§12.9 Pythagorean Theorem.

Annoucements.

① Exam 2 is Due Sunday 04/11 at 11:59 Pm.

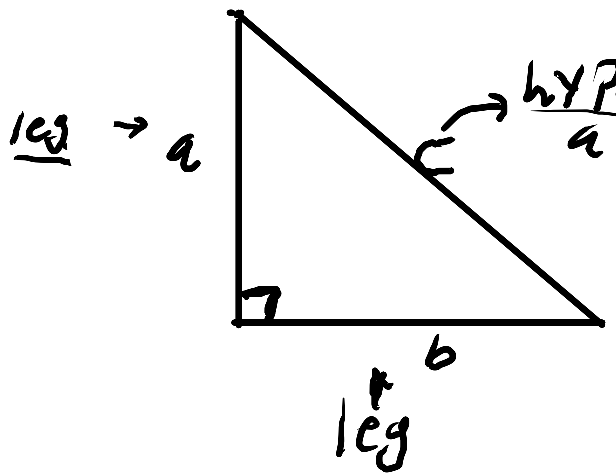
② Tentetive Final Due Date 05/19* Subject to Change.

③ Exam 3 Coming Soon!

"In a right Δ, the square of then length of the hypotenuse is equal to the sum of the squares of the lengths of the two other sides"

$$c^2 = a^2 + b^2$$

$$a^2 + b^2 = c^2$$



hypotenuse: longest side of a right Δ.

Examples Pg 573.

1. A garden gate that is 3 feet wide and 4 feet tall needs a diagonal brace to make it stable. How long a piece of wood will be needed for this diagonal brace? See Figure 12.97.

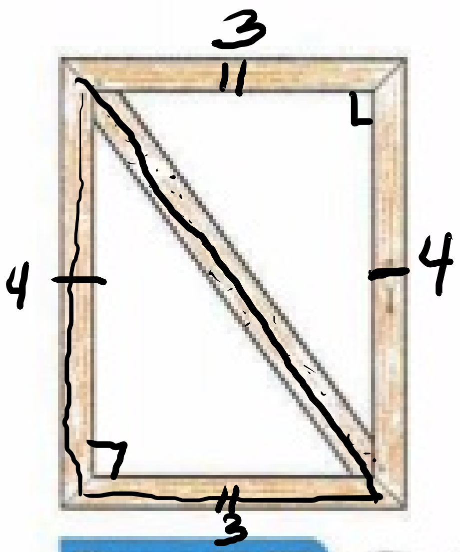
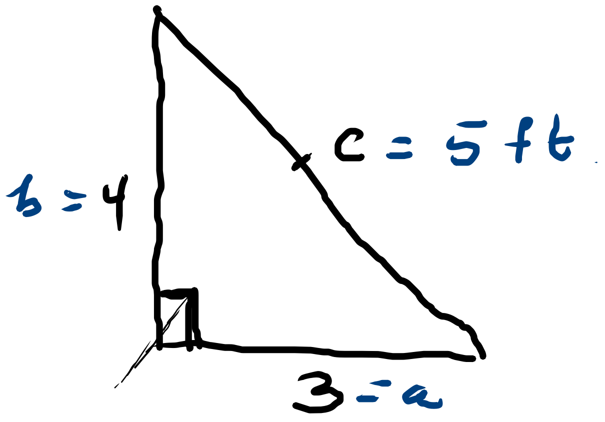


Figure 12.97 Garden gate with diagonal brace.



$$a^2 + b^2 = c^2$$

$$(3)^2 + (4)^2 = c^2$$

$$9 + 16 = c^2$$

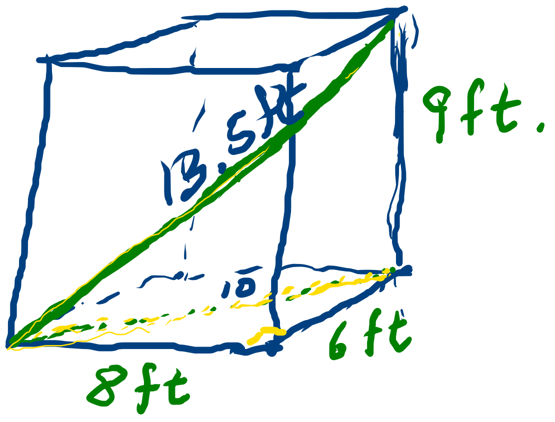
$$25 = c^2$$

$$5^2 = c^2$$

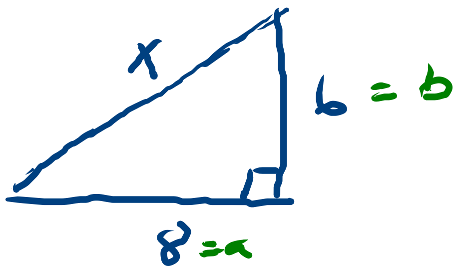
$$c = 5$$

Pg 5 #3

3. An elevator car is 8 feet long, 6 feet wide, and 9 feet tall. What is the longest pole you could fit in the elevator? Explain.



Floor:



$a^2+b^2=c^2$

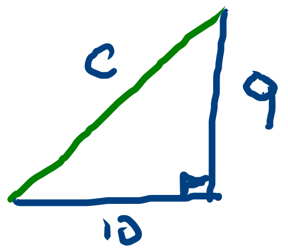
$(8)^2+(6)^2=x^2$

$64+36=x^2$

$100=x^2$ * square root both sides.

$X=10$

Pole



$a^2+b^2=c^2$

$10^2+9^2=c^2$

$100+81=c^2$

$181=c^2$

$\sqrt{181}=\sqrt{c^2}$

$\sqrt{181}=c$

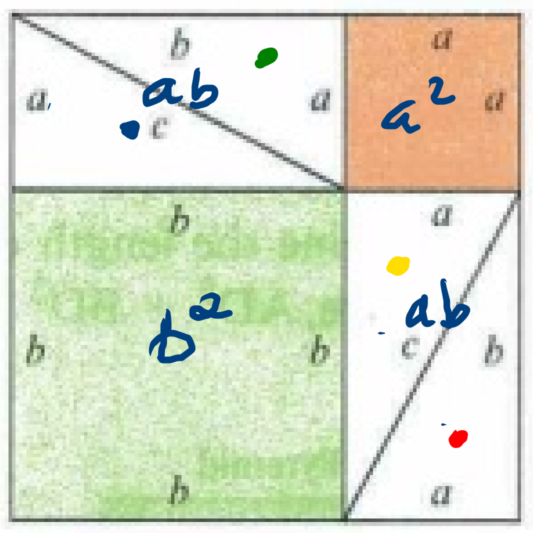
$c \approx 13.5$ ft.

Prove Pythagorean Theorem.

1st way: #6 Pg 573

6. Given a right triangle with short sides of lengths a and b and hypotenuse of length c, use Figure 12.99 to explain why

$$a^2 + b^2 = c^2$$

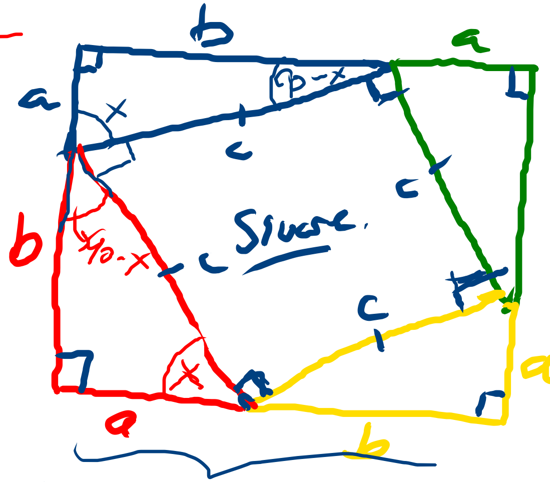


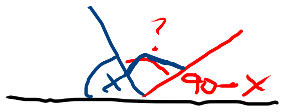
Next, take the triangles and rearranged them.

$90 + x + ? = 180$
$-90 \qquad\quad -90$

$x + ? = 90$
$-x \qquad -x$

$? = 90 - x$





$x + ? + 90 - x = 180$

$? + 90 = 180$

$? = 90$

Area $a+b$

What's the Area?

$a^2 + b^2 + ab + ab$

$a^2 + b^2 + 2ab$

$(a+b)(a+b) \Rightarrow$ FOIL.

$a^2 + ab + ab + b^2$

$a^2 + 2ab + b^2$

$(a+b)(a+b) =$ Area □ + 4 · Area △'s.

$a^2 + 2ab + b^2 = c^2 + 4 \cdot \frac{1}{2} ab$

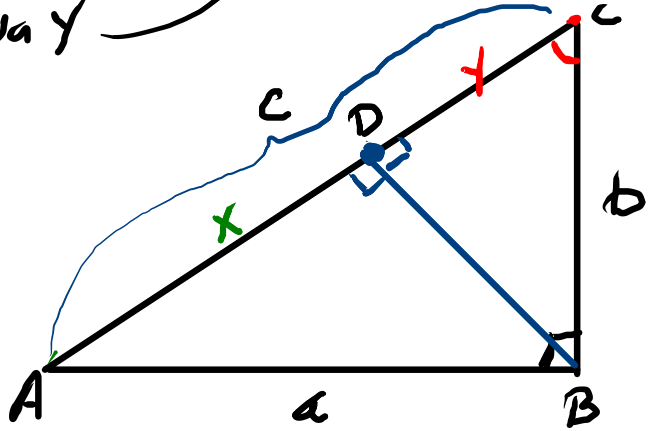
$a^2 + \cancel{2ab} + b^2 = c^2 + \cancel{2ab}$
$\quad -2ab \qquad\qquad -2ab$

$\boxed{a^2 + b^2 = c^2}$

DONE!

2nd way: using Similar Δ's

2nd way

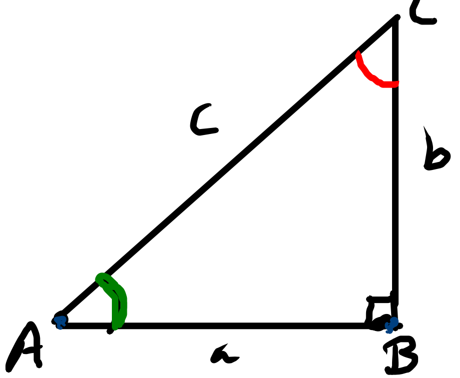


First, make a line segment from the right ∡ to c. Such that the line segment is perpendicular to side c. Now they're three right Δ's.

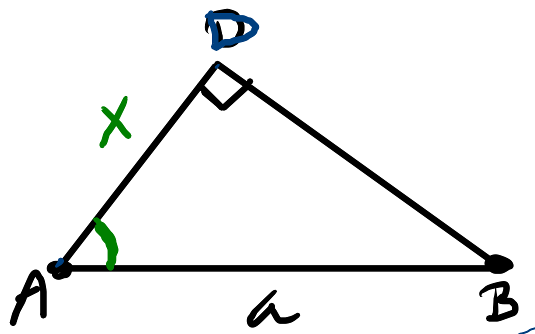
Def: Two triangles are similar:
1. share the same ∡
2. side lengths are proportional.

$\frac{x+y=c}{2}$

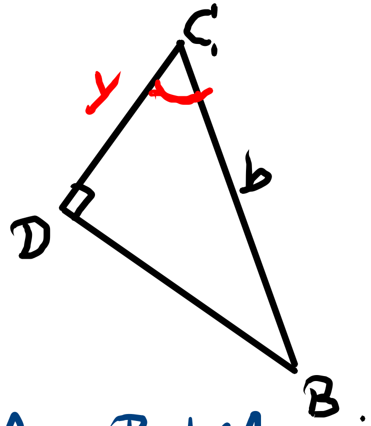
Big



Medium



Baby



The Big Δ is similiar to both Medium and Baby Δ's. Their sides must be propotional.

BigΔ ~ MediumΔ

$$\frac{AB}{AC} = \frac{AD}{AB}$$

$$\frac{a}{c} = \frac{x}{a}$$: cross multiply

$$\boxed{a^2 = cx}$$

BigΔ ~ BabyΔ

$$\frac{AC}{BC} = \frac{BC}{DC}$$

↓

$$\frac{c}{b} = \frac{b}{y}$$: cross multiply

$$\boxed{b^2 = cy}$$

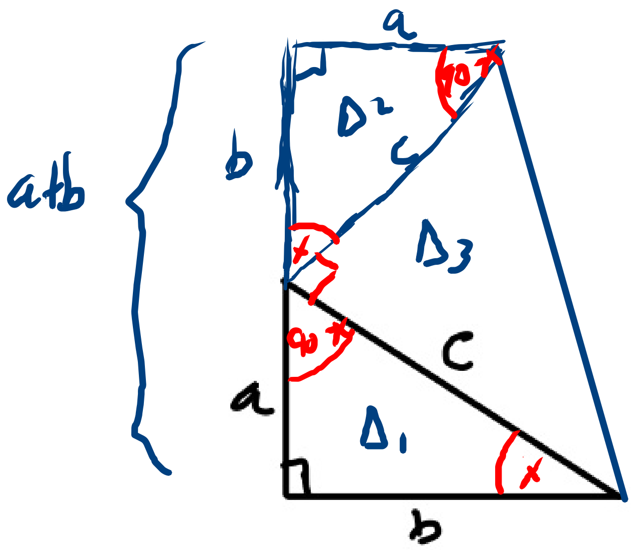
From the two eqs. add them together.

$$a^2 = cx$$
$$+\ b^2 = cy$$

$$a^2+b^2 = cx+cy \text{ * Factor}$$
$$a^2+b^2 = c(x+y)$$
$$a^2+b^2 = c \cdot c$$
$$a^2+b^2 = c^2$$

DONE!

Third way: President Garfield.



Trapezoid and inside are 3 triangles. that are all right Δ's.

$$\text{Area Trapezoid} = \text{Area } \Delta_1 + \text{Area } \Delta_2 + \text{Area } \Delta_3.$$

$$\tfrac{1}{2}(a+b)(a+b) = \tfrac{1}{2}ab + \tfrac{1}{2}ab + \tfrac{1}{2}c \cdot c.$$

$$2\left[\tfrac{1}{2}(a+b)(a+b) = \tfrac{1}{2}ab + \tfrac{1}{2}ab + \tfrac{1}{2}c^2\right] \quad || \text{ Multiply by 2.}$$

$$(a+b)(a+b) = ab + ab + c^2$$

FOIL. || $(a+b)(a+b) = 2ab + c^2.$

$$a^2 + ab + ab + b^2 = 2ab + c^2$$

$$a^2 + \cancel{2ab} + b^2 = \cancel{2ab} + c^2$$

$$-2ab \qquad -2ab$$

$$a^2 + b^2 = c^2$$