Statistical Inference Course Project: Exploring the Central Limit Theorem

By Matthew Sedlar

Overview

This report explores the Central Limit Theorem using a set of randomly generated exponentials. It will look at the sample mean and how it relates to the theoretical mean as well as the sample variance and the overall distribution.

Simulations

Let's start by setting the number of simulations we want to run as well as the number of exponentials and rate for each simulation.

```
nosims <- 1000
nexps <- 40
rate <- 0.2
```

I'm going to run 1,000 simulations on the exponentials using R's replicate function. I am using the replicate function because it takes up less lines than sapply or apply iterations and it's easier to understand if you're just taking a glance at my script.

Once I have the simulation results in the form of a matrix, I will store the averages of those results in a data frame along with the number of simulations. This will come in handy later.

```
# using set.seed so the results can be replicated
# also, 13 is my lucky number
set.seed(13)

# 1000 simulations on 40 random exponentials
sims <- replicate(nosims,rexp(nexps,rate))

# storing the means of those simulations in a data frame
# with the size of the simulations
simsdf <- data.frame(averages = apply(sims,2, mean),
                     size = "1000")
```

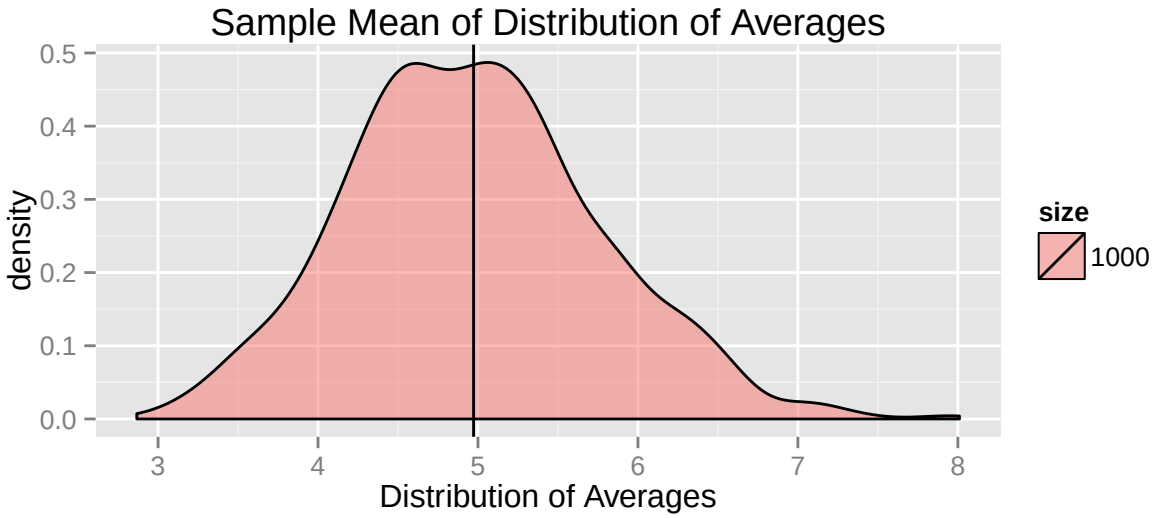
Sample Mean versus Theoretical Mean

Let's examine the mean of our sample distribution:

```
samplemean <- mean(simsdf$averages)
samplemean
```

```
## [1] 4.972512
```

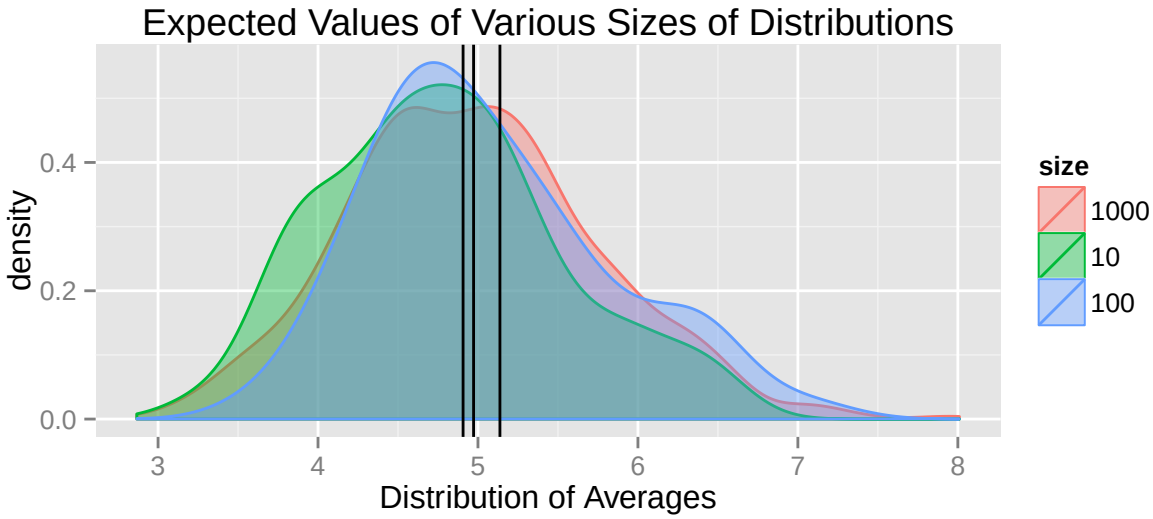
And plot it on the distribution, looking at the density.



We know given the rate (0.2) that our expected value is 5 based on the following formula.

$$E[X] = \frac{1}{\lambda} = \beta$$

We also know from the Central Limit Theorem that the sample mean from unbiased data approximates the population mean. To illustrate this, I'm going to run two more simulations – 100 and 10 times, respectively – and then plot those distributions against the original simulation distribution.



No matter the sample size, the sample mean clusters around the theoretical mean.

Sample Variance versus Theoretical Variance

Next, let's look at our sample variance.

We know that our theoretical variance is 25 because the mean and standard deviation for our exponential distribution is:

$$\frac{1}{\lambda}$$

That makes our variance:

$$Var[X] = \frac{1}{\lambda^2}$$

```
samplevariance <- samplemean^2
samplevariance
```

```
## [1] 24.72587
```

As with the mean, we see the same behavior with the variance. I will use dplyr's group_by function on my simulations data frame to create a data frame that only displays the means and variances for each size of distributions.

```
## Source: local data frame [3 x 3]
##
##   size     mean      var
## 1 1000 4.972512 24.72587
## 2   10 4.784766 22.89398
## 3  100 5.071031 25.71535
```

Distribution

There are three ways to tell if our distribution is normal.

- mean = median = mode
- The distribution is symmetric
- 50% of values are less than the mean and 50% are greater

Two of these are easy to test.

mean = median = mode.

In R, this is a simple logical comparison. Since we're trying to prove if it is "approximately" normal, let's use the signif function on our values to the first decimal point. See appendix for how I generate the mode.

```
signif(samplemean,1) == signif(median(originalsim),1)
```

```
## [1] TRUE
```

```
signif(samplemean,1) == samplemode
```

```
## [1] TRUE
```

50% of values are less than/above the mean

To find out if our values are split 50/50 at the mean, let's use the qexp function to find the 50th quartile.

```r
qexp(0.50,1/samplemean)
```

```
## [1] 3.446683
```

Uh-oh. If we use the pexp function on our mean, we find our distribution is actually split with 37% of values occurring above the mean and 63% of values occuring below. That's hardly symmetric.

```r
# probability of a value occuring above the mean
pexp(samplemean, 1/samplemean, lower.tail=FALSE)
```

```
## [1] 0.3678794
```

```r
# probability of a value occuring below the mean
pexp(samplemean, 1/samplemean)
```

```
## [1] 0.6321206
```

We're looking for "approximately" normal, not normal, however. While the distribution does not fit the strict definition of normal, one could argue that because it meets 1/3 of the criteria, it could be considered approximate.