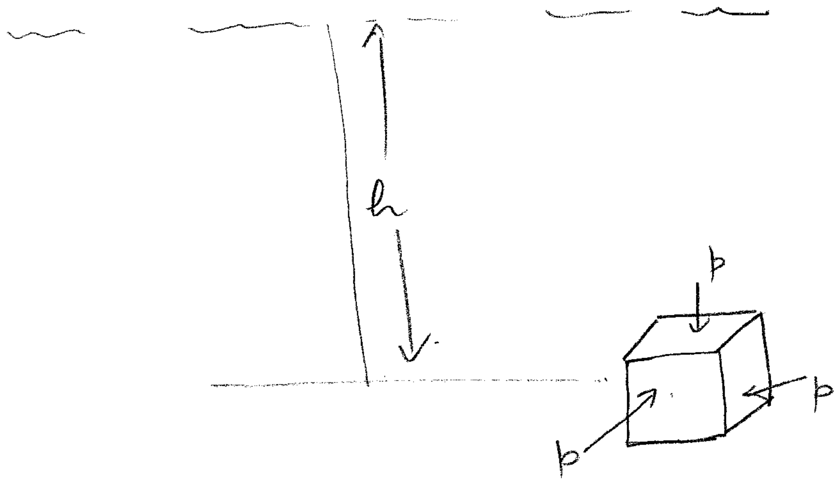
HW 7 - Answers

1

A cube deep at sea



cube dimensions: $a \times a \times a$ outside of the water.

Isotropic linear elastic material

E, ν are known.

density of seawater: ρ

Find the change in the volume of the cube after it settles at the bottom of the sea.

solution

Hydrostatic pressure at depth 'h':

$$p = \rho g h$$

$$\underset{\sim}{T} = -p \underset{\sim}{I}$$

$$\Rightarrow \quad T_{ij} = \begin{bmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{bmatrix}$$

From. $\underset{\sim}{T} = \lambda(\mathrm{tr}\,\underset{\sim}{E})\underset{\sim}{I} + 2\mu\underset{\sim}{E}$,

$$E_{11} = E_{22} = E_{33} = \frac{1}{E}\left(T_{11} - \nu(T_{22} + T_{33})\right)$$

$$= \frac{p}{E}(2\nu - 1)$$

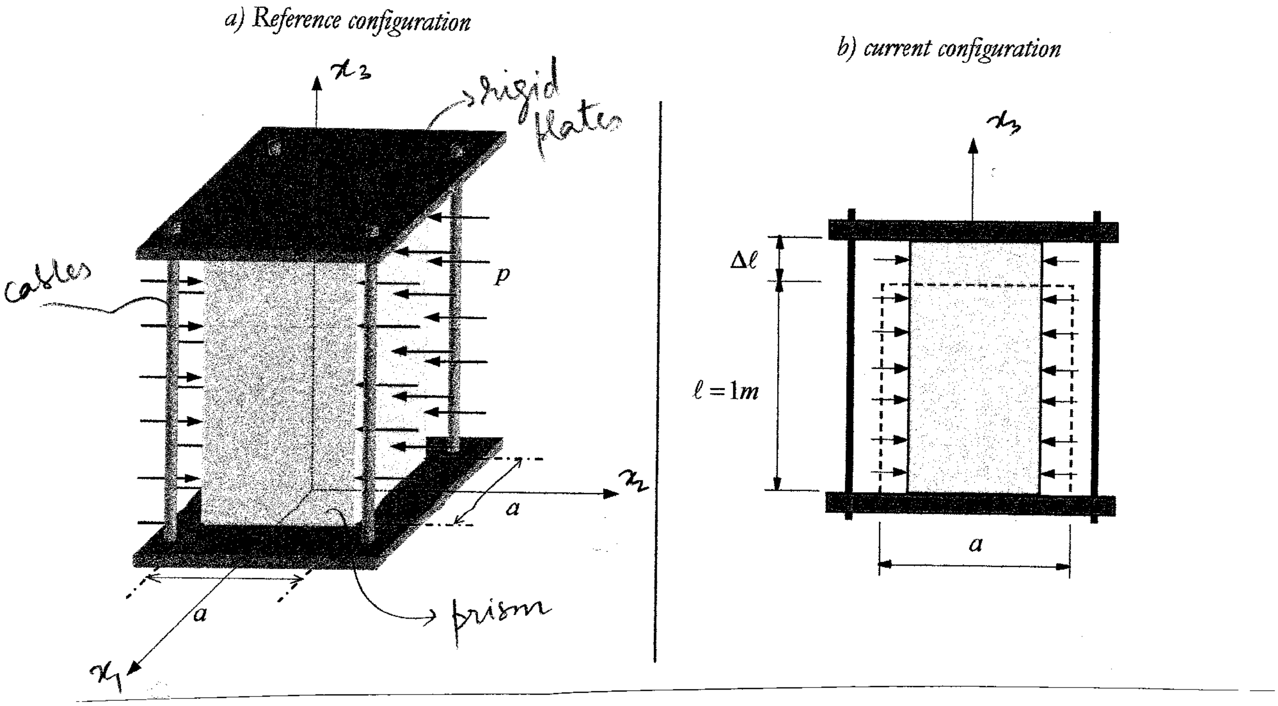
change in volume $\equiv \Delta V$

$$\frac{\Delta V}{V} = \mathrm{tr}(\underset{\sim}{E})$$

$$\Rightarrow \boxed{\Delta V = \frac{3p}{E}(2\nu - 1)\,a^3}$$

2

Prismatic block under compression held by cables



Both the prismatic block and the cables are made of isotropic linear elastic materials of known E_p, E_c, ν.

Find:

- ⟶ Stress on the cable: σ_c
- ⟶ Stress field in the prism
- ⟶ Change in volume of the prism.

In the x_3-direction,

Strain, $E^{p}_{33} = E^{c}_{33}$ $\rightarrow$ due to boundary condition

On the cable,

$$T^{c}_{33} = \sigma_c = E_c E^{c}_{33}$$

$$\Rightarrow \quad E^{c}_{33} = \frac{\sigma_c}{E_c}$$

Stress field in the prism:

A_c: area of cross-section of cables

$$\underset{\sim}{T}^{p} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & \dfrac{-4\sigma_c A_c}{a^2} \end{bmatrix}$$

T^{p}_{33} stress on the prism $\equiv$ force supported by the cables

Strain in the prism in x_3-direction:

$$E^{p}_{33} = \frac{1}{E_p}\left(T^{p}_{33} - \nu\left(T^{p}_{11} + T^{p}_{22}\right)\right)$$

$$E^{p}_{33} = \frac{1}{E_p}\left(-\frac{4\sigma_c A_c}{a^2} + \nu p\right)$$

Applying:

$$E_{33}^{p} = E_{33}^{c}$$

$$\Rightarrow \quad \frac{1}{E_p}\left(-\frac{4\sigma_c A_c}{a^2} + \nu p\right) = \frac{\sigma_c}{E_c}$$

$$\Rightarrow \quad \sigma_c = \frac{\nu E_c p a^2}{\left(E_p a^2 + 4 E_c A_c\right)}$$

And, $$T_{33}^{p} = -\frac{4\nu E_c p A_c}{\left(E_p a^2 + 4 A_c E_c\right)}$$ $\Rightarrow$ stress field in the prism.

$$\Delta V = tr(\underset{\sim}{E}^{p})\, V_0^{p}$$

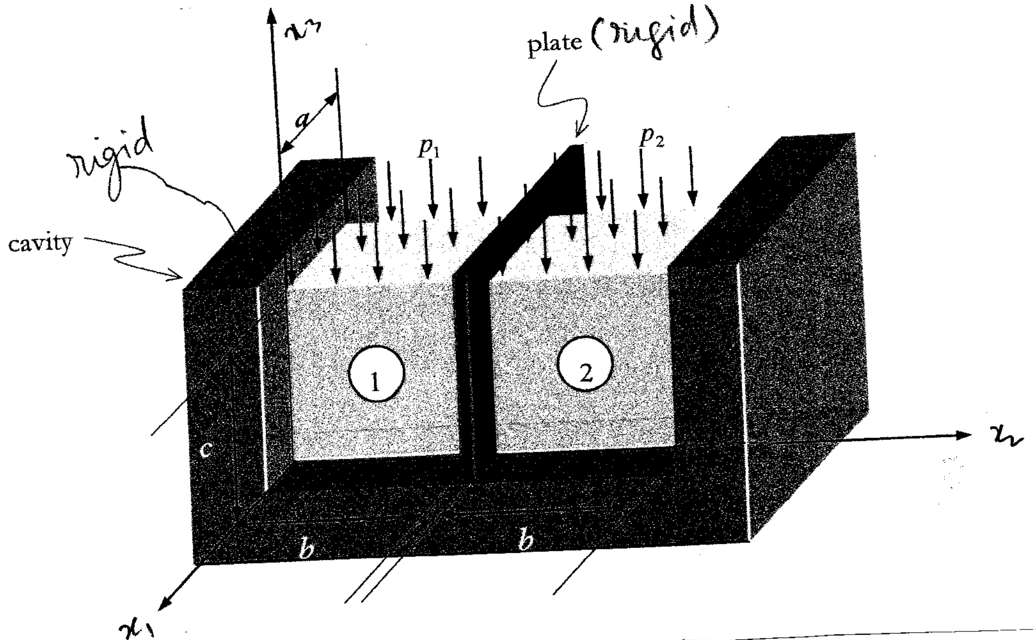
$$\Delta V = tr(\underset{\sim}{T}^{p})\, \frac{(1-2\nu)}{E_p}\, V_0^{p}$$

where, $V_0^{p} = l a^2$

and $$tr(\underset{\sim}{T}^{p}) = -p - \frac{4\nu E_c p A_c}{\left(E_p a^2 + 4 A_c E_c\right)}$$

(3)

Material blocks in rigid cavity under pressure, separated by a rigid plate:



Both material blocks ① & ② are made of the same material: E, ν. Assume 'no shear'.

Find:

→ ① stress field in both material blocks

→ ② Change in edge lengths

solution

For block ①: $T_{11}^{(1)} = 0, \quad T_{33}^{(1)} = -p_1$

For block ②: $T_{11}^{(2)} = 0, \quad T_{33}^{(2)} = -p_2$

At the interface, compatibility ⇒

$$T_{22}^{(1)} = T_{22}^{(2)} = T_{22}$$

also, due to rigid cavity,

$$E_{22}^{(1)} + E_{22}^{(2)} = 0$$

$$\Rightarrow \frac{1}{E}\left[T_{22} - \nu\left(T_{11}^{(1)} + T_{33}^{(1)}\right)\right] + \frac{1}{E}\left[T_{22} - \nu\left(T_{11}^{(2)} + T_{33}^{(2)}\right)\right] = 0$$

$$\Rightarrow \quad 2T_{22} - \nu\left(T_{33}^{(1)} + T_{33}^{(2)}\right) = 0$$

$$\Rightarrow \quad T_{22} = \frac{-\nu(p_1 + p_2)}{2}$$

Thus, for block ①

$$\underset{\sim}{T}^{①} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{-\nu(p_1+p_2)}{2} & 0 \\ 0 & 0 & -p_1 \end{bmatrix}$$

For block ②

$$\underset{\sim}{T}^{(2)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{-\nu(p_1+p_2)}{2} & 0 \\ 0 & 0 & -p_2 \end{bmatrix}$$

Strains in ①:

$$E_{11}^{1} = \frac{1}{E}\left[T_{11}^{1} - \nu\left(T_{22}^{1} + T_{33}^{1}\right)\right] = \frac{\nu}{2E}\left(\nu(p_1+p_2) + 2p_1\right)$$

$$E_{22}^{2} = \frac{1}{E}\left[T_{22}^{1} - \nu\left(T_{11}^{1} + T_{33}^{1}\right)\right] = \frac{\nu}{2E}\left(p_1 - p_2\right)$$

$$E_{33}^{3} = \frac{1}{E}\left[T_{33}^{1} - \nu\left(T_{11}^{1} + T_{22}^{1}\right)\right] = \frac{1}{2E}\left(\nu^2(p_1+p_2) - 2p_1\right)$$

For block ②:

$$E_{11}^{(2)} = \frac{1}{E}\left[T_{11}^{(2)} - \nu\left(T_{22}^{(2)} + T_{33}^{(2)}\right)\right] = \frac{\nu}{2E}\left[\nu(p_1+p_2)+2p_2\right]$$

$$E_{22}^{(2)} = \frac{1}{E}\left[T_{22}^{(2)} - \nu\left(T_{11}^{(2)} + T_{33}^{(2)}\right)\right] = \frac{\nu}{2E}\,(p_2 - p_1)$$

$$E_{33}^{(2)} = \frac{1}{E}\left[T_{33}^{(2)} - \nu\left(T_{11}^{(2)} + T_{22}^{(2)}\right)\right] = \frac{\nu}{E}\left[\nu^2(p_1+p_2) - 2p_2\right]$$

Change in edge lengths:-

For block ①:	For block ②:
$\Delta a^{(1)} = a\,E_{11}^{(1)}$	$\Delta a^{(2)} = a\,E_{11}^{(2)}$
$\Delta b^{(1)} = b\,E_{22}^{(1)}$	$\Delta b^{(2)} = b\,E_{22}^{(2)}$
$\Delta c^{(1)} = c\,E_{33}^{(1)}$	$\Delta c^{(2)} = c\,E_{33}^{(3)}$