MEC 411: Lab Number 2

PID Speed Control of a Turntable

Group # 12

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Table of Contents

Abstract

The objective of this lab is to determine the design input values for a PID controller with an analog circuit that will cause a system with a turntable motor to perform according to design specifications, to compare the results from a P, PI, and PID controller, and to compare the results from this experiment to the previous experiment that used a digital circuit. A PID controller uses selected input variables that can be adjusted to optimize a feedback system. The variables are a factor that is proportional to the error function, the derivative of the error function with respect to time, and the integral of the error function with respect to time. Before proceeding with the experiment, the transfer function of the system must be obtained (Eq. 17), the transfer function of the PID (Eq. 5), as well as the transfer function of the plant (Eq. 15). For the P controller the resistance value obtained was $R_{p2} = 5.675\ k\Omega$, which gives $K_p = 5.675$ (Eq. 19). For the PI controller the resistance values obtained were $R_{p2} = 5.625\ k\Omega$ and $R_i = .5664\ k\Omega$ which gave $K_p = 5.625$ (Eq. 19) and $K_i = 1.7655 * 10^5$ (Eq. 23), respectively. For the PID controller the resistance values obtained were $R_{p2} = 5.625\ k\Omega$, $R_i = .5664\ k\Omega$, and $R_{d2} = .282\ \Omega$ which gave $K_p = 5.625$ (Eq. 19), $K_i = 1.7655 * 10^5$ (Eq. 23), and $K_d = 2.82 * 10^{-10}$ (Eq. 28), respectively. All systems matched the theoretical results. The PID system gave a percent overshoot of 16%, a rise time of 9.5ms, and a settling time of 157.6ms. The rise time and settling time both meet the design specifications. Once the percent overshoot is corrected to 4% due to the amount of noise in the system, it also meets the requirements. Lab 2 was found to have a quicker response time while lab 1 was found to be more accurate, leading to the conclusion that the lab 1 system had a better performance.

I. Introduction

A closed control system has an input signal will go through a plant of some kind such that an output signal is produced. In addition, there is a device that allows for feedback between the input and the output, which is specifically what makes the system closed. Closed control systems can be seen in numerous applications in real life, such as in guided missiles and a thermostat. One of the most interesting uses of close control systems is known as a Proportional Integral Derivative (PID) Controller. The use of a PID controller in a system is based on the idea that the input signal goes through the PID controller, then the plant, which produces the output and provides feedback. The PID modifies the signal with regards to it’s error function, which is defined as the input signal minus the output signal. The PID controller gives the user the ability to modify an input signal with a term that is proportional to the error function in order to describe what is currently happening in the system, a term that relates to the derivative of the error function with respect to time to describe the future of the system, and a term that relates to the integral of the error function with respect to time to take into account the past of the system.

The objective of this laboratory experiment is to implement a PID turntable speed controller for a DC motor. In this case, the PID controller will be created with an analog circuit, or in other words via a circuit instead of digitally on a computer. The experimental results found from this setup will then be compared to values for a theoretical setup found through MATLAB software. Design specifications for the PID controller include a percentage overshoot (PO) of less than 10%, a 2% settling time of no more than 500ms, and a rise time of less than 200ms.

A necessary and vital step for this experiment is to first perform a thorough analysis of the control system itself. First, the system must be broken up into a block diagram. This can be seen in the image below.

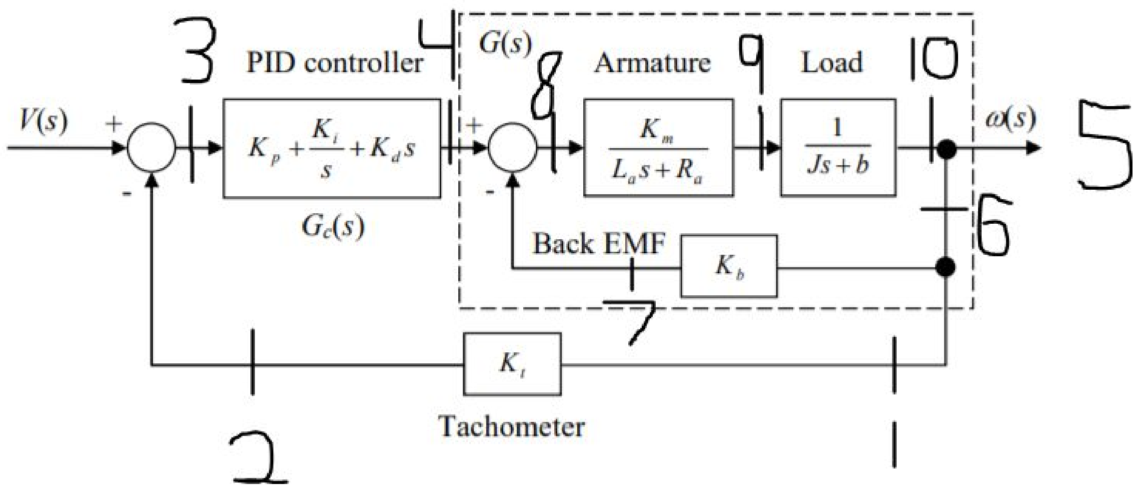

FIG. I. Block diagram of the turntable PID speed control system[1]

From this diagram, one is now able to derive the transfer function for the system. The transfer function can be defined as the ratio of output to input. To obtain this, the block diagram must be analyzed. Starting at the labeled point 1, it is determined that

$$\omega(s) = point\ 1 \tag{1}$$

In which $\omega(s)$ is the output signal and s is the laplace variable. From here we go to point 2, in which it is found, using rules of analysis for a block diagram, that

$$K_t\omega(s) = point\ 2 \tag{2}$$

For which K_t is a constant used to define the tachometer. Next the signal is followed through the summer to get to point 3, and in doing so the analysis yields

$$V(s) - K_t\omega(s) = point\ 3 \tag{3}$$

Where V(s) is the input signal. Next the signal is followed through the PID controller itself, which results in the equation becoming

$$G_c(s)V(s) - G_c(s)K_t\omega(s) = point\ 4 \tag{4}$$

$G_c(s)$ can be defined as

$$G_c(s) = K_p + K_i/s + K_d s \tag{5}$$

For which K_p is the proportional coefficient, K_i is the integral coefficient, and K_d is the derivative coefficient. This derivation will be further explored a little later. From here, the signal moves on through the armature and load, which produces the equation

$$G(s)G_c(s)V(s) - G(s)G_c(s)K_t\omega(s) = point\ 5 \tag{6}$$

G(s) is a transfer function for the armature-load system that needs to be derived. To find this, point 6 is first analyzed, as seen below.

$$\omega(s) = point\ 6 \tag{7}$$

In which point 6 is essentially the same point as point 1, and hence is just the output signal. Point 7 is then analyzed.

$$K_b\omega(s) = point\ 7 \tag{8}$$

K_b is a constant that defines the back emf of the system. From here, the summer of this subsystem is then analyzed.

$$V_{al}(s) - K_b\omega(s) = point\ 8 \tag{9}$$

Where $V_{al}(s)$ is the input to the armature-load system, which is different from the input to the overall system. In fact, $V_{al}(s)$ can be written as

$$V_{al}(s) = G_c(s)V(s) - G_c(s)K_t\omega(s) \tag{10}$$

Which is a direct result of it being located at point 4. From here, the signal is traced through the armature to become

$$G_a(s)V_{al}(s) - G_a(s)K_b\omega(s) = point\ 9 \tag{11}$$

In which $G_a(s)$ is the function of s for the armature. It can be defined as

$$G_a(s) = K_m/(L_a s + R_a) \tag{12}$$

For this equation, K_m, L_a, and R_a are all constants used to define the system. Lastly, through analysis of point 10, it is found that

$$G_l(s)G_a(s)V_{al}(s) - G_l(s)G_a(s)K_b\omega(s) = point\ 10 \tag{13}$$

Where $G_l(s)$ is defined as a function of s for the motor itself, and can be written as

$$G_l(s) = 1/(Js + b) \tag{14}$$

Which has J and b as constants. Through rearranging equation 13 and replacing $G_a(s)$ and $G_l(s)$ with their derived values, the transfer function of the armature load system can be found to be

$$G(s) = \omega(s)/V_{al}(s) = G(s) = K_m/((L_a + R_a)(Js + b) + K_b K_m) \tag{15}$$

And from finding this transfer function, the overall transfer function can have $G(s)$ and $G_c(s)$ subbed out for their actual equations, and then algebraically rearranged to yield the transfer function for the whole system. First, written with $G_c(s)$ and G(s), it is obtained to be

$$T = \omega(s)/V(s) = \frac{G(s)G_c(s)}{1+K_t G(s)G_c(s)} \tag{16}$$

And by subbing in the actual equations it is obtained to be

$$T = \left(\frac{(K_m/((L_a+R_a)(Js+b)+K_b K_m))(K_p+K_i/s+K_d s)}{1+K_t(K_m/((L_a+R_a)(Js+b)+K_b K_m))(K_p+K_i/s+K_d s)}\right) \tag{17}$$

It is possible to derive the coefficients of the PID controller from the circuit diagram given below

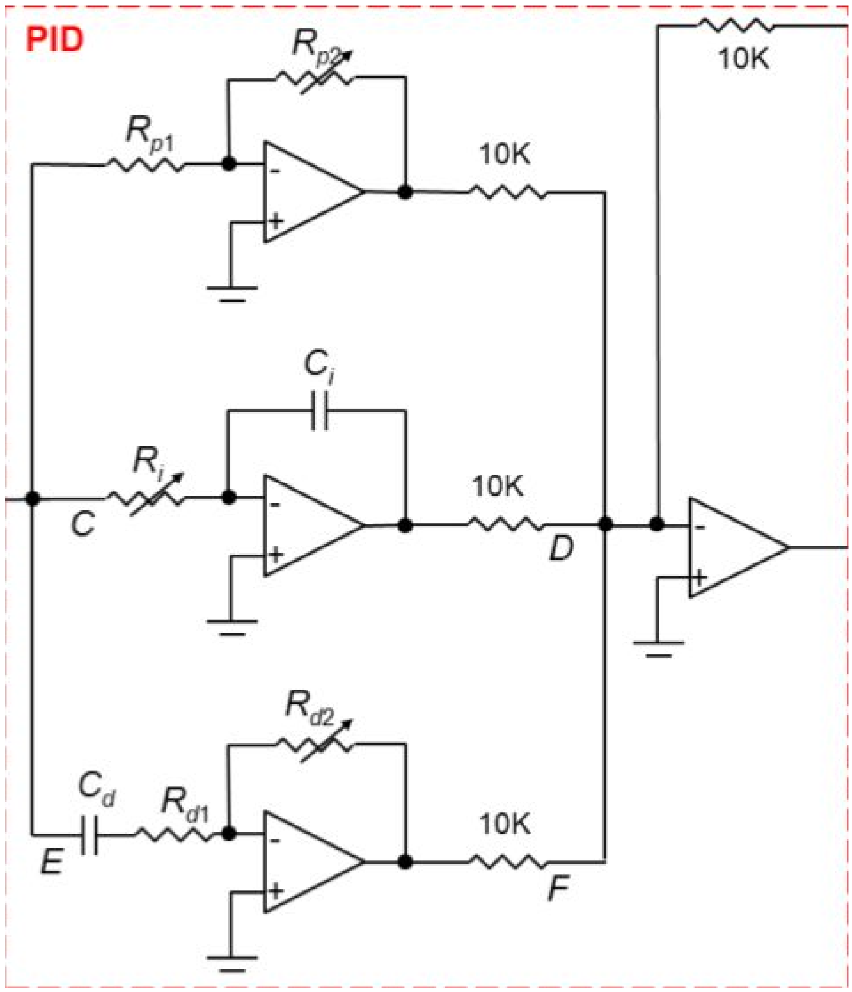

FIG. II. Circuit diagram of the PID controller[1]

The input voltage to this system will be denoted V_o. For the operational amplifier on the top of the circuit, the current in resistor R_{p1} and current through resistor R_{p2} can be set equal to each other to obtain the following equation.

$$V_o/R_{p1} = V_p/R_{p2} \tag{18}$$

In which V_p is the voltage at the output of the operational amplifier. To solve for the transfer function of output over input voltage, equation 18 can be rearranged algebraically. This is seen below.

$$K_p = V_p/V_o = R_{p2}/R_{p1} \tag{19}$$

Where K_p is the transfer function for the proportional amplifier. Next, the transfer function for the integral amplifier can be found, which can be seen in the center of the circuit. This can be done by equating the currents in the middle operational amplifier, which goes through resistor R_i and C_i. The equation for equal currents can be seen below.

$$V_o/R_i = C_i(dV_i/dt) \tag{20}$$

For which V_i is the voltage through the output of the integral amplifier and t is time. By taking note of the following formula in the laplace domain.

$$d/dt = s \tag{21}$$

The transfer function for the integral operational amplifier can be found, as seen below.

$$V_i/V_o = 1/(R_iC_i) * 1/s = K_i/s \tag{22}$$

Where K_i is the transfer function for the integral amplifier, excluding s terms. It is equal to

$$K_i = 1/(R_iC_i) \tag{23}$$

Lastly, the transfer function for the derivative amplifier, or the op amp on the bottom, must be found to fully define the PID controller. The input voltage and output voltage of this op amp are equal to

$$(1/C_{d)}\int idt + iR_{d1} = V_o \tag{24}$$

$$iR_{d2} = V_d \tag{25}$$

Where i is the current going through the derivative amplifier, V_d is the voltage output of this op amp, and C_d is the capacitor used in the derivative op amp. R_{d1} and R_{d2} are the two resistors for this op amp. By remembering that

$$\int dt = 1/s \tag{26}$$

it is possible to find and rearrange the transfer function V_d/V_o This transfer function is seen below.

$$V_d/V_o = C_d(R_{d2}/R_{d1})s = K_d s \tag{27}$$

Where K_d is the transfer function of the derivative op amp, excluding all s terms. It is equal to

$$K_d = (R_{d2}/R_{d1})C_d \tag{28}$$

Since percent overshoot needs to be calculated, the equation needed is

$$PO = (M_{tp} - F_v)/F_V * 100\% \tag{29}$$

In which PO is percent overshoot, f_v is the accepted value of the system, which is one volt for the PID controller, and M_{tp} is the maximum amplitude of the system. Rise time and settling can be found by eye and linear interpolation from the plots and experimental data values.

From here, all relevant derivations for the system have been found, and by extension completion of the rest of this experiment may now begin.

II. Experimental Procedure

Equipment List:

- Waveform Generator
- Oscilloscope

- Resistors
- Op Amps
- Capacitors
- Diodes
- Transistors
- Breadboard
- Wires
- DC Turntable Motor
- Data Recording Software
- Alligator Clips

Build the circuit as shown in the diagram below:

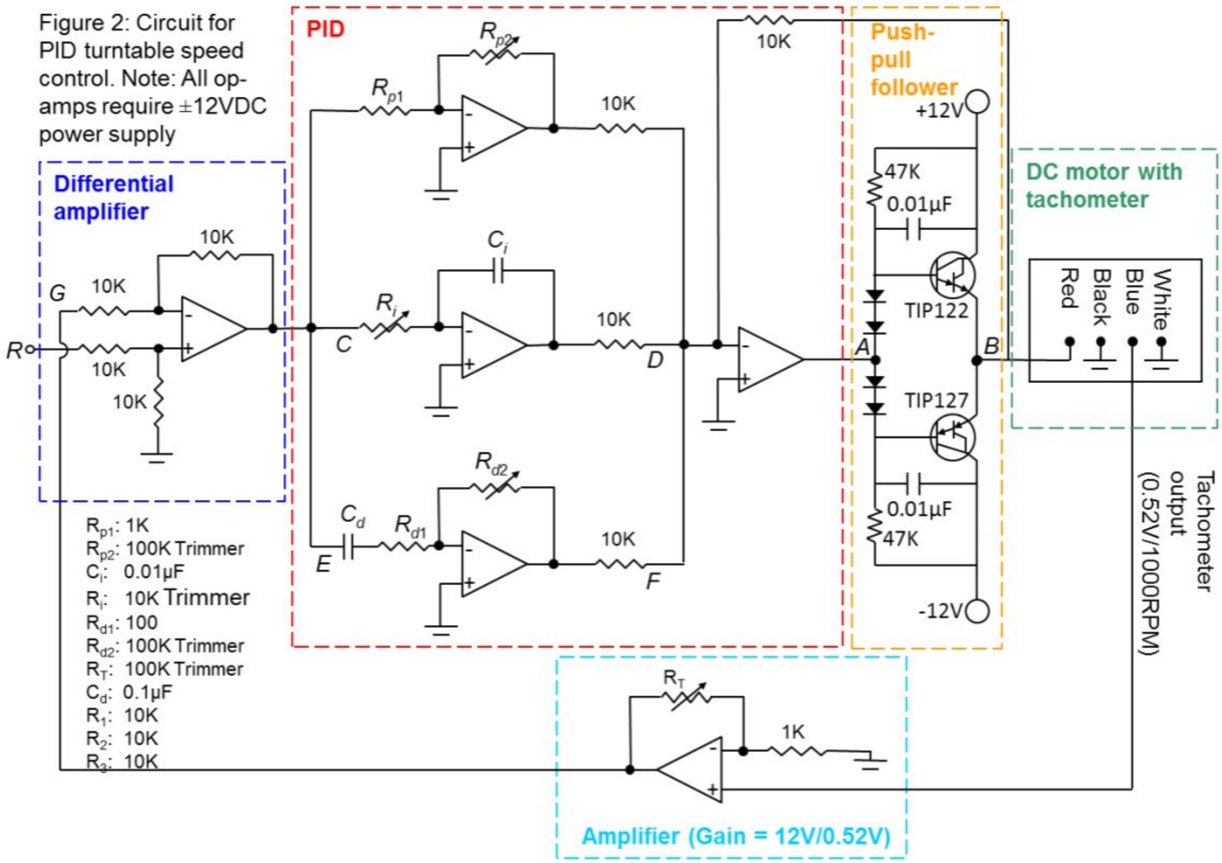

FIG. III. Circuit Diagram of the turntable PID controller[1].

Connect the wave function generator to the input at R. Then, connect channel 1 of the oscilloscope to the input at R and channel 2 of the oscilloscope to the output at point G. The power source should be connected directly to the breadboard.

This experiment requires one full sized breadboard, one TIP122 transistor, one TIP127 transistor, three 0.01 μF capacitors, one 0.1 μF resistor, two 47 kΩ resistors, eleven 10 kΩ resistors, two 1 kΩ resistors, one 100 Ω resistor, three 100 kΩ potentiometers, one 10 kΩ potentiometer, six operational amplifiers, and wires of varying length and color. In addition, a wave function generator, an adjustable power source, a DC motor, a multimeter, an oscilloscope, and a program that can be used to record data. Before proceeding, ensure that all the lab equipment is turned on. Do not connect the circuit to the power supply until the output of both the power supply and the waveform generator are set.

Calculate the resistance value of the potentiometer that will give a gain of 12V/0.52V. Recall that the rules of op-amps are as follows: no current flows through the op-amp, and the potential difference of both the positive and negative sides must be equal[2].

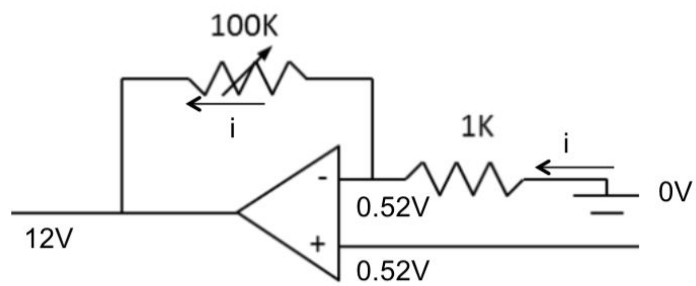

FIG. IV. Circuit Diagram of the operational amplifier, used for an amplifier gain, with a potentiometer[1].

Use the fact that the ground has a potential of 0V to calculate the current through the potentiometer.

$$V = iR$$

$$i = \frac{\Delta V}{R} = \frac{0.52V - 0V}{1k\Omega} = 0.52mA$$

Then, use the current and the gain to calculate the resistance of the potentiometer.

$$V = iR$$

$$R = \frac{\Delta V}{i} = \frac{12V - 0.52V}{0.52mA} = 22k\Omega$$

Set the potentiometer to this calculated resistance using the multimeter.

Set the power supply to output 12V and the wave function generator to output a square wave with an amplitude of 1V and a frequency of 1 Hz. Then disconnect the circuit at points C, D, E, and F. The system is now just a proportional controller. Power the circuit and observe the input and output functions displayed on the oscilloscope. Adjust R_{p2} so that the output function matches the input function as closely as possible without driving the system unstable. Once this is achieved, obtain data from the system and record the resistance value of R_{p2}.

Reconnect the circuit at points C and D so that the system is a proportional-integral controller. Adjust R_{p2} and R_i in order to bring the output function closer to the input function. Record the system data and the resistance values of R_{p2} and R_i.

Reconnect the circuit at points E and F so that the system is a proportional-integral-derivative controller. Adjust R_{p2}, R_i, and R_{d2} to reduce the amount of noise in the system. Record the system data and the resistance values of R_{p2}, R_i, and R_{d2}.

III. Results

The input of a system was compared to the output of a system for a proportional controller, a proportional-integral controller, and a proportional-integral-derivative controller. For the proportional controller, the response of the system was optimal when the value of R_{p2} was equal to 5.675 kΩ. Therefore, it is obtained that $K_p = 5.675$ (Eq. 19). This can be seen in the plot below, which was created using google sheets.

Proportional Controller:

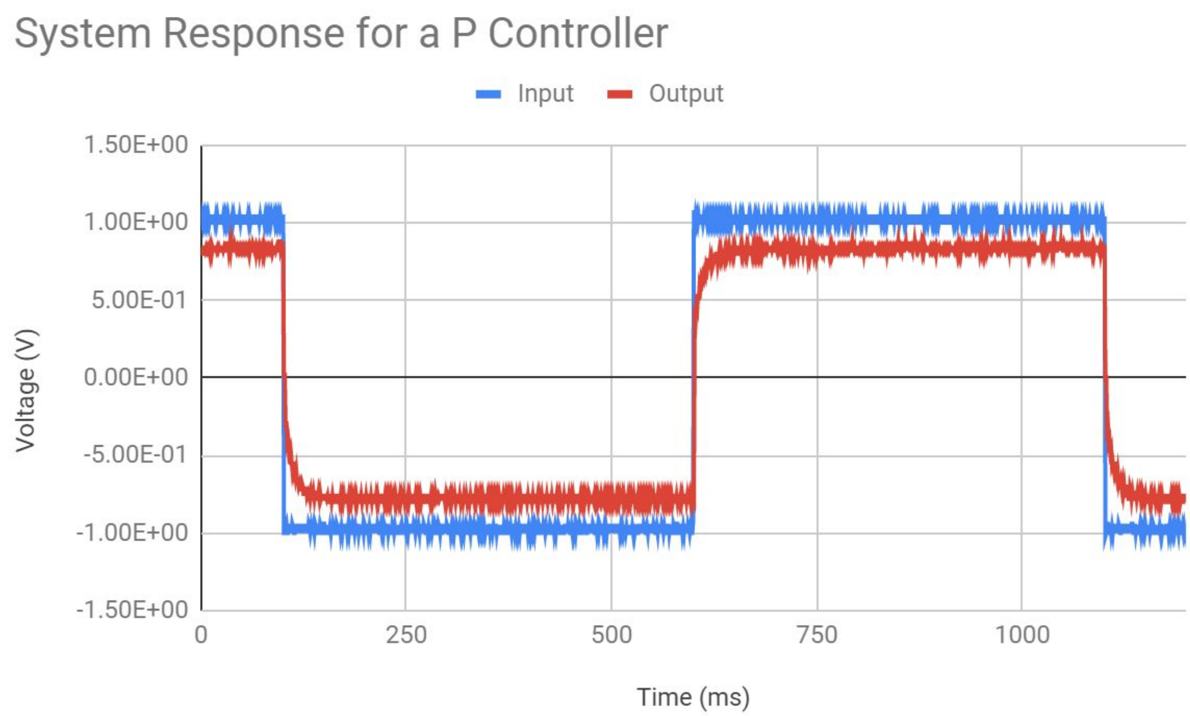

Fig. V. System Response of a Proportional Controller

The output of the P-controller is significantly less than the input with an amplitude for the output of 0.810 volts and an amplitude for the input of 1.04 volts (both of these values were obtained by averaging the data points at the top of the first square wave). This was as expected, since the system would become unstable if driven any harder.

For the proportional-integral (PI) controller the potentiometer values that optimized the output as much as possible were found to be $R_{p2} = 5.625\ k\Omega$ and $R_i = .5664\ k\Omega$. Therefore it is obtained that $K_p = 5.625$ (Eq. 19) and $K_i = 1.7655 * 10^5$ (Eq. 23) for this operational amplifier. The plot comparing the outputs to inputs for the PI controller can be seen below.

Proportional-Integral Controller:

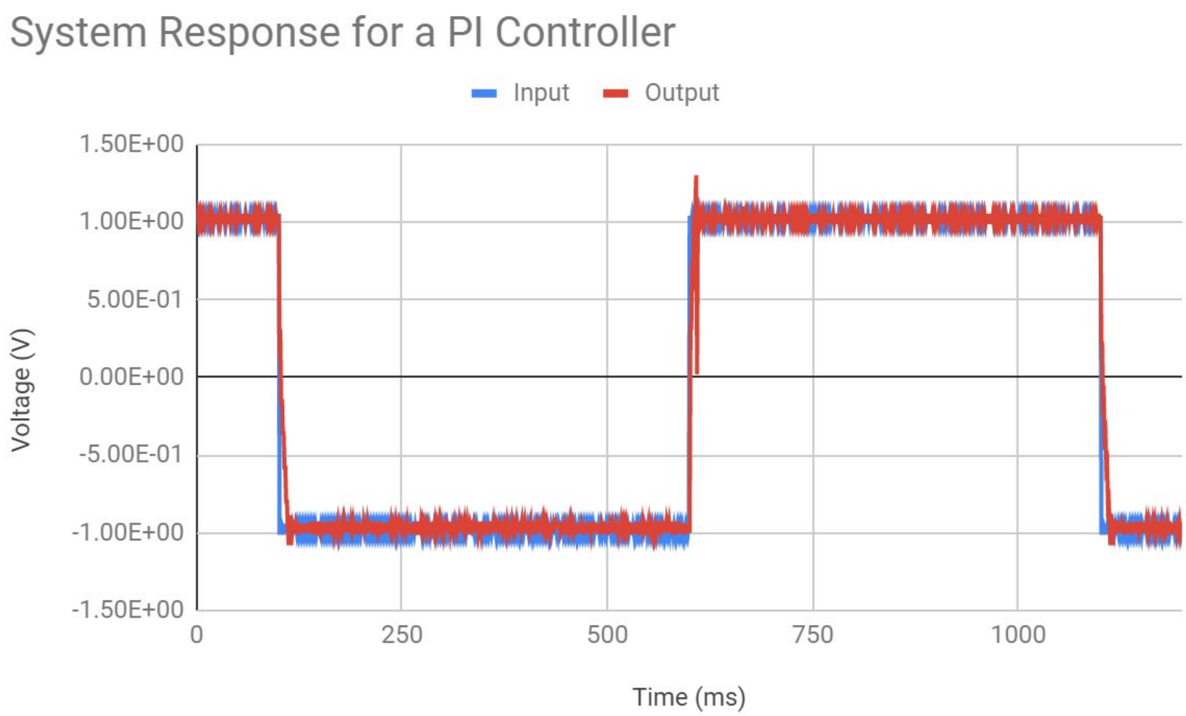

Fig. VI. System Response of a Proportional-Integral Controller

The amplitude for the output is 1.02 volts and the amplitude for the input is 1.04 volts (both of these values were obtained by averaging the data points at the top of the first square wave). The output is very close to the input and has a reasonable amount of noise. As expected, the system can be controlled to perform more optimally with a PI controller than with a PID controller.

For the proportional-integral-derivative (PID) controller the output was further optimized using potentiometer resistance values of $R_{p2} = 5.625\ k\Omega$, $R_i = .5664\ k\Omega$, and $R_{d2} = .282\ \Omega$. As such, it can be found that for the PID controller, $K_p = 5.625$ (Eq. 19), $K_i = 1.7655 * 10^5$ (Eq.

23), and $K_d = 2.82 * 10^{-10}$ (Eq. 28). The plot below shows how well the output matched the input.

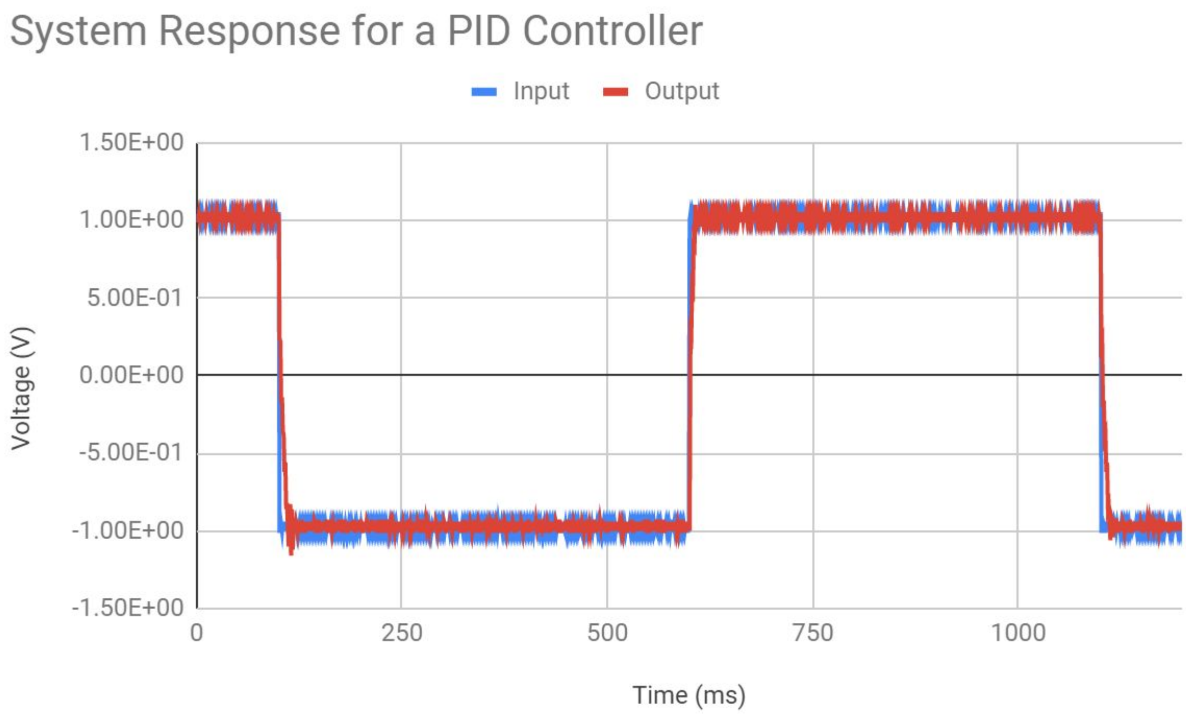

Fig. VII. System Response of a Proportional-Integral-Derivative Controller

The output amplitude is 1.04 volts and the input amplitude is also 1.04 volts (both of these values were obtained by averaging the data points at the top of the first square wave). This shows that a PID controller greatly increases the efficiency of a system while maintaining a reasonable level of noise. The results above are, as expected, better than the results from the P controller and slightly better than the PI controller.

It is also necessary to compare the experimental values to a theoretical data set. In this case, this was done by analyzing the previously derived transfer functions using MATLAB software. Below is a MATLAB plot for the proportional controller that compares the theoretical output with the experimental input and output along the entire waveform.

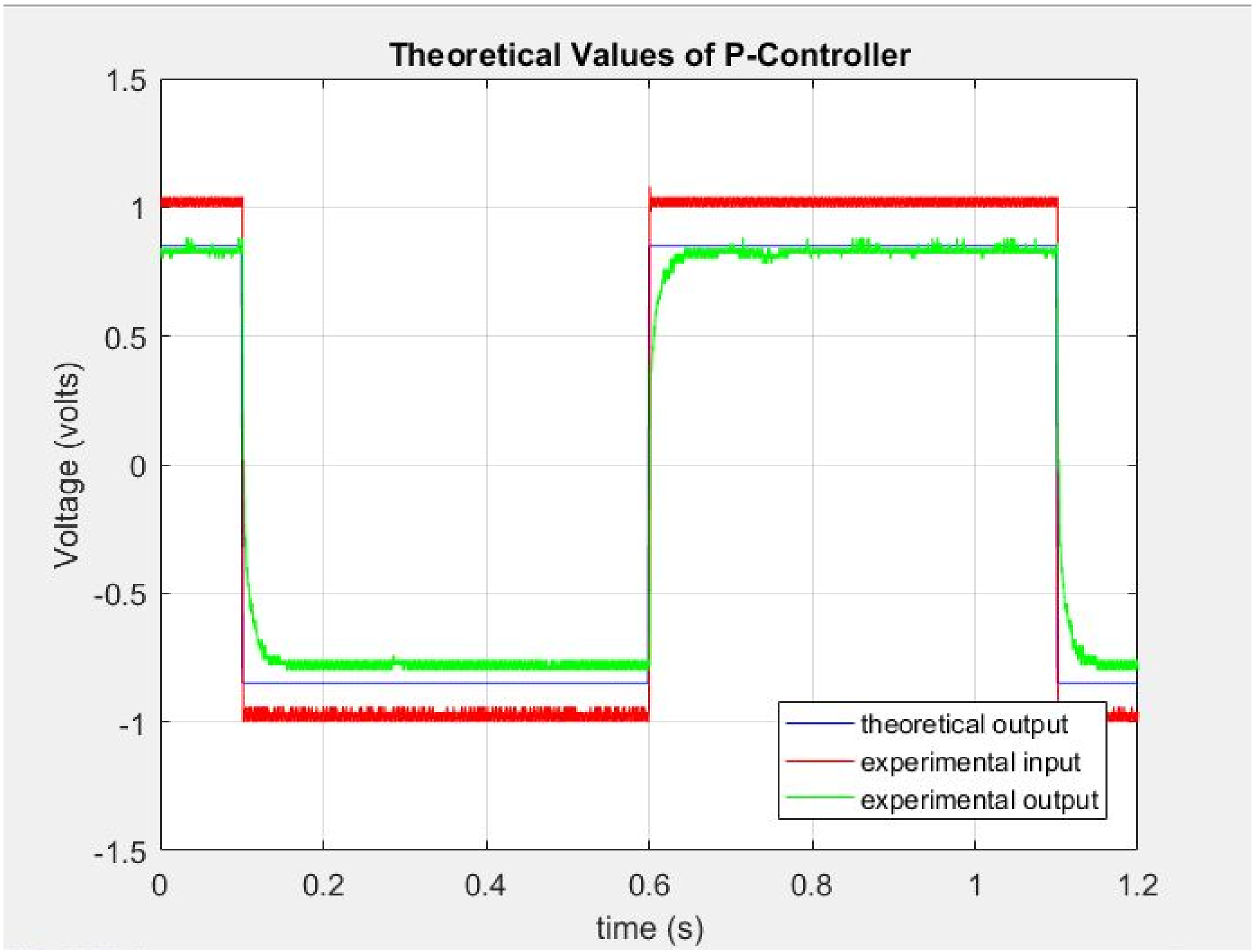

Fig. VIII. Comparing Theoretical and Experimental Values of a P-Controller

As is clearly visible in this plot, the experimental output does not perfectly agree with the theoretical output. Possible sources of this will be explored and verified in the discussion section of this lab report. Additionally, for the P-controller, a plot of just the theoretical results demonstrate that over time the voltage should settle down to a specific value. This plot can be seen as follows.

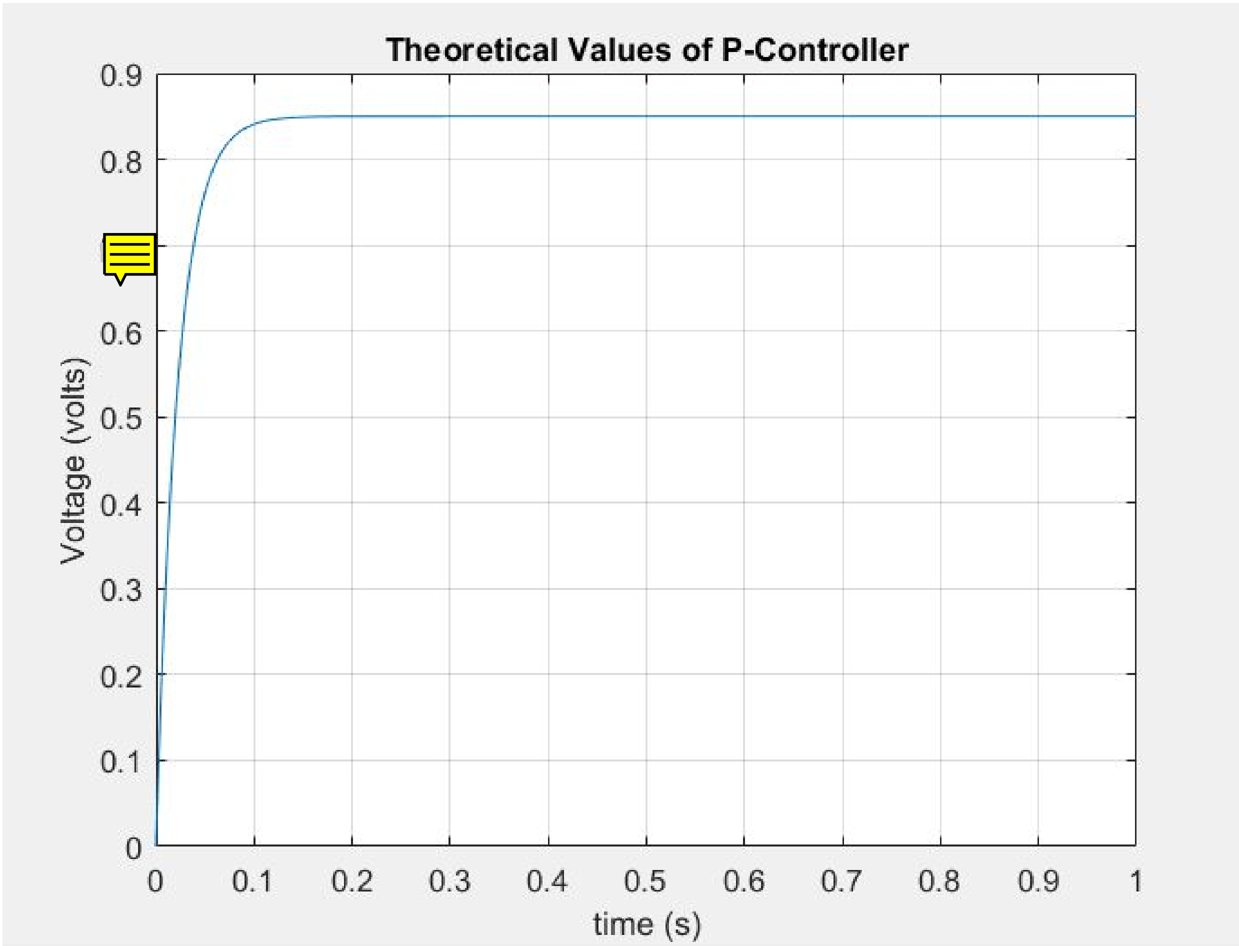

Fig. IX. Theoretical Values of a P-Controller

Where the theoretical voltage output settles at 0.8502 volts.

Next, for the PI controller, MATLAB was again used to produce a plot that compares theoretical output to experimental input and output. This can be seen below.

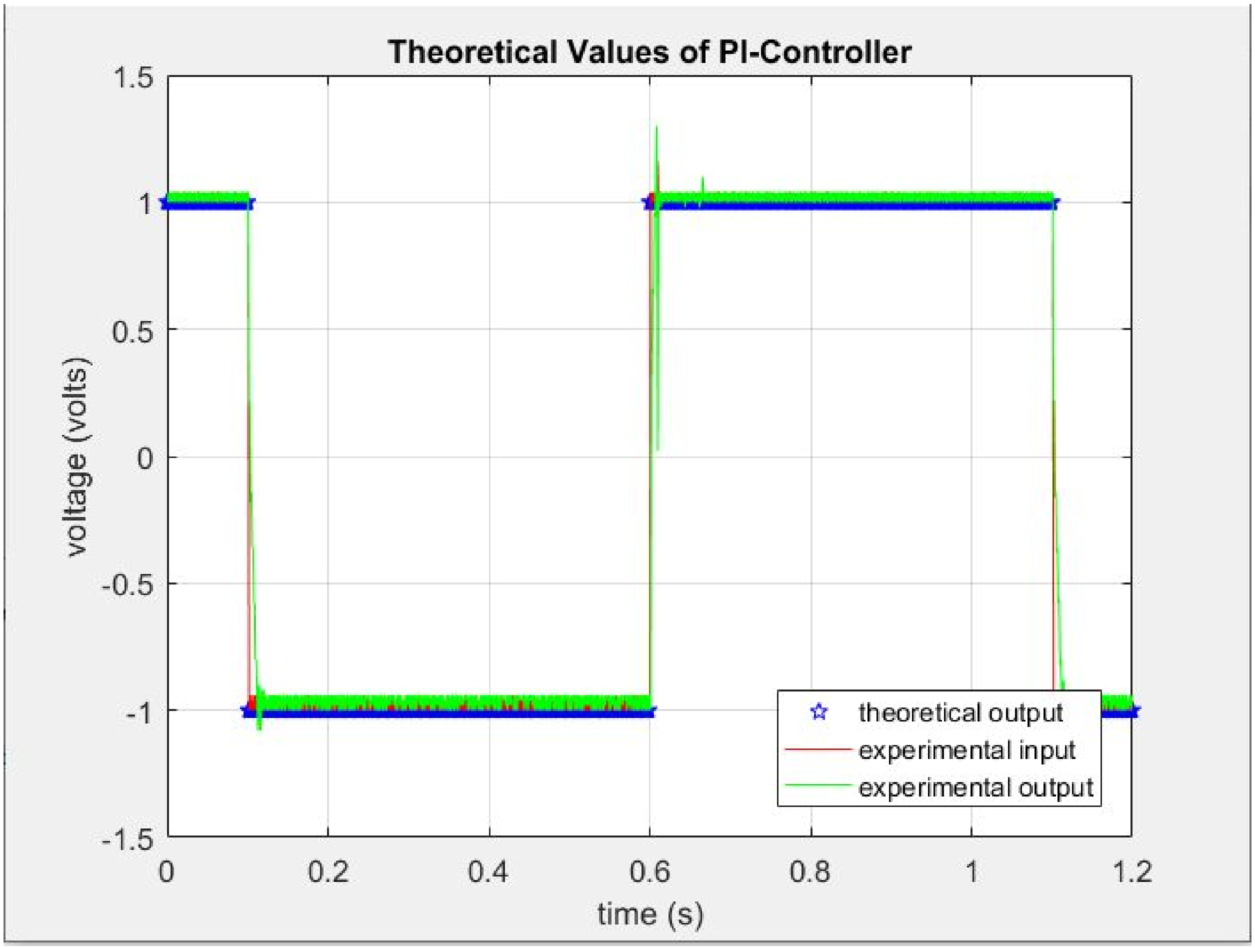

Fig. X. Comparing Theoretical and Experimental Values of a PI-Controller

For the PI-controller, it is abundantly clear that the theoretical and experimental output agree quite well, with them and the input lining up almost perfectly. There are a few fluctuations that will be discussed in the discussion however. Also, note that the theoretical output was modeled as starts so that it would be more visible in the plot, as a straight line it was covered by the experimental output too much to see. A plot demonstrating where the PI-controller can be seen below.

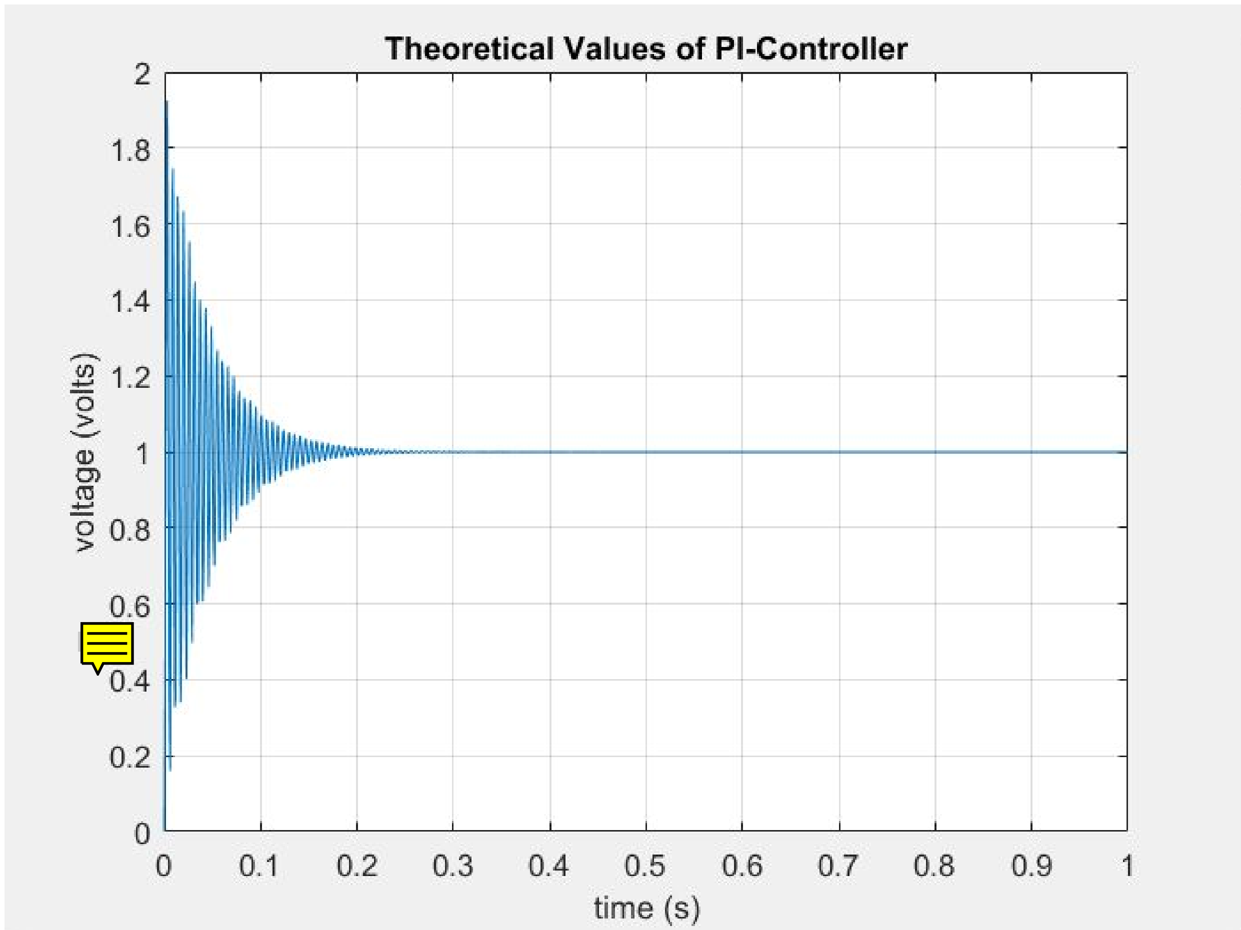

Fig. XI. Theoretical Values of a PI-Controller

In this plot, it is clear that the theoretical voltage of the PI controller settles at approximately 1 volt, which is the same as the input voltage. Rise time can be calculated by determining how long it takes the system to go from 0V to 1V, or from no input to the desired value.

Lastly, below is a plot comparing the experimental input and output voltages to the theoretical output, all of the PID controller.

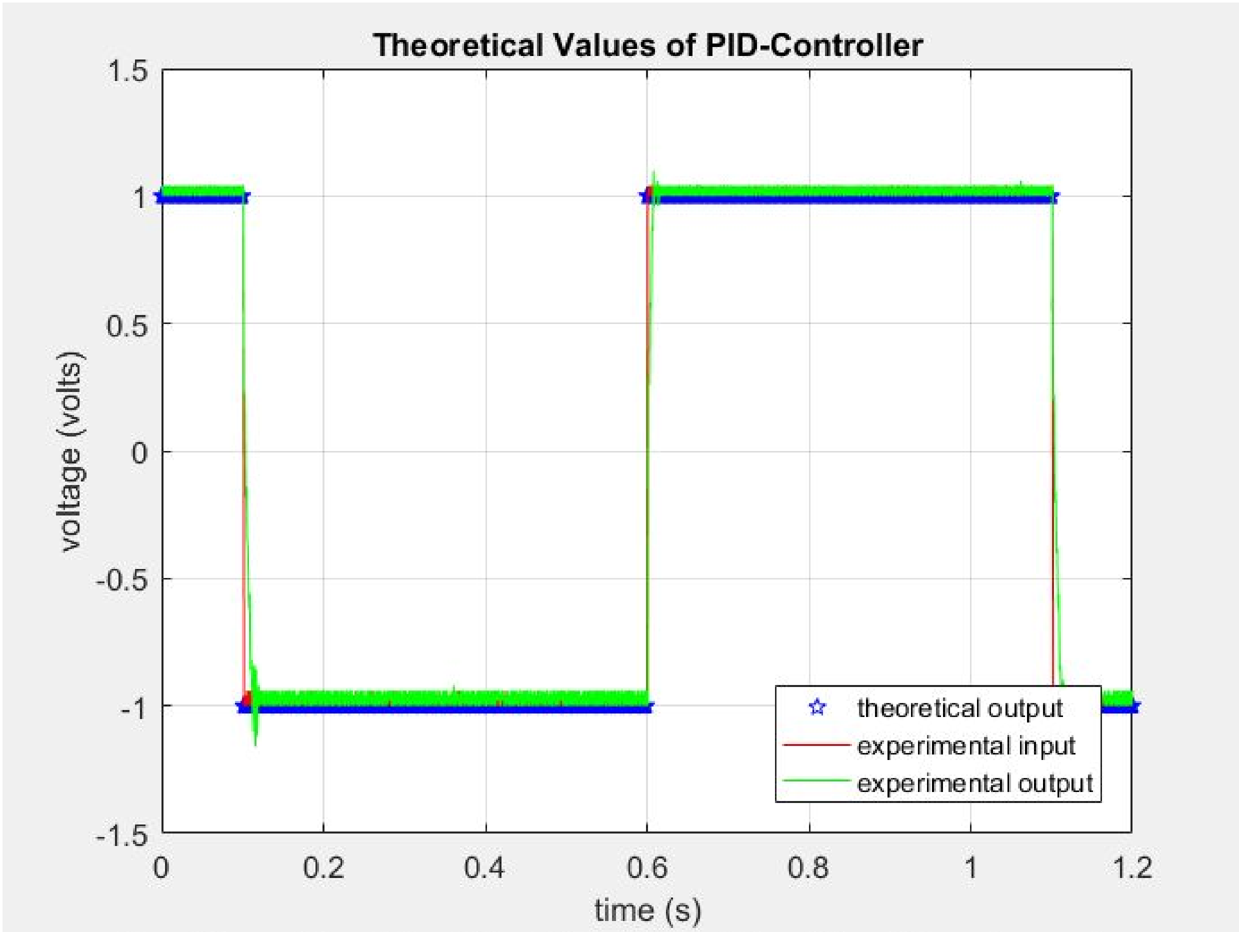

Fig.XII. Comparing Theoretical and Experimental Values of a PID-Controller

Just like the PI controller, the theoretical and experimental values match each other and the input values rather well for the PID controller. It would make sense that the PID controller would be at least as good as the PI controller, given that it is more advanced. This plot will be discussed in more detail in the discussion. However, once again take note of the fact that the theoretical output was plotted as stars to make it easier to see beneath the experimental values. Below is the MATLAB plot of just the theoretical output of the PID controller.

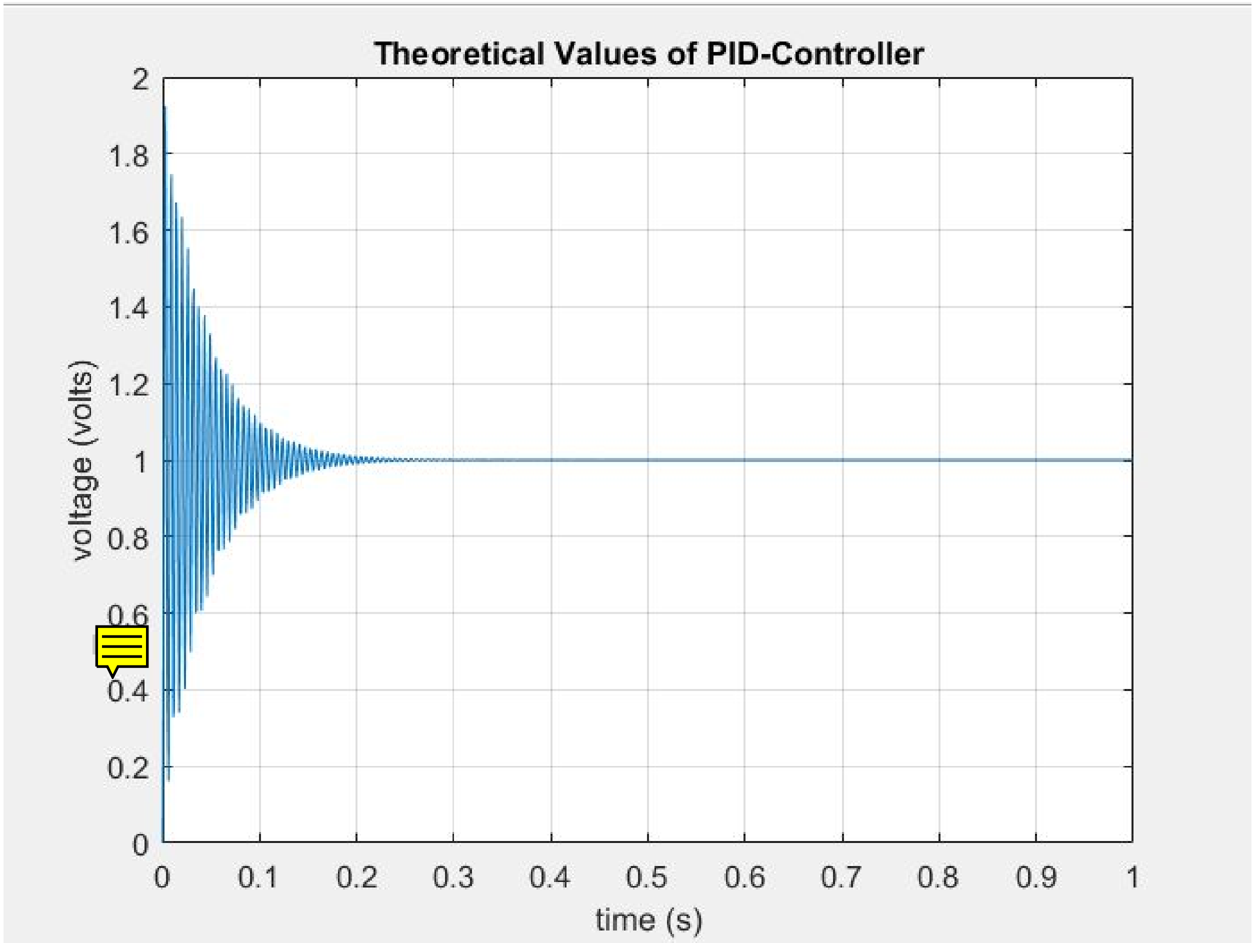

Fig. XIII. Theoretical Values of a PID-Controller

It settles to a value of about 1 volt, as expected from the PI plot as well as the input of the system being at 1 volt.

From these plots it is also relatively simple to calculate the percent overshoot, rise time, and settling time. Percent overshoot can be found from the excel plot of experimental values for the PID controller. The max amplitude of the experimental data is equal to -1.16V, and the accepted value is 1.00V. Using the percent overshoot equation, the following calculation happens.

PO=(1.16-1)/1*100%=16%

So as such the percent overshoot of the system is 16%, which is greater than the maximum allowable percent overshoot of 10%. Reasons for this will be discussed in the discussion. The rise time can be found by noting it is the time it takes for the motor to reach the accepted value. In this case, it takes the motor 9.5ms, so as such the rise time meets the rise time specification of less than 200ms. Our settling time was obtained from finding the point at which the data begins to stay within 2% of the final value, or in other words stays between 1.02V and 0.98V. The point at which this refers to is 157.6ms, which falls within the 2% settling time specification of less than 500ms.

Error Analysis:

In order to calculate the error of the output voltage of the system, the error associated with the amplifier gain, proportional, integral, and derivative op amps will all be considered. First, error in the electrical resistors and capacitors were determined, assuming their percent error to be the standard 5%. There values can be seen below.

- Error in 0.1K resistor
    - $u_{.1K} = 0.05(100\Omega) = 5\Omega$
- Error in 1K resistor
    - $u_{1K} = 0.05(1000\Omega) = 50\Omega$
- Error in 10K resistor
    - $u_{10K} = 0.05(10000\Omega) = 500\Omega$
- Error in 100K resistor
    - $u_{100K} = 0.05(100000\Omega) = 5000\Omega$

- <u>Error in 0.01 microfarad capacitor</u>
    - $u_{.01\mu} = 0.05(.01 * 10^{-6}f) = 5 * 10^{-10}f$
- <u>Error in the 0.1 microfarad capacitor</u>
    - $u_{.1\mu f} = 0.05(.1 * 10^{-6}f) = 5 * 10^{-9}f$
- <u>Error in the Amplifier Gain Potentiometer</u>
    - $u_{potentiometer} = 0.05(22000\ \Omega) = 1100\ \Omega$

From here, it is also required to know the uncertainty in each of the potentiometers used in the PID controller. These can be seen in the table below.

	P-Control error	PI-Control error	PID-Control error
R_{p2}	283.75 Ω	281.25 Ω	281.25 Ω
R_i	0 Ω	28.32 Ω	28.32 Ω
R_d	0 Ω	0 Ω	0.0141 Ω

Table 1. Error Values in the P, PI, and PID Potentiometers

From here it is easy to calculate the error of each op amp.

- Error in the Amplifier gain
    - $\Delta\frac{V_{out}}{V_{in}} = (\frac{V_{out}}{V_{in}})\sqrt{(\Delta R_{potentiometer}/R_{potentiometer})^2 + (\Delta R_{1K}/R_{1K})}$
    - $\Delta\frac{V_{out}}{V_{in}} = (\frac{12\ V}{0.52\ V})\sqrt{(1100\ \Omega/22000\ \Omega)^2 + (50\ \Omega/1000\ \Omega)^2} = 1.632\ V$
- Error in P-Controller Proportional Op amp
    - $\Delta\frac{V_{out}}{V_{in}} = (\frac{V_{out}}{V_{in}})\sqrt{(\Delta R_{p2}/R_{p2})^2 + (\Delta R_{p1}/R_{1p1})}$
    - $\Delta\frac{V_{out}}{V_{in}} = (\frac{0.81\ V}{0.8502\ V})\sqrt{(283.75\ \Omega/5675\ \Omega)^2 + (50\Omega/1000\ \Omega)^2} = 0.067\ V$

- Error in PI-Controller Proportional Op amp
  - $\Delta\frac{V_{out}}{V_{in}} = (\frac{V_{out}}{V_{in}})\sqrt{(\Delta R_{p2}/R_{p2})^2 + (\Delta R_{p1}/R_{1p1})}$
  - $\Delta\frac{V_{out}}{V_{in}} = (\frac{1.02\ V}{1.04\ V})\sqrt{(281.25\ \Omega/5625\ \Omega)^2 + (50\Omega/1000\ \Omega)^2} = 0.0694\ V$
- Error in PI Controller Integral Op amp
  - $\Delta\frac{V_{out}}{V_{in}} = (\frac{V_{out}}{V_{in}})\sqrt{(\Delta R_i/R_i)^2 + (\Delta C_i/C_i)}$
  - $\Delta\frac{V_{out}}{V_{in}} = (\frac{1.02\ V}{1.04\ V})\sqrt{(28.32\ \Omega/566.4\ \Omega)^2 + (5 * 10^{-10}f/.01 * 10^{-6}\ f)^2} = 0.0694\ V$
- Error in PID Controller Proportional Op amp
  - $\Delta\frac{V_{out}}{V_{in}} = (\frac{V_{out}}{V_{in}})\sqrt{(\Delta R_{p2}/R_{p2})^2 + (\Delta R_{p1}/R_{1p1})}$
  - $\Delta\frac{V_{out}}{V_{in}} = (\frac{1.04\ V}{1.04\ V})\sqrt{(281.25\ \Omega/5625\ \Omega)^2 + (50\Omega/1000\ \Omega)^2} = 0.0707\ V$
- Error in PID Controller Integral Op amp
  - $\Delta\frac{V_{out}}{V_{in}} = (\frac{V_{out}}{V_{in}})\sqrt{(\Delta R_i/R_i)^2 + (\Delta C_i/C_i)}$
  - $\Delta\frac{V_{out}}{V_{in}} = (\frac{1.04\ V}{1.04\ V})\sqrt{(28.32\ \Omega/566.4\ \Omega)^2 + (5 * 10^{-10}f/.01 * 10^{-6}\ f)^2} = 0.0707V$
- Error in PID Controller Derivative Op amp
  - $\Delta\frac{V_{out}}{V_{in}} = (\frac{V_{out}}{V_{in}})\sqrt{(\Delta R_{d1}/R_{d1})^2 + (\Delta C_d/C_d) + (\Delta R_{d2}/R_{d2})^2}$
  - $\Delta\frac{V_{out}}{V_{in}} = (\frac{1.04\ V}{1.04\ V})\sqrt{(5\ \Omega/100\ \Omega)^2 + (5 * 10^{-9}f/.1 * 10^{-6}\ f)^2 + (0.0141\ \Omega/.282\ \Omega)^2} = 0.0867\ V$

After this, the only error remaining is the error in the Laplace Transform, however that is extremely complicated to calculate and can be assumed to be minor anyway, and thus it can be ignored. Additionally, below are some errors in the provided motor parameters, using tolerances found in the lab manual[1].

- $u_{K_b} = 0.10(12\ V/KRPM) = 1.2\ V/KRPM$
- $u_{K_m} = 0.10(16.2\ OZ\ IN/A) = 1.62\ OZ\ IN/A$
- $u_{R_a} = 0.15(11.5\ \Omega) = 1.725\ \Omega$

And this is all of the error analysis needed for this lab report.

IV. Discussion

Below are terms that briefly describe $G_c(s)$, all previously found in the results section of the lab report.

Value	P-Controller	PI-Controller	PID-Controller
K_p	5.675	5.625	5.625
K_i	N/A	$1.7655 * 10^5$	$1.7655 * 10^5$
K_d	N/A	N/A	$2.82 * 10^{-10}$

Table 2. Values of the Coefficients of a PID Controller

For the proportional controller, the value of $K_p = 5.675$ resulted from the potentiometer resistance value of $R_{p2} = 5.675k\Omega$. The amplitude of the resulting square wave from the proportional controller was previously stated to be 0.81V, while the input was at an amplitude of 1.04V. This means that the input and output do not match, which was as expected for a proportional controller because it only takes the present time into account, and not the past and/or future like the PI and PID controllers do. If the value of the potentiometer R_{p2} was set any higher, the system would have been unstable and thus undesirable. Additionally, based on the plot comparing the results, made in MATLAB, it was found that the theoretical and experimental results do not meet exactly, specifically seen below the x-axis.The average theoretical amplitude for this controller was 0.8502V, while the average of the experimental was a maximum amplitude of 0.81V, which is reasonably close, and a minimum amplitude of 0.78V, which can be considered too far away from the accepted value. There are numerous cases for why this may be the case, ranging from the waveform generator not working properly to the proportional op amp not being set up correctly. The current theory however, is that the potentiometer used for the amplifier gain was set to a value of 21.12 kilohms instead of the necessary 22 kilohms. This would have occurred if the potentiometer knob was moved a little bit during troubleshooting attempts at fixing the circuit. The effect of this is that the amplifier gain would be 11.4V instead of the typical 12V. If this new amplifier gain is accounted for in the theoretical and experimental comparison, the following plot occurs.

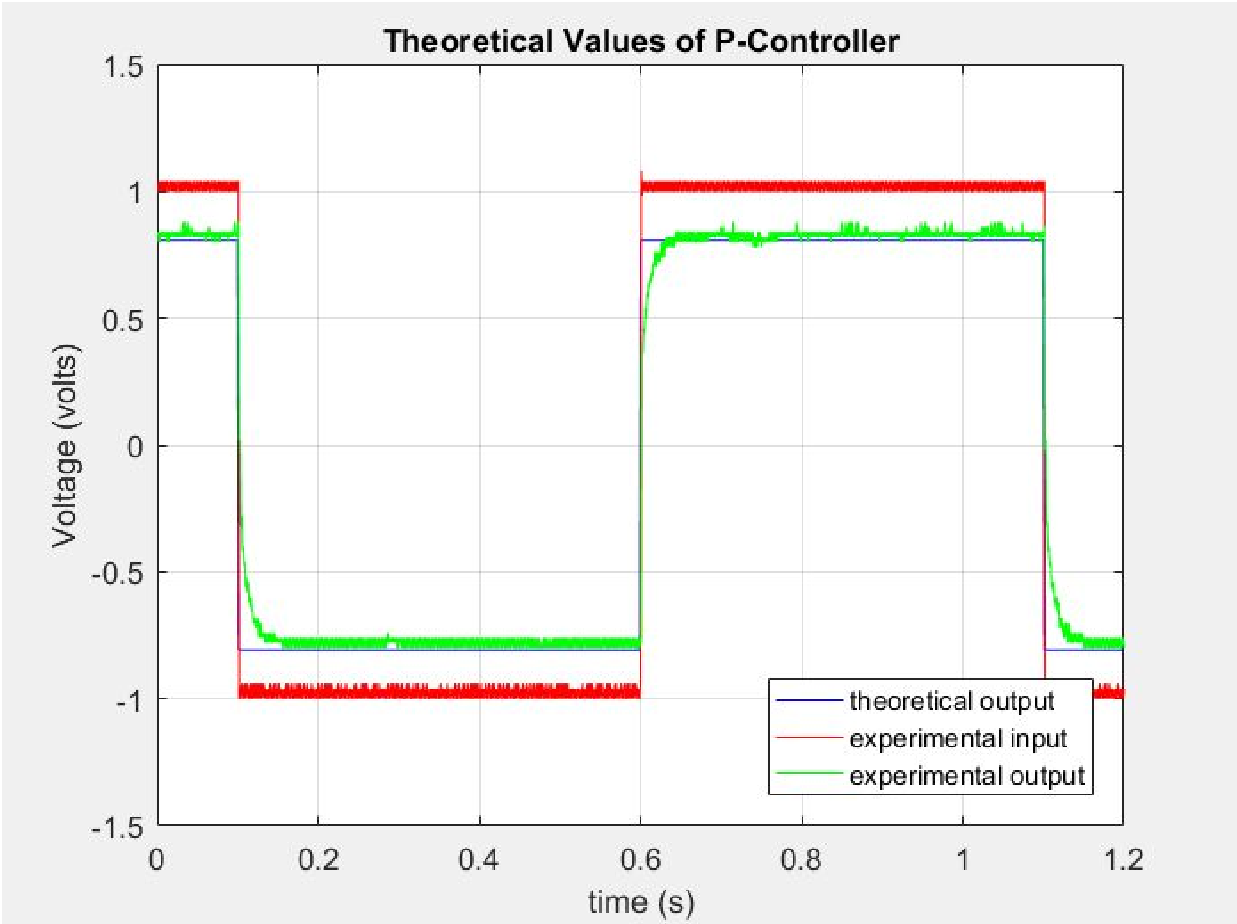

Fig. XIV. Improved Comparison of Theoretical and Experimental Values of a P-Controller

In which the theoretical and experimental output voltage values match each other much more closely as compared to when the voltage gain of the system was set to 12V. Therefore, this seems like a reasonable explanation of the problem, and as such by taking this change into account the results seem valid.

The proportional-integral circuit had values of $K_p = 5.625$ and $K_i = 1.7655 * 10^5$. These values resulted from measuring potentiometer values of $R_{p2} = 5.625k\Omega$ and $R_i = 566.4\Omega$. The data had an average amplitude of 1.02V, while the input had an average amplitude of 1.04V, so it is safe to say that the input and output match fairly well, which is as desired. By comparing the experimental and theoretical outputs, it is seen that their amplitudes are 1.02V and 1.00V respectively, which seems quite reasonable. Therefore it can be said that the results of the PI controller are reasonably correct.

As for the PID controller, the values were $K_p = 5.625$, $K_i = 1.7655 * 10^5$, and $K_d = 2.82 * 10^{-10}$. The potentiometer values were $R_{p2} = 5.625k\Omega$, $R_i = 566.4\Omega$, and $R_{d2} = 0.282k\Omega$. The experimental output amplitude was found to be 1.04V from the excel plots, and the experimental input amplitude was also found to be 1.04V. Therefore, these results agree. The theoretical output was found to be 1.0V, so therefore it does not meet the experimental results perfectly, but is still close enough to be considered reasonable by being a 4% error.

In addition, the PID controller had several specifications that had to be met. Restated, these specifications were to have a percent overshoot of less than 10%, a rise time of no more than 200ms, and a 2% settling time of 500ms. The values for these specifications were found in the results section, but can be seen in the table below for convenience in the discussion.

Percent Overshoot	Rise Time	Settle time (2%)
16%	9.5ms	157.6ms

Table 3. Specifications of the Motor System

The rise time was found to be 9.5ms, which is notably less than the 200ms specification. Additionally, the 2% settling time was found to be at a value of 157.6ms, which is less than the 500ms specification . Therefore, the calculated value of settling time and rise time are less than their respective specifications, and as such can be considered a success. Where issues begin to arise however, is with the percent overshoot, which is greater than the specificatio. This can be most likely attributed to noise, as by looking at the Excel plot of the PID controller, sudden spikes all occur irregularly. If noise is ignored, and thus the 1.16V max amplitude is ignored, then the max amplitude can be found to be 1.04V. This value is found because it occurs regularly at each time the motor changes its direction of rotation.. If percent overshoot is found with this max amplitude, the equation is as follows.

PO=(1.04-1)/1*100%=4%

The percent overshoot is found to be 4% by taking the no noise scenario into account, and as such the percent overshoot of 10% or less is met. Therefore, it is reasonably safe to assume that noise was the cause of the discrepancy in the data.

To get a full grasp of how this PID control system works, it is necessary to compare these values to that of the lab 1 values. Comparing the terms of the PID function can be seen below.

	K_p	K_i	K_d
Lab 1: P-Controller	3	N/A	N/A
Lab 2: P-Controller	5.675	N/A	N/A
Lab 1: PI-Controller	3	$6.5646 * 10^3$	N/A
Lab 2: PI-Controller	5.625	$1.7655 * 10^5$	N/A
Lab 1: PID Controller	3	$6.5646 * 10^3$	$9.39 * 10^{-10}$
Lab 2: PID Controller	5.625	$1.7655 * 10^5$	$2.82 * 10^{-10}$

Table 4. Comparing Lab 1 and Lab 2 PID Coefficients

From this table, it is clear that the values between labs as a whole were relatively close to each other. Both labs K_p and K_d terms are reasonably close to each other, and are both on the same order of magnitude. The main difference between the labs is the K_i terms. These terms are $6.5646 * 10^3$ and $1.7655 * 10^5$ for lab 1 and 2 respectively. Also, note that these terms are the same for each lab's PI and PID controller. The difference between the integral coefficient for the two controllers is on an order of magnitude of 2, which is a fairly large difference. As such, the difference in these values should result in notable differences in how well each system met the design specifications. A table of these values for each PID controller can be seen below. Also, note that this table consists of only original values obtained, not of the values found via mathematical justifications.

	Percent Overshoot	Rise Time	Settling Time
Lab 1	2.6166%	155.2ms	429.2ms
Lab 2	16%	9.5ms	157.6ms

Table 5. Comparing Specifications Between Lab 1 and 2

The percent overshoot of the motor in lab 1 was relatively small at 2.6166%, easy fulfilling the specification, while the overshoot of lab 2 reached a value of 16%, which is well over the 10% maximum. As such, it can be said that lab 1 had the more desirable percent overshoot. As for the rise time, lab 1 had a rise time of 155.2ms, while lab 2 had a significantly shorter rise time of 9.5ms. Both rise times fulfill the 200ms specification. As for the 2% settling time, lab 1 has a value of 429.2ms, while lab 2 has one of 157.6ms. Both of these settling times meet the specification of less than 500ms, with lab 1 having a much longer settling time than lab 2. Based on these comparisons, it makes logical sense to say that the digital lab 1 system overall has a slower response time, but by extension is also more accurate. This can be explained because despite a rather long rise and settling time, the percent overshoot is rather small. On the other hand, the lab 2 analog system is a faster, but much less accurate system. Proof of this is seen in the very fast response and settling time, but very high percent overshoot. Due to the fact that the lab one design meets all three design specifications, while the lab two design does not, it seems to be that the digital circuit is overall better than the analog circuit in this case. The issues with the analog circuit in lab 2 may be a result of the circuit being much larger and more complex, resulting in more noise and as such a less accurate response. In addition, the digital circuit may be slower because LabView has to obtain, process, and produce the data which takes more time than the data collection in lab 2, which can be done quickly through the use of the oscilloscope.

Any other error in this lab is small enough to where it does not affect the data in any significant way. It can be attributed to factors such as noise, the natural resistance of the wires and breadboards, and the resistors dissipating heat over time. In addition, friction from the rotation of the motor can also be a source of error. There also could have been an error in the signal produced by the waveform generator and data read by the oscilloscope. In the future, it would be possible to improve these uncertainties by performing repeated trials and by using newer, higher quality equipment.

Conclusion

- The resistance of the potentiometer that optimized the performance of the P controller was $R_{p2} = 5.675\ k\Omega$, which gives $K_p = 5.675$ (Eq. 19). The resistance values that optimized the performance of the PI controller were $R_{p2} = 5.625\ k\Omega$ and $R_i = .5664\ k\Omega$ which gave $K_p = 5.625$ (Eq. 19) and $K_i = 1.7655 * 10^5$ (Eq. 23), respectively. The resistance values that optimized the performance of the PID controller were found to be $R_{p2} = 5.625\ k\Omega$, $R_i = .5664\ k\Omega$, and $R_{d2} = .282\ \Omega$ which gave $K_p = 5.625$ (Eq. 19), $K_i = 1.7655 * 10^5$ (Eq. 23), and $K_d = 2.82 * 10^{-10}$ (Eq. 28), respectively.
- The experimental results yielded a rise time (9.5ms) and a settling time (157.6ms) for the PID speed control system that fell with the constraints of the design specifications, which were less than 200ms and 500ms, respectively. The percent overshoot, however came out to be 16%, which is significantly higher than the allowed overshoot of 10%.
- Based on the Excel data and the MATLAB plots, the PID system had a more optimal performance than the P and PI systems and all systems reasonably matched the theoretical results.
- The error in the percent overshoot can most likely be attributed to noise in the system and is recalculated to be 4%, which is within the specified 10%.
- Comparing lab 1 and lab 2 results, the control variables all reasonably match, excluding K_d, which had a difference on the order of 2 magnitudes between the two experiments. This points to the differences in how well each system met the design specifications. The system from lab 2 had a quicker response time, but the system from lab 1 was more accurate.

- Since the system from lab 1 met all the design specifications while lab 2 did not, the digital circuit can be said to have a better performance than the analog circuit.
- Any other error in this lab can be attributed to uncertainties associated with the voltage gain due to factors such as the resistance in the electrical components, the DAQ, and any other losses within the system.

References

[1] Machtay, Noah, *PID Speed Control of a Turntable - MEC 411 Lab 2*, Stony Brook University (2019).

[2] Machtay, Noah. *MEC 220.01 Practical Electronics Mechanical Engineers,* Stony Brook University, Stony Brook, NY. Lecture (2017).

[3] Franklin, Gene F., David J. Powell and Abbas Emami-Naeini, *Feedback Control of Dynamic Systems,* Upper Saddle River: Pearson Higher Education (2010).

Appendix

```
close all
clear
clc
excelp=xlsread('Lab2_p');
exceli=xlsread('Lab2_pi');
exceld=xlsread('Lab2_pid');
%general constants, as given in lab manual
Km=16.2; %oz-in/A
Ra=11.5; %ohm
La=0;
J=2.5; %oz-in2
```

```
b=0;

Kb=12; %V/KRPM

Kt=12; %V/KRPM


%resistance values

Rp1=1000; %ohms

Rpp=5675; %ohm

Rpi=5625; %ohm

Rii=566.4; %ohm

Rpd=Rpi; %ohm

Rid=Rii; %ohm

Rdd1=100; %ohm

Rdd2=.282; %ohm


%capacitors

Ci=0.01*10^-6; %farads

Cd=0.1*10^-6; %farads


%K-values

Kpp=Rpp/Rp1; %proportional Kp

Kpi=Rpi/Rp1; %integral Kp

Kii=1/(Rii*Ci); %integral Ki

Kpd=Rpd/Rp1; %differential Kp

Kid=1/(Rid*Ci); %differential Ki

Kdd=(Rdd1^-1*Rdd2)*Cd; %differential Kd


syms s

G=Km/(Ra*(J*s+b)+Kb*Km); %value of G(s)

t=0:.001:1.2; %time range
```

```
tp=1:.001:2.2;
[c,r]=size(t);
%p-controller
Gcp=Kpp; %proportional TF
T=(Gcp*G)/(1+(Kt*Gcp*G)); %transfer function of the system
yp=(1/s)*T; %output
yp_val=ilaplace(yp)

%loop for plot
wp=zeros(c,r); %zero matrix for omega
win=excelp(:,2);
wot=excelp(:,3);
for i=1:r
    if tp(i)>.1+1 && tp(i)<.6+1
        wp(i)=-11.4*(227/3204 - (227*exp(-(129762*tp(i))/2875))/3204);
    elseif tp(i)>1.1+1
        wp(i)=-11.4*(227/3204 - (227*exp(-(129762*tp(i))/2875))/3204);

    else
        wp(i)=11.4*(227/3204 - (227*exp(-(129762*tp(i))/2875))/3204);

    end
end
figure(1)
plot(t,wp,'b', t,win,'r', t,wot,'g')
legend('theoretical output', 'experimental input', 'experimental output', 'location', 'southeast')
grid on
xlabel('time (s)')
ylabel('Voltage (volts)')
```

```
title('Theoretical Values of P-Controller')


%pi controller
Gci=Kpi+Kii/s; %Gc of integral controller
T=(Gci*G)/(1+(Kt*Gci*G)); %transfer function of the system
yi=(1/s)*T; %output
yi_val=ilaplace(yi)


%loop for plot
wi=zeros(r,c); %zero matrix for omega
win=exceli(:,2);
wot=exceli(:,3);
for i=1:r
    if tp(i)>.1+1 && tp(i)<.6+1


        wi(i)=-12*(1/12 - (exp(-(12879*tp(i))/575)*(cos((9*5230017748622982111^(1/2)*tp(i))/18841600) -
(10911744*5230017748622982111^(1/2)*sin((9*5230017748622982111^(1/2)*tp(i))/18841600))/174333924954099 4037))/12);
    elseif tp(i)>1.1+1
        wi(i)=-12*(1/12 - (exp(-(12879*tp(i))/575)*(cos((9*5230017748622982111^(1/2)*tp(i))/18841600) -
(10911744*5230017748622982111^(1/2)*sin((9*5230017748622982111^(1/2)*tp(i))/18841600))/1743339249540994037))/12);


    else
        wi(i)=12*(1/12 - (exp(-(12879*tp(i))/575)*(cos((9*5230017748622982111^(1/2)*tp(i))/18841600) -
(10911744*5230017748622982111^(1/2)*sin((9*5230017748622982111^(1/2)*tp(i))/18841600))/1743339249540994037))/12);


    end
end
figure(2)
plot(t,wi,'bp', t,win,'r', t,wot,'g')
```

```
legend('theoretical output', 'experimental input', 'experimental output', 'location', 'southeast')

grid on

xlabel('time (s)')

ylabel('voltage (volts)')

title('Theoretical Values of PI-Controller')


%PID controller

Gcd=Kpd+Kid/s+Kdd*s; %Gc of integral controller

T=(Gcd*G)/(1+(Kt*Gcd*G)); %transfer function of the system

yd=(1/s)*T; %output

yd_val=ilaplace(yd)


%loop for plot

wd=zeros(r,c); %zero matrix for omega

win=exceld(:,2);

wot=exceld(:,3);

for i=1:r

   if tp(i)>.1+1 && tp(i)<.6+1


      wd(i)=-12*(1/12 -
(1325859730297874022400*exp(-(1187878084626526576312332*tp(i))/5303798537161556519299)*(cos((589824*9648334356
586832968309938221641267027 8^(1/2)*tp(i))/5303798537161556519299) -
(1576779800050963835678 7*964833435658683296830993822164126702 78^(1/2)*sin((589824*964833435658683296830993
82216412670278^(1/2)*tp(i))/5303798537161556519299))/10820836703158718574744555476004529573273600))/159113956
11484669557897);

   elseif tp(i)>1.1+1

      wd(i)=-12*(1/12 -
(1325859730297874022400*exp(-(1187878084626526576312332*tp(i))/5303798537161556519299)*(cos((589824*9648334356
586832968309938221641267027 8^(1/2)*tp(i))/5303798537161556519299) -
```

```
(1576779800050963835678 7*96483343565868329683099382216412670278^(1/2)*sin((589824*96483343565868329683099382216412670278^(1/2)*tp(i))/5303798537161556519299))/10820836703158718574744555476004529573273600))/15911395611484669557897);

    else
        wd(i)=12*(1/12 -
(13258597302978740224 00*exp(-(1187878084626526576312 32*tp(i))/5303798537161556519299)*(cos((589824*96483343565868329683099382216412670278^(1/2)*tp(i))/5303798537161556519299) -
(1576779800050963835678 7*96483343565868329683099382216412670278^(1/2)*sin((589824*96483343565868329683099382216412670278^(1/2)*tp(i))/5303798537161556519299))/10820836703158718574744555476004529573273600))/15911395611484669557897);
    end
end
figure(3)
plot(t,wd,'bp', t,win,'r', t,wot,'g')
legend('theoretical output', 'experimental input', 'experimental output', 'location', 'southeast')
grid on
xlabel('time (s)')
ylabel('voltage (volts)')
title('Theoretical Values of PID-Controller')
```

```
yp_val =

227/3204 - (227*exp(-(129762*t)/2875))/3204

yi_val =
```

```
1/12 - (exp(-(12879*t)/575)*(cos((9*5230017748622982111^(1/2)*t)/18841600) -
(10911744*5230017748622982111^(1/2)*sin((9*5230017748622982111^(1/2)*t)/18841600))/1743339249540994037))/12
```

```
yd_val =
```

```
1/12 -
(86891543284801471932006400*exp(-(7784877815408404570520420352*t)/347566173801948693447322129)*(cos((1660944
384*52253747978631089591047288262600449269618^(1/2)*t)/347566173801948693447322129) -
(7233720972495037286865864039*52253747978631089591047288262600449269618^(1/2)*sin((1660944384*522537479786
31089591047288262600449269618^(1/2)*t)/347566173801948693447322129))/11551985490192992974960471948602388132
8456315699200))/1042698521405846080341966387
```

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Comment Summary

Page 3

1. Digital circuit for PID controller.
2. Analog PID controller should perform faster than the digital PID controller.

Page 12

3. The analysis for differential amplifier and voltage follower is missing.

Page 19

4. Needs plot of a square wave.

Page 21

5. Plot for square wave.

Page 23

6. Plot for a square wave.

Page 35

7. Please provide proper recommendations.

   Conclusion can be more concise.