10.1
p 465
Problem

9. Given that the lines in Figure 10.34 marked with arrows are parallel, determine the sum of the angles $a + b + c$ without measuring. 360 degrees.

Parallel lines Z-rule

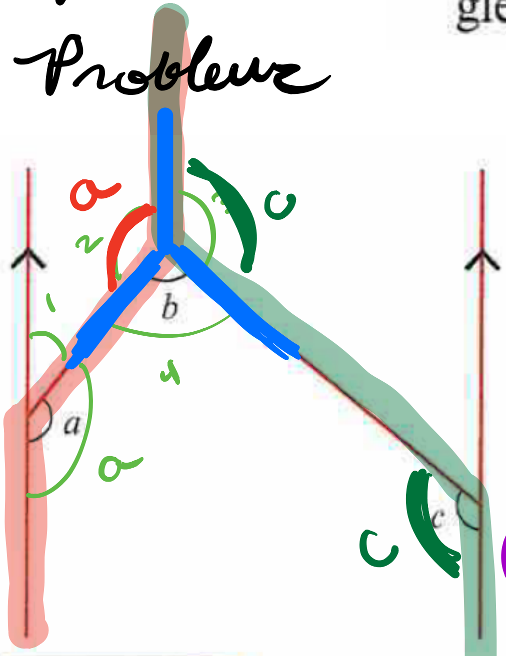
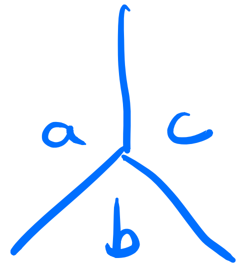
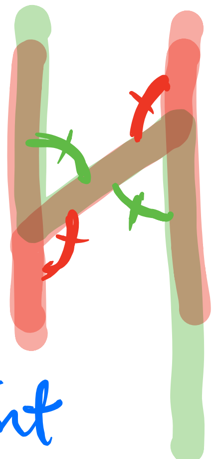


Figure 10.34 What is the sum of the marked angles?





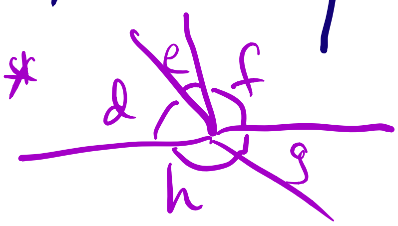
(*) full turn about the same point

So $a+b+c = 360°$

$l+m=360°$

*

*



$d+e+f+g+h = 360°$

10. Determine the sum $a + b + c + d + e + f$ of the angles in the 6-sided shape (hexagon) in Figure 10.35 without measuring. Explain your reasoning. See drawn-in lines. The sum is 720 degrees. b/c it is 6 triangles, each 120 degrees.

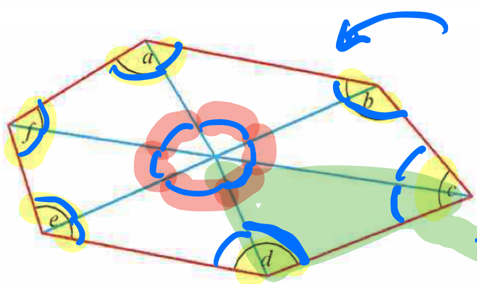
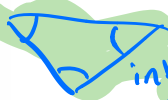


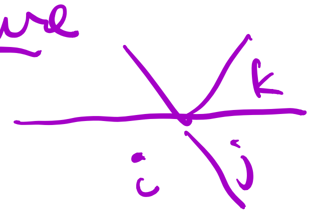
Figure 10.35 What is the sum of the angles in a 6-sided figure?

Sum of these 18 angles is 6 triangles * 180° each = 1080°



interior angles of triangle add ~~to~~ 180°

Compare



$i+j+k \neq 360°$

Those 18 angles 1080° − extra angles in center 360° = $\boxed{a+b+c+d+e+f = 720°}$

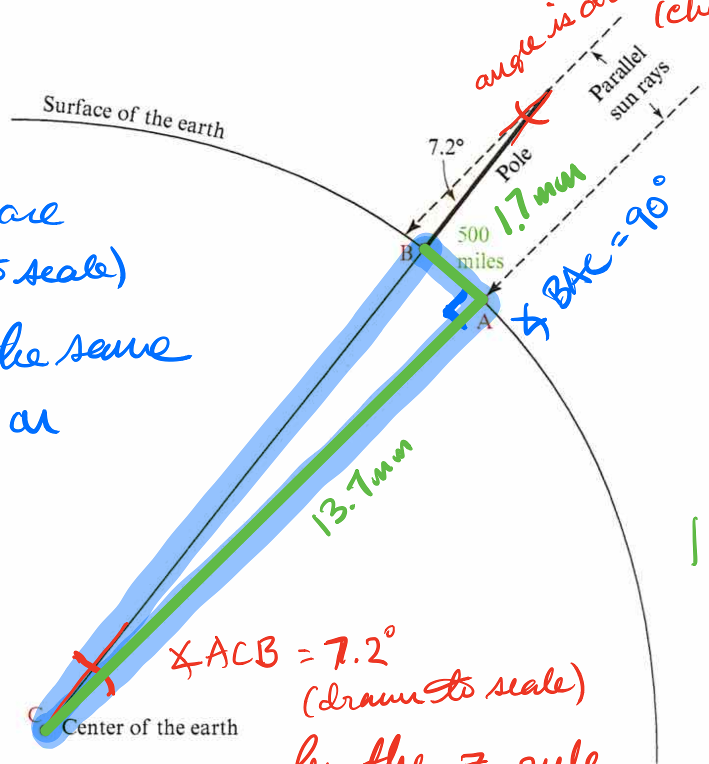
10.2 Angles and Phenomena in the World

Class Activity 10I Eratosthenes's Method for Determining the Circumference of the Earth

CCSS CCSS SMP4

The figure below shows a cross-section of the earth. At noon on June 21, the sun is directly overhead at location A, so that the sun's rays are perfectly vertical there. At the same time, 500 miles away at location B, the sun's rays make a 7.2° angle with the tip of a vertical pole (shown not to scale), which was determined by considering the shadow that the pole casts. Because the sun is far away, sun rays at the earth are (approximately) parallel. Use this information to determine the circumference of the earth, explaining your reasoning.

angle is drawn to scale (checked with protractor)



Sure the angles are accurate (drawn to scale) the sides have the same relationships as on the paper

1.7 mm

$\measuredangle BAC = 90°$

13.7mm

$\measuredangle ACB = 7.2°$ (drawn to scale) by the Z-rule

Scale is

500 miles = 1.7mm

$\frac{500 \text{ miles}}{1.7\text{mm}} = 1$

$13.7\,\text{mm} * \frac{500 \text{ miles}}{1.7\text{mm}}$

= 4030 miles

= radius of earth

C = circumference

r = radius

$\boxed{\text{Circumference of earth}}$

$C = 2\pi r = 2\pi(4030 \text{ miles})$

$\boxed{= 25300 \text{ miles}}$

may get slightly different numbers from rounding when using calculator

from internet: 24900 miles

so, we were pretty close!

(This problem can also be done using "right triangle trig.")

CLASS ACTIVITY

10J Why Do Spoons Reflect Upside Down?, p. CA-210

10K How Big Is the Reflection of Your Face in a Mirror?, p. CA-210

SECTION SUMMARY AND STUDY ITEMS

Section 10.2 Angles and Phenomena in the World

Angles arise in real-world situations. For example, angles are formed by the sun's rays and angles describe reflected light rays.

Key Skills and Understandings

1. Apply the laws of reflection to explain what a person looking into a mirror will see.

see activity 10J below

Practice Exercises for Section 10.2

1. Make math drawings to show the relationship between the angle that the sun's rays make with horizontal ground and the length of the shadow of a telephone pole.

2. Figure 10.40 shows several math drawings (from the point of view of a fly looking down from the ceiling) of a person standing in a room, looking into a mirror on the wall. The direction of the person's gaze is indicated with a dashed line. What place in the room will the person see in the mirror?

bird's eye view

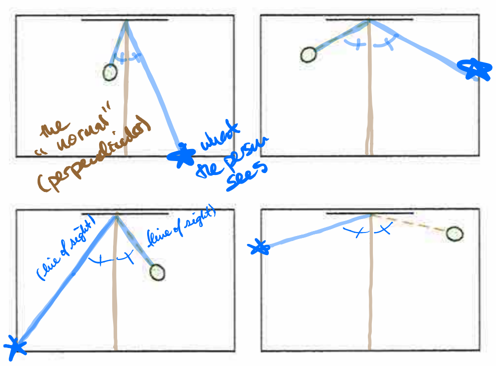


Figure 10.40 Looking into a mirror.

3. Figure 10.41 shows a mirror seen from the top, and a light ray hitting the mirror. Draw a copy of this picture on a blank piece of paper. Use the following paper-folding method to show the location of the reflected light ray:

- Fold and crease the paper so that the crease goes through the point where the light ray hits the mirror and so that the line labeled "mirror" folds onto itself. (This crease is perpendicular to the line labeled "mirror." You'll be asked to explain why shortly.)
- Keep the paper folded, and now fold and crease the paper again along the line labeled "light ray."
- Unfold the paper. The first crease is the normal line to the mirror. The second crease shows the light ray and the reflected light ray.

a. Explain why your first crease is perpendicular to the line labeled "mirror."

b. Explain why your second crease shows the reflected light ray.

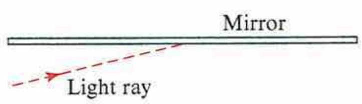


Figure 10.41 Using paper folding to find the reflected ray.

Answers to Practice Exercises for Section 10.2

1. Figure 10.42 shows that when the sun's rays make a smaller angle with horizontal ground, a telephone pole makes a longer shadow than when the sun's rays make a larger angle with the ground.

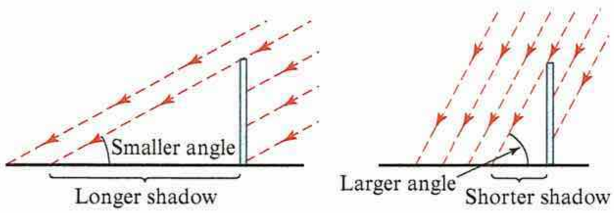


Figure 10.42 Sun rays hitting a telephone pole.

2. Figure 10.43 shows that the person will see locations A, B, C, and D, respectively, which are on various walls. Location C is in a corner. As the drawings show, the incoming light rays and their reflections in the mirror make the same angle with the normal line to the mirror at the point of reflection.

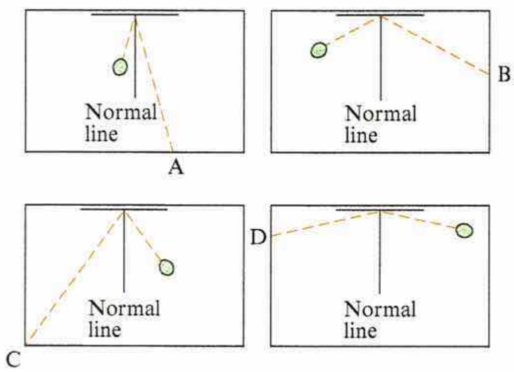


Figure 10.43 What a person sees, looking in a mirror.

3. **a.** The first crease is made so that angles a and b shown in Figure 10.44 are folded on top of each other and completely aligned. Therefore, angles a and b are equal. But these angles must add up to 180° because the angle formed by a straight line is 180°. Hence, angles a and b must both be half of 180°, which is 90°. Therefore, this first crease is the normal line to the mirror at the point where the light ray hits the mirror.

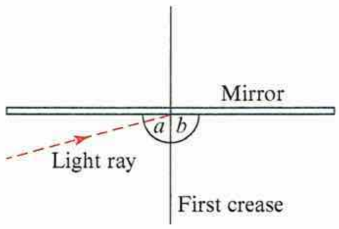


Figure 10.44 First crease.

b. The second crease is made so that angles c and d, shown in Figure 10.45 are folded on top of each other and completely aligned. Therefore, angle c is equal to angle d. Since the first crease is a normal line, by the reflection laws, the second crease shows the reflected light ray.

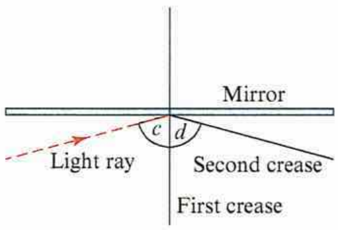


Figure 10.45 Second crease.

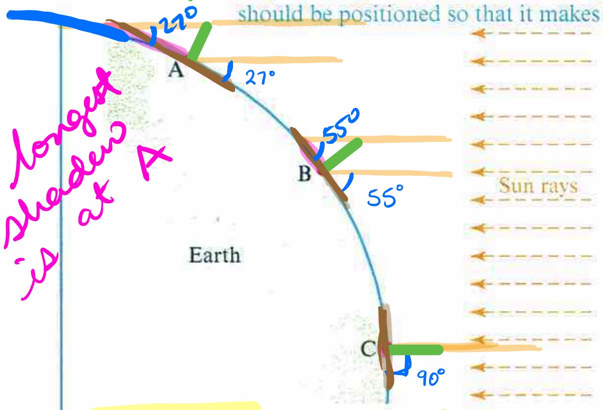
PROBLEMS FOR SECTION 10.2

1. Figure 10.46 depicts sun rays traveling toward 3 points on the earth's surface, labeled A, B, and C. Assume that these sun rays are parallel to the plane of the page.

a. Use a protractor to determine the angle that the sun's rays make with horizontal ground at points A, B, and C.

b. Suppose there are telephone poles of the same height at locations A, B, and C. Make drawings like the ones in Figure 10.47 on which the vertical lines represent telephone poles, and show on your drawings the different lengths of the shadows of these telephone poles, using the information from part (a). Angle at A is about 35 degrees, at B is about 55 degrees, and at C is about 85 degrees.

2. See figures. Joey sees light reflected off the mirror at point A but that light came from an inside wall of the periscope (see figure (a)). So in fact, no light (or extremely little light) is reflected, and Joey sees darkness. The mirror should be positioned so that it makes a 45° angle with the walls of the periscope (see figure (b)).



longest shadow is at A

Figure 10.46 Sun rays traveling to the earth.

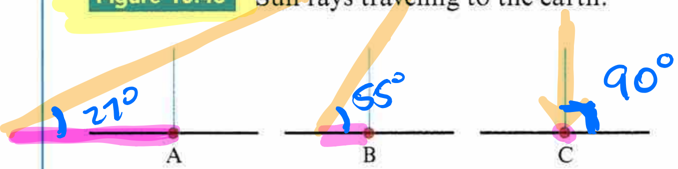


Figure 10.47 Show the shadows produced by the telephone poles.

2. Figure 10.48 shows a cross-section of Joey's toy periscope. What will Joey see when he looks in the periscope? Explain, using the laws of reflection (trace the periscope). What would be a better way to position the mirror in the telescope?

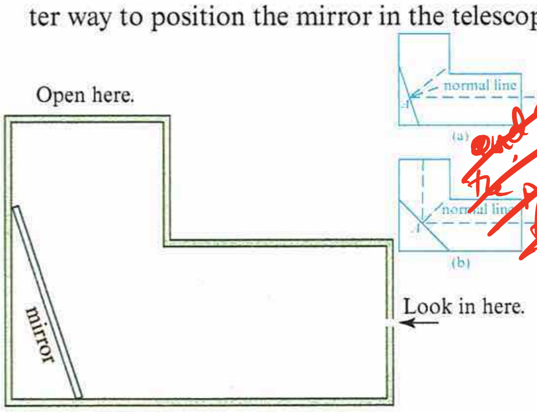


Figure 10.48 Joey's periscope.

~~and 90 - 23.5° from the perpendicular to the sun's rays~~

3. Figure 10.49 shows a sketch of a room that has a pair of perpendicular mirrors, drawn from the point of view of a fly on the ceiling. A person is standing in the room, looking into one of the mirrors at the indicated spot. On a separate piece of paper, draw a large sketch that looks (approximately) like Figure 10.49. Using your sketch, determine what the person will see in the mirror. (You may want to use paper folding, which is described in Practice Exercise 2.) Explain your answer.

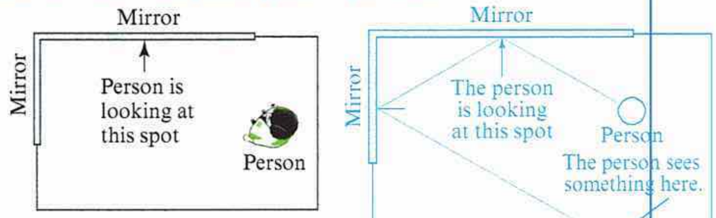


Figure 10.49 What will the person see, looking in the mirror? See figure.

does not contribute to season

the earth is actually (a little) closer to sun in winter!

***4.** Many people mistakenly believe that the seasons are caused by the earth's varying proximity to the sun. In fact, the distance from the earth to the sun varies only slightly during the year, and the seasons are caused by the tilt of the earth's axis. As the earth travels around the sun during the year, the tilt of the earth's axis causes the northern hemisphere to vary between being tilted toward the sun to being tilted away from the sun.

earth
sun

Figure 10.50 shows the earth as seen at a certain time of year from a point in outer space located in the plane in which the earth rotates about the sun. The diagram shows that the earth's axis is tilted 23.5° from the perpendicular to the plane in which the earth rotates around the sun. Throughout this problem, assume that the sun rays are parallel to the plane of the page.

the plane that the orbit lies on

at this point in the year in fig 10.50, the northern hemisphere is tilted towards the sun

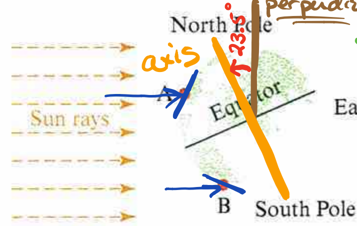


Figure 10.50 Earth and sun.

sunlight
sun is higher in sky at noon at A
sunlight
sunlight more intense
A
B
sun provides less intense sunlight.

a. Locations A and B in Figure 10.50 are shown at noon. Is the sun higher in the sky at noon at location A or at location B? Explain how you can tell. Point A b/c the sun rays' angle w/ surface is closer to 90 degrees

b. During the day, locations A and B will rotate around the axis through the North and South Poles. Compare the amount of sunlight that locations A and B will receive throughout the day. Which location will receive more sunlight during the day? Point A b/c the sun is already lower in the sky for point B at noon (its highest point) it will set sooner at point B. So point A receives more sunlight.

Figure 10.50. Earths northern hemisphere tilted towards sun: it's summer there. Southern hemisphere away from sun: it's winter there.

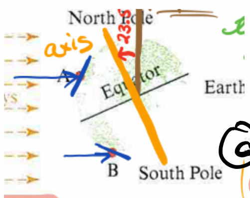
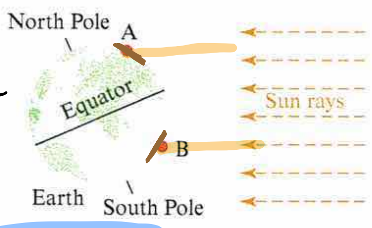


Figure 10.50 Earth and sun.

c. Based on your answers to parts (a) and (b), what season is it in the northern hemisphere, and what season is it in the southern hemisphere in Figure 10.50? Explain. Northern hemisphere: summer; Southern hemisphere: winter.

d. At another time of year, the earth and sun are positioned as shown in Figure 10.51. At that time, what season is it in the northern and southern hemispheres? Why? Winter in the north and summer in the south.

10.51: It's the opposite of 10.50, so northern hemisphere tilted away from sun southern hemisphere tilted towards the sun, so southern hemisphere gets the higher angle sunlight (more intense)



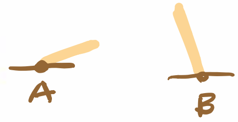


Figure 10.51 Earth and sun at another time of year.

e. At yet another time of year, the earth and sun are positioned as shown in Figure 10.52. At that time, what season is it in the northern and southern hemispheres? Why? Give an either-or-answer. (Notice that Figure 10.51 still shows the tilt of the earth's axis.)

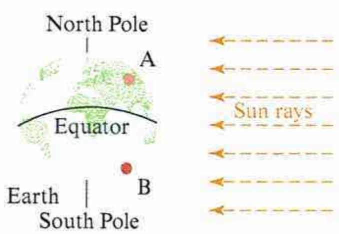


Figure 10.52 Earth and sun at yet another time of year.

Either fall in N & spring in S or vice versa. But since the earth travels counter-clockwise around the sun, then the northern hemisphere will soon be tilted toward the sun, so it is spring there and fall in the southern hemisphere

f. Refer to Figures 10.50, 10.51, and 10.52 and the results of the previous parts of this problem to answer the following: During which seasons are the sun's rays most intense at the equator? Look carefully before you answer—the answer may surprise you. Spring and fall. b/c the sun's rays are directly over the equator then.

*5. Refer to Figures 10.50, 10.51, 10.52, and 10.53 to help you answer the following: There are only certain locations on the earth where the sun can ever be seen *directly* overhead. Where are these locations? How are these locations related to the Tropic of Cancer and the Tropic of Capricorn? Explain.

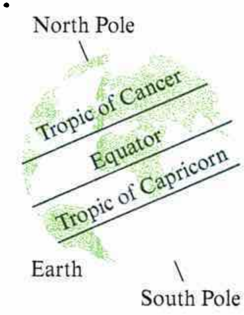


Figure 10.53 The Tropic of Cancer and the Tropic of Capricorn.

The sun can only be seen directly overhead between the Tropic of Cancer and the Tropic of Capricorn—these being the locations at which the sun is directly overhead at the summer solstice and winter solstice, respectively

*6. Department store dressing rooms often have large mirrors that actually consist of three adjacent mirrors, put together as shown in Figure 10.54 (as seen looking down from the ceiling). Use the laws of reflection to show how you can stand in such a way as to see the reflection of your back. Make a careful drawing, using an enlarged version of Figure 10.54, that shows clearly how light reflected off your back can enter your eyes. Your drawing should show where you are standing, the location of your back, and the direction of the gaze of your eyes. (You may wish to experiment with paper folding before you attempt a final drawing. See Practice Exercise 2.) See figure.

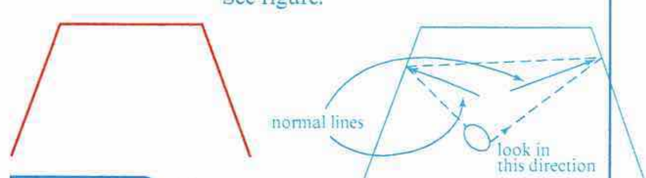
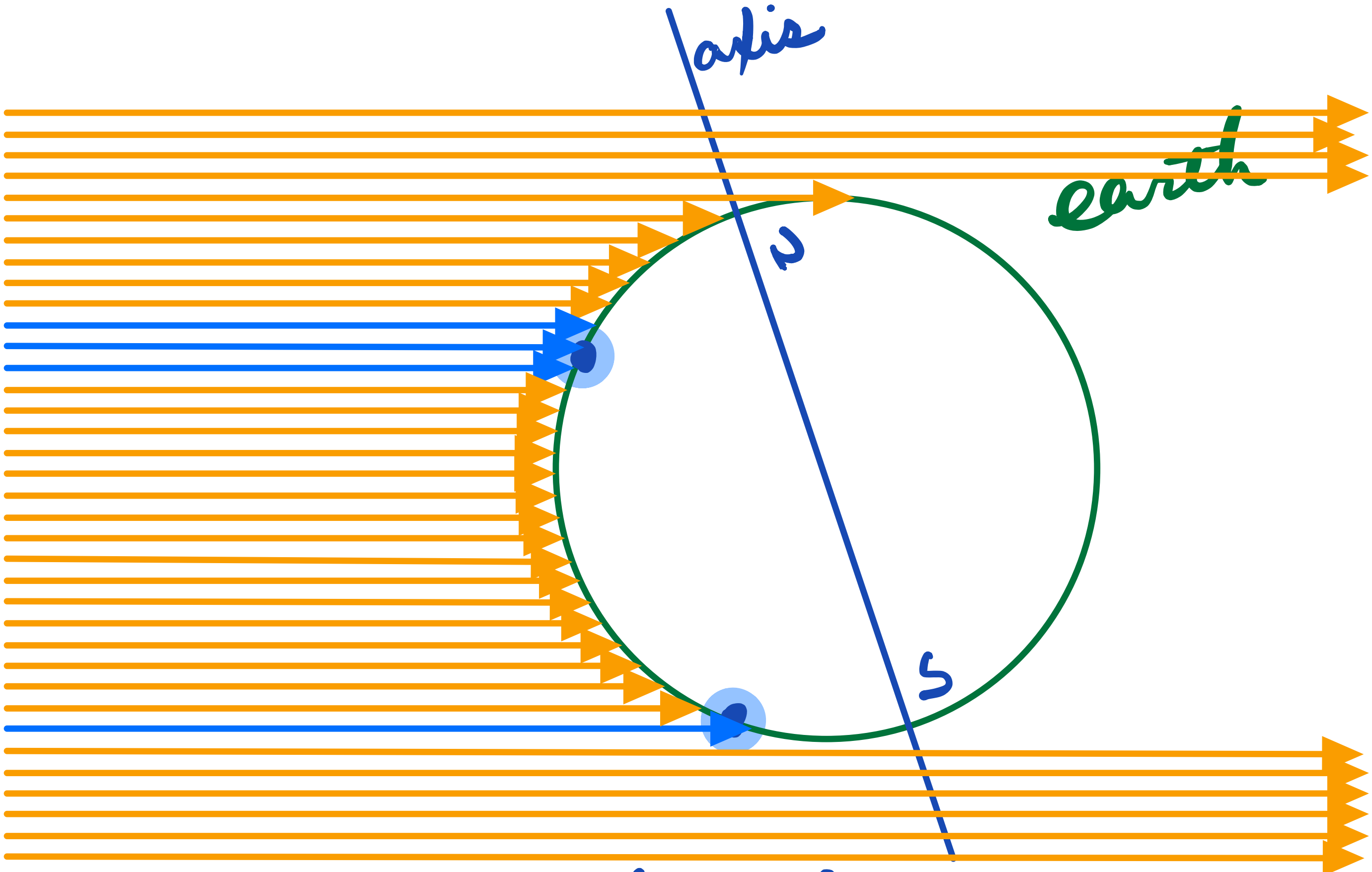


Figure 10.54 Department store mirror.

*7. A **concave** mirror is a mirror that curves in, like a bowl, so that the normal lines on the reflective side of the mirror point toward each other. Makeup mirrors are often concave. The left side of Figure 10.55 shows an eye looking into a concave makeup mirror. The right side of Figure 10.55 shows an eye looking into an ordinary flat mirror. Trace the two diagrams in Figure 10.55 and, in each case, show where a woman applying eye makeup sees her eye in the mirror. Use the laws of reflection to show approximately where the woman sees the top of her reflected eye and where she sees the bottom of her reflected eye. (Assume that the woman sees light that enters the center of her eye.) Notice that the figure shows the normal lines to the concave mirror. Based



The same amount of land gets way more sunlight at A, because the sunlight comes in at a high angle, so sunlight is "closer together"

It's summer at A.

The same size city A in N hemisphere gets "more sunbeams" than B in S hemisphere so it's summer in north & winter in south.