562_HW_1

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#Question 3 Corroboration

```r
# Parameters
m <- 5          # Degrees of freedom for X1; must be > 0
n <- 10         # Degrees of freedom for X2; must be > 4 for Var[Y] to exist
set.seed(123)   # For reproducibility
N <- 1e7        # Number of simulations

# Simulate chi-square random variables
X1 <- rchisq(N, df = m)
X2 <- rchisq(N, df = n)

Y <- (n * X1) / (m * X2)

# Compute empirical mean and variance
empirical_mean_Y <- mean(Y)
empirical_var_Y <- var(Y)

# Compute theoretical mean and variance
theoretical_mean_Y <- n / (n - 2)
theoretical_var_Y <- (2 * n^2 * (m + n - 2)) / (m * (n - 2)^2 * (n - 4))

# Print results
cat("Empirical E[Y]:", empirical_mean_Y, "\n")

## Empirical E[Y]: 1.250003

cat("Theoretical E[Y]:", theoretical_mean_Y, "\n\n")

## Theoretical E[Y]: 1.25

cat("Empirical Var[Y]:", empirical_var_Y, "\n")

## Empirical Var[Y]: 1.353302

cat("Theoretical Var[Y]:", theoretical_var_Y, "\n")

## Theoretical Var[Y]: 1.354167
```

#Question 5

##a

```r
percentile_t12_p95 <- qt(0.95, df = 12)
percentile_t12_p95

## [1] 1.782288
```

##b

```r
percentile_F45_10 <- qf(0.10, df1 = 4, df2 = 5)
percentile_F45_10

## [1] 0.2468783
```

##c

```r
prob_F45_gt_4.6 <- pf(4.6, df1 = 4, df2 = 5, lower.tail = FALSE)
prob_F45_gt_4.6

## [1] 0.06259346
```

##d

```r
prob_t10_gt_3 <- pt(3, df = 10, lower.tail = FALSE)
prob_t10_gt_3

## [1] 0.006671828
```

##e

```r
percentile_chi2_3_95 <- qchisq(0.95, df = 3)
percentile_chi2_3_95

## [1] 7.814728
```

##f

```r
prob_chi2_3_gt_2.6 <- pchisq(2.6, df = 3, lower.tail = FALSE)
prob_chi2_3_gt_2.6

## [1] 0.4574895
```

#Question 7

```r
df1 <-1
df2 <- 3

critical_value <- 2*3

prob_Y <- 1 - pf(critical_value,df1,df2)

cat("Probability P(Y > 2) from f dist:", prob_Y, "\n")

## Probability P(Y > 2) from f dist: 0.09172111
```

##Validate from Origional distribution

```r
# Parameters
mu <- 0          # Mean used for corroboration does not affect the end result
sigma <- 1       # Standard deviation for X_1, X_2, X_3, X_4
N <- 1e7         # Number of simulations for accuracy

X1 <- rnorm(N, mean = mu, sd = sigma)
X2 <- rnorm(N, mean = 0, sd = sigma)
X3 <- rnorm(N, mean = 0, sd = sigma)
X4 <- rnorm(N, mean = 0, sd = sigma)

# Compute the original value Y = (X1 - mu)^2 / (X2^2 + X3^2 + X4^2)
Y <- (X1 - mu)^2 / (X2^2 + X3^2 + X4^2)

# Compute the probability P(Y > 2)
prob_Y_gt_2 <- mean(Y > 2)

# Print the simulation result
cat("Probability P(Y > 2) from origional dist:", prob_Y_gt_2, "\n")
```

```
## Probability P(Y > 2) from origional dist: 0.0916704
```