Numerical Methods for Partial Differential Equations
A.Y. 2023/2024

Laboratory 03
Finite Element method for the Poisson equation in 2D

Exercise 1.

Let $\Omega = (0,1) \times (0,1)$, and let us decompose its boundary $\partial\Omega$ as follows (see Figure 1):

$$\begin{aligned}
\Gamma_0 &= \{x = 0, y \in (0,1)\} \ , \\
\Gamma_1 &= \{x = 1, y \in (0,1)\} \ , \\
\Gamma_2 &= \{x \in (0,1), y = 0\} \ , \\
\Gamma_3 &= \{x \in (0,1), y = 1\} \ .
\end{aligned}$$

Let us consider the following Poisson problem with mixed Dirichlet-Neumann boundary conditions:

$$\begin{cases}
-\boldsymbol{\nabla} \cdot (\mu \ \boldsymbol{\nabla} u) = f & \mathbf{x} \in \Omega, \quad (1a) \\
u = g & \text{on } \Gamma_0 \cup \Gamma_1, \quad (1b) \\
\mu \boldsymbol{\nabla} u \cdot \mathbf{n} = h & \text{on } \Gamma_2 \cup \Gamma_2, \quad (1c)
\end{cases}$$

where $\mu(\mathbf{x}) = 1$, $f(\mathbf{x}) = -5$, $g(\mathbf{x}) = x + y$ and $h(\mathbf{x}) = y$.

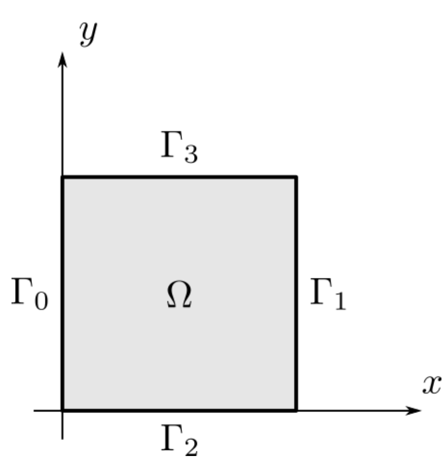

Figure 1: (a) Domain Ω and partition of its boundary into Γ_0, Γ_1, Γ_2 and Γ_3. (b) Mesh obtained by generating a square with 20 subdivisions in `deal.II` and then converting it to a triangular mesh.

1.1. Write the weak formulation and the Galerkin formulation of problem (1).

1.2. Implement in `deal.II` a finite element solver for (1), using triangular elements.