Numerical Methods for Partial Differential Equations
A.Y. 2023/2024

Laboratory 10
Finite Element method and domain decomposition algorithms

Exercise 1.

Let $\Omega \subset \mathbb{R}^2$ be the domain shown in Figure 1. Let us consider the following Poisson problem:

$$\begin{cases} -\Delta u = 0 & \text{in } \Omega \ , \\ u = 0 & \text{on } \Gamma_0 \ , \\ u = 1 & \text{on } \Gamma_1 \ , \\ \nabla u \cdot \mathbf{n} = 0 & \text{on } \Gamma_2 \cup \Gamma_3 \ . \end{cases} \tag{1}$$

We partition the domain into two subdomains Ω_0 and Ω_1, as shown in Figure 2.

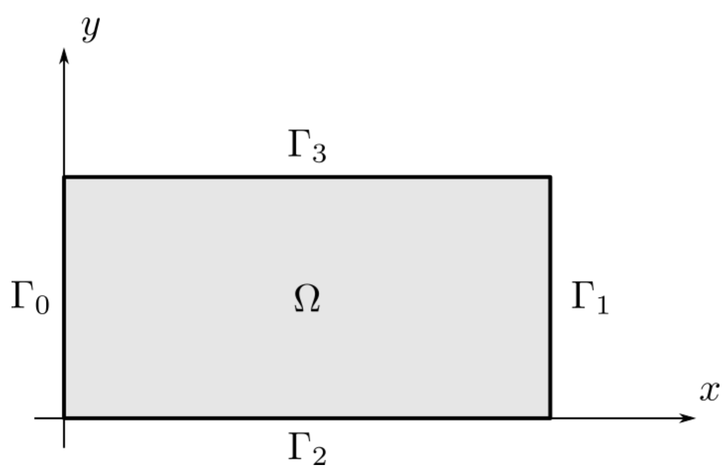

Figure 1: Domain Ω and its boundary portions.

1.1. Write an iterative algorithm for the solution of problem (1) based on the decomposition of Ω into Ω_0 and Ω_1 shown in Figure 2, using Dirichlet interface conditions for the first subdomain and Neumann interface conditions for the second subdomain.

1.2. Implement a solver for problem (1) based on the algorithm derived at previous point.

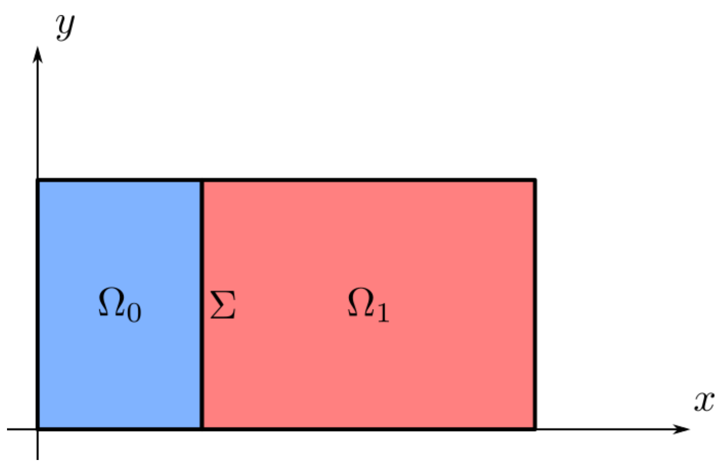

Figure 2: Decomposition of the domain Ω into Ω_0 and Ω_1 along the interface Σ.