HW # 1 Solutions

(1) $y_i = A_{ij} x_j + B_{ij} z_j$

(2) $C_1 = (a_{11} + \delta_{11}) b_1 + (a_{12} + \delta_{12}) b_2 + (a_{13} + \delta_{13}) b_3$

$= (a_{11} + 1) b_1 + (a_{12}) b_2 + (a_{13}) b_3$

$C_1 = a_{11} b_1 + a_{12} b_2 + a_{13} b_3 + b_1$

Similarly,

$C_2 = b_2 + a_{21} b_1 + a_{22} b_2 + a_{23} b_3$

$C_3 = b_3 + a_{31} b_1 + a_{32} b_2 + a_{33} b_3$

(3) $\delta_{ij} \delta_{ik} \delta_{jk} = \delta_{kj} \delta_{jk} = \delta_{kk} = \delta_{11} + \delta_{22} + \dots + \delta_{nn} = n$

(4) $e_{ijk} e_{kji} = e_{ijk} e_{jik} = (\delta_{ii} \delta_{jj} - \delta_{ij} \delta_{ji}) = \delta_{ii} \delta_{jj} - \delta_{ii}$

$(kji \equiv ikj \equiv jik)$

$= 3 \times 3 - 3$
$= 6.$

(5) $S_{kk} = P_{kk} - P_{kk} \underbrace{\delta_{mm}}_{3} / 3 = 0$

(6) $Q_{ij} x_j = a_i$

multiply by Q_{ik} to exploit the properties of the orthogonal matrix $\equiv Q_{rp} Q_{rq} = \delta_{pq}$)

$Q_{ik} Q_{ij} x_j = Q_{ik} a_i$

$\delta_{kj} x_j = Q_{ik} a_i$

$x_k = Q_{ik} a_i \Rightarrow \{x\} = [Q]^T \{a\}$

$\Rightarrow [Q]^{-1} = [Q]^T$

HW #1 Sol^{ns}

7

(i) 90°

(ii) $(\underline{a}+\underline{b})\cdot[(\underline{a}*\underline{b})\times(\underline{c})] = 0$

$$\underbrace{\underline{a}\cdot(\underline{a}\times\underline{c})}_{0} + \underline{a}\cdot(\underline{b}\times\underline{c}) + \underline{b}\cdot(\underline{a}\times\underline{c}) + \underbrace{\underline{b}\cdot(\underline{b}\times\underline{c})}_{0} =$$

$$(\underline{a}\times\underline{c} = -\underline{c}\times\underline{a})$$

$$\underline{a}\cdot(\underline{b}\times\underline{c}) = \underline{b}\cdot(\underline{c}\times\underline{a})$$

8 → done in class

9 Expand!

$$\begin{aligned}R_{ik}R_{jk} &= \left(\cos\theta\delta_{ik} + n_i n_k(1-\cos\theta) - \sin\theta \in_{ikp} n_p\right)\left(\cos\theta\delta_{jk} + n_j n_k(1-\cos\theta) - \sin\theta \in_{jkq} n_q\right)\\ &= \cos^2\theta\delta_{ik}\delta_{kj} + n_i n_k n_j n_k(1-\cos\theta)^2 + \sin^2\theta \in_{ikp} n_p \in_{jkq} n_q\\ &\quad + \cos\theta(1-\cos\theta)(\delta_{ik}n_j n_k + n_i n_k\delta_{jk}) - \cos\theta\sin\theta(\delta_{ik}\in_{jkq} n_q + \delta_{jk}\in_{ikp} n_p)\\ &\quad - (1-\cos\theta)\sin\theta(n_i n_k \in_{jkq} n_q + n_j n_k \in_{ikp} n_p)\end{aligned}$$

Now simplify by noting $n_k n_k = 1$ and $\delta_{ij}n_j = n_i$

$$\begin{aligned}R_{ik}R_{jk} &= \cos^2\theta\delta_{ij} + n_i n_j(1-\cos\theta)^2 + \sin^2\theta \in_{ikp} n_p \in_{jkq} n_q\\ &\quad + \cos\theta(1-\cos\theta)(n_j n_i + n_i n_j) - \cos\theta\sin\theta(\in_{jiq} n_q + \in_{ijp} n_p)\\ &\quad - (1-\cos\theta)\sin\theta(n_i n_k \in_{jkq} n_q + n_j n_k \in_{ikp} n_p)\end{aligned}$$

Note that $n_k \in_{jkq} n_q = 0$ (expand out in full, or note that this represents **n** crossed with itself) and $(\in_{jiq} n_q + \in_{ijp} n_p) = (\in_{jiq} n_q - \in_{jip} n_p) = 0$, and recall $\in_{ijk}\in_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{mk}$, so that

$$\begin{aligned}R_{ik}R_{jk} &= \cos^2\theta\delta_{ij} + n_i n_j(1-\cos\theta)^2 + \sin^2\theta(\delta_{ij}\delta_{pq} - \delta_{ip}\delta_{jq})n_p n_q + 2\cos\theta(1-\cos\theta)n_j n_i\\ &= \delta_{ij}(\sin^2\theta + \cos^2\theta) + n_i n_j(1 - 2\cos\theta + \cos^2\theta - \sin^2\theta + 2\cos\theta - 2\cos^2\theta) = \delta_{ij}\end{aligned}$$

This verifies that R_{ij} is indeed orthogonal.

Homework - I

(8)

$$\underset{\sim}{A} \cdot \underset{\sim}{B} = A_{ij} B_{kl} (\underline{e}_i \otimes \underline{e}_j) \cdot (\underline{e}_k \otimes \underline{e}_l)$$

$$= A_{ij} B_{kl} \delta_{jk} (\underline{e}_i \otimes \underline{e}_l)$$

$$= A_{ik} B_{kl} \underline{e}_i \otimes \underline{e}_l = A_{ij} B_{jk} \underline{e}_i \otimes \underline{e}_k$$

$$\underset{\sim}{A}' \cdot \underset{\sim}{B}' = \underbrace{(M_{ip} A_{pq} M_{jq})}_{A'_{ij}} \underbrace{(M_{jr} B_{rs} M_{ks})}_{B'_{jk}} \underbrace{(\underline{e}_t M_{it})}_{e'_i} \otimes \underbrace{(\underline{e}_u M_{ku})}_{e'_k}$$

$$= \underbrace{M_{ip} M_{it}}\ \underbrace{M_{jq} M_{jr}}\ \underbrace{M_{ks} M_{ku}}\ A_{pq} B_{rs} (\underline{e}_t \otimes \underline{e}_u)$$

$$= \delta_{pt} \quad \delta_{qr} \quad \delta_{us} \qquad A_{pq} B_{rs} \ \underline{e}_t \otimes \underline{e}_u$$

$$= (\delta_{pt} A_{pq})\ \delta_{qr} (\delta_{us} B_{rs})\ \underline{e}_t \otimes \underline{e}_u$$

$$= A_{tq} \delta_{qr} B_{ru} \ \underline{e}_t \otimes \underline{e}_u$$

$$= A_{tr} B_{ru} \ \underline{e}_t \otimes \underline{e}_u$$

↑

equivalent to $A_{ij} B_{jk} \underline{e}_i \otimes \underline{e}_k$