Derivatives and Fixed Income — Lecture Notes

Programme Grande Ecole - Master 1 (PGE M1)
Quantitative Finance Track
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Options: Fundamentals and Payoff Structures at Maturity

1. Introduction

In this lecture, we introduced the foundations of **options**, a core class of derivative instruments.

Whereas forwards and futures constitute *obligations* for both the buyer and the seller of such a contract, options are *rights without obligation* for the buyer of the option. This asymmetry is what makes them versatile and more complex instruments.

Important note: For now we only study the *payoff at maturity* of an option. The value of an option *during its life before expiry* is more complex, since it depends also on other variables such as time, volatility, and interest rates. We will return to this in later lectures.

2. Definitions

- A **Call option** gives the holder the right to *buy* the underlying asset at the strike price K at maturity T.
- A **Put option** gives the holder the right to *sell* the underlying asset at the strike price K at maturity T.
- Options require the payment of a **premium** (the upfront cost of buying the option).
- **American option:** can be exercised at any time before maturity.
- **European option:** can only be exercised at maturity.

Explanation. Calls and puts are the two building blocks of options. A call is a right to buy, while a put is a right to sell. Importantly, both rights are paid for through the premium. This premium reflects the cost of having flexibility. American and European options differ only in their exercise style: American can be used anytime before expiry, while European options lock you in until the end.

Analogy (used in class): A call option is like a concert ticket reservation: you pay a fee today to secure the right to buy later. A put option is like car insurance: you pay a premium and only exercise if something bad happens. These analogies highlight that options provide *choices*, not obligations.

3. Call Options

Payoff and P&L formulas.

$$\text{Value of Call} = \max(S_T - K, 0)$$

$$\text{P\&L of Call} = \max(S_T - K, 0) - C$$

where S_T = price at maturity, K = strike price, C = call premium.

Properties.

- **Long Call:** right to buy. Unlimited upside, maximum loss = premium.
- **Short Call:** obligation to sell. Maximum gain = premium, unlimited downside.

Explanation. Buying a call is a bullish strategy: you believe the asset price will rise. The worst case is you lose the premium if the price stays below K. Selling (writing) a call is the opposite: you earn the premium upfront, but you expose yourself to potentially unlimited losses if the price skyrockets.

Break-even.

$$S_T = K + C$$

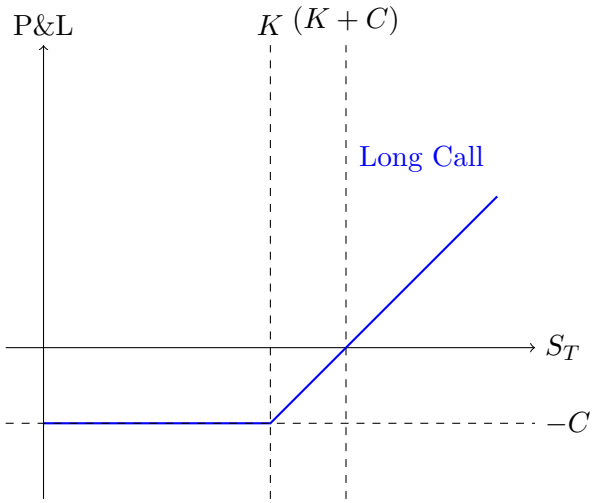
The break-even shows the price level at which the call buyer neither gains nor loses, after accounting for the premium.

Worked Example. Suppose today the stock is \$50. You buy a call with strike $K = 55$ for a premium of $C = 2$.

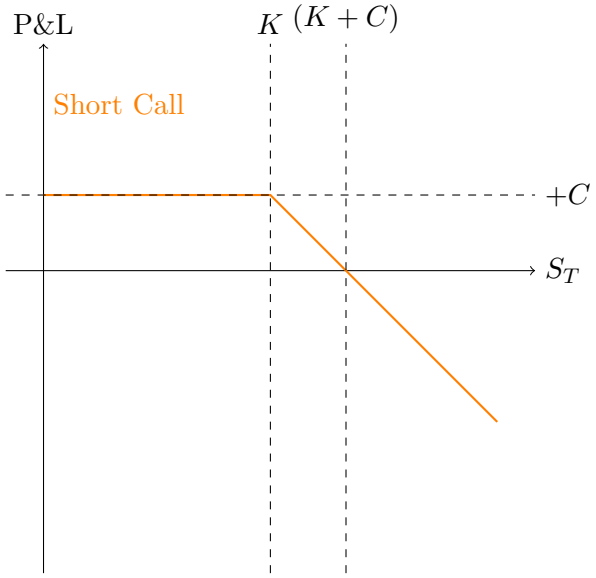
- If $S_T = 70$: payoff = 15, P&L = 13.
- If $S_T = 52$: payoff = 0, P&L = -2.

This illustrates the asymmetric profile: limited loss, unlimited upside.

Payoff diagrams.



Explanation of the Long Call Diagram. The blue line represents the profit and loss of buying a call option. To the left of the strike price K, the line is flat at $-C$, meaning that if the option expires out of the money the buyer simply loses the premium paid. At the strike price K, the option starts to gain value as the underlying rises. The break-even point is at $K + C$, where the profits from exercising the option exactly offset the premium. Beyond this level, the line slopes upward without limit, showing that a long call has *unlimited upside potential* while its downside is strictly limited to the premium C.



Explanation of the Short Call Diagram. The orange line shows the profit and loss profile of writing (selling) a call option. To the left of the strike price K, the option expires worthless for the buyer, so the seller keeps the premium $+C$ as profit. At the strike price, the payoff begins to decline because the seller is now obliged to deliver the underlying below market value. The break-even point is $K + C$, where the premium received is exactly offset by the losses from the rising underlying. Beyond this level, the line slopes downward without bound, illustrating that a short call has a *limited maximum gain* equal to the premium, but *unlimited potential losses* if the underlying price continues to rise.

4. Put Options

Payoff and P&L formulas.

$$\text{Value of Put} = \max(K - S_T, 0)$$

$$\text{P\&L of Put} = \max(K - S_T, 0) - P$$

Properties.

- **Long Put:** right to sell. Max profit $= K - P$, max loss = premium.
- **Short Put:** obligation to buy. Max profit = premium, downside limited if asset falls to zero.

Explanation. Buying a put is a bearish strategy: you expect the price to fall. The put acts like insurance. Selling a put is bullish: you collect the premium but must buy if the asset crashes, which exposes you to significant downside.

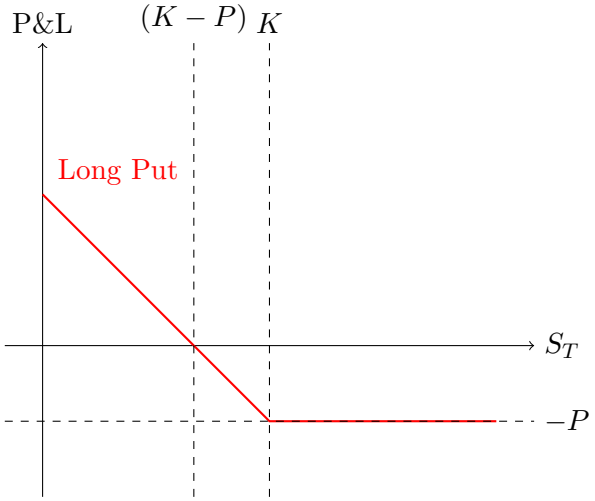
Break-even.

$$S_T = K - P$$

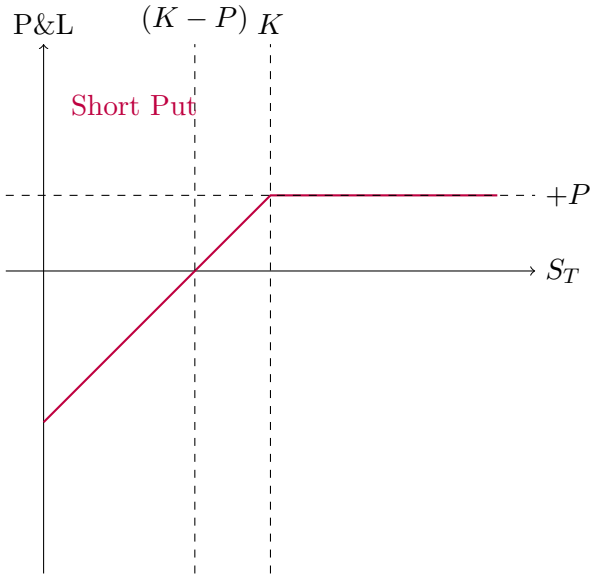
Worked Example. Suppose today the stock is \$50. You buy a put with strike $K = 45$ for a premium of $P = 3$.

- If $S_T = 30$: payoff = 15, P&L = 12.
- If $S_T = 48$: payoff = 0, P&L = -3.

Payoff diagrams.



Explanation of the Long Put Diagram. The red line represents the profit and loss of buying a put option. When the underlying price is above the strike K, the option expires worthless, and the buyer's loss is limited to the premium P. As the price falls below K, the put gains value because the holder can sell the asset at the higher strike price. The break-even point is $K - P$, where the gains from exercising exactly offset the premium. If the underlying price were to fall all the way to zero, the maximum profit would be $K - P$. This payoff profile shows that a long put is a *bearish strategy*: it benefits from falling prices, with limited loss (the premium) and capped upside.



Explanation of the Short Put Diagram. The purple line shows the profit and loss of selling a put option. If the underlying price stays above the strike K, the option expires worthless and the seller keeps the premium $+P$ as profit. As the price falls below K, the seller must buy the asset at the strike price, even though it is worth less in the market. The break-even point is $K - P$, where the premium received is exactly offset by losses. If the underlying were to fall all the way to zero, the maximum loss would be $K - P$. This payoff profile shows that a short put is a *bullish strategy*: it earns steady income if the price holds or rises, but it exposes the seller to potentially large losses if the asset collapses.

5. Comparative Payoff Structures

Summary Table.

	Long Call	Short Call	Long Put	Short Put
Right/Obligation	Right to Buy	Obligation to Sell	Right to Sell	Obligation to Buy
Profit Potential	Unlimited	Premium Only	Limited $(K - P)$	Premium Only
Loss Potential	Premium	Unlimited	Premium	Large (to $S = 0$)
Break-even	$K + C$	$K + C$	$K - P$	$K - P$

Explanation. This table condenses the essence of option strategies. The symmetry is striking: what the long side gains, the short side loses, and vice versa. Long positions sacrifice a limited premium for asymmetric payoffs, while short positions gain a fixed premium but risk substantial losses.

6. Determinants of Option Prices (Preview)

- Price of underlying asset (S).
- Strike price (K).
- Time to maturity (T).
- Volatility of the underlying asset.
- Risk-free interest rate (r).

- Dividends (if any).

Explanation. Option prices are not arbitrary; they depend on a small set of drivers. If the underlying asset is volatile, the option is more valuable. If there is more time to maturity, the option is worth more, since there is more time for favorable movements. Interest rates and dividends also play subtle roles, which we will analyze later.

7. Key Takeaways

Summary

- We are studying only *payoffs at maturity*. The valuation of options during their life is more complex.
- Calls = rights to buy, Puts = rights to sell.
- Long call = right to buy; Short call = obligation to sell.
- Long put = right to sell; Short put = obligation to buy.
- Long positions have limited losses (the premium) and potential for substantial profit.
- Short positions earn the premium but face significant downside risks.
- A long call has **unlimited upside**, while a short call has **unlimited downside**.
- A long put has **limited (but potentially large) upside**, while a short put has **limited (but potentially large) downside**.
- Break-even points: Long Call = $K + C$, Long Put = $K - P$.
- Options are best understood through analogies: calls as ticket reservations, puts as insurance.

	Long Call	**Short Call**	**Long Put**	**Short Put**
Right / Obligation	Right to Buy	Obligation to Sell	Right to Sell	Obligation to Buy
Maximum Gain	Unlimited Upside	Premium Only	Limited (up to $K - P$)	Premium Only
Maximum Loss	Premium	Unlimited Downside	Premium	Limited (down to $K - P$)
Break-even	$K + C$	$K + C$	$K - P$	$K - P$

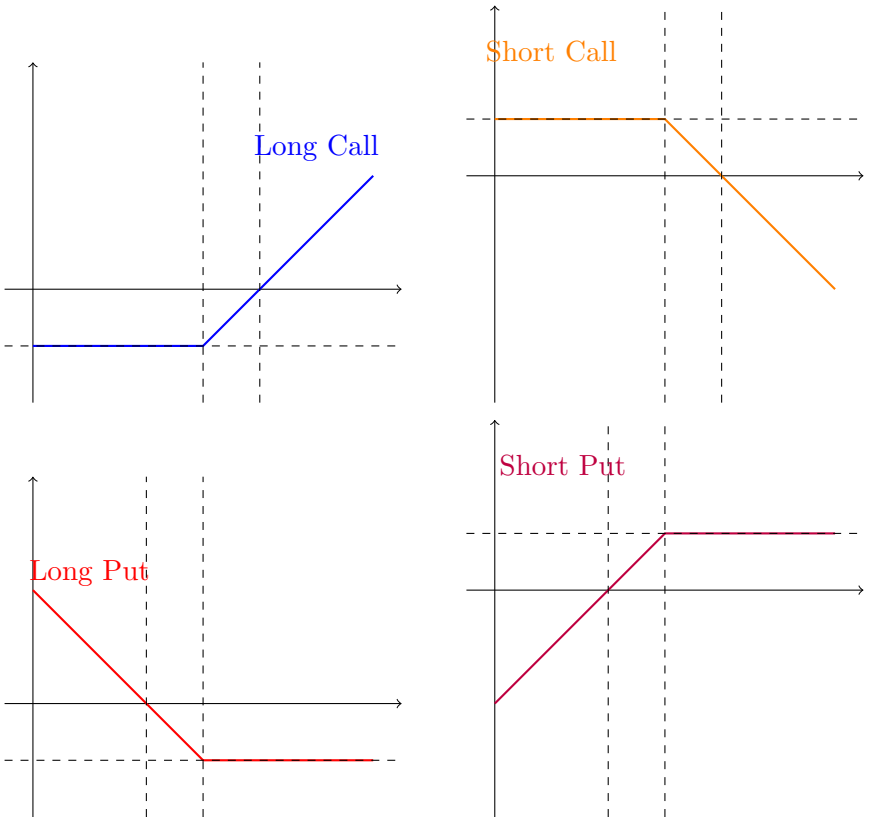


Figure 1: Payoff diagrams at maturity for long and short calls and puts.