MEMS 5501 Mechanics of Continua
Pathak

Homework 1

(Due Monday, Sep 16, 6PM)

1. If [A] and [B] are *n* x *n* square matrices and $\{x\}$, $\{y\}$, $\{z\}$ are *n* x *1* column matrices, express the following matrix equation in index form:
   $\{y\}=[A]\{x\}+[B]\{z\}$ [1 point]

2. Expand the following equation for an index range of three:
   $c_i=\left(a_{ij}+\delta_{ij}\right)b_j$ [2 points]

3. Evaluate the expression: $\delta_{ij}\delta_{ik}\delta_{jk}$ [2 points]
   (Note: Assume an index range of '*n*')

4. Evaluate the expression: $e_{ijk}e_{kji}$, [3 points]
   where e_{ijk} is the permutation symbol (alternator).
   (Of course, in this case the index range is three, based on the definition of the alternator.)
   (Hint: Use the identity, $e_{ijk}e_{pqk}=\delta_{ip}\delta_{jq}-\delta_{iq}\delta_{jp}$ *.)*

5. Let $S_{ij}=P_{ij}-P_{kk}\delta_{ij}/3$. Calculate S_{kk}. [2 points]

6. Given an orthogonal matrix [Q], use indicial notation to solve the matrix equation [Q]{x}={a} for {x}.
   (Hint: Recall the properties of an orthogonal matrix and use them.) [3 points]

7. Given three vectors $\underline{a}$, $\underline{b}$, $\underline{c}$
   (i) What is the angle between vectors $\left(\underline{a}+\underline{b}\right)$ and $\left(\underline{a}+\underline{b}\right)\times\underline{c}$? [1 point]
   (ii) Show that $\underline{a}.\left(\underline{b}\times\underline{c}\right)=\underline{b}.\left(\underline{c}\times\underline{a}\right)=\underline{c}.\left(\underline{a}\times\underline{b}\right)$ [3 points]
   (Hint: The part (i) of the question is a hint for the part (ii). Use the properties of dot and cross products.)

8. Prove that the inner product of two 2nd-order tensors is invariant to a change of coordinate system.
   [3 points]

9. Let $R_{ij}=\delta_{ij}\cos\theta+n_in_j\left(1-\cos\theta\right)-e_{ijk}n_k\sin\theta$, where n_k are the components of a unit vector, and e_{ijk} is the permutation symbol (alternator).
   Calculate $R_{ik}R_{jk}$. [5 points]
   (Hints: $\underline{n}\times\underline{n}\equiv n_ke_{jkq}n_q=0$ *;* $e_{jiq}n_q+e_{ijp}n_p=0$ *;* $e_{ijk}e_{pqk}=\delta_{ip}\delta_{jq}-\delta_{iq}\delta_{jp}$ *)*