3. Volatility Arbitrage & Advanced Strategies

The Volatility Arbitrage Framework

Volatility arbitrage profits from inconsistencies between implied and realized volatility. This strategy relies on the principle that while implied volatility represents the market's expectation of future price fluctuations, realized volatility represents what actually occurs. Traders typically take long positions when implied volatility it appears underpriced, and short when it appears overpriced. The core profit comes from volatility mean-reversion, which is the tendency of volatility spreads to converge over time.

Core Strategies

Volatility Risk Premium

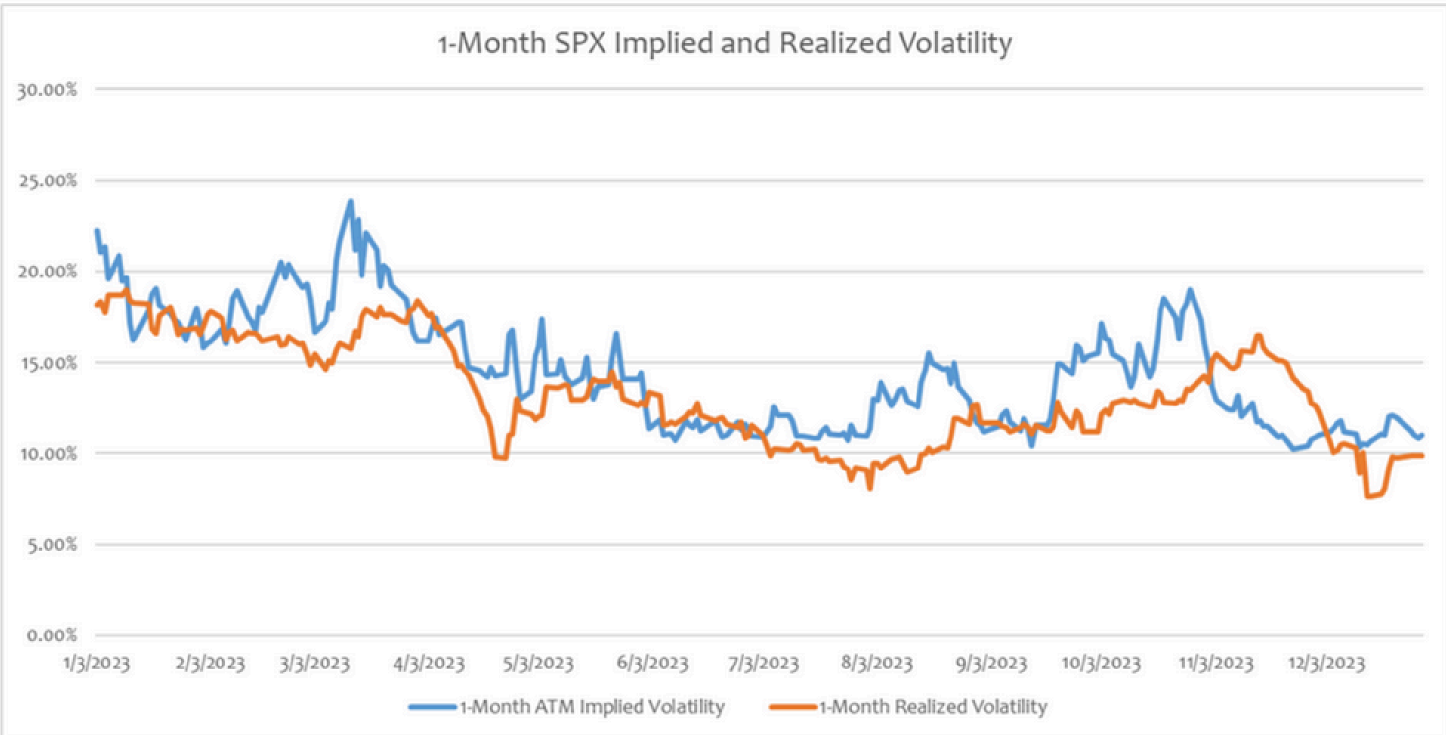
The Volatility Arbitrage or Volatility Risk Premium (VRP) is the persistent difference between Implied Volatility and Realized Volatility :

$$VRP = Implied\, Volatility - Realized\, Volatility$$

Where, IV>RV, creating a premium. The reason behind the existence of the VRP consists of two aspects. One is Investor Behaviour, where Investors want protection so they overpay for options to hedge against crashes, which inflates the Implied Volatility. The other is the Market Structure, because there are more natural buyers than sellers, and the volatility sellers charge extra since they are bearing gamma and vega risks.

Of course, such strategy bears **risks**, and such are:

1. Tail Risk Sudden volatility spikes can wipe out profits.
2. Mean-reversion failure VRP can disappear over extended periods.
3. High execution costs Bid-ask spreads and margin requirements erode profits.



Implied vs Realized volatility of the SPX index in 2023: on the graph we can see the difference between the two volatility types, where IV is mostly greater than RV.

Dispersion Trading

Dispersion trading exploits the difference between the Index Implied Volatility (like the SPX) and the Average Single-Stock Implied Volatility (like companies that are part of the SP 500)

$$DispersionP/L = (Index\ IV - Avg.\ Stock\ IV) \times Correlation$$

Where, Index IV < Avg. Stock IV because of diversification benefits. This strategy works because of market inefficiency, meaning that options markets usually overestimate stock correlations during calm periods and that single-stock options demand inflates their implied volatility.

Risks this strategy has are:

1. **Correlation Risk** : Systemic events (e.g., Fed meetings) spike correlations.
2. **Liquidity Mismatch** : Single-stock options less liquid than indexes.
3. **Gamma Overhead** : Managing 500+ delta hedges is operationally intense. This strategy also uses quantitative tools like forecast dispersion P&L , which uses Dispersion = $f(\sigma_{index}, \sigma_i, \rho_{ij}, \omega_i)$ and Principal Component Analysis (PCA) to identify dominant correlation drivers.

Gamma Scalping

Gamma scalping is a delta-hedging strategy, mainly used to profit from the Gamma of their options positions and from realised volatility exceeding implied volatility. The P&L of deltahedged options position is approximately :

$$\text{P\&L} \approx \underbrace{\Theta \cdot \Delta t}_{\text{Time decay}} + \underbrace{\frac{1}{2}\Gamma \cdot (\Delta S)^2}_{\text{Gamma P\&L}} + \underbrace{\nu \cdot \Delta\sigma}_{\text{Vega impact}}$$

where,

- Γ = Option convexity (change in delta per $1 move)
- Θ = Time decay per day
- ΔS = Underlying price change

Profit Drivers

- *Volatility Over-realization*: When σrealized > σimplied
- *Autocorrelation Breakdown*: Trending markets enhance gamma profits
- *Liquidity Asymmetry*: Exploiting bid-ask spreads during rebalancing

Risk Considerations

<u>1. *Gamma Risk (Convexity Risk)*</u>

Gamma measures how quickly delta changes as the underlying moves. High gamma means frequent rebalancing is needed.

<u>2. *Theta Decay (Time Risk)*</u>

Gamma scalping profits rely on selling options (negative theta) and hedging dynamically.

<u>3. *Volatility Mispricing (Vega Risk)*</u>

Gamma scalping assumes implied vol (IV) > realized vol (RV).

<u>4. *Liquidity & Slippage*</u>

Frequent delta hedging requires liquid markets.

<u>5. *Dividend & Corporate*</u>

Action Risk Unexpected dividends or stock splits alter option pricing.

<u>6. *Pin Risk (Expiration Risk)*</u>

Near expiration, delta becomes unstable if the underlying is near the strike.

Stochastic Volatility (Heston Model)

As mentioned in the introduction, the Heston model is a mathematical model that describes the evolution of the volatility of an underlying asset. Since it is a stochastic volatility model, it assumes that the volatility of the asset follows random paths. The model also assumes that S_t (asset price) is extracted from a stochastic process,

$$dS_t = \mu S_t dt + \sqrt{\nu_t} S_t dW_t^1$$

where √vt (volatility) given by a Feller square-root

$$d\nu_t = \kappa(\theta - \nu_t)dt + \xi\sqrt{\nu_t}dW_t^2$$

and W^S_t, W^v_t are Wiener processes (a.k.a. continuous random walks) with correlation ρ. The parameters are :

- v_0 : the initial variance.
- θ : the long-run average variance of the price; as t tends to infinity, the expected value of νt tends to θ.
- ρ : the correlation of the two Wiener processes.
- κ : the rate at which νt reverts to θ.
- ξ : the volatility of the volatility, or 'vol of vol', which determines the variance of νt .

The Heston model is without a doubt one of the most useful models when it comes to Volatility Trading due to it's stochastic processes

Local Volatility (Dupire's Model)

Dupire's model, is an option pricing model that treats volatility as a function of asset level and time. As we mentioned before, it is a generalisation of the BS model, but this time the volatility is not constant. Instead of random paths here the volatility is following a stochastic differential equation,

$$dS_t = (r_t - d_t)S_t d_t + \sigma_t S_t dW_t$$

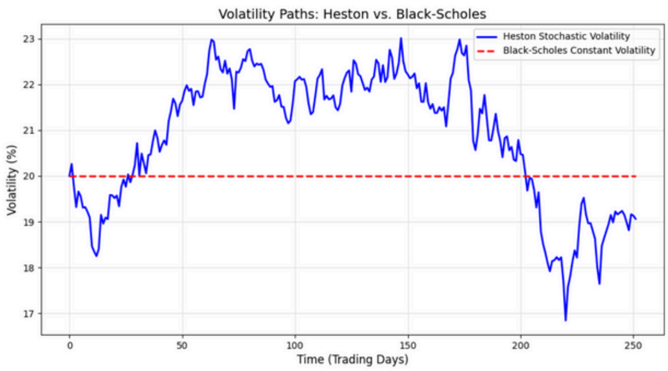
where,

1. Drift $(r_t - d_t)$:
   - r_t = risk-free rate
   - d_t = dividend yield
2. Diffusion $\sigma_t dW_t$:
   - σ_t = instantaneous volatility
   - W_t = Brownian motion (randomness driver)

Volatility Modeling Approaches

Volatility (σ_t) can be modeled in three main ways:

1. **Black-Scholes**: Assumes volatility is constant, making pricing tractable but unrealistic for most markets.
2. **Local Volatility**: Treats volatility as a deterministic function of the asset price and time.
3. **Stochastic Volatility**: Models volatility as a random process (e.g., mean-reverting) driven by its own noise.



Volatility Paths of the Black-Scholes model and the Heston model: on the graph we can see how dynamic the volatility is in the Heston model (blue), while the BS(red) is a constant function. This is the reason why the Heston model is much more flexible than the BS.