Numerical Methods for Partial Differential Equations A.Y. 2023/2024

Laboratory 05

Finite Element method for the diffusion-reaction equation in 3D

Exercise 1.

Let $\Omega = (0,1)^3$, be the unit cube and let us consider the following diffusion-reaction problem:

$$\begin{cases}
-\nabla \cdot (\mu \nabla u) + \sigma u = f & \mathbf{x} \in \Omega, \\
u = 0 & \text{on } \partial\Omega,
\end{cases}$$
(1a)
(1b)

where $\mathbf{x} = (x, y, z)^T$, $\sigma(\mathbf{x}) = 1$, $f(\mathbf{x}) = 1$ and

$$\mu(\mathbf{x}) = \begin{cases} 100 & \text{if } x < \frac{1}{2}, \\ 1 & \text{if } x \ge \frac{1}{2}. \end{cases}$$

- 1.1. Starting from the code of Laboratories 3 and 4, implement a finite element solver for problem (1). The solver should read the mesh from file (four differently refined meshes are provided as mesh/mesh-cube-*.msh).
- 1.2. Compute the solution on the mesh mesh/mesh-cube-40.msh (i.e. N+1=40), with linear finite elements.