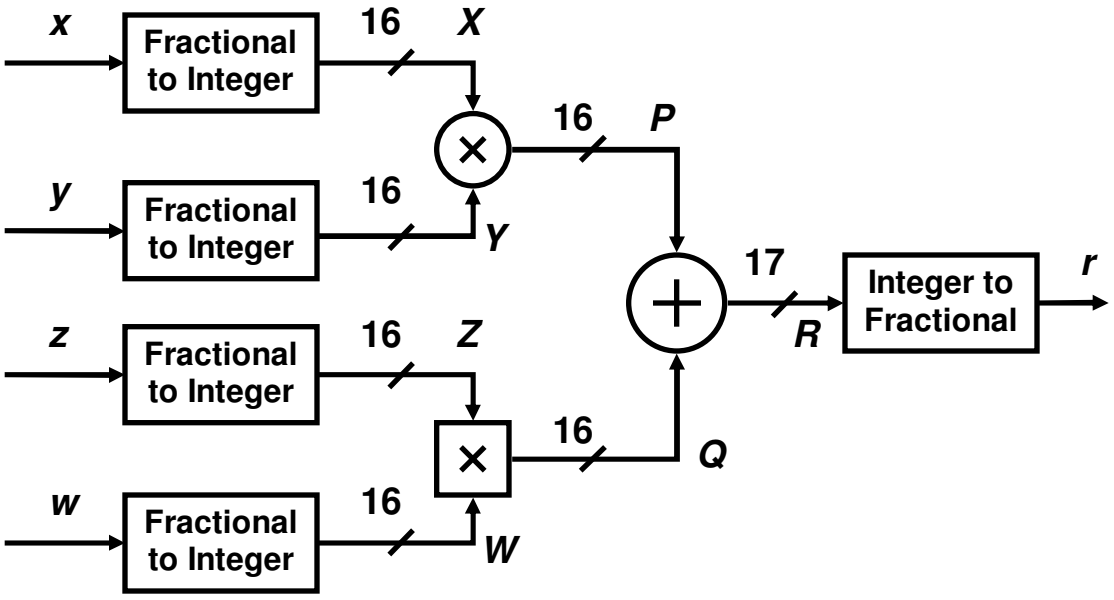
Fixed-point arithmetic

Assume the structure below in which:

- ***x*** is a real values ranging from **–0.5 to +0.5**
- ***y*** is a real value ranging from **–1.0 to +1.0**
- ***z*** is a real values ranging from **–2.0 to +2.0**
- ***w*** is a real value ranging from **–4.0 to +4.0**

The circuit performs as follows.

- First, a conversion from fractional representation to integer representation is carried out (capital letters signify integer values).
- Then ***X*** is multiplied by ***Y*** using a fractional multiplier. The product ***P*** is represented over a 16-bit signed field.
- ***Z*** is multiplied by ***W*** using an integer multiplier. The product ***Q*** is represented over a 16-bit signed integer.
- ***P*** is added to ***Q*** to yield a 17-bit signed integer ***R***.
- Finally, a conversion from integer representation to fractional representation is carried out.



1. Assuming that the fractional-to-integer conversions are performed to ensure the highest numerical accuracy over 16-bit fields, specify the scale factors which ***x***, ***y***, ***z***, ***w*** are multiplied with.
2. Assuming ***x*** **= –0.3**, ***y*** **= 0.7**, ***z*** **= 1.6**, and ***w*** **= –3.1**, compute the integers ***X***, ***Y***, ***Z***, ***W***, ***P***, ***Q***, and **R**. Assume von Neumann rounding.
3. Perform the final integer-to-fractional conversion and provide the value of ***r***.