Due: 19th of October 2017 at 11:59pm

COMP 2007 – Assignment 4

All submitted work must be done individually without consulting someone elses solutions in accordance with the Universitys Academic Dishonesty and Plagiarism policies.

IMPORTANT! Questions 1a–b and 2a–c should be submitted via Blackboard as pdf (no handwriting!). The implementation required for Questions 2d should be done in Ed, and submitted via Ed.

Questions

Christmas is coming up and you have decided to invest in a Christmas tree production company. The company has k different forests growing Christmas trees, let us call these $F = \{\text{forest } 1, \text{forest } 2, \ldots, \text{forest } k\}$. Your task is to plan an optimal cutting tree schedule for the next Y years. To be able to sell a Christmas tree it has to be a mature tree. You estimate that each forest i, will mature $w_{i,j}$ Christmas trees in year j. Christmas trees have a limited lifetime: a tree which matures in year j can only be cut down and sold in that year, or in the $\delta_j - 1$ years afterwards. After that the tree will be too old to sell and will fall down naturally.

The economic predictions also show that if the company harvests more than u_j Christmas trees in year j, the market would be flooded and the Christmas tree market would crash...you don't want that to happen.

Additionally, cutting too many trees from a single forest destabilises the local ecosystem. Since you are environmentally conscious, you cannot harvest more than τ_i trees total from forest i over the entire Y years.

Your task is to develop an algorithm that determines a Christmas tree harvesting schedule that maximizes the number of Christmas trees sold (you should only return the number of trees that should be sold).

To aid you in your task you have been provided with an implementation of the Ford-Fulkerson algorithm. You may assume without proof that this algorithm correctly returns the maximum flow of a given flow network G in $O(m^2 \log C)$ time using $O(n + m)$ space, where C is maximum flow in G.

1. **[20 points]** Consider the case when $Y = 3$, $k = 2$, $\delta_1 = \delta_2 = 2$ and $\delta_3 = 1$.
   (a) Formulate the problem of determining a schedule with maximum number of Christmas trees sold as a network flow problem. [10 points]
   (b) Argue why your algorithm is correct. [10 points]
2. **[80 points]** In this question your task is to generalise your solution to k forests, Y years and variable tree lifespans.
   (a) Formulate the problem of determining a schedule with maximum profit (maximum number of Christmas trees sold) as a network flow problem for a given Y, k and $\delta_1, \ldots, \delta_Y$. [15 points]
   (b) Argue why your formulation is correct. [15 points]
   (c) Prove an upper bound on the time complexity of your algorithm. [20 points]
   (d) Implement your algorithm (in Ed) and test it on the provided instances. Each instance is using the following format:

k
Y
δ_1
...
δ_Y
τ_1
...
τ_k
$w_{1,1}$
...
$w_{1,Y}$
$w_{2,1}$
...
$w_{k,Y}$
u_1
...
u_Y

You may assume that all the values given are non-negative integers. The output should be the maximum number of Christmas trees that can be sold.

[30 points]