Homework 4

(Due Monday, October 7, 6pm)

1. For the Cauchy stress tensor with components

$$[T] = \begin{pmatrix} 100 & 250 & 0 \\ 250 & 200 & 0 \\ 0 & 0 & 300 \end{pmatrix} \text{(MPa)},$$

calculate the following:

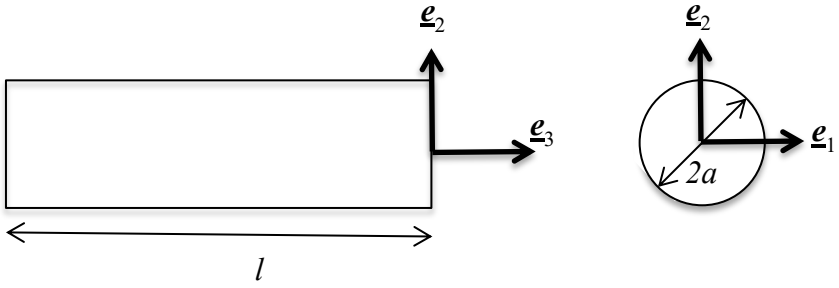
i. The traction vector acting on an internal material plane with normal $\underline{n} = (\underline{e}_1 - \underline{e}_2)/\sqrt{2}$. [2 points]

ii. The principal stresses. [2 points]

iii. The hydrostatic stress. [2 points]

iv. The deviatoric stress tensor. [2 points]

2. Suppose that *R*, the region occupied by a certain body in its current configuration, is a cylinder of length *l* and radius *a*.



Suppose that the matrix of components of the Cauchy stress tensor field in the cylinder is

$$[T] = \begin{pmatrix} 0 & 0 & -\alpha x_2 \\ 0 & 0 & \alpha x_1 \\ -\alpha x_2 & \alpha x_1 & \beta + \gamma x_1 + \delta x_2 \end{pmatrix}$$

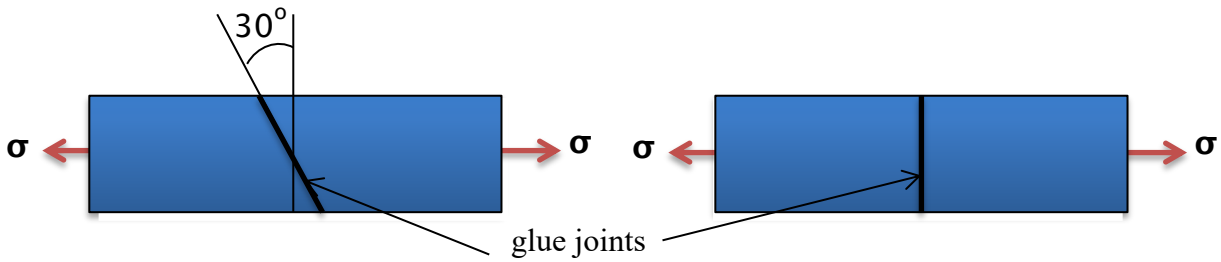
where $\alpha, \beta, \delta, \gamma$ are constants.

i. Verify that this stress field satisfies the equilibrium equations in the absence of body forces. [2 points]

ii. Verify that the curved surface of the cylinder is traction-free. [3 points]

iii. Calculate the traction on the end $x_3 = 0$. Hence calculate the resultant force acting on the cylinder at the end $x_3 = 0$. [5 points]

3. The figure shows two designs for a glue joint. The glue will fail if the stress acting normal to the joint exceeds 60 MPa, or if the shear stress acting parallel to the plane of the joint exceeds 300 MPa. Assume a coordinate system of your choice.



i. Calculate the normal and shear stress acting on each joint, in terms of the applied stress σ. [4 points]

ii. Hence, calculate the value of σ that will cause each joint to fail. [3 points]