Appendix A

Tensors and Tensor Algebra

At the time of this writing, the majority of applications of FEA and IGA involve structural analysis. In the past, the theory of structural analysis always made me nervous because of its reliance on *tensors*. I had to study electromagnetic tensors in graduate school, so I came to associate tensors with relativity and multi-dimensional electromagnetism.

Relativity still scares me, but the tensors in this book are much simpler than the ones I had to grapple with in grad school. As this appendix will make clear, tensors are fairly easy to understand and work with. In fact, you've probably already used tensors in your professional or academic work.

This chapter starts by explaining what tensors are, and then discusses tensor notation and operations. I'll discuss arithmetic operations like X and Y, but in the interest of preserving your sanity, I won't present any tensor calculus.

A.1 What are Tensors?

If you've worked with image processing or graphical rendering, you're familiar with vectors and matrices. A vector contains its data in a one-dimensional structure and a matrix stores data in a two-dimensional structure. In graphical rendering, vectors frequently contain the coordinates of a point or the components of a color, and matrices frequently represent transformations of a point in space.

If you're already familiar with vectors and matrices, then you'll be happy to know that they're both simply types of tensors. That is, a tensor is a generalization of matrices, vectors, and similar structures. But while vectors are one-dimensional and matrices are always two-dimensional, a tensor can have zero or more dimensions.

I'm not going to present the formal definition used by physicists and mathematicians. Instead, because this is (mostly) a programming book, I'm going to treat tensors as data structures: *a tensor is an array of elements containing zero or more dimensions*.

If you're familiar with object-oriented programming, you can think of a tensor as a superclass of the matrix class and vector class. The tensor class has a set of operations that can be performed on all tensors, including matrices and vectors. But matrices and vectors also have operations that aren't available to other tensors.