Figure 10.62

Three ways that two spheres can meet.

These 2 spheres don't meet.

These 2 spheres meet at a single point.

These 2 spheres meet along a circle.

CLASS ACTIVITY

10N The Global Positioning System (GPS), p. CA-214

10O Circle Designs, p. CA-215

If you did Class Activity 10M, you saw some applications of circles. We will use circles again in the next section to construct special triangles and 4-sided figures. The mathematical definition of circle will allow us to explain why our constructions work.

SECTION SUMMARY AND STUDY ITEMS

Section 10.3 Circles and Spheres

A circle is the collection of all the points in a plane that are a fixed distance away from a fixed point in the plane. A sphere is the collection of all the points in space that are a fixed distance away from a fixed point in space. Two circles in a plane can either not meet, meet at a single point, or meet at two points. Two spheres in space can either not meet, meet at a single point, or meet along a circle.

Key Skills and Understandings

[handwritten annotations: all point in the plane / all points in space]

1. Give the definitions of circles and spheres.

[handwritten: a collection of points all the same fixed distance from the center.]

2. Given a situation that involves distances from one or more points, use circles or spheres to describe the relevant locations.

[handwritten: Circle lives in 2 dimensional space / Sphere lives in 3 dimensional space]

Practice Exercises for Section 10.3

[handwritten: p475]

1. Give the (mathematical) definitions of the terms *circle* and *sphere*.

2. Points P and Q are 4 centimeters apart. Point R is 2 cm from P and 3 cm from Q. Use a ruler and a compass to draw a precise picture of how P, Q, and R are located relative to each other.

3. A radio beacon indicates that a certain whale is less than 1 kilometer away from boat A and less than 1 kilometer away from boat B. Boat A and boat B are 1 kilometer apart. Assuming that the whale is swimming near the surface of the water, draw a map showing all the places where the whale could be located.

4. Suppose that during a flight an airplane pilot realizes that another airplane is 1000 feet away. Describe the shape formed by all possible locations of the other airplane at that moment.

5. An airplane is in radio contact with two control towers. The airplane is 20 miles from one control tower and 30 miles from another control tower. The control towers are 40 miles apart. Is this information enough to pinpoint the exact location of the airplane? Why or why not? What if you know the altitude of the airplane—do you have enough information to pinpoint the location of the airplane?

2. Points P and Q are 4 centimeters apart. Point R is 2 cm from P and 3 cm from Q. Use a ruler and a compass to draw a precise picture of how P, Q, and R are located relative to each other.
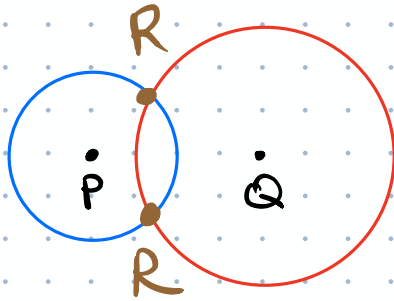
two possible locaters fur R (at the intersection of circles)

Collection of all the points in the plane the fixed distance 2 cm from the center P makes up a circle.

all the points __3__ cm from Q is another circle

4. Suppose that during a flight an airplane pilot realizes that another airplane is 1000 feet away. Describe the shape formed by all possible locations of the other airplane at that moment.

in space

The shape is a sphere.

Answers to Practice Exercises for Section 10.3

1. Be sure to describe these shapes in terms of a fixed distance to a fixed point.

2. See Figure 10.63. Start by drawing points P and Q on a piece of paper, 4 centimeters apart. Now open a compass to 2 cm, stick its point at P, and draw a circle. Since point R is 2 cm from P, it must be located somewhere on that circle. Open a compass to 3 cm, stick its point at Q, and draw a circle. Since point R is 3 cm from Q, it must also be located somewhere on this circle. Thus, point R must be located at one of the two places where your two circles meet. Plot point R at either one of these two locations.
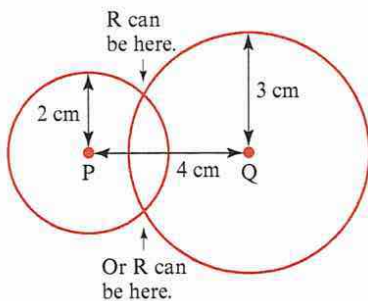

Figure 10.63 Locating points P, Q, and R.

3. Since the whale is less than 1 km from boat A, it must be *inside* a circle of radius 1 km, centered at boat A. Similarly, the whale is *inside* a circle of radius 1 km, centered at boat B. The places where the insides of these two circles overlap are all the possible locations of the whale, as shown in Figure 10.64.

4. The other airplane could be at any of the points that are 1000 feet away from the airplane. These points form a sphere of radius 1000 feet.
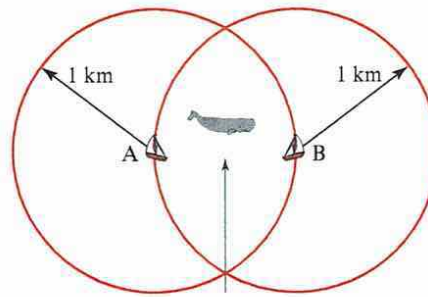
The whale could be anywhere in the shaded region.

Figure 10.64 Two boats and a whale.

5. The locations that are 20 miles from the first control tower form a sphere of radius 20 miles, centered at the control tower. (Some of these locations are underground, and therefore are not plausible locations for the airplane.) Similarly, the locations that are 30 miles from the second control tower form a sphere of radius 30 miles. The airplane must be located in a place that lies on *both* spheres (i.e., at some place where the two spheres meet). The spheres must meet either at a single point (which is unlikely) or in a circle. So, most likely, the airplane could be anywhere on a circle. Therefore, we can't pinpoint the location of the airplane without more information.

Now suppose that the altitude of the airplane is known. Let's say it's 20,000 feet. The locations in the sky that are 20,000 feet from the ground form a very large sphere around the whole earth. This large sphere and the circle of locations where the airplane might be located either meet in a single point (unlikely) or in two points. So, even with this information, we still can't pinpoint the location of the airplane, but we can narrow it down to two locations.

PROBLEMS FOR SECTION 10.3

1. 🍎 Smallville is 7 miles south of Gotham. Will is 8 miles from Gotham and 6 miles from Smallville. Draw a map showing where Will could be. Be sure to show a scale for your map. Explain why you draw your map the way you do. Can you pinpoint Will to one location, or not? No, see figure above. You can only narrow it down to two possible points.
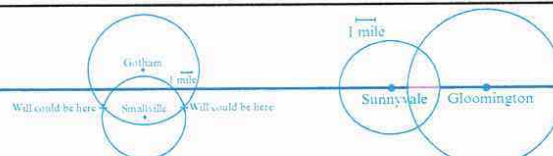
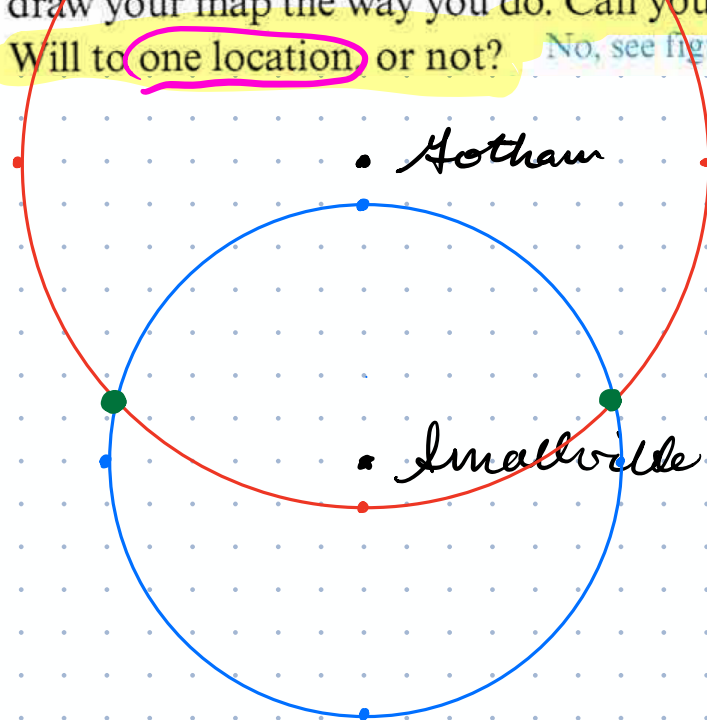2. 🍎 A new mall is to be built to serve the towns of Sunnyvale and Gloomington, whose centers are 6 miles apart. The developers want to locate the mall not more than 3 miles from the center of Sunnyvale and also not more than 5 miles from the center of Gloomington. Draw a simple map showing Sunnyvale, Gloomington, and *all* potential locations for the new mall, based on the given information. Be sure to show the scale of your map. Explain how you determined the possible locations for the mall.

See figure to the above right. The mall must be w/i the circle around Sunnyvale, b/c it is not more than 3 mi. from there. Similarly, the mall must be w/i the circle around Gloomington, b/c it is not more than 5 mi. from there. The shaded region shows all the points that satisfy both conditions.

1. Smallville is 7 miles south of Gotham. Will is 8 miles from Gotham and 6 miles from Smallville. Draw a map showing where Will could be. Be sure to show a scale for your map. Explain why you draw your map the way you do. Can you pinpoint Will to one location or not? No, see figure above. Y

Radius 8 Circle about Gotham

N

1 mile

• Gotham

• Smallville

Will can be at either of the 2 points above where the circles meet.

No, we can't pinpoint will to 1 location.

2. ⧗ A new mall is to be built to serve the towns of Sunnyvale and Gloomington, whose centers are 6 miles apart. The developers want to locate the mall not more than 3 miles from the center of Sunnyvale and also not more than 5 miles from the center of Gloomington. Draw a simple map showing Sunnyvale, Gloomington, and *all potential locations for the new mall* based on the given information. Be sure to show the scale of your map. Explain how you determined the possible locations for the mall.

S   G

(p 476)

Scale
⌐ = 1 mi.

all places
within 3 mi.
from S

all possible
locations
for new
mall

• S

• G

all
locations not more
than 5 miles from
the center of Gloomington

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3. A radio beacon indicates that a certain dolphin is less than 1 mile from boat A and at least $1\frac{1}{2}$ miles from boat B. Boats A and B are 2 miles apart. Draw a simple map showing the locations of the boats and *all* the places where the dolphin might be located. Be sure to show the scale of your map. Explain how you determined all possible locations for the dolphin.

Assume the dolphin is on the surface of the water (which is a plane)

Map (bird's eye view, as always)

Scale:
⌐ = 1 mile

• A

• B