562\_HW\_1

Matthew Stoebe

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#Question 3 Corroboration

# Parameters  
m <- 5 # Degrees of freedom for X1; must be > 0  
n <- 10 # Degrees of freedom for X2; must be > 4 for Var[Y] to exist  
set.seed(123) # For reproducibility  
N <- 1e7 # Number of simulations  
  
# Simulate chi-square random variables  
X1 <- rchisq(N, df = m)  
X2 <- rchisq(N, df = n)  
  
Y <- (n \* X1) / (m \* X2)  
  
# Compute empirical mean and variance  
empirical\_mean\_Y <- mean(Y)  
empirical\_var\_Y <- var(Y)  
  
# Compute theoretical mean and variance  
theoretical\_mean\_Y <- n / (n - 2)  
theoretical\_var\_Y <- (2 \* n^2 \* (m + n - 2)) / (m \* (n - 2)^2 \* (n - 4))  
  
# Print results  
cat("Empirical E[Y]:", empirical\_mean\_Y, "\n")

## Empirical E[Y]: 1.250003

cat("Theoretical E[Y]:", theoretical\_mean\_Y, "\n\n")

## Theoretical E[Y]: 1.25

cat("Empirical Var[Y]:", empirical\_var\_Y, "\n")

## Empirical Var[Y]: 1.353302

cat("Theoretical Var[Y]:", theoretical\_var\_Y, "\n")

## Theoretical Var[Y]: 1.354167

#Question 5

##a

percentile\_t12\_p95 <- qt(0.95, df = 12)  
percentile\_t12\_p95

## [1] 1.782288

##b

percentile\_F45\_10 <- qf(0.10, df1 = 4, df2 = 5)  
percentile\_F45\_10

## [1] 0.2468783

##c

prob\_F45\_gt\_4.6 <- pf(4.6, df1 = 4, df2 = 5, lower.tail = FALSE)  
prob\_F45\_gt\_4.6

## [1] 0.06259346

##d

prob\_t10\_gt\_3 <- pt(3, df = 10, lower.tail = FALSE)  
prob\_t10\_gt\_3

## [1] 0.006671828

##e

percentile\_chi2\_3\_95 <- qchisq(0.95, df = 3)  
percentile\_chi2\_3\_95

## [1] 7.814728

##f

prob\_chi2\_3\_gt\_2.6 <- pchisq(2.6, df = 3, lower.tail = FALSE)  
prob\_chi2\_3\_gt\_2.6

## [1] 0.4574895

#Question 7

df1 <-1  
df2 <- 3  
  
critical\_value <- 2\*3  
  
prob\_Y <- 1 - pf(critical\_value,df1,df2)  
  
cat("Probability P(Y > 2) from f dist:", prob\_Y, "\n")

## Probability P(Y > 2) from f dist: 0.09172111

##Validate from Origional distribution

# Parameters  
mu <- 0 # Mean used for corroboration does not affect the end result  
sigma <- 1 # Standard deviation for X\_1, X\_2, X\_3, X\_4  
N <- 1e7 # Number of simulations for accuracy  
  
X1 <- rnorm(N, mean = mu, sd = sigma)  
X2 <- rnorm(N, mean = 0, sd = sigma)  
X3 <- rnorm(N, mean = 0, sd = sigma)  
X4 <- rnorm(N, mean = 0, sd = sigma)  
  
# Compute the original value Y = (X1 - mu)^2 / (X2^2 + X3^2 + X4^2)  
Y <- (X1 - mu)^2 / (X2^2 + X3^2 + X4^2)  
  
# Compute the probability P(Y > 2)  
prob\_Y\_gt\_2 <- mean(Y > 2)  
  
# Print the simulation result  
cat("Probability P(Y > 2) from origional dist:", prob\_Y\_gt\_2, "\n")

## Probability P(Y > 2) from origional dist: 0.0916704