MEMS 5501 Mechanics of Continua
Pathak

Homework 6

(*Due Monday, Nov 4, 6pm*)

1. A cylinder is made up of an isotropic linear elastic solid material, and is subjected to a strain:

$$E_{rr} = E_{\theta\theta} = a\sin\theta$$
$$E_{r\theta} = \frac{a\cos\theta}{2}$$
$$E_{\theta z} = E_{rz} = E_{zz} = 0$$

where E_{ij} are the components of the infinitesimal strain tensor, and a is an arbitrary constant. Calculate the traction vector on the lateral boundary surface of the cylinder, in cylindrical coordinates.
[5 points]

2. A displacement field in an isotropic linear elastic material is given by:

$$\underline{u} = \alpha\left(x_1^2 - 5x_2^2\right)\underline{e}_1 + \left(2\alpha x_1 x_2\right)\underline{e}_2$$

   i. Obtain the infinitesimal strain tensor. [2 points]
   ii. Obtain the principal strains. [2 points]
   iii. If the shear modulus of the material is known, find the Young's modulus (in terms of shear modulus *G*) that will ensure equilibrium at any point. [7 points]

3. Consider a slab made of an isotropic linear elastic material. At a given point in the slab, the volumetric deformation (dilatation) is $\Delta = -2\times10^{-3}$, the shear deformation is $E_{12} = -\sqrt{3}\times10^{-3}$ and the normal strain is $E_{11} = 0$. The material slab is subjected to a state of plane strain (in $x_1 - x_2$ plane).
   i. Obtain the infinitesimal strain tensor and the principal strains. [3 points]
   ii. Assuming that the elastic constants are Young's modulus $E = 50MPa$ and Poisson's ratio $\nu = 0.25$, obtain the stress tensor components and principal stresses. [3 points]
   iii. Obtain the strain energy density (*W*). [3 points]

Note: Assume small deformations and theory of linear elasticity for all problems.