

575_HW4

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#Question 1

```
library(coda)

set.seed(123)

n_iter <- 1500  #Number of iterations
burn_in <- 500  # Burn-in period

rho_values <- c(0.25, 0.65, 0.95)
results <- list()

for(rho in rho_values) {

  X <- numeric(n_iter)
  Y <- numeric(n_iter)

  X[1] <- 2
  Y[1] <- 4

  for(t in 2:n_iter) {
    mean_X <- 2 + (rho / 2) * (Y[t - 1] - 4)
    var_X <- 1 - rho^2
    X[t] <- rnorm(1, mean = mean_X, sd = sqrt(var_X))

    mean_Y <- 4 + 2 * rho * (X[t] - 2)
    var_Y <- 4 * (1 - rho^2)
    Y[t] <- rnorm(1, mean = mean_Y, sd = sqrt(var_Y))
  }

  X_samples <- X[(burn_in + 1):n_iter]
  Y_samples <- Y[(burn_in + 1):n_iter]

  E_X <- mean(X_samples)
  E_Y <- mean(Y_samples)
  Var_X <- var(X_samples)
  Var_Y <- var(Y_samples)

  acf_X <- acf(X_samples, plot = FALSE)$acf[2]
  acf_Y <- acf(Y_samples, plot = FALSE)$acf[2]

  ess_X <- effectiveSize(X_samples)
```

```

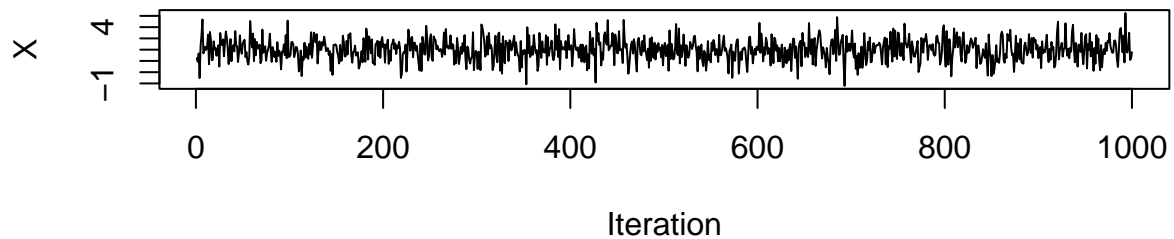
ess_Y <- effectiveSize(Y_samples)

results[[paste0("rho_", rho)]] <- list(
  rho = rho,
  E_X = E_X,
  E_Y = E_Y,
  Var_X = Var_X,
  Var_Y = Var_Y,
  acf_X = acf_X,
  acf_Y = acf_Y,
  ess_X = ess_X,
  ess_Y = ess_Y
)

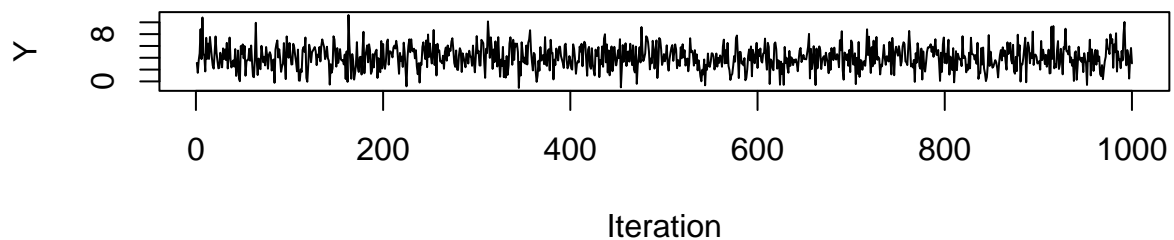
par(mfrow = c(2, 1))
plot(X_samples, type = "l", main = paste("Trace Plot for X (rho =", rho, ")"),
     xlab = "Iteration", ylab = "X")
plot(Y_samples, type = "l", main = paste("Trace Plot for Y (rho =", rho, ")"),
     xlab = "Iteration", ylab = "Y")
par(mfrow = c(1, 1))
}

```

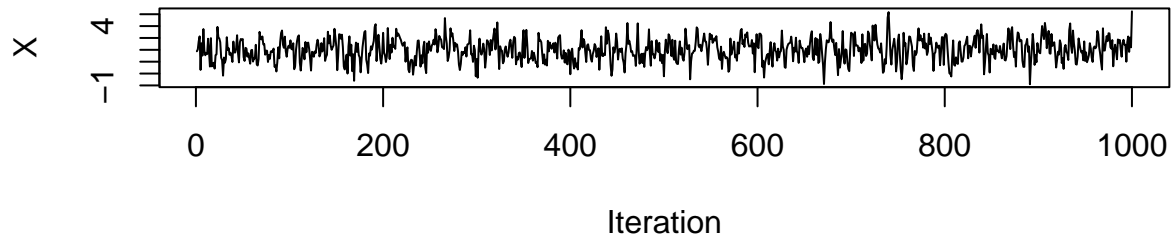
Trace Plot for X (rho = 0.25)



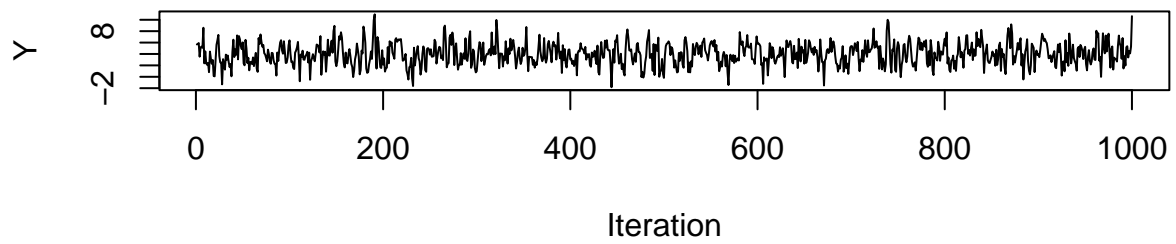
Trace Plot for Y (rho = 0.25)



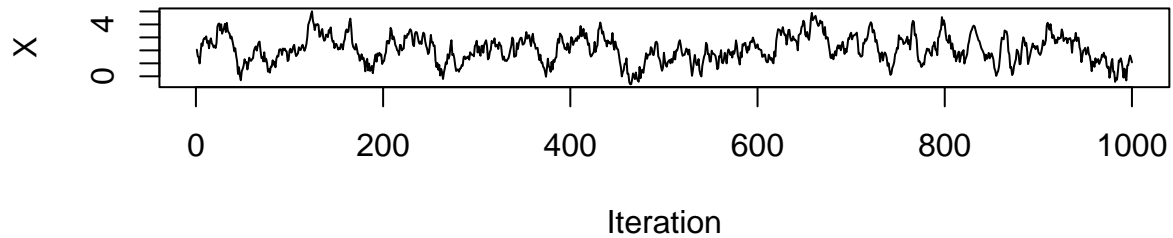
Trace Plot for X (rho = 0.65)



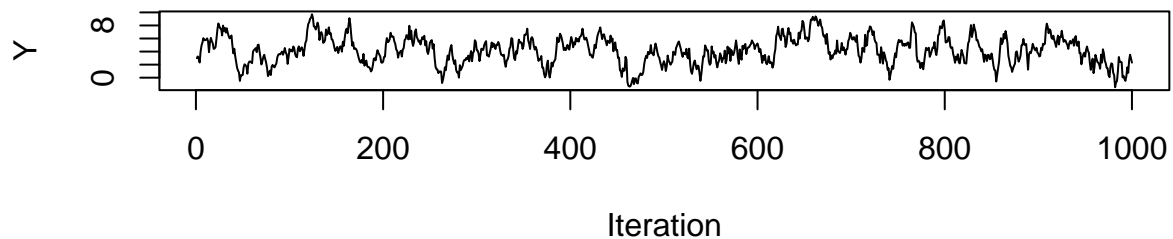
Trace Plot for Y (rho = 0.65)



Trace Plot for X ($\rho = 0.95$)



Trace Plot for Y ($\rho = 0.95$)



```
stat_names <- c("E[X]", "Var[X]", "E[Y]", "Var[Y]", "ACF_X", "ACF_Y", "ESS_X", "ESS_Y")
rho_labels <- c("0.25", "0.65", "0.95")

summary_table <- matrix(NA, nrow = length(stat_names), ncol = length(rho_labels))
rownames(summary_table) <- stat_names
colnames(summary_table) <- rho_labels

for (rho in rho_labels) {
  res <- results[[paste0("rho_", rho)]]
  summary_table["E[X]", rho] <- res$E_X
  summary_table["Var[X]", rho] <- res$Var_X
  summary_table["E[Y]", rho] <- res$E_Y
  summary_table["Var[Y]", rho] <- res$Var_Y
  summary_table["ACF_X", rho] <- res$acf_X
  summary_table["ACF_Y", rho] <- res$acf_Y
  summary_table["ESS_X", rho] <- res$ess_X
  summary_table["ESS_Y", rho] <- res$ess_Y
}

print(summary_table)
```

```
##           0.25           0.65           0.95
## E[X]    2.002495e+00    2.0127162    2.0908938
## Var[X]   9.740391e-01    0.9135344    1.0638474
## E[Y]     4.045121e+00    3.9846203    4.1649047
```

```
## Var[Y] 3.963372e+00 3.8864420 4.3218763
## ACF_X 2.237973e-02 0.3887208 0.9076168
## ACF_Y 5.817089e-02 0.4090377 0.9067950
## ESS_X 1.000000e+03 507.8375577 48.3801028
## ESS_Y 8.891629e+02 579.9959727 48.8315211
```

#Question 2

```
n_iter <- 1000

theta <- numeric(n_iter)
gamma <- numeric(n_iter)

yA <- c(12, 9, 12, 14, 13, 13, 15, 8, 15, 6)
yB <- c(11, 11, 10, 9, 9, 8, 7, 10, 6, 8, 8, 9, 7)
nA <- length(yA)
nB <- length(yB)
sumA <- sum(yA)
sumB <- sum(yB)

a_theta <- 120
b_theta <- 10
a_gamma <- 1
b_gamma <- 1

theta[1] <- 1
gamma[1] <- 1

for (t in 2:n_iter) {
  shape_theta <- a_theta + sumA + sumB
  rate_theta <- b_theta + nA + nB * gamma[t - 1]
  theta[t] <- rgamma(1, shape = shape_theta, rate = rate_theta)

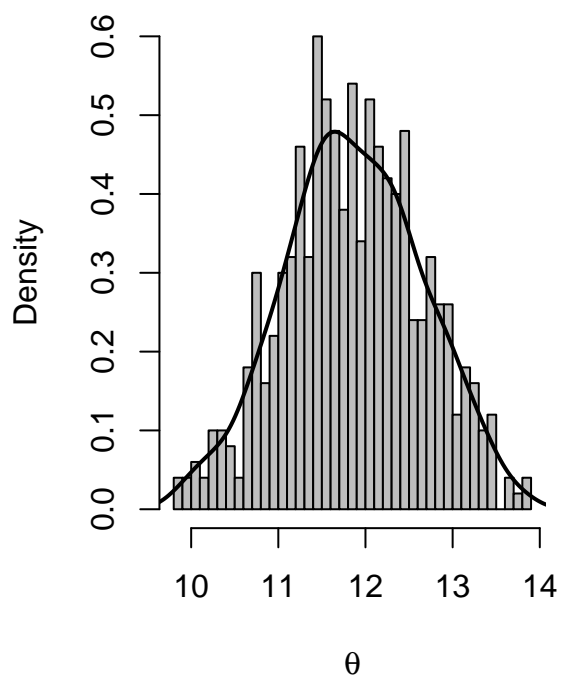
  shape_gamma <- a_gamma + sumB
  rate_gamma <- b_gamma + nB * theta[t]
  gamma[t] <- rgamma(1, shape = shape_gamma, rate = rate_gamma)
}

burn_in <- 500
theta_samples <- theta[(burn_in + 1):n_iter]
gamma_samples <- gamma[(burn_in + 1):n_iter]

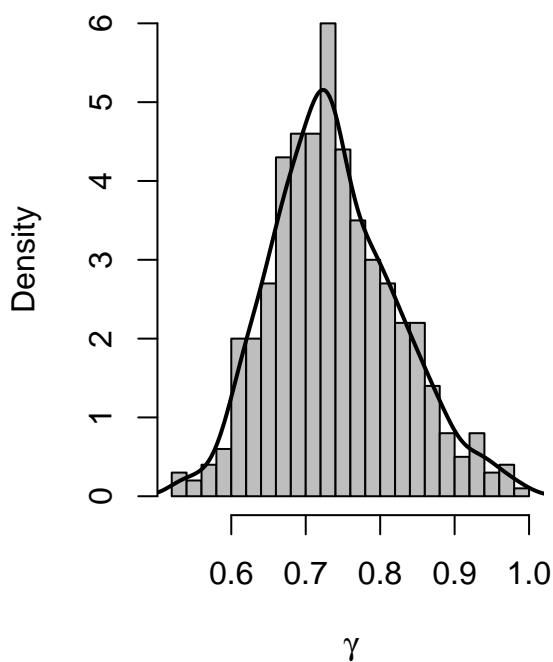
par(mfrow = c(1, 2))
hist(theta_samples, breaks = 30, probability = TRUE,
     main = "Posterior Distribution of theta", xlab = expression(theta),
     col = "grey")
lines(density(theta_samples), col = "black", lwd = 2)

hist(gamma_samples, breaks = 30, probability = TRUE,
     main = "Posterior Distribution of gamma", xlab = expression(gamma),
     col = "grey")
lines(density(gamma_samples), col = "black", lwd = 2)
```

Posterior Distribution of theta



Posterior Distribution of gamma



```
par(mfrow = c(1, 1))

E_theta <- mean(theta_samples)
Var_theta <- var(theta_samples)
E_gamma <- mean(gamma_samples)
Var_gamma <- var(gamma_samples)

cat("Posterior mean of theta:", E_theta, "\n")

## Posterior mean of theta: 11.85906

cat("Posterior variance of theta:", Var_theta, "\n")

## Posterior variance of theta: 0.6193738

cat("Posterior mean of gamma:", E_gamma, "\n")

## Posterior mean of gamma: 0.7386707

cat("Posterior variance of gamma:", Var_gamma, "\n")

## Posterior variance of gamma: 0.007067903
```

#I couldnt get RJAGS to install for some reason si i cant do the final part here