Derivatives and Fixed Income — Lecture Notes

Program: PGE M1 — Quantitative Finance Track
Skema Business School
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Lecture 5: Option Strategies

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1 Recap and Context

Up to this point, we have covered:

- The time value of money: discrete and continuous compounding.
- Forward and futures pricing, arbitrage logic, and margin mechanics.
- Vanilla options: calls and puts, their payoffs and P&L.
- Option pricing models: analytical (Black–Scholes) and numerical (Monte Carlo).
- Sensitivity measures — the Greeks: Δ, Γ, Θ, ν, ρ.

We now move beyond single options to *option combinations*, or structured positions built from multiple vanilla options.

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2 The Long Straddle

A **long straddle** combines a long call and a long put with the same strike K and expiration T. It profits from large price movements in either direction.

Construction

$$\text{Long Straddle} = +1\ \text{Call}(K,T) + +1\ \text{Put}(K,T)$$

At inception, the total premium is the sum of the two option premiums:

$$\text{Total Cost} = C_0 + P_0$$

Payoff at Expiration

$$\Pi_{\text{straddle}}(S_T) = \max(S_T - K, 0) + \max(K - S_T, 0)$$

This creates a "V"-shaped payoff: losses if the price stays near K, and profits if it moves far above or below K.

Numerical Illustration

Given:

$$K = 50, \quad C_0 = 10, \quad P_0 = 10$$

Substitution:

$$\text{Total Premium} = 10 + 10 = 20$$

Breakeven Points:

$$S_T = K \pm \text{Total Premium} = 50 \pm 20 \Rightarrow \boxed{S_T = 30 \text{ or } 70}$$

So at maturity the strategy earns profits if $S_T < 30$ or $S_T > 70$.

	Max Profit	**Max Loss**	**P&L at Maturity**
Symbols	Left: $K - (C_0{+}P_0)$; Right: ∞	$C_0{+}P_0$	$\lvert S_T{-}K\rvert - (C_0{+}P_0)$
Numbers (K=50, C_0=10, P_0=10)	Left: 30; Right: ∞	20	$\begin{cases} 30{-}S_T, & S_T{\le}50, \\ S_T{-}70, & S_T{\ge}50 \end{cases}$

Table 1: Long Straddle: bounded profit on the downside, unlimited on the upside, and limited loss equal to total premium.

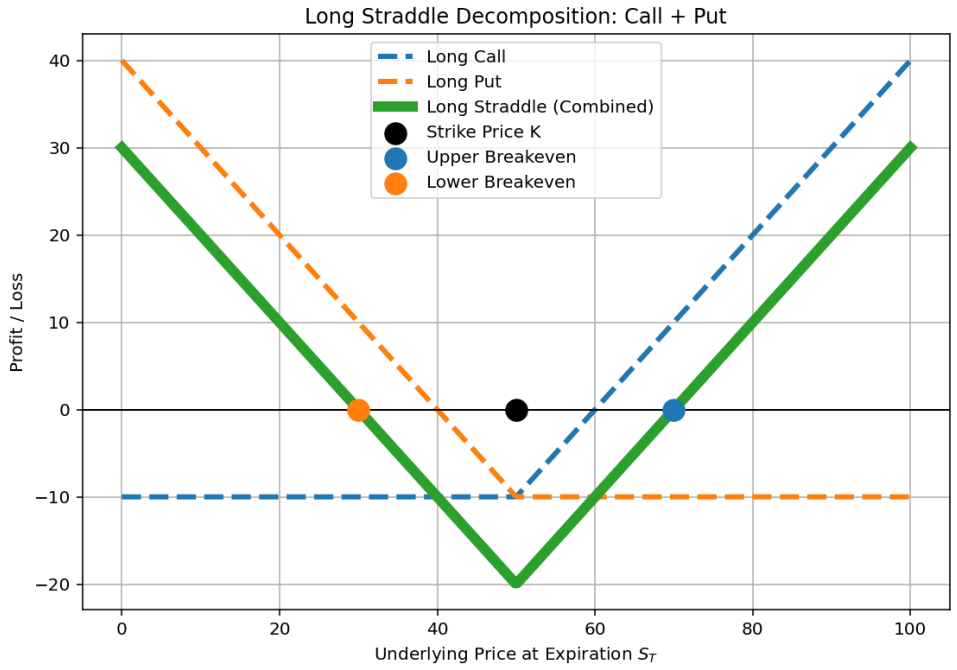

Figure 1: Long Straddle: Combination of Long Call and Long Put

Listing 1: Long Call, Long Put, and Long Straddle Payoffs

```python
import numpy as np
import matplotlib.pyplot as plt

# Parameters
K = 50          # Strike price
C0 = 10         # Call premium
P0 = 10         # Put premium
total_premium = C0 + P0

# Range of possible prices at expiration
S_T = np.linspace(0, 100, 400)

# Individual payoffs
call_payoff = np.maximum(S_T - K, 0) - C0
put_payoff = np.maximum(K - S_T, 0) - P0
straddle_payoff = call_payoff + put_payoff

# Plot
plt.figure(figsize=(9,6))

# Plot payoffs
plt.plot(S_T, call_payoff, '--', lw=3, label='Long Call')
plt.plot(S_T, put_payoff, '--', lw=3, label='Long Put')
plt.plot(S_T, straddle_payoff, lw=6, label='Long Straddle (Combined)')

# Reference lines
plt.axhline(0, color='black', lw=1)

# Mark key x-axis points
plt.scatter([K], [0], color='black', s=150, label='Strike Price K',
    zorder=5)
plt.scatter([K + total_premium], [0],  s=150, label='Upper Breakeven',
    zorder=5)
plt.scatter([K - total_premium], [0], s=150, label='Lower Breakeven',
    zorder=5)

# Labels and title
plt.title('Long Straddle Decomposition: Call + Put')
plt.xlabel('Underlying Price at Expiration $S_T$')
plt.ylabel('Profit / Loss')
plt.legend(loc='best')
plt.grid(True)
plt.show()
```

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Greeks Intuition for the Straddle

	Δ	Θ	ν (Vega)
Long Straddle	Neutral near $S_0 = K$	$-$	$+$

- **Delta:** The call's positive Delta and the put's negative Delta offset each other near-the-money, making the initial Delta roughly zero.
- **Theta:** Both options lose time value, so the overall position has negative Theta.

- **Vega:** Both options gain from higher volatility, so the position is Vega positive.

Interpretation: A long straddle is a bet on volatility — not on direction. You expect a major move, but you don't know whether it will be up or down.

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3 The Short Straddle

A **short straddle** is the exact opposite position:

$$\text{Short Straddle} = -1\ \text{Call}(K,T) + -1\ \text{Put}(K,T)$$

You receive both premiums up front, and you profit if the stock stays near the strike price.

Payoff at Expiration

$$\Pi_{\text{short straddle}}(S_T) = -\max(S_T - K, 0) - \max(K - S_T, 0)$$

The payoff is an inverted "V" shape — profits if prices remain stable, heavy losses if they move too far.

	Max Profit	**Max Loss**	**P&L at Maturity**
Symbols	$C_0{+}P_0$	Unbounded as $S_T{\to}0$ or $S_T{\to}\infty$	$-(\lvert S_T{-}K\rvert{-}(C_0{+}P_0))$
Numbers (K=50, C_0=10, P_0=10)	20	Unbounded below (loss grows without limit)	$\begin{cases} S_T{-}30, & S_T{\le}50, \\ 70{-}S_T, & S_T{\ge}50 \end{cases}$

Table 2: Short Straddle: limited profit (total premium received) and unbounded losses on both sides.

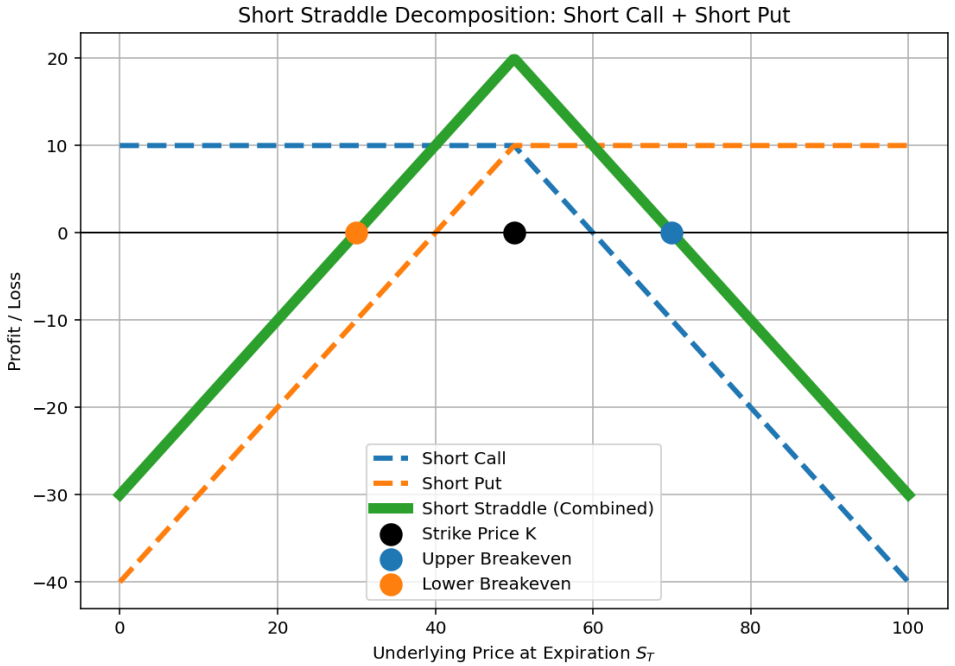

Figure 2: Short Straddle: Selling a Call and a Put with the Same Strike

Listing 2: Short Call, Short Put, and Short Straddle Payoffs

```python
import numpy as np
import matplotlib.pyplot as plt

# Parameters
K = 50          # Strike price
C0 = 10         # Call premium
P0 = 10         # Put premium
total_premium = C0 + P0

# Range of possible prices at expiration
S_T = np.linspace(0, 100, 400)

# Individual payoffs (short positions)
short_call_payoff = -np.maximum(S_T - K, 0) + C0
short_put_payoff = -np.maximum(K - S_T, 0) + P0
short_straddle_payoff = short_call_payoff + short_put_payoff

# Plot
plt.figure(figsize=(9,6))

# Plot payoffs
plt.plot(S_T, short_call_payoff, '--', lw=3, label='Short Call')
plt.plot(S_T, short_put_payoff, '--', lw=3, label='Short Put')
plt.plot(S_T, short_straddle_payoff, lw=6, label='Short Straddle (
    Combined)')

# Reference line
plt.axhline(0, color='black', lw=1)

# Mark key x-axis points
plt.scatter([K], [0], color='black', s=150, label='Strike Price K',
    zorder=5)
plt.scatter([K + total_premium], [0], s=150, label='Upper Breakeven',
    zorder=5)
plt.scatter([K - total_premium], [0], s=150, label='Lower Breakeven',
    zorder=5)

# Labels and title
plt.title('Short Straddle Decomposition: Short Call + Short Put')
plt.xlabel('Underlying Price at Expiration $S_T$')
plt.ylabel('Profit / Loss')
plt.legend(loc='best')
plt.grid(True)
plt.show()
```

Greeks Intuition

	Δ	Θ	ν (Vega)
Short Straddle	Neutral near K	$+$	$-$

- **Delta:** Initially neutral, as opposite Deltas offset.
- **Theta:** Positive — time decay now works in your favor.
- **Vega:** Negative — increased volatility hurts this position.

Interpretation: A short straddle is a bet on stability. It benefits when markets are quiet and volatility drops (for instance, after earnings announcements).

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4 Greeks Behavior Across Strategies

Strategy	Δ	Θ	ν	Market View
Long Straddle	0	$-$	$+$	Expect volatility increase, big move
Short Straddle	0	$+$	$-$	Expect stability, volatility drop

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