

## Announcements.

- ① Course evaluations open May 3<sup>rd</sup> - May 17<sup>th</sup> on CSI-Email.
- ② Exam 3 , will open on April 28<sup>th</sup>. ~ 9-10 Problems.
- ③ Final Exam Coming soon! About 9-10 Problems , go over old exams.
- ④ Next week going over Exams, and Q&A .

## §16.1 Principles of Probability.

A scientific way to measure uncertainty.

Sample Space: The set of different possible individual outcomes.

- Ex:
- Flipping a coin.  $\{H, T\}$
  - Die :  $\{1, 2, 3, 4, 5, 6\}$

Event: Collection of outcomes; Sub set of the sample space.

- Ex:
- Flipping a coin as achieving a head  $\{H\}$ .
  - In a die, the event of rolling an even #  $\{2, 4, 6\}$
  - In a die, the event of rolling a # bigger than 3 :  $\{4, 5, 6\}$

## Definition of Probability

- ① The likelihood of a certain outcome quantified.
- ② The fraction or percent of times an outcome is expected to occur.

$$Pr(\text{Event}) = P(\text{Event}) = \frac{\# \text{ of outcomes in an Event}}{\# \text{ of all possible outcomes}}$$

- Probability of achieving heads on a flip?

$$P(H) = \frac{1}{2} = .5 = 50\%$$

- Probability of rolling a die and achieve an even #.

$$S = \{1, 2, 3, 4, 5, 6\} \quad Pr(\text{Even } \#) = \frac{3}{6} = \frac{1}{2} = 50\%$$

$$\text{Even } \# = \{2, 4, 6\}$$

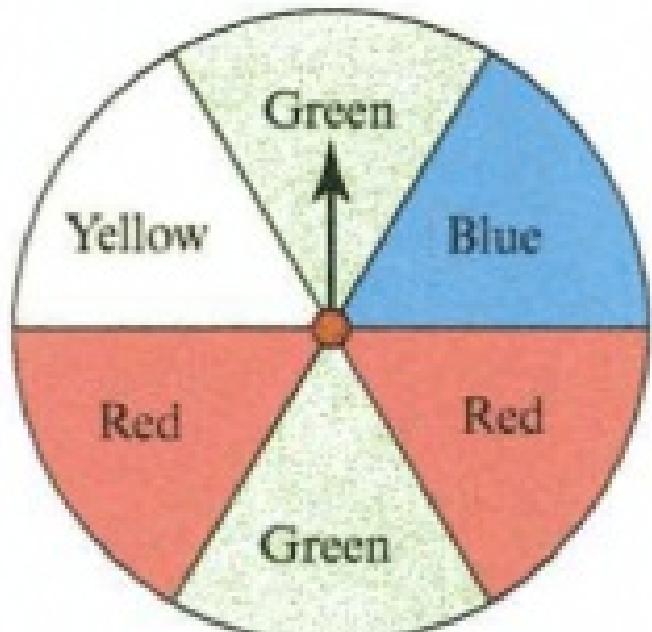
Keep in mind

E: Event.

$P(E)$  is Positive  
and always between  
0 and 1.

PJ 728 #1

1. Determine the probability of spinning either a red or a yellow on the spinner shown in **Figure 16.2**.  
Explain briefly.



6 total outcomes:  $\{G, G, R, R, B, Y\}$ .

Event of Red or Yellow =  $\{R, R, Y\}$ .

$$P(\text{Red or Yellow}) = \frac{3}{6} = \frac{1}{2} = .50 = 50\%$$

1. Some games have spinners. When the arrow in a spinner is spun, it can land in any one of several different colored regions. Determine the probability of spinning each of the following on a spinner like the one in **Figure 16.4**: The spinner is

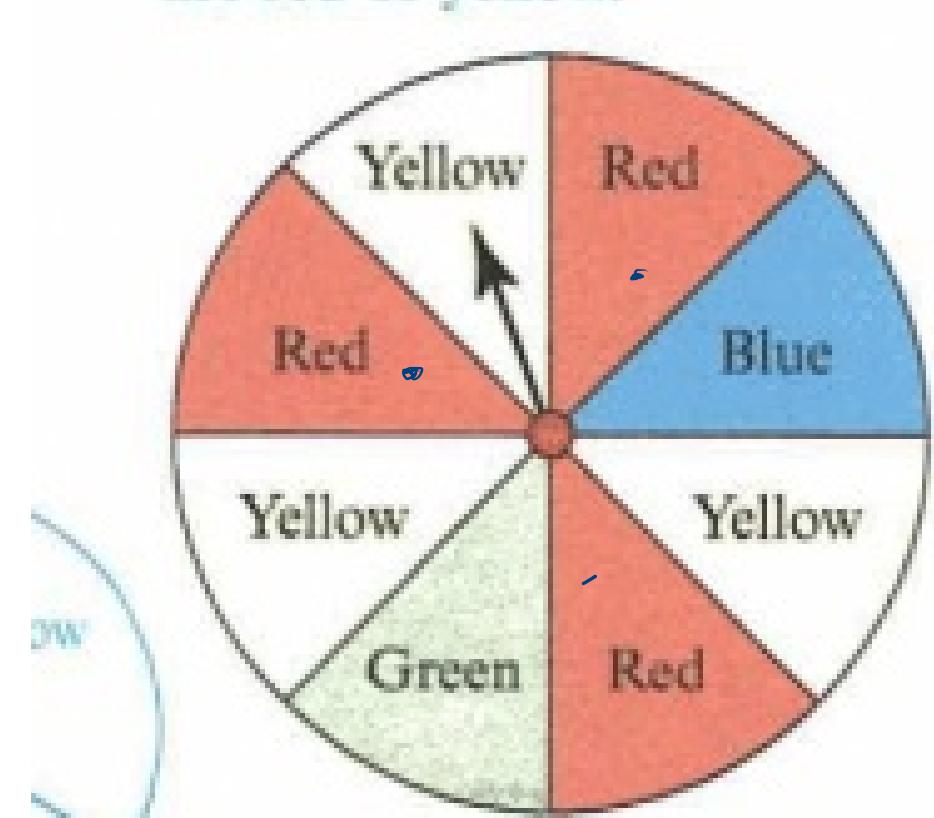
- Red
- Either red or green
- Either red or yellow

$$\textcircled{a} \quad P(\text{Red}) = \frac{3}{8} = .375 = 37.5\%$$

$$\textcircled{b} \quad P(\text{Red or Green}) = \frac{4}{8} = \frac{1}{2} = 50\%$$

$$\textcircled{c} \quad P(\text{Red or Yellow}) = \frac{6}{8} = \frac{3}{4} = .75 = 75\%$$

16.4



2. a. Draw a spinner such that the probability of landing on red is  $\frac{1}{3}$ , the probability of landing on green is  $\frac{1}{4}$ , and the only other color that the spinner could land on is yellow. Answers will

vary. One possible answer is in Figure to above left.

- b. For your spinner in part (a), what is the probability of landing on yellow? Explain. The proba

- c. Now draw a different spinner that has the same probabilities as in part (a). Explain briefly.

$$\textcircled{b} \quad P(\text{Yellow}) = \frac{5}{12}; \text{ see diagram.}$$

$$P(\text{Red}) + P(\text{Green}) + P(\text{Yellow}) = 1$$

$$\frac{1}{3} + \frac{1}{4} + x = 1$$

$$\frac{4}{12} + \frac{3}{12} + x = 1$$

$$\frac{7}{12} + x = 1$$

$$x = \frac{5}{12}$$

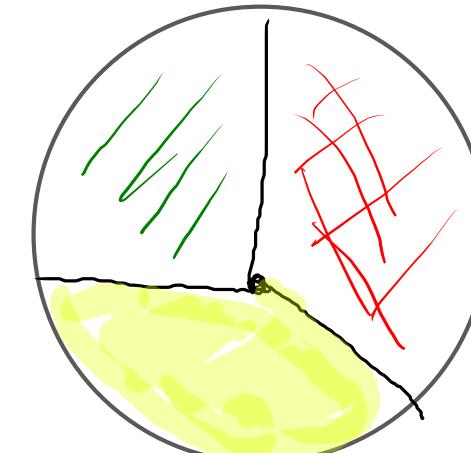
$$P(\text{Red}) = \frac{1}{3} \cdot \frac{3}{4} = \frac{4}{12}$$

$$P(\text{Green}) = \frac{1}{4} \cdot \frac{3}{4} = \frac{3}{12}$$

$$P(\text{Yellow}) = ?$$



(c)



## §16.2 Counting Number of Outcomes.

Flipping a Coin :  $\{H, T\}$ .

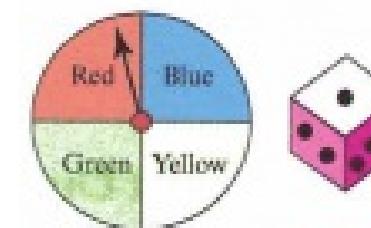
Flip two coins : Quarter, Nickel :  $\{HH, HT, TT, TH\}$ .  
 $\{H, T\} \quad \{H, T\} = \text{Total outcomes } 4$ .

### Two-Stage Experiment.

Spinner :  $\{\text{Red, blue, yellow, green}\} = 4$

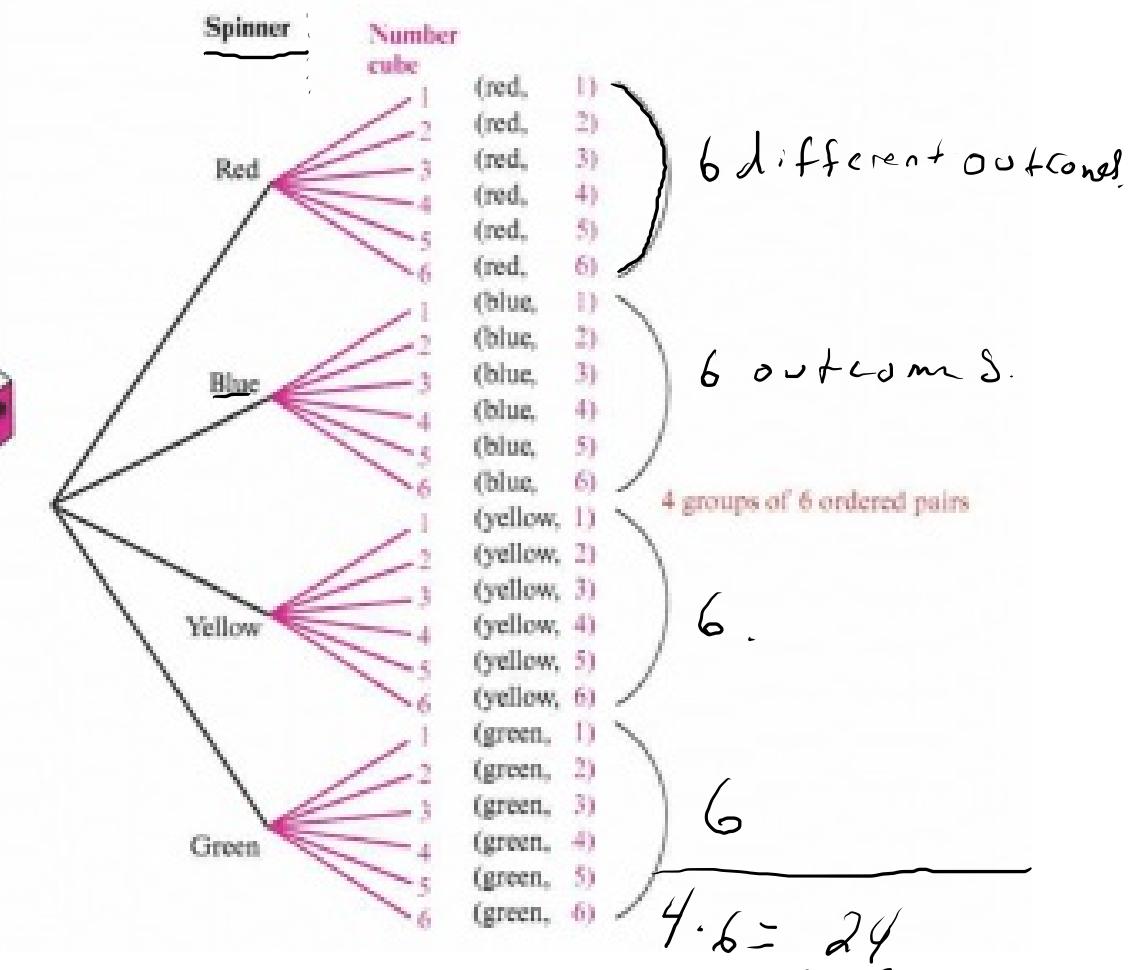
Dice :  $\{1, 2, 3, 4, 5, 6\} = 6$

$$4 \cdot 6 = 24$$



A two-stage experiment:  
spin the spinner.  
roll the number cube

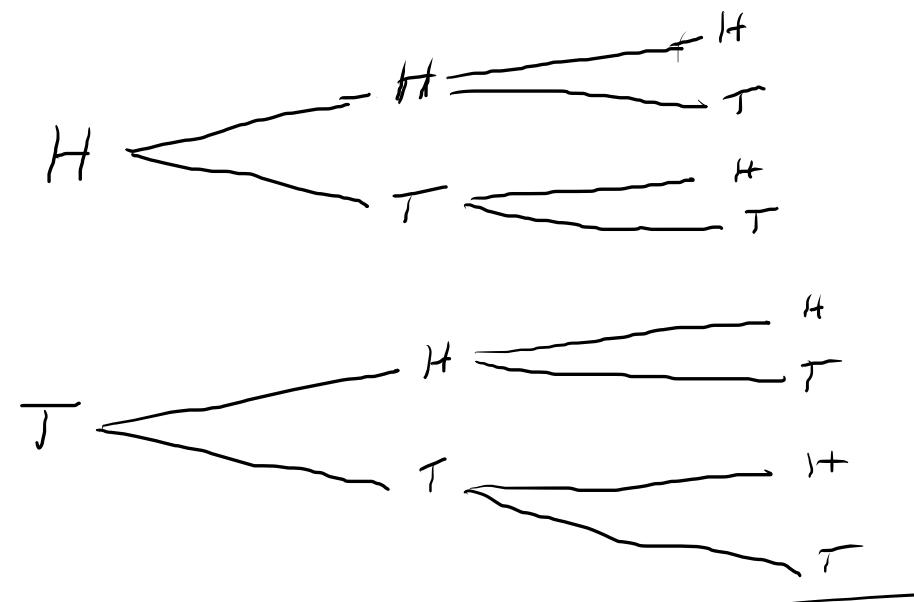
Pg 781.



To find total number of different outcomes in a multi-stage experiment just multiply each of the individual outcomes.

Ex] Flipping a coin 3 times.

$$\left\{ \begin{array}{c} 2 \\ \hline 1^{\text{st}} \end{array} \cdot \begin{array}{c} 2 \\ \hline 2^{\text{nd}} \end{array} \cdot \begin{array}{c} 2 \\ \hline 3^{\text{rd}} \end{array} = 8 \text{ different outcomes.} \end{array}$$



8 outcomes

Ex] Rolling two dice.

$$\frac{6}{\text{Blue die}} * \frac{6}{\text{Green die}} = 36.$$

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2						
3						
4						
5						
6						(6, 6)

Pg 733

1. How many different 3-digit numbers can you write using only the digits 1, 2, and 3 if you do not repeat any digits (so that 121 and 332 are not counted)? Show how to solve this problem with an organized list and with a tree diagram. Explain why this problem can be solved by multiplying.

①  $\underline{3} \cdot \underline{2} \cdot \underline{1} = 6 \text{ outcomes}$

1  $\begin{array}{c} 2 - 3 \\ 3 - 2 \end{array}$   
2  $\begin{array}{c} 1 - 3 \\ 3 - 1 \end{array}$   
3  $\begin{array}{c} 1 - 2 \\ 2 - 1 \end{array}$

2. How many different 3-digit numbers can be made by using only the digits 1, 2, and 3, where repeated digits are allowed (so that 121 and 332 are counted)? Show how to solve this problem with an organized list and with a tree diagram. Explain why this problem can be solved by multiplying.

②  $\underline{3} \cdot \underline{3} \cdot \underline{3} = 27 \text{ outcomes}$

4. Annette buys a wardrobe of 3 skirts, 3 pants, 5 shirts, and 3 sweaters, all of which are coordinated so that she can mix and match them any way she likes. How many different outfits can Annette create from this wardrobe? (Every day Annette wears either a skirt or pants, a shirt, and a sweater.)

$$\frac{6}{\text{Skirt + pants.}} \cdot \frac{5}{\text{Shirt}} \cdot \frac{3}{\text{Sweater}} = \underline{\underline{90 \text{ different outfits.}}}$$

**3. Explain your answers to the following:**

- a. How many 9-digit numbers are there that use only the digits 1, 2, 3, ..., 8, 9? (Repetitions are allowed, so, for example, 123211114 is allowed.)  
<sup>9<sup>9</sup></sup>

b. How many 9-digit numbers are there that use each of the digits 1, 2, 3, ..., 8, 9 exactly once?

$$= 9^9 = 387,420,489 \text{ different outcomes}$$

$$\textcircled{b} \quad \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{= 362,880} = 9$$

# Factories

$n!$ : number of ways to arrange  $n$  different objects.

$$4! = 4 \cdot 3 \cdot 2 \cdot 1$$

$$\#S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

5. Most Georgia car license plates currently use the format of 3 numbers followed by 3 letters (such as 123 ABC). How many different license plates can be made this way? \* Assume repetition is allowed

$$\underbrace{\underline{10} \cdot \underline{10} \cdot \underline{10}}_{\#^5} + \underbrace{\underline{26} \cdot \underline{26} \cdot \underline{26}}_{\text{Letters.}}$$

$$= 10^3 \cdot 26^3 =$$

6. a. A 40-member club will elect a president and then elect a vice-president. How many possible outcomes are there? 1560. We have 40 choices for Pres, leaving 39 choices for V.P.

b. A 40-member club will elect a pair of co-presidents. How many possible outcomes are there?

$$\begin{array}{r} \textcircled{a} \\ \underline{40.} \quad \underline{39} \\ \text{Prcz} \quad \text{V.P.} \\ = \boxed{1560} \end{array}$$

$$\frac{40}{P_1} \quad \frac{39}{P_2} = 1560, \quad \frac{1560}{2} = \boxed{780}$$

outcomes.