Options: Fundamentals and PnL at Maturity

Derivatives and Fixed Income Class

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Introduction to Options

- Forwards/Futures = *obligations* for both sides.
- Options = *rights without obligation* for the buyer.
- Asymmetry $\Rightarrow$ flexibility but higher complexity.
- Focus today: **payoff at maturity**.

Options differ from forwards and futures in one key respect: buyers have rights, not obligations. This asymmetry is powerful: it means you can walk away if the trade is unfavorable. For now, we only study the payoff at maturity — the cleanest way to see who gains or loses. Later, we will extend this to valuation before maturity, which requires considering time, volatility, and other factors.

Option Definitions

- **Call option**: right to *buy* at strike K at time T.
- **Put option**: right to *sell* at strike K at time T.
- Buyer pays a **premium** upfront.
- **American option**: exercisable any time until T.
- **European option**: exercisable only at T.

Calls and puts are the building blocks. Both require paying a premium, which is the cost of flexibility. American-style options let you act early; European-style options keep you waiting until expiry. Analogies help: a call is like paying a reservation fee for concert tickets, while a put is like insurance on your car. Both grant choices, not obligations.

Call Options: Payoff and P&L

$$\text{Payoff} = \max(S_T - K, 0), \quad \text{P\&L} = \max(S_T - K, 0) - C$$

- **Long Call:** right to buy. Unlimited upside, max loss = premium.
- **Short Call:** obligation to sell. Max gain = premium, unlimited downside.

Buying a call is bullish: you profit if the underlying rises. If it does not, the option expires worthless, and you only lose the premium. Selling a call is the mirror image: you collect the premium, but risk unlimited losses if prices surge. This asymmetry makes short calls dangerous.

Call Options: Break-even

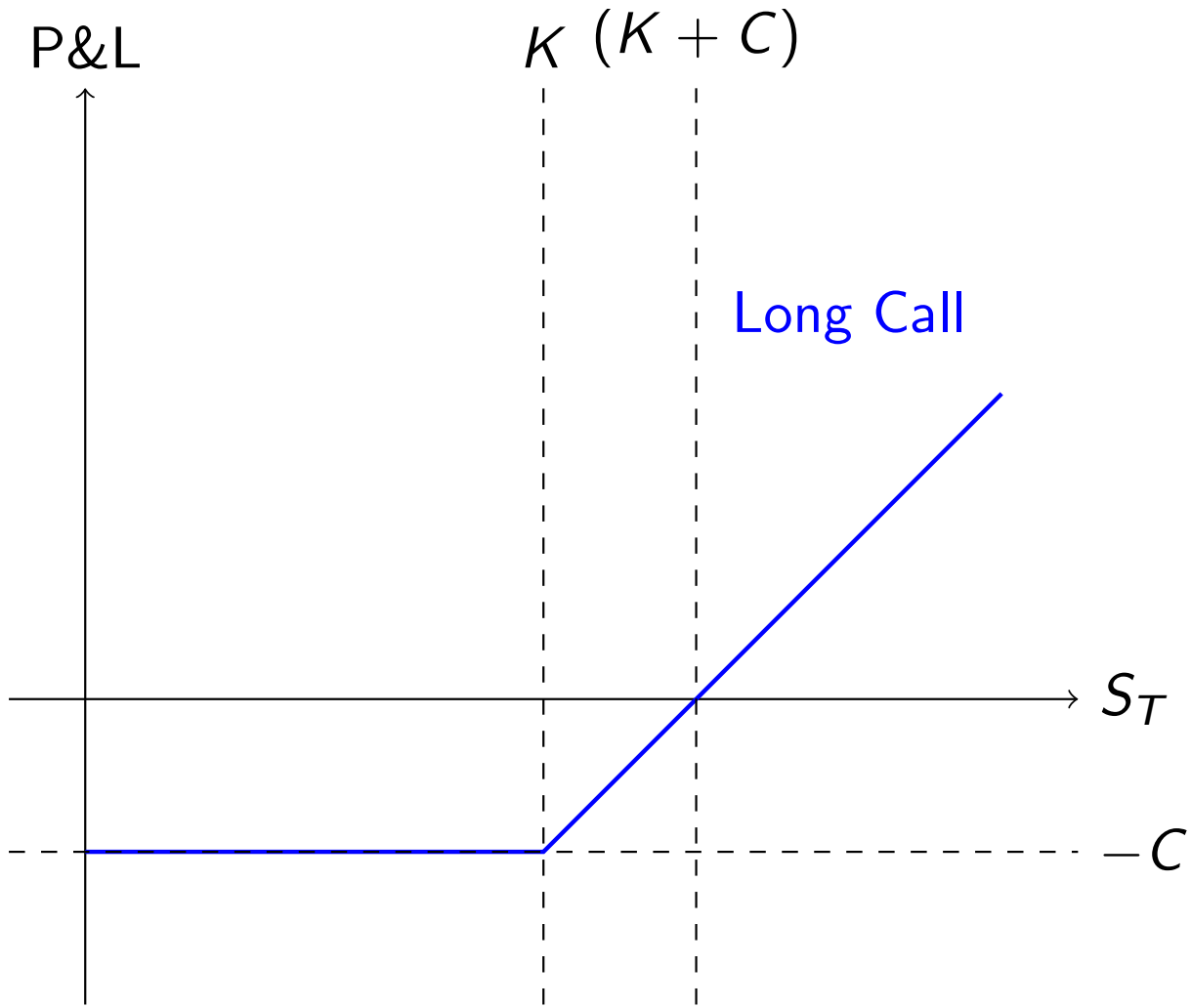
$$\text{Break-even: } S_T = K + C$$

Worked Example: $S_0 = 50$, strike $K = 55$, premium $C = 2$.

- If $S_T = 70$: payoff $= 15$, P&L $= 13$.
- If $S_T = 52$: payoff $= 0$, P&L $= -2$.

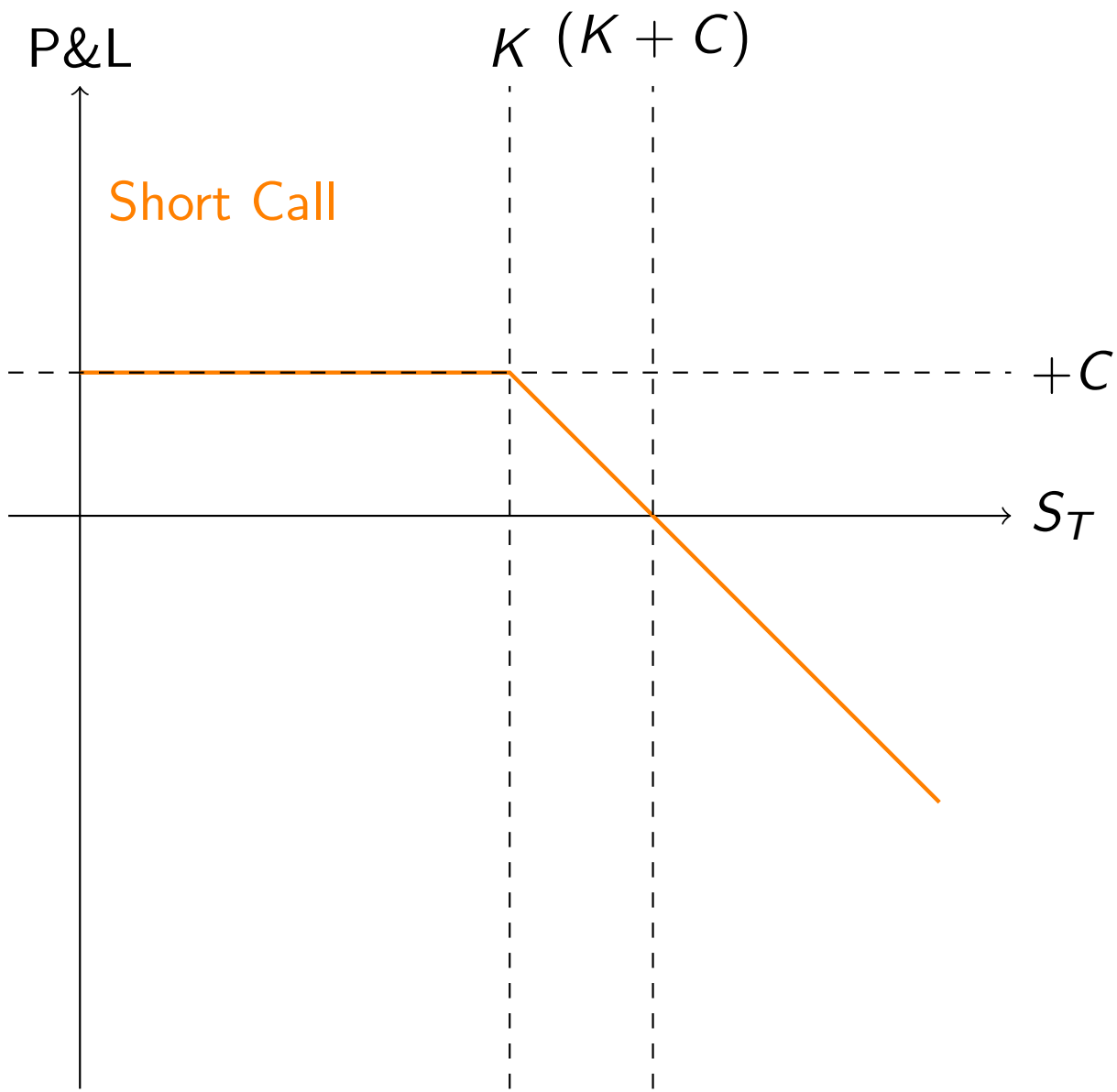
The break-even tells us where the call buyer's profits offset the initial premium. In the example, you need the stock to rise above 57 before the position becomes profitable. Below that, the premium creates a net loss.

Long Call: Payoff Diagram



The line is flat at $-C$ until the strike, showing that the premium is the only loss if the option expires worthless. Past $K + C$, the call turns profitable and rises without limit. This makes the long call attractive for bullish investors seeking leveraged upside with limited downside.

Short Call: Payoff Diagram



Put Options: Payoff and P&L

$$\text{Payoff} = \max(K - S_T, 0), \quad \text{P\&L} = \max(K - S_T, 0) - P$$

- **Long Put:** right to sell. Max profit $= K - P$, max loss $= P$.
- **Short Put:** obligation to buy. Max gain $= P$, loss if $S_T \to 0$.

A long put is bearish: like buying insurance, it pays off if prices fall. A short put is bullish: you profit from collecting the premium if the price holds up, but you risk losses if the market collapses.

Put Options: Break-even & Example

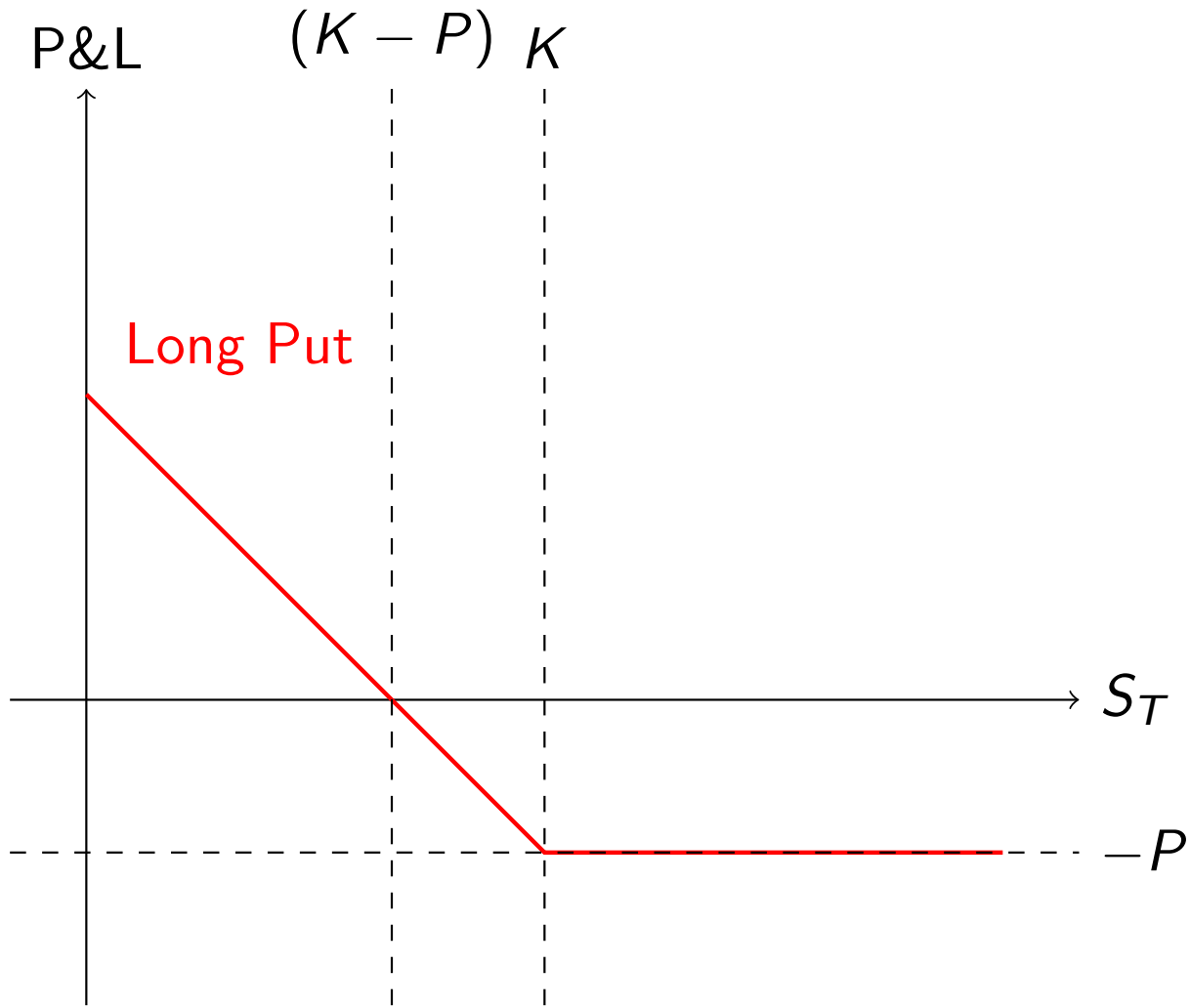
$$\text{Break-even: } S_T = K - P$$

Worked Example: $S_0 = 50$, strike $K = 45$, premium $P = 3$.

- If $S_T = 30$: payoff $= 15$, P&L $= 12$.
- If $S_T = 48$: payoff $= 0$, P&L $= -3$.

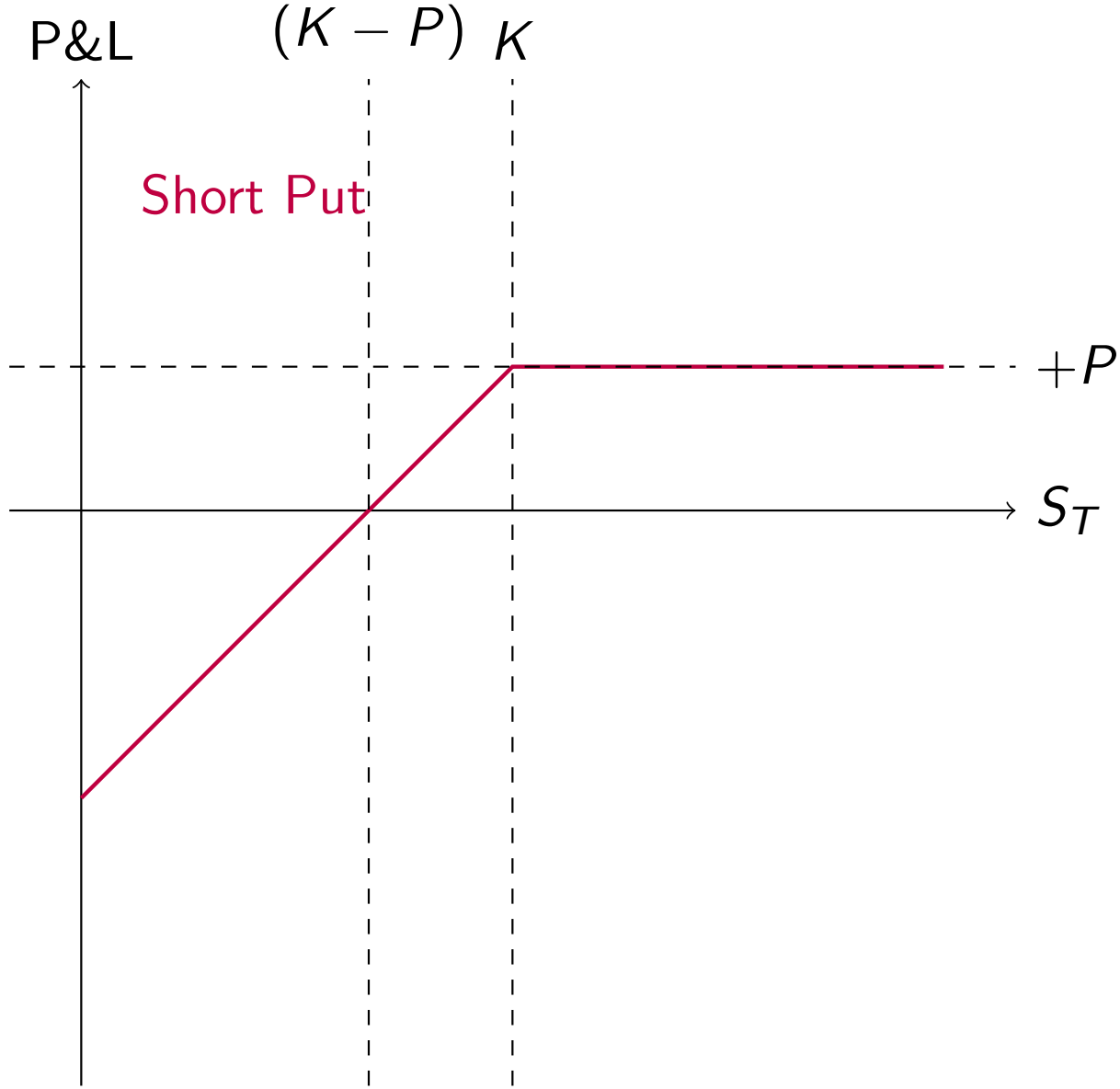
Just like with calls, the premium shifts the break-even point. For this put, the position only profits when the asset falls below 42 ($K - P$). Otherwise, the buyer loses the premium.

Long Put: Payoff Diagram



Above the strike, the option expires worthless, leaving a flat loss equal to the premium. Below K, the put gains value as the asset falls, until profits cap at $K - P$ (if $S_T = 0$). This makes the long put a natural hedge against falling markets.

Short Put: Payoff Diagram



Comparative Payoff Structures

	Long Call	Short Call	Long Put	Short Put
Right/Obligation	Right to Buy	Obligation to Sell	Right to Sell	Obligation to Buy
Profit Potential	Unlimited	Premium Only	Limited $(K - P)$	Premium Only
Loss Potential	Premium	Unlimited	Premium	Large (to $S = 0$)
Break-even	$K + C$	$K + C$	$K - P$	$K - P$

This table captures the symmetry: longs risk a small, known premium for asymmetric payoffs, while shorts collect a small, fixed premium but face large risks. Calls give unlimited upside to the buyer but unlimited downside to the seller; puts offer limited gains but still significant risks for the seller.

Determinants of Option Prices (Preview)

- Underlying price S.
- Strike price K.
- Time to maturity T.
- Volatility of underlying.
- Risk-free interest rate r.
- Dividends.

Options don't have arbitrary prices — they are shaped by a handful of key factors. Higher volatility increases the chance of large movements, raising option value. More time to maturity increases optionality. Interest rates and dividends tilt the balance between calls and puts. These drivers will be central to option pricing models.

Recap: Key Takeaways

- Options = rights without obligations (asymmetry).
- Calls = rights to buy; puts = rights to sell.
- Long = buyer of right (pays premium, limited loss).
- Short = seller of right (collects premium, faces risk).
- Break-evens: $K + C$ for calls, $K - P$ for puts.
- Payoff diagrams show the asymmetry clearly.

The essence of options is choice. Buyers trade limited, known losses (the premium) for asymmetric exposure. Sellers earn income but accept substantial risk. Diagrams and break-evens provide intuition, and later lectures will extend from payoffs-at-maturity to full option valuation.

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