573_FinalExam

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Question 1

\mathbf{a}

Which plot (Left or Right) of the LJung-box statistics suggests autocorrelation exists in the time series. Justify your answer. (2 pts)

The left plot indicates that there is autocorrelation as the different lag values have signifficance below the p value line.

b

What are the null and alternative hypotheses corresponding to the p-value the arrow points to? (3 pts) - Null Hypothesis: there is no autocorrelation up to a specific lag - Alternative hypothesis: there is some autocorrelation at one or more legs

Question 2

Im assuming here that "quantities" refer to either predicting, filtering, smoothing, and forecasting. It seems that the quantities of interest are defined in the question itself?

\mathbf{a}

current position and velocity would be a filtering problem

c (why is this out of order?)

Predicting during the huricane wold be a forecasting problem

b

this would be a smoothing operation

Question 3

Fit HHMM model with two hidden states

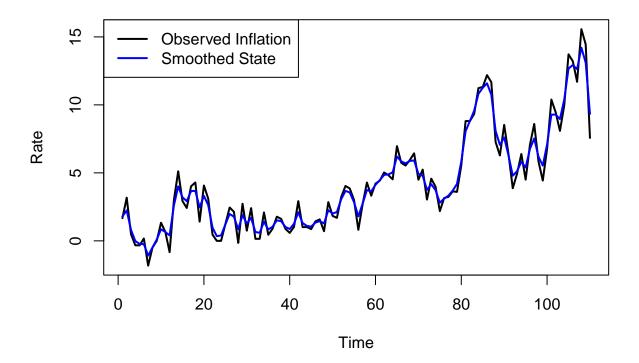
```
library(depmixS4)
## Warning: package 'depmixS4' was built under R version 4.4.2
## Loading required package: nnet
## Loading required package: MASS
## Loading required package: Rsolnp
## Warning: package 'Rsolnp' was built under R version 4.4.2
## Loading required package: nlme
library(astsa)
data(qinfl)
# Build HMM with 2 states and Gaussian emissions
mod <- depmix(response = qinfl ~ 1,</pre>
              data = data.frame(qinfl=qinfl),
              nstates = 2,
              family = gaussian())
fitmod <- fit(mod)</pre>
## converged at iteration 18 with logLik: -243.8824
# Show parameter estimates
summary(fitmod)
## Initial state probabilities model
## pr1 pr2
## 0 1
## Transition matrix
          toS1 toS2
## fromS1 0.976 0.024
## fromS2 0.029 0.971
## Response parameters
## Resp 1 : gaussian
      Re1.(Intercept) Re1.sd
         7.860 3.124
## St1
               1.902 1.578
## St2
```

Fit linear Gaussian state-space model

```
library(MARSS)
## Warning: package 'MARSS' was built under R version 4.4.2
data(qinfl)
inflation <- qinfl</pre>
Y <- matrix(inflation, nrow = 1)
model.list <- list(</pre>
 Z = matrix(1),
 A = matrix(0),
 B = matrix("b1"),
 U = matrix("u1"),
 Q = matrix("q1"),
 R = matrix("r1"),
 x0 = matrix("x0"),
 V0 = matrix("v0")
)
fit_initial <- MARSS(Y, model = model.list, method = "kem")</pre>
## Warning! Abstol convergence only. Maxit (=500) reached before log-log convergence.
##
## MARSS fit is
## Estimation method: kem
## Convergence test: conv.test.slope.tol = 0.5, abstol = 0.001
## WARNING: Abstol convergence only no log-log convergence.
## maxit (=500) reached before log-log convergence.
## The likelihood and params might not be at the ML values.
## Try setting control$maxit higher.
## Log-likelihood: -215.6751
## AIC: 443.3501 AICc: 444.1656
##
##
        Estimate
## R.r1 7.84e-01
## B.b1 9.36e-01
## U.u1 3.46e-01
## Q.q1 1.67e+00
## x0.x0 1.15e-05
## V0.v0 4.38e-03
## Initial states (x0) defined at t=0
## Standard errors have not been calculated.
## Use MARSSparamCIs to compute CIs and bias estimates.
## Convergence warnings
## Warning: the x0.x0 parameter value has not converged.
## Type MARSSinfo("convergence") for more info on this warning.
```

```
fit_final <- MARSS(Y, model = model.list, inits = fit_initial, method = "BFGS")</pre>
## Success! Converged in 10 iterations.
## Function MARSSkfas used for likelihood calculation.
## MARSS fit is
## Estimation method: BFGS
## Estimation converged in 10 iterations.
## Log-likelihood: -215.191
## AIC: 442.3819
                  AICc: 443.1975
##
##
         Estimate
## R.r1 8.33e-01
## B.b1 9.44e-01
## U.u1 3.02e-01
## Q.q1 1.57e+00
## x0.x0 1.58e+00
## V0.v0 4.56e-06
## Initial states (x0) defined at t=0
## Standard errors have not been calculated.
## Use MARSSparamCIs to compute CIs and bias estimates.
summary(fit_final)
## m: 1 state process(es) named X.Y1
## n: 1 observation time series named Y1
##
##
     term
               estimate
## 1 R.r1 8.328455e-01
## 2 B.b1 9.438911e-01
## 3 U.u1 3.019168e-01
## 4 Q.q1 1.570584e+00
## 5 x0.x0 1.581683e+00
## 6 V0.v0 4.557436e-06
states = fit final$states
time_index <- seq_along(qinfl)</pre>
states <- as.numeric(fit_final$states)</pre>
qinfl_data <- as.numeric(qinfl)</pre>
plot.ts(qinfl_data, type = "1", col = "black", lwd = 2,
        ylab = "Rate",
        main = "Observed Inflation vs. Smoothed State")
lines(states, col = "blue", lwd = 2)
legend("topleft", legend = c("Observed Inflation", "Smoothed State"),
       col = c("black", "blue"), lwd = 2)
```

Observed Inflation vs. Smoothed State



This is similar to stochastic regression as it allows for the relationships to vary with time instead of being set.

\mathbf{c}

HMM assumes that the process swaps between discrete spaces with specific feature sets while gaussian state assumes that the evolution is continual.

\mathbf{d}

Stochastic Regression

```
library(MARSS)
data(qinf1)
data(qintr)

len <- length(qinf1)
Z <- array(NA, c(1, 2, len))
Z[1, 1, ] <- 1
Z[1, 2, ] <- qintr

model_list <- list(
    B = diag(2),
    U = "zero",
    A = matrix(0,1,1),</pre>
```

```
Q = "diagonal and unequal",
  Z = Z,
  R = matrix("r",1,1),
  x0 = matrix(c("x01", "x02"), 2, 1),
  V0 = diag(0.01, 2),
  tinitx = 0
inits <- list(</pre>
  x0 = matrix(c(0, 0), 2, 1)
fit_init <- MARSS(qinfl, model=model_list, method="kem", inits=inits)</pre>
## Warning! Abstol convergence only. Maxit (=500) reached before log-log convergence.
##
## MARSS fit is
## Estimation method: kem
## Convergence test: conv.test.slope.tol = 0.5, abstol = 0.001
## WARNING: Abstol convergence only no log-log convergence.
## maxit (=500) reached before log-log convergence.
## The likelihood and params might not be at the ML values.
## Try setting control$maxit higher.
## Log-likelihood: -201.4763
## AIC: 412.9526
                  AICc: 413.5295
##
##
            Estimate
## R.r
            1.467358
## Q.(X1,X1) 0.004689
## Q.(X2,X2) 0.013585
## x0.x01
           0.000149
             0.000686
## x0.x02
## Initial states (x0) defined at t=0
## Standard errors have not been calculated.
## Use MARSSparamCIs to compute CIs and bias estimates.
## Convergence warnings
## Warning: the Q.(X1,X1) parameter value has not converged.
## Type MARSSinfo("convergence") for more info on this warning.
fit_final <- MARSS(qinfl, model=model_list, method="BFGS", inits=fit_init)</pre>
## Success! Converged in 12 iterations.
## Function MARSSkfas used for likelihood calculation.
##
## MARSS fit is
## Estimation method: BFGS
## Estimation converged in 12 iterations.
## Log-likelihood: -197.801
## AIC: 405.602 AICc: 406.179
##
```

```
##
              Estimate
## R.r
              1.53e+00
## Q.(X1,X1)
              5.65e-11
## Q.(X2,X2)
              8.11e-03
## x0.x01
             -8.43e-01
## x0.x02
              1.12e+00
## Initial states (x0) defined at t=0
##
## Standard errors have not been calculated.
## Use MARSSparamCIs to compute CIs and bias estimates.
```

summary(fit_final)

```
##
## m: 2 state process(es) named X1 X2
## n: 1 observation time series named Y1
##
##
                    estimate
          term
## 1
           R.r
               1.534416e+00
## 2 Q.(X1,X1)
               5.648406e-11
## 3 Q.(X2,X2) 8.109504e-03
## 4
        x0.x01 -8.430630e-01
## 5
        x0.x02 1.121526e+00
```

\mathbf{e}

in model d, inflation is described by another observed variable with a time varying slope. this is not available in model b.

\mathbf{f}

This really depends on how you view inflation and its relation to time. I would personally choose the final model as it has a better view of the general macroeconomic environment with the incorporation of interest rate.