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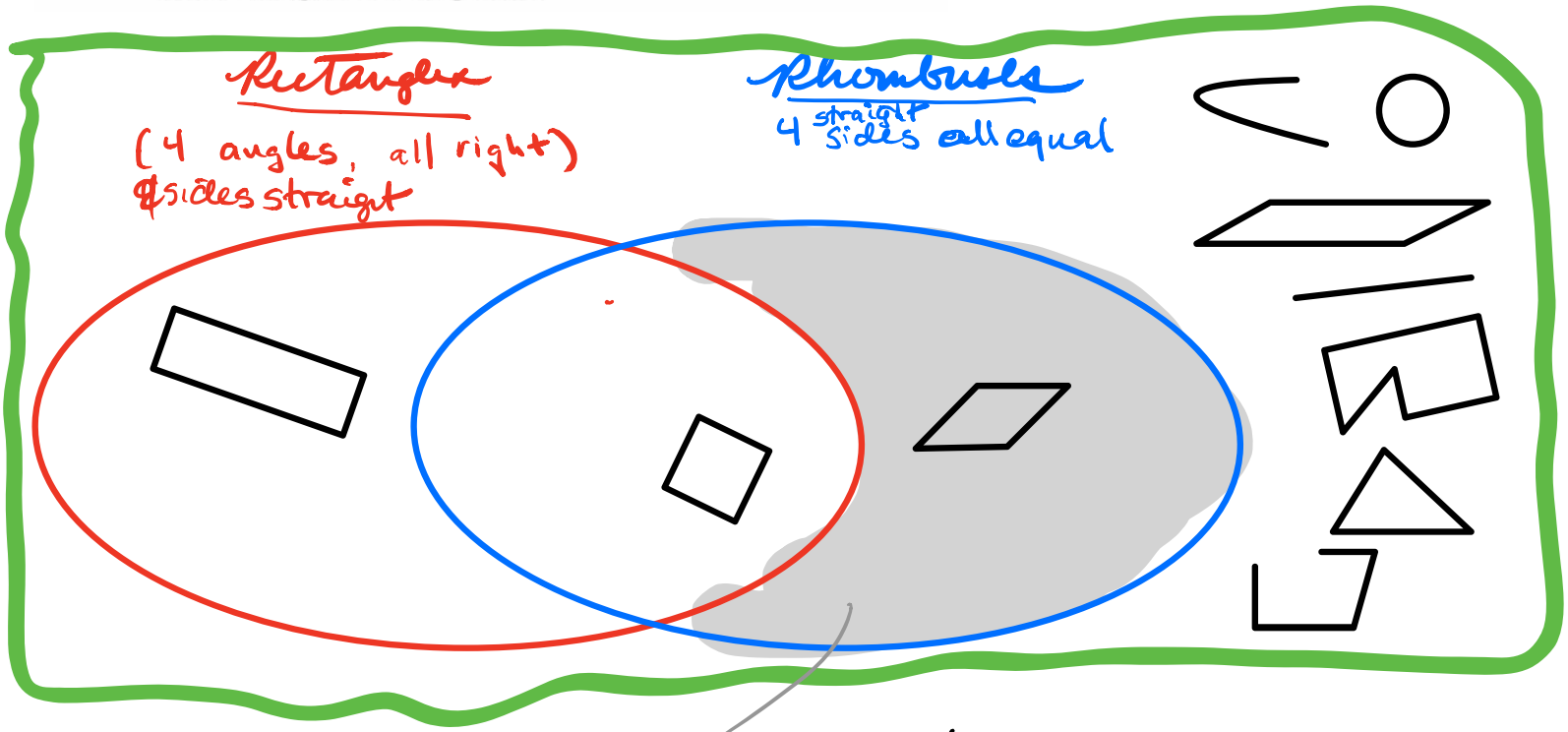
## Practice Exercises for Section 10.4

1. Draw a Venn diagram (or other clear diagram) showing the relationship between the categories of rectangles and rhombuses. Which shapes are in both categories? Explain.




Definition of a square: four sides all same length, & 4 right angles

"blue only"
"blue but not red"
"rhombus but not rectangle"

- Polygon
- Quadrilateral

## How Can We Use Short Lists of Properties to Define Special Quadrilaterals?

What's the difference between the following three categories?

- Quadrilaterals that have 4 right angles
- Quadrilaterals that have 4 right angles and opposite sides parallel
- Quadrilaterals that have 4 right angles and opposite sides of the same length

The surprising answer is that there is no difference. It turns out that each of the three descriptions specifies the exact same collection of shapes—namely, rectangles. If a 4-sided shape has 4 right angles, then *automatically* its opposite sides are parallel and have the same length. How can that be, and why is that so? For now, we leave this as a mystery, to be explored in Chapter 14.

Because we can specify the same category of shapes by listing different collections of properties, we have some choice in how to define shapes. It's worthwhile to pick a short list of properties to define a category of shapes. Why? To decide if a shape is or isn't a rectangle, or some other type of shape, students will check to confirm that all the defining properties hold for the shape. So a short list of properties means less checking.

Look again at the categories in Class Activity 10R, some of which consist of the same shapes. Based on what you found, what *short* lists of properties would you use to define the sets of squares, rectangles, rhombuses, and parallelograms? Compare your definitions with these standard definitions of some special categories of quadrilaterals, which we will use from now on:

**square** square—quadrilateral with 4 right angles whose sides all have the same length

**rectangle** rectangle—quadrilateral with 4 right angles

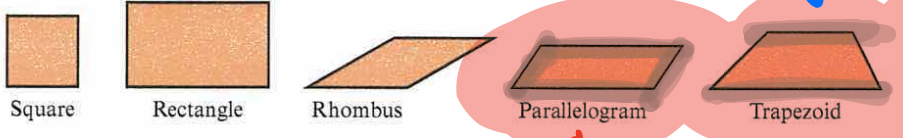
**rhombus** rhombus—quadrilateral whose sides all have the same length. The name **diamond** is sometimes used instead of rhombus

**parallelogram** parallelogram—quadrilateral for which opposite sides are parallel

**trapezoid** trapezoid—quadrilateral for which at least one pair of opposite sides are parallel. (Some books define a trapezoid as a quadrilateral for which *exactly one* pair of opposite sides are parallel.)

Figure 10.67 illustrates some examples.

**Figure 10.67**
Some special quadrilaterals.




## How Can We Classify Special Quadrilaterals in a Hierarchy?

CCSS
3.G.1
4.G.2
5.G.3
5.G.4

In Class Activity 10R, you probably noticed that some categories of quadrilaterals are subcategories of other categories. An important consequence of this is that *a given shape can belong to more than one category*. For example, the shape on the left in Figure 10.68 is a square because it has four sides of the same length and four right angles. But it is *also* a rectangle because it has four sides and four right angles. To help young students understand this idea, teachers can point out that squares are special kinds of rectangles, and they can call squares "square rectangles" to make that point.

How do hierarchies of shape categories come about? Think about this question in terms of lists of properties. If the list has only one property, such as "4 straight sides," then many will satisfy just that one property. If we add another property to that list, so it becomes "4 straight sides and 4 right angles," then only some of the shapes in the initial category will satisfy both of those properties. Usually, the more properties we add to the list, the fewer shapes satisfy all those

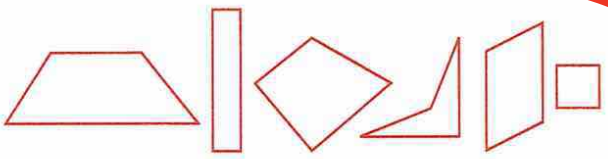
To categorize shapes, we usually look first for how many straight sides the shapes have. Next, we study the especially interesting category of 4-sided shapes.

## What Are Quadrilaterals?

quadrilateral

A quadrilateral is a closed shape in a plane consisting of 4 line segments that do not cross each other. Figure 10.65 shows examples of quadrilaterals and Figure 10.66 shows figures that are not quadrilaterals. The name *quadrilateral* makes sense because it means *four sided* (quad = four, lateral = side).

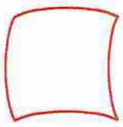
**Figure 10.65**
Quadrilaterals.



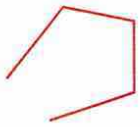
These shapes are quadrilaterals.

quadrilaterals

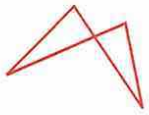
**Figure 10.66**
Figures that are not quadrilaterals.



This shape is not made out of line segments.



This figure is not closed.



This shape has sides that cross.

Within the category of quadrilaterals, there are several subcategories, for example, squares and rectangles. To describe a category of shapes, we will need to focus on properties. In Class Activity 10Q, think about which properties of shapes are relevant for identifying and categorizing shapes.

CCSS
1.G.1
2.G.1

**CLASS ACTIVITY**

10Q What Properties Do These Shapes Have?, p. CA-217

When we identify and classify shapes mathematically, we consider the numbers of sides and angles, the lengths of the sides, the sizes of the angles, and the relationships between sides (such as whether they are parallel or not). We may also consider symmetry, which we will study in Chapter 14. But we don't consider some attributes that can be important in artwork or daily life and may stand out to children, such as the color, the orientation, or the overall size of the shape.

In Class Activity 10R, see how considering the parts of shapes and their properties allows us to classify shapes into categories.

**CLASS ACTIVITY**

10R How Can We Classify Shapes into Categories Based on Their Properties? p. CA-218

When you classified shapes into categories in Class Activity 10R, you probably noticed that some categories contain exactly the same shapes even though they were described by different properties. This leads to our next topic: defining categories of shapes.

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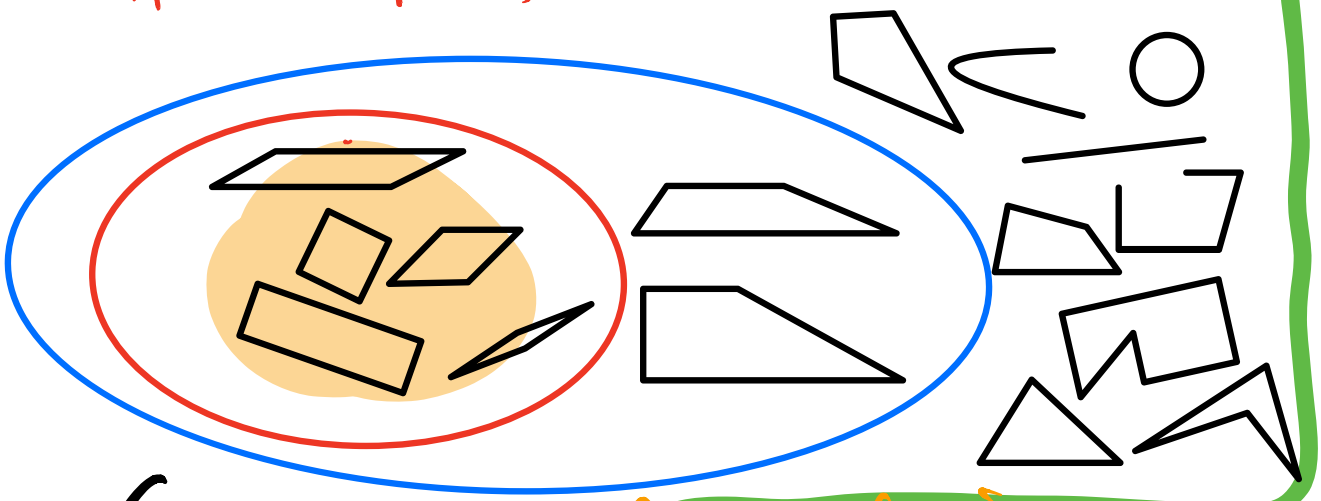
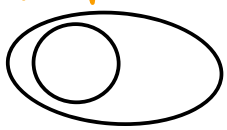
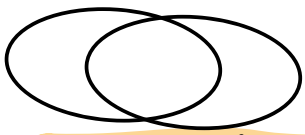
2. Draw a Venn diagram (or other clear diagram) showing the relationship between the categories of parallelograms and trapezoids. Explain.

parallelogram (4 straight sides, both pairs of opposite sides parallel)

trapezoid 4 straight sides & at least 1 pair of opposite sides parallel



$x$ = the number of pairs of sides
$x = 2$ & $x \geq 1$ — no problem.

We draw our (Venn) diagram like  instead of like  to show that parallelograms are a subcategory of trapezoids (a special case of trapezoids)

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**trapezoid**—quadrilateral for which at least one pair of opposite sides are parallel. (Some books define a trapezoid as a quadrilateral for which *exactly one* pair of opposite sides are parallel.)

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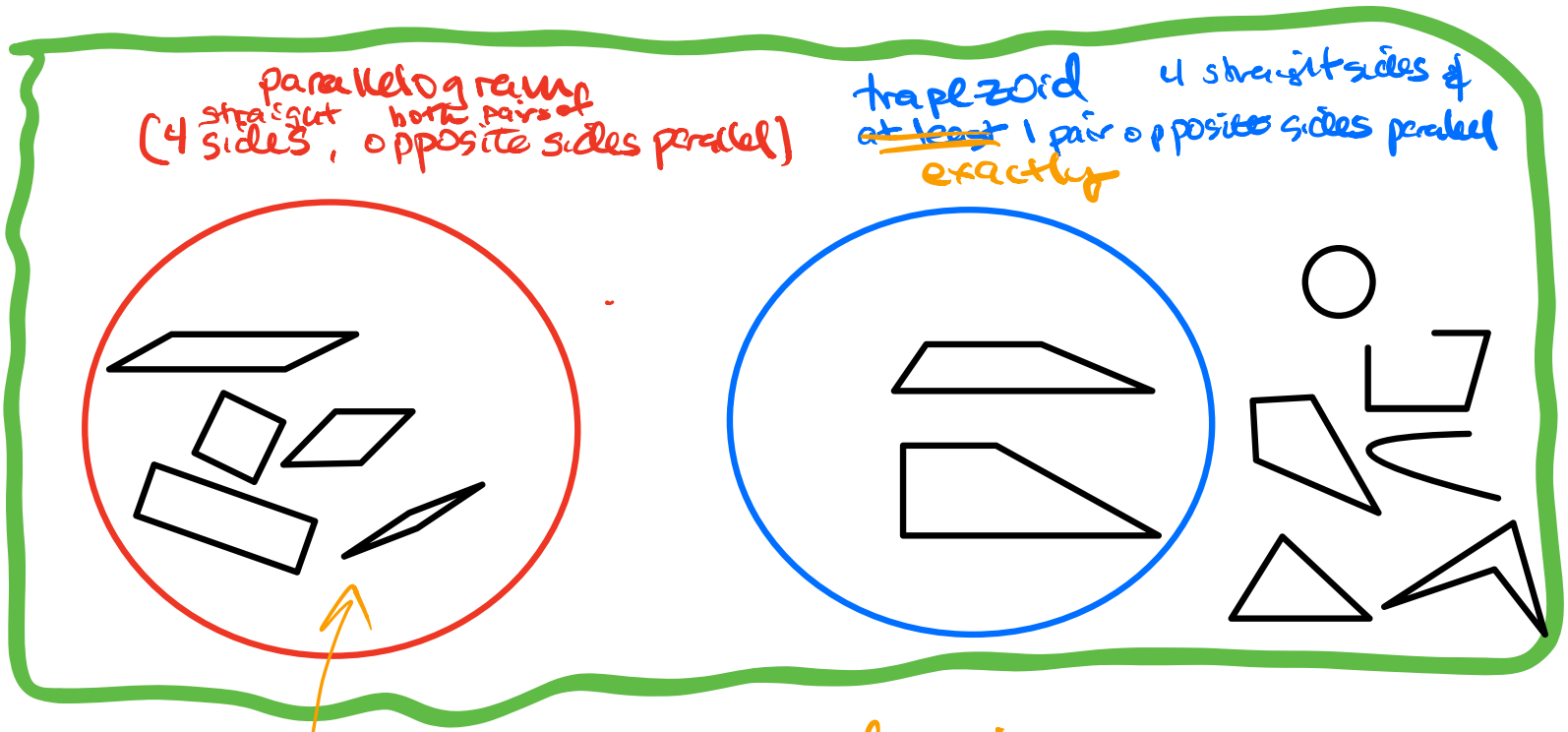
3. Some books define trapezoids as quadrilaterals that have *exactly one* pair of parallel sides. Draw a Venn diagram (or other clear diagram) showing how the categories of parallelograms and trapezoids are related when this alternate definition of trapezoid is used. Explain.




is a parallelogram
$x = 2$

is not an (alternate) trapezoid
$x \neq 1$

The number of pairs
$x = 1$ and $x = 2$ impossible.

4 straight sides that close up (not )

square—quadrilateral with 4 right angles whose sides all have the same length

rectangle—quadrilateral with 4 right angles

rhombus—quadrilateral whose sides all have the same length. The name **diamond** is sometimes used instead of rhombus

parallelogram—quadrilateral for which opposite sides are parallel

all (2 pairs)

trapezoid—quadrilateral for which at least one pair of opposite sides are parallel. (Some books define a trapezoid as a quadrilateral for which *exactly one* pair of opposite sides are parallel.)

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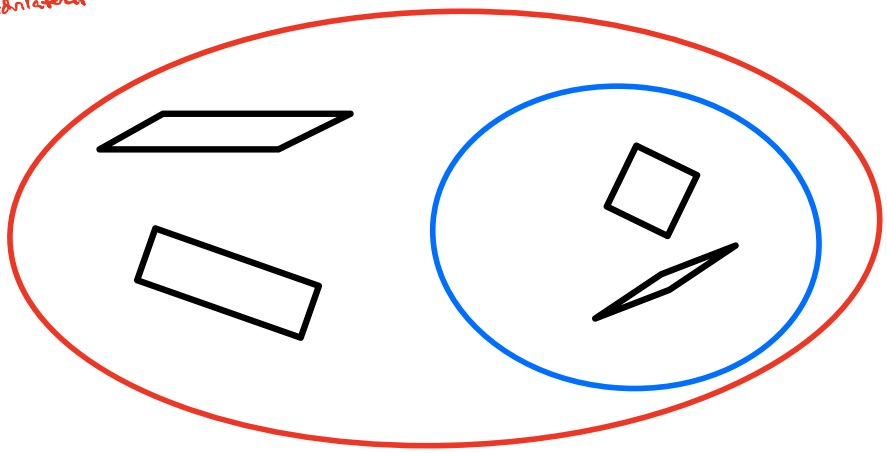
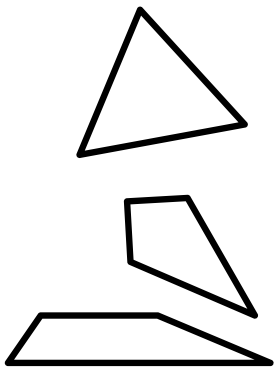
4. Draw a Venn diagram (or other clear diagram) showing the relationship between the categories of rhombuses and parallelograms.

Parallelogram
Quadrilateral with all (2) pairs opposites sides parallel ~~equal~~

Rhombus
Quadrilateral with all sides equal (length)





- a rhombus is always a parallelogram
- all rhombuses are parallelograms
- rhombus is a subcategory of parallelogram

← 3 ways of getting the same point across