# HW6_567

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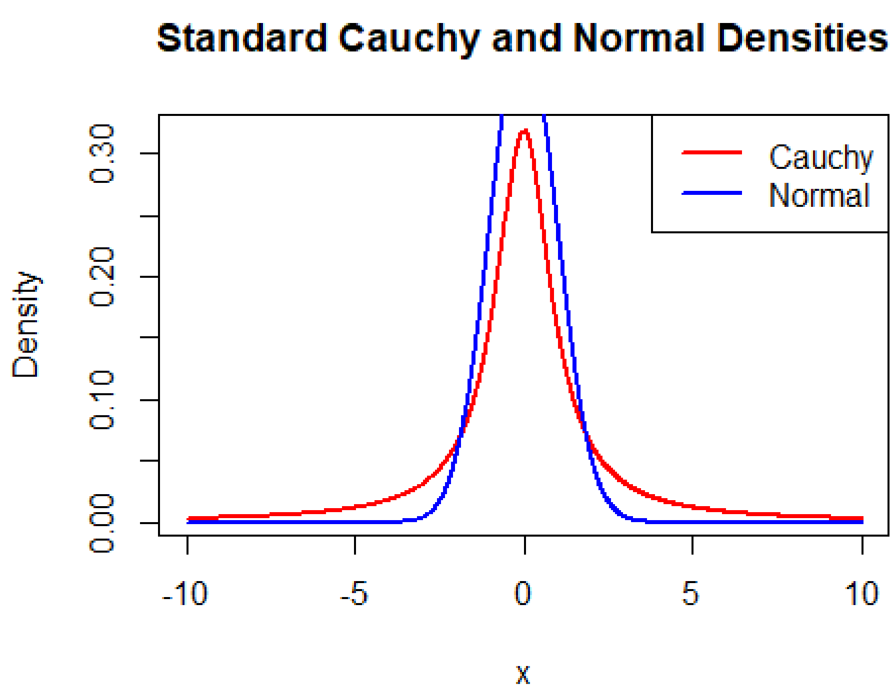
#Question 1 ##a Start by visualizing the Cauchy density using a red curve for $-10 \le x \le +10$. One option is to use dcauchy(). For comparison, also overlay a normal density curve in blue.

```
x <- seq(-10, 10, length.out = 1000)


cauchy_density <- dcauchy(x, location = 0, scale = 1)
normal_density <- dnorm(x, mean = 0, sd = 1)


plot(x, cauchy_density, type = "l", col = "red", lwd = 2,
     ylab = "Density", main = "Standard Cauchy and Normal Densities",
     xlab = "x")
lines(x, normal_density, col = "blue", lwd = 2)
legend("topright", legend = c("Cauchy", "Normal"),
       col = c("red", "blue"), lwd = 2)
```

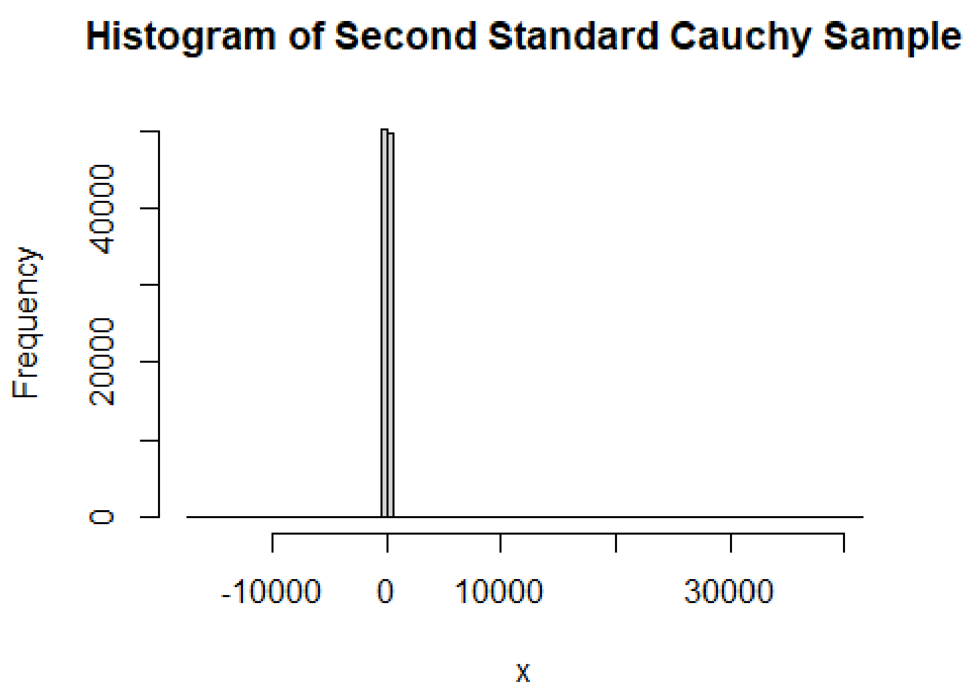



##b To prepare to use the inverse transform method, find $F^{-1}(u)$.

```r
set.seed(490)
n <- 100000

u <- runif(n, min = 0, max = 1)
x_cauchy <- tan(pi * (u - 0.5))
```

##c (4 pts) Use the inverse transform method to generate a random sample of 10,000 observa- tions from the standard Cauchy distribution. Show a histogram of your sample using a large number of bins for your histogram (e.g., 100). In order to show all the data, do NOT restrict your x axis. What striking features do you notice about your histogram?

```r
set.seed(123)

u <- runif(n, min = 0, max = 1)
x_cauchy <- tan(pi * (u - 0.5))

hist(x_cauchy, breaks = 100, main = "Histogram of Second Standard Cauchy
Sample", xlab = "x")
```

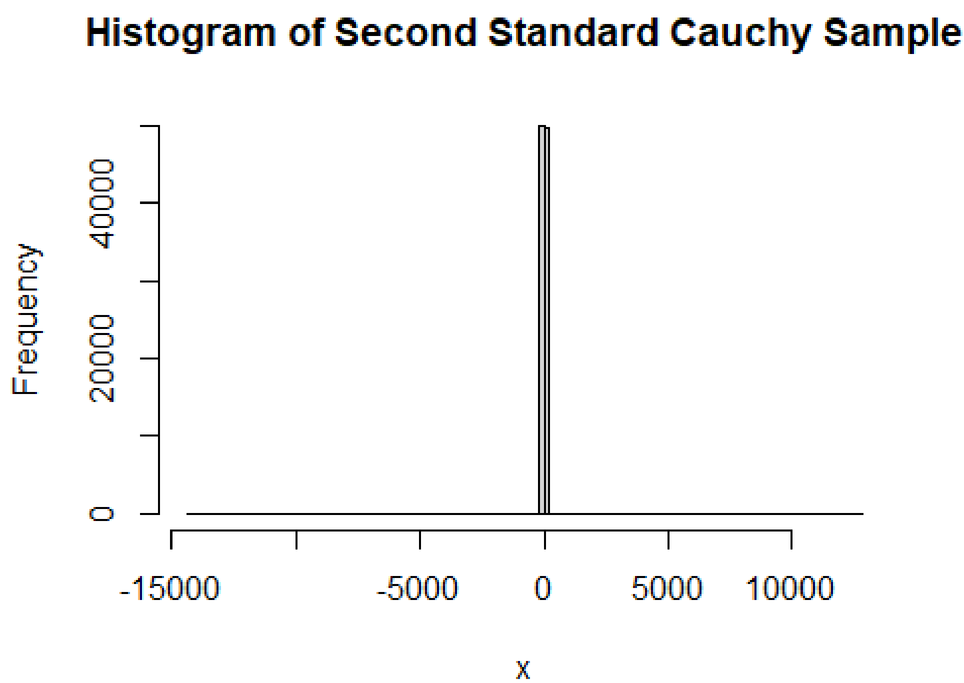



There are some insane outliers every time this is run

##d

```r
# Set a different seed
set.seed(999)

u2 <- runif(n, min = 0, max = 1)
```

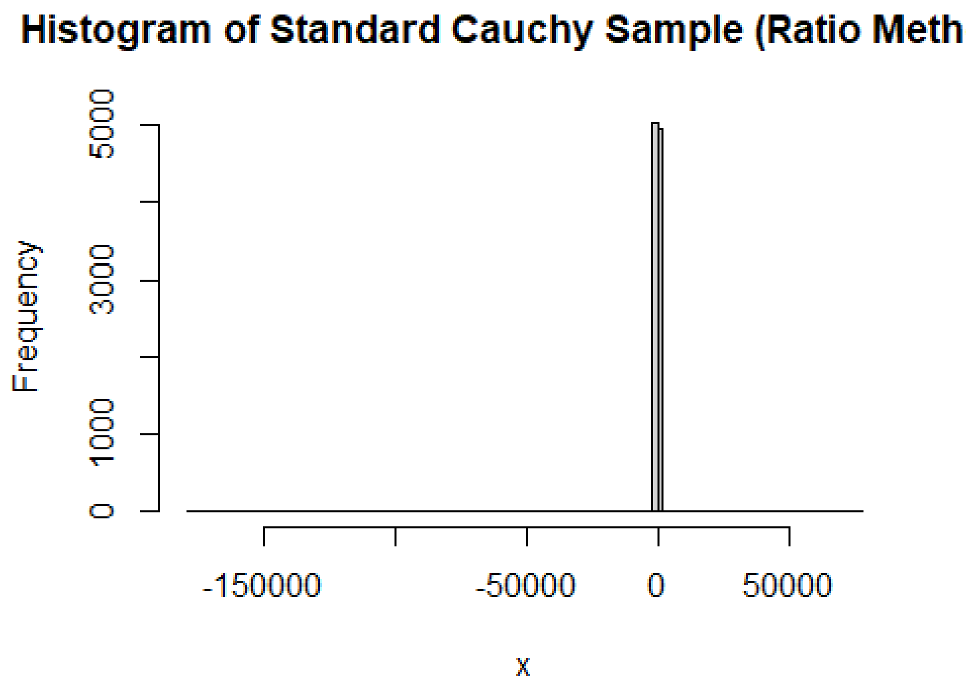
```
x_cauchy2 <- tan(pi * (u2 - 0.5))


hist(x_cauchy2, breaks = 100, main = "Histogram of Second Standard Cauchy
Sample", xlab = "x")
```




More insane outliers which wont even show up on a plot

##e The ratio of two independent standard normal (mean = 0, sd = 1) random variables is distributed standard Cauchy. Use this information and rnorm() to generate a random sample of 10,000 observations. Show the histogram.

```
set.seed(789)

n <- 10000
Z1 <- rnorm(n, mean = 0, sd = 1)
Z2 <- rnorm(n, mean = 0, sd = 1)

# Compute the ratio
x_cauchy_ratio <- Z1 / Z2

# Plot histogram
hist(x_cauchy_ratio, breaks = 100, main = "Histogram of Standard Cauchy
Sample (Ratio Method)",  xlab = "x")
```

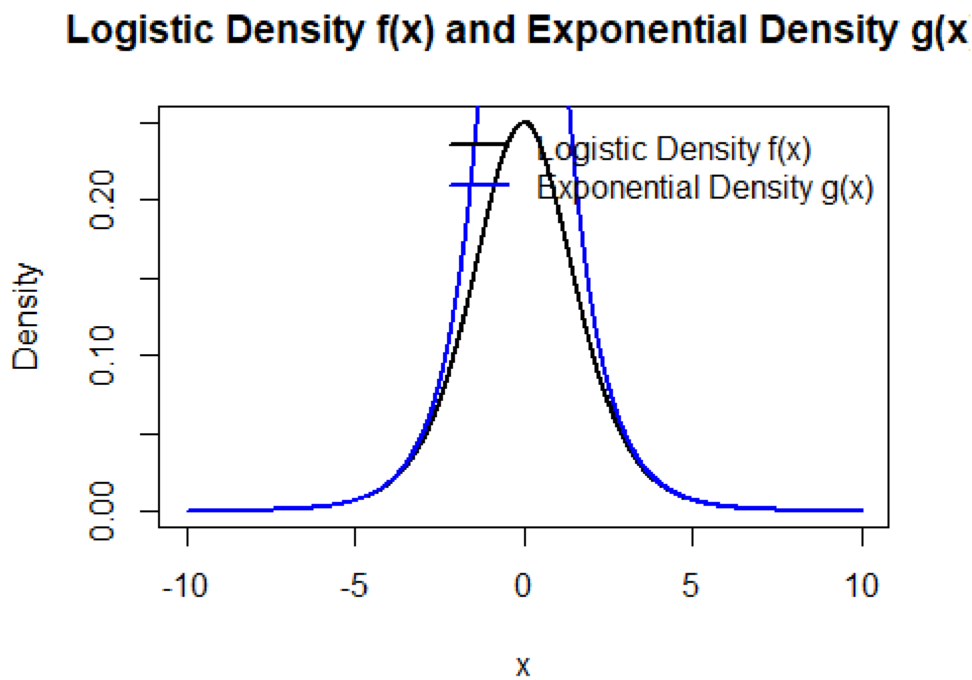



Again, Because of the outliers this distribution is totally shifted from the one above.

#Question 2

##a

```
x_vals <- seq(-10, 10, length.out = 1000)


f_x <- exp(-x_vals) / (1 + exp(-x_vals))^2
g_x <- ifelse(x_vals >= 0, exp(-x_vals), exp(x_vals))


plot(x_vals, f_x, type = "l", col = "black", lwd = 2,
     xlab = "x", ylab = "Density",
     main = "Logistic Density f(x) and Exponential Density g(x)")

lines(x_vals, g_x, col = "blue", lwd = 2)


legend("topright", legend = c("Logistic Density f(x)", "Exponential Density
g(x)"),
       col = c("black", "blue"), lwd = 2, bty = "n")
```




```r
# PARAMS
set.seed(420)
N <- 10000
samples <- numeric(N)
accepted <- 0
total_trials <- 0

while (accepted < N) {
  x <- rexp(1, rate = 1)
  alpha <- (1 / (1 + exp(-x)))^2

  u <- runif(1)

  if (u <= alpha) {
    # Assign random sign
    s <- sample(c(-1, 1), 1)
    samples[accepted + 1] <- s * x
    accepted <- accepted + 1
  }
  total_trials <- total_trials + 1
}


acceptance_rate <- (N / total_trials) * 100
cat("Acceptance Rate:", round(acceptance_rate, 2), "%\n")
```

```
## Acceptance Rate: 50.08 %

hist(samples, breaks = 50, probability = TRUE,
     main = "Histogram of Samples from Logistic Distribution",
     xlab = "Value", xlim = c(-10, 10), col = "lightblue", border = "white")

x_vals <- seq(-10, 10, length.out = 1000)
logistic_density <- exp(-x_vals) / (1 + exp(-x_vals))^2
lines(x_vals, logistic_density, col = "red", lwd = 2)

legend("topright", legend = c("Sample Histogram", "Logistic Density"),
       col = c("lightblue", "red"), lwd = c(10, 2), bty = "n")
```