MEMS 5501 Mechanics of Continua
Pathak

Homework 2

(*Due Monday, Sep 23, 6pm*)

1. A 2nd-order tensor is given as $\underset{\sim}{A} = \underline{a} \otimes \underline{a} + \underline{e}_3 \otimes \underline{e}_3$, where $\underline{a} = \underline{e}_1 - 2\underline{e}_2$.
   i. Find out whether the component matrix [A] of the tensor $\underset{\sim}{A}$ has an inverse. [2 points]
   ii. Calculate the eigenvalues of the matrix [A]. [2 points]

2. If base vectors $\underline{e}_i^{'}$ are given by

$$\underline{e}_1^{'} = \frac{\underline{e}_1 + 2\underline{e}_2}{\sqrt{5}}; \quad \underline{e}_2^{'} = \frac{-2\underline{e}_1 + \underline{e}_2 - \underline{e}_3}{\sqrt{6}}; \quad \underline{e}_3^{'} = \frac{-2\underline{e}_1 + \underline{e}_2 + 5\underline{e}_3}{\sqrt{30}}$$

   i. Verify that the base $\underline{e}_i^{'}$ is orthonormal. [2 points]
   ii. Write the transformation matrix [M] = (M_{ij}) for the transformation of the coordinate system from bases $\underline{e}_i$ to $\underline{e}_i^{'}$. [3 points]

3. Consider vectors

$$\underline{a} = 3\underline{e}_1 + 2\underline{e}_2$$
$$\underline{b} = 2\underline{e}_1 - 3\underline{e}_2$$
$$\underline{c} = -2\underline{e}_3$$

   i. Form the tensors $\underset{\sim}{D} = \underline{a} \otimes \underline{b} + \underline{b} \otimes \underline{a} + \underline{c} \otimes \underline{c}$ and $\underset{\sim}{F} = \underline{a} \otimes \underline{b} \otimes \underline{c}$. [2 points]
   ii. Calculate the contractions D_{ii} and F_{ii3}. [3 points]

4. Consider the motion

$$x_1(X_r, t) = X_1$$
$$x_2(X_r, t) = \frac{1}{2}e^{t/\tau}(X_2 + X_3) + \frac{1}{2}e^{-t/\tau}(X_2 - X_3)$$
$$x_3(X_r, t) = \frac{1}{2}e^{t/\tau}(X_2 + X_3) - \frac{1}{2}e^{-t/\tau}(X_2 - X_3)$$

   where τ is a constant time scale.
   i. From these equations, write $X_r(x_i, t)$. [3 points]
   ii. Calculate the components of velocity in both their material and spatial description. [4 points]

5. Consider the velocity field

$$v_1(x_i) = Va^2 x_2 x_3$$
$$v_2(x_i) = -Va^2 x_1 x_3$$
$$v_3(x_i) = Vax_3$$

   where *a* and *V* are constants. Determine the acceleration in the spatial description. [4 points]