Robotics Lecture Notes **Spring 2018**

Lecture 6: Preliminaries of Grasping

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6.1 Introduction

In this lecture, we will go over some preliminary topics in robotic grasping. We consider a multi-fingered hand grasping an object. The purpose of grasping may be to immobilize the object when transferring from one configuration to another or to further manipulate the object with respect to the hand. We will assume both the object and the fingers to be rigid bodies. Furthermore, we will assume that the contact between the object and the finger is a point contact. Given the geometry of the object, the link lengths, types of joints, and geometry of the fingers, we will try to answer two fundamental questions in grasp analysis (a) What is the relation of the net force and moment acting on the object to the torques applied at the finger joints? (b) What is the relation of the net linear and angular velocity of the grasped object to the joint rates of the fingers?

6.2 Relation between Contact Wrench and Body Wrench

A wrench is force-moment pair, i.e., $\mathbf{F} = \begin{bmatrix} \mathbf{f} \\ \tau \end{bmatrix}$ where $\mathbf{f} \in \mathbb{R}^3$ is a force and $\tau \in \mathbb{R}^3$ is a moment. Consider, the rigid body in Figure 6.1 with a force-moment pair or wrench acting at the origin of frame $\{C_i\}$. Note that for now, we will not think about how the wrench arises (in our context it may arise from the robot finger pushing against a object). In grasping, the frame $\{C_i\}$ is called the contact frame at the ith contact, where the number of contact points is n_c, i.e., $i = 1, \ldots, n_c$. Let $\mathbf{g}_{bc_i} = \begin{bmatrix} \mathbf{R}_{bc_i} & \mathbf{p}_{bc_i} \\ \mathbf{0} & 1 \end{bmatrix}$ be the transformation matrix of the contact frame $\{C_i\}$ with respect to the body frame $\{B\}$, where $\mathbf{R}_{bc_i}$ is the rotation matrix of the contact frame $\{C_i\}$ with respect to the body frame $\{B\}$ and $\mathbf{p}_{bc_i}$ is the position vector of origin of $\{C_i\}$ with respect to the body frame $\{B\}$. Let $\mathbf{f}_{c_i}$ be the force acting at the origin of frame $\{C_i\}$ and expressed in the frame $\{C_i\}$. Let $\boldsymbol{\tau}_{c_i}$ be the moment acting on the body about the axis of frame $\{C_i\}$ and expressed in the frame $\{C_i\}$. In short, we will write $\mathbf{F}_{c_i} = \begin{bmatrix} \mathbf{f}_{c_i} \\ \boldsymbol{\tau}_{c_i} \end{bmatrix}$ is the wrench acting at the origin of $\{C_i\}$ and expressed in the frame $\{C_i\}$. Let $\mathbf{F}_{b_i} = \begin{bmatrix} \mathbf{f}_{b_i} \\ \boldsymbol{\tau}_{b_i} \end{bmatrix}$ be the wrench acting at the origin of $\{B\}$ and expressed in the frame $\{B\}$ whose effect is equivalent to the wrench $\mathbf{F}_{c_i}$. Then,

$$\mathbf{f}_{b_i} = \mathbf{R}_{bc_i} \mathbf{f}_{c_i} \tag{6.1}$$

$$\begin{aligned} \boldsymbol{\tau}_{b_i} &= \mathbf{R}_{bc_i} \boldsymbol{\tau}_{c_i} + \mathbf{p}_{bc_i} \times \mathbf{R}_{bc_i} \mathbf{f}_{c_i} \\ &= \mathbf{R}_{bc_i} \boldsymbol{\tau}_{c_i} + \hat{\mathbf{p}}_{bc_i} \mathbf{R}_{bc_i} \mathbf{f}_{c_i} \end{aligned} \tag{6.2}$$

The Equations (6.1) and (6.2) can be written compactly in a matrix vector form as

$$\mathbf{F}_{b_i} = \begin{bmatrix} \mathbf{f}_{b_i} \\ \boldsymbol{\tau}_{b_i} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{bc_i} & \mathbf{0} \\ \hat{\mathbf{p}}_{bc_i} \mathbf{R}_{bc_i} & \mathbf{R}_{bc_i} \end{bmatrix} \begin{bmatrix} \mathbf{f}_{c_i} \\ \boldsymbol{\tau}_{c_i} \end{bmatrix} \tag{6.3}$$

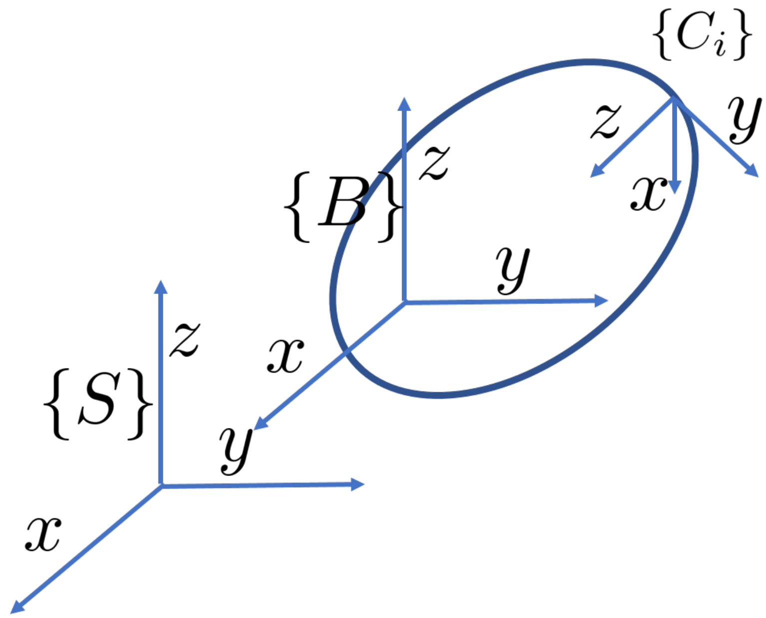

Figure 6.1: Schematic sketch of an object with the body frame, $\{B\}$, spatial frame, $\{S\}$, and the contact frame $\{C_i\}$

By direct calculation, it can be seen that $Ad^T_{(\mathbf{g}^{-1}_{bc_i})} = \begin{bmatrix} \mathbf{R}_{bc_i} & \mathbf{0} \\ \hat{\mathbf{p}}_{bc_i}\mathbf{R}_{bc_i} & \mathbf{R}_{bc_i} \end{bmatrix}$. Therefore,

$$\boxed{\mathbf{F}_{b_i} = Ad^T_{(\mathbf{g}^{-1}_{bc_i})}\mathbf{F}_{c_i}} \tag{6.4}$$

Let $\mathbf{F}_b = \begin{bmatrix} \mathbf{f}_b \\ \boldsymbol{\tau}_b \end{bmatrix}$ be the net body wrench due to the contact wrench, i.e., net wrench acting at the origin of $\{B\}$ and expressed in the frame $\{B\}$ whose effect is equivalent to all the contact wrenches. Therefore,

$$\boxed{\mathbf{F}_b = \sum_{i=1}^{n_c} Ad^T_{(\mathbf{g}^{-1}_{bc_i})}\mathbf{F}_{c_i}} \tag{6.5}$$

where n_c is the number of contact points.

Up to this point we have just assumed that there is some contact wrench acting at a contact point (without making any assumptions about how the contact wrench is produced). Furthermore, we have assumed that there is some way in which the contact frame is defined without going into the details. We will now look at the common restrictions that are there on the contact wrench as well as one way of choosing the contact frame.

6.2.1 Choice of Contact Frame

The ease with which the contact frame can be chosen depends on our knowledge about the object. Frequently, it is assumed that the equation of the surface of the object is given (either as an implicit function or as a parametric function). If the equation of the object surface is known, a reference frame can be chosen with the aid of basic differential calculus. If instead, we assume that we know a $3D$ point cloud (or a $2D$ image of the object, as obtained by a camera), the choice of the contact frame requires more work. For simplicity, we will assume here that the object is described an implicit surface model, i.e., the equation of the surface of the object in the body frame $\{B\}$ is given by $h(x, y, z) = \beta$, where β is some constant. Let $\mathbf{q} = (x, y, z)$ be any point on the object and $\mathbf{q}_c = (x_c, y_c, z_c)$ be the coordinates of a contact point on the object in the body frame $\{B\}$. Then the gradient of the object at the point $\mathbf{q}_c$, denoted by $\nabla_{\mathbf{q}} f(\mathbf{q}_c)$ is

$$\nabla_{\mathbf{q}} f(\mathbf{q}_c) = \begin{bmatrix} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \\ \frac{\partial h}{\partial z} \end{bmatrix}_{\mathbf{q}=\mathbf{q}_c}$$

where the subscript $\mathbf{q} = \mathbf{q}_c$ denotes that the partial derivatives are evaluated at the contact point $\mathbf{q}_c$. The z-axis of the ith contact frame is chosen as

$$\mathbf{n}_i = -\frac{\nabla_{\mathbf{q}} f(\mathbf{q}_c)}{\|\nabla_{\mathbf{q}} f(\mathbf{q}_c)\|}$$

To choose an orthogonal frame, note that, any vector $\mathbf{a}$ that is orthogonal to $\mathbf{n}_i$ will satisfy $\mathbf{n}_i^T \mathbf{a}$. This implies

$$n_{ix} a_x + n_{iy} a_y + n_{iz} a_z = 0.$$

where $\mathbf{n}_i = \begin{bmatrix} n_{ix} \\ n_{iy} \\ n_{iz} \end{bmatrix}$ and $\mathbf{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$. Without loss of generality, we can choose a_x and a_y, and then compute a_z from the equation above (note that the components of $\mathbf{n}_i$ are known). Normalizing the vector a gives us another axis, which we call $\mathbf{t}_i$. The third axis $\mathbf{o}_i$ can be obtained by $\mathbf{o}_i = \mathbf{n}_i \times \mathbf{t}_i$. Thus

$$\mathbf{R}_{bc_i} = \begin{bmatrix} \mathbf{t}_i \ \mathbf{o}_i \ \mathbf{n}_i \end{bmatrix}$$

6.2.2 Constraints on Wrench that can be applied by a finger

A robotic finger that is in contact with an object can usually push on the object and cannot pull. This implies that the magnitude of the contact force along the contact normal is always non-negative. In the rest of this lecture, without loss of generality, we will assume that the contact force is applied along the normal. Note that whether the applied contact force from the finger is entirely along the contact normal is determined by the orientation of the end effector or finger tip frame. If the finger has at least 3 DoF, we can ensure that the finger pushes along the contact normal by properly orienting the end effector or finger tip frame. There are different models of the forces that can be applied at the finger object contact depending on the modeling assumptions made. There are four types of finger-object friction model, namely (a) Frictionless Point Contact (b) Point Contact with Friction (c) Soft Finger Contact (d) Anisotropic Soft Finger Contact. In the subsequent discussion (f_{ti}, f_{oi}, f_{ni}) are the components of the contact force and $(\tau_{ti}, \tau_{oi}, \tau_{ni})$ are the components of the contact moment at contact C_i.

6.2.2.1 Frictionless Point Contact

In this model, in the contact frame, $f_{ni} \geq 0$, $f_{ti} = f_{oi} = 0$, and $\tau_{ti} = \tau_{oi} = \tau_{ni} = 0$. Let us define the wrench basis as $\mathcal{B}_{c_i} = \begin{bmatrix} \mathbf{e}_z \\ \mathbf{0} \end{bmatrix}$, where the unit vector $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ is denoted by $\mathbf{e}_z$ and $\mathbf{0}$ is 3×1 column vector with all entries as zero. Thus the contact wrench can be expressed as

$$\mathbf{F}_{c_i} = \mathcal{B}_{c_i} f_{ni} = \begin{bmatrix} \mathbf{e}_z \\ \mathbf{0} \end{bmatrix} f_{ni} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} f_{ni} \tag{6.6}$$

6.2.2.2 Point Contact with Friction

In this model, in the contact frame, $f_{ni} \geq 0$, f_{ti} and f_{oi} are non-zero in general, and $\tau_{ti} = \tau_{oi} = \tau_{ni} = 0$. Assuming that Coulomb's friction law is valid, the tangential forces must satisfy the following constraint

$$\sqrt{f_{ti}^2 + f_{oi}^2} \leq \mu f_{ni} \tag{6.7}$$

where μ is the coefficient of friction. In this case, the wrench basis is $\mathcal{B}_{c_i} = \begin{bmatrix} \mathbf{I}_{3\times 3} \\ \mathbf{0}_{3\times 3} \end{bmatrix}$, where $\mathbf{I}_{3\times 3}$ is the 3×3 identity matrix and $\mathbf{0}_{3\times 3}$ is the 3×3 matrix with all entries as zero. The contact wrench at each contact is

$$\mathbf{F}_{c_i} = \mathcal{B}_{c_i} \begin{bmatrix} f_{ti} \\ f_{oi} \\ f_{ni} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{3\times 3} \\ \mathbf{0}_{3\times 3} \end{bmatrix} \begin{bmatrix} f_{ti} \\ f_{oi} \\ f_{ni} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_{ti} \\ f_{oi} \\ f_{ni} \end{bmatrix} \tag{6.8}$$

6.2.2.3 Soft Finger Contact

In a soft finger contact, apart from the tangential frictional force, it is assumed that a frictional moment about the contact normal acts on the object. Since we are essentially approximating a surface contact with a point contact, the soft finger model tries to model the effect of the distributed friction force on the surface by considering a moment about the contact normal due to the distributed tangential forces. In this model, in the contact frame, $f_{ni} \geq 0$, f_{ti}, f_{oi}, and τ_{ni} are non-zero in general, and $\tau_{ti} = \tau_{oi} = 0$ The friction model used is a generalization of Coulomb's friction law and is given by

$$\sqrt{f_{ti}^2 + f_{oi}^2} \leq \mu f_{ni} \quad \text{and} \quad |\tau_{ni}| \leq \gamma f_{ni} \tag{6.9}$$

where γ is a constant, which has the units of length. In this case, the wrench basis is $\mathcal{B}_{c_i} = \begin{bmatrix} \mathbf{I}_{3\times 3} & \mathbf{0}_{3\times 1} \\ \mathbf{0}_{3\times 3} & \mathbf{e}_z \end{bmatrix}$. Thus $\mathcal{B}_{c_i}$ is a 6×4 matrix. The contact wrench at each contact is

$$\mathbf{F}_{c_i} = \mathcal{B}_{c_i} \begin{bmatrix} f_{ti} \\ f_{oi} \\ f_{ni} \\ \tau_{ni} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{3\times 3} & \mathbf{0}_{3\times 1} \\ \mathbf{0}_{3\times 3} & \mathbf{e}_z \end{bmatrix} \begin{bmatrix} f_{ti} \\ f_{oi} \\ f_{ni} \\ \tau_{ni} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{ti} \\ f_{oi} \\ f_{ni} \\ \tau_{ni} \end{bmatrix} \tag{6.10}$$

6.2.2.4 Anisotropic Soft Finger Contact

The anisotropic soft finger contact assumes that the frictional resistance may depend on the direction of motion. The friction law is a generalization of Coulomb's friction law and is derived by using the *principle of maximum power dissipation*. The tangential forces and moment si subject to the following constraint

$$\sqrt{\left(\frac{f_{ti}}{e_{ti}}\right)^2 + \left(\frac{f_{oi}}{e_{oi}}\right)^2 + \left(\frac{\tau_{ni}}{e_{ni}}\right)^2} \leq \mu f_{ni} \tag{6.11}$$

where e_{ti} and e_{oi} are dimensionless constants and e_{ni} is a constant with the dimensions of length. The wrench basis and the contact wrench is the same as for soft finger contact model.

6.2.3 Grasp Matrix

Any grasp can be characterized by the grasp matrix and the restrictions on the contact forces given by the choice of the friction model. In this section, we derive the grasp matrix for each of the different types of contact friction model. Let $\boldsymbol{\beta}_{c_i}$ be the vector formed by concatenating the non-zero force/moment components in any friction/contact model. Thus $\mathbf{F}_{c_i} = \mathcal{B}_{c_i}\boldsymbol{\beta}_{c_i}$ and using Equation (6.5) we obtain

$$\mathbf{F}_b = \sum_{i=1}^{n_c} Ad^T_{(\mathbf{g}^{-1}_{bc_i})}\mathbf{F}_{c_i} = \sum_{i=1}^{n_c} Ad^T_{(\mathbf{g}^{-1}_{bc_i})}\mathcal{B}_{c_i}\boldsymbol{\beta}_{c_i} \tag{6.12}$$

We define $G_i = Ad^T_{(\mathbf{g}^{-1}_{bc_i})}\mathcal{B}_{c_i}$. Note that the dimension of G_i is $6 \times k$, where k is the number of non-zero components of the contact force/moment. Also, the size of $\boldsymbol{\beta}_{c_i}$ is $k \times 1$. Then the body wrench is

$$\mathbf{F}_b = \sum_{i=1}^{n_c} Ad^T_{(\mathbf{g}^{-1}_{bc_i})}\mathcal{B}_{c_i}\boldsymbol{\beta}_{c_i} = \sum_{i=1}^{n_c} G_i\boldsymbol{\beta}_{c_i} \tag{6.13}$$

Let $\boldsymbol{\beta}$ be the $kn_c \times 1$ vector formed by concatenating all the $\boldsymbol{\beta}_{c_i}$. Then Equation (6.13) can be written as a matrix vector product

$$\mathbf{F}_b = G\boldsymbol{\beta} \tag{6.14}$$

where

$$G = \begin{bmatrix} G_1 & G_2 & \cdots & G_{n_c} \end{bmatrix}$$

is a $6 \times kn_c$ matrix. We will now look at the specific form of the grasp matrix G for each of the friction models.

6.2.3.1 Frictionless Point Contact

Here, $k = 1$, $\boldsymbol{\beta}_{c_i} = f_{ni}$. The matrix G_i has dimension 6×1 and is given by

$$G_i = Ad^T_{(\mathbf{g}^{-1}_{bc_i})}\mathcal{B}_{c_i} = \begin{bmatrix} \mathbf{R}_{bc_i} & \mathbf{0} \\ \hat{\mathbf{p}}_{bc_i}\mathbf{R}_{bc_i} & \mathbf{R}_{bc_i} \end{bmatrix} \begin{bmatrix} \mathbf{e}_z \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{bc_i}\mathbf{e}_z \\ \hat{\mathbf{p}}_{bc_i}\mathbf{R}_{bc_i}\mathbf{e}_z \end{bmatrix}$$

Note that

$$\mathbf{R}_{bc_i}\mathbf{e}_z = \begin{bmatrix} \mathbf{t}_i & \mathbf{o}_i & \mathbf{n}_i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \mathbf{n}_i.$$

Therefore $G_i = \begin{bmatrix} \mathbf{n}_i \\ \hat{\mathbf{p}}_i\mathbf{n}_i \end{bmatrix}$, where we have abbreviated $\hat{\mathbf{p}}_{bc_i}$ as $\hat{\mathbf{p}}_i$ for notational convenience. Thus the grasp matrix is

$$\mathbf{G} = \begin{bmatrix} \mathbf{n}_1 & \mathbf{n}_2 & \cdots & \mathbf{n}_{n_c} \\ \hat{\mathbf{p}}_1\mathbf{n}_1 & \hat{\mathbf{p}}_2\mathbf{n}_2 & \cdots & \hat{\mathbf{p}}_{n_c}\mathbf{n}_{n_c} \end{bmatrix} \tag{6.15}$$

6.2.3.2 Point Contact with Friction

Here, $k = 3$, $\boldsymbol{\beta}_{c_i} = \begin{bmatrix} f_{ti} \\ f_{oi} \\ f_{ni} \end{bmatrix}$. The matrix G_i has dimension 6×3 and is given by

$$G_i = Ad^T_{(\mathbf{g}^{-1}_{bc_i})} \mathcal{B}_{c_i} = \begin{bmatrix} \mathbf{R}_{bc_i} & \mathbf{0} \\ \hat{\mathbf{p}}_{bc_i}\mathbf{R}_{bc_i} & \mathbf{R}_{bc_i} \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{bc_i} \\ \hat{\mathbf{p}}_{bc_i}\mathbf{R}_{bc_i} \end{bmatrix}$$

Therefore $G_i = \begin{bmatrix} \mathbf{R}_i \\ \hat{\mathbf{p}}_i \mathbf{R}_i \end{bmatrix}$, where we have abbreviated $\hat{\mathbf{p}}_{bc_i}$ as $\hat{\mathbf{p}}_i$ and $\mathbf{R}_{bc_i}$ as $\mathbf{R}_i$ for notational convenience. Thus the grasp matrix is

$$\mathbf{G} = \begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_2 & \cdots & \mathbf{R}_{n_c} \\ \hat{\mathbf{p}}_1\mathbf{R}_1 & \hat{\mathbf{p}}_2\mathbf{R}_2 & \cdots & \hat{\mathbf{p}}_{n_c}\mathbf{R}_{n_c} \end{bmatrix} \tag{6.16}$$

6.2.3.3 Soft Finger Contact

Here, $k = 4$, $\boldsymbol{\beta}_{c_i} = \begin{bmatrix} f_{ti} \\ f_{oi} \\ f_{ni} \\ \tau_{ni} \end{bmatrix}$. The matrix G_i has dimension 6×4 and is given by

$$G_i = Ad^T_{(\mathbf{g}^{-1}_{bc_i})} \mathcal{B}_{c_i} = \begin{bmatrix} \mathbf{R}_{bc_i} & \mathbf{0} \\ \hat{\mathbf{p}}_{bc_i}\mathbf{R}_{bc_i} & \mathbf{R}_{bc_i} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{e}_z \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{bc_i} & \mathbf{0} \\ \hat{\mathbf{p}}_{bc_i}\mathbf{R}_{bc_i} & \mathbf{R}_{bc_i}\mathbf{e}_z \end{bmatrix}$$

Therefore $G_i = \begin{bmatrix} \mathbf{R}_i & \mathbf{0} \\ \hat{\mathbf{p}}_i \mathbf{R}_i & \mathbf{n}_i \end{bmatrix}$, where we have abbreviated $\hat{\mathbf{p}}_{bc_i}$ as $\hat{\mathbf{p}}_i$ and $\mathbf{R}_{bc_i}$ as $\mathbf{R}_i$ for notational convenience. Thus the grasp matrix is

$$\mathbf{G} = \begin{bmatrix} \mathbf{R}_1 & \mathbf{0} & \mathbf{R}_2 & \mathbf{0} & \cdots & \mathbf{R}_{n_c} & \mathbf{0} \\ \hat{\mathbf{p}}_1\mathbf{R}_1 & \mathbf{n}_1 & \hat{\mathbf{p}}_2\mathbf{R}_2 & \mathbf{n}_2 & \cdots & \hat{\mathbf{p}}_{n_c}\mathbf{R}_{n_c} & \mathbf{n}_{n_c} \end{bmatrix} \tag{6.17}$$

In the above matrix all the zeros are 3×1 column vectors with all entries as zero.

6.2.3.4 Summary

To summarize, the discussion above a grasp can be characterized by the grasp map $\mathbf{G}$ along with the constraints on the components of the contact force given by the friction law. Given any external wrench $\mathbf{F}_e$, if the grasp has to keep the object in equilibrium, then the contact wrenches can be obtained by solving the equation:

$$\mathbf{G}\boldsymbol{\beta} = -\mathbf{F}_e \tag{6.18}$$

along with the constraints that $f_{ni} \geq 0$, and the constraints arising from the friction law, given by Equation (6.7) or (6.9), or (6.11), depending on the choice of the friction model. Note that to solve for the contact wrench $\boldsymbol{\beta}$, we are not solving a system of linear equations. Technically, we are solving a system of linear equations along with linear and quadratic (second order cone) inequalities. The solution of these problems falls under the general topic of optimization. In fact, the problem of finding $\boldsymbol{\beta}$ is a *convex optimization* problem. Technically, (a generalization of) the problem can be posed as a second order cone program (SOCP), which is a special type of convex optimization problem and there are known techniques to solve them (although the field matured in the last fifteen years).