

Artificial Life Summer 2015

Cellular Automata 2D Conway's Game of Life

Master Computer Science [MA-INF 4201]

Mon 8:30 – 10:00, LBH, Lecture Hall III.03a

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Department of Computer Science, University of Bonn

1

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Artificial Life [MA-INF 4201], Mon, April 27, 2015

Overview:

- Cellular Automata in 2 dimensions
- Examples and Applications of CAs
- Conway's Game of Life
- Computational Universality
- Is Information == Structure ?

2

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Cellular Automata in 2 dimensions

Although S.Wolfram has investigated the 1-dim Cellular Automata very intensively, the original idea from Stanislaw Ulam (1940) and J. von Neumann was a 2 dimensional Cellular Automaton.

In the 50ies and 60ies the idea of Cellular Automata were the basis for a series of analogue computers.

The famous German computer scientist Konrad Zuse published in 1969 an idea (from the 40ies) „*Rechnender Raum*“ where he supposed that the law of natures are discrete, working like a CA.

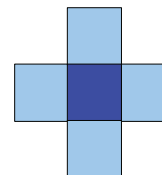
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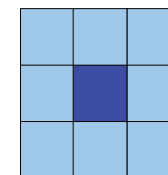
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Neighborhood in 2 dimensions

For a 2 dimensional rectangular grid there are two variants to define the neighborhood with $r=1$:



$r = 1$, von Neumann



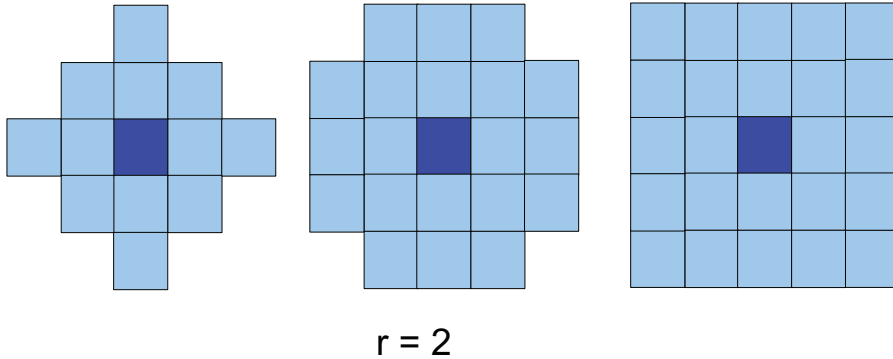
$r = 1$, Moore

4

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Neighborhood in 2 dimensions

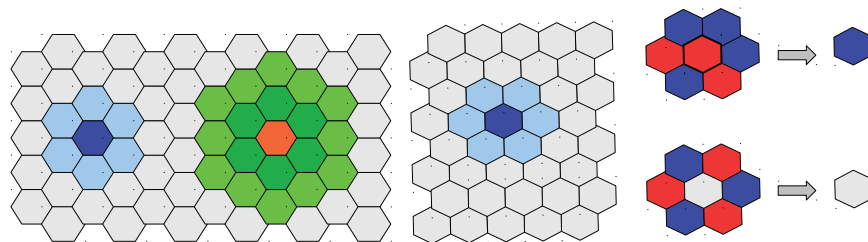
A larger neighborhood radius r requires a more precise definition of the neighboring cells:



5

Cellular Automata extended

Cellular Automata can be easily extended to higher dimensions, (e.g. 3-dim) to a different tiling of the workspace (e.g. triangles or hexagons in 2-dim) or even to a non-uniform neighborhood (e.g. graph). Only the definition of the neighborhood and the rule has to be adjusted accordingly.



6

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- Cellular Automata in 2 dimensions
- **Examples and Applications of CAs**
- Conway's Game of Life
- Computational Universality
- Is Information == Structure ?

7

Examples and Applications of CAs

- Growth of crystals
- Population dynamics
- Modeling, and predicting traffic situations
- Modeling urban city development
- Modeling diffusion process
- Generation of „close to real“ patterns
- Modeling forest fires
- ...

8

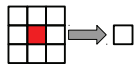
Cellular Automata in 2 Dimensions

Majority-Voting CA

$d=2$, rectangular grid, $r=1$, Moore, $k=4$,



The state of the cell is changing to the majority of states present in the neighborhood.



9

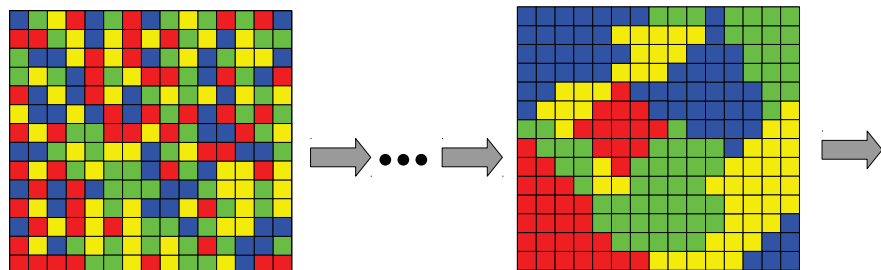
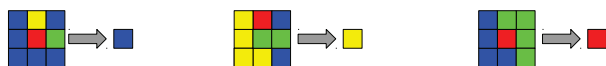
Cellular Automata in 2 Dimensions

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$d=2$, rectangular grid, $r=1$, Moore, $k=4$,



The state of the cell is changing to the majority of states present in the neighborhood.

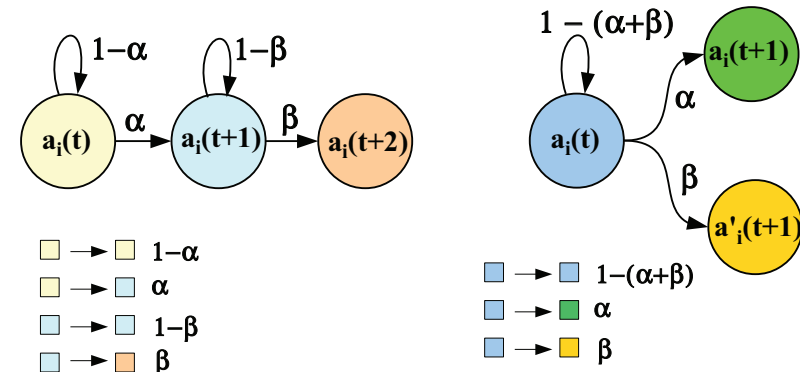


10

Cellular Automata in 2 Dimensions

Non Deterministic Cellular Automata

As an extension to the deterministic rule of a classically defined Cellular Automaton, the transition from one state to the next state can have a stochastic component.

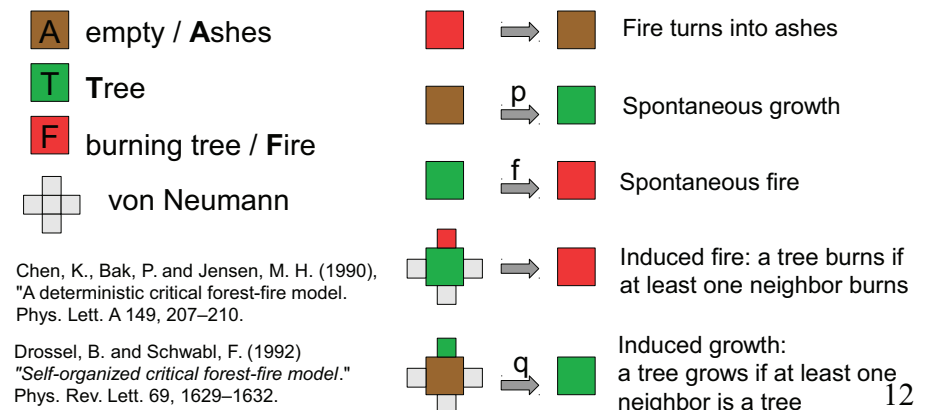


11

Cellular Automata in 2 Dimensions

Forest Fire CA

$d=2$, rectangular grid, $r=1$, von Neumann, $k=3$
non deterministic cellular automaton,



12

Cellular Automata in 2 Dimensions

Forest Fire CA

$d=2$, rectangular grid, $r=1$, von Neumann, $k=3$
non deterministic cellular automaton,

The behavior of the complete system is determined by the underlying micro-behavior of the cells.

The control parameters are:

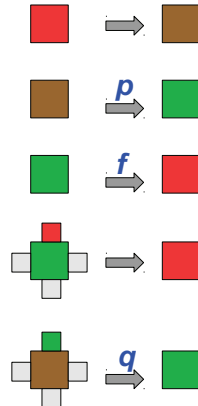
p rate of spontaneous growth

f rate of spontaneous fire

q rate of induced growth

A setting of $q=0$ (no induced growth) is a good setting to start from.

Interesting (fractal) behavior arises when $f \ll p$ (e.g. $p/f = 100$).

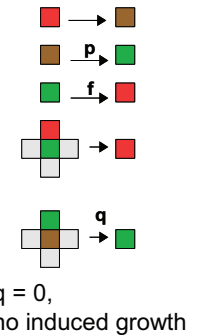
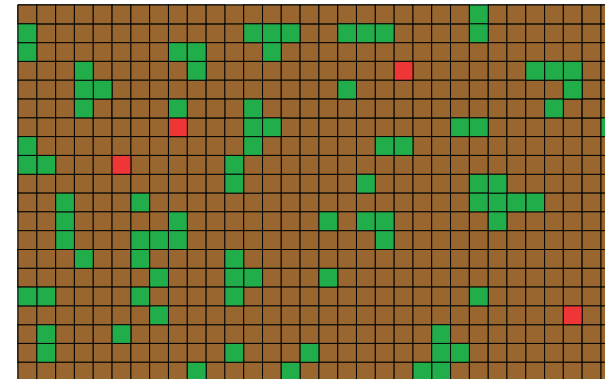


13

Cellular Automata in 2 Dimensions

Forest Fire CA

$d=2$, rectangular grid, $r=1$, von Neumann, $k=3$
non deterministic cellular automaton,



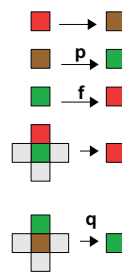
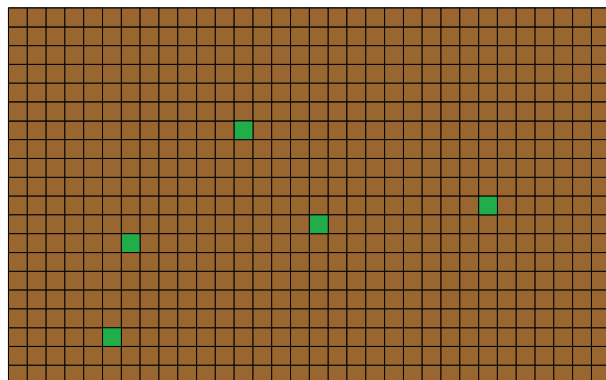
$q = 0$,
no induced growth

15

Cellular Automata in 2 Dimensions

Forest Fire CA

$d=2$, rectangular grid, $r=1$, von Neumann, $k=3$
non deterministic cellular automaton,

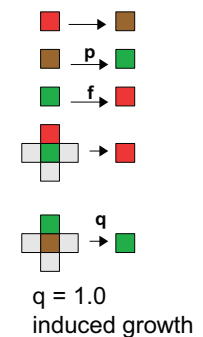
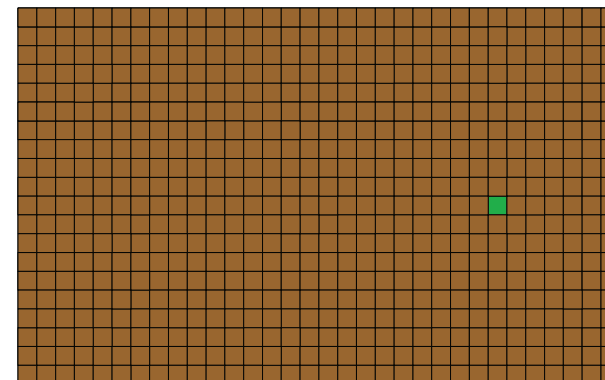


14

Cellular Automata in 2 Dimensions

Forest Fire CA

$d=2$, rectangular grid, $r=1$, von Neumann, $k=3$
non deterministic cellular automaton,



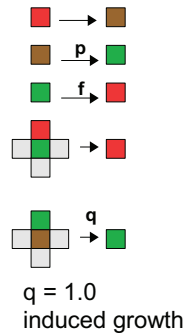
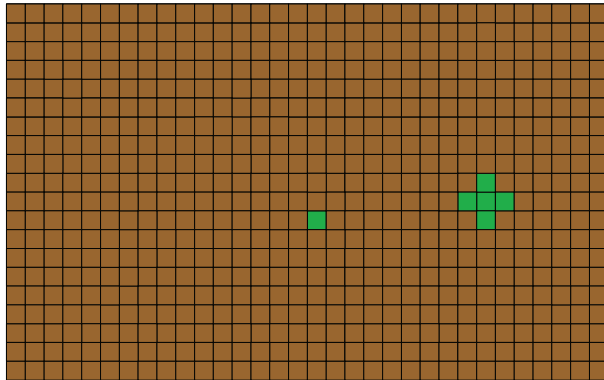
$q = 1.0$
induced growth

16

Cellular Automata in 2 Dimensions

Forest Fire CA

$d=2$, rectangular grid, $r=1$, von Neumann, $k=3$
non deterministic cellular automaton,

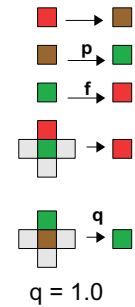
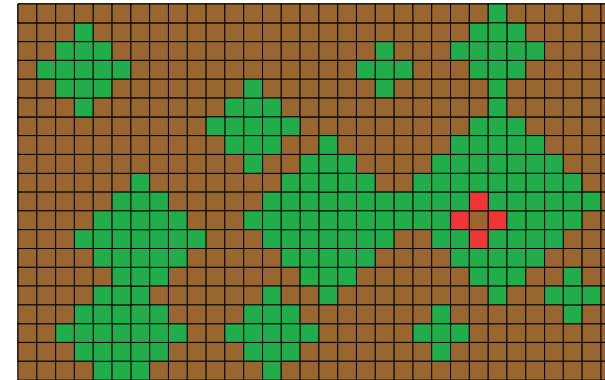


17

Cellular Automata in 2 Dimensions

Forest Fire CA

$d=2$, rectangular grid, $r=1$, von Neumann, $k=3$
non deterministic cellular automaton,

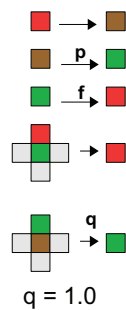
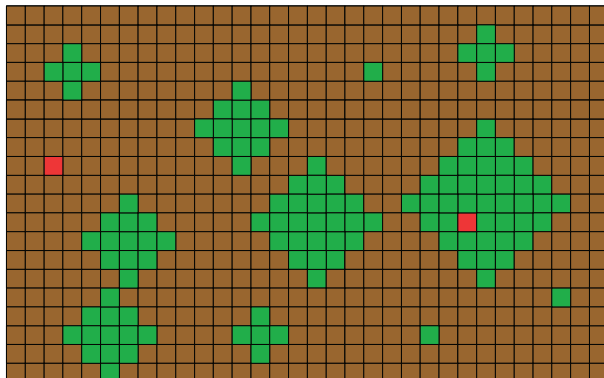


19

Cellular Automata in 2 Dimensions

Forest Fire CA

$d=2$, rectangular grid, $r=1$, von Neumann, $k=3$
non deterministic cellular automaton,

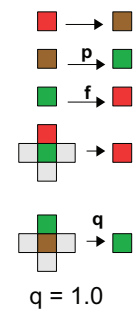
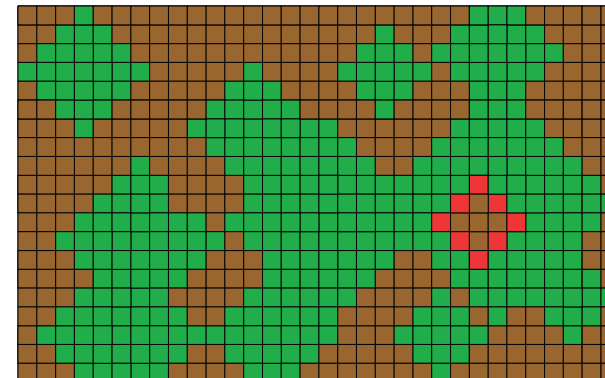


18

Cellular Automata in 2 Dimensions

Forest Fire CA

$d=2$, rectangular grid, $r=1$, von Neumann, $k=3$
non deterministic cellular automaton,

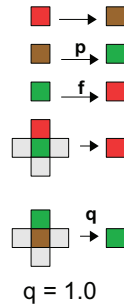
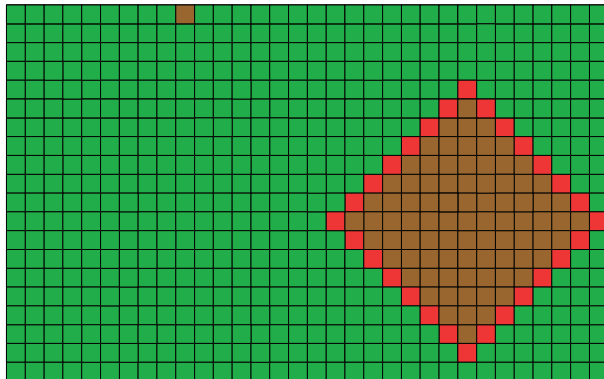


20

Cellular Automata in 2 Dimensions

Forest Fire CA

$d=2$, rectangular grid, $r=1$, von Neumann, $k=3$
non deterministic cellular automaton,



21

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Overview:

- Cellular Automata in 2 dimensions
- Examples and Applications of CAs
- **Conway's Game of Life**
- Computational Universality
- Is Information == Structure ?

22

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Conway's Game of Life

In 1970 the British professor for mathematics John H. Conway proposed a 2 dim cellular automaton:

Game of Life or *Conway's Game of Life*

Conway's Game of Life is probably the most popular 2-dimensional cellular automaton.

23

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Conway's Game of Life

Conway's Game of Life:
 is a Cellular Automaton defined on a

$d=2$, 2-dimensional rectangular grid, using a

$r=1$, Moore-Neighborhood, **Moore-periphery** and has

$k=2$, binary states for each cell: **O** dead, **I** alive.

The rule is legal.

The rule is implementing concepts from population dynamics:
birth, *survival*, death from *overcrowding* or from *loneliness*.

24

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Conway's Game of Life: 23/3 rule

The rule for Conway's Game of Life was inspired by observations from population dynamics.

Survival:

A living cell survives if 2 or 3 neighboring cells are alive.

Birth:

A cell is born if exactly 3 neighboring cells are alive.

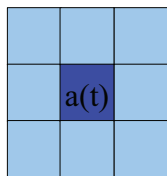
Death:

A living cell dies from overcrowding if more than 3 neighboring cells are alive or from loneliness if less than 2 neighboring cells are alive.

25

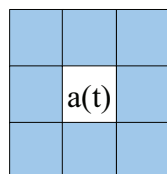
Conway's Game of Life: 23/3 rule

If cell $a(t)$ is **alive**



$\sum a_p(t)$	8	7	6	5	4	3	2	1	0
$a(t+1)$	O	O	O	O	O	I	I	O	O
overcrowded						survival		loneliness	

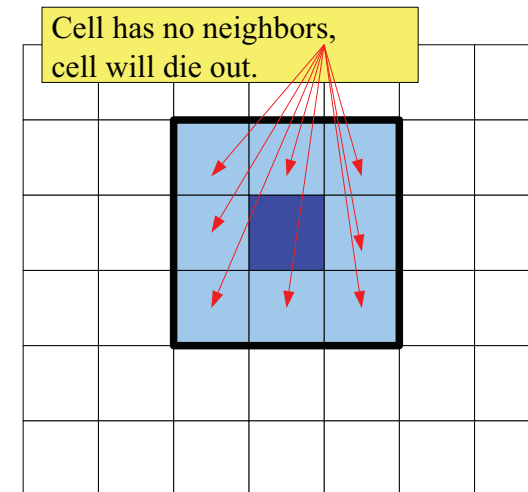
If cell $a(t)$ is **dead**



$\sum a_p(t)$	8	7	6	5	4	3	2	1	0
$a(t+1)$	O	O	O	O	O	I	O	O	O
birth									

26

Conway's Game of Life



Rule: 23/3

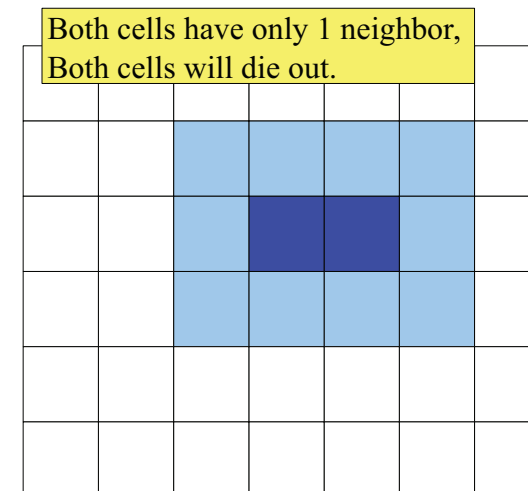
A cell survives with 2 or 3 neighbors.

A cell is born if it has 3 neighbors.

Cell dies out.

27

Conway's Game of Life



Rule: 23/3

A cell survives with 2 or 3 neighbors.

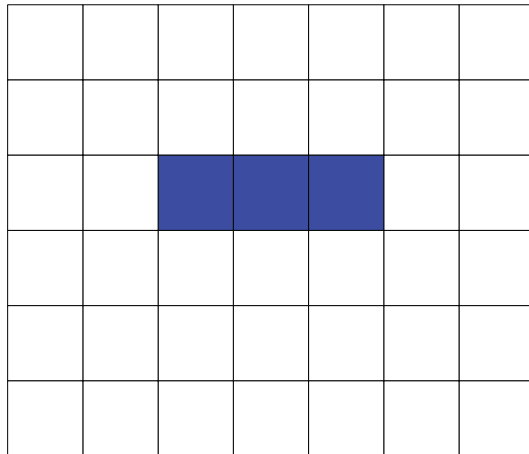
A cell is born if it has 3 neighbors.

Both cells die out.

28

Conway's Game of Life

time step t



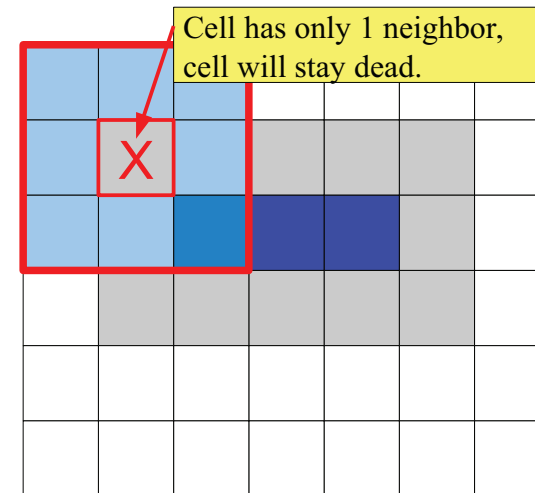
Rule: 23/3

A cell survives with
2 or 3 neighbors.

A cell is born if it has 3
neighbors.

29

Conway's Game of Life



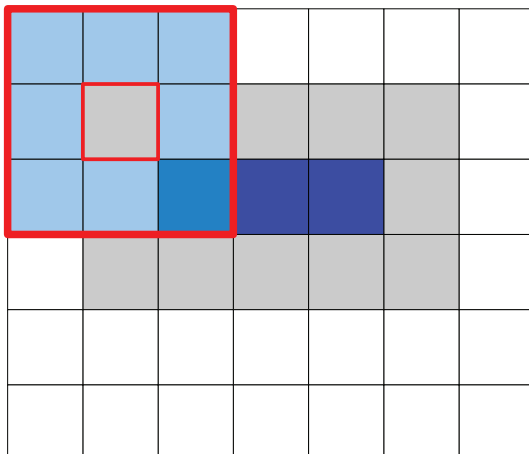
Rule: 23/3

A cell survives with
2 or 3 neighbors.

A cell is born if it has 3
neighbors.

31

Conway's Game of Life



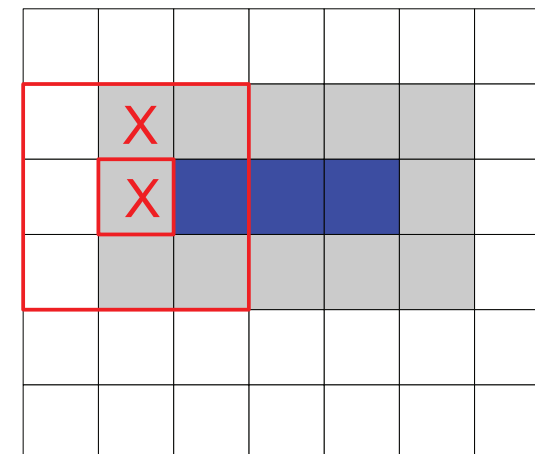
Rule: 23/3

A cell survives with
2 or 3 neighbors.

A cell is born if it has 3
neighbors.

30

Conway's Game of Life



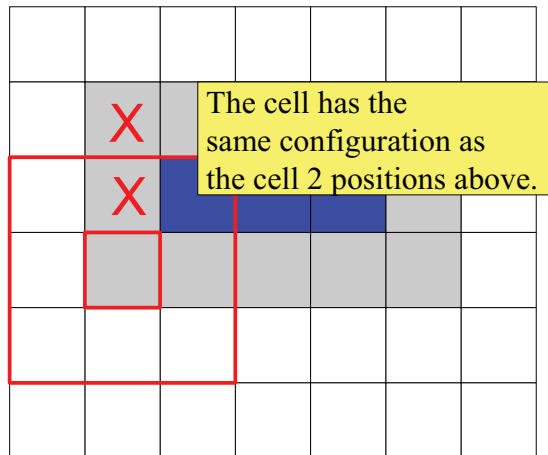
Rule: 23/3

A cell survives with
2 or 3 neighbors.

A cell is born if it has 3
neighbors.

32

Conway's Game of Life



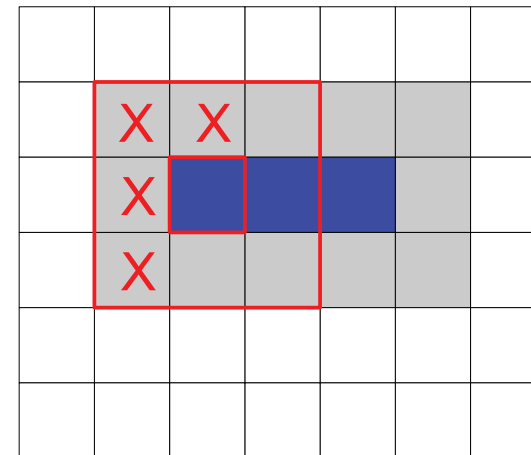
Rule: 23/3

A cell survives with 2 or 3 neighbors.

A cell is born if it has 3 neighbors.

33

Conway's Game of Life



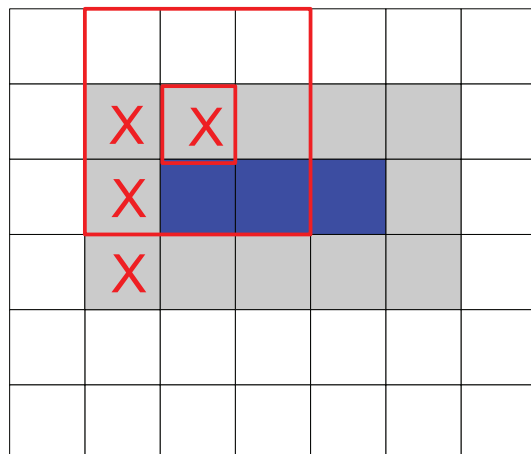
Rule: 23/3

A cell survives with 2 or 3 neighbors.

A cell is born if it has 3 neighbors.

35

Conway's Game of Life



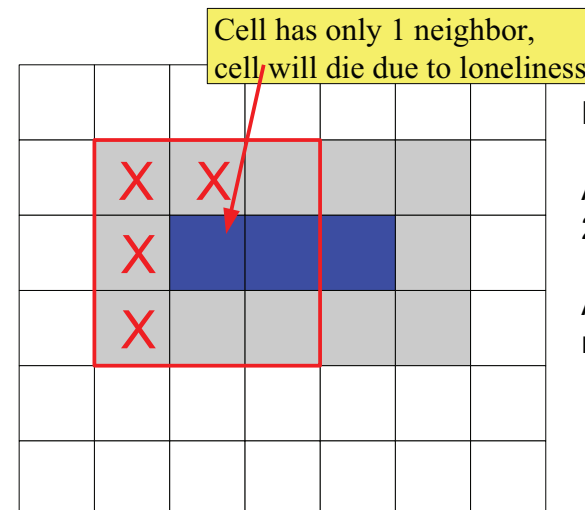
Rule: 23/3

A cell survives with 2 or 3 neighbors.

A cell is born if it has 3 neighbors.

34

Conway's Game of Life



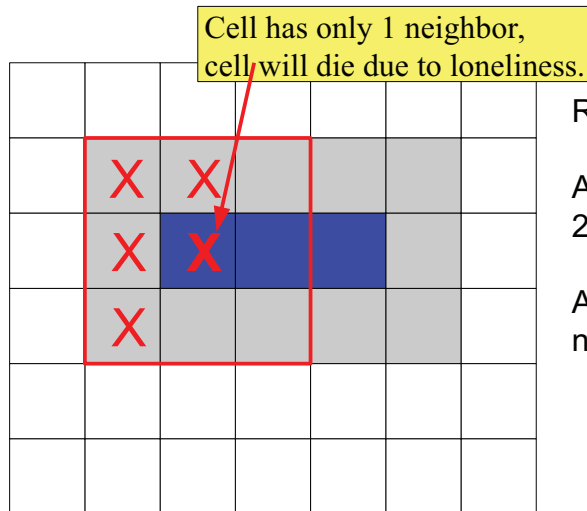
Rule: 23/3

A cell survives with 2 or 3 neighbors.

A cell is born if it has 3 neighbors.

36

Conway's Game of Life



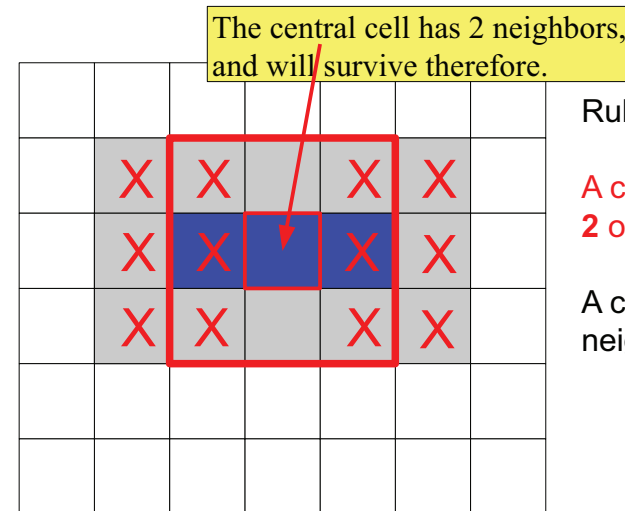
Rule: 23/3

A cell survives with
2 or 3 neighbors.

A cell is born if it has 3
neighbors.

37

Conway's Game of Life



Rule: 23/3

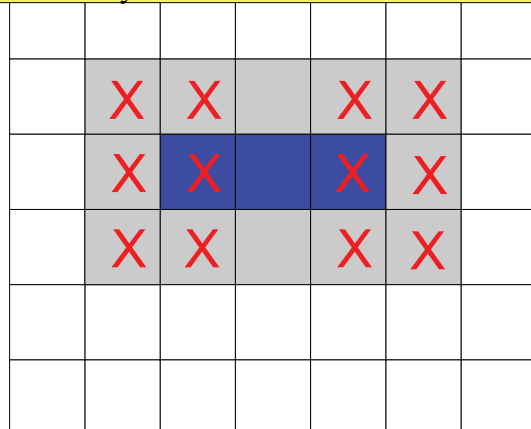
A cell **survives** with
2 or 3 neighbors.

A cell is born if it has 3
neighbors.

39

Conway's Game of Life

Since the rule is symmetric,
we directly have the state of these other cells.



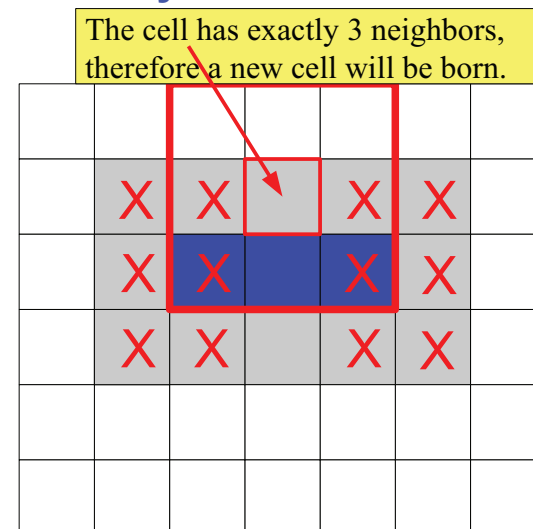
Rule: 23/3

A cell survives with
2 or 3 neighbors.

A cell is born if it has 3
neighbors.

38

Conway's Game of Life



Rule: 23/**3**

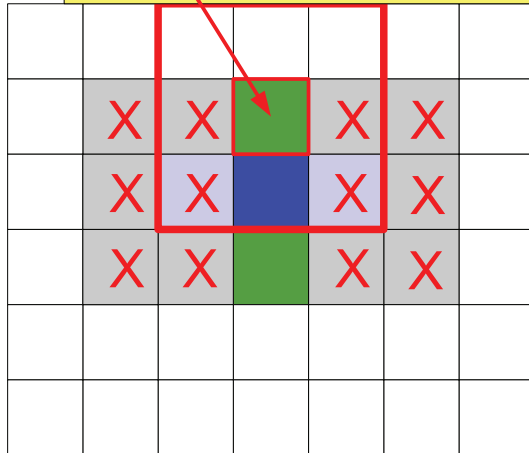
A cell survives with
2 or 3 neighbors.

A cell is **born** if it has **3**
neighbors.

40

Conway's Game of Life

The cell has exactly 3 neighbors, therefore a new cell will be born.



Rule: 23/3

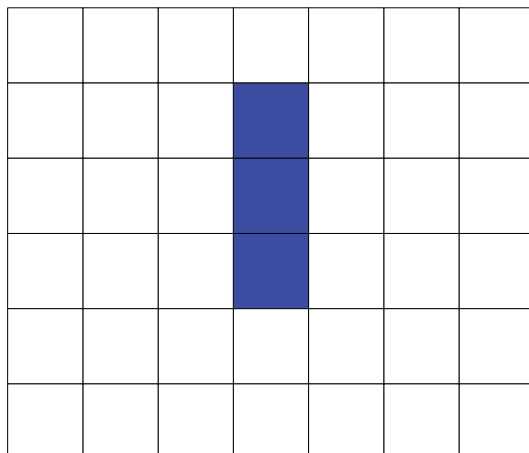
A cell survives with 2 or 3 neighbors.

A cell is **born** if it has 3 neighbors.

41

Conway's Game of Life

time step $t+1$



Rule: 23/3

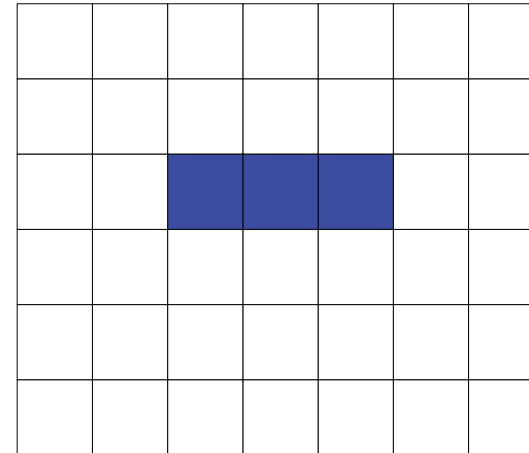
A cell survives with 2 or 3 neighbors.

A cell is born if it has 3 neighbors.

42

Conway's Game of Life

time step $t+2$ has the same pattern as time step t



Rule: 23/3

A cell survives with 2 or 3 neighbors.

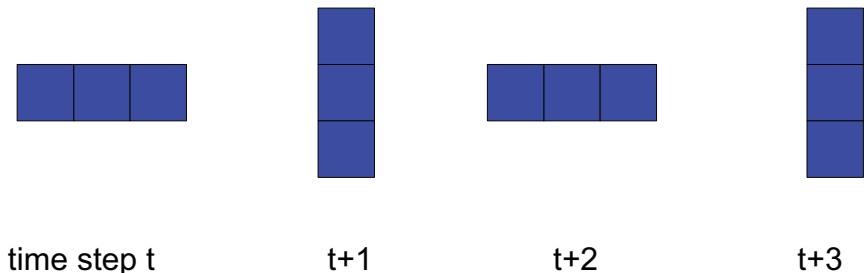
A cell is born if it has 3 neighbors.

43

Conway's Game of Life

Three cells in a row, in Conway's Game of Life yield a periodic structure, oscillating with period 2, called: **Blinker**

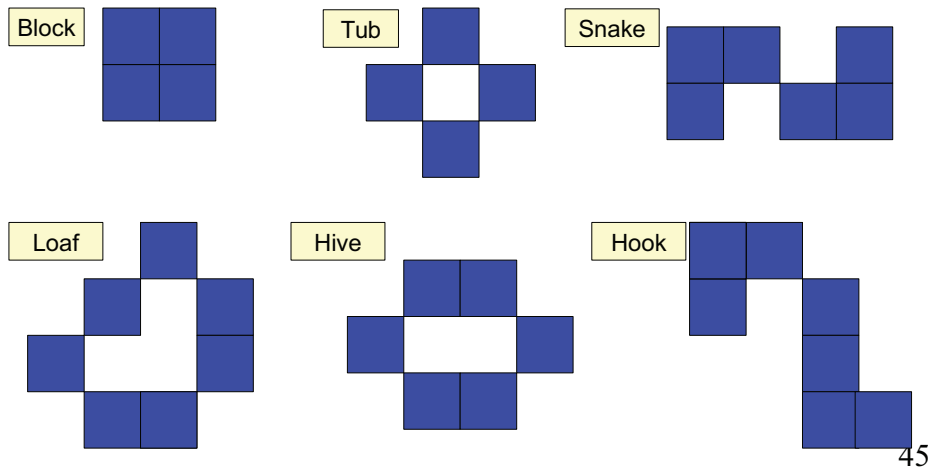
It is the archetype of a *periodic class II* behavior.



44

Conway's Game of Life

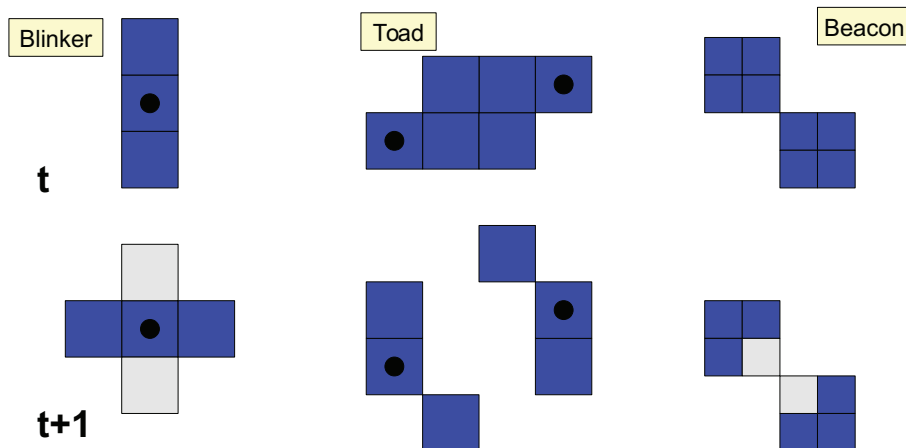
Examples of stationary **class II** patterns:



45

Conway's Game of Life

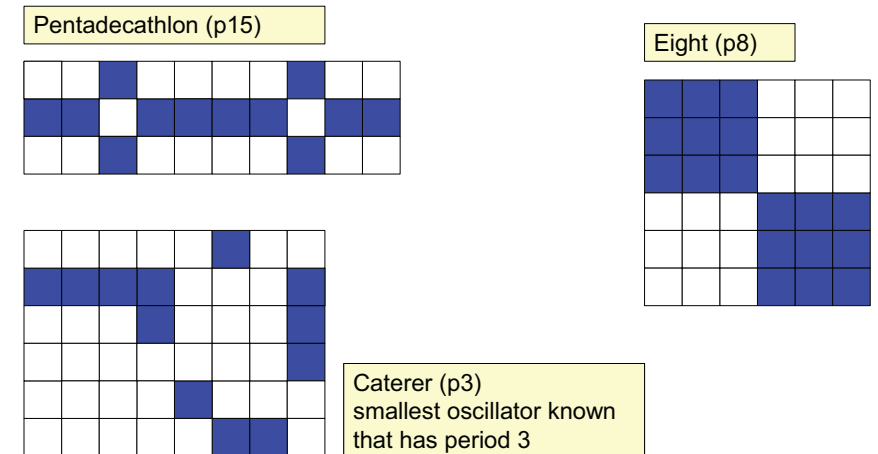
Examples of oscillating **class II** patterns, period 2



46

Conway's Game of Life

Examples of **class II** oscillators.

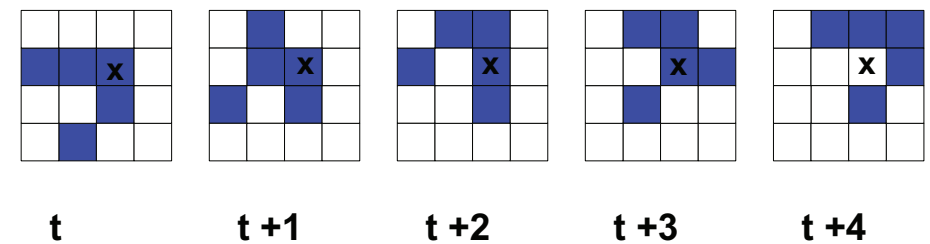


47

Conway's Game of Life

A special Game of Life pattern became very famous: it is reproducing its shape in 4 steps.

Please notice: although the exact shape is reproduced, the pattern is **NOT** periodic.

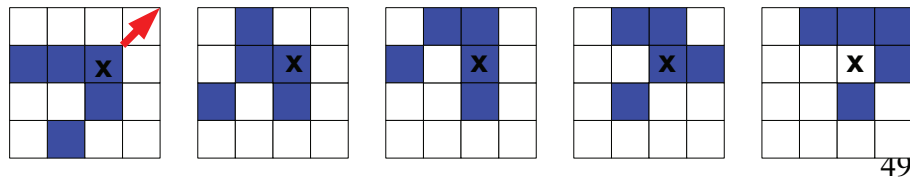


48

Conway's Game of Life: *Glider*

This pattern is called *Glider*:

- the *Glider* is a prototypic **class IV** pattern
- a *Glider* consists of 5 living cells
- in 4 steps it moves one cell in diagonal direction.



49

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Conway's Game of Life: *Glider*

A *Glider* is moving over the underlying grid. After 4 time steps the original shape is reconstructed in an adjacent position and all five cells have changed their state meanwhile.

*Would you consider it to be the **same** Glider ?*



50

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Conway's Game of Life

Conway doubted that it is possible to create a Game of Live pattern that can grow infinitely, he offered 50\$ for the first one to find one.

In 1970, Bill Gosper (a MIT mathematician) found such a pattern, and was rewarded by Conway with 50\$.

Bill Gosper's pattern is an oscillator, constantly changing its shape, with a cycle length of 30 steps. During each cycle, a 5 cell sub-structure is remaining. The nice thing is, that this left over is a *Glider*. Thus, every cycle a *Glider* is produced.

51

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Conway's Game of Life: *Glidergun*

This structure is called: *Glidergun*. It is composed by 4 elements, consisting of 36 cells in a 36X9 bounding box, cycle length 30 time steps.

Two blocks, one on the left side, one on the right implement two stoppers, limiting the structure.

The two other elements approach each other, collide, turn around, and separate again. When they reach the stoppers they are reflected and the cycle starts over.

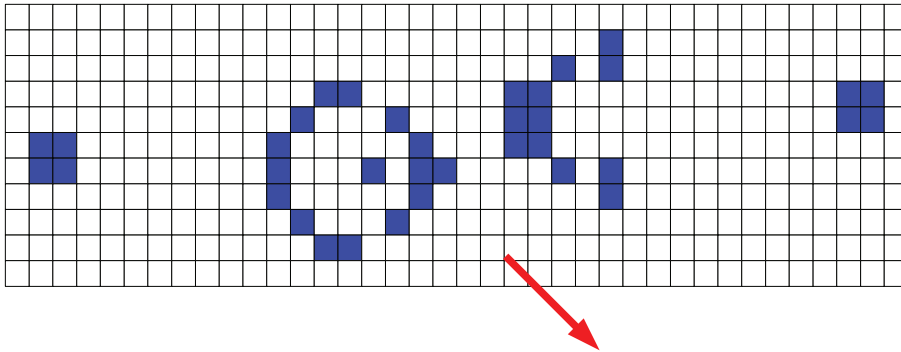
The collision produces the 5 cell remainder which is shaped like a glider. This *Glider* starts to move, and leaves the *Glidergun*.

52

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Conway's Game of Life: *Glidergun*

Glidergun by Bill Gosper, 36 cells, 36 x 9 bounding box.



This, Gosper Glidergun, is producing a glider leaving to the lower right every cycle (30 time steps).

53

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Conway's Game of Life: *Glidergun*

Meanwhile, a wide variety of other *Gliderguns* have been found, invented or constructed with different characteristics like periodicity, size or type of gliders. Research and development is still ongoing today.

A smart combination of simple Game of Life elements can be used to construct complex machineries.

Gliderguns are the *motors* of complex Game of Life constructions.

Streams of Gliders are the *information carriers* and the *tools* of Game of Life machineries.

54

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Conway's Game of Life

Streams of Gliders can be regarded as information streams, a single Glider would be a single bit.

Gliders can interfere when they collide with other Gliders and with specially shaped structures.

Depending on the individual phase during the collision they can be erased, delayed, reflected or doubled.

By this means, it is possible to construct Game of Life structures that act as *Boolean gates*, AND, OR, NOT, NAND, NOR, XOR, ...

55

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Conway's Game of Life

Whenever a *Glider* collides with an other Glider or with an other structure an intermediate cell structure arises. This intermediate structure is of course undergoing further changes following the Game of Life rule. The final result is typically hard to predict.

With a lot of effort and a lot of Game of Life experience it is possible to influence the shape of the remainder. Carefully arranged, the collision can yield a structure that is useful for further processing.

Colliding Gliders are the *universal tools* to *construct* and *destroy* other structures in Game of Life.

56

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Conway's Game of Life: *Gliders*

GF: 30, Cells: 54



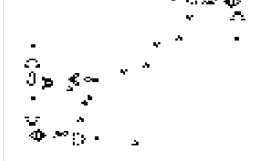
GF: 144, Cells: 132



GF: 44, Cells: 298



GF: 570, Cells: 260



GF: 9990, Cells: 339



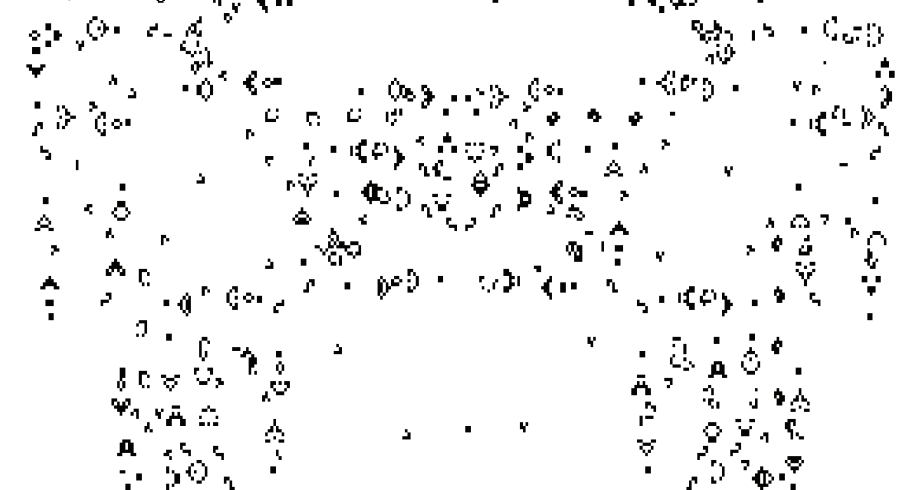
GF: 36, Cells: 1973



From: <http://www.radical-eye.com/lifepage/patterns/gunprev.html> 57

Conway's Game of Life: *Gliders*

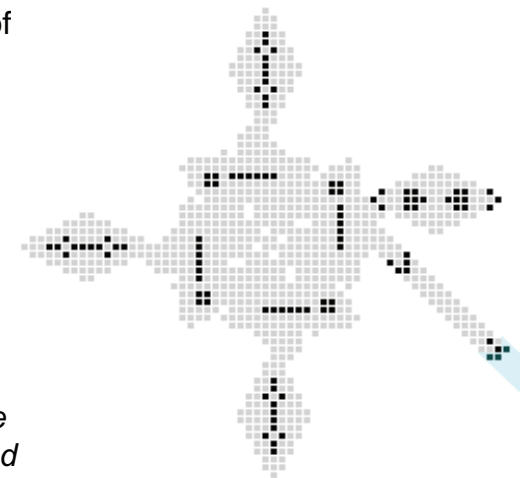
GF: 400, Cells: 2426



From: <http://www.radical-eye.com/lifepage/patterns/gunprev.html> 58

Conway's Game of Life: *Gliders*

The research on Game of Life structures is still ongoing:
e.g. in 29-April-2010 a new Glidergun has been published, with a 45 time step cycle.



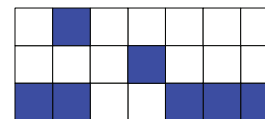
„Period 45 is currently the shortest known odd period for a true-period gun.“

From: <http://pentadecathlon.com/lifeNews/index.php> 59

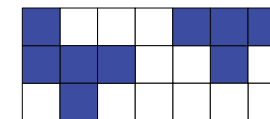
Conway's Game of Life

Long lasting patterns (Methuselah patterns)

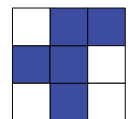
Acorn



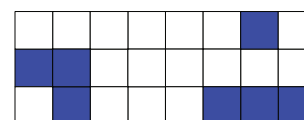
Rabbits



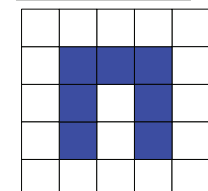
r-pentomino



Diehard



Pi-heptomino



Other Game of Life rules

Conway's Game of Life has the rule **23/3** or (**S23/B3**) with a cell **Surviving** if it has **2** or **3** neighbors, and a new cell is **Born** having **3** neighbors.

There are other rules, that lead to interesting behavior:

2 / 3
3 / 3
13 / 3
23 / 3
34 / 3
35 / 3
236 / 3
135 / 35
1357 / 1357

61

Game of Life Simulators

A wide variety of simulation tools for Conway's Game of Life exist today for all typical computing architectures and operating systems, and most of them are freely available.

Some of them are accompanied with a large library of patterns.

Some of them are designed to simulate cellular automata, and "just" include Conways Game of Life.

The list on the next page is is neither complete, nor should it be considered as being a judgment; these are just some of my personal favorites.

62

Game of Life Simulators

The list is neither complete, nor should it be considered as being a judgment; these are just my personal favorites.

Mirek's Celebration, MCell 4.20

1D and 2D Cellular Automata explorer by Mirek Wojtowicz

<http://psoup.math.wisc.edu/mcell/>

Winlife32, Version 2.3

A system for manipulating Conway's Game of Life

<http://www.winlife32.com/>

Golly, Version 2.6, Mac OS X 10.6 or later.

An open source, cross-platform Game of Life Simulator

<http://www.macupdate.com/info.php/id/19622/golly>

63

Overview:

- Cellular Automata in 2 dimensions
- Examples and Applications of CAs
- Conway's Game of Life
- **Computational Universality**
- Is Information == Structure ?

64

Conway's Game of Life

Streams of Gliders can be regarded as information streams, a single Glider would be a single bit.

Gliders can interfere when they collide with other Gliders and with specially shaped structures.

Depending on the individual phase during the collision they can be erased, delayed, reflected or doubled.

By this means, it is possible to construct Game of Life structures that act as *Boolean gates*, AND, OR, NOT, **NAND**, NOR, XOR, ...

65

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Overview:

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- **Is Information == Structure ?**

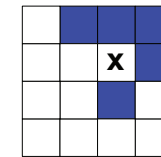
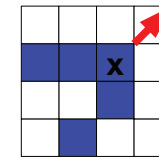
66

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Is Information == structure ?

A *Glider* is moving over the underlying grid. After 4 time steps the original shape is reconstructed in an adjacent position and all five cells have changed their state meanwhile.

Would you consider it to be the **same** Glider ?



67

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Cellular Automata 2D Conway's Game of Life

Thank you for your attention

Dr. Nils Goerke, Autonomous Intelligent Systems,
Department of Computer Science, University of Bonn

68

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