FinalExam

Question 1

a

```
prop_var  <- c(2.90,1.24,0.85,0.72,0.22,0.07)/6
cumsum(prop_var)
```

```
## [1] 0.4833333 0.6900000 0.8316667 0.9516667 0.9883333 1.0000000
```

We would keep the first 3 principal components accounting for 83.2% of the variance of the scaled data

b

The fourth eigenvalue still represents a meaningful amount of information taking explained variance from .83 to .95. It is significantly above the standard cutoff which for 6 eigenvalues would be 1/6 = .17. This woudl provide a much fuller picture and is likely worth the added complexity.

c

The first principal component appears to represent overall team strength. where positive pc1 means a more complete team and negative pc1 is a more struggling team. We can look at loading and see that DefPts and DefRush are both negative (lower is better) and the Offenseive poitns, Offenseive rush yards and offensive pass yards are positive (higher is better). The only exception is that the Defensive pass loading is slightly positive which may indicate some nuance on defense.

d

Principal component 2 contrasts run heavy offesnses against pass dominant teams with strong pass defense. this is seen with the positive loading on offensive rushing and strong negative loads on defensive points, pass defense, and offensive passing. Here a positive score indicates a lot of rushing output, weaker passing offense and stronger defense while a neagtive score indicates a pass focused offense with a weaker pass defense.

e

26 has a slightly above average pc1 indicating that both their offense and defense were above average, and a very strong and high pc2 indicating that they were a rush first team

f

Point 16 representing the chiefs hovers in the middle of the plot for both PC1 and PC2. They are just above average in PC1 indicating that the overal team is slightly above average, and they also hover right around 0 for PC2 indicating a balanced team between rushing and passing.

g

```
load("Data/nfl2024.RData")

vars <- c("DefPts","DevRush","DevPass","OffPts","OffRush","OffPass")

X   <- scale(nfl[ , vars])

pca <- prcomp(X, center = FALSE, scale. = FALSE)
scores <- pca$x

team <- "Arizona Cardinals"
row  <- which(nfl$Team == team)
z    <- scores[row, 1:4]

x_std_hat <- as.numeric(z %*% t(pca$rotation[ , 1:4]))

means <- colMeans(nfl[ , vars])
sds   <- apply(nfl[ , vars], 2, sd)

x_hat <- means + x_std_hat * sds
names(x_hat) <- vars

print("Reconstructed Values")
```

```
## [1] "Reconstructed Values"
```

```
round(x_hat, 2)
```

```
##  DefPts DevRush DevPass  OffPts OffRush OffPass
##   22.97  122.19  215.58   24.47  140.58  209.65
```

```
actual <- nfl[nfl$Team == team, vars]

#diff <- actual - x_hat
#diff
```

h

```
library(MASS)

lda_fit <- lda(Playoff ~ ., data = nfl[ , c("Playoff", vars)],
```

```
              prior = c(0.5625, 0.4375))

round(lda_fit$scaling, 3)
```

```
##            LD1
## DefPts  -0.186
## DevRush -0.027
## DevPass  0.001
## OffPts   0.423
## OffRush -0.021
## OffPass -0.042
```

i

```
scores <- predict(lda_fit)$x
playoff_teams   <- nfl$Team[nfl$Playoff == 1]
playoff_scores  <- scores[nfl$Playoff == 1, 1]

team_low <- playoff_teams[ which.min(playoff_scores) ]
team_low
```

```
## [1] "Los Angeles Rams"
```

```
playoff_scores[ which.min(playoff_scores) ]
```

```
##        19
## -0.861632
```

j

Our prior should be the proportion of teams making the playoffs from the data we have as shown below.

```
prop.table(table(nfl$Playoff))
```

```
##
##      0      1
## 0.5625 0.4375
```

k

```
xnew <- data.frame(DefPts = 21.89,
                   DevRush = 124.85,
                   DevPass = 208.82,
                   OffPts = 23.69,
                   OffRush = 132.15,
                   OffPass = 189.54)

predict(lda_fit, newdata = xnew)
```

```
## $class
## [1] 1
## Levels: 0 1
##
## $posterior
##            0         1
## 1 0.03855635 0.9614437
##
## $x
##        LD1
## 1 1.297546
```

l

Im a little confused about what is being asked here so i have recreated the heirarchical clustering appraoch and am using it to produce two groups to evaluate the results.

```
hc_cent <- hclust(dist(scale(nfl[ , vars])), method = "centroid")
groups2 <- cutree(hc_cent, k = 2)

table(Cluster = groups2, Playoff = nfl$Playoff)
```

```
##        Playoff
## Cluster  0  1
##       1 17 14
##       2  1  0
```

Here the result is that we identify 17 /18 non playoff teams correctly, but we do not identify any of the playoff teams correctly. Generally, all teams are in cluster 1 with a single team in cluster two. This is because i think Carolina is such a strong outlier that it creates its own cluster. This is a terrible classification but may not be exactly what the question is asking for

m

We must standardize because our metrics are on vary different scales. Points range up to maybe 40 and yards range in the hundreds. without standardizing, the euclidean distance used for clustering would be dominated by the variable with larger variance and would not really represent what we want it to. By putting our information on the common scale of standard deviations, euclidean distance is now more appropriate and considers all variables equally.

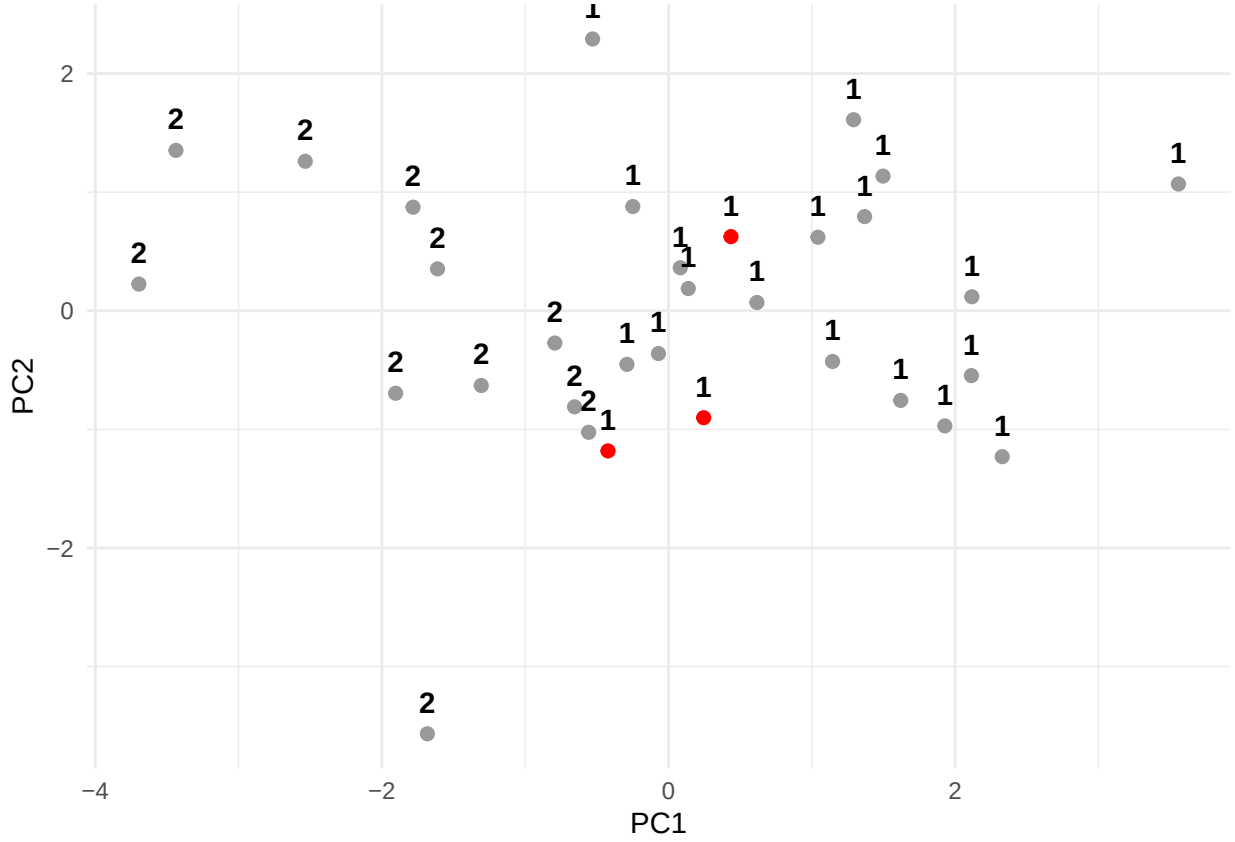
n

```
library(ggplot2)
hc_ward <- hclust(dist(scale(nfl[ , vars])), method = "ward.D2")
groups2w<- cutree(hc_ward, 2)
table(Cluster = groups2w, Playoff = nfl$Playoff)
```

```
##        Playoff
## Cluster  0  1
##       1 18  3
##       2  0 11
```

```r
scores_df <- data.frame(
  Team    = nfl$Team,
  PC1     = pca$x[, 1],
  PC2     = pca$x[, 2],
  Cluster = factor(groups2w),
  Playoff = factor(nfl$Playoff,
                   levels = c(0, 1),
                   labels = c("Non-playoff", "Playoff"))
)
scores_df$Pred <- ifelse(scores_df$Cluster == "2", "Playoff", "Non-playoff")
scores_df$Color <- ifelse(scores_df$Pred == scores_df$Playoff, "correct", "wrong")

ggplot(scores_df, aes(PC1, PC2)) +
  geom_point(aes(colour = Color), size = 2) +
  geom_text(aes(label = Cluster), vjust = -1, fontface = "bold") +
  scale_colour_manual(values = c(correct = "grey60", wrong = "red")) +
  labs(x = "PC1", y = "PC2") +
  theme_minimal() +
  theme(legend.position = "none")
```




```r
misclassified <- subset(scores_df, Color == "wrong")
print(misclassified$Team)
```

```
## [1] "Houston Texans"        "Los Angeles Rams"      "Washington Commanders"
```

Hierarchical clustering at level 2 performs quite well only getting three teams incorrect (accuracy = .906). These three teams are the rams, texans, and commanders and on visual inspection do appear cluster more closely with the non playoff teams than the playoff teams. It also looks like my axis are flipped for this analysis from what was done in the assignment but The interpretation remains the same.

o

It seems cluster 2 is a strong playoff team as they are the most positive on pc1. because PC2 talks more about play style, it is subjective and we focus on PC1 which leads us to cluster 2.

p

```
library(Hotelling)
```

```
## Loading required package: corpcor
```

```
hot <- hotelling.test(
        x = nfl[nfl$Playoff == 1 , vars],
        y = nfl[nfl$Playoff == 0 , vars])

hot
```

```
## Test stat:  78.168
## Numerator df:  6
## Denominator df:  25
## P-value:  5.901e-06
```

q

```
pvals <- sapply(vars, \(v)
           t.test(nfl[[v]] ~ nfl$Playoff)$p.value)

alpha <- 0.05 / length(vars)

data.frame(Variable = vars,
           p.value  = round(pvals, 5),
           Significant = pvals <= alpha)
```

```
##         Variable p.value Significant
## DefPts    DefPts 0.00000        TRUE
## DevRush  DevRush 0.00011        TRUE
## DevPass  DevPass 0.58277       FALSE
## OffPts    OffPts 0.00004        TRUE
## OffRush  OffRush 0.00674        TRUE
## OffPass  OffPass 0.26419       FALSE
```

r

LDA and the Hotelling means test would have been invalidated if we have this problem. PCA and Clsutering should stay valid.

Question 2

a

PCA and Factor analysis may help identify outliers via dimension reduction making it more easily to visually identify them on a plot

Clustering especially heirarchical will give us an indication of how "difficult" it is for some of these things to cluster together which can also help us identify outliers

Hotelling test is designed for comparing groups so is not useful for identifying individual outliers.

b

PCA and Factor analysis will be great for exploration here, but is likely not used directly for modeling.

Clustering is the primary useful approach for the traditional customer segmentation.

If labels are provided which is uncommon for segmentation but common for other similar use cases for this type of analysis, LDA could be used. this would be relevant for target variables like customer churn.

Question 3

```
mu     <- c(1,2,3)

Sigma  <- matrix(c(3,1,-1, 1,2,0, -1,0,3), 3,3)

A      <- matrix(c(2,-1,0, 0,3,1), nrow = 2, byrow = TRUE)

mu_Y   <- A %*% mu
Sigma_Y<- A %*% Sigma %*% t(A)

mu_Y
```

```
##      [,1]
## [1,]    0
## [2,]    9
```

```
Sigma_Y
```

```
##      [,1] [,2]
## [1,]   10   -2
## [2,]   -2   21
```

$$\mathbf{Y} = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \sim \mathcal{N}_2\left(\begin{pmatrix} 0 \\ 9 \end{pmatrix}, \begin{pmatrix} 10 & -2 \\ -2 & 21 \end{pmatrix} \right).$$