MEMS 5501 Mechanics of Continua
Pathak

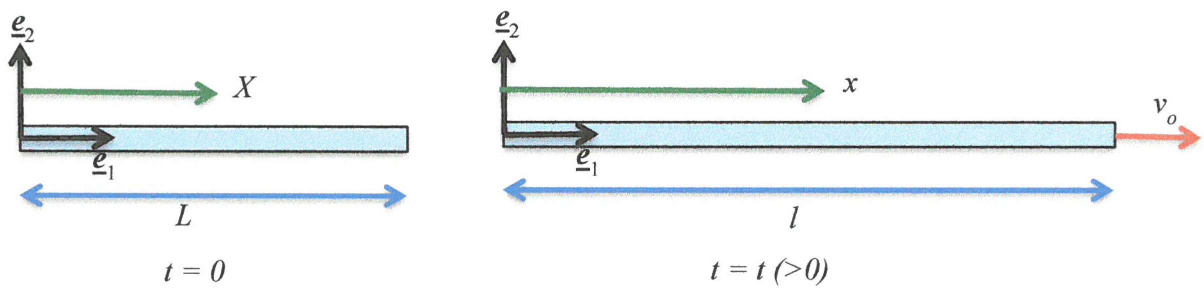
Your Name: Answers

Mid-term Examination

October 11, 2017

[Total: 28 points]

1. A rubber band has initial length L. One end of the band is held fixed. For time $t > 0$, the other end is pulled at constant speed v_o. Let X denote position on the band in the reference configuration. And, let x denote position in the deformed configuration. Assume one dimensional, homogeneous deformation.



1.1. Write down the (horizontal component) position x of a material particle as a function of its initial position X and time t. [3 points]

1.2. Determine the (horizontal) velocity distribution as a function of x. [2 points]

1.3. Find the deformation gradient. [2 points]

1.4. Calculate the acceleration of a generic point on the band as a function of time and other relevant variables in spatial coordinates. [3 points]

1.1

In material description:

$v(X=0) = 0$

$v(X=L) = v_o$

$v = \dfrac{v_o X}{L}$

$v = \dfrac{\partial x}{\partial t}$

$\partial x = \dfrac{v_o X}{L}\partial t$

$x = \dfrac{v_o X}{L} t + C$ integration constant.

Since $x = X$ at $t = 0$

$$c = X$$

$$\Rightarrow \quad x = X + \frac{v_0 X}{L} t$$

(1-2)

$$X = \frac{x}{\left(1 + \frac{v_0 t}{L}\right)}$$

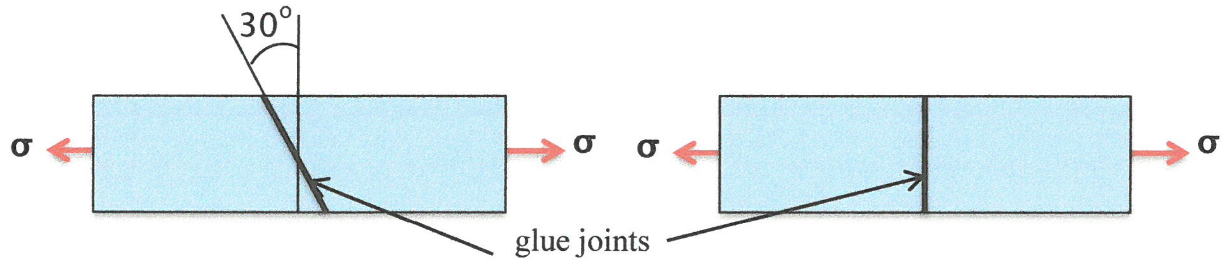
Since, $v = \frac{v_0 X}{L} \Rightarrow v = v_0 x / (L + v_0 t)$.

(1-3) $F_{11} = \frac{\partial x_1}{\partial X_1} = \frac{\partial x}{\partial X} \Rightarrow F_{11} = 1 + \frac{v_0 t}{L}$

(1-4)

$$f = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} v = -\frac{v_0^2 x}{(L + v_0 t)^2} + \frac{v_0}{(L + v_0 t)} \left(\frac{v_0 x}{L + v_0 t}\right)$$

2. The figure shows two designs for a glue joint.



Write expressions for (in terms of the applied stress σ)

2.1 Normal component of traction on each joint. [4 points]

2.2 Shear component of traction on each joint. [4 points]

(2)

Assuming $\underline{e}_1$ to be horizontal.

(2.1)

$\underline{n}_1$ (normal to first joint) $= \cos 30^\circ \, \underline{e}_1 + \sin 30^\circ \, \underline{e}_2$

$\underline{n}_2$ (normal to second joint) $= \underline{e}_1$

$\underline{t}_1$ (Traction on first joint) $= \sigma \cos 30^\circ \, \underline{e}_1$

$\underline{t}_2$ (Traction on second joint) $= \sigma \, \underline{e}_1$

<u>Normal components</u>:

$$|\underline{t}_1^n| = \underline{t}_1 \cdot \underline{n}_1 = \sigma \cos^2(30^\circ)$$

$$|\underline{t}_2^n| = \sigma$$

(2.2)

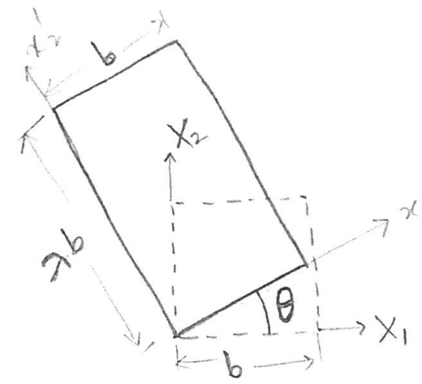
<u>Shear components</u>:

$$|\underline{t}_1^t| = |\underline{t}_1 - |\underline{t}_1^n| \underline{n}_1| = \sigma \cos 30 \left| (\underline{e}_1 \cos 30) \cdot (\underline{e}_1 \cos 30 + \underline{e}_2 \sin 30) \right|$$

$$= \sigma \sin 30 \cos 30$$

$$|\underline{t}_2^t| = 0$$

3. A square of edge length 'b' undergoes a rotation (θ) and a stretch (λ in the x_2 direction) to result in the following configuration:



3.1. Calculate the deformation gradient tensor, in terms of 'θ' and 'λ'. [4 points]
3.2. Demonstrate that this motion is real for all values of 'θ'. [3 points]
3.3. Can any condition be imposed on either 'θ' or 'λ' to make this motion impossible (mathematically not real)? Write those conditions. [3 points]

3.1

$$\underset{\sim}{F} = \underset{\sim}{R} \cdot \underset{\sim}{U}$$ (using Polar decomposition into rotation & deformation)

$\underset{\sim}{R}$ → Rotation, $\underset{\sim}{U}$ → stretch

$$R_{ij} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$U_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F_{ij} = R_{ik} U_{kj} = \begin{bmatrix} \cos\theta & -\lambda\sin\theta & 0 \\ \sin\theta & \lambda\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3.2

Motion real if $\det(\underset{\sim}{F}) \neq 0$

$$\det(\underset{\sim}{F}) = \cos\theta\,(\lambda\cos\theta) + \lambda\sin\theta\,\sin\theta$$

$$= \lambda(\cos^2\theta + \sin^2\theta) = \lambda.$$

↳ independent of 'θ'

3.3 $\det(\underset{\sim}{F}) = 0$ if $\lambda = 0$

no condition on 'θ'

↳ only defines the rigid body motion, not deformation.