Autonomous Intelligent Systems,
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Exercises for Artificial Life (MA-INF 4201), SS11

Exercises sheet 5, due: Mon 23.05.2011

16.5.2011

Group	Name	29	30	31	32	33	34	$\sum$

Assignment 29 (2 Points)

Prove or disprove the following sentence in a formal way:
The Fibonnaci sequence is rising faster than the exponential function.

Assignment 30 (2 Points)

Draw a diagram (on your own) that shows two sets of spirals: one set is curving clockwise, one set is counter clockwise, with consecutive Fibonacci numbers of spirals in the two sets.
Remark: copying a picture from the internet is not sufficient.

Assignment 31 (4 Points)

Please determine the long-term behaviour of the following iterated function:
$x(i+1) = a * x(i) * (1 - x(i))$ for $a = 3.3$ and for $a = 3.5$
for the two starting conditions $x(i = 0) = 0.228734167$ and $\hat{x}(i = 0) = 0.228734168$.

Assignment 32 (1 Point)

What is called the Golden Angle?

Assignment 33 (2 Points)

Suppose a netto growth of a population of 3% per year.
How long will it take until this population has doubled its size?
Please derive a formula for the number of years necessary to reach the doubling.

Assignment 34 (4 Points)

Calculate the limit α of the ratios of successive terms of the Fibonacci sequence using a program and analytically.

$$\alpha = \lim_{n\to\infty} \frac{F_{n+1}}{F_n} \qquad F_{n+2} = F_n + F_{n+1} \qquad F_0 = 0, \quad F_1 = 1$$

Artificial Life
Exercise 5

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May 22, 2011

Assignment 29

$$
\begin{aligned}
exp(0) &= 1,\ exp(1) = e \\
exp(x) &= e * exp(x-1) \quad \text{with} \quad x > 1 \\
fib(0) &= 1,\ fib(1) = 1 \\
fib(x) &= fib(x-2) + fib(x-1)
\end{aligned}
$$

We try to prove the following by induction.

$$exp(x) \overset{?}{>} fib(x)$$

Basis:

$$
\begin{aligned}
exp(2) &\approx 7.38 \\
fib(2) &= 2 \\
exp(2) &> fib(2)
\end{aligned}
$$

Induction:

$$
\begin{aligned}
\frac{exp(x)}{exp(x-1)} &\overset{?}{>} \frac{fib(x)}{fib(x-1)} \\
\frac{exp(x)}{exp(x-1)} &\overset{?}{>} \frac{\overbrace{fib(x-2)}^{<fib(x-1)} + fib(x-1)}{fib(x-1)} \\
\frac{e * exp(x-1)}{exp(x-1)} &\overset{?}{>} \frac{fib(x-1) + fib(x-1)}{fib(x-1)} \\
e &\overset{?}{>} \frac{2 * fib(x-1)}{fib(x-1)} \\
e &\overset{!}{>} 2
\end{aligned}
$$

Assignment 30

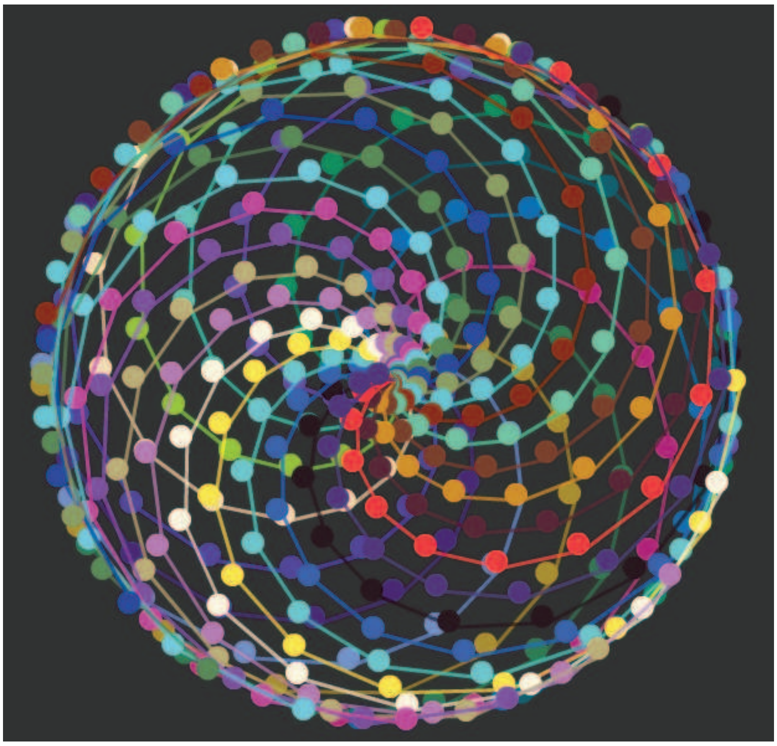

Figure 1: Two sets of spirals with 13 clockwise and 21 counter clockwise spirals

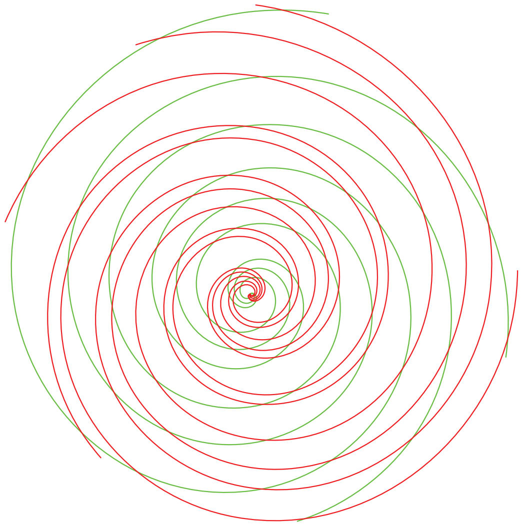

Figure 2: Two sets of spirals with 3 clockwise and 5 counter clockwise spirals

Assignment 31

All four variants start oscillating between two (resp. four) values after a couple of steps. See Figure 3.

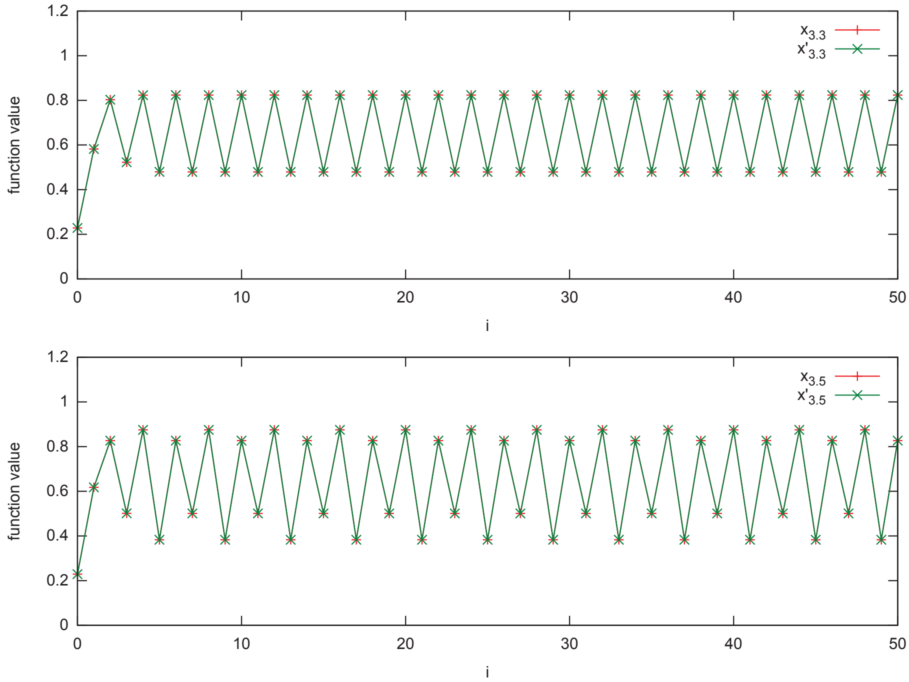

Figure 3: Diagram of the given iterated function with two different starting criterias and two different values a

Assignment 32

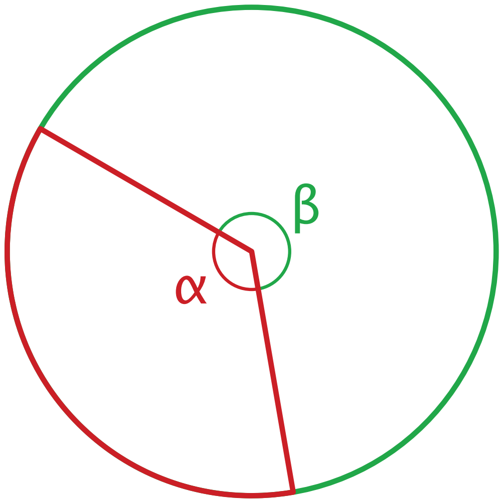

Figure 4: Golden angle α

Figure 4 shows the golden angle α in the circle. The ratio between the golden angle α and the remaining angle β is the same as the ratio between β and the angle of the whole circle $(\alpha + \beta)$.

$$
\begin{aligned}
\frac{\alpha}{\beta} &= \frac{\beta}{\alpha+\beta} \\
\alpha &= 360° - \beta \\
\frac{360° - \beta}{\beta} &= \frac{\beta}{360° - \beta + \beta} \\
\frac{360° - \beta}{\beta} &= \frac{\beta}{360°} \\
360°(360° - \beta) &= \beta^2 \\
\beta^2 + 360° * \beta - 360°^2 &= 0 \\
\beta_1 &= 180°(\sqrt{5} - 1) \approx 222.492° \\
\alpha_1 &= 360° - \beta_1 = 180°(3 - \sqrt{5}) \approx 137.5°
\end{aligned}
$$

Assignment 33

$$
\begin{aligned}
x(i+1) &= 0.03 * x(i) + x(i) \\
x(i+1) &= 1.03 * x(i) \\
x(i+n) &= x(i) * 1.03^n
\end{aligned}
$$

$$
\begin{aligned}
x(i+n) &\stackrel{?}{=} 2 * x(i) \\
x(i) * 1.03^n &= 2 * x(i) \\
1.03^n &= 2 \\
n &= log_{1.03}(2) \approx 23.45
\end{aligned}
$$

It will take roughly 24 years until the population has doubled.

Assignment 34

The limit of the Fibonacci sequence equals the Golden ratio.

$$\alpha = \lim_{n \to \infty} \frac{F_{n+1}}{F_n}$$
$$\alpha = \frac{1+\sqrt{5}}{2} \approx 1.618$$

Proof:

$$\begin{aligned}
\alpha &= \lim_{n \to \infty} \frac{F_{n+1}}{F_n} = \lim_{n \to \infty} \frac{F_n}{F_{n-1}} \\
F_{n+1} &= F_n + F_n - 1 \\
\lim_{n \to \infty} \frac{F_{n+1}}{F_n} &= \lim_{n \to \infty} \frac{F_{n-1} + F_n}{F_n} \\
\lim_{n \to \infty} 1 + \underbrace{\frac{F_{n-1}}{F_n}}_{\alpha^{-1}} &= 1 + \frac{1}{\alpha} \\
\iff \quad \alpha &= 1 + \frac{1}{\alpha} \\
0 &= \alpha^2 - \alpha - 1 \\
\alpha &= \frac{1 \pm \sqrt{5}}{2}
\end{aligned}$$