Homework 3

Due Monday, Sep 30 (6pm)

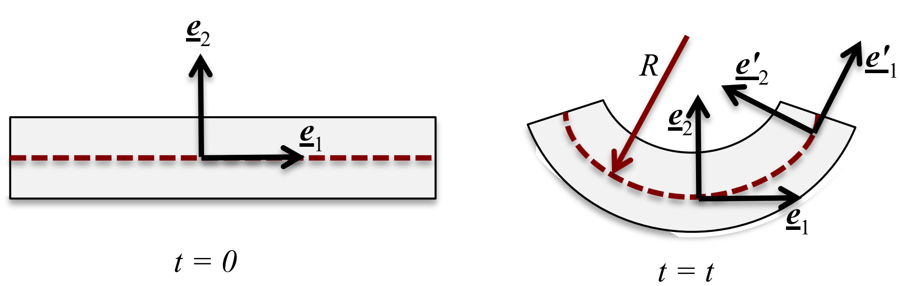
1. A motion of a fluid is given by the equations

$$x_1 = X_1 + \frac{1}{3}X_2 at + \frac{1}{9}X_3 a^2 t^2$$
$$x_2 = X_2 + \frac{1}{3}X_3 at + \frac{1}{9}X_1 a^2 t^2$$
$$x_3 = X_3 + \frac{1}{3}X_1 at + \frac{1}{9}X_2 a^2 t^2$$

where a is a constant. Find, as a function of time, the velocity and acceleration of the particle that was at the point (l, $2l$, $3l$) at the reference time $t = 0$, where l is a constant with the dimension of length.
[4 points]

2. An initially straight beam is bent into a circle with radius R, as shown in the figure below, after homogeneous deformation. Beam's thickness and the length remain unchanged. Such a deformation can be described as

$$x_1 = (R - X_2)\sin(X_1/R)$$
$$x_2 = R - (R - X_2)\cos(X_1/R)$$
$$x_3 = X_3$$

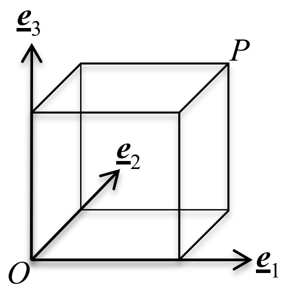


i. Calculate the deformation gradient field in the beam. Express the answer in the material coordinates. [2 points]
ii. Calculate the Lagrange strain field in the beam. [2 points]
iii. Calculate the infinitesimal strain field in the beam. [3 points]

3. Suppose that the region R_o occupied by a body in a reference configuration is a unit cube. The body undergoes the homogeneous deformation described by

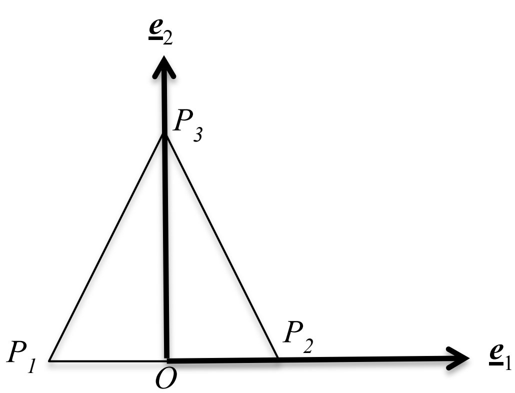
$$x_1 = \lambda_1 X_1; \qquad x_2 = \lambda_2 X_2; \qquad x_3 = \lambda_3 X_3$$

where the components have been taken with respect to an orthonormal basis $\underline{\boldsymbol{e}}_i$ aligned with the axes of the cube, as shown below.

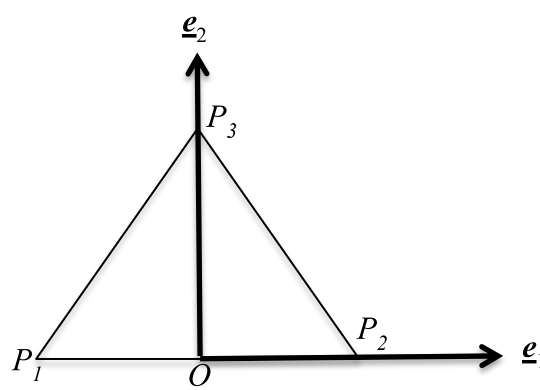


i. What are the principal stretches corresponding to this deformation? [1 point]
ii. Derive the relationships between the principal stretches if the body is composed of an incompressible material. [2 points]
iii. Derive the relationships between the principal stretches if the length of the fiber *OP* remains unchanged by the deformation. [4 points]

4. Consider a body *B* undergoing deformation over time *t*. Three particles P_1, P_2, P_3 (that belong to *B*) are located 1m apart, forming an equilateral triangle, before the deformation, as shown below. After the deformation, P_1 and P_2 move in such a way that their distance from P_3 remains 1m, but the distance between them is now 1.2m, as shown below. Assuming homogenous deformation for the entire body, compute the components of the Lagrange strain tensor associated with this deformation, expressing your answer as components in the $\underline{\boldsymbol{e}}_i$ base (indicated in the figure below). [7 points]



Before deformation *(t* = 0*)*



After deformation *(t = t)*

(*Hint: Treat the sides of the triangle as material line element under stretch. Find and use the relationship between stretch ratio and the components of Lagrangian strain tensor.*