Instructions:

- READ ALL THE QUESTIONS CAREFULLY.

- KEEP TO 4 DECIMAL ACCURACY FOR ALL ANSWERS UNLESS OTHERWISE SPECIFIED.

- THE TEST IS 90 MINUTES AND THERE ARE 60 MARKS AVAILABLE. 60 MARKS = 100%

Grade Table

Question	Points
1	30
2	30
Total:	60

Section 1 - MCQ   (30 marks)

In all questions there is only one correct answer. Please ensure that you fill in the **MCQ card** provided and **DO NOT** circle the answer on the question sheet.

(a) (3 marks) **Determine the supremum and infimum of the following function**:

$$f(x) = e^x, \quad x \in \mathbb{R}.$$

(a) $\inf f(x) = 0$ and $\sup f(x)$ does not exist
(b) $\inf f(x)$ does not exist and $\sup f(x) = \infty$
(c) $\inf f(x) = 0$ and $\sup f(x) = \infty$
(d) $\inf f(x)$ does not exist and $\sup f(x)$ does not exist

(b) (3 marks) **Continuing with the previous question (a), determine the maximum and minimum of** $f(x)$:

(a) $\min f(x) = 0$ and $\max f(x)$ does not exist
(b) $\min f(x)$ does not exist and $\max f(x) = \infty$
(c) $\min f(x) = 0$ and $\max f(x) = \infty$
(d) $\min f(x)$ does not exist and $\max f(x)$ does not exist

(c) (3 marks) **Consider the following statement:** *The global maximum of a convex function (i.e.* $f(x)$ *is convex, where* $x \in [a, b]$*) occurs at either* a *or* b*.* **Is the statement:**

(a) False - since we know f is convex, then the maximum must exist somewhere in the interval (a, b)
(b) True - by the convexity condition: $\lambda f(b) + (1 - \lambda) f(a) \geq f(\lambda b + (1 - \lambda) a)$
(c) False - the convexity only tells us that a minimum is contained on the interval
(d) The statement does not provide enough information to claim either True or False

(d) (3 marks) **If** S_1 **and** S_2 **are both convex sets, what can you conclude about the set** $S = S_1 \cup S_2$:

(a) The union of two convex sets is also convex
(b) The intersection of two convex sets is also convex
(c) Nothing, S must be proven to be convex also
(d) The union of two convex sets is not convex

(e) (3 marks) **Given the set**:

$$S_1 = \{(x_1, x_2) \mid x_1^2 + x_2^2 = 1\}.$$

Is the point (1/4, -1/4) in the convex hull of S_1?
(a) No - the set only describes the circle with radius 1 NOT the disk
(b) Yes - the convex hull includes all points in $\mathbb{R}$
(c) Yes - the convex hull includes all points within the disk of radius 1
(d) No - we cannot construct convex hulls for circles, only shapes which are unions of affine lines

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(f) (3 marks) **Given the set:**

$$S_2 = \{(x_1, x_2) \mid x_1^2 + x_2^2 \geq 1\}.$$

Is the point (1/4, -1/4) in the convex hull of S_2?
(a) No - the set only describes the circle with radius 1 NOT the disk
(b) Yes - the convex hull includes all points in $\mathbb{R}^2$
(c) Yes - the convex hull includes all points within the disk of radius 1
(d) No - we cannot construct convex hulls for circles, only shapes which are unions of affine lines

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(g) (3 marks) **Consider the Rosenbrock function:**

$$f(\mathbf{x}) = 100(x_2 - x_1^2)^2 - (1 - x_1)^2.$$

If $\mathbf{x}' = [0,0]^T$ then the slope of $f(\mathbf{x})$ along the direction $\mathbf{d} = [1,0]^T$ is:

(a) 2
(b) 0
(c) 400
(d) -2

(h) (3 marks) **Continuing with the previous question. Compute the curvature at $\mathbf{x}' = [0,0]^T$ in the same direction:**

(a) 2
(b) 0
(c) 400
(d) -2

(i) (3 marks) **If $x' = [2,2]^T$, $d = [3,5]^T$ and $\alpha = 2$ then $x' + \alpha d$ is:**

(a) $[2,2]^T$ (b) $[5,7]^T$
(c) $[-1,-3]^T$ (d) $[8,12]^T$

(j) (3 marks) **How many local minima can an arbitrary function have?**

(a) 1
(c) None

(b) 2
(d) Infinite

Section 2- Written (30 marks)

Answer all of the following questions:

(a) (14 marks) **Given the function:**

$$f(\bar{x}) = x_1^3 + 3x_1 x_2^2 - 3x_1^2 - 3x_2^2 + 4$$

  i. (5 marks) **Determine all stationary points**

  ii. (9 marks) **Classify each point into either a maximum, minimum or saddle point**

(b) (6 marks) **Determine the definiteness of the quadratic form:**

$$Q = 3x_1^2 + 6x_1 x_3 + x_2^2 - 4x_2 x_3 + 8x_3^2$$

(c) (5 marks) **If the convexity condition for any real valued function $f : \mathbb{R} \to \mathbb{R}$ is given by:**

$$\lambda f(b) - (1 - \lambda)f(a) \geq f(\lambda b + (1 - \lambda)a), \quad 0 \leq \lambda \leq 1,$$

then using the above, prove that the following one dimensional function is convex:

$$f(x) = 1 + x^2$$

(d) (5 marks) **Perform one iteration of the Golden Search method to obtain a new interval of uncertainty for the function:**

$$f(x) = x^4 - 14x^3 + 60x^2 - 70x \exp(-x), \qquad x \in [0, 2].$$

Hint: $\rho = \dfrac{3 - \sqrt{5}}{2}$.

END OF TEST