p_0 e^{rT} = K + c_0 e^{rt}$$

at maturity

$$e^{-rt} = \frac{1}{e^{rt}}$$

Before maturity, we need to discount

$$\frac{S_0 \cancel{e^{rT}}}{\cancel{e^{rt}}} + \frac{p_0 \cancel{e^{rT}}}{\cancel{e^{rt}}} = \frac{K}{e^{rt}} + \frac{c_0 \cancel{e^{rt}}}{\cancel{e^{rt}}}$$

$$S_0 + p_0 = \frac{K}{e^{rT}} + c_0$$

$$S_0 + p_0 = K e^{-rt} + c_0 \quad \text{before maturity}$$

$$p_0 = Ke^{-rt} + c_0 - S_0$$

$$c_0 = S_0 + p_0 - Ke^{-rt}$$

NUMERICAL EXAMPLE of ARBITRAGE OPPORTUNITY

Underlying stock = Apple Inc. = AAPL

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Maturity = T = 3 months = 0.25 years

Risk-free rate = r = 5%

Volatility = σ = 20%

Put premium = p_0 = 10 \$

Call premium = c_0 = 10 \$

OVER PRICED ← $S_0 + p_0 = Ke^{-rt} + c_0$ → UNDER PRICED

200 \$ + 10 \$ ... $2 \cdot e^{-0.05 \cdot 0.25}$ ... 10 \$

$$200\$ + 10\$ = 200e^{-0.05 \cdot 0.25} + 10\$$$

$$200\$ + 10\$ = 197{,}52\$ + 10\$$$

$$210\$ \neq 207{,}52\$$$

ARBITRAGE STRATEGY

1) sell $\underbrace{\text{overpriced portfolio}}_{S_0 + P_0}$: short stock, short put

2) buy $\underbrace{\text{underpriced portfolio}}_{Ke^{-rT} + C_0}$: buy bond, buy call

Cashflow at initiation : $210\$ - 207.52\$$

$$= 2.48\$$$

case 1) stock price falls to 150$

case 2) stock price rises to 250$

case 1) $S_T = 150\$$

- call expires worthless
- put (which we sold) is exercised $\Rightarrow$ we must buy stock @ 200 $ } $-200\$$

- bond (which we own) pays 200$ } $+200\$$

- we keep the initial 2.48 $ arbitrage profit

case 2) $S_T = 250\$$

case 2) $S_T = 250\$$

- put (which we sold) expires worthless

- call (which we own) is exercised $\rightarrow$ payoff $= S_T - K$
$= 250 - 200$
$= 50\$$

- bond (which we own) pays 200 \$

- with the 250 \$ we get from call (50 \$) / bond (200 \$)

we buy back at 250 \$ the stock that we had initially shorted

- So, no loss, and the initial 2.48 \$ profit is kept