2.6 page 211

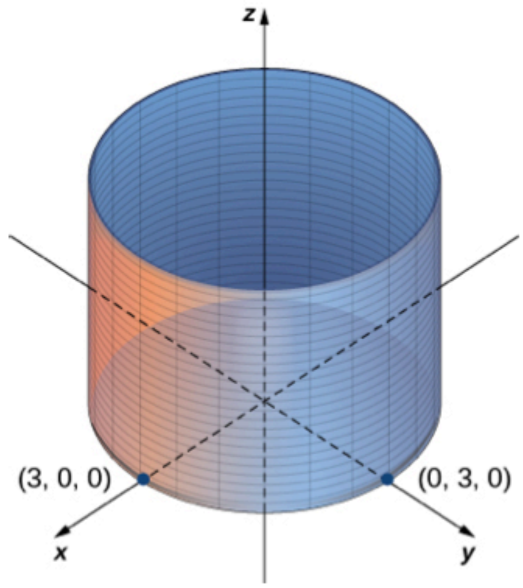


Figure 2.75 In three-dimensional space, the graph of equation $x^2 + y^2 = 9$ is a cylinder with radius 3 centered on the z-axis. It continues indefinitely in the positive and negative directions.

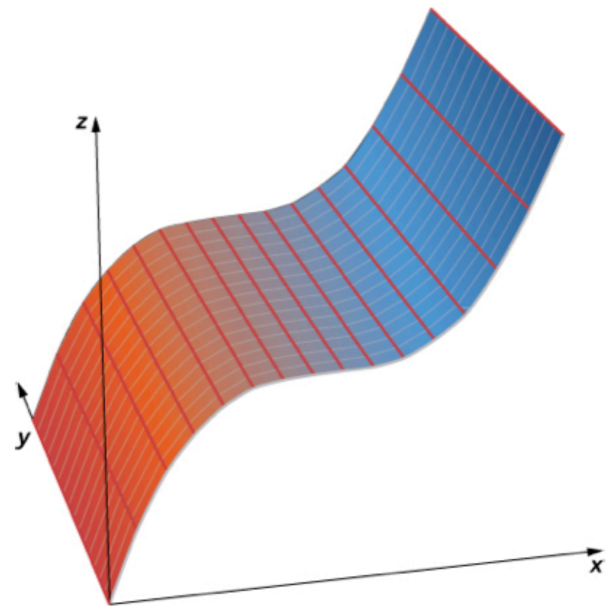


Figure 2.76 In three-dimensional space, the graph of equation $z = x^3$ is a cylinder, or a cylindrical surface with rulings parallel to the y-axis.

2.6 page 211

Definition

A set of lines parallel to a given line passing through a given curve is known as a cylindrical surface, or **cylinder**. The parallel lines are called **rulings**.

2.6 page 214

Definition

The **traces** of a surface are the cross-sections created when the surface intersects a plane parallel to one of the coordinate planes.

2.6 page 212

Example 2.55

Graphing Cylindrical Surfaces

equations using 2 variable when drawn in 3D space.

Sketch the graphs of the following cylindrical surfaces.

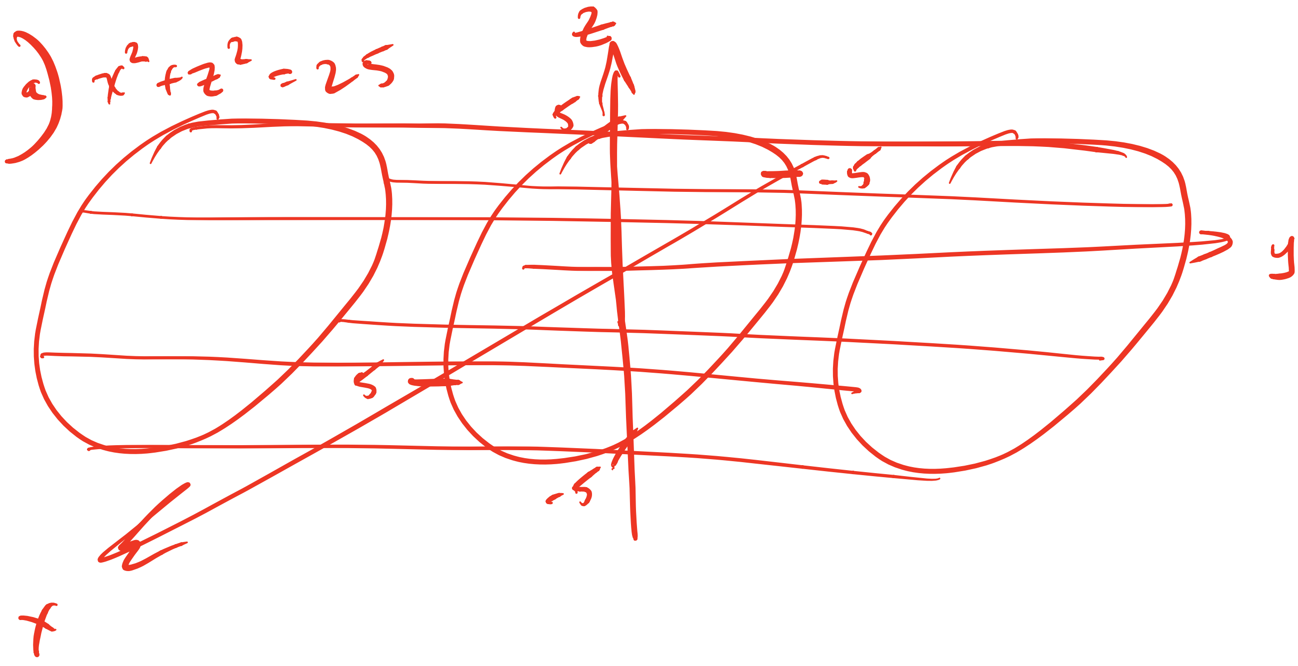
a. $x^2 + z^2 = 25$

~~b. $z = 2x^2 - y$~~

c. $y = \sin x$

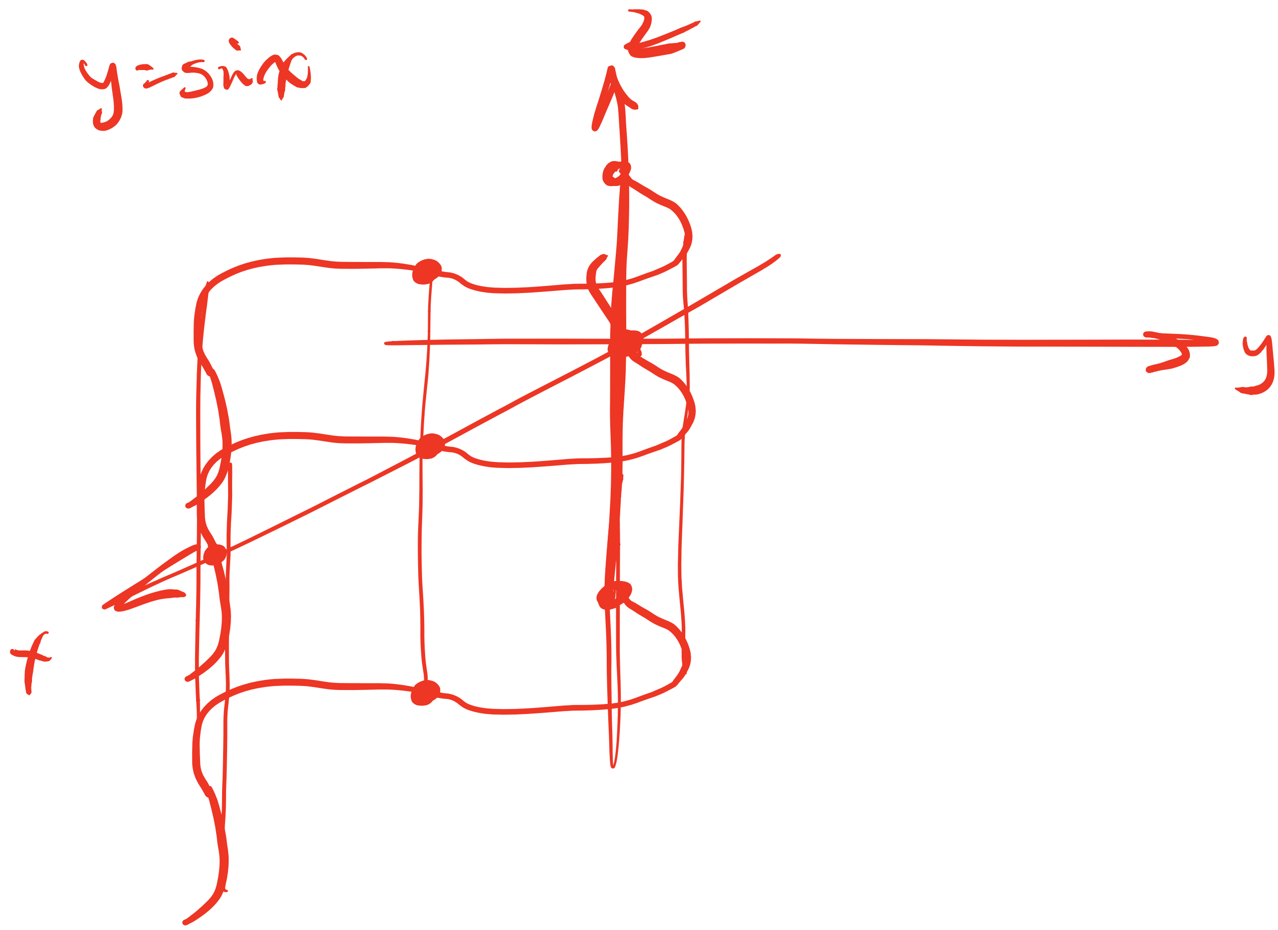
Draw curve in xy-plane then extend in the z direction.

a) $x^2+z^2=25$



This cylinder uses x & z. Draw the trace in xz-plane. Extend in the remaining direction (y).

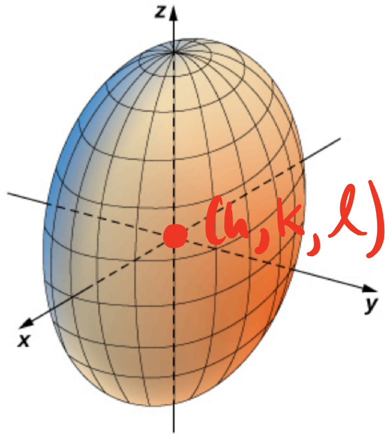
c) $y=\sin x$



Definition

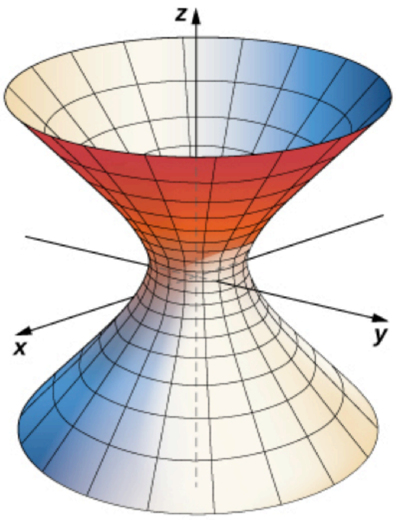
Quadric surfaces are the graphs of equations that can be expressed in the form

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Jz + K = 0.$$



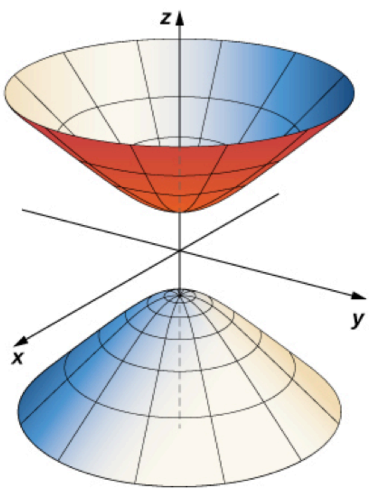
Ellipsoid (if $a=b=c$ then sphere)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} + \frac{(z-l)^2}{c^2} = 1$$



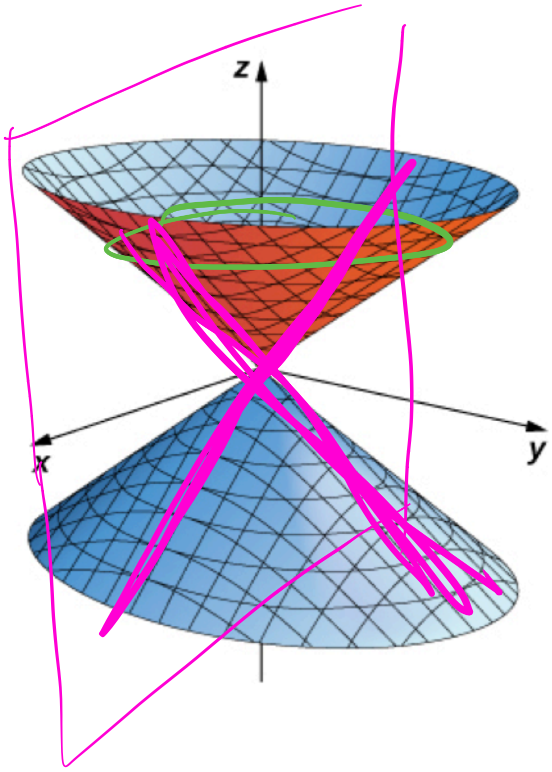
Hyperboloid of one sheet

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} - \frac{(z-l)^2}{c^2} = 1$$



Hyperboloid of two sheets

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} - \frac{(z-l)^2}{c^2} = -1$$



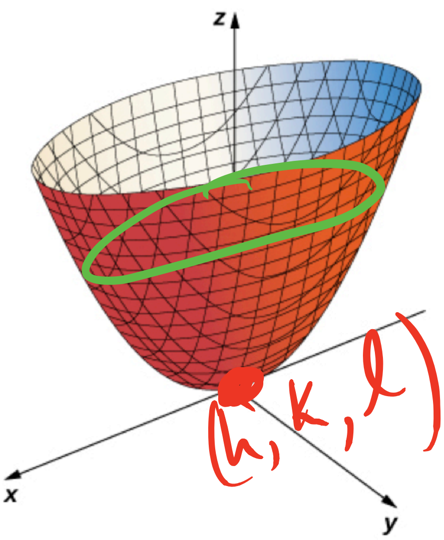
Cone

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}-\frac{(z-l)^2}{c^2}=0$$

z-trace (hold z constant) get $x^2+y^2=c$

y=0 trace $x^2-z^2=0$ $x=\pm z$
(y=k)

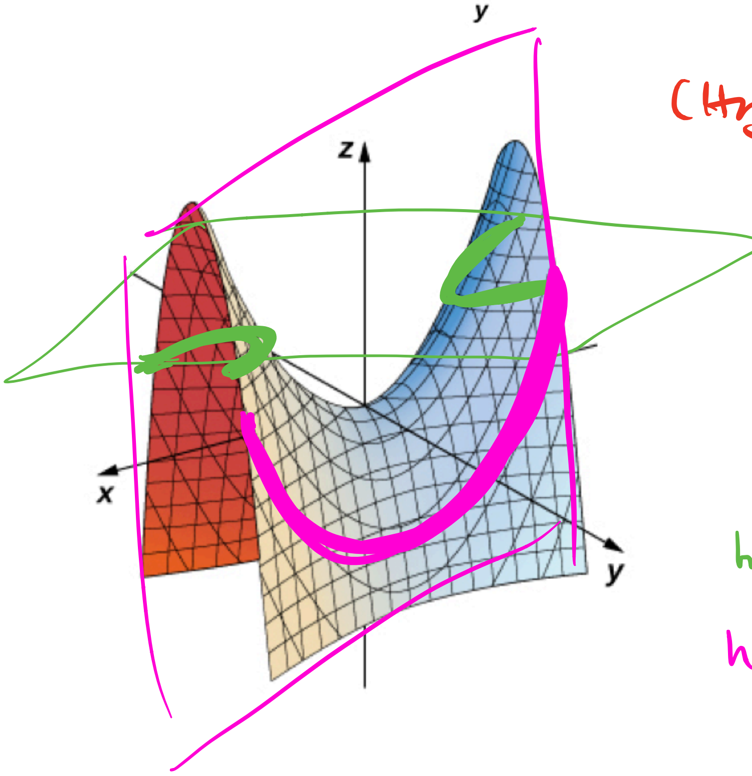
(look at p 221-222 for traces)



Paraboloid

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=z-l$$

z-Trace (hold z constant), get $x^2+y^2=c$ → circle



(Hyperbolic paraboloid)
Saddle

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=z-l$$

holding z constant, we get $x^2-y^2=c$
(hyperbola trace)

holding y constant, we get $x^2+c=z$
(parabola trace)

Example 2.57

Identifying Traces of Quadric Surfaces

Describe the traces of the elliptic paraboloid $x^2 + \frac{y^2}{2^2} = \frac{z}{5}$.

Holding x constant we get

$y^2 = z$ parabola.

Holding y constant we get

$x^2 = z$ parabola

Holding z constant we get

$x^2 + y^2 = c$ circle

Example 2.59

Identifying Equations of Quadric Surfaces

Identify the surfaces represented by the given equations.

a. $16x^2 + 9y^2 + 16z^2 = 144$

b. $9x^2 - 18x + 4y^2 + 16y - 36z + 25 = 0$

a) like $x^2 + y^2 + z^2 = 1$ $\rightarrow$

ellipsoid

~~hyperboloid 1 sheet~~

~~hyperboloid 2 sheet~~

~~cone~~

~~paraboloid~~

~~saddle~~

b) $x^2 + y^2 = z$ $\rightarrow$

~~ellipsoid~~

~~hyperboloid 1 sheet~~

~~hyperboloid 2 sheet~~

~~cone~~

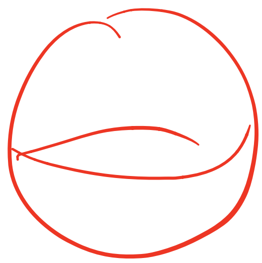
paraboloid

~~saddle~~

<u>Most simple quadrics.</u>

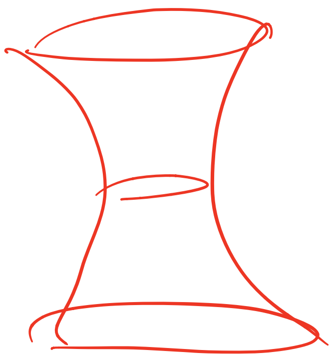
(symmetric/unit width, centered at origin (0,0,0))

Ellipsoid



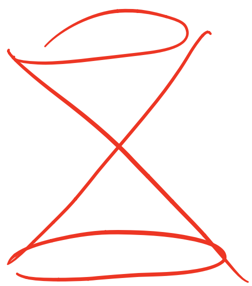
$$x^2+y^2+z^2=1$$

Hyperboloid - 1 sheet



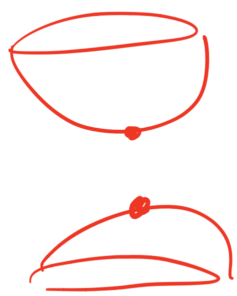
$$x^2+y^2-z^2=1$$

Cone



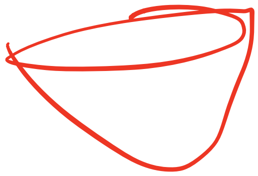
$$x^2+y^2-z^2=0$$

Hyperboloid - 2 sheets



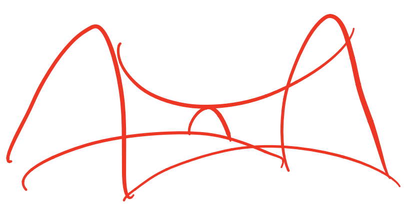
$$x^2+y^2-z^2=-1$$

Paraboloid



$$x^2+y^2=z$$

Saddle



$$x^2-y^2=z$$

a curve

Definition

A **vector-valued function** is a function of the form

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} \quad \text{or} \quad \mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}, \tag{3.1}$$

plane curve   space curve

where the **component functions** f, g, and h, are real-valued functions of the parameter t. Vector-valued functions are also written in the form

$$\mathbf{r}(t) = \langle f(t), g(t) \rangle \quad \text{or} \quad \mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle. \tag{3.2}$$

In both cases, the first form of the function defines a two-dimensional vector-valued function; the second form describes a three-dimensional vector-valued function.

Example 3.1

Evaluating Vector-Valued Functions and Determining Domains

For each of the following vector-valued functions, evaluate $\cancel{\mathbf{r}(0), \mathbf{r}\left(\frac{\pi}{2}\right), \text{ and } \mathbf{r}\left(\frac{2\pi}{3}\right)}$. Do any of these functions have domain restrictions?

$\vec{r}\left(\frac{\pi}{3}\right)$ ✓✓

a. $\mathbf{r}(t) = 4\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j}$

b. $\mathbf{r}(t) = 3\tan t\,\mathbf{i} + 4\sec t\,\mathbf{j} + 5t\,\mathbf{k}$

$$\vec{r}\left(\frac{\pi}{3}\right) = 3\tan\frac{\pi}{3}\,\mathbf{i} + 4\overset{2}{\sec\frac{\pi}{3}}\,\mathbf{j} + 5\,\frac{\pi}{3}\,\mathbf{k}$$

$$= \boxed{3\sqrt{3}\,\mathbf{i} + 8\,\mathbf{j} + 5\frac{\pi}{3}\,\mathbf{k}}$$

Domain restrictions

b/c "tan t" → $t \neq 90° + 180° n$

$$\boxed{t \neq \frac{\pi}{4} + \frac{\pi}{2} n}$$

b/c "sec t" → $\cos t \neq 0$

$t \neq 90° + 180° n$

$$\boxed{t \neq \frac{\pi}{4} + \frac{\pi}{2} n}$$

Formal

Definition

A vector-valued function $\mathbf{r}$ approaches the limit $\mathbf{L}$ as t approaches a, written

$$\lim_{t \to a} \mathbf{r}(t) = \mathbf{L},$$

provided

$$\lim_{t \to a} \|\mathbf{r}(t) - \mathbf{L}\| = 0.$$

This is a rigorous definition of the limit of a vector-valued function. In practice, we use the following theorem:

Theorem 3.1: Limit of a Vector-Valued Function

Let f, g, and h be functions of t. Then the limit of the vector-valued function $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$ as t approaches a is given by

$$\lim_{t \to a} \mathbf{r}(t) = \left[\lim_{t \to a} f(t)\right]\mathbf{i} + \left[\lim_{t \to a} g(t)\right]\mathbf{j}, \tag{3.3}$$

provided the limits $\lim_{t \to a} f(t)$ and $\lim_{t \to a} g(t)$ exist. Similarly, the limit of the vector-valued function $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$ as t approaches a is given by

$$\lim_{t \to a} \mathbf{r}(t) = \left[\lim_{t \to a} f(t)\right]\mathbf{i} + \left[\lim_{t \to a} g(t)\right]\mathbf{j} + \left[\lim_{t \to a} h(t)\right]\mathbf{k}, \tag{3.4}$$

provided the limits $\lim_{t \to a} f(t)$, $\lim_{t \to a} g(t)$ and $\lim_{t \to a} h(t)$ exist.

just limit each component.

Example 3.3

Evaluating the Limit of a Vector-Valued Function

Limits of curves

For each of the following vector-valued functions, calculate $\lim_{t \to 3} \mathbf{r}(t)$ for

a. $\mathbf{r}(t) = \left(t^2 - 3t + 4\right)\mathbf{i} + (4t + 3)\mathbf{j}$

b. $\mathbf{r}(t) = \frac{2t - 4}{t + 1}\mathbf{i} + \frac{t}{t^2 + 1}\mathbf{j} + (4t - 3)\mathbf{k}$

$$\lim_{t \to 3} \left(\left(t^2 - 3t + 4\right)\mathbf{i} + (4t + 3)\mathbf{j} \right) = \left((3)^2 - 3(3) + 4\right)i + \left(4(3) + 3\right)j$$
$$= (9 - 9 + 4)i + (12 + 3)j$$
$$= \boxed{4i + 15j}$$

Definition

Let f, g, and h be functions of t. Then, the vector-valued function $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$ is continuous at point $t = a$ if the following three conditions hold:

1. $\mathbf{r}(a)$ exists
2. $\lim_{t \to a} \mathbf{r}(t)$ exists
3. $\lim_{t \to a} \mathbf{r}(t) = \mathbf{r}(a)$

Similarly, the vector-valued function $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$ is continuous at point $t = a$ if the following three conditions hold:

1. $\mathbf{r}(a)$ exists
2. $\lim_{t \to a} \mathbf{r}(t)$ exists
3. $\lim_{t \to a} \mathbf{r}(t) = \mathbf{r}(a)$

component functions

Theorem ~~Corollary~~

$r(t) = f(t)i + g(t)j + h(t)k$

is cts at $t = a$

iff f, g, h all cts at $t = a$

Definition

formally

The **derivative of a vector-valued function** $\mathbf{r}(t)$ is

$$\mathbf{r}'(t) = \lim_{\Delta t \to 0} \frac{\mathbf{r}(t + \Delta t) - \mathbf{r}(t)}{\Delta t}, \tag{3.5}$$

provided the limit exists. If $\mathbf{r}'(t)$ exists, then $\mathbf{r}$ is differentiable at t. If $\mathbf{r}'(t)$ exists for all t in an open interval (a, b), then $\mathbf{r}$ is differentiable over the interval (a, b). For the function to be differentiable over the closed interval $[a, b]$, the following two limits must exist as well:

$$\mathbf{r}'(a) = \lim_{\Delta t \to 0^+} \frac{\mathbf{r}(a + \Delta t) - \mathbf{r}(a)}{\Delta t} \text{ and } \mathbf{r}'(b) = \lim_{\Delta t \to 0^-} \frac{\mathbf{r}(b + \Delta t) - \mathbf{r}(b)}{\Delta t}.$$

Theorem 3.2: Differentiation of Vector-Valued Functions

Let f, g, and h be differentiable functions of t.

i. If $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$, then $\mathbf{r}'(t) = f'(t)\mathbf{i} + g'(t)\mathbf{j}$.

ii. If $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$, then $\mathbf{r}'(t) = f'(t)\mathbf{i} + g'(t)\mathbf{j} + h'(t)\mathbf{k}$.

Example 3.5

Calculating the Derivative of Vector-Valued Functions

Use **Differentiation of Vector-Valued Functions** to calculate the derivative of each of the following functions.

a. $\mathbf{r}(t) = (6t + 8)\mathbf{i} + \left(4t^2 + 2t - 3\right)\mathbf{j}$

b. $\mathbf{r}(t) = 3\cos t\,\mathbf{i} + 4\sin t\,\mathbf{j}$

c. $\mathbf{r}(t) = e^t \sin t\,\mathbf{i} + e^t \cos t\,\mathbf{j} - e^{2t}\,\mathbf{k}$

$r'(t) = (-3\sin t)i + (4\cos t)j$

Theorem 3.3: Properties of the Derivative of Vector-Valued Functions curves

Let **r** and **u** be differentiable vector-valued functions of t, let f be a differentiable real-valued function of t, and let c be a scalar.

i.	$\frac{d}{dt}[c\mathbf{r}(t)] = c\mathbf{r}'(t)$	Scalar multiple
ii.	$\frac{d}{dt}[\mathbf{r}(t) \pm \mathbf{u}(t)] = \mathbf{r}'(t) \pm \mathbf{u}'(t)$	Sum and diffe ence
iii.	$\frac{d}{dt}[f(t)\mathbf{u}(t)] = f'(t)\mathbf{u}(t) + f(t)\mathbf{u}'(t)$	Scalar product
iv.	$\frac{d}{dt}[\mathbf{r}(t) \cdot \mathbf{u}(t)] = \mathbf{r}'(t) \cdot \mathbf{u}(t) + \mathbf{r}(t) \cdot \mathbf{u}'(t)$	Dot product
v.	$\frac{d}{dt}[\mathbf{r}(t) \times \mathbf{u}(t)] = \mathbf{r}'(t) \times \mathbf{u}(t) + \mathbf{r}(t) \times \mathbf{u}'(t)$	Cross product
vi.	$\frac{d}{dt}[\mathbf{r}(f(t))] = \mathbf{r}'(f(t)) \cdot f'(t)$	Chain rule
vii.	If $\mathbf{r}(t) \cdot \mathbf{r}(t) = c$, then $\mathbf{r}(t) \cdot \mathbf{r}'(t) = 0$.	

special case where $u=r$

position • velocity = 0

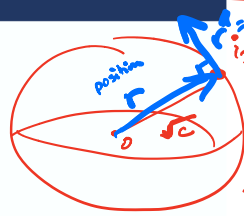
if we're stuck on a sphere

position vector ⊥ velocity vector

$$\frac{d}{dt}\left(r(t)\cdot r(t)\right) = r'(t)\cdot r(t) + r(t)\cdot r'(t) = 2\,r(t)\cdot r'(t)$$

Example 3.6

Using the Properties of Derivatives of Vector-Valued Functions



r' = velocity

if $r \cdot r = c$ then curve is stuck on a sphere

Given the vector-valued functions

$$\mathbf{r}(t) = (6t+8)\mathbf{i} + \left(4t^2+2t-3\right)\mathbf{j} + 5t\mathbf{k}$$

and

$$\mathbf{u}(t) = \left(t^2-3\right)\mathbf{i} + (2t+4)\mathbf{j} + \left(t^3-3t\right)\mathbf{k},$$

calculate each of the following derivatives using the properties of the derivative of vector-valued functions.

a. $\frac{d}{dt}[\mathbf{r}(t) \cdot \mathbf{u}(t)]$ $= (\ r'\ )\cdot(\ u\ ) + (\ r\ )\cdot(\ u'\ )$

b. $\frac{d}{dt}[\mathbf{u}(t) \times \mathbf{u}'(t)]$

$(r\cdot u)' = r'\cdot u + r\cdot u'$

$$r = \langle 6t+8,\ 4t^2+2t-3,\ 5t \rangle$$
$$r' = \langle 6,\ 8t+2,\ 5 \rangle$$
$$u = \langle t^2-3,\ 2t+4,\ t^3-3t \rangle$$
$$u' = \langle 2t,\ 2,\ 3t^2-3 \rangle$$

$$r'\cdot u = 6(t^2-3) + (8t+2)(2t+4) + 5(t^3-3t)$$
$$r\cdot u' = (6t+8)(2t) + (4t^2+2t-3)2 + 5t(3t^2-3)$$

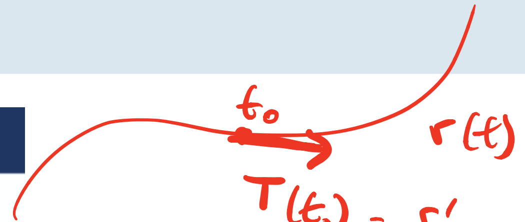
a) $(r\cdot u)' = \boxed{6(t^2-3) + (8t+2)(2t+4) + 5(t^3-3t) + (6t+8)(2t) + (4t^2+2t-3)2 + 5t(3t^2-3)}$

Definition

Let C be a curve defined by a vector-valued function $\mathbf{r}$, and assume that $\mathbf{r}'(t)$ exists when $t = t_0$. A tangent vector $\mathbf{v}$ at $t = t_0$ is any vector such that, when the tail of the vector is placed at point $\mathbf{r}(t_0)$ on the graph, vector $\mathbf{v}$ is tangent to curve C. Vector $\mathbf{r}'(t_0)$ is an example of a tangent vector at point $t = t_0$. Furthermore, assume that $\mathbf{r}'(t) \neq \mathbf{0}$. The **principal unit tangent vector** at t is defined to be

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}, \tag{3.6}$$

provided $\|\mathbf{r}'(t)\| \neq 0$.



$T(t_0) = \frac{r'}{|r'|}$

(so T has unit length)

Example 3.7

Finding a Unit Tangent Vector

Find the unit tangent vector for each of the following vector-valued functions:

a. $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j}$

b. $\mathbf{u}(t) = \left(3t^2 + 2t\right)\mathbf{i} + \left(2 - 4t^3\right)\mathbf{j} + (6t + 5)\mathbf{k}$

$u' = \langle 6t+2,\ -12t^2,\ 6 \rangle$

$$\boxed{T = \frac{(6t+2)i - 12t^2 j + 6k}{\sqrt{(6t+2)^2 + (12t^2)^2 + 6^2}}}$$

Definition

Let f, g, and h be integrable real-valued functions over the closed interval $[a, b]$.

1. The **indefinite integral of a vector-valued function** $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$ is

$r = fi + gj$

$$\int [f(t)\mathbf{i} + g(t)\mathbf{j}]dt = \left[\int f(t)dt\right]\mathbf{i} + \left[\int g(t)dt\right]\mathbf{j}. \quad (3.7)$$

$\int r = \int f\, i + \int g\, j$

The **definite integral of a vector-valued function** is

$$\int_a^b [f(t)\mathbf{i} + g(t)\mathbf{j}]dt = \left[\int_a^b f(t)dt\right]\mathbf{i} + \left[\int_a^b g(t)dt\right]\mathbf{j}. \quad (3.8)$$

2. The indefinite integral of a vector-valued function $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$ is

$$\int [f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}]dt = \left[\int f(t)dt\right]\mathbf{i} + \left[\int g(t)dt\right]\mathbf{j} + \left[\int h(t)dt\right]\mathbf{k}. \quad (3.9)$$

Example 3.8

Integrating Vector-Valued Functions

Calculate each of the following integrals:

a. $\int \left[\left(3t^2 + 2t\right)\mathbf{i} + (3t - 6)\mathbf{j} + \left(6t^3 + 5t^2 - 4\right)\mathbf{k}\right]dt$

b. $\int \left[\langle t, t^2, t^3 \rangle \times \langle t^3, t^2, t \rangle\right]dt$

compute cross first

c. $\int_0^{\pi/3} \left[\sin 2t\,\mathbf{i} + \tan t\,\mathbf{j} + e^{-2t}\,\mathbf{k}\right]dt$

Theorem 3.4: Arc-Length Formulas

$r = fi + gj$

i. *Plane curve*: Given a smooth curve C defined by the function $\mathbf{r}(t) = f(t)\,\mathbf{i} + g(t)\,\mathbf{j}$, where t lies within the interval $[a, b]$, the arc length of C over the interval is

$len = \int_{t=a}^{b} |r'|$

$$s = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2}\,dt = \int_a^b \|\mathbf{r}'(t)\|\,dt. \tag{3.11}$$

ii. *Space curve*: Given a smooth curve C defined by the function $\mathbf{r}(t) = f(t)\,\mathbf{i} + g(t)\,\mathbf{j} + h(t)\,\mathbf{k}$, where t lies within the interval $[a, b]$, the arc length of C over the interval is

$$s = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2}\,dt = \int_a^b \|\mathbf{r}'(t)\|\,dt. \tag{3.12}$$

Example 3.9

Finding the Arc Length

Calculate the arc length for each of the following vector-valued functions:

a. $\mathbf{r}(t) = (3t - 2)\,\mathbf{i} + (4t + 5)\,\mathbf{j},\ 1 \le t \le 5$

b. $\mathbf{r}(t) = \langle t\cos t,\ t\sin t,\ 2t \rangle,\ 0 \le t \le 2\pi$

usually $|r'|$ is a functn of t

$r' = 3i + 4j \qquad |r'| = 5$

$$\text{arclength} = \int_a^b |r'|$$

$$= \int_1^5 5\,dt = 5t\Big]_1^5$$

$$= 5(5-1)$$

$$= \boxed{20}$$

b) ($|r'(t)|$ will be an expressn with t in it)

Theorem 3.5: Arc-Length Function

Let $\mathbf{r}(t)$ describe a smooth curve for $t \geq a$. Then the arc-length function is given by

$$s(t) = \int_a^t \| \mathbf{r}'(u) \| \, du. \tag{3.14}$$

Furthermore, $\frac{ds}{dt} = \| \mathbf{r}'(t) \| > 0$. If $\| \mathbf{r}'(t) \| = 1$ for all $t \geq a$, then the parameter t represents the arc length from the starting point at $t = a$.

Example 3.10

Finding an Arc-Length Parameterization

Find the arc-length parameterization for each of the following curves:

a. $\mathbf{r}(t) = 4\cos t\,\mathbf{i} + 4\sin t\,\mathbf{j},\ t \geq 0$

b. $\mathbf{r}(t) = \langle\, t+3,\ 2t-4,\ 2t \,\rangle,\ t \geq 3$

The Normal and Binormal Vectors

curve velocity $\vec{r}$, $\vec{r}'$

We have seen that the derivative $\mathbf{r}'(t)$ of a vector-valued function is a tangent vector to the curve defined by $\mathbf{r}(t)$, and the unit tangent vector $\mathbf{T}(t)$ can be calculated by dividing $\mathbf{r}'(t)$ by its magnitude. When studying motion in three dimensions, two other vectors are useful in describing the motion of a particle along a path in space: the **principal unit normal vector** and the **binormal vector**.

torsion

unit tangent vector $\boxed{T = \frac{r'}{|r'|}}$

Definition

Let C be a three-dimensional **smooth** curve represented by $\mathbf{r}$ over an open interval I. If $\mathbf{T}'(t) \neq \mathbf{0}$, then the principal unit normal vector at t is defined to be

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}. \tag{3.18}$$

$\boxed{N = \frac{T'}{|T'|}}$

The binormal vector at t is defined as

(torsion)

$$\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t), \tag{3.19}$$

where $\mathbf{T}(t)$ is the unit tangent vector.

$\boxed{B = T \times N}$

thus N is $\perp$ to T

(to prove use Properties 3.3 viii)

T is already "unit speed" b/c $T = \frac{r'}{|r'|}$

Note that, by definition, the binormal vector is orthogonal to both the unit tangent vector and the normal vector. Furthermore, $\mathbf{B}(t)$ is always a unit vector. This can be shown using the formula for the magnitude of a cross product

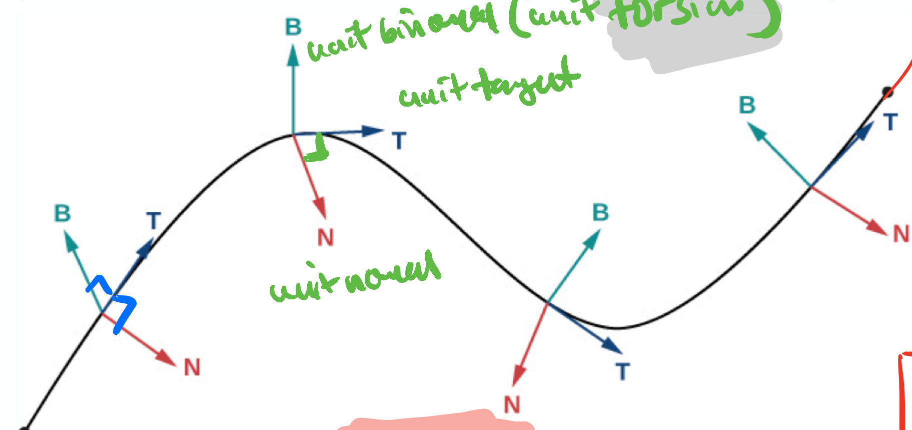


Figure 3.7 This figure depicts a Frenet frame of reference. At every point P on a three-dimensional curve, the unit tangent, unit normal, and binormal vectors form a three-dimensional frame of reference.

Frenet frame: T, N, B

See: differential geometry 1

Example 3.12

Finding the Principal Unit Normal Vector and Binormal Vector

For each of the following vector-valued functions, find the principal unit normal vector. Then, if possible, find the binormal vector.

a. $\mathbf{r}(t) = 4\cos t\,\mathbf{i} - 4\sin t\,\mathbf{j}$

b. $\mathbf{r}(t) = (6t+2)\,\mathbf{i} + 5t^2\,\mathbf{j} - 8t\,\mathbf{k}$

Find Frenet Frame.

$r' = \langle 6, 10t, -8 \rangle$

$|r| = \sqrt{36 + 100t^2 + 64} = \sqrt{100 + 100t^2} = \boxed{10\sqrt{1+t^2}}$

$T = \left\langle \frac{6}{10\sqrt{1+t^2}}, \frac{10t}{10\sqrt{1+t^2}}, \frac{-8}{10\sqrt{1+t^2}} \right\rangle$

$N =$

... the rest left as exercise for reader.

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Definition

Let $\mathbf{r}(t)$ be a twice-differentiable vector-valued function of the parameter t that represents the position of an object as a function of time. The **velocity vector** $\mathbf{v}(t)$ of the object is given by

$$\text{Velocity} = \mathbf{v}(t) = \mathbf{r}'(t). \tag{3.20}$$

r'

The **acceleration vector** $\mathbf{a}(t)$ is defined to be

$$\text{Acceleration} = \mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t). \tag{3.21}$$

r''

The *speed* is defined to be

$$\text{Speed} = v(t) = \|\mathbf{v}(t)\| = \|\mathbf{r}'(t)\| = \frac{ds}{dt}. \tag{3.22}$$

$|r'|$

Quantity	Two Dimensions	Three Dimensions
Position	$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$	$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$
Velocity	$\mathbf{v}(t) = x'(t)\mathbf{i} + y'(t)\mathbf{j}$	$\mathbf{v}(t) = x'(t)\mathbf{i} + y'(t)\mathbf{j} + z'(t)\mathbf{k}$
Acceleration	$\mathbf{a}(t) = x''(t)\mathbf{i} + y''(t)\mathbf{j}$	$\mathbf{a}(t) = x''(t)\mathbf{i} + y''(t)\mathbf{j} + z''(t)\mathbf{k}$
Speed	$v(t) = \sqrt{(x'(t))^2 + (y'(t))^2}$	$v(t) = \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}$

Table 3.4 Formulas for Position, Velocity, Acceleration, and Speed

Example 3.14

Studying Motion Along a Parabola

A particle moves in a parabolic path defined by the vector-valued function $\mathbf{r}(t) = t^2\mathbf{i} + \sqrt{5 - t^2}\,\mathbf{j}$, where t measures time in seconds.

a. Find the velocity, acceleration, and speed as functions of time.
b. Sketch the curve along with the velocity vector at time $t = 1$.

$$r' = \left\langle 2t, \frac{-\cancel{2}t}{\cancel{2}\sqrt{5-t^2}} \right\rangle$$

speed at time t.

$$|r'| = \boxed{\sqrt{4t^2 + \frac{t^2}{5-t^2}}} = \text{speed as a fn of } t.$$