

$$f(x, y) =$$

$$\begin{cases} 0, & \text{se } y = 0 \\ x + f(x, y - 1), & \text{caso contrário} \end{cases}$$

Seja $T := \lambda r m n. IFTE \ ISZERO \ (n) \ 0 \ SOMA(m)(r \ (m) \ (pred \ n))$

A Função multiplicação recursiva é: $MULT_REC := M \ T$

$$MULT_REC \ 3 \ 2 = M \ T \ 3 \ 2$$

$$= T \ (M \ T) \ 3 \ 2$$

$$\triangleright \lambda r m n. IFTE_lazy \ ISZERO_lazy \ (n) \ 0 \ SOMA(m)(r \ (m) \ (PRED \ n)) \ (M \ T) \ 3 \ 2$$

$$\triangleright \lambda m n. IFTE_lazy \ ISZERO_lazy \ (n) \ 0 \ SOMA(m)((M \ T) \ (m) \ (PRED \ n)) \ 3 \ 2$$

$$\triangleright \lambda n. IFTE_lazy \ ISZERO_lazy \ (n) \ 0 \ SOMA(3)((M \ T) \ (3) \ (PRED \ n)) \ 2$$

$$\triangleright IFTE_lazy \ ISZERO_lazy \ (2) \ 0 \ SOMA(3)((M \ T) \ (3) \ (PRED \ 2))$$

$$\triangleright SOMA(3)((M \ T) \ (3) \ (PRED \ 2))$$

$$\triangleright SOMA(3)(T \ (M \ T) \ 3 \ 1)$$

$$\triangleright SOMA(3)(\lambda r m n. IFTE_lazy \ ISZERO_lazy \ (n) \ 0 \ SOMA(m)(r \ (m) \ (PRED \ n)) \ (M \ T) \ 3 \ 1)$$

$$\triangleright SOMA(3)(\lambda m n. IFTE_lazy \ ISZERO_lazy \ (n) \ 0 \ SOMA(m)((M \ T) \ (m) \ (PRED \ n)) \ 3 \ 1)$$

$$\triangleright SOMA(3)(\lambda n. IFTE_lazy \ ISZERO_lazy \ (n) \ 0 \ SOMA(3)((M \ T) \ (3) \ (PRED \ n)) \ 1)$$

$$\triangleright SOMA(3)(IFTE_lazy \ ISZERO_lazy \ (1) \ 0 \ SOMA(3)((M \ T) \ (3) \ (PRED \ 1)))$$

$$\triangleright SOMA(3)(SOMA(3)((M \ T) \ 3 \ 0))$$

$$\triangleright SOMA(3)(SOMA(3)(T \ (M \ T) \ 3 \ 0))$$

$$\triangleright SOMA(3)(SOMA(3)(\lambda r m n. IFTE_lazy \ ISZERO_lazy \ (n) \ 0 \ SOMA(m)(r \ (m) \ (PRED \ n)) \ (M \ T) \ 3 \ 0))$$

$$\triangleright SOMA(3)(SOMA(3)(\lambda m n. IFTE_lazy \ ISZERO_lazy \ (n) \ 0 \ SOMA(m)((M \ T) \ (m) \ (PRED \ n)) \ 3 \ 0))$$

$$\triangleright SOMA(3)(SOMA(3)(\lambda n. IFTE_lazy \ ISZERO_lazy \ (n) \ 0 \ SOMA(3)((M \ T) \ (3) \ (PRED \ n)) \ 0))$$

$$\triangleright SOMA(3)(SOMA(3)(IFTE_lazy \ ISZERO_lazy \ (0) \ 0 \ SOMA(3)((M \ T) \ (3) \ (PRED \ 0))))$$

$$\triangleright SOMA(3)(SOMA(3)(0))$$

$$\triangleright SOMA(3)(3)$$

$$\triangleright^* 6$$