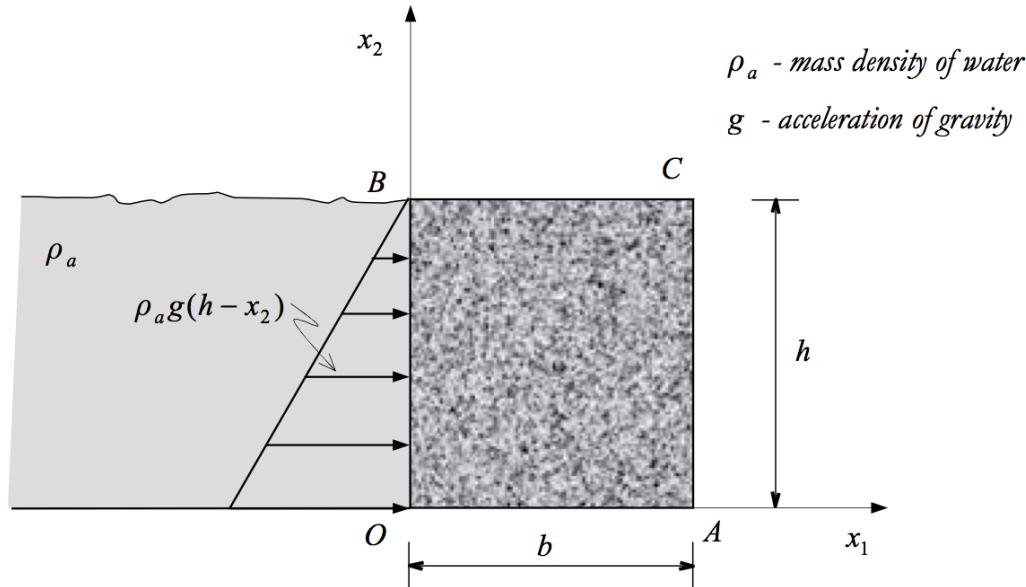## Homework 5

(*Due Monday, October 28, 6pm*)

1. A prismatic dam is subjected to water pressure, as pictured below. The dam has thickness b and height h. Obtain the restrictions of the stress tensor components (in Cartesian coordinates) on the faces BC, OB, and AC.

[7 points]

2. Show that

$$\frac{D}{Dt}\int_V \rho P_{ij..}(\underline{x},t)dV = \int_V \rho \frac{DP_{ij..}(\underline{x},t)}{Dt}dV$$

where $P_{ij..}(\underline{x},t)$ is a continuum property per unit mass, which can be a scalar, vector, or a tensor of any degree.

(Hint: Refer to the methods covered in class for the derivation of "The material time derivative of a volume integral" for a generic physical quantity per unit mass. You will also need to use the "Equation of continuity" along the way.)   [7 points]

3. Given the velocity field: $v_1 = x_1 x_3$; $v_2 = x_2^2 t$; $v_3 = x_2 x_3 t$

    and the stress field:

$$T_{ij} = \alpha \begin{pmatrix} x_2 x_1 & -x_2 x_3 & 0 \\ -x_2 x_3 & x_2^2 & -x_2 \\ 0 & -x_2 & x_3^2 \end{pmatrix}$$

    where α is a constant. Obtain the body force (per unit volume) that will guarantee the principal of conservation of the linear momentum.   [11 points]