GUIDE TO SEASONAL ADJUSTMENT WITH X-13ARIMA-SEATS©

TIME SERIES ANALYSIS BRANCH

Office for National Statistics

METHODOLOGY AND QUALITY

ONS: *Guide to Seasonal Adjustment with X-13ARIMA-SEATS©*

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PURPOSE OF THIS GUIDE

X-13Arima -Seats

Seasonal adjustment is widely used in official statistics as a technique to allow the timely interpretation of time series data. X-13ARIMA-SEATS has been chosen from the many available seasonal adjustment software packages as the standard package for use in official statistics in the United Kingdom (UK). This was agreed by the Statistical Policy and Standards Committee in 2012.

X-13ARIMA-SEATS incorporates the X-11 and SEATS methods of seasonal adjustment, two alternative methods that are widely used by many of the leading national statistical institutes National Statistics Institute (NSI) across the world.

X-13ARIMA-SEATS was developed by the United States Census Bureau (USCB) and replaces X-12-ARIMA, the previously recommended software for seasonal adjustment of official statistics in the UK. The software is comprehensive, with many options to tailor seasonal adjustment to each individual series. It therefore requires many choices to be made by its users.

The main purpose of this guide is to provide practical guidance on seasonal adjustment using X-13ARIMA-SEATS. The guide explains what seasonal adjustment is and addresses many of the issues and problems associated with seasonal adjustment.

A detailed explanation of the X-11 method can be found in Ladiray and Quenneville (2001) while an explanation of the SEATS method can be found in Gómez and Maravall (2001).

What is in this guide

This guide starts with a brief introduction to seasonal adjustment and to some of the main associated issues (Chapter 1). It then provides an overview of the X-11 and SEATS methods (Chapter 2), and a description of how to run the programme (Chapter 3).

Chapter 25 discusses how to download, install, and use the software. Chapter 26 provides a useful quick guide to seasonal adjustment that gives a list of solutions to common seasonal adjustment problems.

Chapter 4 to Chapter 24 address issues to consider when performing seasonal adjustment.

- For making simple updates on existing spec files due to new data, see Chapter 11
- For an explanation of the regARIMA model, see Chapter 8

- For understanding and implementing regressors, see Chapter 9, Chapter 10, and Chapter 11
- For writing a new spec file, see Chapter 3, Chapter 12, Chapter 13, and Chapter 15
- For understanding output from X13ARIMA-SEATS, see Chapter 16, and Chapter 18
- For analysing revisions patterns, see Chapter 7, and Chapter 19
- For information about software X-13ARIMA-SEATS can be used with, see Chapter 3, Chapter 16, and Chapter 25
- For measures of uncertainty, see Chapter 24

How to use this guide

This guide is intended to be used as a reference guide. Users are advised to read chapters 1-3 before attempting any seasonal adjustment, and to refer to other parts of the guide as appropriate. Seasonal adjustment practitioners should attend a training course. This guide complements, but is not a substitute for such a training course. ONS provides a short training course for users of seasonal adjustment. The University of Southampton provides an advanced module in time series analysis and seasonal adjustment. More information on training courses can be obtained from Time Series Analysis Branch (TSAB).

What this guide is not

This guide does not describe the detailed workings of X-13ARIMA-SEATS, nor the underlying mathematics. The interested reader is referred to the X-13ARIMA-SEATS user manual (USCB 2017) as a good starting point. Additional technical references and papers are listed in the references in the bibliography.

What does TSAB do?

The ONS Time Series Analysis Branch (TSAB) provides expertise in time series analysis and in particular seasonal adjustment. TSAB has produced this guide, and provides courses in seasonal adjustment. TSAB is responsible for the quality of all ONS seasonal adjustment and manages a rolling programme of annual seasonal adjustment reviews of statistical outputs across ONS. TSAB also provides support for practitioners and users of seasonal adjustment across the Government Statistical Service and is the main point of contact for any questions or queries regarding seasonal adjustment and time series analysis in official statistics.

Training Courses

The Office for National Statistics has a programme of training courses including seasonal adjustment. Further information can be found from Government Analysis Function website. A good suggestion for beginners could be the Awareness in Seasonal Adjustment course. Contact TSAB for more information on training.

Useful websites

- Australian Bureau of Statistics (ABS), ABS website, methods; classifications; concepts & standards, Time Series Analysis: The Basics.
- European Union, Eurostat website, methodology, Seasonal Adjustment
- Government Analysis Function, Government Analysis Function website, GSS methodology support, Information on Specific Statistical Methods
- Office for National Statistics (ONS), ONS website, methodology topics and statistical concepts, Seasonal Adjustment
- United States Census Bureau (USCB), released 11 July 2022, USCB website, software, X-13ARIMA-SEATS Seasonal Adjustment Program

Contact details

If you have a query related to time series analysis, seasonal adjustment or if you have any comments on this guide we can be contacted at: Time Series Analysis Branch, Office for National Statistics, Cardiff Road, Newport, NP10 8XG or by email.

ACKNOWLEDGEMENTS

This guide has been developed with the input and assistance of the following people: Claudia Annoni, Folasade Ariyibi, Simon Compton, Jennifer Davies, Francis Dunnett, Duncan Elliott, Giorgia Galeazzi, Mark Hogan, Juliane Haarmann, Lee Howells, Fida Hussain, Cathy Jones, Peter Kenny, Jim Macey, Craig McLaren, Ross Meader, Dimitrios Nikolakis, Neil Parkin, Chris Payne, Nigel Stuttard, Anthony Szary, Anita Visavadia, Matt Whipple and Ping Zong.

Please bring any errors or omissions to our attention using the contact details above.

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ACRONYMS

ACF Autocorrelation Function

AIC Akaike Information Criterion

AICC Akaike Information Criterion Corrected

ANOVA Analysis of Variance

AR Autoregressive

ARIMA Autoregressive Integrated Moving Average

ARMA Autoregressive Moving Average

BIC Bayesian Information Criterion

CAT Comparison of Annual Totals

CPI Consumer Price Index

CTQ Contingency Table Q

ESS European Statistical System

GDP Gross Domestic Product

GSS Government Statistical Service

IDS IDentifiable Seasonality Test Result

MA Moving Average

MCD Months for Cyclical Dominance

MM Month-on-Month

NSA Non Seasonally Adjusted Series

NSI National Statistics Institute

ONS Office for National Statistics

PACF Partial Autocorrelation Function

QQ Quarter-on-Quarter

regARIMA regression+ARIMA

SA Seasonally Adjusted Series

SEATS Signal Extraction in ARIMA Time Series

SI Irregular to Seasonal Ration (also I/S)

STAR Stability of Trend and Adjusted Series Rating

SPSC UK Statistical Policy and Standards Committee

TC Temporary Change

TRAMO Time Series Regression with ARIMA Noise, Missing Observations and Outliers

TSAB Time Series Analysis Branch

USCB United States Census Bureau

YY Year-on-Year

PEEIs Principal European Economic indicators

1

INTRODUCTION TO SEASONAL ADJUSTMENT

1.1 WHAT IS SEASONAL ADJUSTMENT?

A time series is defined as *"...a collection of observations made sequentially in time*[1]*"*. Many of the most well known statistics published by the Office for National Statistics (ONS) are regular time series including: the claimant count; the Consumer Price Index (CPI); Balance of Payments and Gross Domestic Product (GDP). Users of these time series typically seek to understand the general pattern of the data, for example the long term movements, and whether any unusual occurrences have had major effects on the series. This type of analysis is complicated because there will normally be regular effects associated with the time of the year and the arrangement of the calendar that obscure movements. For example, retail sales rise each December because ofChristmas and this may obscure underlying movements in the retail sales trend. The purpose of seasonal adjustment is to remove variation associated with the time of the year and the arrangement of the calendar. This helps users to interpret movement in the series between consecutive time periods. Figure 1.1 below is a good example of a series with large seasonal peaks and troughs and also the seasonally adjusted series with these peaks and troughs removed.

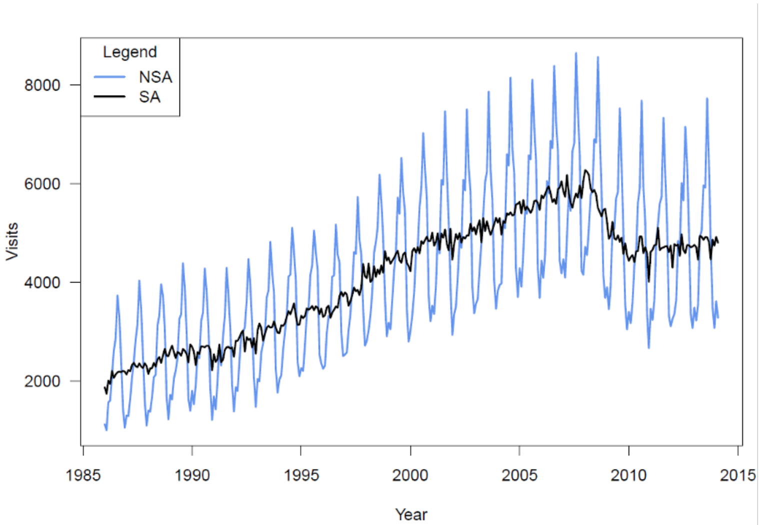

Figure 1.1: Non seasonally adjusted series (NSA) and seasonally adjusted (SA) data for United Kingdom visits abroad

1 Chatfield (2016)

1.2 COMPONENTS OF A TIME SERIES

Time series can be thought of as combinations of three distinctly different types of behaviour, each representing the impact of certain types of real world events on the data. These three components are: seasonal and calendar-related effects, irregular fluctuations, and trend behaviour. Seasonal and calendar effects are usually grouped into the same component, but sometimes are reported separately.

Seasonal effects are patterns that repeat approximately a whole number of times per year; they may evolve or change suddenly. The seasonal effects could be caused by various factors, such as weather patterns[2], administrative measures[3], and social, cultural and religious events[4].

Effects caused by the arrangement of the calendar include:

- variation in the length of months and quarters caused by due to the nature of the calendar
- trading day effects, which are caused by months having different numbers of each day of the week from year to year. For example, spending in hardware stores is likely to be higher in the same month when it has five weekends rather than four
- moving holidays, which may fall in different months from year to year: for example Easter, which can occur in either March or April.

Irregular fluctuations may occur because of due to a combination of unpredictable or unexpected factors, such as unseasonable or extreme weather, natural disasters or strikes. The contribution of the irregular fluctuations will generally change in magnitude and/or direction from period to period.

The trend represents the underlying behaviour and direction of the series. It captures the long-term behaviour of the series as well as the medium-term business cycle for socio-economic time series. The long-term and medium-term behaviour can be separated into separate trend and cycle components. In practice these are usually combined into a trend-cycle component. This guide will refer to a single trend-cycle component.

2 e.g. the increase in energy consumption with the onset of winter
3 e.g. the start and end dates of the school year
4 e.g. retail sales increasing in the run up to Christmas

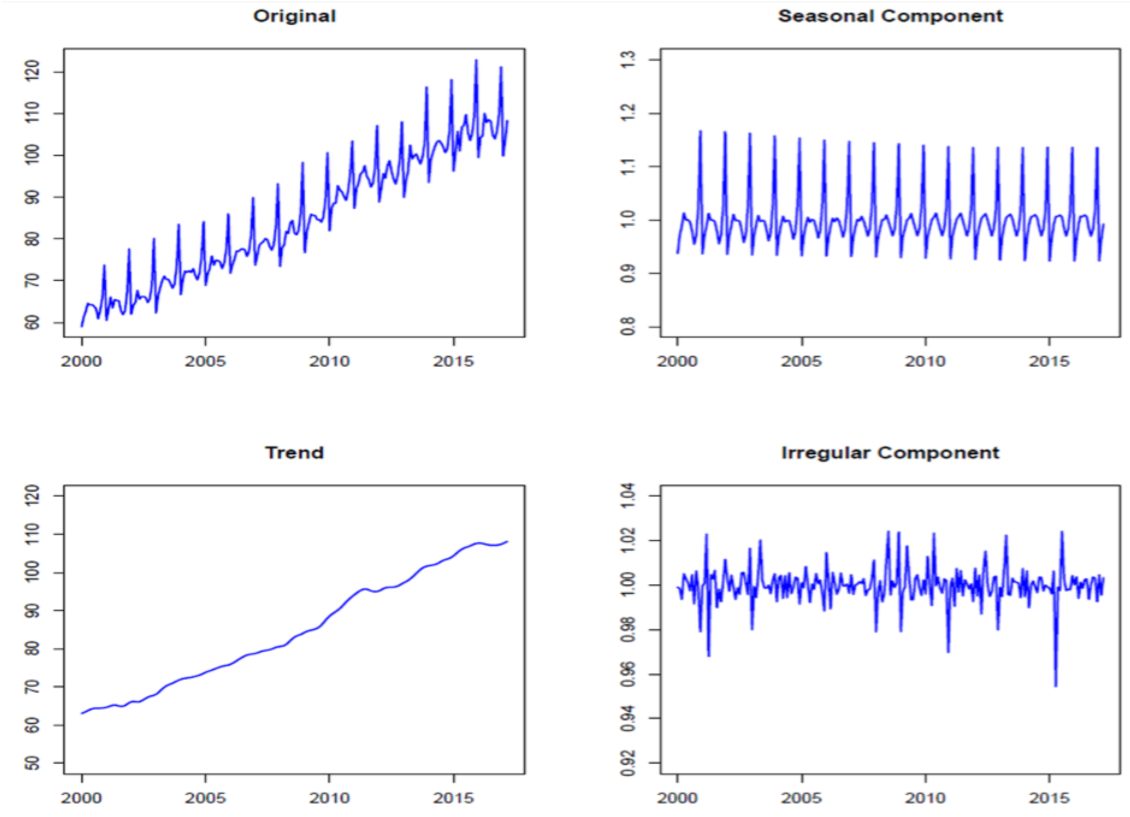

Figure 1.2: Components of a time series, with multiplicative decomposition

1.3 THE SEASONAL ADJUSTMENT PROCESS

Although there are many ways in which these components could fit together in a time series, in practice we use one of two models:

1. Additive model: $Y_t = C_t + S_t + I_t$
2. Multiplicative model: $Y_t = C_t \times S_t \times I_t$

where Y_t is the observed original time series at time point t, C_t is the trend-cycle, I_t is the irregular component, and $S_t$is the seasonal component and the calendar effects. There are occasions when calendar effects might need to be removed separately; in those cases the notation can be extended in a natural way, with a redefinition of S_t to be just the seasonal component. The seasonally adjusted series is formed by estimating and removing the seasonal and calendar effects. The seasonally adjusted series are

1. Additive model: $Y_t - S_t = C_t + I_t$
2. Multiplicative model: $\frac{Y_t}{S_t} = C_t \times I_t$

In a multiplicative decomposition, the seasonal effects change proportionately with the trend. If the trend rises, the seasonal effects increase in magnitude, while if the trend moves downward the seasonal effects diminish. In an additive decomposition the seasonal effects remain broadly constant regardless of which direction the trend is moving in. In practice most economic time series exhibit a multiplicative relationship and hence the multiplicative decomposition often provides the best fit. A multiplicative decomposition cannot be used in its most basic form if any zero or negative values appear in the time series, however it could be used with the temporary addition of a constant value to the time series.

1.4 OTHER ISSUES THAT AFFECT SEASONAL ADJUSTMENT

There are a variety of issues that can impact on the quality of the seasonal adjustment. These include:

- Extreme values (usually modelled as additive outliers). These usually have identifiable causes, such as strikes, war, or extreme weather conditions. They are normally considered to be part of the irregular component
- Trend breaks (usually modelled as level shifts) where the trend component suddenly increases or decreases sharply. Possible causes include changes in definitions relating to the series that is being measured for example, to take account of a reclassification of products or a change in the rate of taxation
- Seasonal breaks are sudden and sustained changes in the seasonal pattern. An example would be the change in the seasonal pattern of car sales when the registration numbers moved from an annual to six-month cycle

These issues need to be addressed as part of the seasonal adjustment process in order to obtain the most reliable estimate of the seasonal component.

1.5 INTERPRETING TIME SERIES OUTPUTS

Current practice at ONS is to publish the original non-seasonally adjusted and seasonally adjusted estimates. In ONS publications the focus is usually on the level of the SA series and the period-to-period change or rate of change in the level of the SA series. A comparison with the same period in the previous year is also often published, but this gives a historical picture of the growth rate and can result in a delay in the identification of turning points in the series. A very crude form of seasonal adjustment is to consider the growth rate from the same period in the previous year for the non-seasonally adjusted series, and sometimes it is assumed that this growth rate would be the same for the seasonally adjusted and the original unadjusted series. However, this will only be the case if the seasonal component is stable and there are no other calendar related effects. In practice seasonality often evolves over time, and the seasonal factors should reflect this. Currently the measure of Year-on-Year (YY) change recommended by ONS is the YY change in the seasonally adjusted series, as this takes into account the impact of a change in seasonal patterns over time and calendar effects.

In general, annual totals of seasonally adjusted estimates will not sum to the annual totals of the non-seasonally adjusted estimates. This is caused by a various factors, including moving seasonality, incomplete cycles, calendar effects and outlier treatment. Annual totals of the seasonally adjusted series can be constrained to annual totals of the unadjusted series. For more information [5].

In addition to removing seasonality, it may also be of interest to remove the impact of the irregular component from a time series in order to produce a trend (also known as a trend-cycle, or a short-term trend)[6].

Seasonally adjusted and trend estimates are subject to revision as additional raw data become available. As the seasonal component cannot be directly observed, it is estimated and these estimates can change as new data points are added to the series. These revisions are an important part of the process, and revisions should be incorporated into the production process to ensure appropriate interpretation of the seasonally adjusted and trend estimates.

1.6 X-13ARIMA-SEATS

X-13ARIMA-SEATS was developed by United States Census Bureau in 2012. The program broadly runs through the following steps:

- The series is modified for user-defined prior adjustments
- The program fits a regression+ARIMA (regARIMA) model to the series in order to: detect and adjust for outliers and other distorting effects, to improve forecasts and seasonal adjustment; detect and estimate additional components such as calendar effects; and extrapolate forwards (forecast) and backwards (backcast) an extra one to three years of data

5 see Chapter 5
6 currently ONS does not publish trends

- The program decomposes the modified series to estimate and remove the seasonal component. X-13ARIMA-SEATS includes two methods for decomposing the modified series:
  - The X-11 method is a non-parametric approach that uses a series of moving averages to decompose the time series into the three components (trend, seasonal and irregular). It does this in three iterations, successively improving the estimates of the three components and producing a wide range of diagnostic statistics to describe the final seasonal adjustment
  - The SEATS method is a model-based approach that decomposes the identified ARIMA model into models for components, and also produces a wide range of diagnostic statistics.

1.7 KNOW YOUR SERIES

In addition to being able to use a seasonal adjustment program and understand the outputs, a proficient user should have an appreciation of factors that are likely to affect the series being seasonally adjusted. Knowing the background to a series will give clues as to where to look for likely problems. Some examples follow:

- Is the way in which the data are collected likely to lead to any unusual effects? Are they collected on a non-calendar basis, or is there a lag between the activity being measured and when this is recorded?
- Has there been any change to the method or timing of data collection? This may cause trend or seasonal breaks
- Is the series likely to be affected by trading days or Easter effects?
- Have there been any events which are likely to cause breaks in the series or large outliers? These could include war, the budget moving from March to November, Britain dropping out of the Exchange Rate Mechanism, extreme weather, industrial disputes, or other events which may affect individual series.

2 OVERVIEW OF X-13ARIMA-SEATS

2.1 INTRODUCTION

When seasonal adjustment is embedded in to the production process, producers of official statistics can regard seasonal adjustment as a black box process; raw data go in and seasonally adjusted data are automatically derived. This chapter provides an overview of the X-13ARIMA-SEATS software and outlines some of the concepts and methods used to derive seasonally adjusted estimates.

X-13ARIMA-SEATS (2017) is the successor to X-12-ARIMA and represents the next generation of seasonal adjustment software developed by the U.S. Census Bureau. X-13ARIMA-SEATS can perform seasonal adjustment using either the non-parametric X-11 algorithm method or the model-based Signal Extraction in ARIMA Time Series (SEATS) method.

In 2012, the UK Statistical Policy and Standards Committee (SPSC) recommended that X-13ARIMA-SEATS be the standard software for seasonal adjustment in the Government Statistical Service (GSS). This chapter will give an overview of the X-13ARIMA-SEATS concepts and methods. Further resources on the use of X-13ARIMA-SEATS can be found format the website of the U.S. Census Bureau.

2.1.1 *Brief review of X-13ARIMA-SEATS software development*

Before introducing X-13ARIMA-SEATS, we give a brief review of the development of X-13ARIMA-SEATS software in terms of seasonal adjustment methods. The X-11 seasonal adjustment method (Shiskin, Young, and Musgrave, 1965) was developed by the U.S. Census Bureau and has formed the basis for most seasonal adjustment of official statistics in the UK.

The X-11-ARIMA program (Dagum, 1980) is an improved version of the X-11 program and was developed by Statistics Canada. It introduced ARIMA modelling to extend a time series which will often reduce revisions to seasonally adjusted series at the current end.

The next development in the X-11 family of software for seasonal adjustment was X-12-ARIMA. This was developed by the U.S. Census Bureau and uses a modelling strategy called regARIMA (Findley et al., 1998). Algorithms in the software that use regARIMA modelling can be used to identify, estimate and adjust for factors which may distort estimation of the seasonal component. In 2012, the U.S. Census Bureau launched X-13ARIMA-SEATS as a replacement for X-12-ARIMA.

2.1.2 *New features of X-13ARIMA-SEATS*

X-13ARIMA-SEATS combines X-12-ARIMA and SEATS, and enables seasonal adjustment using either the X-11 algorithm or the SEATS decomposition. The new software uses the pre-processing method of X-12-ARIMA (regARIMA modelling) and incorporates both the X-11 algorithm of X-12-ARIMA and the SEATS decomposition of TRAMO-SEATS[1]. As the main pre-processing method of X-12-ARIMA is largely based on Time Series Regression with ARIMA Noise, Missing Observations and Outliers (TRAMO), the results of running the SEATS decomposition in X-13ARIMA-SEATS will often be close to the results of running TRAMO-SEATS.

The fundamental difference between the X-11 algorithm and the SEATS decomposition is that X-11 is (essentially) non-parametric, while SEATS is parametric. The use of models in SEATS means that standard errors are available for the seasonally adjusted series (and indeed for all components of the time series), unlike the filter-based X-11 algorithm.

2.2 METHODS IN X-13ARIMA-SEATS

The seasonal adjustment process is summarised by the flow chart in Figure 2.1

2.2.1 *regARIMA models – prior adjustment*

X-13ARIMA-SEATS uses the pre-processing method of X-12-ARIMA which is largely based on TRAMO and uses regARIMA models. This approach models the time series and estimates prior adjustments before seasonal adjustment is carried out. For example, regARIMA models are able to adjust for outliers and other distorting effects to improve the forecasts and estimation of the components of the time series. The regARIMA model can be thought of as a straightforward regression model but one in which the error term is appropriately modelled to deal with the correlation structure that is often present in time series. The problem of autocorrelation in a time series for ordinary least squares is that it breaks the independence assumptions, which can lead to invalid inference. The ARIMA part of the model is therefore important for making informed decisions on the inclusion of regression variables.

1 see Chapter 25 for details

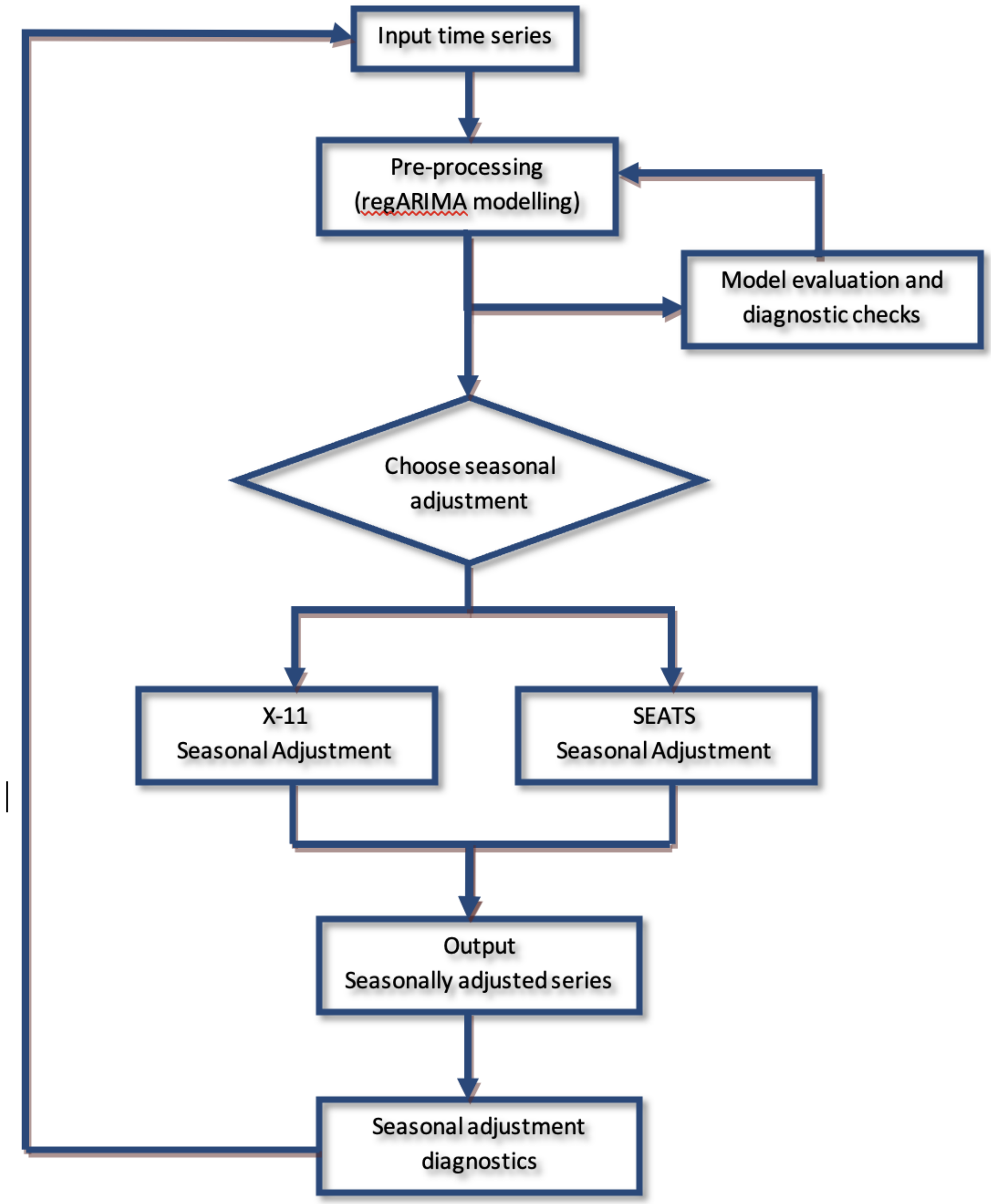

Figure 2.1: Seasonal adjustment in X-13ARIMA-SEATS

There are a range of built-in types of regression variables available in X-13ARIMA-SEATS including:

- Outlier and trend change effects: additive (or point) outliers, Temporary Change (TC) outliers, level shifts, ramps

- Seasonal effects: calendar month indicators, trigonometric seasonal (sines and cosines)
- Calendar effects: trading day (flows or stocks), leap-year February, length of month, moving holidays (eg. for example, Easter)
- User-defined effects

Diagnostics to evaluate the fit of the model or the significance of the regression variables are available in the X-13ARIMA-SEATS output. More information on the regARIMA model can be found in Chapter 8.

2.2.2 *Seasonal adjustment – the X-11 Method*

X-13ARIMA-SEATS can use the X-11 method to perform seasonal adjustment. The X-11 method uses a series of moving averages for seasonal adjustment. The following is a brief outline of the X-11 method. It provides an overview of how the original series Y_t, can be decomposed into a trend-cycle C_t, a seasonal component S_t and an irregular component I_t.

Trend-cycle C_t: is defined as the underlying level of the series. It is a reflection of the medium-long term movement in a series and it is typically the result of influences such as population growth, general economic development and business cycles. It refers to the generally smooth deterministic movement in a time series.

Seasonal component S_t: includes variations which repeat approximately periodically with a period of one year and which evolve more or less smoothly from year to year. Calendar variations, such as Easter effects, are also included within the seasonal component.

Irregular component I_t: contains those parts of the time series that cannot be predicted and are effectively the residual component after the identification of the trend and seasonal components. It may include sampling errors and unpredictable events like strikes and floods.

The default decomposition of a time series in X-13ARIMA-SEATS is the multiplicative decomposition; $Y_t = C_t \times S_t \times I_t$. An alternative decomposition is the additive decomposition where all the components are related additively. In the multiplicative model, S_t and I_t are scale-free numbers varying about a level of 1 (or 100%); in the additive model S_t and I_t are in the same units as the original series and vary about a level of zero[2].

The X-11 process can be described by the following sequence of steps[3].

Assuming that there is a multiplicative relationship between the components, the steps are as follows:

2 for more information on the choice of decomposition see Chapter 12
3 for a full description see Ladiray and Quenneville (2001)

1. A preliminary trend-cycle (C'_t) is obtained by applying a trend moving average to the original series (Y_t)

2. This initial estimate of the trend is removed from the original estimate to give a de-trended time series denoted by: $SI_t = S_t \times I_t = Y_t(C'_t)$

3. Extreme values are then identified by an automatic process and replaced in the SI_t time series

4. A seasonal moving average is then applied to the modified SI_t time series for each month (or quarter) separately to give a preliminary estimate of the seasonal component $\hat{S}_t$

5. Dividing Y_t by $\hat{S}_t$ gives a preliminary seasonally adjusted series, $SA'_t = \frac{Y_t}{\hat{S}_t}$

6. This process is then repeated, using a Henderson moving average (Henderson 1916) to estimate the trend-cycle in step 1

For an additive decomposition, the division in step 2 and step 5 is replaced with subtraction.

The detailed steps of the X-11 seasonal adjustment process mentioned above are described as follows.

Step 1: Preliminary estimate of the trend

Moving averages can have a smoothing effect and some of these can remove seasonality in data. These are applied to the original series to give a preliminary estimate of the trend. For a monthly series a 2x12 moving average is applied while for a quarterly series a 2x4 moving average is used. Moving averages are described in more details in Chapter 13. A 2x12 moving average is centred at the month t and uses the previous six months, the current month (t) and the subsequent six months. The weights of the moving average are symmetrical. The same weight is given to months which are the same distance from the current month. The effect of a 2x12 average on a monthly series is to remove stable annual variation and reduce purely random variance by a factor of 12.5 while leaving any linear trend unchanged.

Step 2: estimate the seasonal and irregular components

Once the preliminary estimate of the trend C_t is known, it is possible to estimate the remaining SI_t component. This is done by dividing the original data (Y_t) by the first estimate of C_t if a multiplicative model ($Y_t = C_t x S_t x I_t$) is used or by subtracting C_t from the original data if an additive model ($Y_t = C_t + S_t + I_t$) is used. The Irregular to Seasonal Ration (also I/S) (SI) may be plotted for each quarter/month separately. Note that they are presented in the output from the software as percentages not ratios for a multiplicative decomposition and differences for an additive decomposition. We will refer to them as SI ratios. Figure 2.2 shows an example of SI ratios for January.

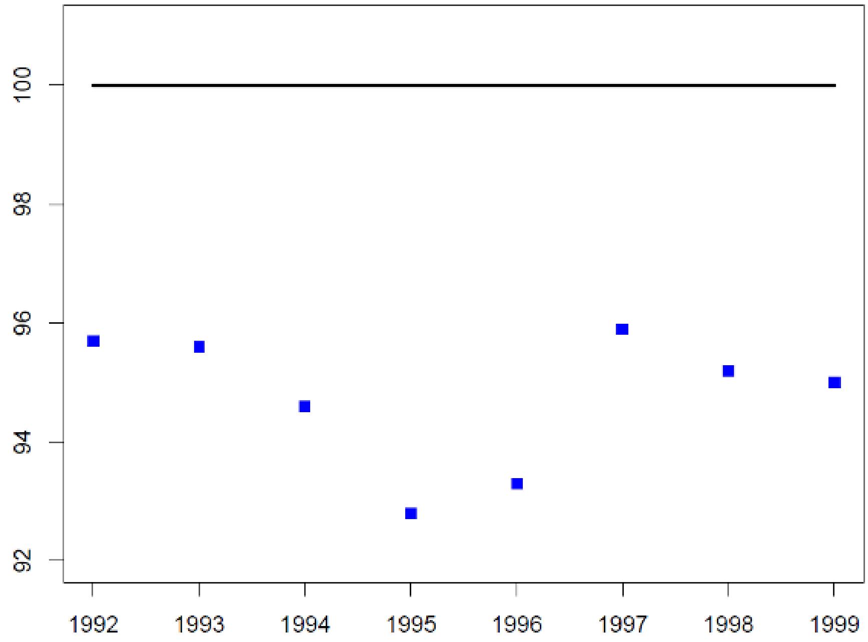

Figure 2.2: January SI ratios for retail sales in non-specialised stores (predominantly food)

The 100% line at the top of the graph represents the new trend of the series once C_t has been removed from Y_t. This diagram clearly shows that the January SI values are all below trend, a typical example of a seasonal effect. The graph of the SI ratios is a useful tool that should be used in analysing the seasonal behaviour, especially to identify breaks in the series.

Step 3: Extreme values are then identified and replaced in the "SI" series

X-13ARIMA-SEATS identifies and temporarily replaces outliers in the SI graphs for each month. This reduces any distortion of the seasonal adjustment. In Figure 2.3 the outliers are temporarily replaced by the points shown in pale blue.

The routine to identify and replace outliers is used several times to get progressively better estimates of the size of the outliers. The following steps are used:

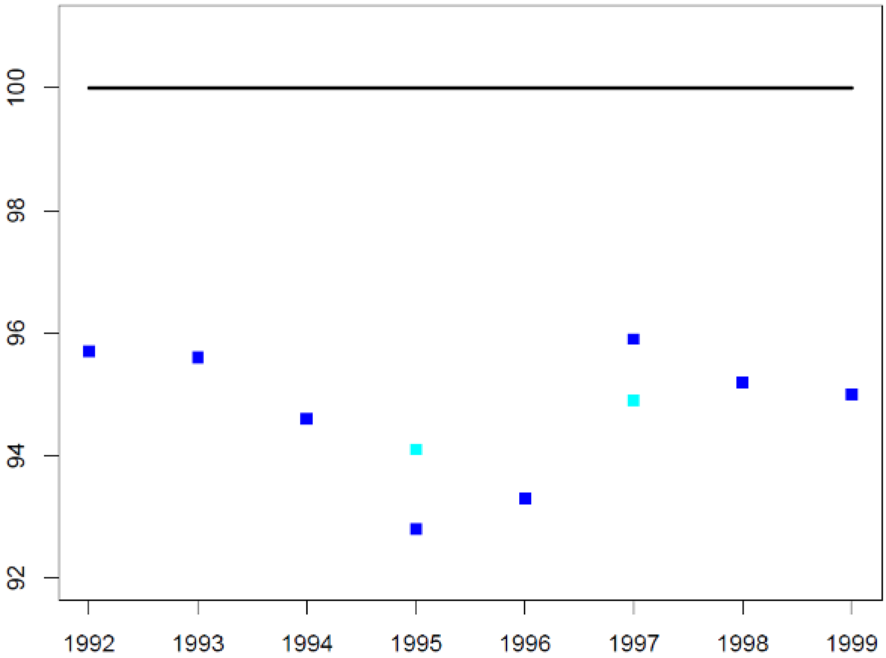

Figure 2.3: January SI ratios with replacement values

- A rough estimate of the irregular is made, from the combined seasonal-irregular component
- A standard deviation is calculated for each 5 year moving span of the irregular component and used as the standard deviation for the middle year of the five. For the first two and last two years the standard deviation of the nearest available five year span is used
- Where any irregular is more than 1.5 standard deviations from zero (or from 100% for a multiplicative model), the SI value for that point is considered extreme and partially or fully replaced. The extreme value is given a weight which depends upon how extreme the irregular is, as shown in Figure 2.4. The default boundaries on the standard deviation (known in the software as sigma limits) are 1.5 and 2.5. These limits can be changed by the user.

The modified SI ratios are used for estimating the seasonal component so that extreme values do not distort the estimate. However, outliers remain in the final seasonally adjusted estimate.

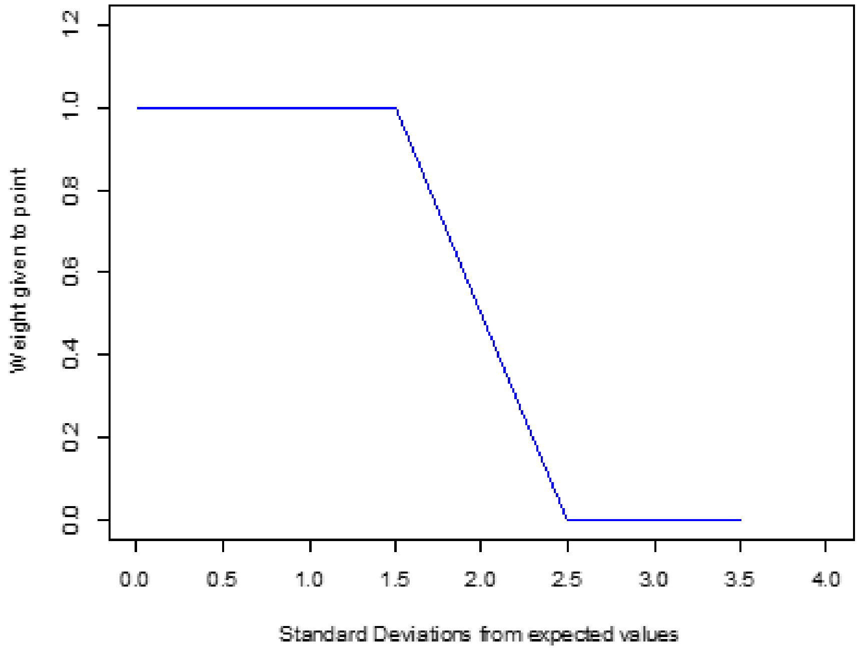

Figure 2.4: Outlier identification and weighting

Step 4: A seasonal moving average is applied to the modified SI series for each month/quarter separately to give a preliminary estimate of S and hence I

A seasonal moving average (a 3×3 in the first part of iteration B and a 3×5 in the second part of iteration B) is then applied to the SI series to estimate the seasonal factors. If the derived moving averages represent the seasonal component, the irregular component is defined by the deviation of each point from the moving averages.

This process is repeated for each month/quarter. An example of the January SI ratios and moving average values is shown in Figure 2.5.

Figure 2.5 illustrates the results of the application of a 3x3 moving average (pale blue line) to the modified January SI ratios. It shows the graphical representation of the separation of the seasonal and irregular components.

Step 5: Dividing Y by S gives a preliminary seasonally adjusted series, denoted SA1

After the seasonal factors have been estimated, the seasonally adjusted series can be derived. The original series Y_t is divided by S_t to give a preliminary seasonally adjusted series $C_t \times I_t$ (if a multiplicative model is used). In case of an additive model, the original series Y_t minus the seasonal component S_t gives an estimate of the seasonally adjusted series

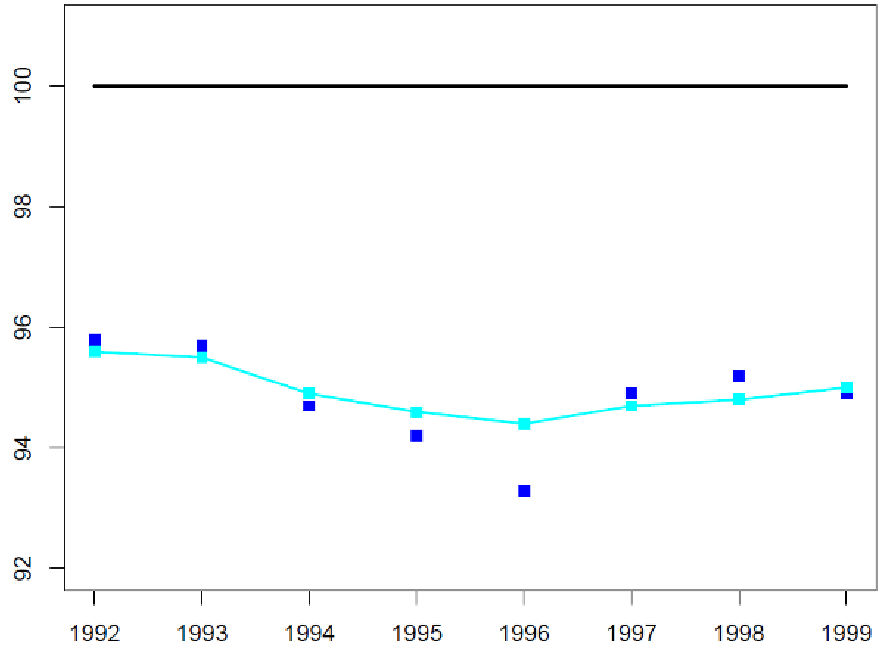

Figure 2.5: January SI ratios with moving average values

$C_t + I_t$. This seasonally adjusted series is referred to as SA1 since it is just the result of the first iteration.

Step 6: Repeat seasonal estimation with improved trend estimates

This process from steps 1 to 5 is then repeated, but this time the trend is estimated from the SA1 series by applying a Henderson moving average (such moving averages are very good at estimating trends, but can only be used on series which do not exhibit seasonality).

Two more iterations X-11: parts C and D

The output of the steps described above is not regarded as the final set of estimates. X-13ARIMA-SEATS carries out this entire process (that is i.e steps 1 to 6, including two estimations of the seasonal component) twice more. For these estimations, however, the starting point is not the raw series Y_t but a modified Y_t with extremes removed or reduced. The final output of the third iteration (Part D) is the final set of time series estimates.

2.2.3 *Seasonal adjustment - the SEATS method*

X-13ARIMA-SEATS can also use the SEATS method to perform seasonal adjustment. In contrast to the non-parametric X-11 adjustments, SEATS is a model based methodology for seasonal adjustment and is used for decomposing a time series in to trend-cycle, seasonal, transitory and irregular components. The SEATS method was developed from the work of Cleveland and Tiao (1976), Box, Hillmer, and Tiao (1978), Burman (1980), Hillmer and Tiao (1982), Bell and Hillmer (1984), Maravall and Pierce (1987) and Maravall (1988) in the context of seasonal adjustment of economic time series. The TRAMO-SEATS software has been developed primarily by Gómez and Maravall (2001) of the Bank of Spain.

X-13ARIMA-SEATS uses the pre-processing method of X-12-ARIMA, which is largely based on TRAMO. After prior adjustment, the Auto Regression (AR) and Integrated (I) terms are allocated to ARIMA component models. The AR process relates recent values of trend and seasonality to past values. Terms are allocated according to their magnitude and frequency. If the effect is large enough and at the correct frequency, the effect is allocated to the model for trend or seasonal, otherwise it is allocated to the model for the transitory component. Identifying Moving Average (MA) orders of component models via partial fraction decomposition is a difficult part of the process. The required condition for this process is that variances of components cannot be negative ("admissible" decomposition). One of the problems is that no unique admissible decomposition can distribute noise amongst components in any way. The solution for this is to allocate all white noise to the irregular component ("canonical" decomposition), then components can be estimated from their models. The main method is to use standard signal extraction techniques, for example, the Wiener-Kolmogorov filter. The filter is symmetric which requires forecasts (and backcasts) of observed series and allows standard errors of components to be obtained (confidence intervals).

2.3 DIFFERENCES BETWEEN X-12-ARIMA AND X-13ARIMA-SEATS

The inclusion of the SEATS method of seasonal adjustment is the only substantial change between X-12-ARIMA and X-13ARIMA-SEATS.

However, results from running the X-11 algorithm in X-12-ARIMA and X-13ARIMA-SEATS may differ because of additional functionality. The extra functionality and improvements include:

- new **spectrum** spec with options to – change criteria for identifying peaks, change span, control print and save options
- new regressors – stock calendar regressors (constrained stock TD, end-of-month stock Easter), and up to five groups of user-defined holiday regressors
- extension of regressor set tested by – **aictest=td** in the regression spec from {no effect, **td**} to {no effect, **td**, **td1coef**}, **types=all** in the outlier spec from {**ao**, **ls**, **tc**} to {**ao**, **ls**, **tc**, **so**} where so is a seasonal outlier

- optional lognormal adjustment for forecasts
- new diagnostics – model-based F-tests for stable seasonality (more reliable than M7) and for trading day, Akaike Information Criterion Corrected (AICC) test for length of month and leap year regressors
- improved performance of sliding spans and history specs in SEATS, and incorporation of HTML output from SEATS.

2.4 EXAMPLE: DECOMPOSING THE ORIGINAL TIME SERIES

Figure 2.6 shows an original series and the trend-cycle, seasonal and irregular components. The purpose of seasonal adjustment is to estimate and remove the seasonal component from the original data to give seasonally adjusted estimates.

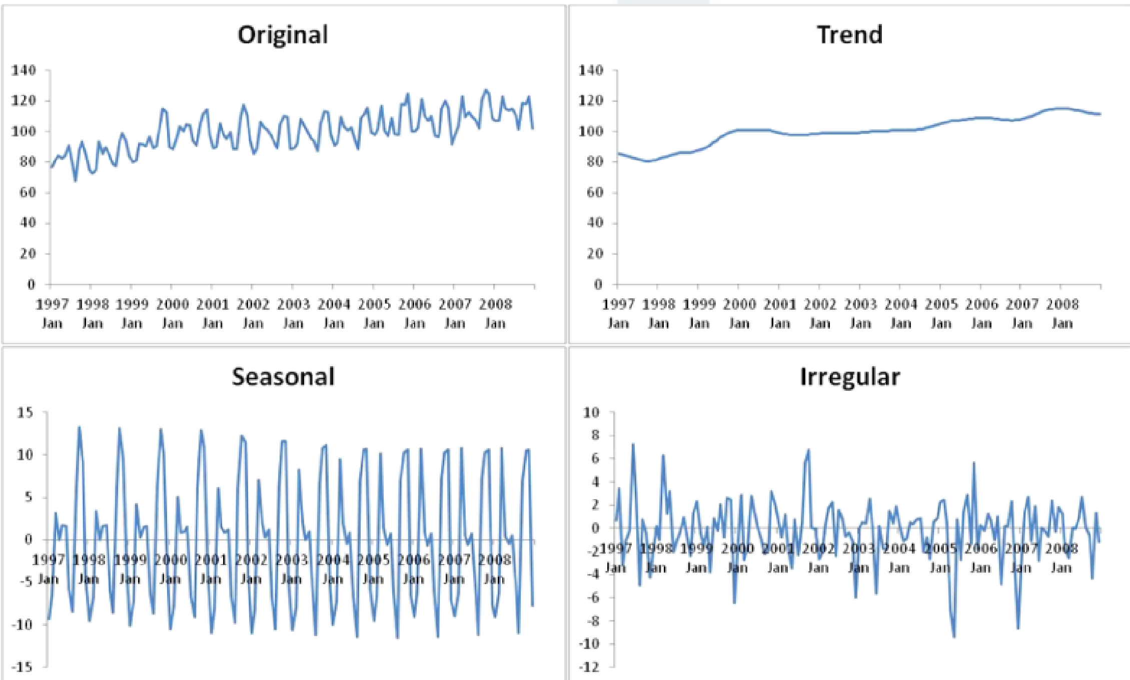

Figure 2.6: Graphical representation of time series components

2.5 DIAGNOSTIC CHECKING

As indicated in the flowchart in Figure 2.1 diagnostic checking is important at the pre-processing stage and following seasonal adjustment. Both the regARIMA modelling and the X-11 or SEATS decomposition should be checked by the user for accuracy and improved where necessary. There should not be any residual seasonal or calendar effects in the published seasonally adjusted series or in the irregular component. If there are residual seasonal or calendar effects, as indicated by relevant diagnostics, the model and regressor options should be checked in order to remove seasonality. Note that the issue of residual seasonality equally applies to directly and indirectly seasonally adjusted estimates[4].

Even if no residual effects are detected, the adjustment may be unsatisfactory if the seasonally adjusted estimates are subject to large revisions when they are recalculated as new data become available. Such instability should be measured and checked. X-13ARIMA-SEATS includes two types of stability diagnostics: sliding spans and revisions history

X-13ARIMA-SEATS provides many useful diagnostics that are discussed in more detail in subsequent chapters of this guide.

4 see Chapter 6

3 HOW TO USE X-13ARIMA-SEATS WITH WIN X-13

3.1 DOWNLOADING AND INSTALLING X-13ARIMA-SEATS AND WIN X-13

The files for X-13ARIMA-SEATS and the front end Win X-13 can be downloaded from the website of the U.S. Census Bureau.

X-13ARIMA-SEATS can be run in an interactive file editor such as Win X-13 or through other software which can call the X-13ARIMA-SEATS executable, for example R. It is recommended to use X-13ARIMA-SEATS through Win X-13 as Win X-13 automatically generates diagnostic summaries and graphical output.

Creation of spec files, running of X-13ARIMA-SEATS and analysis of the output can all be performed using Win X-13. Win X-13 is a commonly used graphical user interface for X-13ARIMA-SEATS.

NOTE: The installation of programs to government department computers is governed by departmental policy which must be adhered to.

3.2 USING X-13ARIMA-SEATS WITH WIN X-13

Detailed instructions on how to use X-13ARIMA-SEATS are available from the X-13ARIMA-SEATS Reference Manual. The following is a condensed guide. To use X-13ARIMA-SEATS you will need an executable file. Two additional files must also be created. It is often simpler to have these in the same folder but not necessarily the same folder as the folder with the executable file:

1. One file containing the data that are to be seasonally adjusted using X-13ARIMA-SEATS named, for example, "myfile.dat".

2. The second file contains the instructions to seasonally adjust the data series, and is known as the specification file or spec file. It is given a name such as "myfile.spc".

Both files can be in text format. They can easily be created using either the Notepad text editor or, in the case of the spec file, WinX13 (the front end to the X-13ARIMA-SEATS software). Once these files have been created, X-13ARIMA-SEATS can be used to perform seasonal adjustment.

3.2.1 *The data file*

The data file is simply the raw data that have to be seasonally adjusted. The data can be in a number of different formats as described in the user manual X-13ARIMA-SEATS (2017). Below we briefly describe **free** format, **datevalue** format, and **x13save** format.

Free format

This is a simple format. It comprises a single column of data with new numbers of a new line. There are no labels or headers in this format. An example of **free** format data is given below:

```
89814.18
85582.54
85059.04
85880.88
97711.15
78266.99
84479.21
79941.65
```

If the data are in free format, then a **start** statement with the start date of the **series** is required in the series specification.

Datevalue format

A file in **datevalue** format includes dates. The delimiter that separates each field may be a tab or a space. An example of **datevalue** data is provided below. In this example the delimiter between the year and the month, and between the month and the value, is a tab.

```
1991	1	89814.18
1991	2	85582.54
1991	3	85059.04
1991	4	85880.88
1991	5	97711.15
1991	6	78266.99
1991	7	84479.21
```

In this case the **start** argument in the series specification is not required. Labels are not required within the data file, but they are permitted.

x13save format

A file in **x13save** format includes dates. This may also be referred to as **x12save** format. A file may only be produced in **x13save** format by X-13ARIMA-SEATS. An example of **x13save** data is given below:

date	mydata.a1
199101	8.98E+04
199102	8.56E+04
199103	8.51E+04
199104	8.59E+04
199105	9.77E+04
199106	7.83E+04
199107	8.45E+04
199108	7.99E+04

In this case the **start** argument in the **series** specification is not required. Labels are not required within the data file, but are allowed.

3.2.2 *The specification file*

To use X-13ARIMA-SEATS a command file called the specification file or spec file, must be created. A spec file is a text file used to specify program options. The name of the spec file must end with a ".spc" extension. A spec file contains a set of functional units called specifications or specs that X-13ARIMA-SEATS reads to obtain required information for the time series, such as the decomposition model to be used, the analysis to be performed and the desired output. Each spec controls options for a specific function. For example, the **series** spec specifies the original data to be input and the span to be used in the analysis, whereas the transform spec specifies any **transformation** and any prior adjustments to be applied.

An X-13ARIMA-SEATS spec file must begin with a **series** or a **composite** spec[1], whilst the other spec can be entered in any order. Note that not all specs are required in a spec file. A general input syntax of a spec file is presented below:

```
specificationname{
```

1 for more information about the composite spec see Chapter 20

```
   argument1 = value
   argument2 = (value1 value2 value3)
   argument3 = "A string value"
   argument4 = 2003.9 # The dates 2003.9 and 2003.SEP are equivalent
       for monthly series
   }
# This symbol is followed by a comment that is not meant to be
    executed.
```

When writing a spec file, one should note that:

- Dates are in the format *yyyy.period* (for example 2003.9 or 2003.*sep*)
- If an argument has 2 or more values, these must be enclosed in parentheses
- Character values, such as titles and file names, should be enclosed in quotation marks
- Everything on the line following the "#" symbol is treated by X-13ARIMA-SEATS as a comment
- The maximum number of characters read on a line is 132

Arguments can be set within each spec. Arguments define the function that X-13ARIMA-SEATS uses, and if an argument is not specified, a default value is assigned. Arguments can be written in any order within the spec, using upper, lower, or mixed case as X-13ARIMA-SEATS is not case sensitive. The spec file also includes the name and the path of any optional files containing data for the time series being modelled, data for user-defined and predefined regressors, values for any user-defined prior adjustments, and model types to try with the automatic model procedure. These names and paths are listed in appropriate specs. An example of a simple spec file is shown below:

```
series{
   title =  "Example Specification"
   start =  1994.1
   period = 4
   name = "Default"
   file = "c:\research\test.dat"
   }

transform{
   function = log
   file = "c:\research\testtp.tpp"
```

```
   type = temporary
   }

arima{
   model = (0,1,1)(0,1,1)
   }

regression{
   aictest  = (Easter)
   }

x11{}
```

The above spec file includes the **series**, **transform**, **arima**, **regression** and **x11** specifications. The series spec provides X-13ARIMA-SEATS with a file name and path, indicating where the original data are stored. The **transform** spec takes the logarithms of the series, and provides the name of a file containing the temporary priors to prior adjust the series. The file specifies an ARIMA (0 1 1)(0 1 1) model, using the **arima** spec. The **regression** spec tests the significance of regression variables, known or suspected to affect the series, as indicated in the regression specification. Here {aictest=(Easter)} tests whether an Easter variable should be included within the regARIMA model specified. Finally, the **x11** spec generates seasonal adjustments from the default selection of seasonal and trend filters. To include additional options in a specification file, please refer to appropriate chapters in this guide or for comprehensive details see the X-13ARIMA-SEATS (2017) or contact TSAB for further guidance.

3.3 EXAMPLE OF USE OF X-13ARIMA-SEATS WITH WIN X-13

In the following example we will use the time series of visits abroad made by UK residents over the period January 1986 to August 2013 inclusive from the International Passenger Survey. The series is saved into a text file in **free** format; that is only the values are saved with no headers or labels. The data are copied and pasted into a Notepad document and then saved in the working folder (here the My Documents folder) as "UKVisitsAbroad.dat". Note that the quotation marks are used so that the file can be saved directly as a ".dat" file rather than first saving as a Notepad file and then changing the file extension to ".dat".

Now that the data file has been saved in the correct format the spec file must be written. Open Notepad and write the settings as below. Notice that the spec file is saved as "UKVisitsAbroad.spc", by saving the file using the double quotation marks, the format can be set as ".spc" the correct format for spec files.

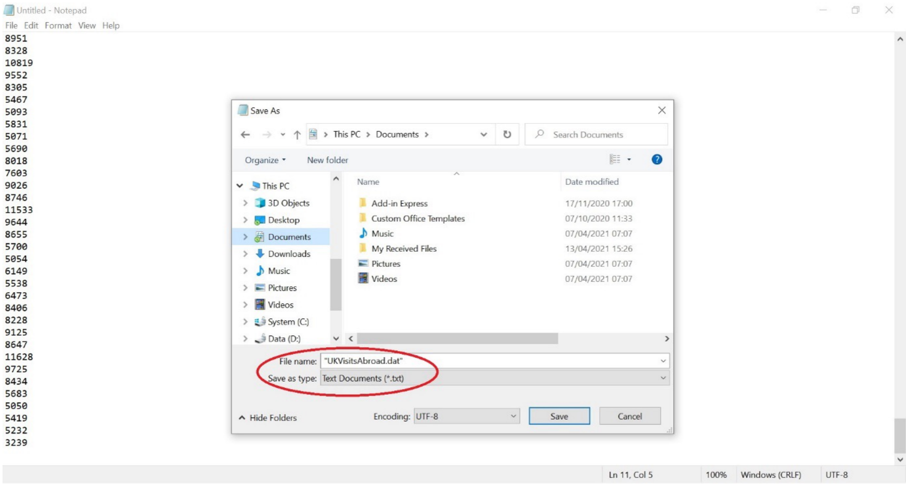

Figure 3.1: Saving the ".dat." file

The spec file used in this example is shown in Figure 3.2. The series spec tells X-13ARIMA-SEATS where the ".dat" file is, what format the file is and when the series starts. The transform spec tells X-13ARIMA-SEATS to choose the decomposition automatically. The regression and outlier specs tell X-13ARIMA-SEATS to test, and were found significant, to adjust for prior adjustments. The pickmdl spec chooses the best ARIMA model from the list of ARIMA models in the "x12.mdl" file. Alternatively, the automdl spec can be used. The x11 spec performs the seasonal adjustment and history and slidingspans specs perform diagnostic checks. For more information on specs see section 7 of the X-13ARIMA-SEATS manual (USCB 2017)

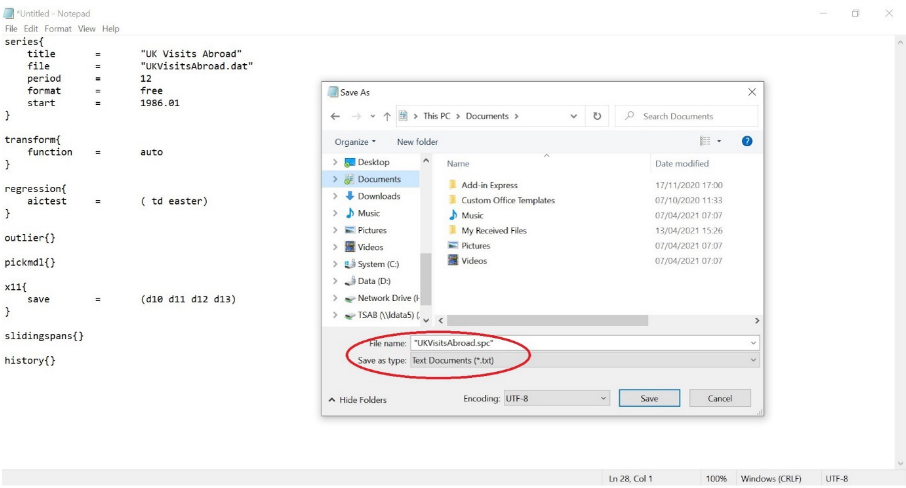

Figure 3.2: Automatic spec file

Once the spec file has been written and saved, Win X-13 can be opened to begin the seasonal adjustment process. Once Win X-13 has been opened, navigate to the working folder and select the spec file. Also ensure that the option to view output files and diagnostics is selected. It is also useful to run Win X-13 in graphics mode.

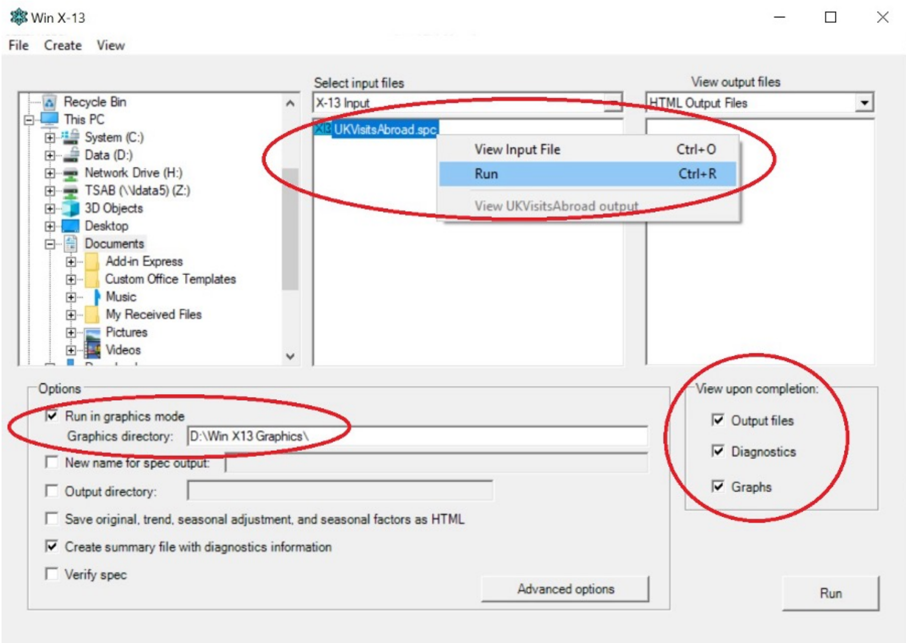

Figure 3.3: Running the automatic spec file

Once the spec file has been right clicked and the option “Run” selected, X-13ARIMA-SEATS will perform the automatic seasonal adjustment and provide charts, output and diagnostics. These are shown below from Figure 3.4 to Figure 3.6.

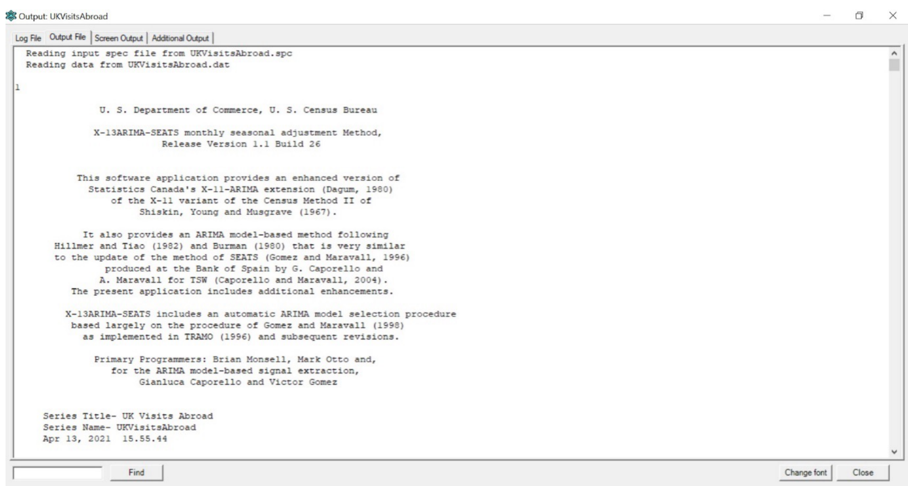

Figure 3.4: Output file

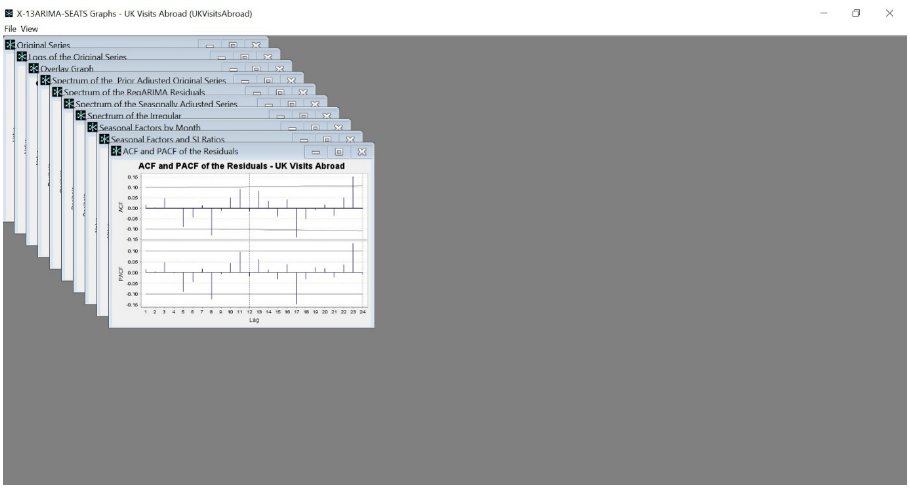

Figure 3.5: Charts for automatic spec file

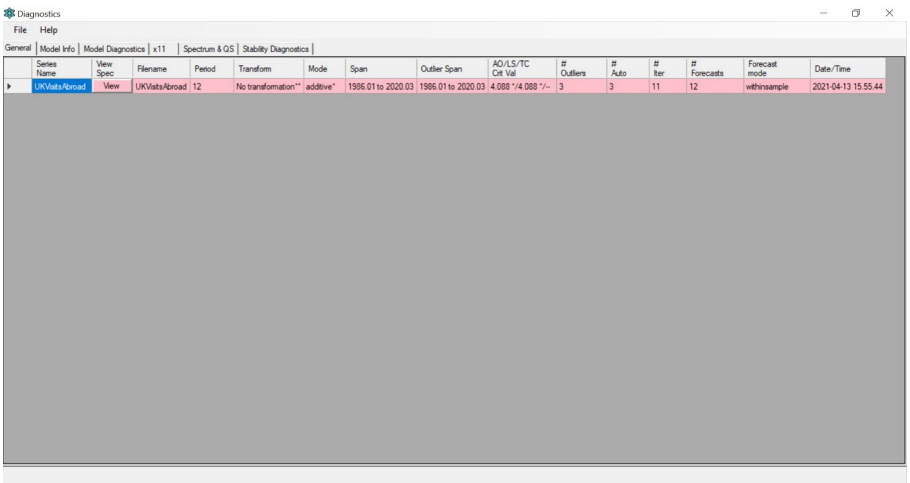

Figure 3.6: Diagnostics for automatic spec file

The automatic spec file recommends performing seasonal adjustment on this series using:

- an additive decomposition (General tab in the diagnostics window)
- (0 1 1)(0 1 1) ARIMA model (not shown in figure Figure 3.6 but found in the "Model info" tab in the diagnostics window)
- prior adjustment for Easter (Easter[15]) and trading days (td1coef) (not shown in Figure 3.6 but found in the "Model info" tab in the diagnostics window)
- a 3x5 seasonal moving average and a 13-term trend moving average (not shown in Figure 3.6 but found in the "x11" tab in the diagnostics window).

To fix these settings open the spec file and change the individual specs as below. The changes are highlighted. The user should check the diagnostics to determine whether the seasonal adjustment is performing adequately and make any necessary adjustments. Once adjustment have been made and the quality of the seasonal adjustment is adequate, this spec file is now ready for use in production for the next 12 months until the next seasonal adjustment review is due. If there are any changes to or shocks in the series before the next scheduled review, the parameters should be reviewed to reflect this.

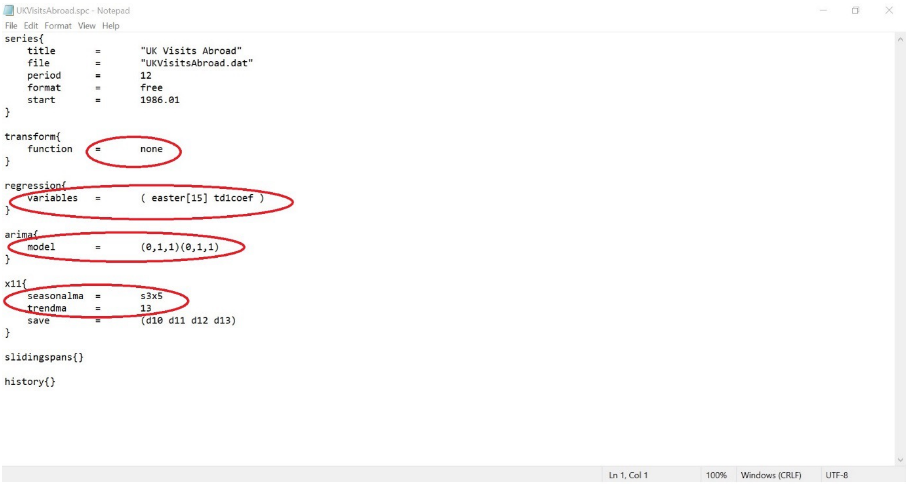

Figure 3.7: Final spec file

3.4 SUMMARY

- The seasonal adjustment program, X-13ARIMA-SEATS, and the software front end, Win X-13, can be downloaded from the website of the U.S. Census Bureau
- A spec file can be used to specify parameters for seasonally adjusting a time series (advice on setting parameters is provided in the remainder of this guide).

4 LENGTH OF THE SERIES

4.1 INTRODUCTION

Any series of more than three years in length can be seasonally adjusted by X-13ARIMA-SEATS. However, if the series is very short, the program does not have a large number of data points to work with, so it may have problems finding significant evidence of, for example, seasonality, Easter effects or trading days. Very short series are likely to have large revisions as new data have the capacity to greatly change the estimates of the seasonal factors. On the other hand, a very long series will have data which will not relate to the pattern of the current end of the series. For example, how much would 1980s data reflect the current pattern? What new information would it give which would still be accurate?

4.2 HOW LONG A TIME SERIES DOES X-13ARIMA-SEATS NEED?

X-13ARIMA-SEATS has a few absolute minimum requirements for certain functions to work:

- 3 years of data is the minimum for X-13ARIMA-SEATS to model or do any seasonal adjustment
- 5 years and 3 months (5 years and 1 quarter) of data are needed for X-13ARIMA-SEATS to automatically fit an ARIMA model and to calculate correctly all the criteria to test the models (especially the average forecast error for the last 3 years). If there are fewer data points, then the user can impose their own model
- There must be at least 5 years of data including the forecast for the automatic selection of the seasonal moving average to work, and to allow for evolving seasonality. Constant seasonality is used if there is less than 5 years of data
- X-13ARIMA-SEATS uses all of the data available when fitting an ARIMA model unless the **modelspan** argument is used.

4.3 SEASONALLY ADJUSTING SHORT SERIES

Short series are considered to be those where less than 5 complete calendar years of data are available. X-13ARIMA-SEATS will seasonally adjust the series assuming strict stable seasonality, so that the seasonal factors do not change over the span of the series. If a short series needs to be seasonally adjusted, please refer to the following guidelines:

- If less than 3 years of data are available, the seasonally adjusted version of the series should not be published until more observations are available. At least 3 years of data are required for X-13ARIMA-SEATS to operate. It is important to monitor the behaviour of the series, and some personal judgement will probably be required. Use external information or patterns in related series if possible to seasonally adjust the series

- If 3-5 years of data are available, consider not publishing the seasonally adjusted version of the series until more observations are available. X-13ARIMA-SEATS can use ARIMA modelling or provide trading day and Easter adjustments, but they are generally of very poor quality and subject to large revisions as future observations become known. The option of constraining of seasonally adjusted series to the raw annual totals is not available.

4.3.1 *Methods available to seasonally adjust short series*

Three options are available to seasonally adjust short series with 3 to 5 years of data. These are:

1. To use the arima spec in X-13ARIMA-SEATS to forecast and backcast the series in order to have enough data to run the seasonal adjustment
2. To seasonally adjust the actual data with no ARIMA modelling (using x11 spec only)
3. To link the series with another that covers a longer time period and then seasonally adjust the resulting linked series

The ARIMA extrapolation method

The first method is to use the **arima** spec to forecast and backcast the data. Because the automatic model selection operates only with more than 5 years of data, a simple form of model, such as a (0,1,1)(0,1,1) with a log transformation if appropriate, should be fixed in the arima spec. The forecast spec should have 1 to 2 years of maxlead in order to have a series of 5 to 6 years of data. The x11 spec can be run with the automatic selection of the seasonal moving average to make sure that the selected moving average is the most appropriate for the behaviour of the seasonal component. Trading day and Easter regressors can be used to estimate those effects, but it should be kept in mind that the adjustments might be of very poor quality and subject to large revisions as future observations become known.

The X11 spec without ARIMA modelling

The ARIMA extrapolation method has some limitations that need to be taken into account. The use of the (0 1 1)(0 1 1) model means that the forecasts will have a stable seasonal pattern, which will necessarily be the average seasonal pattern of the actual data. Therefore, the effect of adjusting the extrapolated data will be much the same as adjusting the actual data without extrapolation. In this situation, it makes sense simply to use the **x11** spec without any modelling. The only drawback to this method is that it is not possible to use ARIMA modelling to identify outliers or calendar effects; however, such identifications are unreliable in short series, as mentioned above.

Linking

The third option available is to link two series together to provide a longer series of 5 years in length. Linking involves using a related series (the indicator series), which has the same general behaviour as the short series, to extrapolate the series back. This option is appropriate only where an indicator series is available for months/quarters before the start of the short series. Where an overlap is available between the two series, they can be linked to provide a longer series that can be adjusted using X-13ARIMA-SEATS. Advice on the linking of series is available from TSAB.

Choice of methods

The choice of which approach to follow depends upon a number of factors.

- Options 1 and 2 may be easier to implement; thus if time is limited one of these options may be more appropriate
- Option 3 can be used only if there is an indicator series which can be used to extend the short series further back. The effectiveness of this method will depend upon how similar the behaviour of the indicator series is to the short series being adjusted

More advice is available from TSAB.

Seasonal adjusted short series are likely to be subject to large revisions as new data become available. Outliers and breaks will have a particularly large effect on short series. It is especially important that, if they exist, these effects are identified (usually through knowledge of the series) and removed.

4.4 WHAT IS THE RECOMMENDED LENGTH OF A TIME SERIES FOR SEASONAL ADJUSTMENT?

Although anything under 5 years is considered short, it is desirable to have more than 5 years to ensure acceptable stability when the series is updated with new data. At least 10 years is necessary to ensure that the adjustment of the first year is unlikely to be revised extensively. If the series has between 5 and 10 years of data, it may be modelled and adjusted in the same way as for a longer series, but it must be recognised that there may well be revisions (perhaps because of changes of model, recognition of previously undetected breaks, revisions to the existing data, or very unseasonal new data affecting the moving averages) when more data are added.

At the other extreme, a long series may have discontinuities or large changes in seasonal pattern. Regressors should be included in the regARIMA model if there is a trend or seasonal break in the series history. One criterion for using these regressors is whether there are enough data before and after the break to enable reliable identification. Among other things, this depends on how irregular the series is: the greater the volatility, the more data will be needed.

Because there are so many variable factors, it is not possible to lay down any hard and fast rules as to how many years of data are necessary to obtain a publishable adjustment. Anything under 5 years is very risky, and should be published with strong warnings and only if the series is of such high importance that some guidance on its evolution is essential. Another factor to consider is how clean the data are. If the data appear to show strong seasonality, there is more reason to trust the models suggested by X-13ARIMA-SEATS. Between 5 and 10 years the series becomes more reliable, but there are still risks. It is best to provide additional warnings if the series is published and there are large revisions on update. With 10 or more years the presumption is that adjustments should be published; this will be overridden only if there is some serious problem indicated by the diagnostics, for instance an indication of serious instability from the sliding spans analysis.

5 CONSISTENCY ACROSS TIME

5.1 INTRODUCTION

The pattern of seasonality and other cyclical effects such as trading days and moving holidays changes from year to year. When these effects are removed from the time series to produce the best seasonally adjusted estimates, the changing patterns cause an inconsistency: the annual totals of the seasonally adjusted series are different from the annual totals of the original series.

A separate inconsistency can sometimes arise when monthly and quarterly versions of the same series are seasonally adjusted independently, resulting in a difference between the totals of the monthly seasonally adjusted series and the quarterly series.

In both cases it may be necessary to remove the inconsistencies by constraining, that is forcing the totals to match up. If so, the seasonally adjusted series can be perturbed slightly using a mathematical method called benchmarking. The perturbations will reduce the quality of the seasonally adjusted estimates slightly, so should not be applied unless absolutely necessary.

Constraining is most often necessary for the following reason. When the seasonally adjusted estimates are used as a small part of a larger system, such as in National Accounts, then it is often essential that the annual totals of the seasonally adjusted estimates be forced to be equal to the totals for the original series. The differences between constrained and unconstrained are smoothly spread out, and non seasonal. Therefore, the quality of the seasonal adjustment is not removed in this process, but there are minor tweaks to the trend to the estimates to be incorporated into the larger system of accounts.

In order to constrain the totals of the seasonally adjusted estimates to the annual totals of the original series the **force** spec should be included within the spec file. If it is necessary to constrain monthly estimates to quarterly, then a separate program must be used because this type of constraining cannot (at the time of writing) be accomplished within X-13ARIMA-SEATS.

5.2 ANNUAL CONSTRAINING

5.2.1 *What is annual constraining?*

In general, the annual total of seasonally adjusted estimates is not exactly equal to the annual total of the original estimates. There are procedures in the X-13ARIMA program that ensure that the seasonally adjusted estimates and the original estimates have a similar average level over any 12 month or four quarter period, but the agreement is rarely exact. If exact agreement is required, X-13 ARIMA-SEATS can use a routine that constrains the seasonally adjusted estimates so that the annual totals equal those of the original estimates. It does this in a way that closely preserves the period-to-period changes observed in the unconstrained seasonally adjusted estimates.

There is no mathematical reason why the annual totals of the original estimates and seasonally adjusted estimates should agree. Indeed, if trading day adjustments are made, there are good reasons why they should not. This is because the seasonally adjusted estimates have been scaled to an average pattern of trading days, while the original estimates correspond to the actual pattern of days in the year. However, these constraints are often applied, usually to make series more consistent for users.

Occasionally it may be desirable to constrain the seasonal adjustment so that financial years, or some other yearly totals, of the seasonally adjusted estimates and the original estimates are equal. This can also be achieved using X-13ARIMA-SEATS.

5.2.2 *When should annual constraining be applied?*

The main reason for applying annual constraining is so that users are presented with a consistent set of annual totals for a time series, regardless of whether they are looking at the original or seasonally adjusted estimates. The trade-off is that there is a loss of optimality in the seasonal adjustment, particularly when seasonality is evolving quickly, or when trading day effects are large. The following four criteria should be used when deciding whether or not to apply annual constraining.

1. *Use*: Consider how the data are used and any stated preferences of users. This criterion will sometimes override the other criteria. For example, if the series are part of the National Accounts dataset they will probably need to be constrained

2. *Concept*: Decide whether annual constraining makes conceptual sense for your time series. Consider, for example, how the timing of the observations in your time series relate to calendar years, think carefully

about the way the data are collected and compiled and what they represent. It may be apparent from this that annual constraining is inappropriate for your series. Consider the following examples: the observations in the Labour Force Survey are a sequence of overlapping 3 month averages; Retail Sales data are collected on a 4, 4, 5 week pattern rather than a calendar month basis; prices, earnings, claimant count and many other series are stock series measured at a point in time, for which averaging to the same annual averages might have little presentational importance

3. *Continuity*: If a time series is already being seasonally adjusted, then strong reasons are needed for changing the approach to constraining. It would be a poor policy to allow switching between applying and not applying annual constrain every few years

4. *Effect*: Look at the E4 table of the X-13ARIMA-SEATS output to see if the differences, or ratios if multiplicative, between the constrained and unconstrained data are large. If they are negligible and the other criteria are satisfied, then annual constraining may be preferable.

5.2.3 *Annual constraining and revisions*

When annual constraining is applied, it is important to ensure that the revision policy takes this into account. For example, a revision policy that revises the last month every time a new monthly observation is added to a series would change the additivity of the previous year when January data are published and December is revised. It is primarily for this reason that the revision policy for the National Accounts datasets only ever involves revising observations in the current year.

5.2.4 *Annual constraining in X-13ARIMA-SEATS*

By default, X-13ARIMA-SEATS will not adjust the seasonally adjusted values to force agreement. If the user wants to force the annual totals, the force specification should be used. At least five years of data is required to use the force specification. Care should be taken when using the force specification in conjunction with the composite specification[1].

A typical application of annual constraining is shown in the spec file below. Some values for the series, transform, arima, forecast and x11 specifications are shown for completeness but are not relevant to this discussion. The force specification is shown with typical options; at the time of writing these are the values used by ONS when constraining.

1 see Chapter 20

If the parameter **round** is specified as round=no, it is guaranteed that the totals agree, but the totals as printed in the output may not due to rounding effects. If it is required that the totals should agree as rounded and as printed then use round=yes. Unless it is known in advance how many figures will be required in the final published figures, the recommended option to use is round=no.

```
Example specification file, using the force specification to
   constrain annual totals.

series{
   title = "Example of constraining the annual totals"
   start = 1994.1
   period = 4
   file = "mydata.txt"
   }

transform{
   function = log
   }

arima{
   model = (0,1,1)(0,1,1)
   }

forecast{}

force{
   type = denton
   round = no
   usefcst = no
   mode = ratio
   save = saa
   }

x11{
   appendfcst = yes
   trendma = 7
   seasonalma = s3x5
   save = d11
   }
```

5.2.5 *Financial year constraining*

For some series it may be desirable to constrain the seasonal adjustment so that financial year totals (or some other yearly totals) for the seasonally adjusted and the original data are equal. The default operation of the force specification uses calendar years, but there is an optional parameter ***start*** which allows the use of any other year. To use the standard UK financial year for a monthly series, include the specification *start=april* or *start=apr*; for a quarterly series, use start=q2.

If calendar and financial year constraints are required, two separate constrained time series could be produced using X-13ARIMA-SEATS. This requires two separate seasonal adjustment runs, with specification files differing only in the value of the option start. The peculiarity of producing two different constrained series starting from the same unconstrained series should lead to questioning whether such a requirement is sensible. Note that it is mathematically possible to constrain seasonally adjusted estimates to agree with the original estimates over both calendar and financial years. This would require the use of an alternative program. The user should be careful when adding additional constraints as this reduced the degrees of freedom in the optimisation problem and could lead to undesirable effects. Again, the user should question whether such a requirement is sensible.

The user should consult section 7.6 of the X-13ARIMA-SEATS manual (USCB 2017) for other options when constraining annual totals.

5.3 STATISTICS

If the **force** spec is used, an extra table, D11A, appears in the output. This contains the seasonally adjusted series constrained to annual totals. An extract from a run of X-13ARIMA-SEATS is shown below. Compare table A1 – the original time series data, with table D11A – the seasonally adjusted series with constrained yearly totals, and note that the total for each year is the same, except the last year, which has data for only three quarters so cannot be constrained.

	1st	2nd	3rd	4th	TOTAL
2009	1856.93	1963.16	2074.00	1923.90	7817.98
2010	1777.60	1600.36	1852.92	1795.65	7026.54
2011	1727.91	1911.78	2025.32	1871.16	7536.17
2012	1984.75	1809.97	1812.81		5607.53

Table 5.1: A1 time series data (for the span analysed)

	1st	2nd	3rd	4th	TOTAL
2009	1889.94	2031.42	2043.76	1852.87	7817.98
2010	1808.59	1670.71	1818.57	1728.66	7026.54
2011	1757.79	1979.68	1991.44	1807.26	7536.17
2012	2015.25	1875.00	1776.94		5667.18

Table 5.2: D11.A final seasonally adjusted series with forced yearly totals denton method used.

5.4 CONSISTENCY

Another way in which non-additivity over time can arise in seasonal adjustment is when monthly and quarterly series are seasonally adjusted independently. The seasonally adjusted quarterly estimates and the sum of the seasonally adjusted estimates of the correspondent months can be very different, giving rise to extreme non-additivity between seasonally adjusted components and totals.

This inconsistency can result in a phenomenon similar to the one outlined in Chapter 6. The key difference is that rather than a seasonal feature moving between components of a total, it moves between months of a quarter.

For example, if all firms brought forward payment of bonuses to February rather than March in a particular year, it would be recognised as seasonality in the quarterly seasonal adjustment, but would not in the monthly seasonal adjustment. A similar phenomenon may be observed in energy series, where peaks in energy consumption switch between months according to the weather patterns. The weather also accounts for similar timing differences in harvests, affecting many agricultural statistics.

Under such circumstances there are three methods to seasonally adjust monthly and corresponding quarterly series:

- Indirect seasonal adjustment of the quarterly series by summing the corresponding seasonally adjusted months, which may result in sub-optimal seasonal adjustment of the quarterly series
- Direct seasonal adjustment of the quarterly and monthly series, which may result in a loss of additivity
- Direct seasonal adjustment with constraining to ensure additivity, which may lead to a distortion of the monthly series

A decision about which method to use should be based on careful consideration of the uses of the series and analysis of the technical characteristics of the series involved. It is not possible to constrain monthly to quarterly series within X-13ARIMA-SEATS. For advice on the use of the force specification in the context of annual constraining, contact TSAB.

6 AGGREGATE SERIES

6.1 INTRODUCTION

An aggregate series, also known as a composite series, is a series composed of two or more other (component) series. The component series can be combined in a variety of ways to form the aggregate series. An aggregate series itself may also be a component series of another aggregate series, therefore it is possible to have different levels of aggregation.

Many of the time series that are seasonally adjusted by the Office for National Statistics are aggregate series, and therefore it is important to understand how to deal with the seasonal adjustment of them. For example:

- The number of visitors to the UK is the sum of the number of visitors from North America, the number from Western Europe and the number from other countries
- Total unemployment is the sum of male and female unemployment and also the sum of unemployment by age groups

In practice, as well as in theory, component series can be combined in various ways to form aggregate series. There are two distinct approaches to the seasonal adjustment of aggregate series:

1. *Direct Seasonal Adjustment* - this involves seasonally adjusting the aggregate series without reference to the component series
2. *Indirect Seasonal Adjustment* - this method involves seasonally adjusting the individual component series, and then combining the resulting seasonally adjusted components to obtain the seasonally adjusted aggregate

The two methods usually do not produce the same results. Direct and indirect methods produce equivalent results only under very restrictive assumptions, such as when no calendar or outlier adjustment is made, the decomposition is additive and no forecasts are used. In practice, such conditions are rarely met, and the differences in the series produced under the two approaches can be significant depending on the series concerned.

This chapter discusses seasonal adjustment for aggregate series. Section 6.2 discusses the problem of inconsistency between the approaches in more detail. Section 6.3 provides a checklist of the issues that need to be considered when choosing between the options used to perform a seasonal adjustment of an aggregate series. Section 6.4 discusses other topics related to seasonal adjustment for aggregate series.

6.2 ADDITIVITY OF COMPONENTS AND SEASONAL ADJUSTMENT

The issue of consistency between the component parts and the aggregate after seasonal adjustment is usually referred to as one of additivity, even though the aggregate may be formed from the components by operations other than addition. It is important to realise that there is an inherent contradiction between the quality of the seasonal adjustment and the consistency across series. The goal of additivity can conflict with the primary purpose of seasonal adjustment, that of helping users to interpret the behaviour of a time series. For example, it seems reasonable to assume that, for each point in time, seasonally adjusted male unemployment and seasonally adjusted female unemployment should sum to the seasonally adjusted total unemployment. In practice, using the direct approach to seasonal adjustment – seasonally adjusting the male, female and total series separately – will not normally achieve this additivity. The indirect approach of deriving seasonally adjusted unemployment by adding the seasonally adjusted male and female series will guarantee additivity but will lead to an inferior seasonal adjustment of the total series.

This problem of loss of additivity with the direct seasonal adjustment is particularly striking in situations where a seasonal feature in a time series switches between its components. An example of this in the ONS is the car production series, where seasonal peaks in production can be switched between cars for the home market and cars for export markets according to market conditions and car manufacturers' international production plans. The differences between the direct and indirect seasonal adjustments for these series are large and suggest that this might be happening. A direct seasonal adjustment is therefore used for total car production and no constraining procedures are introduced to reconcile total production with production for the home market and production for export. This lays open the possibility of each of the component series for example, increasing, but the total series decreasing in a particular month; this is a consequence of the non-additivity and optimising the interpretation of each individual series.

One likely problem of inconsistency with indirect seasonal adjustment is that the total series can be combined in many different ways for example, in the case of labour market statistics, the unemployed, can be split by sex, age, region, duration of unemployment, ethnic origin, educational attainment, etc. If one were to seasonally adjust series for unemployment by age band and add them together, this would result in a different indirect seasonal adjustment of total unemployment to that derived indirectly from the sex (male and female) series and both would be different from that obtained by directly seasonally adjusting the total. Under such circumstances additivity can only be achieved using one of following three standard seasonal adjustment methods:

1. Indirect seasonal adjustment at the lowest level of multidimensional disaggregation, for example, seasonally adjusting males and females separately in each age band

2. Constraining out any non-additivity in the seasonally adjusted series

3. Restricting the method of seasonal adjustment so that it is entirely linear and results in a completely additive series

Each of these is theoretically problematic and in some cases difficult in practice as well. All are potentially distortionary in their effects on the seasonal adjustment of one or more series in the dataset. However, examples of applying each of these exist in official statistics: the first characterises Eurostat's approach to seasonal adjustment of European industrial production, resulting in the seasonal adjustment of thousands of series; there are many examples of the second in the ONS, including extensive constraining of the Labour Force Survey series; and the Bank of England currently uses the third to adjust the monetary aggregates dataset.

The analyst faces a difficult choice. The direct approach is most likely to deliver a seasonally adjusted series that allows users to understand its underlying behaviour. However, for presentational and analytical reasons, users may require seasonally adjusted results that preserve the identities that are present in the non seasonally adjusted data. In these circumstances the analyst has to choose between three options:

1. *Indirect seasonal adjustment*, which may result in sub-optimal seasonal adjustment of the total series

2. *Direct seasonal adjustment*, which may result in a loss of additivity

3. *Direct seasonal adjustment with constraining to ensure additivity*, which may lead to a distortion of the component series[1].

6.3 What type of adjustment should be used

The decision on the level at which to seasonally adjust an aggregate series needs to be taken on a case-by-case basis. Listed below are some factors that should be considered when making that decision.

1 see Section 6.4.1

6.3.1 *Factors favouring indirect seasonal adjustment*

The indirect adjustment may be preferred for one or more of the following reasons:

1. It enables information about series to be used at the level at which it is known, that is to say the lower levels (for example, the estimation and application of prior adjustments for seasonal breaks for motor car series in the Index of Services dataset)

2. It enables appropriate filtering of different types of data within a time series dataset (for example, Trade in Services where many different data sources, some monthly, some quarterly, some annual, need to be treated differently)

3. It ensures consistency across different data-sets (for example, where a component is used in two different parts of the national accounts)

4. It guarantees additivity between components and totals. Therefore, an indirect adjustment will ensure that the seasonally adjusted components combine to equal the seasonally adjusted aggregate

5. Disaggregated data often need to be seasonally adjusted anyway to satisfy user needs

6. Disaggregated data are sometimes more important to users than an aggregate (for example, unemployment is more important than the total Labour Force or total working age population)

One concern about aggregation using indirect seasonal adjustment is related to the quality of seasonal adjustment, in particular the possibility of residual seasonality. One of the likely cases is that if the time series are too volatile to identify a seasonal component, the seasonal component may only become apparent as you move up the aggregation structure. This can result in an identifiable seasonal effect in indirectly seasonally adjusted time series, known as residual seasonality.

6.3.2 *Factors favouring direct seasonal adjustment*

Indirect seasonal adjustment does not necessarily result in a good quality seasonal adjustment at the aggregate level, possible reasons for this might be:

1. Adjustments or other processes occurring between seasonal adjustment and production of the final headline aggregate might re-introduce seasonality

2. Component time series are not independent of each other and their multivariate properties generate different seasonal dynamics at an aggregate level, the essential problem is that non- seasonal component series can combine to form a highly seasonal aggregate

3. Similar to the previous point, is the case where a dataset is all part of the same sample survey. The further the series is disaggregated, the greater the contribution of sampling variability to movements in the series. In this case, seasonality is harder to estimate, with greater potential for revisions

4. The more series there are to seasonally adjust, the more time-consuming the monitoring and reanalysing becomes. In extreme cases this becomes completely unmanageable and no attempt can be made to do anything other than run the adjustments on default settings

5. A potential consequence of these revisions is that seasonal adjustment at a disaggregated level results in much higher I/S and I/C ratios and therefore longer moving averages are used than would be the case at an aggregate level. The result is that the seasonal adjustment is more sluggish than it should be. Seasonal adjustment is performed to help users interpret short-term movements in the time series ONS presents and indirect adjustment might build in a much slower response to changes than is necessary, resulting in a more volatile seasonally adjusted series and potentially lagging users' ability to perceive signals inherent in the data.

6.3.3 *Direct vs indirect methods*

Whether it is more appropriate to use direct or indirect seasonal adjustment is still an open question. Neither theoretical nor empirical evidence uniformly favours one approach over the other.

However, as ESS guidance (Mazzi et al., 2015) states, the following should be taken into serious consideration before we choose between the direct and the indirect seasonal adjustment:

- Descriptive statistics on the quality of the indirect and direct seasonally adjusted estimates, for example, the smoothness of the component time series, residual seasonality tests on the indirect seasonally adjusted estimates, and measures of revision

- Characteristics of the seasonal pattern in the component time series

- User demand for consistent and coherent outputs, especially where they are additively related

- The level of aggregation.

6.4 OTHER RELATED TOPICS

6.4.1 *Constraining to preserve additivity*

For aggregate series where it is deemed essential to use direct seasonal adjustment and where users demand that additivity be preserved, it is necessary to constrain the seasonally adjusted component series so that they are consistent with the seasonally adjusted total series. One method of ensuring that the difference between the direct and indirect seasonal adjustment is small is to use the same model and consistent prior adjustments when seasonally adjusting the total series and the component parts. The chosen model, the one most appropriate for the total series will not necessarily be the most appropriate model for component series so the seasonal adjustment of the component series may be sub-optimal. Even if this approach is used, the total may still not equal the sum of the components. In these circumstances the difference (total – sum of components) needs to be distributed across the component series. There are many ways of doing this including:

1. All of the difference is attributed to the largest series or to the least significant series
2. The difference allocated to all series, in proportion to the size of each series
3. The difference allocated to all series, in proportion to the size of the irregular components of each series
4. Multivariate regression-based benchmarking

Constraining would need to be applied outside of X-13ARIMA-SEATS.

6.4.2 *Other considerations*

In considering whether or not to undertake a composite analysis of an aggregate series, as well as considering the points discussed in Section 6.3 as they relate to the particular series or data set concerned, a couple of other issues may be important, particularly as each case will be different. Size of the data set and the time available may determine what is feasible in terms of seasonal adjustment. Does the indirect adjustment have residual seasonality? If so, then a simple solution could be direct adjustment? The composite adjustment may also help to identify problems in component series. Are there any outliers in the aggregate, and/or the component series that have been identified, and replaced that could be adversely affecting the indirect seasonal adjustment of the aggregate series, or are there outliers that have been identified in the direct adjustment that have not been identified in the adjustments of component series?

6.4.3 *Using X-13ARIMA-SEATS to choose the seasonal adjustment*

If it is unclear which type of adjustment is needed, and the decision is to be based on the quality of the seasonal adjustment, the X-13ARIMA-SEATS program can produce diagnostics to aid in the decision between the direct and indirect methods. In order to compare direct and indirect adjustments in the X-13ARIMA-SEATS program, the **composite** spec can be used to produce diagnostics to evaluate both methods. There is an example and more information about the composite spec in Chapter 20.

7 REVISIONS AND UPDATES

7.1 INTRODUCTION

When a new data point for the original estimates becomes available, more information is available concerning the seasonal pattern and the underlying trend of the time series. This additional information may lead to a change in published seasonally adjusted and trend estimates. This change in seasonally adjusted and trend estimates is known as the revision. This chapter explains how seasonal adjustments can be updated, and the rationale for each method. The second part looks at issues that may influence revisions policies.

7.2 TYPES OF UPDATING

The following methods are widely used for updating the time series outputs from a seasonal adjustment:

- **Annual updating** involves an annual review and assessment of each directly seasonally adjusted time series. The seasonal adjustment settings and prior corrections are analysed and improved where possible. X-13ARIMA-SEATS diagnostics, such as the M and Q statistics, Sliding Spans, Revisions history and diagnostics for the regARIMA model are used to assess the suitability of the seasonal adjustment parameters. TSAB is responsible for reviewing the seasonal adjustment parameters for all ONS directly seasonally adjusted time series

- **Current updating** involves running X-13ARIMA-SEATS every month or quarter with the latest available time series data to derive the latest seasonally adjusted estimates. When each of these seasonal adjustment runs is performed, the seasonal adjustment options used are not the defaults, but those determined during the most recent annual update. The lengths of the moving averages; prior adjustments for Easter effects and trading days; and the type of ARIMA model used, should all be specified in the specification file. This type of updating is used across the ONS to obtain the latest seasonally adjusted estimates

- **Forward Factors** involves using forecasted seasonal factors (Table D10a of the X-13ARIMA-SEATS output) derived at the time of the annual update. This results in revisions to the seasonally adjusted estimates being applied only once a year, when the new forward factors are estimated. This option may be adopted where there are system constraints

Within ONS annual updates are carried out by TSAB, while individual branches responsible for the original data carry out current updating.

7.3 WHICH SEASONAL ADJUSTMENT PARAMETERS SHOULD BE FIXED OR RE-ESTIMATED

In general, everything should be fixed when a seasonal adjustment review is complete. There should be no automatic procedures in order to prevent changes to the seasonal adjustment specifications and therefore reduce the size of revisions in subsequent publications. Outlined below is a general guide to which seasonal adjustment parameters should be fixed and which should be re-estimated each time a current update is run:

- The ARIMA model should be fixed in the X-13ARIMA-SEATS arima specification
- The appropriate transformation should be fixed in the transformation specification
- Easter and trading day regressors should be included in the regression specification as variables, and not left to the automatic selection using the AICC test
- The dates for additive outliers and level shifts defined by the automatic outlier detection should be converted into fixed regressors in the regression specification. In this way the effect of the regressors will be fixed in the model whilst the parameter adjustment is re-estimated every month/quarter. An alternative approach for known impacts is to also fix the parameter estimate so that it is not re-estimated at each time period
- User-defined seasonal regressors should be used in the regression specification to correct for a seasonal break. These regressors should be converted into permanent priors only if the time series have multiple seasonal breaks[1]
- The trend and seasonal moving averages should be fixed in the x11 specification for smaller revisions.

1 for more information see Chapter 14

7.4 REVISIONS

Each new data point that becomes available will impact on the estimates of the seasonal and trend component for previous periods. Each seasonal adjustment update will potentially cause revisions along the length of the seasonally adjusted estimates. Figure 7.1 shows the impact on the seasonally adjusted series of adding additional data points.

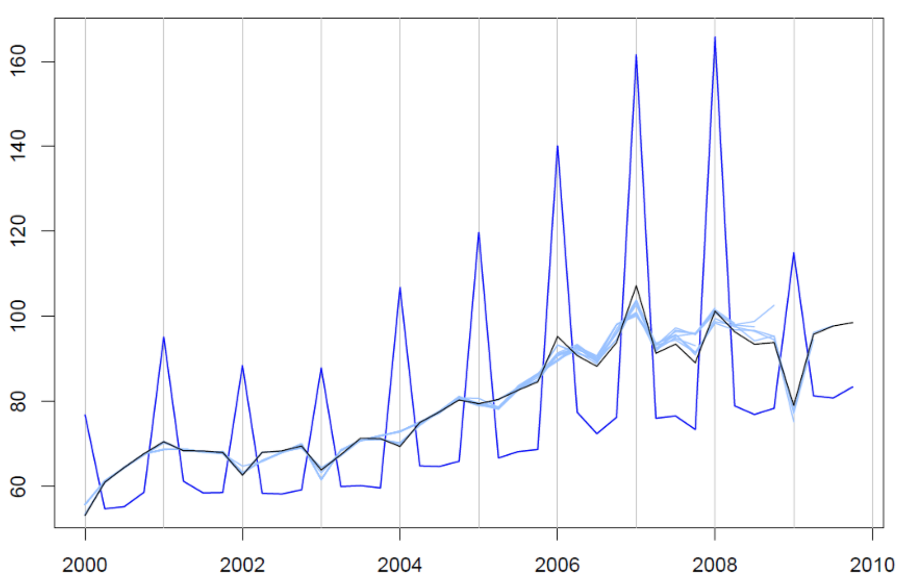

Figure 7.1: Revisions of seasonally adjusted estimates over time

Major revisions are typically applied after the addition of new data to the immediately preceding period and to the corresponding period one year prior. For example, when new original data became available in December 2006, the seasonally adjusted and trend estimates were revised from November 2006 backwards in time, with potentially larger revisions for November 2006 and December 2005. However, if the annual totals are constrained[2], revisions cannot be made without revising the whole year, and so the revision would not generally be made.

Often, the nature of the seasonal adjustment will imply that revisions should occur at other time lags. For example, time lags of two and three periods might also be revised, if these show large changes when a new data point is added. In general, the seasonal adjustment must be revised back to any periods where the raw data has to be revised. If revisions to raw data are large, then the seasonal adjustment of neighbouring points may also need to be revised. A graph of the adjustment, before and after the revisions, should be checked to see if this is generally the case.

2 see Chapter 5

Reference Period	April-2019	July-2019	Oct-2019	Jan-2020
2019 Q1	100	102	101	101
2019 Q2		104	105	105
2019 Q3			102	101
2019 Q4				110

Table 7.1: Published time series at different publication dates

If a problem with seasonal adjustment is found between annual updates, this should be corrected as soon as possible. This will mean that seasonally adjusted and trend estimates may be revised. If a seasonally adjusted series is particularly smooth, then in the annual update it may be necessary to revise merely the previous two years, or even the previous year only. Conversely, if the adjustment is dominated by the irregular component, it may be necessary to revise up to four years prior.

A revision policy should be determined taking user requirements and revisions of raw data into account. The revisions history diagnostic[3] and the revision triangle method are useful indicators for determining which revision policy is best from a seasonal adjustment perspective.

Table 7.1 shows an example of a real time database where the columns denote the date of publication of a time series and the rows denote the time period that the observation refers to. Reading across a row shows how a particular time point has changed at different publication dates. The latest estimate of all time points is shown in the final column which was published in January 2020.

A revisions triangle can be calculated from the real time data base simply by calculating how a time point is revised between consecutive publication dates. Table 7.2 is a revisions triangle calculated from the real time database in Table 7.1 and shows that between the time series published in October 2019 and January 2020, there were no revisions to 2019 Q1 or 2019 Q2 but the estimate of 2019 Q3 was revised down by 1 in January 2020. Reading down a diagonal shows how a time series has revised at a particular lag.

Reference Period	April-2019	July-2019	Oct-2019	Jan-2020
2019 Q1	-	2	-1	0
2019 Q2		-	1	0
2019 Q3			-	-1
2019 Q4				-

Table 7.2: QQ revision to published time series

3 see Chapter 19

The decision as to when to revise, and how many data to revise, is important. Revisions policies can have a major impact on user confidence and the quality of published data. Therefore, a revisions policy that takes into account all the factors detailed above is an important element for producing high quality seasonally adjusted series. In general, revisions and revisions policies for seasonally adjusted and trend estimates will depend on the nature of the time series and the area responsible for publishing the data.

For more information see also the European Statistical System (ESS)[4] guidelines on revision policy for Principal European Economic indicators (PEEIs) from the European Commission.

4 Mazzi et al. (2015)

8 THE REG-ARIMA MODEL

8.1 INTRODUCTION

The regARIMA part of the X-13ARIMA-SEATS program precedes seasonal adjustment. It modifies the time series so that the seasonal adjustment process will produce higher quality estimates. The time series is modified by forecasts, removal of effects caused by the arrangement of the calendar, and by temporary removal of outliers and similar effects.

- *Series extension* - the series is extended forwards by adding forecasts, and backwards by adding backcasts. This produces a longer span of data to input in the seasonal adjustment process leading to better quality seasonal adjustment, particularly at the ends of the series. Consequently revisions are lower when the time series is augmented with new observations
- *Calendar effects* – effects associated with the arrangement of the calendar are removed from the series. This improves the estimation of seasonal effects and makes the series easier to interpret
- *Outliers, breaks and other changes* – the series can be adjusted for unusual and disruptive features such as a sudden and sustained drop in the level of a series. Removing such features makes the seasonal adjustment more robust by preventing them from distorting the subsequent estimation of seasonality. However, these features typically represent the real world behaviour of whatever the time series is measuring so the features are returned to the series after seasonal adjustment is complete.

8.2 OVERVIEW OF REGARIMA

regARIMA is the name of the statistical modelling facility in X-13ARIMA-SEATS. It enables two types of models to be fitted to a time series: an ARIMA model and a regression model.

Autoregressive Integrated Moving Average (ARIMA) models, are models for time series that take account of trend and seasonality in the data. The program X-13ARIMA-SEATS© will choose the most appropriate form of ARIMA model for an individual series using the model fitting criteria built into the program, or the user can specify the form of ARIMA model to be applied.

The regression part of regARIMA refers to the options that enhance the ARIMA model with variables that can represent, for example, outliers or calendar effects. In technical terms, the regARIMA is as a linear regression where the error terms follow an ARIMA process rather than a white noise process:

$$y_t = x_t^{'}\beta + z_t \tag{1}$$

here x_t is a vector of regression variables, β a vector of regression parameters, and the error term z_t follows an ARIMA process.

Once the model has been specified it can be used to produce forecasts and backcasts. The variables of the regression part of the model give estimates of calendar and other effects that are removed before the series goes through the seasonal adjustment process.

This chapter shows how the regARIMA model can be set up. There are three topics:

1. Transformation of the series
2. Specification of the ARIMA part
3. Specification of the regression part

Although it is convenient to separate headings 2 and 3 in this way, they are not really separate sequential processes. If we do not include appropriate regression variables (particularly for effects like outliers and level shifts), it may be impossible to produce a satisfactory ARIMA model. There is often an iterative cycle of adding new regression variables and re-specifying the ARIMA model until a satisfactory set of diagnostics is obtained, as discussed in more detail in Section 8.6.

8.3 TRANSFORMATION OF THE SERIES

The first step in fitting a regARIMA model is to transform the series. X-13ARIMA-SEATS allows the following transformation: none, log, logistic, square root, inverse and Box-Cox. Most commonly the regARIMA model is fitted either to the original series or to the log transformed series, the choice depends on whether the series is additive or multiplicative. When the series is additive, the regARIMA model is fitted directly to the original series. When the series is multiplicative, the model is fitted to the log transformed series. The effect of a log transformation is to change the scale of a series and to turn multiplicative effects into additive ones. This is done purely for the purpose of fitting the model. Guidance on choosing whether a multiplicative or additive decomposition is most appropriate for a series can be found in Chapter 12.

8.4 specification of the arima part of the model

The purpose of ARIMA modelling is to identify systematic structural features in the history of the series. We assume that these features will continue to be present in the future and will use them to forecast future values. The ARIMA method provides a wide range of possible models, which have been found very effective in modelling typical socio-economic series showing trends, seasonality and business cycle effects.

This section gives a brief and largely non-technical explanation of ARIMA modelling. TSAB should be consulted if more technical detail is needed. We start by discussing the model for non-seasonal series, because this is simpler. This model is extended in a straightforward way to apply it to seasonal series. For non-seasonal series the form of the ARIMA model is that the value at a given time point is modelled as some combination of previous values plus a random quantity called the innovation. Innovations are modelled as independent samples from a normal distribution with zero mean and constant variance. Within this general framework we distinguish the three types of effect included in the ARIMA model.

- *Autoregressive (AR)*: the value depends on some linear combination of previous series values
- *Moving average*: the value depends on some linear combination of previous innovations
- *Integrated*: the autoregressive and moving average effects apply to differences of the values, rather than the values themselves

In each of these categories, the number of lags involved is referred to as the order of the effect. The order of a model is abbreviated in the form (p, d, q), where p is the order of the autoregressive component, d is the order of differencing and q is the order of moving average. the example models below are represented as (1 0 0), (0 1 1) and (2 0 2) respectively. In what follows, x_t is the series value at time t the innovation is ε_t. ϕ_i are coefficients to be estimated on the autoregressive lags and θ_i are coefficients to be estimated on the moving average lags.

1. A first order autoregressive model (1 0 0):
   $x_t - \phi_1 x_{t-1} = \varepsilon_t$
2. A first order integrated moving average model (0 1 1):
   $x_t - x_{t-1} = \varepsilon_t - \theta_1 \varepsilon_{t-1}$
3. A second order autoregressive and second order moving average model (2 0 2):
   $x_t - \phi_1 x_{t-1} - \phi_2 x_{t-2} = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$

Note that all terms involving x have been moved to the left of the equals sign and all terms involving ε to the right; this is the conventional arrangement.

If there is a seasonal component then there is an effect that repeats at annual intervals. This is modelled in the same way as a non-seasonal effect, except that the dependence is on values occurring one, two,... years ago rather than one, two,... periods ago. For example, for a monthly series, a seasonal autoregressive component will involve a relationship between x_t and x_{t-12}, x_{t-24},... and similarly for seasonal moving averages or differences.

A seasonal series will usually have a non-seasonal component, defined by (p d q) as above, and a seasonal component with separate parameters and written as (P D Q). The combination of these two components is indicated by putting the two brackets together, with a subscript on the seasonal indicating the seasonal period. For example, the seasonal model called the airline model (so called because Box and Jenkins used it to model monthly numbers of airline passengers), is written (0 1 1)(0 1 1) 12 – though the subscript is usually omitted because its value is clear from the context.

Sampling variation or irregularity in the observed series means it is often possible to find different ARIMA models that will fit satisfactorily. We recommend choosing the simplest model which will give a satisfactory fit, where simplest means having the smallest number of parameters. This is known as parsimonious parameterisation, or parsimony for short. The process of searching for a model often involves adding parameters to avoid specification errors and then de-selecting previously chosen parameters that can be removed in the interests of parsimony.

The principal tool in testing for a satisfactory fit is the autocorrelation of the residuals, which are the estimates of the innovations. The innovations should be independent, so any significant serial correlations in the residuals could be an indication of a deficiency in the model. There are guidelines to indicate how a model should be changed to remove particular patterns of serial correlation, though these will not normally be needed, because the automatic choices of X-13ARIMA-SEATS are almost always adequate.

There are two approaches to automatic model selection in X-13 ARIMA-SEATS.

1. Search through a predetermined list of candidate models to find a satisfactory fit. The candidates are usually arranged in order of increasing complexity

2. Starting with a simple model, allow the program to add terms successively up to predetermined limits on the number of terms, until

the fit meets some criterion of goodness of fit. The terms added at each stage being determined by the deficiencies in the previous fit

The first approach is based on work by Statistics Canada. The second approach is based on the TRAMO method of Gomez and Maravall at the Bank of Spain. The user of the latest X-13ARIMA-SEATS has the choice of either method, or may impose a choice of model if the automatic choice is not satisfactory. The first method is provided by the **pickmdl** spec, the second by the **automdl** spec. The two approaches are outlined in the following sections, Section 8.4.1 and Section 8.4.2

8.4.1 *Automatic model selection with the automdl spec*

The **automdl** spec proceeds by successively improving the first simple model. Terms are added or modified when the specification tests show that the current model is inadequate and the modified model is better; terms are removed when they are no longer significant. This continues automatically within user-specified limits on the complexity of permitted models.

The limits on complexity are specified in terms of the maximum values of the parameters p, d, q, P, D and Q. There are default values for these maxima which are built into the program, but these may be specified by the user if necessary. The maxima for d and D are specified by the **maxdiff** argument, the maxima for p, q, P and Q by the **maxorder** argument. The default values are $d \leqslant 2$, $D \leqslant 1$, $(p, q) \leqslant 2$, $(P, Q) \leqslant 1$, which may be represented in terms of the arguments as **maxdiff=(2 1)**, **maxorder=(2 1)**. For normal purposes these default values will be adequate; they should not be overridden unless the program fails to produce a satisfactory model using them.

If no satisfactory model is produced with the default values, alternatives may be tried. The only alternative with maxdiff is (2 2), which should not be tried until all other possibilities are exhausted.

For maxorder, Table 8.1 shows alternative values which empirical research has shown may sometimes be preferable. Any model produced using these limits should be carefully scrutinised to see that all its diagnostics are satisfactory.

Sometimes the program will fail to select an ARIMA model. There are other parameters that can be varied to improve the outcome in difficult cases, but their use requires care and experience. It is recommended that TSAB be consulted if problems arise.

Series length	Monthly Series	Quartely Series
< 6 years	(2 1)	(2 1)
6-10 years	(3 1)	(3 1)
10-15 years	(4 1)	(3 1)
> 15 years	(4 2)	(4 2)

Table 8.1: Alternative values for the maxorder argument

8.4.2 *Automatic model selection with the pickmdl spec*

When the pickmdl spec is used X-13ARIMA-SEATS selects a model from a limited list of models, subject to certain model selection tests. The program uses as default a list based on the original research by Statistics Canada, which consists of five models:

(0 1 1)(0 1 1)
(0 1 2)(0 1 1)
(2 1 0)(0 1 1)
(0 2 2)(0 1 1)
(2 1 2)(0 1 1)

If necessary, the user may add to this list or provide a completely different list; this requires careful testing before an alternative can be considered safe, and it is recommended that TSAB be consulted before undertaking any change.

The selection tests are as follows:

1. A test that the residuals are not serially correlated
2. Testing the MA part of the model for evidence of over differencing
3. Within-sample forecast error. For this test to pass the absolute average percentage error of within-sample forecasts in the last three years should be less than 15%

The first two tests are aimed at misspecification, which would give false forecasts even if the fit of the data is satisfactory. The third test is a quantitative measure of within-sample forecast accuracy in the last three years of the series. The models are tested sequentially in the order shown above, which is clearly that of increasing complexity. If no model passes all tests then the first and simplest one, the (0 1 1)(0 1 1) model, is used. However, it is used only for the purpose of estimating the regression effects; no forecasts are generated if no model passes the tests. The default operation of the method is that the first model tested which passes all the tests is accepted. This can be specified explicitly by putting **method=first**. The alternative is **method=best**, which means that all models are tested and the acceptable model with the lowest forecast error is accepted. It is recommended to use the default.

If the program also performs automatic identification of outliers using the **outlier** spec, the default is to identify them simultaneously with the estimation of the first ARIMA model that is tested and take them as given for the estimation of the other models. This is the recommended approach.

Other optional arguments are available, which are detailed in section 7.12 of the X-13ARIMA-SEATS manual (USCB 2017). We recommend leaving them at their default values.

8.4.3 *Fixing the ARIMA model*

X-13ARIMA-SEATS also provides the option of imposing directly the lag structure of the ARIMA, and even the values of some or all of its parameters. This is done by using the **arima** spec. The model that is specified in this way can be used in any action performed on the series, for example outlier detection, regression, forecasting. This is likely to be useful in two circumstances.

1. When the user has specific knowledge about the real world processes underlying the production of the data

2. When we want to ensure that the model identified by the automatic process is not updated during concurrent adjustment

These are discussed in more detail in the following paragraphs. When the user has specific knowledge about the real world processes underlying the production of the data, and this knowledge will not be captured by the automatic modelling procedure. For example, in the case of monthly financial data, it may be known that there is a quarterly cycle because of the way institutions operate as well as the annual cycle. Similarly, the quarterly cycle in the operation of the Labour Force Survey might impose an extra structure on the LFS unemployment series.

Generally concurrent adjustment uses all available data to adjust the current value, but experience shows that frequent switching between models caused by updating can introduce undesirable instability in the seasonally adjusted output. It is therefore general practice to keep the form of the chosen model fixed between annual re-analyses, and re-estimate the parameter values each period.

The appropriate form of specification for the standard arima spec is: **arima{model=(p d q)(P D Q)}** where p, d, q are the orders of the regular part of the model and P, D, Q are the orders of the seasonal part. The seasonal part may optionally be followed by the seasonal period, either 4 or 12, and if necessary the model may be extended with other brackets followed by an appropriate period. (In common with other X-13ARIMA-SEATS specs, the commas in the inner brackets may be omitted provided the figures are separated by spaces.) The other possible arguments of **arima** are concerned with pre-specifying parameter values, and should not be used. Note that if **arima** is used in a spec file there must not also be an automatic modelling spec: **automdl** or **pickmdl**.

8.4.4 *Identifying a model manually*

In most cases the automatic modelling options will produce satisfactory models. It should seldom be necessary for users of X-13ARIMA-SEATS to undertake manual model identification from scratch. The program can produce the necessary autocorrelation outputs, and there are textbooks which describe the procedure, but it requires considerable experience to produce reliable results.

The one situation in which manual identification may be justified is when automatic identification has produced a model that, while satisfying the tests, still has some unsatisfactory features. For example, although the combined test on the serial correlations of the residuals may be passed, there may still be some individual significant correlations at fairly low lags. It may then be justifiable to try manual refinement of the automatic model. In the example case, it could be worthwhile adding an extra coefficient at the appropriate lag to the AR or MA component; if the extra coefficient is significant and the significant serial correlation has been removed, the extra term may be justified.

The process of manual refinement needs a degree of skill and care, and advice should be sought if in any doubt. The program provides a number of diagnostics which are helpful in this process, for example the model identification statistics AICC and Bayesian Information Criterion (BIC). Details of the use of these may be found in X-13ARIMA-SEATS (2017).

8.5 SPECIFY THE REGRESSION PART

In addition to the relations within a time series that are captured by the ARIMA model, the series may also be affected by external deterministic factors, such as outliers, trend or seasonal breaks, calendar effects. In this case the only satisfactory approach is first to subtract these deterministic effects, and then fit an ARIMA model to the linearised series. However, the exact size of the deterministic effects is usually unknown and needs to be estimated, which cannot be done until the ARIMA structure is determined. The most effective way of doing this is by use of a regARIMA model, which encompasses both the deterministic effects and the ARIMA structure and estimates them simultaneously. The previous section described how to specify the ARIMA part of the model.

This section describes how the regression part should be specified. Specifying the regression part of the model essentially means deciding which variables to include. The recommended option is that a regression variable is included only if theory or knowledge of the series indicates that a regression effect is possible, and this is proved with statistical tests. What is meant by theory or knowledge of the series is whether or not the user expects a particular variable to be important given any prior knowledge for the same or a similar series. This theoretical question is answered in the relevant chapters. For example, the trading day chapter describes in which cases a trading day effect is possible.

This chapter concentrates on the statistical tests that should be used to decide whether a particular effect is indeed significant or not. The rest of this section is organised as follows: Section 8.5.1 considers the question of types of regression variable. Section 8.5.2 describes the built-in regression variables that are available to users and Section 8.5.3 how to specify other variables which are not available in the program. Section 8.5.4 describes how to include variables optionally, basing the decision on statistical tests of significance in X-13ARIMA-SEATS.

8.5.1 *Types of regression variables*

X-13ARIMA-SEATS has a wide range of program-specified variables, which will cover many of the common situations; those most often used are described below. Users can also include variables of their own to capture effects which are not provided by the built-in types. Values for user-defined variables must be input into the program in the same format as series values; this includes values for any forecast periods as well as those corresponding to actual observations. All the built-in variables are assigned to a variable type, and users must specify an appropriate type for any user-defined variables. The concept of variable type is important in two contexts.

- The program output in some areas gives a significance test for the joint effect of all variables of a given type, rather than each variable separately; this is explained in more detail in the discussion of estimation and inference (Section 8.6)
- The type of a variable determines whether its effect is combined with the seasonal effect in adjusting the series

The latter point needs further explanation. All the regression variables are removed from the original series in order to estimate the seasonal component, but for many variables this removal is not permanent, since we wish to regard their effects as part of the information about real world effects that the adjusted series conveys. For example, we could identify a trading day effect and a level shift in the same series. Both can distort seasonal patterns, and both are removed before estimating the seasonal. However, the trading day effect is part of the variation which we wish to remove, since we regard it as a calendar effect, so it is not replaced when obtaining the seasonally adjusted series. The level shift represents some real world effect which we wish to be able to observe in the seasonally adjusted series, and perhaps try to explain. Its estimated effect is therefore replaced in the final adjusted series.

As a general rule, all calendar-related variables, for example trading day, holiday, length of month, leap year, are removed from the final seasonally adjusted series, while seasonal and outlier variables are not. User-defined variables that do not fit with any of the pre-specified types can be assigned the non-specific type **user**. They are not removed from the final series unless **final=user** is specified in the **x11** spec.

It should be noted that some effects which might be dealt with through regression variables could in some circumstances be handled instead by prior adjustment. For example, if there is a series break of known size, a multiplicative prior adjustment would remove it. The effect of this on the adjustment process is the same as that of a level shift regression variable of the same size. The prior adjustment is removed from the series before seasonal estimation, and is restored in the final adjusted series.

8.5.2 *Program-specified regression variables*

The program-specified variables include the following.

1. A *constant term*. This variable will rarely need to be used, as its presence implies a deterministic trend of order equal to the total number of differences, which will in most cases be difficult to justify. A constant term might be used though if the regular part of the ARIMA model includes a zero difference term. Another case is when the ARIMA model was selected with the **automdl** method, and it was found that a constant is significant. On the other hand, it is not advisable to use a constant together with the **pickmdl** model selection routine

2. *Deterministic seasonality*. This can be in the form of either seasonal constants or of trigonometric regression variables, but one cannot include both. The advantage of trigonometric variables is that their use requires fewer parameters. Although deterministic or almost deterministic seasonality will be the case for many series, the use of

seasonal constants is not particularly recommended. This is because deterministic seasonality is equivalent to a seasonal MA coefficient of 1, in a seasonal (0 1 1) ARIMA model. Although in this case it is more efficient to reduce the order of seasonal differencing and capture the seasonal fluctuations with seasonal constants and a stable seasonal Autoregressive Moving Average (ARMA), for the purpose of seasonal adjustment this is not likely to add much value; in practice a seasonal (0 1 1) ARIMA with a MA coefficient equal to, for example 0.95 will be indistinguishable from deterministic seasonality. In addition, if one has to reduce a seasonal overdifferencing to a stable seasonal ARMA model with seasonal means as just described this has to be done manually, as X-13ARIMA-SEATS does not do it automatically, as it does for non-seasonal overdifferencing. For these reasons the use of seasonal constants is discouraged. Trigonometric regression variables on the other hand can be used occasionally, for example in short monthly series where the alternatives of using seasonal dummies or seasonal differencing cost 12 observations.

It should be noted that there is a difference between constants and deterministic seasonals and all other regression variables, in that these variables are not adjusted out of the original series before estimating the seasonal component through the X-11 algorithm; these variables are there simply to make it possible to forecast the series

3. *Trading day variables*. There is a wide range of alternative variables, which correspond to the various different ways in which the number of occurrences of the days of the week within the month can affect economic activity. Details on the trading day effect, the alternative variables, and when it is appropriate to use each of them are given in Chapter 9. What should be emphasised though is that only one trading day variable can be included in a regression

4. *Easter effect*. There is a range of built-in variables that correspond to the different ways in which Easter affects economic activity, detailed in Chapter 10. Other holiday variables, aiming in accounting for the effects on economic activity from other holidays, can be used as well

5. If the users know that an important event has happened that is likely to affect the quality of the seasonal adjustment they should include a regression variable that takes into account this effect. Depending on the nature of the event and the duration of its effects this variable may be an additive outlier, level shift, temporary change, or ramp. Automatically detected outliers can also be included in the regression, if so requested by the outlier spec. In contrast to the above in-

tervention variables, these outliers are unknown and can be detected only from the data

6. Finally, there may be cases where a variable does not affect a series in the same way throughout the sampling period. This can happen in cases of policy changes or other important events. In such cases change of regime variables can be used to account for the change. In particular, change of regime variables can be used for either trading day regressors or deterministic seasonality. The case for deterministic seasonality with change of regime is when a seasonal break is suspected[1].

8.5.3 *User-specified variables*

The variables that are built into X-13ARIMA-SEATS will in most cases cover the user's needs, but there will be some special cases for which the program has not provided. In such cases the users can define and use their own regression variables. User-defined variables are necessary in situations such as the following.

1. *Non-calendar data*, that is data collected on a different basis from that dictated by the monthly calendar. If for instance monthly flow data are collected on the last Sunday of every month, then one has months with 4 or 5 complete weeks, rather than months with 4 or 5 Wednesdays, for example, as assumed by the standard trading day variable. A specially designed variable should be used instead of a trading day one

2. If the series is affected by a program-specified variable but with a delay then one needs to input the lagged program-specified variable as a user-specified one, because X-13ARIMA-SEATS does not have an option to do it automatically

3. A *change of regime* that cannot be captured by the built-in change of regime variables. To specify user-defined variables, the variables must be named and their names listed in the user argument of the regression spec. The types must be listed in the usertype argument in matching order to the names in the user argument, unless all user variables are to have the same type. The values of the variables for all periods, including forecast ones, must be listed in a table. The table may be embedded as data in the spec file, but is better stored in a text file which is referenced by a **file** argument.

1 for more details on seasonal breaks see Chapter 14

8.5.4 *Optional variables and statistical tests*

The procedure implied above is that variables may be included speculatively in the model, tested for significance and then removed if not significant. This then requires another run to re-estimate the model without the variables. There is an alternative to this, which is to specify that certain variables are to be tested to see if their inclusion improves the model fit, included if there is improvement and excluded if not. The only variables which may be treated in this way are trading day and Easter and variables of type user.

The program will estimate the model with and without the optional variables and select the version which gives the smaller value of AICC. If several variables are specified for this treatment, they are tested in up to three groups: first any trading day variables, next any Easter variables, finally any user variables. All of the variables in a group are tested and included or excluded together. The variables to be tested in this way are specified in the aictest argument; they must normally also be listed in either the variables or user arguments, depending on whether or not they are built-in variables. Exceptionally, if **Easter** is included in the aictest argument then it may be omitted from the variables argument. In this case, the program automatically considers different lengths w = 1, 8, 15 for the Easter[w] effect, and compares the best of these with the option of no Easter effect.

Another type of optional variable controlled by statistical tests is an automatic outlier variable. If the spec file includes an outlier spec, the program will search each possible time point to see if an outlier included at that point would be significant. All outliers identified a as result of this proves are added to the model[2].

8.6 ESTIMATION AND INFERENCE WITH REGARIMA

Three issues are relevant to the estimation of a regARIMA model: the order of differencing, regression effects and the ARMA specification of the stationary series. This order corresponds to their order of importance in a seasonal adjustment. If the series is differenced fewer times than required, the regression effects and their statistical significance may not be reliable. Mis-specifying the ARMA of the stationary series, on the other hand, is not as harmful for the purpose of seasonal adjustment as omitting a regression variable. Once the model is estimated one needs to check whether it is properly fitted or not. In most cases this can be taken for granted for the ARIMA part of the model, as it has most probably been automatically se-

2 for more details of types of outlier and options on inclusion see Chapter 11

lected. With regard to the regression part of the model though, one needs to test whether the regression variables are significant or not and whether additional variables should be included. The usual t-statistic, chi-squared and Akaike Information Criterion (AIC) tests are used.

8.6.1 *Automatically run tests*

A lot of tests are run by default and do not need to be asked for in the spec. In particular, the program produces t-statistics for all regression coefficients; it also provides chi-squared values for groups of variables wherever they are more meaningful as a group. For example, it is more meaningful to test whether the six trading day variables together show a significant effect, rather than testing if, say, the number of Tuesdays in a month has any significant effect.

When a variable is tested individually, it should be considered statistically significant if its t-statistic is beyond the critical value for the significance level required. As a rule of thumb, a t-statistic greater than 2 in absolute value is significant at the 5% level. If, on the other hand, a group of variables such as trading day regressors are jointly tested, then the chi-squared test statistic is appropriate. For this statistic the program automatically gives the p-value, and any value less than 0.05 is usually considered significant.

In some cases, it is not necessary to carry out an explicit test, since the terms have been included automatically because they are significant. This applies to level shifts and other outlier terms selected by the outlier spec. In other cases, the appropriate type of test should be carried out on the individual terms or groups of terms, and the terms retained or deleted as appropriate. For example, it is good practice to verify that the p-value or the t-value associated with trading day or Easter terms selected by the aictest are statistically significant.

8.6.2 *Non-automatic tests*

Although in most cases the automatic tests will be sufficient, one might often wish to test a hypothesis that cannot be tested automatically. This can often be done by some manipulation or combination of the outputs. There are several such examples.

1. Alternative models may require separate runs of X-13ARIMA-SEATS, because it is not possible to specify one as a specialised version of another (these are referred to as non-nested models). In such cases the model selection statistics AIC, AICC, BIC may be used. The model with the smallest value of the chosen criterion is preferred. There

are limitations on the use of these statistics. In particular, the dependent variable in the regression must be identical in all the cases being compared, meaning that the series span, order of differencing, transformation and the outliers included in the model must be the same

2. The autoregressive and moving average parameters do not have t-statistics attached. They do have a standard error attached to their values, however, and the t value is the ratio of the two. Normally the automatic model selection will have ensured that only significant terms are retained, but if a model has been manually refined such checks are needed

3. If we wish to test whether the parameters attached to two variables are equal, we can rearrange the model to provide a direct test. We define a new variable which is the sum of the two variables, and include this in the model in place of one of the two. The coefficient of the remaining single variable now represents the difference in the two parameters, and its significance is a test of the difference

4. If we wish to test the joint significance of a group of variables, and we are not able to define them as a group by type for which a chi-squared value is produced, we can carry out runs with and without the variables concerned and compare the results. The simplest comparison is to use one of the model selection statistics AICC or BIC. An alternative, which can give a numerical level of significance rather than a yes-no result, is to compare the log likelihood values from the same table as the model selection statistics. The difference in log likelihood is distributed as chi-squared with degrees of freedom equal to the number of extra parameters

These are merely examples of the ways in which hypotheses about the model fitting may be constructed and tested to meet particular circumstances. The principles exemplified here may be adapted to other situations. If difficulties arise which users cannot easily solve, in the first instance TSAB should be consulted.

8.7 SUMMARY OF IMPLEMENTATION INSTRUCTIONS

The analyst should keep parsimony in mind. Regression variables should be included in the model only if they are significant and improve the quality of the seasonal adjustment. The analyst should implement the model differently depending on whether the time series is being analysed for the first time, at the regular seasonal adjustment review, or is being used in production.

8.7.1 *Time series analysed for the first time*

- Consider whether there are special effects which might suggest that automatic model selection is inappropriate. If not, use an automatic model selection with an appropriate forecast horizon. Backcasts may also be used unless the series is very long
- If the **pickmdl** method is used, backcasts can be produced by specifying **mode=both**
- Include in the regression spec all the variables that might have an effect on the series. For example, consider whether the process being measured is likely to be affected by calendar effects like Easter or days of the week; see if there are any known events in the history of the series which might have caused breaks; see if any external variables like weather might affect the series
- Run X-13ARIMA-SEATS to generate model selection and regression statistics
- If the chosen model selection procedure does not give a satisfactory model, consider varying the selection procedure[3] or manually refining the best model produced by the automatic procedure. As a last resort use the default (0 1 1)(0 1 1) model
- Check the statistical significance of the regression variables, as explained above. If the tests show that the set of variables that should be included is different from the one that was actually included, amend accordingly and run X-13ARIMA-SEATS again, repeating the process of ARIMA model selection
- The simultaneous estimation of the regression and ARIMA parameters may, rarely, lead to convergence problems. If the maximum number of iterations is run without convergence having been achieved, one should increase this maximum number of iterations and try

3 see Section 8.4

again. This can be done with the **maxiter** argument in the **estimate** spec. If the estimation has not converged, it is wrong to use the output of the last iteration as the estimate. If convergence cannot be achieved, more options are provided in the X-13ARIMA-SEATS manual (USCB 2017), or consult TSAB.

8.7.2 *Model set up for regular seasonal adjustment review*

- All changes to the model should be considered before introduction to see what effect they have on historical data. Stability may often be more important than a marginal improvement in fit
- Use the automatic model selection procedure that was used in the previous analysis. If a different model is chosen, examine the test statistics to see whether the new model is substantially better than the previous one. If the difference is marginal and the statistics for the previous model are still satisfactory, consider imposing the previous model in the interests of stability
- Include in the regression spec all the variables that were found significant in the previous run, plus any excluded but found to be on the margin
- Run X-13ARIMA-SEATS to generate model selection and regression statistics
- Check the significance of the regression variables as before. If any which were previously included are now strongly non-significant, with a t-value lower than 1, they can be dropped. If any regression variables that were previously included have a t-value in between 1 and 2 it is better to keep the variable in the model to avoid unacceptable revisions of previously published seasonally adjusted data. That variable should be tested again at the following annual re-analysis.

8.7.3 *Model set up for production running*

- Using the arima spec, impose the ARIMA model that was selected at the time of the last re-analysis
- Use the forecast spec to specify the appropriate forecast and backcast horizons
- Use the regression variables that were selected at the time of the re-analysis. Any effects selected for inclusion by aictest should instead be fixed in the variables argument, and aictest should be removed

- Fix neither the ARIMA nor the regression parameters
- Run X-13ARIMA-SEATS.

9 TRADING DAY

9.1 INTRODUCTION

Trading day effects are those parts of the movements in a time series that are attributable to the arrangement of days of the week in calendar months. For example, a month containing 5 Saturdays is likely to show a higher level of sales than a month containing 4 Saturdays. As with seasonal effects, it is desirable to estimate and remove trading day effects from time series to help interpretation.

X-13ARIMA-SEATS estimates trading day effects by adding regressors to the regARIMA model. Section 9.2 discusses the problems of the arrangement of the calendar. Section 9.3 describes when to adjust for trading day effects. Section 9.4 provides details of the different types of regressors used by X-13ARIMA-SEATS to adjust for trading day effects. Section 9.5 describes the recommended procedure to adjust for trading day effects and explains how to implement the results in a production environment. Section 9.6 gives more details of related options and topics. Finally, Section 9.7 gives guidance on dealing with non-calendar data.

9.2 THE ARRANGEMENT OF THE CALENDAR

Trading day effects arise because the number of occurrences of each day of the week in a month differs from year to year. An example of the arrangement of the calendar problem is shown in Table 9.1, where the number of occurrences for June is calculated across three years, 2019-2021.

These differences will cause regular effects in some series. For example, a production series where no work takes place on Saturday or Sunday will have two fewer working day in June 2019 than in June 2020, which will have the same working days as June 2021. Thus, it is likely that the series has a slightly lower value in June 2019 than in June 2020 or 2021 without reflecting the long-term trend.

Those differences are not genuine movements of the production, but are just because the numbers of working days in a factory are 20, 22 and 22 respectively. This regular effect can be identified and removed by the use of regressors in the regARIMA model.

Trading day effects may also reflect how data are recorded more than when the event happened, for example sales for the weekend may be

	Number of days		
Year	2019	2020	2021
Monday	4	5	4
Tuesday	4	5	5
Wednesday	4	4	5
Thursday	4	4	4
Friday	4	4	4
Saturday	5	4	4
Sunday	5	4	4

Table 9.1: Day of week composition for June 2019, 2020 and 2021

recorded on the following Monday. This will cause a regular effect that should be removed from the series.

For quarterly series there is less variation in the possible arrangement of the days of the week.

- There are between 90 and 92 days per quarter with a minimum of 12 occurrences of a particular day and maximum of 14 in any one quarter
- In non-leap years, quarter 1 has 90 days, 1 short of 13 full weeks
- In leap years, quarter 1 has 13 full weeks so is not affected by trading day variation
- Quarter 2 is never affected by trading day variation, since the total number of each day of the week is always equal to 13
- Quarters 3 and 4 have 92 days, 13 full weeks plus one day

As the quarters are all very close to 13 weeks in length, it is difficult to detect and estimate trading day effects in quarterly data. For the majority of series trading days effects are insignificant for quarterly series.

Another effect of the arrangement of the calendar is the leap year effect. A leap year is a year with one extra day inserted into February. The leap year is 366 days with 29 days in February as opposed to the normal 28 days. This effect can cause regular variation in some series and therefore needs to be removed to make a proper comparison between Februarys or quarter ones.

X-13ARIMA-SEATS enables effects associated with arrangement of the calendar to be removed from the series using regressors (for trading days) or constant variables (for leap year). This adjustment is done before the seasonal adjustment takes place. The adjusted series is usually referred to as trading day adjusted series if the series is adjusted for trading days and leap year only or calendar adjusted series if the series is adjusted for trading days, leap year and moving holidays (for example, Easter). Trading day adjusted series and calendar adjusted series are sometimes required by Eurostat and are routinely published in some European countries for their national accounts.

Another type of calendar-related effect, usually grouped with trading day effects, is the effect of variation in the length of calendar periods (months or quarters). To understand the use of this, we can take industrial output in June and July as an example for comparison. The industrial output in June and July may differ systematically for two reasons: firstly, the average output per day may differ from June to July because of differences in demand or other factors; secondly, July has one more day than June. With standard seasonal adjustment both these effects are regarded as part of the seasonal pattern, and the June and July seasonal factors will reflect both. For some purposes it could make the analysis easier to interpret if we regard the first effect as the true seasonal and the second as a calendar effect. If we apply a length of month adjustment[1], the seasonal factors will reflect this true seasonal. The overall seasonal adjustment will be almost unchanged, because length of month and true seasonal together come to the standard seasonal. Note that the length of month adjustment treats all days as equal, so it is inconsistent with trading day adjustment. Also this is not length of working month but length of calendar month, so the example above is only realistic for an industry working seven days a week; in other cases an appropriate length of month variable could be defined as a user variable.

9.3 WHEN TO ADJUST FOR TRADING DAY

X-13ARIMA-SEATS provides a number of diagnostics to test a series for the presence of trading day effects. Testing the statistical significance of trading day regressors is discussed in greater detail in Section 9.5, which describes the recommended procedure for testing and adjusting for trading day effects. In general, where trading day effects are found to be statistically significant, the series should be adjusted to remove these effects from the final seasonally adjusted series. However, it is always important to look at the results of the trading day regression and try to relate it to the time series itself. If the results are counterintuitive (for example, they suggest most car production takes place on Saturdays) then it is worth investigating whether there is anything in the recording of the data which is

1 see Section 9.6 for details

causing this result. It is better not to implement the trading results if they are counterintuitive.

Trading day effects should not be estimated for the following types of data:

- **Data that are collected on a 4, 4, 5 week pattern**. That is to say, the recording periods in a year consists of a four times repeated pattern of a four week recording period, followed by another four week recording period, followed by a five week recording period. This means that the year is divided differently from the calendar periods described by months. This system of data collection does not, in general, exhibit trading day effects as each collection period contains a full number of weeks

- **Data that are collected at a point in time**, for example the third Thursday of a month. However, if the collection day can occur on different days of the week, for example, the first day of a month, it may be that there is an effect depending on the day concerned. This can be estimated using a stock trading variable (see tdstock below)

- **Data that are not collected in strict calendar months**. There may, in this case, be some sort of trading day effect, but this effect should not be estimated for using the regressors provided by X-13ARIMA-SEATS. For further information about adjusting such data contact TSAB

- **Quarterly data**. Whilst it is possible to estimate trading day effects for quarterly flows data using X-13ARIMA-SEATS, trading day effects will, in general, cancel out within the quarter

X-13ARIMA-SEATS provides different diagnostics to test for the statistical significance of trading day regressors.

9.4 OPTIONS AVAILABLE TO ADJUST FOR TRADING DAY EFFECTS

As previously noted X-13ARIMA-SEATS can fit a regression to model certain effects that result from the arrangement of the calendar, such as trading day effects. It is possible for the user to define regressors in the regression spec. X-13ARIMA-SEATS contains a number of predefined variables, contained in the variables argument, to adjust for a variety of calendar effects, including five different regressors that specifically adjust for the effects of trading days; four specifically designed for flow series and one for stock series. Three other options not specifically designed to adjust for trading day effects but related to trading day effects are discussed in Section 9.6; they allow the user to adjust for length of month (lom option), length of quarter (loq option) and leap year (lpyear option) effects.

Using the variables argument in the regression spec allows you to specify one of the following regressors to adjust for trading day effects,

- tdnolpyear this is used to estimate flow trading day effects
- td this is used to estimate flow trading day effects and is a combination of the tdnolpyear variable and the lpyear variable
- td1nolpyear this is used to estimate flow trading day effects
- td1coef this is used to estimate flow trading day effects and is a combination of the td1nolpyear variable and the lpyear variable
- tdstock [w] this estimates a day-of-week effect for stock data or inventories that are reported on the day of each month

Each of these options is discussed in more detail in the following table.

Variable name	X-13ARIMA-SEATS Command	Comments
tdnolpyear	regression{variables=tdnolpyear}	It includes 6 day-of-week contrast variables and it is used for flow series only. The variable compares the number of each weekday to the number of Sundays in the month. Coefficients are estimates for Monday to Saturday and Sunday can be derived as the negative of the sum of the other days. tdnolpyear assumes that each day has a different effect.
td	regression{variables=td}	Includes the tdnolpyear regressor as well as estimating the effects of a leap year. The leap year effect is handled either by re-scaling (for transformed series) or by including the lpyear regression variable (for untransformed series). The td regressor cannot be used in conjunction with tdnolpyear, td1coef, td1nolpyear or tdstock[w] regressors in the regression spec, or the adjust=lpyear, adjust=lom, adjust=loq
td1nolpyear	regression{variables=td1nolpyear}	A weekday-weekend contrast variable that can be used for flow series only. This is more parsimonious than the tdnolpyear option, as there is only one variable in the regression. The difference between the tdnolpyear is that td1nolpyear assumes the same effect for all the weekdays and another for Saturdays and Sundays rather than an effect for each day individually. The td1nolpyear regressor cannot be used in conjunction with td, tdnolpyear, td1coef or tdstock[w] regressors in the regression spec.

Variable name	X-13ARIMA-SEATS Command	Comments
td1coef	regression{variables=td1coef}	Similar to the td regressor in the same way that td1nolpyear is similar to tdnolpyear. This means that td1coef includes the td1nolpyear regressor as well as estimating the effects of a leap year. The leap year effect is handled either by re-scaling (for transformed series) or by including the lpyear regression variable (for untransformed series). If the td1coef regressor is used, neither td, tdnolpyear, td1nolpyear or tdstock[w] regressors can be used in the regression spec.
tdstock[w]	regression{variables=tdstock[31]}	Estimates day-of-week effects for inventories or other stocks that are recorded on the w-th day of the month. This allows the user to specify a value for w (from 1 to 31), where specifying 31 will mean that it is an end of month variable, as it will take this to be the last day of the month for those months with fewer than 31 days. Research suggests that this variable is rarely significant. If the tdstock[w] regressor is used neither td, tdnolpyear, td1nolpyear, td1coef, lom nor loq regressors can be used in the regression spec. Furthermore the tdstock[w] variable cannot be used with quarterly data.

Table 9.2: Trading day regressor options

For further information and description of handling trading day adjustment with regression models used in X-13ARIMA-SEATS[2].

9.5 HOW TO ADJUST FOR TRADING DAY EFFECT

Section 9.2 and Section 9.3 described when to adjust for trading day effects, whilst Section 9.4 introduced the options available in X-13ARIMA -SEATS to adjust for different trading day effects. This section describes, firstly, how to identify the presence of trading day effects in a series, and secondly, the generally recommended process to adjust for trading day effects.

The general order of testing the significance of regressors is described in Chapter 8, which discusses the regARIMA model. Chapter 8 explained how the chi-squared test should be used to test for the significance of trading day effects. However, other methods also exist for detecting the presence of trading day effects such as the AIC test.

This section will describe three ways of identifying whether or not trading day effects are present in a series, and then the procedure for adjusting

2 see Findley et al., 1998

for trading day effects in a production run. Two different scenarios will be outlined to set up the seasonal adjustment for the production run:

- Firstly producing prior adjustments that can be fixed for a year in the production run, in X-13ARIMA-SEATS, X-12-ARIMA and X-11-ARIMA based programs
- Secondly the recommended procedure of setting up the spec file for a production run using a regression variable, rather than permanent priors, which is an option that can be used only in X-12-ARIMA and X-13ARIMA-SEATS based software.

9.5.1 *Testing for trading day effects with X-13ARIMA-SEATS*

Spectral plot

The spectral analysis reveals whether or not significant trading day peaks are found in a seasonal adjustment. Two spectral plots are produced, one from the first differences of the adjusted series, adjusted for extreme values from Table E2 of the output and a second of the final irregular component, adjusted for extreme values from Table E3. From these plots, X-13ARIMA-SEATS will estimate whether any of the peaks at predetermined frequencies (the frequencies are determined by the cyclical nature of the trading day pattern) are significantly different from that of the neighbouring peaks. If the program finds that peaks exist at the trading day cyclical frequency[3] for further information) it will return a warning message in the command prompt such as,

WARNING: At least one visually significant trading day peak has been found in one or more of the estimated spectra

This test is very sensitive and has a tendency to show trading day effects when other diagnostics don't (and, no doubt, sometimes when they don't actually exist). Nevertheless if this warning message is returned, the series should be tested to estimate the significance of trading day effects. If this warning message is returned when a particular trading day regressor has been used, it may be necessary to test a different trading day regressor to see if that performs better. For example if the td regressor has been used, it is possible that using the td1coef regressor will perform better and could remove the trading day peaks. The performance of different trading day regressors can be assessed with the two tests described below but can also be assessed by their impact on the overall performance of the seasonal adjustment. If a warning message is still returned after a significant td

3 see Findley et al., 1998

variable has been included and if resources are limited, it may be better not to examine the series in more detail but just to retain the td variable.

The AIC test The AIC test can be activated in the regression spec to evaluate whether or not a particular regressor is preferred, compared to not having that regressor in the model. For example, the following may be specified,

```
regression{aictest=(td1coef)}

This will generate the following likelihood statistics in the output,

Likelihood statistics for model without td1coef
Likelihood Statistics
 -----------------------------------------------------------------
 Effective number of observations (nefobs)      138
 Number of parameters estimated (np)              3
 Log likelihood (L)                           130.8304
 Transformation Adjustment                   -980.3013
 Adjusted Log likelihood (L)                 -849.4709
 AIC                                         1704.9417
 AICC (F-corrected-AIC)                      1705.1209
 Hannan Quinn                                1708.5104
 BIC                                         1713.7235
 -----------------------------------------------------------------

Likelihood statistics for model with td1coef
Likelihood Statistics
 -----------------------------------------------------------------
 Effective number of observations (nefobs)      138
 Number of parameters estimated (np)              4
 Log likelihood (L)                           130.6730
 Transformation Adjustment                   -980.3013
    Adjusted Log likelihood (L)              -849.6283
 AIC                                         1707.2565
 AICC (F-corrected-AIC)                      1707.5573
 Hannan Quinn                                1712.0148
 BIC                                         1718.9656
 -----------------------------------------------------------------
**** AICC (with aicdiff=0.00) prefers model without td1coef ****
```

In the above example trading day effects do not appear to be present in this particular series, and so the td1coef would not be used to adjust this series for trading day effects. The aictest argument compares the AICC statistics (these are in bold in the above example, only to highlight which statistics are compared) and depending on which model has the lower

AICC statistic, will return a line that states which model it prefers. The above example gives an AICC statistic of 1707.5573 for the model with a td1coef regressor and an AICC statistic of 1705.1209 for the model without the td1coef regressor. Therefore the model without td1coef is preferred. Note that the user should compare AICC values, only when two models differ only with one regressor, such as td1coef.

Note that there is an additional option with aictest, namely the argument aicdiff mentioned in the tables above. The purpose of this is to avoid extreme sensitivity to minor changes in the AICC criterion. If aicdiff is different from the default value of zero, there is a bias in favour of the simpler model (in this case excluding trading day variables); the tested variable is included only if it improves the AICC criterion by at least the amount aicdiff. This could be used in a spec which is routinely re-run in the annual re-analysis, to reduce the risk of instability on update.

Only one trading day regressor can be tested at a time with this option, and therefore to compare the performance of different trading day regressors the AICC statistic of each model would have to be saved and the program re-run using the different regressor. The results of the aictest can be saved in the log file using the savelog argument. For example, the following regression spec would test the td variable against no td variable and save the results of this test in the log file.

```
regression{
   aictest=(td)
   savelog=aictest
   }
```

The AICC statistics and other specified diagnostics that could help evaluate the performance of the seasonal adjustment, with (or without) particular trading day regressors could be compared by saving the log file of each run into another file, for example an Excel spreadsheet.

The aictest can be used to test the following regressors, td, tdnolpyear, td1coef, td1nolpyear, tdstock, Easter, and user-defined regressors. NB the aictest cannot test a specific tdstock[w] variable, only tdstock, which defaults to an end of month variable. If more than one type of variable is tested, the order in which these tests are carried out is first trading day regressors, second Easter regressors, finally user-defined regressors.

When resources are sufficient it is recommended to undertake a detailed analysis of the series to be taken into account as well as the nature of the series and whether or not a particular regressor would seem appropriate. For example, in the case of a flow series, the four regressors, td, tdnolpyear, td1coef, and td1nolpyear should be tested.

If resources are slightly more limited, the td regressor should be tested, and where the aictest prefers the model with the td variable to use the td variable. If the spectral analysis returns a warning that trading day peaks are present when the td variable is specified, the user should, if resources permit, test other trading day regressors in particular the td1coef regressor. The AICC statistic is one of the diagnostics that should be considered in choosing between regressors, where they are found to be significant.

The χ^2 test ant t-values

The chi-squared (χ^2) test is used to test the joint significance of a group of trading day regressors. Therefore the chi-squared test statistic will only be produced in the cases where the td, tdnolpyear and the tdstock[w] variables are used, as these variables include six day-of-week contrast variables. The td1coef and td1nolpyear variables are, for the purposes of trading day effects, only using one variable, the weekday, weekend contrast variable and hence the t-value will provide an indication of the significance of these variables. When a variable has been specified in the variables argument of the regression spec then t-values will be estimated for the individual regressors and the chi-squared test will test for the joint significance of those variables that have six day-of-week contrast variables. In the following example the variables included in the variables argument are td and Easter[1]. The chi-squared test is testing the joint significance of the trading day variables only.

```
  Regression Model
-----------------------------------------------------------------------
                         Parameter    Standard
  Variable               Estimate       Error    t-value
-----------------------------------------------------------------------
  Trading Day
     Mon                  0.0089     0.00581     1.54
     Tue                  0.0013     0.00591     0.22
     Wed                  0.0256     0.00601     4.26
     Thu                  0.0109     0.00590     1.84
     Fri                 -0.0012     0.00609    -0.19
     Sat                 -0.0177     0.00605    -2.92
    *Sun (derived)       -0.0278     0.00594    -4.68

  Easter[1]               0.0281     0.01160        2.42
----------------------------------------------------------------------
 *For full trading-day and stable seasonal effects, the derived
  parameter estimate is obtained indirectly as minus the sum
  of the directly estimated parameters that define the effect.

  Chi-squared Tests for Groups of Regressors
```

```
-----------------------------------------------------------------
   Regression Effect       df    Chi-Square  P-Value
-----------------------------------------------------------------
   Trading Day             6       167.19     0.00
```

As the above example shows, the t-values for some days are not significant, that is the coefficients are not significantly different from zero. However, the chi-squared test for the trading day effect gives a p-value of 0.00 which is within the recommended 5% limit (less than or equal to 0.05) for accepting significant trading day effects.

The t-values should be used as a test of significance for the td1coef and td1nolpyear variables, whereas the chi-squared test should be used to check the overall significance of either the td, tdnolpyear, or the tdstock[w] regressors. If a variable is found to be significant, it should, in general, be included in the model. If all the trading day regressors are found to be significant, the AICC test and diagnostics for assessing the quality of the seasonal adjustment should be used to determine which of them should be selected.

9.5.2 *Adjust for trading day effects*

Use one of the following three methods to adjust for trading day effects. Regression Variable with Test of Significance

Once it has been decided which trading day regressor should be used to adjust for the trading day effect, the aictest described in Section 10.5.1.2 can be used to adjust for the effect. This means that every time X-13ARIMA-SEATS runs to estimate the seasonally adjusted series, AIC values are derived for models with and without the specified trading day variable and the optimal model will be used for forecasting. This option will provide users with the optimal seasonally adjusted series, but will generate higher revisions than the other two methods which will be described in the following sections.

This option should not be used for production run because the trading day regressor variable is not fixed and frequent switching between models caused by updating can cause undesirable instability in the seasonally adjusted output. If there are some special reasons as to why this option must be used, a nonzero value of the aicdiff argument should be used to reduce instability.

Regression variables fixed in the model

If a particular regression variable has been identified as significant and the best one to use for the seasonal adjustment, then the regression spec should contain in the variables argument all the regressors that should be included. For example, if, as in the previous example, Easter[1] and td are found to be significant, the spec file, along with the other parameters that have been fixed should include the following arguments in the regression spec.

```
regression{
   variables=(Easter[1] td)
   }
```

This is the recommended option for the production run because the form of the chosen regressor is fixed though the parameter values, the trading day factors, are re-estimated each period. When a particular trading day regressor is fixed in the model, this should be included even if it appears no longer to be significant. The decision of whether to remove the variable or not should be taken at the point of a seasonal adjustment review, to reduce the revisions.

If a strict revisions policy is in place it is suggested not to use this option since the estimated trading day factors will change when new data become available.

Permanent Prior Adjustments

Once a particular regressor has been chosen see above and Chapter 8, on regARIMA then the trading day factors can be saved to obtain permanent prior adjustments through the following five steps.

1. Run the spec file with your chosen settings, for example, decomposition, ARIMA model etc. Include in the regression spec with the variables and save arguments activated (NB, in order to save the trading day factors obtained from any trading day variable the appropriate argument is save = td for all variables). For example, if the td regressor was found to be the most appropriate the regression spec would look like this:

   ```
   regression{
      variables=(td)
      save=(td)
      }
   ```

2. The save argument will save a text file with the trading day factors in the same directory that the log and output files are saved. If the

name of the spec file was "filename.spc" this would mean that the trading day factors would be saved in a file called "filename.td"

3. If permanent priors are required for more than one year, then the forecast spec should be used to set the number of forecast periods and appendfcst=yes option should be included in the "x11" spec. For example, the following, assuming monthly data, would provide trading day factors for three years into the future:

```
regression{
   variables=(td)
   save=(td)
   }

forecast{
   maxlead=36
   }

x11{
   appendfcst=yes
   }
```

4. It is essential to save the ".td" file with a different name (for example: "[filename]pp.td") otherwise it will be overwritten the next time the spec file would be run with the variables=(td) option

5. In order to set up the spec file for a production run, remove the regression spec and use the prior adjustments that have been saved in step 4 by using the transform spec. Therefore, in the final spec file, used for production runs, the regression spec is not used and the transform spec is activated, as in the following:

```
transform{
   file=(filenamepp.td)
   format=x12 save
   type=(permanent)
   }
```

It should be noted that the above example has given a certain method of saving and using the permanent priors in a particular format and so on. There are various ways in which permanent priors can be saved and used to transform the original series. For further information on the different options available see section 7.18 of the X-13ARIMA-SEATS manual (USCB 2017).

If holiday (such as Easter) or user-defined regressor variables are used to estimate other effects that are required to be used as permanent priors, then the factors must be multiplied or added (depending on the transformation used) together so that they are all in one file to be used in the transform spec. For example, if Easter[1][4] and td variables have been estimated and the holiday and trading day factors have been saved, save=(hol td) is the appropriate argument in the regression spec, which saves the respective factors in separate files with the name of the spec file and the extension ".hol" and ".td" respectively, then these factors should be multiplied or added together and following step 4 and step 5 will mean that the original series is adjusted by the prior adjustment file so that the seasonal adjustment is then performed on a series with these calendar effects removed, which should improve the quality of the seasonal adjustment.

Problems:

- With the permanent priors option, the adjustment for trading day factors is fixed as they are based only on the data that are available at the point in time when the trading day effects are being estimated (during the annual review). If the trading day pattern changes between annual reviews, the permanent priors will not capture this modification so the seasonally adjusted series will not be optimal.
- It is very complicated to set up and keep up to date..

Criteria for deciding which of the three methods should be used are as follows.

1. Revisions policy: how strict the revisions policy is determines which approach to use in the production runs. The use of permanent priors is the method that gives the minimum revisions, but also gives the less optimal seasonally adjusted series and it is very complicated to set up and keep up to date. The second option in terms of revision size is fixing the regression variables in the model but allowing the coefficients to be re-estimated when new data are available. This is the recommended method since it balances a fairly good quality of seasonal adjustment with revision and practicality. Contrary, the use of a regression variable with a test for significance will provide the optimal seasonal adjustment but with the biggest revisions of the three methods

2. Seasonal Adjustment Review: for reviewing the parameters the regression variable with a test for significance should be used. It provides users with the optimal model to run the seasonal adjustment.

4 see Chapter 10

9.6 RELATED TOPICS

Section 9.3 gave specific details on the options available in X-13ARIMA - SEATS which allow the user to adjust for trading day effects. Three related options that in effect also make adjustments related to average daily effects in flow series are: the length of month lom, length of quarter loq and leap year lpyear. Each of these options is discussed in the following table. These options are very rarely specified in practice.

Variable Name	X-13ARIMA-SEATS Command	Comments
lom	regres-sion{variables=lom}	Can be used to adjust a series for effects resulting from the differing of length of a month. The regression includes a length-of-month regression variable. Cannot be used with the td, td1coef or tdstock[w] variables.
loq	regres-sion{variables=loq}	Same as the lom option, but is used in the case of quarterly data. Cannot be used with the td, td1coef or tdstock[w] variables.
lpyear	regres-sion{variables=lpyear}	Contrast variable for leap year effects. This variable can only be used with flow series and cannot be used in conjunction with the td or td1coef variables.

Table 9.3: Length of time regressors used in flow series

Adjustments for length of month, length of quarter and leap years can also be made in the transform spec[5].

9.7 NON-CALENDAR DATA

Some data do not align strictly to calendar months and some common problems that can arise under these circumstances are:

1. Bank holidays can switch between months. For example, in a 4-4-5 week pattern of collecting data, the late May Bank Holiday might fall in the "May" collection period in some years and "June" in others. These can sometimes be modelled in X-13ARIMA-SEATS. TSAB can be contacted for more details
2. Conventional trading day adjustments are usually inappropriate, but, particularly if the recording periods are not the same length between years, they might have a large impact on the series

5 for more details see section 7.18 of the X-13ARIMA-SEATS (2017)

3. Sometimes survey questionnaires ask for respondents to record the exact period the response refers to, and crude adjustments are made to calendarise the data at respondent level in the ONS. This is a difficult area; TSAB has some experience of dealing with this problem and can be contacted for advice

4. "*Phase shift*" effects might occur, particularly within a 4-4-5 week data collection pattern. This is because a pattern of 4 weeks, 4 weeks, 5 weeks repeated throughout the year adds up to only 52 weeks, or 364 days, but there are 365 days in a normal calendar year and 366 in a leap year. This means that the collection periods will be earlier and earlier in each successive year. Indeed, in surveys which operate under this regime, such as the Retail Sales Inquiry or the Labour Force Survey, either a "survey holiday week" is taken or an extra week's data collection is added to a 4 week collection period every 5 or 6 years in order to roughly realign the collection periods with calendar months. However, these moving reference periods can have an impact on the time series, particularly in the presence of strong seasonal patterns. These can sometimes be modelled in X-13ARIMA-SEATS and removed from the data. Ask TSAB for more details, if you encounter this problem.

10 EASTER AND OTHER MOVING HOLIDAYS

10.1 INTRODUCTION

Easter Sunday is a moving holiday that can occur in either March or April, for monthly data, or in the first or second quarter for quarterly data. The effect on series caused by movements in the date of Easter needs to be removed from them seasonally adjusted series that we produce. X-13ARIMA-SEATS estimates calendar effects, such as Easter effects, by adding regressors to the regARIMA model. This chapter explains when and how to adjust for the effects of Easter. Section 10.2 illustrates the Easter effects, Section 10.3 describes when to adjust for Easter effects, Section 10.4 provides details on the options available in the X-13ARIMA-SEATS program to adjust for Easter effects, Section 10.5 explains the recommended procedure to adjust for Easter effects and Section 10.6 lists some related topics.

10.2 THE EASTER EFFECTS

The date of Easter Sunday is the first Sunday after the first full moon after the 21 of March, and based on the Gregorian calendar can be anywhere between the 22 March and 25 April. As with seasonal effects, it is desirable to estimate and remove Easter effects from time series to help interpretation. The effects of Easter can be best understood by considering an example. We would expect sales of chocolate to be higher in the days and weeks before Easter. If Easter occurs in March all of the additional expenditure will occur in March. If however, Easter falls on the 25 April, the sales in March will be lower than the previous case and higher in April. These effects need to be removed from the seasonally adjusted series. A similar but opposite effect may be present in series of industrial production, where there may be lower production in months in which Easter falls because fewer days are worked. This effect can be removed by the X-13ARIMA-SEATS program.

If no Easter correction is made by X-13ARIMA-SEATS, the additional sales will initially join the SI ratios. The monthly SIs are then smoothed, leaving the Easter effect in the irregular component. The final seasonally adjusted series will thus show systematic peaks and troughs caused by the effects of Easter.

In this respect, Easter effects can be considered similar to trading day effects, and removing the Easter effect from the seasonally adjusted series improves the quality of the seasonal adjustment. As with trading day effects, Easter effects can be estimated in the X-13ARIMA-SEATS program using the **regression** spec. There are three options within this spec that provide pre-defined regressors that will adjust for the effects of Easter. The regression spec also allows one further option, a user-defined **regressor**, which allows users to define their own Easter holiday regressor.

10.3 WHEN TO ADJUST FOR EASTER EFFECT

This section examines some methods that can be used to identify the presence of Easter effects. An advantage of X-13ARIMA-SEATS over X-11-ARIMA is the use of the AIC test in the regression spec that provides diagnostics for assessing whether an Easter regressor should be included or not. However, there are other indicators that can show the presence of an Easter effect, such as a graphical analysis of the non seasonally adjusted and seasonally adjusted (without an Easter adjustment) series, the SI ratios plot, and the E5 and E6 tables given in the output. Each of these approaches is discussed in turn. The AIC test for Easter regressors is discussed in more detail in Section 10.5, where a brief introduction to the test is provided.

The X-13ARIMA-SEATS program provides a number of diagnostics to test a series for the presence of Easter effects. Testing the statistical significance of Easter regressors is discussed in greater detail in Section 10.5, which describes the recommended procedure for testing and adjusting for Easter effects. In general, where Easter effects are found to be statistically significant, the series should be adjusted to remove these effects from the final seasonally adjusted series. However, it is always important to look at the results of the build up period (w) and try to relate it to the time series itself. If the results are counterintuitive (for example, they suggest most passengers travel the day before Easter, where w=1) then it is worth investigating whether there is anything in the recording of the data which is causing this result. It is better not to implement the Easter results if they are counterintuitive.

Easter effects should not be estimated with the default X-13ARIMA-SEATS regressors for the following types of data:

- Data that are collected on a 4-4-5 week pattern. That is to say, the recording periods in a year consists of a four times repeated pattern of a four week recording period, followed by another four week recording period, followed by a five week recording period. This means that the year is divided differently to the calendar periods described by months. There may, in this case, be some sort of Easter effect; however, this effect should not be estimated for using the regressors provided by X-13ARIMA-SEATS

- Data that are collected at a point in time for example the third Thursday of a month or the 1st of the month. There may, in this case, be some sort of Easter effect; however, this effect should not be estimated for using the regressors provided by X-13ARIMA-SEATS. There is a default regressor for the case when data are collected on the last day of each month
- Data that are not collected in strict calendar months. Also, in this case there may be some sort of Easter effect; however, this effect should not be estimated for using the regressors provided by X-13ARIMA-SEATS. For further information about adjusting such data contact TSAB

X-13ARIMA-SEATS provides different diagnostics to test for the statistical significance of Easter regressors.

10.4 OPTIONS AVAILABLE TO ADJUST FOR EASTER EFFECTS

The X-13ARIMA-SEATS program fits a regARIMA model to estimate Easter effects and trading day effects. It is possible for the user to define regressors in the regression spec, using the user and usertype arguments. However, the X-13ARIMA-SEATS program also contains pre-defined variables in the variables argument, to adjust for a variety of calendar effects, including 3 different regressors that specifically adjust for the effects of Easter. The 3 regressors are Easter[w], SCEaster[w] and EasterStock[w]. Each of these regressors has a parameter that is required which details the length of the build-up period.

Variable Name	X-13ARIMA-SEATS Command	Comments
Easter[W]	regression{ variables=Easter[W]}	The value of w must be supplied and states the number of days before Easter for which the level of daily activity changes, due to caused by the effects of Easter. The new level of activity remains the same from the w^{th} day to the day before Easter. w can take any value from 1 to 25. Hence, Easter[1] would mean that the holiday effect is estimating the change in the level of daily activity for the Saturday before Easter, whereas Easter[8] assumes the change in the level of activity occurs for the 8 days before Easter Sunday. It is possible to specify more than one of these variables, with different values for w in order to estimate complex effects
SCEaster[w]	regression{ variables=SCEaster[w]}	The regression variable in this case is a Statistics Canada holiday regression variable. This regression variables assumes that the level of daily activity changes on the (w-1)th day and remains at the new level through to Easter Sunday. w must be supplied and can take any value from 1 to 24. It is possible to specify more than one of these variables, with different values for w in order to estimate complex effects
Easterstock[w]	regression{ variables=Easterstock[w]}	End of month stock Easter holiday regressor. This is generated from the Easter[w] regressor. The value of w can range from 1 to 25
Easterstock[w]	regression{ variables=Easterstock[w]}	Similar to the td regressor in the same way that td1nolpyear is similar to tdnolpyear. This means that td1coef includes the td1nolpyear regressor as well as estimating the effects of a leap year. The leap year effect is handled either by re-scaling (for transformed series) or by including the lpyear regression variable (for untransformed series). If the td1coef regressor is used, neither td, tdnolpyear, td1nolpyear or tdstock[w] regressors can be used in the regression spec
user	regression{ user=Easter file="Easter.txt"}	The user argument must be used in conjunction with the usertype and data or file arguments in the regression spec. This allows a user-defined regression variable to be used, such asie the UK equivalents to the pre-defined regressors in X-13ARIMA-SEATS. If the file is held in a different directory then the path should also be specified, The values in this file should cover the time frame of data including forecasts and backcasts specified in the forecast spec

Table 10.1: Predefined Easter regressor options

For further information and description of handling Easter adjustments with the regression model used in X-13ARIMA-SEATS see Findley et al., 1998.

10.5 HOW TO ADJUST FOR EASTER EFFECT

Section 10.2 and Section 10.3 described when to adjust for Easter effects, whilst Section 10.4 introduced the different options available in X-13ARIMA-SEATS to adjust for Easter effects. This section describes in more detail the process of using the aictest argument in the regression spec to test the Easter[w] and user options. This section also describes the generally recommended procedure to adjust for Easter effects.

The general order for testing the significance of regressors is described in Chapter 8, which discusses the regARIMA model. Chapter 10 explained how the t-value should be used to test for the significance of the chosen Easter regressor. However, other methods, such as the AIC test, also exist for detecting the presence of Easter effects and determining the build up period (the value of w in the Easter[w] option). This section will describe three ways of identifying whether or not Easter effects are present in a series, and then the procedure for adjusting for Easter effects in a production run. Two different scenarios will be outlined for setting up the seasonal adjustment for the production run:

- Firstly, the recommended procedure of setting up the specification file for a production run using a regression variable which is an option that can only be used in X-12-ARIMA based programs
- Secondly, producing prior adjustments that can be fixed for a year's production run in both X-12-ARIMA and X-11-ARIMA based programs.

10.5.1 *Testing for Easter effects with X-13ARIMA-SEATS*

The AIC test

The AIC test can be activated in the regression spec to evaluate whether or not a particular regressor is preferred, compared to not having that regressor in the model. For example the following may be specified:

```
regression{
   aictest=(Easter)
   }
```

This will test four models: no Easter regressor, Easter[1], Easter[8] and Easter[15].The model with the lowest AICC value will be the one that is preferred.

This generates the following tables in the output:

```
MODEL ESTIMATION/EVALUATION
 Exact ARMA likelihood estimation
 Max total ARMA iterations     1500
 Max ARMA iter s w/in an IGLS iterati 40
 Convergence tolerance      1.00E-05

Likelihood statistics for model without Easter
Likelihood Statistics
-----------------------------------------------------------------
Number of observations (nobs)                  255
Effective number of observations (nefobs)      242
Number of parameters estimated (np)              9
Log likelihood                             727.8055
Transformation Adjustment                -1064.5352
Adjusted Log likelihood (L)               -336.7297
AIC                                        691.4593
AICC (F-corrected-AIC)                     692.2352
Hannan Quinn                               704.1086
BIC                                        722.8598
-----------------------------------------------------------------

Likelihood statistics for model with Easter[1]
Likelihood Statistics
-----------------------------------------------------------------
Number of observations (nobs)                  255
Effective number of observations (nefobs)      242
Number of parameters estimated (np)             10
Log likelihood                             730.4800
Transformation Adjustment                -1064.5352
Adjusted Log likelihood (L)               -334.0551
AIC                                        688.1103
AICC (F-corrected-AIC)                     689.0626
Hannan Quinn                               702.1650
BIC                                        722.9996
-----------------------------------------------------------------

Likelihood statistics for model with Easter[8]
Likelihood Statistics
-----------------------------------------------------------------
Number of observations (nobs)                  255
Effective number of observations (nefobs)      242
Number of parameters estimated (np)             10
Log likelihood                             735.5112
Transformation Adjustment                -1064.5352
```

```
Adjusted Log likelihood (L)                -329.0240
AIC                                         678.0480
AICC (F-corrected-AIC)                      679.0004
Hannan Quinn                                692.1027
BIC                                         712.9374
----------------------------------------------------------------


Likelihood statistics for model with Easter[15]
Likelihood Statistics
----------------------------------------------------------------
Number of observations (nobs)                   255
Effective number of observations (nefobs)       242
Number of parameters estimated (np)              10
Log likelihood                              740.3271
Transformation Adjustment                 -1064.5352
Adjusted Log likelihood (L)                -324.2081
AIC                                         668.4161
AICC (F-corrected-AIC)                      669.3685
Hannan Quinn                                682.4708
BIC                                         703.3055
----------------------------------------------------------------
*** AICC (with aicdiff= 0.0000) prefers model with Easter[15] ***
```

The models are compared on the basis of the AICC values, and the one with the lowest chosen - in the above example Easter[15] - returns the lowest AICC value, at 669.3685. In the above example aicdiff= 0.00 means that the regressor selected is the regressor with the lowest AICC value.

It is possible to specify that the AICC has to be at least a given amount lower than the AICC for no regressor before it is selected. This is achieved by using the **aicdiff = n** argument where n can take any value. For example if **n** = 100 the AICC value of any Easter regressor must be at least 100 less than the AICC of no regressor before it is included. In the above example if **aicdiff = 100** then none of these Easter regressors would have been chosen.

This test checks only the three Easter regressors described above. If other regressors are also specified in the aictest argument, then the tests are performed sequentially; trading day regressors, then Easter regressors and then user-defined regressors. In order to test a specific Easter[w] or SCEaster[w] regressor, X-13ARIMA-SEATS should be run once with and once without the regressor, each time saving the AICC values using the savelog argument in the **regression** spec as shown below:

```
regression{
   variables=Easter[w]
```

```
  savelog=aictest
  }
```

The AICC values can then be compared across different regressors or no regressor. This option is significantly more time consuming and would be necessary only if further analysis was deemed appropriate, if for example, the regressor automatically chosen appeared to perform poorly.

The recommended procedure is to use the aictest=Easter argument and use the chosen regressor either to estimate permanent priors or to set regression variables for a production run (see below).

If the preferred model is any of Easter[1], Easter[8] or Easter[15] then, in general the chosen regressor should be used to adjust the series for Easter effects. A final check should be done to verify if the t-value of the selected regressor is statistically significant.

The AIC test for a user-defined regressor variable is similar to that described above, but the AICC values compared are for a model with and for a model without the specified regressor. The AIC test is activated as shown below:

```
regression{
  aictest=(user)
  }
```

This should be used if a user-defined regressor has been constructed to adjust for Easter effects. Again, the model with the lowest AICC value is chosen, and again it is possible to use the aicdiff argument. If the user-defined Easter regressor is preferred, then it should be used to adjust for the effects of Easter. If a series has been seasonally adjusted without any adjustment marginparGraphical analysis of the non seasonally and seasonally adjusted datafor the effects of Easter, a plot of the non seasonally adjusted (NSA) and seasonally adjusted (SA) series may reveal the presence of an Easter effect. Figure 10.1 provides an example of a time series exhibiting an Easter effect, which has not been corrected for, when seasonally adjusted. The seasonally adjusted series contains systematic effects around Easter. The effect is particularly noticeable in 1997, where there is an early Easter, 30 of March. But as the graph shows this effect is compensated for in the March of other years, which results in residual seasonality in the adjusted (SA) series. If an adjustment were made to account for the Easter effects this would significantly improve the quality of seasonal adjustment.

Graphical analysis of the unmodified SI ratios

If a series has been seasonally adjusted , without any adjustment for the effects of Easter, a plot of the unmodified SI ratios for March and April

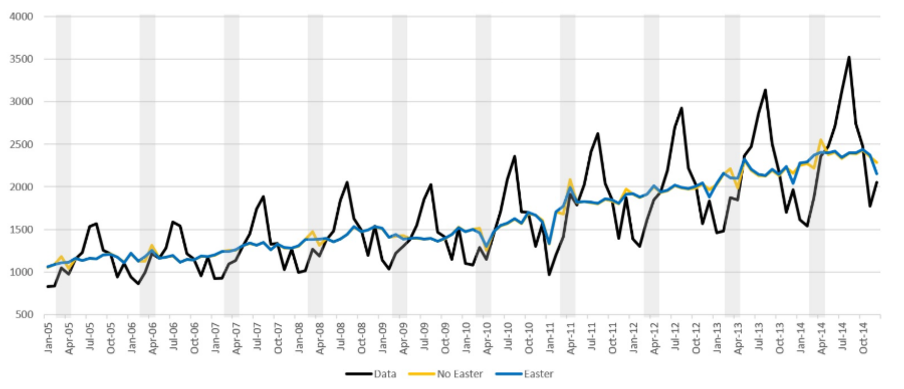

Figure 10.1: Seasonally adjusting with and without an Easter effect

may reveal the presence of an Easter effect. The graph below show the SI ratios for March and April. In this example, the March SI ratio is lower

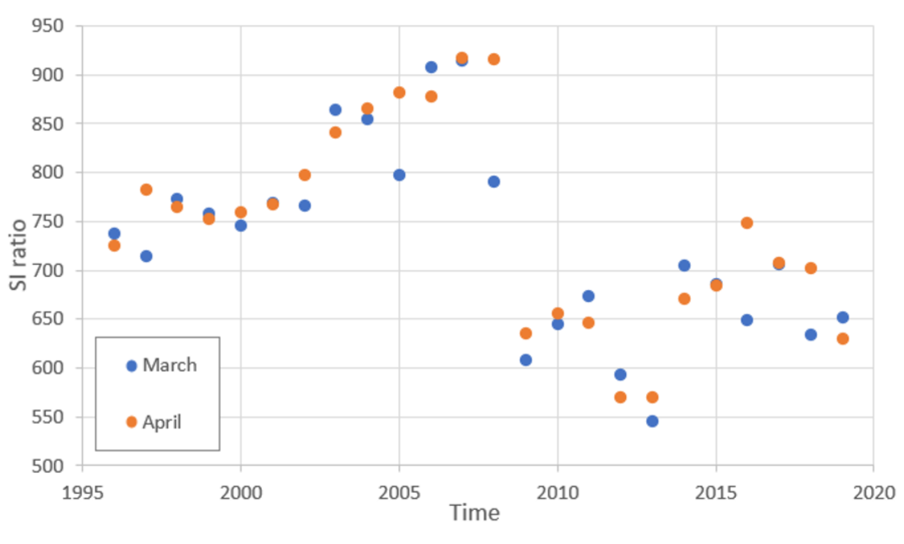

Figure 10.2: SI ratios for March and April for the series showing Easter effects

than the April SI ratio when Easter is in March (1997, 2002, 2005, 2008, 2013 and 2016). Note also that in 2018, Easter fell on 1 April, so that Good Friday was in March.

E5 and E6 output tables

The E5 table produced in the output gives the month-to-month or quarter-to-quarter percentage change in the original series. Therefore, if an Easter effect is present, it may be possible to see different changes in March and April (or quarter 1 and quarter 2) of those years where there is an early or late Easter. For example, Figure 10.3 shows the figures from an E5 table. Table E5 shows the percentage change from month to month or quarter to quarter in the seasonally adjusted series. In the same way as above for the E5 table, it may be possible to see a change in the values for March or

April in years where there is an early Easter relative to years without an early Easter. There is a noticeable change, over 6, in the month-to-month changes for April in 2000 and 2011, which have late Easters. In other years the April value tends to be just around 2 or 3. April also has negative values for 2012 and 2013, when Easter fell earlier on 8 April and 31 March respectively.

E 5 Month-to-month percent change in the original series
From 1996.Feb to 2017.Mar
Observations 254

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	AVGE
1996		4.4	2.1	2.6	-1.7	2.8	0.4	-2.4	2.0	3.5	9.0	16.0	3.5
1997	-26.3	2.1	2.6	2.5	-1.6	3.4	1.7	-3.4	0.2	5.6	8.3	16.9	1.0
1998	-26.0	1.1	0.9	3.3	-0.3	0.2	2.2	-1.7	-0.8	3.2	9.0	15.6	0.6
1999	-25.2	0.7	2.6	1.0	1.8	0.3	1.6	-1.9	-1.0	4.5	9.7	16.2	0.9
2000	-24.1	-0.8	1.0	6.4	-2.3	0.0	1.3	-1.2	-0.5	4.3	9.2	18.7	1.0
2001	-27.6	0.8	1.6	3.9	2.1	-0.7	1.6	-2.5	0.6	3.7	9.1	19.7	1.0
2002	-28.1	2.3	3.0	3.2	-0.7	-2.6	3.4	-3.3	0.9	4.9	9.4	15.0	0.6
2003	-26.9	1.7	2.5	3.6	-2.2	1.4	1.7	-2.6	1.3	4.5	8.8	15.1	0.7
2004	-25.2	0.3	2.8	3.7	0.4	0.5	0.7	-2.2	0.7	4.7	8.4	13.3	0.7
2005	-26.4	1.0	3.4	1.9	0.3	1.3	0.8	-3.2	0.7	4.0	9.5	16.6	0.8
2006	-27.6	1.2	3.0	4.0	0.8	1.1	0.6	-2.2	-0.6	4.0	9.1	18.3	1.0
2007	-29.5	2.7	4.7	3.6	-0.2	0.9	0.0	-1.9	0.4	3.7	9.8	16.3	0.9
2008	-27.0	3.6	1.7	1.7	4.5	-1.4	0.7	-2.6	-0.1	2.5	8.1	12.9	0.4
2009	-25.8	0.0	3.9	3.4	0.5	2.5	0.1	-1.9	0.0	4.5	8.5	13.4	0.8
2010	-28.4	3.9	4.7	1.3	2.3	0.2	0.9	-2.4	-0.6	4.9	9.4	12.3	0.7
2011	-24.4	1.2	3.0	6.0	-1.8	0.4	1.4	-2.8	0.9	4.7	8.2	16.5	1.1
2012	-27.9	1.3	6.1	-0.8	1.7	0.1	0.9	-2.3	1.3	2.9	8.0	16.5	0.6
2013	-28.6	4.0	3.7	-1.2	4.3	1.0	1.6	-3.6	1.3	2.2	8.7	17.7	0.9
2014	-28.6	2.6	3.8	3.2	0.0	1.6	0.1	-2.9	-0.5	5.1	10.4	13.3	0.7
2015	-27.7	0.9	5.1	0.3	2.3	1.0	0.1	-4.0	2.1	3.1	11.2	10.5	0.4
2016	-25.1	0.1	3.5	2.3	3.1	-1.1	3.1	-3.3	0.6	6.8	10.6	11.5	1.0
2017	-28.3	3.6	1.8										-7.7
AVGE	-26.9	1.7	3.1	2.7	0.6	0.6	1.2	-2.6	0.4	4.2	9.2	15.3	

Table Total- 204.63 Mean- 0.81 Std. Deviation- 9.60
Min - -29.48 Max - 19.74

Figure 10.3: Example of E5 table

Table E6 from X13 output shows the month-to-month or quarter-to-quarter percentage change in the seasonally adjusted series. In the same way as above for the E5 table, it may be possible to see a change in the values for March or April in years where there is an early Easter relative to years without an early Easter.

10.5.2 *Adjust for Easter effects*

Use one of the following three methods to adjust for Easter effects:

Regression variable with test of significance

Once it has been decided which Easter regressor should be used to adjust for the Easter effect, the aictest described in Section 10.5.1 can be used to adjust for the effect. This means that every time X-13ARIMA-SEATS is run to estimate the seasonally adjusted series, AIC values are derived for models with and without the specified Easter variable and the optimal model will be used for forecasting. This option will provide users with the optimal seasonally adjusted series, but will generate higher revisions than the other two methods described in the following sections.

Problems:

This option should not be used for a production run because the Easter regressor variable is not fixed and frequent switching between models caused by updating can introduce undesirable instability in the seasonally adjusted output. If there is a special reason why this option must be used, a non-zero value for the **aicdiff** argument should be used to reduce instability

Regression variables fixed in the model

If a particular regression variable has been identified as significant and the best one to use for the seasonal adjustment, then the regression spec should contain in the variable argument all the regressors that should be included. For example, if, as in the previous example, Easter[1] and td are found to be significant, the spec file, along with the other parameters that have been fixed should include the following arguments in the regression spec:

```
regression{
   variables=(Easter[1] td)
   }
```

As with the trading-day regressors, this is the recommended option for the production run because the form of the chosen regressor is fixed though the parameter values, the Easter factors, are re-estimated each period. When a particular Easter regressor is fixed in the model, this should be included even if it appears to no longer be significant. The decision of whether to remove the variable or not should be taken at the point of a seasonal adjustment review, so as to reduce the number of revisions.

Problems:

If a strict revisions policy is in place, it is suggested not to use this option since the estimated Easter factors will change when new data become available

Permanent prior adjustments

Once a particular regressor has been chosen (see above and chapters on regARIMA and on procedures for analysing series with X-13ARIMA-SEATS) then the Easter factors can be saved in order to obtain permanent prior adjustments. The following steps should be taken to obtain permanent prior adjustments.

1. Run the spec file with your chosen settings, for example, decomposition, ARIMA model and so forth. Include in the regression spec with the variable and save arguments activated (NB, in order to save the Easter factors obtained from any Easter variable the appropriate argument is **save = hol** for all variables). For example, if the **Easter[1]** regressor was found to be the most appropriate the regression spec would look like this,

```
regression{
   variables=(Easter[1])
   save=(hol)
   }
```

2. The save argument will save a text file with the Easter factors in the same directory that the log and output files are saved. If the name of the spec file was "filename.spc" this would mean that the Easter factors would be saved in a file called "filename.hol"

3. If permanent priors are required for more than one year, then the forecast spec should be used to set the number of forecast periods and the appendfcst=yes option should be included in the x11 spec. For example, the following, assuming monthly data, would provide Easter factors for two years into the future,

```
regression{
   variables=(Easter[1])
   save=(hol)
   }

forecast{
   maxlead=24
   }
x11{
```

```
   appendfcst=yes
   }
```

4. It is useful to save the ".hol" file with a different name (for example "filenamepp.hol") otherwise it will be overwritten the next time the spec file is run with the variables=(Easter[1]) and save = (hol) options

5. In order to set up the spec file for a production run, remove the regression spec and use the prior adjustments that have been saved in step 4 by using the transform spec. Therefore, in the final spec file, used for production runs, the regression spec is not used and the transform spec is activated, as in the following,

```
transform{
   file=(filenamepp.hol)
   format = x12save
   type = (permanent)
   }
```

It should be noted that the above example has given a certain method of saving and using the permanent priors in a particular format. There are various ways in which permanent priors can be saved and used to transform the original series[1].

If trading day or user-defined regressor variables are used to estimate other effects that are required to be used as permanent priors, then the factors must be multiplied or added together so that they are all in one file to be used in the transform spec. For example, if Easter[1] and td variables have been estimated and the holiday and trading-day factors have been saved (save=(hol td) is the appropriate argument in the regression spec, which saves the respective factors in separate files with the name of the spec file and the extension ".hol" and ".td" respectively) then these factors should be multiplied or added together, and following steps 4 and 5 will mean that the original series is adjusted by the prior adjustment file so that the seasonal adjustment is then performed on a series with these calendar effects removed, which should improve the quality of the seasonal adjustment.

1 for further information on the different options available see section X-13ARIMA-SEATS, 2017

Problems:

with the permanent priors option, the adjustment for Easter factors are fixed as they are only based on the data that are available at the point in time when the Easter effects are being estimated (during the annual review). If the Easter pattern changes between annual reviews the permanent priors will not capture this modification, so the seasonal adjusted series will not be optimal. It is very complicated to set up and keep up to date.

Criteria for deciding which of the three methods should be used are as follows:

1. *Revisions Policy:* How strict the revisions policy is determines which approach to use in the production runs. The use of permanent priors is the method that gives the minimum revisions, but also gives the less optimal seasonally adjusted series and it is very complicated to set up and keep up-to-date. The second option in terms of revision size is fixing the regression variable in the model. This is the recommended method since it balances a fairly good quality of seasonal adjustment with revisions and practicality. Contrary, the use of a regression variable with a test for significance will provide the optimal seasonal adjustment but with the biggest revisions of the three methods

2. *Seasonal Adjustment Review:* for reviewing the parameters the regression variable with a test for significance should be used. It provides users with the optimal model to run the seasonal adjustment.

10.5.3 *The US/UK problem*

As the U.S. Census Bureau developed the X-13ARIMA-SEATS program, the regressors that are defined in the program have been constructed to account for the holiday as it occurs in the United States. When these regressors are used on UK data, they are theoretically wrong. For example, the Easter[1] regressor in X-13ARIMA-SEATS accounts only for Easter Saturday, while the UK one should account for three extra public holidays, Good Friday, Easter Sunday and the bank holiday Monday.

It is possible to construct a regressor that follows the public holidays associated with Easter in the UK. The UK regressor can be computed so as to be a UK equivalent to the Easter[w] regressors discussed above. If a UK regressor is deemed necessary, then it must be specified using the user-defined regressor, as shown above, and can be compared using the aictest=user argument in the regression spec.

As empirical testing suggests that there is little difference between UK and US regressors, it is sufficient to use the pre-defined regressors. However, if a detailed analysis is required, or there is some question over the performance of the pre-defined regressors, then using a UK defined Easter regressor may be appropriate. TSAB have investigated the impact of UK Easter regressors on a number of ONS series, a paper on this can be found in the ONS Survey Methodology Bulletin. For further information contact TSAB.

10.6 RAMADAN EFFECT AND OTHER MOVING HOLIDAYS

Other examples of moving holidays include Chinese New Year, Diwali, and Holi. Within X-13ARIMA-SEATS, only Easter has predefined moving holidays. This section shows how to produce user- defined regressors for seasonally adjusting the effects of moving holidays, using the example of Ramadan. While all these moving holidays have dates based on the lunar calendar, regressors can be created to account for any moving holiday with predictable dates.

Ramadan is a month in the Islamic calendar, which repeats every twelve lunar cycles, and is a moving holiday in the Gregorian calendar. Ramadan starts at the first sighting of the new moon, and lasts for 29 or 30 days, depending on the lunar cycle. Ramadan begins every calendar year approximately 11 days earlier than in the previous year. For example, Ramadan was between 12/04/2021 and 12/05/2021 and between 01/04/2022 and 01/05/2022 in the UK. When recorded in the Gregorian calendar, the effect caused by movements in the date of Ramadan needs to be removed from the seasonally adjusted series.

During Ramadan, in countries with large Islamic populations, many people may have fewer hours sleep, and shorter working hours. There may also be changes in household consumption habits, leading to an increase in consumer prices, especially those of food products. As a result, it may be beneficial to account for the moving holiday of Ramadan as a regression variable when seasonally adjusting.

10.6.1 *How to estimate and adjust for Ramadan effects*

To estimate and adjust for Ramadan effects in X-13ARIMA-SEATS, a user-defined regression variable will need to be created and saved in a file with the extension 'rmx'.

Similar to the choice of regressors patterns for Easter[1,8,15], the daily distribution of regressors within Ramadan need to be decided. The strongest effect will depend on the series being tested, so there is no definitive correct answer to this. A few possible options to consider are weighting every day of Ramadan, just the first week of Ramadan, or the final week of Ramadan and first week after Ramadan, centred around the holiday Eid al-Fitr.

To perform time series analysis over several years, regressors would be required for the length of the span. Ideally, the regressors would be calculated to cover infinite time, but an approximation of a multiple of 33 years is acceptable, because there are approximately 34 lunar years in 33 years.

There are a range of options for calculating the monthly regressors:

- For a crude estimate, use a continuous variable indicating the **percentage of days with Ramadan regressor applied per month** within the period considered. For this option, the average ratio in each month or quarter is computed, and then the average ratio of each month of the year is subtracted from the monthly value. The long-term average of the regressors should sum to zero

- The R function **genhol** can be used to create regressors. For this, the daily pattern of regressors needs to be known. Using the "rmd" argument, **genhol** produces dates relative to the middle day of Ramadan, and with the number of days considered before and after this given in the "start" and "end" arguments

- A **binary 365x365 table**, where each column corresponds to a day of the year, and each row corresponds to a different possible starting date for Ramadan can be used to calculate daily regressors. Apply daily weights to the period you want to consider, and zero for all other dates. Find the average daily value within each calendar month, and then for each month subtract the average value for that month of the year, so that the long-term average sums to zero or near-zero. The INDEX MATCH function in Excel may come in useful. The long-term average of the regressors should sum to zero

The output can be saved as an rmx file, and read in by the spc file following the instructions in Chapter 14. Beware that some years may have two beginning dates for Ramadan, with the first day of Ramadan occurring in both January and December of 1997 and 2030.

11 LEVEL SHIFTS AND ADDITIVE OUTLIERS

11.1 INTRODUCTION

When analysing time series, analysts may need to identify possible inconsistencies or effects in the data. In some cases, these problems will have an identifiable and real-world cause. For example, promotion activities to achieve a sales goal; change in business rules, regulations and policies; weather changes; natural disasters; international conflicts and wars; financial market crashes. The presence of such effects can affect the qualities of the seasonal adjustment. In these situations, the analyst will normally try to remove the inconsistency before the seasonal adjustment of the series. This chapter will deal with two types of effect: level shifts and additive outliers.

11.2 LEVEL SHIFTS

11.2.1 *What is a level shift?*

A level shift (or trend break) is defined as an abrupt but sustained change in the underlying level of the time series. The annual seasonal pattern is not changed. There are many potential causes of level shifts in time series, including change in any of the following: concepts and definitions of the survey population, the collection method, economic behaviour, legislation or social traditions.

11.2.2 *Why adjust for a level shift?*

Level shifts are a problem for seasonal adjustment because they will distort the estimation of the seasonal factors if not corrected for appropriately.

Within X-13ARIMA-SEATS, the initial trend-cycle is calculated by applying a centred 12-term moving average (for monthly series) to the series after trading day effects, Easter, and other prior adjustments have been applied. The trend-cycle is removed from the original estimates to give an estimate of the seasonal times irregular component (SI Ratios) and then the seasonal factors are estimated after the replacement of extreme values. The Henderson filters are then applied to the seasonally adjusted estimates to produce the final trend-cycle estimate.

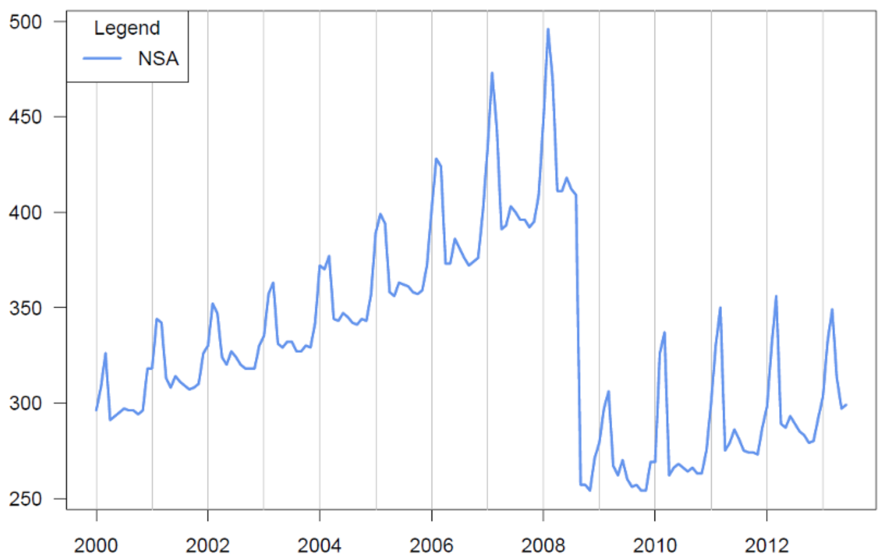

Figure 11.1: Example of a level shift

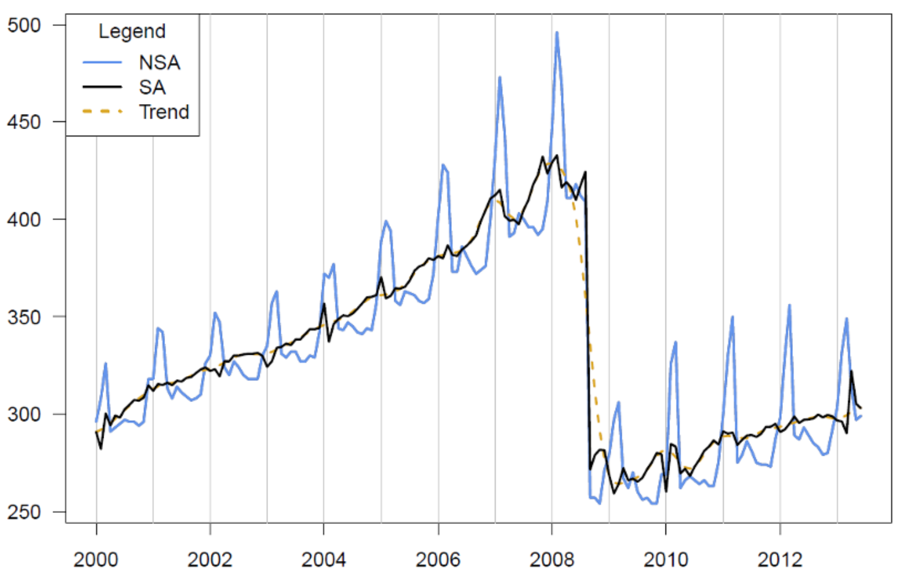

Figure 11.2: No prior adjustment for level shift regressor

Figure 11.2 shows an example where a level shift has not been corrected. If there is an abrupt change in the level of the series, when the moving averages are applied to the series to calculate the preliminary trend-cycle, the estimates will be distorted. As the calculation of the irregular and seasonal components follows on from this initial trend-cycle estimation, they will be distorted. The resulting seasonally adjusted estimates will be more volatile.

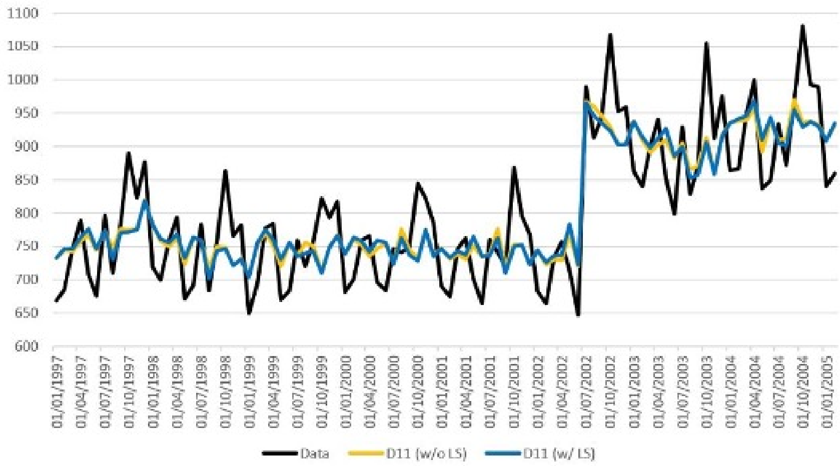

Figure 11.3: Comparing SA values with and without a level shift regressor

The trend-cycle (dashed line) shows a rapid decrease from mid-2008 to the beginning of 2009. The trend estimates before the level shift are lower than expected, and those following the shift are higher than expected. The result for the seasonally adjusted series is an increased level of irregularity around the level shift, increasing the volatility of the seasonally adjusted series. The final seasonally adjusted and trend estimates will be misleading.

11.3 ADDITIVE OUTLIERS

11.3.1 *What is an additive outlier?*

An additive outlier is a data point which falls out of the general pattern of the trend and seasonal component. An outlier may be caused by a random effect, that is an extreme irregular point, or it may have an identifiable cause such as a strike or bad weather. This chapter will deal only with the second type of outlier, where there is an underlying economic reason that explains the unusual behaviour of the data point. For more information about how X-13ARIMA-SEATS deals with extreme values see Chapter 5.

Figure 11.4 provides an example of a series requiring an additive outlier regressor for seasonal adjustment.

11.3.2 *Why adjust for an additive outlier*

Additive outliers are a problem for seasonal adjustment because the method of seasonal adjustment is based on moving averages. These are affected by the presence of extreme values or outliers, which can make the average un-

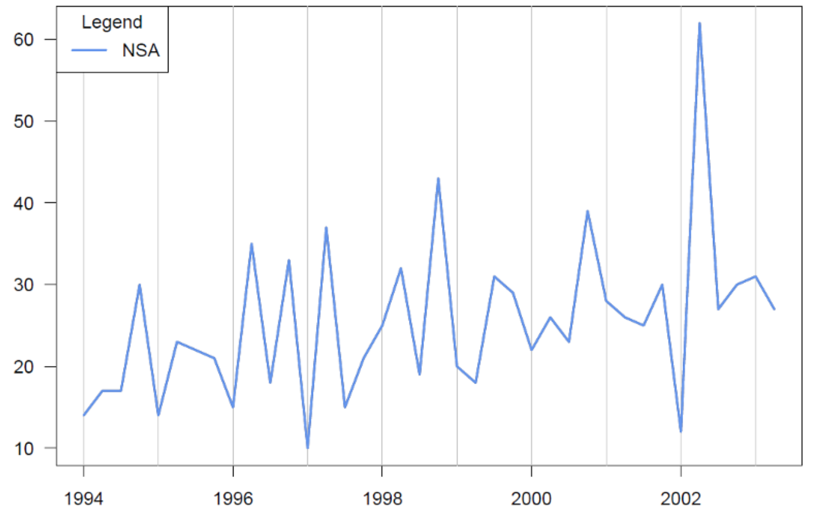

Figure 11.4: Example of series requiring an additive outlier regressor when seasonally adjusting

representative of the pattern of the series. If some adjustment or allowance is not made for outliers, these will cause distortion in the estimates of all the components in a time series. Seasonally adjusting the data above, without allowing for the outliers, gives the following results:

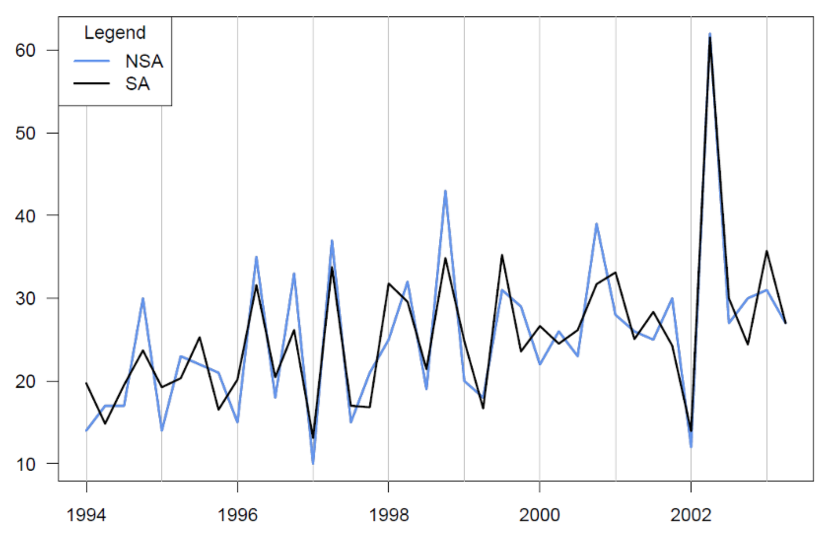

Figure 11.5: Seasonal adjustment with no prior adjustment for outlier

11.4 HOW IDENTIFY AND ADJUST FOR LEVEL SHIFTS AND ADDITIVE OUTLIERS

The following paragraph describes how to identify and adjust for both level shifts and additive outliers, since the procedure to adjust for these two discontinuities is the same. In practice, graphing the series and the results of the regARIMA test for outliers and level shifts will provide the best ways of identifying these features.

11.4.1 *Run the series in X-13ARIMA-SEATS and look at the output diagnostics*

- Run the series using the standard spec file in X-13ARIMA-SEATS without checking for level shifts and additive outliers
- Look at the graph of the NSA and SA series and the X-13ARIMA-SEATS output

The identification of level shifts and additive outliers is not always as obvious as in the examples shown in this chapter. In cases where the level shifts and additive outliers cannot be spotted from the graphical representation of the series, an analysis of the X-13ARIMA-SEATS output can help in the detection of level shifts and additive outliers.

Level shifts can be detected by analysing Tables E5 and E6 in the output. These tables show the Month-on-Month (MM) (or quarter-on-quarter) changes in the original and seasonally adjusted estimates. In fact, a level shift will appear as a sudden large increase which is not followed by a corresponding decrease (or vice versa) in Tables E5 and E6. The adjustment for the level shift effectively attempts to remove this sudden change in the level.

Additive outliers can be detected by analysing Table C17 and D13 in addition to Table E5 and E6 in the output. In fact, an additive outlier will appear as one or more zeros in the period of the outlier in Table C17, which gives the final weights for the irregular component, and as a residual pattern of the irregular component presented in Table D13. In addition to inspecting these four output tables, M1, M2 and M3 statistics should be checked. M1, M2 and M3 each measure the level of the irregular component in the series compared to the trend and the seasonal components. If these fail (values greater than 1), this may indicate that outliers need to be replaced as temporary prior adjustments.

11.4.2 *Length of the series before and after the level shift or outlier*

1. The entire length of the series needs to be at least 5 years to use regARIMA

2. The outlier correction analysis does not have restrictions in terms of data available before and after the outlier

3. The level shift cannot occur on the start date of the series since the level of the series prior to the given data is unknown

4. A level shift at the second data point cannot be distinguished from an outlier at the start date of the series, and a level shift at the last date of the series cannot be distinguished from an additive outlier since the level of the series after the discontinuity is unknown. In these situations the knowledge of the series or the use of external information are of primary importance to prior adjust the original series.

11.4.3 *Testing for level shifts and additive outliers in X13-ARIMA-SEATS*

If the user wants to search simultaneously for level shifts and additive outliers within a span of data, the following spec could be used.

```
series{
   title="Example of level shift and outlier search"
   start=1994.1
   period=4
   file="mydata.txt"
   }

arima{
   model=(0,1,1)(0,1,1)
   }

outlier{
   types=(ao ls)
      span=(2002.2,)
   }

x11{
   mode=mult
   }
```

The outlier specification performs automatic detection of additive outliers and level shifts or any combination of the two using the model specified in the arima specification. After outliers (referring to any of the outlier types mentioned above) have been identified, the appropriate regression variables are incorporated into the model as automatically identified outliers.

It is important to notice that this specification will detect only additive outliers and level shifts with a particularly high critical value. In fact, the value to which the absolute values of the outlier t-statistics are compared depends on the number of observations included in the interval searched for outliers. For example, if the outlier search is run for the last year of data, the critical value will be 3.16 (compared with the standard critical value of 1.96). This means that all the outliers with t-statistic values less than 3.16 will not be detected using this option and that the visual check of the NSA and SA series remains an important diagnostic in the identification of outliers. High critical values are used to overcome the problem of multiple testing. For more information on the default critical values for outlier identification[1]

11.4.4 *Confirming the reason for the level shifts or additive outliers*

If the X-13ARIMA-SEATS test and the graphs of the NSA and SA series lead you to suspect a level shift or an outlier, then check which months/quarters it appears in. Question if there is any evidence for suspecting a level shift or outlier at this time point.

11.5 DEFINITION

X-13ARIMA-SEATS seasonally adjusts a time series by modelling it as at least three unobserved components. The process of breaking down a series into these components is known as decomposition.

There are two basic ways in which X-13ARIMA-SEATS can model seasonality in order to identify and remove it. The first, and most common, is the ***multiplicative model***, which is of the form:

$$Y_t = C_t \times S_t \times I_t \tag{2}$$

where Y_t is the original series, C_t is the trend-cycle, which includes the medium and long term movements in the series, S_t is the seasonal component, which includes repeating movements at annual intervals, and I_t is the irregular component.

This decomposition model is used for most economic time series. However, a multiplicative decomposition model cannot be used when there are negative numbers or zero values in the series.

For other series where the seasonal changes are independent of the level of the series/the trend, the ***additive model*** will be more appropriate. It decomposes the series as follows:

$$Y_t = C_t + S_t + I_t \tag{3}$$

1 see Table 7.22 of X-13ARIMA-SEATS manual (USCB 2017)

Two other decompositions available in X-13ARIMA-SEATS, log-additive and pseudo-additive. The log-additive is an alternative to the multiplicative decomposition but is rarely used when seasonally adjusting series with the X-11 algorithm. The pseudo-additive model is sometimes used for time series where there are non-negative values with regular zero values. For example, this could be applied to agricultural time series.

All prior adjustments will be of the same type as the decomposition model, so for an additive model all prior adjustments are additive, and for a multiplicative model all priors are multiplicative.

11.5.1 *Adjust for the level shifts or the additive outliers*

Use the regARIMA model

If you suspect that a level shift or an outlier is present at a particular time point (either before or after the graphical analysis) then it should be specifically tested for using X-13ARIMA-SEATS by including the following specification in the spec file.

```
regression{
   variables=(ao2002.2 ls1996.4)
   }
```

The *variables = (ao2002.2 ls1996.4)* command instructs X-13ARIMA-SEATS to analyse the level shift or outlier and to create temporary priors to adjust for the discontinuity.

This command allows the user to test and adjust for a level shift and an additive outlier. More than one ao and/or ls may be specified in the model. When the command is included in the regression spec, the output includes t-tests for each regressor (ao and ls regressors) included in the model. The definition of this test is in the regARIMA chapter.

The use of regARIMA ao and ls regressors is usually recommended if the presence of an outlier or level shift has been previously confirmed and the t-test absolute value is greater than 1.96.

The temporary prior adjustments derived by X-13ARIMA-SEATS are shown in Table A8. These prior adjustments are the result of the multiplication (or addition) of all the temporary priors derived by ao and ls regressors included in the model. The temporary priors adjust the level of the series before the point at which the level shift occurs and at each point where an ao occurs without altering the annual seasonal pattern.

The use of temporary priors derived by the regARIMA approach avoids any distortion in the estimation of the components of the series and leads to a seasonally adjusted series that shows the discontinuity.

Figure 11.6 and Figure 11.7 show that the use of regressors to correct the discrepancies in the series improve the quality of the seasonal adjustment.

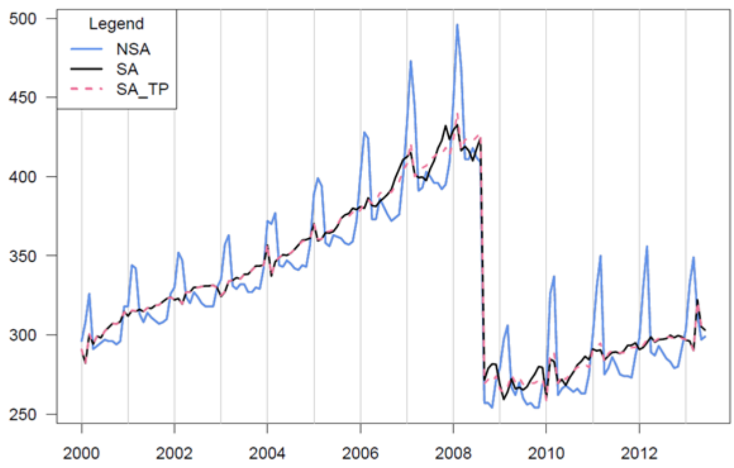

Figure 11.6: Difference between seasonally adjusted series when prior adjusting for level shift (SA_TP) and no prior adjustment (SA)

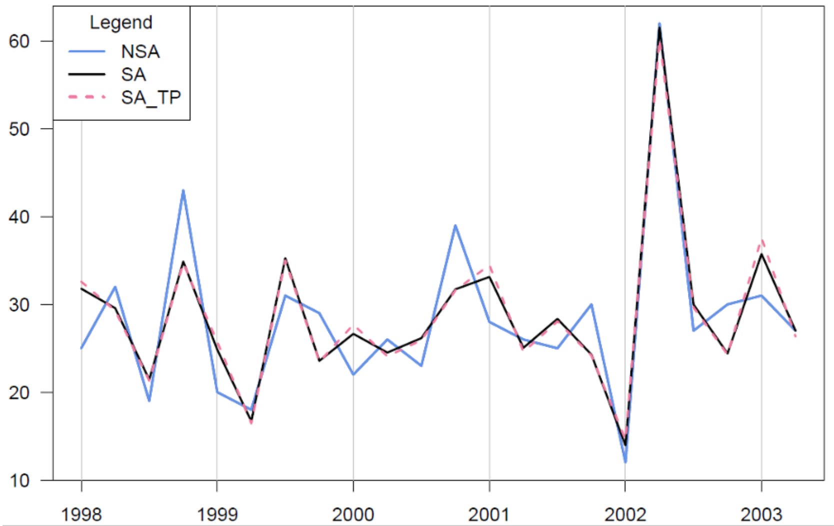

Figure 11.7: Difference between seasonally adjusted series when prior adjusting for additive outlier in 2002 quarter 1 (SA_TP) and no prior adjustment (SA)

Problems:

- High degree of irregularity in the series may hide the presence of a level shift.
- High degree of irregularity can make the estimation of the magnitude and timing of the level shift less accurate.
- When regressors are included in the variables argument they will be corrected for even if they are not significant. The t-value of each regressor (or the chi-squared of a group of regressors) should be checked to be sure of its (their) significance.

Use of automatic outlier detection

If the user wants to search simultaneously for level shifts and additive outliers within a span of data, the following spec should be used.

```
series{
   title="Example of level shift and outlier search"
   start=1994.1
   period=4
   file="mydata.txt"
   }

arima{
   model=(0 1 1)(0 1 1)
   }

outlier{
   types=(ao ls)
   span=(2002.2,)
   lsrun=n
   critical=2.58
   }

x11{
   mode=mult
   }
```

The outlier spec performs automatic detection of additive outliers and level shifts or any combination of the two using the model specified in the arima spec. After outliers (referring to either of the outlier types mentioned above) have been identified, the appropriate regression variables are incorporated into the model as automatically identified outliers and the model is re-estimated. The default critical value for the t-statistic is 1.96, corresponding to a p-value of 0.05, but this can be changed as in the above example, by using the command critical. A value of 2.58 corresponds to a p-value of 0.01. This would find only the most statistically significant variables.

The **lsrun** command computes t-statistics to test null hypotheses that each run of $2, ..., n, ..., 5$ successive level shifts cancels to form a temporary level shift. If one or more level shift t-tests indicate that a run of 2 or more successive level shifts cancel, a user-defined regressor can be used to capture the temporary level shift effect. In this way two or more level shifts can be replaced by one user-defined regressor. The usertype argument should be set to ls for this regressor, so the user-defined regressor is treated as level shift. An example of this is given below:

```
regression{
   user=(myls)
   usertype=ls
   file="myls.rmx"
   format="x12save"
   }
```

where **myls** is the user-defined regressor created to take into account of the temporary level shift. For example, if there are two level shifts that cancel out, one in January 2002 and the other in March 2003, the regressor has the following form:

$$\begin{array}{lr} 0 & t < \text{January 2002} \\ -1 & \text{January 2002} \leqslant t \leqslant \text{March 2003} \\ 0 & t > \text{March 2003} \end{array}$$

where t defines the date of the observation.

The **span** command specifies start and end dates of a span of the time series to be checked for outliers. The two dates must both lie within the series and within the **modelspan** if one is specified. For more information about **modelspan** using the composite or the series spec, see section 7.4 or section 7.15 of the X-13ARIMA-SEATS user manual (USCB 2017), respectively. If one of the two dates is missing, for example, **span = (2002.2,)**, X-13ARIMA-SEATS sets the missing date on the start date or end date of the series. For example, using the outlier specification defined above, X-13ARIMA-SEATS searches for outliers between February 2002 and the last data point of the series.

Problems:

- Automatic outlier detection is not always stable. Sometimes outliers switch from being significant one month to being not significant the following month and going back to being significant the next month. This instability has an effect on the revision pattern. In fact, the revisions history of series with automatic outlier detection in place is more erratic than the revision history without automatic outlier detection or with outlier regression variables in the regARIMA model. This problem does not apply to series with stable seasonality. This stability problem can be reduced if a shorter span is considered in the search of outliers, for example if the outlier specification is used in the last year of data.

- It detects only outliers with a high critical value. Automatic outlier detection does not consider outliers with a t-value between 1.96 and the critical value set by X-13ARIMA-SEATS.
- The test for automatic outlier detection may have less power at the last data point. A level shift at the last time point of the series cannot be distinguished from an additive outlier since the level of the series after the discontinuity is unknown. In these situations the knowledge of the series or the use of external information are of primary importance to prior adjust the original series. Criteria for deciding which of the two methods should be used are the size of the dataset and the importance of the series:
  - The ***size of the dataset*** being analysed influences the decision about which of the two methods to use. If a large dataset is analysed, the automatic search for outliers is preferable as it is less time consuming than testing for individual outliers, easier to update, but still accurate. The use of regressors in the model is preferable for small datasets.
  - Regarding the ***importance of the series***, the manual method is more accurate and produces more stable results. In this situation, though, the automatic method can be used to monitor the series between seasonal adjustment reviews. This will keep users informed of possible problems that have an effect on the seasonal adjustment of the latest data points.

Size of revisions

If the size of revisions is important, then the manual method should be used. Although again in this situation, the automatic method could be used to monitor the series between seasonal adjustment reviews.

11.6 OTHER OUTLIER TYPES

Additive outliers and Level Shifts are the most common outlier types to be used in seasonal adjustment. However, other outlier types exist to handle more intricate corrections.

11.6.1 *Ramps (RP)*

A Ramp is a type of outlier used when a trend is changing too quickly to be considered a natural movement of the trend itself yet is not an instant change where a LS would be a better option. If the trend's change is not instantaneous then it is often proper to do nothing and accept that the trend is simply changing and so Rp regressors are quite a rare choice in seasonal adjustments. The decision to use them is usually based on whether a sharp change in trend level causes problems for the quality of the seasonal adjustment. For example, Rp regressors have been used on some series affected by the 2008/2009 financial crisis.

11.6.2 *Temporary changes (TC)*

A Temporary Change is a type of outlier used when an additive outlier is suspected to take more than one period to have its effects diminished exponentially. The rate of decay can also be set. This is usually not a default outlier tested by X-13ARIMA-SEATS. However, if there is any information that the effects of an AO are diminishing then a case can be made to use a TC instead.

12 DECOMPOSITION MODELS

12.1 THE OPTIONS

Two specs are involved in the selection of the model to be used in the seasonal adjustment decomposition, transform and x11. Those two specs may be written, for a ***multiplicative decomposition***[2], as:

```
transform{
   function = log
   }

x11{
   mode = mult
   }
```

while, in case of an ***additive decomposition***[3], as:

```
transform{
   function = none
   }

x11{
   mode = mult
   }
```

Note that in the first case **{mode=mult}** may be omitted, because **{function=none}** may be omitted, again because it is the default. However, the explicit statement of a default value does no harm and may make the process clearer.

12.2 HOW TO DECIDE WHICH SEASONAL DECOMPOSITION TO USE

There are two ways of identifying which model is the most appropriate: by inspecting the graph or by analytical means.

12.2.1 *Graphical inspection*

In many cases, inspection of the graph of the time series and knowledge of the data will make it clear as to which decomposition model to choose. Under the multiplicative model, the seasonality of the series is affected by the level of the series. So, when the graph of the series shows that the size of the seasonal peaks and troughs increase (or decrease) as the trend rises (or falls), a multiplicative decomposition model is appropriate. Alternatively, if the size of the seasonal peaks and troughs are independent of the level of the trend then an additive decomposition model is more appropriate.

Figure 12.1, for example, shows an case of a multiplicative decomposition. The difference between 2002 Q4 and 2003 Q1 is 95, whereas the difference between 2006 Q4 and 2007 Q1 is 119 and finally the difference between 2009 Q4 and 2010 Q1 is 162. This shows that the seasonal troughs increase as the trend rises and so suggests that a multiplicative decomposition model is appropriate.

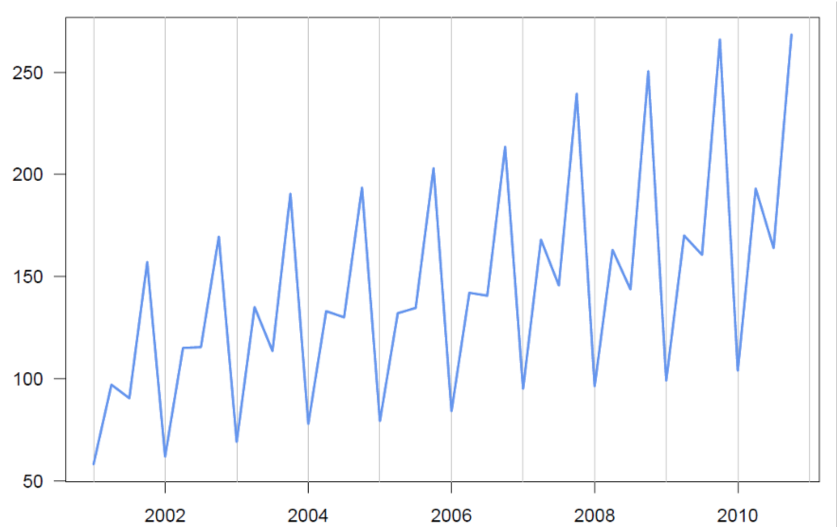

Figure 12.1: Example of a multiplicative decomposition

Figure 12.2 shows an example of an additive decomposition. The difference in all the same periods mentioned above is 8, showing that the size of the seasonal troughs is independent of the level of the trend, hence suggesting an additive decomposition model is appropriate.

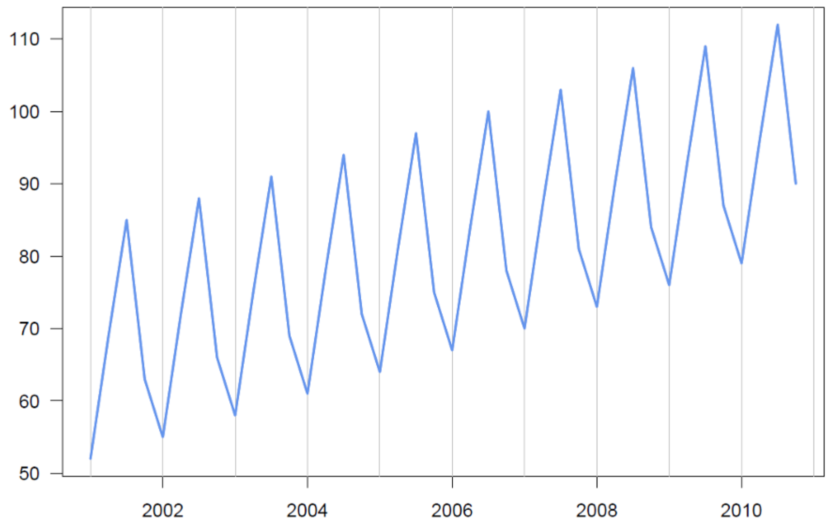

Figure 12.2: Example of an additive decomposition

It may not be clear from a plot of the series which of the two models is the most appropriate; for example, when the trend is fairly flat, both models may produce satisfactory results. A way of identifying the most appropriate model is to use an automatic process in X-13ARIMA-SEATS. This fits both models, calculate a goodness-of-fit statistic, the AICC test, and choose the model with the best (lowest) AICC.

If a series or any prior adjustments contain negative numbers or zeros then the additive model must be used. If a multiplicative model is specified, the program will generate an error and will interrupt the seasonal adjustment run. In this case, if the series contains zero/negative values and seems to call for a multiplicative seasonal adjustment, the constant argument is sometimes useful. The constant argument allows the user to specify a constant to add to the series before modelling which then allows the automatic transformation selection procedure to be used to determine if a log transformation should be used to transform the series. This can be written as shown below (selecting a suitable constant, 0.0001 in this example):

```
transform{
   constant = 0.0001
   function = auto
   }
```

The constant argument is then removed from the series after the X-11 algorithm has been performed. The value of the constant should reflect the scale of the series and can vary in orders of magnitude. A visual check should inform the user if the constant is appropriate.

The user may have some knowledge about the series that may help to identify which model is most likely to be appropriate. In general, all series in a subsystem of accounts will normally be adjusted using the same decomposition model, unless a series clearly follows the other model.

The model used also affects how the seasonal and irregular components are presented. Under a multiplicative model both these components are given as ratios (expressed in percentage terms) which vary around 100. With an additive model the components are differences which vary around zero.

12.2.2 *Analytical approach*

In the case that knowledge of the series and the graph do not help in the decision of which decomposition to use, an automatic procedure can be used. This is done by invoking the automatic transformation in the **transform** spec, using the following argument:

```
transform{
   function=auto
   }
```

Provided the series being processed has all positive values, X-13 ARIMA-SEATS performs an AICC-based selection to decide between a log transformation and no transformation. In this case, the **mode** in the **x11** spec is not necessary, since it is automatically selected by X-13ARIMA-SEATS in order to match the transformation selected for the series (**mult** for the log transformation and **add** for no transformation).

To decide which transformation to use, and therefore which decomposition model to use, X-13ARIMA-SEATS fits a regARIMA model to the untransformed and transformed series. It chooses the log transformation except when

$$AICC_{nolog} - AICC_{log} < \Delta_{AICC} \tag{4}$$

where $AICC_{nolog}$ is the value of AICC from fitting the regARIMA model to the untransformed series $AICC_{l}og$ is the value of AICC from fitting the regARIMA model to the transform series, and Δ_{AICC} is the value entered for the **aicdiff** argument, with a default of -2. Negative values of Δ_{AICC} bias the selection in favour of the log transformation. The default -2 is used not for statistical reasons but for convenience. Multiplicative adjustment is appropriate for the great majority of official statistics series and the use of additive decomposition is suggested only when statistical support for additive adjustment is strong.

If a regARIMA model has been specified in the regression and/or arima specs, then the procedure uses this model to generate the AICC statistics needed for the test. If no model is specified, the program uses the ARIMA model (0 1 1)(0 1 1), to generate the AICC statistics.

12.3 UPDATING

The same decomposition model should be used throughout the year. It should not need to be changed at the annual update except in very exceptional circumstances. If the model does look as if it should be changed then TSAB should be consulted.

13 MOVING AVERAGES

13.1 INTRODUCTION

The X-13ARIMA-SEATS program uses Moving Average (MA) throughout its iterations to decompose the original time series into the trend, seasonal and irregular components, as described in Chapter 1. The program uses moving averages for two different purposes: to estimate the trend component and to estimate the seasonal component when X-13ARIMA-SEATS is run automatically. It uses two different types of moving averages to estimate the trend component – one before the seasonal component has been removed and one after. When the user specifies the trend moving average, that trend moving average is used in all iterations of the X-11 algorithm. This chapter will describe what moving averages are, the different types of moving averages that are used, the options available in the program to select specific types of moving averages and why these may be of use.

13.2 WHAT ARE MOVING AVERAGES?

A moving average is a weighted average of a moving span of fixed length of a time series. They can be used to produce a smoother version of the time series. In the simplest cases the objective is to remove or filter out as much as possible of the irregular component of the series, leaving the smooth trend. Besides removing or reducing irregularity, a moving average may also be chosen because of its ability to remove a fixed seasonal pattern. There are many forms of moving average, which differ in their ability to remove irregularity while preserving the trend as accurately as possible. X-13ARIMA-SEATS uses several of these forms, according to the needs of the different stages of the program.

The simplest moving average is a symmetric moving average (an equal number of data points either side of the target data point are included in the average) with equal weights applied to each data point.

Consider the following time series: $a_1, a_2, ..., a_{t-1}, a_t, a_{t+1}$, where a_t is the value of the series at time point t.

If the span of data for the symmetric moving average is equal to 3 and equal weight is given to each point then the moving average at time point 2 is equal to:

$$u_2 = \frac{a_1 + a_2 + a_3}{3} = \frac{1}{3}a_1 + \frac{1}{3}a_2 + \frac{1}{3}a_3 \tag{5}$$

Where u_2 is the value of the moving average at time period 2. The weights applied to each data point are equal to 1/3. As this is a moving average the average is also calculated for the next time point. Hence the moving average at time point 3 is:

$$u_3 = \frac{a_2 + a_3 + a_4}{3} = \frac{1}{3}a_2 + \frac{1}{3}a_3 + \frac{1}{3}a_4 \tag{6}$$

More generally, the moving average at time point t is:

$$u_t = \frac{a_{t-1} + a_t + a_{t+1}}{3} = \frac{1}{3}a_{t-1} + \frac{1}{3}a_t + \frac{1}{3}a_{t+1} \tag{7}$$

This is called a 3x1 moving average. It cannot be calculated for the first and last points of the series, as we do not have an equal number of data points either side of the target data points. This is known as the end-point problem. The 3x1 moving average can be generalised to take any number of data points, each with an equal weight.

In a 5x1 moving average each data point will have a weight of 1/5 and in a 9x1 each will have a weight of 1/9. The longer the moving average the smoother the resultant series, as it takes information from a greater number of data points. However, with a longer moving average, the value can be estimated for fewer points because of the end point problem. In the case of a 9x1 moving average, the average cannot be calculated for the first four and the last four terms of the series.

A symmetric moving average with equal weights where the span has an even number of points would estimate an average at the mid-point between two time points. By not applying the same weight to each point, it is possible to produce a symmetric moving average with an even length of span. Using the notation introduced earlier, a 4x1 moving average of the first four points of a series

$$\frac{a_1 + a_2 + a_3 + a_4}{4} = \frac{1}{4}a_1 + \frac{1}{4}a_2 + \frac{1}{4}a_3 + \frac{1}{4}a_4 \tag{8}$$

would estimate an average between the second and third time periods and a moving average of the next four points

$$\frac{a_2 + a_3 + a_4 + a_5}{4} = \frac{1}{4}a_2 + \frac{1}{4}a_3 + \frac{1}{4}a_4 + \frac{1}{4}a_5 \tag{9}$$

would estimate an average between the third and fourth time periods.

An average of these two averages would then be centred at the third time period. This is called a 2x4 moving average and takes the general form

$$u_t = \frac{1}{2}\left(\frac{a_{t-2} + a_{t-1} + a_t + a_{t+1}}{4} + \frac{a_{t-1} + a_t + a_{t+1} + a_{t+2}}{4}\right) = \frac{1}{8}a_{t-2} + \frac{1}{4}a_{t-1} + \frac{1}{4}a_t + \frac{1}{4}a_{t+1} + \frac{1}{8}a_{t+2} \tag{10}$$

This sort of average is used to remove the seasonal component from a quarterly series. Although it does not give equal weights to all the data points it is symmetric and gives equal weights to each quarter. For example, the 2x4 moving average for quarter 2 will give a weight of 1/4 to quarters 1, 2 and 3 of the current year, 1/8 to quarter 4 of the current year and 1/8 to quarter 4 of the previous year, so a total weight of 1/4 for quarter 4. This can be used to remove seasonal effects in quarterly data. In the same way, a 2x12 moving average can be used to remove seasonal effects and provide an estimate of the trend for a monthly series.

The X-13ARIMA-SEATS program also uses combinations of simple (equally weighted) moving averages to estimate the seasonal component. An example of this is the 3x3 moving average, which is an average of three consecutive 3x1 moving averages (that is the 3x1 moving averages for $t-1$, t and $t+1$). The structure of weights for this is:

$$u_t = \frac{1}{3}\left(\frac{a_{t-2} + a_{t-1} + a_t}{3} + \frac{a_{t-1} + a_t + a_{t+1}}{3} + \frac{a_t + a_{t+1} + a_{t+2}}{3}\right) = \frac{1}{9}a_{t-2} + \frac{2}{9}a_{t-1} + \frac{3}{9}a_t + \frac{2}{9}a_{t+1} + \frac{1}{9}a_{t+2} \tag{11}$$

Such a moving average gives more weight to observations closer the point of interest (t) and less weight to observations further away from the point of interest.

All these examples have used simple (equally weighted) moving averages or combinations of them. There are other moving averages which use more complex patterns of unequal weights. Different moving average may be derived with particular objectives such as relating to some simple local models of series.

For example, if we suppose the series to be a linear trend with an irregular added, the simple moving averages will give the linear trend plus an irregular which is smaller because of the averaging. The length of average chosen will depend on how much smoothing we want to produce which will in turn depend on how irregular the original series is.

There is a penalty in making the average longer however because the linear trend will seldom continue for a long time, although it may be an adequate approximation over a short period. If the trend has some curvature, using too long an average will distort it. Thus, the choice of length of average will be a matter of balancing the objectives of smoothing and following the trend.

Another objective may be exemplified by the 2x4 average. If a quarterly series has a stable seasonal component, it is obvious that averaging any four successive terms will produce a series in which the seasonal has been cancelled out. As shown above, the 2x4 average is equivalent to taking a 4-term average and then a 2–term average, the first step will remove the seasonal. Thus, this average applied to a seasonal quarterly series will reduce the irregular, remove the seasonal and still preserve the trend (provided it does not curve too much). Similarly, a 2x12 moving average will give an estimate of the trend of a seasonal monthly series.

In many cases the trend of a series has too much curvature to be adequately represented by these simple averages. To avoid this problem, X-13ARIMA-SEATS uses a family of moving averages called Henderson averages. These have the property that they will reproduce exactly a trend which can be represented as a cubic polynomial while producing an output which has maximum smoothness for their given length. However, it should be noted that they cannot remove a seasonal effect. The formulae for the weights of the Henderson averages are rather complex, and so they are not reproduced here. For more information on their design and performance, please consult TSAB.

All the examples provided assume that the moving average is calculated at a point at which enough time series values are available before and after the point to apply the formula. Obviously, if we are calculating an average at or near the end of the series this will not be possible because of the end-point problem. In such cases the X-11 part of the program provides asymmetrical approximations to the symmetric weights, which are generally constructed by assuming that the series may be forecast in some simple way, such as fitting a straight line by regression to the last few points and extrapolating it.

One of the improvements in X-13ARIMA-SEATS is to provide a better way of dealing with the end-point problem. The ARIMA part of the program allows forecasting and backcasting of the series which allows centred moving averages to be calculated for the first and final data points. The moving average for these points would otherwise use asymmetric weights, which gives a greater weight to the point itself and would therefore not generate such stable estimates of the separate components (*trend*, *seasonal* *and irregular*) at those points. In practice, a combination of forecasts and asymmetrical weights are used.

13.3 TREND MOVING AVERAGES

The trend moving averages are weighted arithmetic averages of data along consecutive points, as described in the previous section. There are two types of trend moving averages used by X-13ARIMA-SEATS:

- Centred simple moving averages (for example 2x12 or 2x4), and;
- Henderson moving averages.

When X-13ARIMA-SEATS is used in automatic mode the simple moving average is applied at the first stage of the seasonal adjustment process before the seasonal component has been removed. This gives a preliminary estimate of the trend. Henderson moving averages are used later in the process and are applied to successive estimates of the seasonally adjusted series to give refined estimates of the trend. The lengths of the Henderson moving averages are determined automatically. The user can specify a length of Henderson filter to replace the automatic choice.

13.3.1 *Options for trend moving averages*

Henderson moving averages: There are 50 types of Henderson moving average that can be manually chosen using the x11 spec. Any odd number between 3 and 101 inclusive can be specified. In general 9-, 13- or 23-term averages are used for monthly data and a 5- or 7-term for quarterly data. By default the program will choose one these if the trendma argument is not used in the x11 spec.

The choice of a trend filter is a matter of balancing smoothing power and flexibility of the trend. The choice is based on the I/C ratio, which is the comparative size of the irregular variations (I) relative to those of the trend (C). The default choice should be accepted in most cases; the only common situation where it may be overridden is if it is necessary to ensure that a group of series all use the same trend filter. To choose a specific length of Henderson trend moving average using the x11 spec the following argument should be used:

```
trendma = n
(where n is any odd number from 3 to 101)
```

13.3.2 *Trend moving average selection by X-13ARIMA-SEATS*

If the trend moving average is not specified by the user the default is that the first estimate of the trend will use a centred one year moving average and the Henderson trend moving average is selected by the algorithm using the following criterion.

I/C	**Quarterly Data**	**Monthly Data**
0 to 0.99	5-term	9-term
1 to 3.49	5-term	13-term
3.5 and over	7-term	23-term

The I/C ratio is shown in table D12 of the output (the trend estimate) and in table F2H. The program selects a longer filter when movements in the irregular component are large compared to movements in the trend. Where an ARIMA model provides backcasts and forecasts a symmetric Henderson moving average can be used. If no ARIMA model is applied, or forecasts are shorter than the span of data required for the selected length of moving average, then asymmetric moving averages will be used, the weights of which are available from TSAB.

13.3.3 *When to change the trend moving average*

In general, it is recommended that the trend moving average is selected automatically during a seasonal adjustment review via the default option and then fixed for production run purposes. However, in some situations it may be necessary to change the moving averages manually. If the M4 summary statistic in table F3[1] fails it may be necessary to use a shorter moving average as a longer moving average may be removing some pattern in the irregular. For more information on manually changing the trend moving averages contact TSAB.

13.4 SEASONAL MOVING AVERAGES

Seasonal moving averages are weighted arithmetic averages applied to each month (or quarter) over all the years in the series (a particular seasonal moving average is applied to each column of data, as it is presented in the output). They are used by the X-13ARIMA-SEATS program to estimate the seasonal component of the series. The moving averages are applied to the SI series (seasonal plus irregular) that is the series with the

1 see Chapter 16 for further details

trend component removed; thus, the seasonal factors for January, for example, are obtained by smoothing the SI values for January in successive years. This procedure is a way of estimating both the seasonal and irregular components. As with trend estimation, the choice of moving average is a matter of balancing smoothing power against flexibility. The moving averages that are applied are a combination of simple moving averages, for example a 3x3 moving average or a 3x5 moving average (three applications of a 5-term moving average).

13.4.1 *Options for seasonal moving averages*

As with the trend moving averages the program has a default rule to select the length of average based on the irregularity of the SI series. It is also possible to manually choose the seasonal moving averages applied to each month. The seasonal filters (seasonal moving averages) that are available include 3x1, 3x3, 3x5, 3x9 and a 3x15. If no seasonal filter is specified in the x11 spec then the program will choose the filter automatically. The filter chosen by the program is then applied to every month (or quarter).

The seasonal filter can be fixed using the argument seasonalma in the x11 spec. For a 3x5 moving average would be specified as follows:

x11seasonalma=s3x5

The options available using the seasonalma argument in the x11 spec are shown below.

Name	Description
msr	Applies default option.
s3x1	Applies a 3x1 moving average to all months/quarters.
s3x3	Applies a 3x3 moving average to all months/quarters.
s3x5	Applies a 3x5 moving average to all months/quarters.
s3x9	Applies a 3x9 moving average to all months/quarters.
s3x15	Applies a 3x15 moving average to all months/quarters.
stable	Applies a stable seasonal filter to all months/quarters.

Name	Description
x11default	Applies a 3x3 moving average to all months/quarters to calculate the initial seasonal factors in each iteration, and a 3x5 moving average to calculate the final seasonal factors.
(snxm snxm snxm snxm)	Allows the application of a specific nxm moving average for quarter 1,2,3 and 4). The values of n and m can be different each time, to produce separate moving averages for each quarter.
(snxm snxm snxm snxm snxm snxm snxm snxm snxm snxm snxm snxm)	Allows the application of a specific nxm moving average for month 1,2,...,11 and 12.

The criterion for selection of the seasonal moving average when the default is activated (either by not using the seasonalma argument or by specifying **seasonalma=msr**), is based on the global I/S ratio, which is shown in tables D10 and F2H of the output. The global I/S ratio measures the relative size of irregular movements (I) and seasonal movements (S) averaged over all months or quarters. It is used to determine what seasonal moving average is applied using the following criteria:

I/S	Seasonal moving average applied
0 to 2.5	3x3
3.5 to 5.5	3x5
6.5 and over	3x9

The global I/S ratio is calculated using data that ends in the last full calendar year available. If the global I/S ratio is greater than 2.5 but less than 3.5 or greater than 5.5 but less than 6.5 (so that it does not fall in the bands specified above) then the I/S ratio will be calculated using one year less of data to see if the I/S ratio then falls into one of the ranges given above. If it still does not fall into one of these ranges, the I/S ratio is calculated with another year removed. This is repeated either until the I/S ratio falls into one of the ranges or after five years of data have been removed in the calculation of the I/S ratio. If after five years have been removed and the resulting I/S ratio still does not fall into the above range, then a 3x5 moving average will be used. The purpose of this procedure is to avoid instability on update; for example, if a 3x5 average has been used but the updated I/S ratio falls below the lower limit for 3x5, the choice will be changed to 3x3 only if the ratio falls clearly within the band for 3x3.

As can be seen in the selection criteria, the larger the I/S ratio - indicating that the irregular component is large relative to the seasonal component - the longer the moving average applied.

13.4.2 *Manual selection of seasonal moving averages*

If we know that the volatility of activity in a particular month (or quarter) is smaller or larger than in other periods, then a different seasonal moving average may be applied for that month (or quarter). A good example of this might be an average wage series, which includes bonus payments every April. Bonus payments tend to be very volatile relative to other movements in wages. If the default returned a 3x3 moving average based on the global I/S ratio but from table D9A in the output the I/S ratio for April was greater than 6.5 (indicating that this month is volatile, with a large irregular component relative to the seasonal component) then as there is an identifiable reason for this volatility, it is valid to specify the use of a 3x9 average for April only by including the following line in the x11 spec.

```
x11{
   seasonalma=(s3x3 s3x3 s3x3 s3x9 s3x3 s3x3 s3x3 s3x3 s3x3 s3x3 s3x3
       s3x3)
   }
```

The option of a 3 term moving average (3x1) uses only three years of data, which allows seasonality to change very rapidly over time. This option should not normally be used, as it will usually lead to large revisions as new data become available. If this is to be used it will usually be in one month and because there is a known reason for wanting to track fast changing seasonality. For example, if a new data collection method has been introduced but a seasonal break has not been identified, it may be beneficial for the first few years to track this changing seasonality closely with a shorter moving average.

The option **stable** means that all of the values from the month or quarter are used to calculate the average. This average would only be suitable where the I/S ratio is very high and in general should not be used. If there are less than five full years of data in the series the program will by default use the stable moving average option. It is possible to specify a seasonal moving average for series with less than 5 years of data using the argument sfshort argument. There are two options with this argument either **sfshort=no**(default), which means that a stable seasonal filter will be used, or **sfshort=yes**, which will then use the seasonal moving average specified in the seasonalma argument.

A final option is **x11default** which specifies an option for the program to use moving averages as they were used in previous version of X-11 and X-11-ARIMA. If no seasonal moving average is specified then a 3x3 moving average is used to calculate the initial seasonal factors in the first two iterations, and then a 3x5 moving average is used to calculate the final seasonal factors. This option is not recommended.

13.4.3 *When to change the seasonal moving average*

The default options will in general select the most appropriate moving average. However, there will be occasions when the user will need to specify a different seasonal moving average to that identified by the program. In order to identify those occasions when user intervention is necessary the following guidelines provide some useful indicators.

Some of the quality statistics in table F3 of the output, particularly M4, M6, M8 and M9, can indicate that changing the seasonal moving average may be beneficial[2].

SI ratios given in table D8 and these ratios plotted against the seasonal factors in table D10 can be useful in determining an appropriate moving average. If the SI values do not closely follow the seasonal factors, it may be appropriate to use a different moving average, as that will be a more responsive moving average. For example, the Figure 13.1 and Figure 13.2 plot the SI ratios and the seasonal factors for the month of March. In this case a 3x5 moving average has been selected and this tracks the seasonality closely. If, however a 3x9 moving average is used, this tracks the SI ratios more vaguely and will reduce the quality of the seasonal adjustment. Use of too short a moving average should be avoided, as it will overfit the model.

An alternative to looking at these graphs is to look at the I/S ratios for each month; these are shown in table D9A of the output. Caution should be exercised when selecting a different seasonal moving average for a particular month as this may cause unwanted revisions. In the majority of cases the automatic selection procedure of the X-13ARIMA-SEATS program - based on the global I/S ratio, estimated from the entire series - is sufficient.

2 see Chapter 16 for more details

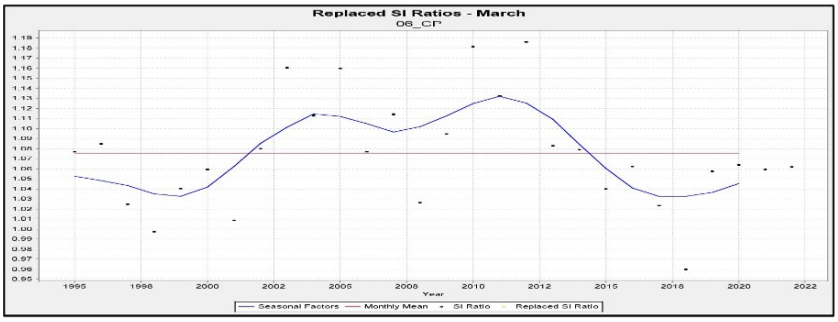

Figure 13.1: 3x5 seasonal moving average

Figure 13.2: E3x9 seasonal moving average

If the size of the irregular component has changed through the series, for example if the sample size of the survey has changed, then different moving averages may be more appropriate for different sections of the series (as highlighted in the previous section). In this case it will usually be best to use the length of moving average most appropriate for the recent section of the series. Seasonal breaks can also distort the automatic selection of the most appropriate seasonal moving average[3].

13.5 UPDATING

When running X-13ARIMA-SEATS for an annual update in the seasonal adjustment review, the following steps are recommended.

1. Run the seasonal adjustment on the default trend moving average options (the program selects the trend and seasonal moving average)

2. From the output find the Henderson trend moving average and the seasonal moving average that are selected

3. Plot the SI ratios and seasonal components, check the I/S ratios in table D9A and ask the producer of the series about any patterns that

3 see Chapter 14

could be adjusted for. Decide whether to change or keep the previous moving averages

4. In the **x11** spec specify the arguments trendma=n seasonalma =name (where **n** is the Henderson trend identified in step 2 and name is the seasonal moving average option chosen from Steps 2 and 3)

5. Use this spec file for seasonal adjustment of the series for the following year.

13.6 SUMMARY

- Moving averages are a weighted average of a moving span of fixed length of a time series
- Trend moving averages aim to eliminate the seasonal and irregular movements to leave just the trend
- The options for a trend moving average are centred moving averages or Henderson moving averages
- The trend moving averages are selected based on the I/C ratio (Irregular movements/Trend movements)
- Seasonal moving averages are applied to successive occurrences of the same month and aim to remove the trend and irregular to leave the seasonal component
- The options for seasonal moving averages are the x11default (the default option), 3x1, 3x3, 3x5, 3x9, 3x15, stable or manually fixed seasonal moving averages for each period
- The seasonal moving averages are chosen based on the I/S (Irregular movements/Seasonal movements) ratio.

14 SEASONAL BREAKS

14.1 WHAT IS A SEASONAL BREAK?

A seasonal break is defined as a sudden and sustained change in the seasonal pattern of a series. There are many potential causes of seasonal breaks in series, including changes in the data source, methodological changes, or administrative changes. Seasonal breaks will often be accompanied by a level shift. Figure 14.1 provides an example of a seasonal break.

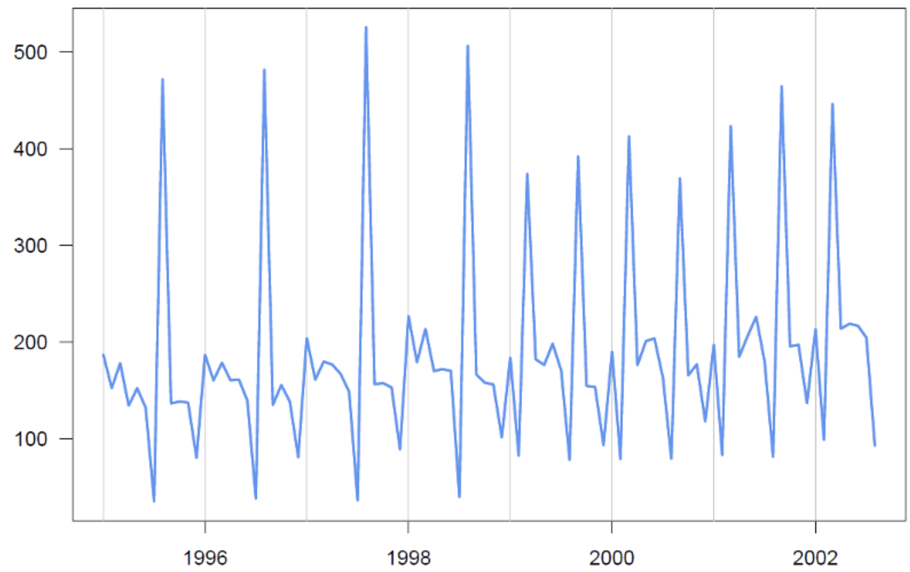

Figure 14.1: Example of a seasonal break: car registrations

There is a stable pattern of one major peak and one major trough per year prior to January 1999. After January 1999, there is a sudden and permanent change in the seasonal pattern to two peaks and two troughs per year. This particular change resulted from an alteration in the car registration system.

Where a seasonal series changes to become non-seasonal, it is only necessary to seasonally adjust the seasonal part.

14.2 WHY ADJUST FOR A SEASONAL BREAK?

Seasonal breaks are a problem for seasonal adjustment because the methodology is based on moving averages. A moving average is applied to the seasonal irregular component (SI ratios, which can be found in the D8 table within X-13ARIMA-SEATS output) of the series to obtain the estimate of the seasonal component (that will be removed by seasonal adjustment). However, the moving average used for this purpose is designed to deal with series which have a smoothly evolving "deterministic" seasonal component plus an irregular component with stable variance. If there is a seasonal break in the series it will be reflected in the SI ratios. When the moving averages are applied to the SI ratios to estimate the seasonal component, the estimate of the seasonal component will be distorted. The result is known as leakage.

Leakage occurs when part of the variation of one component has been incorporated into the variation of another component. The result could be that either some seasonal variation is left in the irregular component (and so not all the seasonal variation is removed from the seasonally adjusted series); or some variation that is not caused by seasonality is removed from the series. The result in both cases is increased volatility in the seasonally adjusted series and potential residual seasonality.

In the case of seasonal breaks, the leakage is of both types. The break in the seasonality causes distortion in the estimation of the seasonal factors (D8 table). Specifically, part of the step change at the break will leak into the irregular component. This has the effect of making the irregular in the vicinity of the break look larger, while turning the step change in the seasonal factor into a smooth transition. Besides the distorted seasonal factors, this has an important effect on the main diagnostic for stable seasonality (the F-test in Table D8A); the artificially large irregulars will inflate the residual mean square, and will reduce the F-value.

Figure 14.2 shows an example of SI ratios in a series with a seasonal break. The figure shows that there has been a sudden drop in the level of the SI ratios for August between 1998 and 1999. All of the SI values from 1998 are being treated as outliers. Moving averages are applied to these ratios in order to estimate the seasonal component of the series for August.

In this example, for the years prior to 1999, the estimates for the seasonal factors will be lower than they would be if the seasonal break was properly accounted for. When these seasonal factors are applied to the original data to produce the seasonally adjusted series, some of the seasonal variation will remain in the irregular component (it has leaked into the irregular series), resulting in residual seasonality in the seasonally adjusted series. Conversely, after 1999 the seasonal factor estimates are higher than they

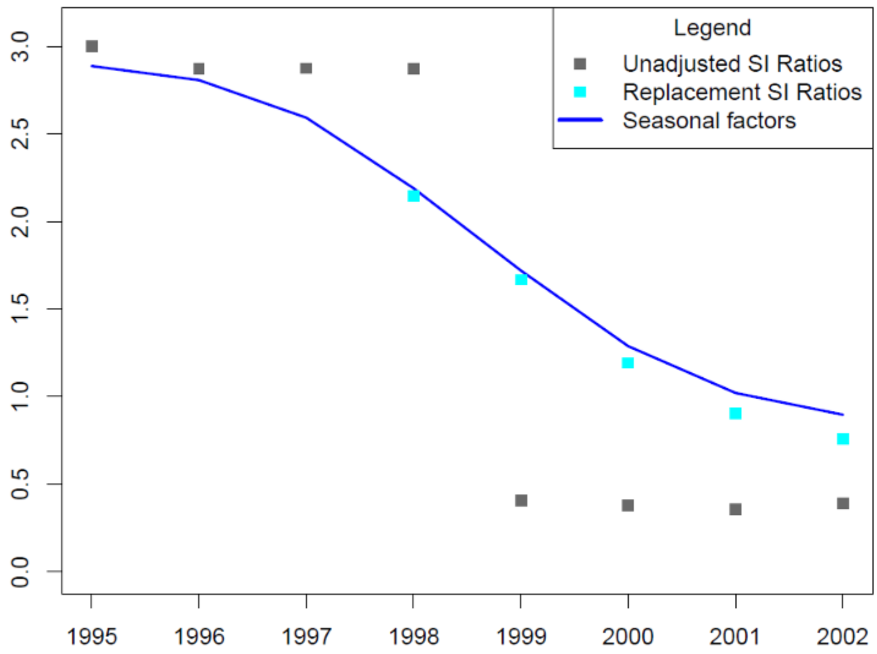

Figure 14.2: SI ratios

should be. The result of this is that variation is removed from the seasonally adjusted series that is not seasonal. This means that the seasonally adjusted series will not be reflecting the economic behaviour of the series. The result in both cases is a higher level of volatility in the seasonally adjusted series (as shown in Figure 14.3), and a greater likelihood of revisions.

Seasonal breaks cause distortion in the estimation of seasonal factors before and after the break. This distortion is often quantitatively significant. It can influence several years of data, depending on the length of the moving average, and, if not corrected for, can generate a seasonal pattern in the seasonally adjusted series. In the example above, where a 3x3 moving average was applied, the estimates of the seasonal factors will be distorted for 5 years. If the seasonal moving average applied to the majority of series were a 3x5 moving average, the distortions in the seasonal factors would occur for seven years. As a result, the seasonally adjusted series would appear to be more volatile near the break and would not reflect the underlying behaviour of the series.

Furthermore, as with trend breaks, seasonal breaks create problems in the identification of trading day and Easter effects and in fitting an ARIMA model. Hence, adjusting for seasonal breaks also improves other parts of the seasonal adjustment process. Figure 14.3 shows how the seasonal adjustment for the Car Registration series is very volatile and does not reflect the underlying behaviour of the series when no adjustment is made for the seasonal break. There are also periodic troughs remaining in the seasonally adjusted series, indicating that there is residual seasonality.

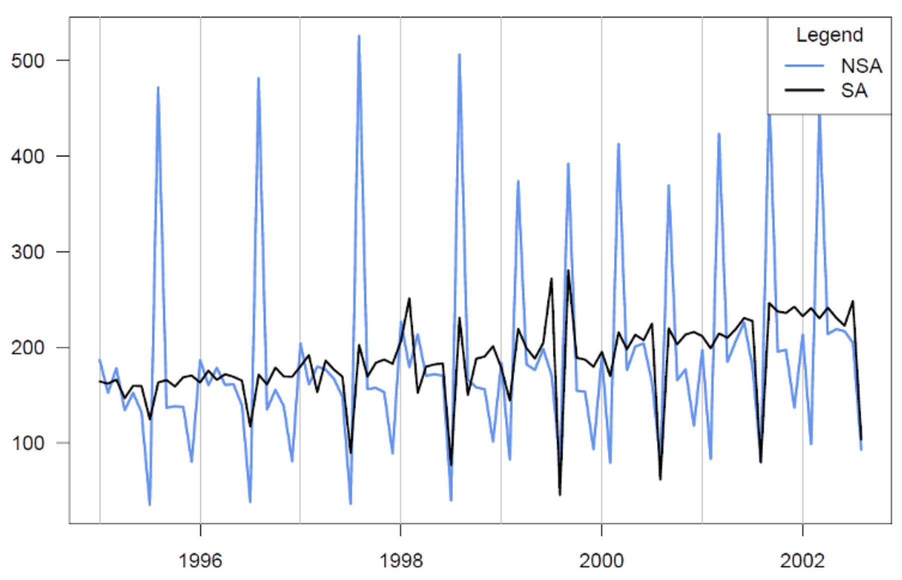

Figure 14.3: No prior adjustment for seasonal break

14.3 A USEFUL TEST FOR SEASONALITY

The most used test for seasonality is the "Combined test for the presence of identifiable seasonality", given after table D8A of the output from X-13ARIMA-SEATS. We will refer to this test as the IDentifiable Seasonality Test Result (IDS). The results discussed in this chapter rely on the assumption that the seasonality tests occur after all other processing that is required. The following are examples of effects that can invalidate seasonality tests.

- Outliers
- Trend or seasonal breaks
- Large calendar, Easter, or trading day effects

All such effects should be investigated individually and any prior adjustments should be made before testing for seasonality or running seasonal adjustment. The result of the IDS test is that one of the following statements will always occur in the output file:

- identifiable seasonality present
- identifiable seasonality probably not present

- identifiable seasonality not present

It is recommended that a series is adjusted in the first two cases and not adjusted in the last one. However, we cannot rely on just one statistic – it is much better to consider all the time series plots and the other statistics produced by X-13ARIMA-SEATS. There are cases when we might need to deviate from the simple recommendation above, for example when there are:

a) contradictory results from other statistics/plots

b) weak seasonality

c) composite time series

Often a) goes hand in hand with b) or c).

14.4 HOW TO IDENTIFY AND ADJUST FOR A SEASONAL BREAK

This section describes how to identify and adjust for a seasonal break. The section has been structured following a logical order that should be used in the process of analysing the seasonal break.

14.4.1 *Run the Series in X-13ARIMA-SEATS and look at the output diagnostics*

In some cases, there will be prior knowledge that a seasonal break is expected. In the case of the car registration series, for example, analysts knew of the administrative change in registration procedures in advance and that its purpose was to change the seasonal pattern. In such cases we can go straight to the tests described in Section 14.4.3. In other cases we should carry out a preliminary screening to see if there is any indication of a seasonal break. The following steps suggest a systematic way of carrying out this screening.

1. Run the series using the standard spec file in X-13ARIMA-SEATS without testing for a seasonal break at any particular point

2. As a first test, look at the diagnostics in Table D8A (F-tests for seasonality). If the stable seasonality tests are significant, but the moving seasonality test is not significant, it is unlikely that there is a seasonal break. If the moving seasonality F-value is large, the seasonality is changing considerably, and it is worth checking to see if some of this change occurs in steps – proceed to the next check

3. Graph the series and look at the graphs of SI ratios against the seasonal factors[1]. A seasonal break will appear as a sudden change in the level of the SI ratios for a particular month/quarter. The adjustment for the seasonal break effectively attempts to remove this sudden change in the level of the SI ratios

4. The seasonal break identification is not always as obvious as the example in this chapter. In those cases where the seasonal break cannot be spotted from the graphical representation of the series, an analysis of the X-13ARIMA-SEATS output can help in the detection of the break. In addition to the inspection of the SI ratios, seasonal breaks can be detected by analysing tables C17 and E5 in the output: when a seasonal break occurs, C17 may show a concentration of outliers within a particular month/quarter and E5 may show a sudden change in the size of the month-to-month/quarter-to-¬quarter changes for one or more months/quarters. In the example table C17 below, there is a suspected seasonal break in 2001, as seen by the lower weights in consecutive months

```
C 17 Final weights for irregular component
 From 1998.1 to 2003.2
 Observations    22
 Lower sigma limit 1.50
 Upper sigma limit 2.50

-------------------------------------------------------------
           1st      2nd      3rd       4th        S.D.
-------------------------------------------------------------
 1998     100.00   100.00   100.00   100.00       2.89

 1999     100.00   100.00   100.00   100.00       2.89

 2000     100.00   100.00   100.00   100.00       2.89

 2001      93.77    50.55    35.04   100.00       2.87

 2002     100.00   100.00   100.00     0.00       2.73

 2003       0.00   100.00
```

1 Chapter 17 - Section 17.5 gives instructions on how to do this

14.4.2 *Testing for seasonal break using X-13ARIMA-SEATS*

The regARIMA modelling stage of the program provides a function called a seasonal change of regime variable, which is specifically designed to model a seasonal break. If there is reason to suspect a seasonal break is present at a particular time point then it can be specifically tested for using X-13ARIMA-SEATS by including the following specification:

```
regression{
   variables=(seasonal/1999.jan//)
   save=(rmx)
   }
```

This command is appropriate to the car registration series above, where the presumed date of the break is January 1999.

The ***variables=(seasonal/1999.jan//)*** argument instructs X-13ARIMA-SEATS to test for a fixed change in seasonal pattern from January 1999. This is achieved by including dummy seasonal regressors prior to the break, which take the value of 0 after the break. When included in the regression spec, the output includes t-tests for each month/quarter regressor and a chi-squared test to verify the significance of the regressors as a group[2]. This command allows the user to test, but not adjust, for a change in the pattern of the seasonal component. This means that the data passed to the x11 spec will still be affected by the break, and so the calculated seasonal factors and the D8A F-test will be distorted in the way described. Hence it is unwise to place any reliance in the output of the x11 spec in the sequence above. Indeed, it may be sensible to run just the regARIMA part at this stage, without an x11 spec, until decisions have been made on whether a seasonal break is present.

The date specified for the change of the seasonal component (January 1999 in the example above) divides the series into two spans. The first span contains the data for periods prior to this date and the second span contains data for periods on and after this date. Including the argument ***variables=(seasonal/1999.jan//)*** in the regression spec means that X-13ARIMA-SEATS has been asked to estimate partial change of regime variables for the early span, where partial means that the change of the seasonal component is restricted to the early span. This type of partial change of regime can only be used in conjunction with another component which models the seasonality over the whole series. This could be either a set of fixed seasonal dummy regressors or a seasonal difference term in the model[3].

2 definition of these tests can be found in Chapter 8
3 for example, $D \geqslant 1$ in the ARIMA (p d q)(P D Q) model

With this combination, the whole series model deals with seasonality on both sides of the break and the partial change of regime regressors estimate the difference between the seasonality in the two parts of the series. This difference can then be used to construct permanent prior adjustments to force the seasonal pattern of the latter part of the series onto the first part. The effect of the change of regime variables is to include in the regARIMA model, a set of regressor variables (11 in the case of a monthly series, 3 for quarterly) which show the contrast between one month and another. If these regressor variables can be saved or generated by some other means, they can be included as user variables with exactly the same effect as the change of regime. In the example above, the **save=(rmx)** command has been included in the regression spec to save the regressor variables with the associated dates so that they can be used later for the seasonal adjustment.

The use of regARIMA regressors is only recommended if the presence of a seasonal break has been previously confirmed and the p-value from the chi-squared test is less than 0.05. This is because the chi-squared test is biased in defining a seasonal break as significant (it has a high type I error). One possible drawback of the ***variables=(seasonal/1999.jan//)*** command is that the estimated effects of the permanent priors do not always balance out, leading to the problem that the level of the series does not remain constant. This means that the permanent priors should be checked to see if they compensate (so that any increase must be matched by an equal decrease), especially in current price series. The change of regime regressors for a seasonal break will be self-compensating automatically for an additive model, because only 11 factors are estimated, the twelfth being calculated to make them sum to zero. For multiplicative this applies to the logs of the seasonal break regressors, which may not cancel out exactly when transformed back to the original scale, but unless the seasonal changes are large this should not be a major effect.

14.4.3 *Confirming a reason for a seasonal break with the host branch*

If the X-13ARIMA-SEATS test and graphics lead you to suspect a seasonal break, then check which months/quarters it appears in. Ask the host branch if there is any reason for suspecting a seasonal break at this time point.

14.4.4 *Length of the series before and after the seasonal break*

1. The entire length of the series in total needs to be at least 5 years to use regARIMA

2. At least 1 year of data either side of the break is needed to be able to use regARIMA as a tool for analysing the break

3. If less than 1 year is available after the break, consider not publishing the seasonally adjusted version of the series until more observations are available. This might not be possible if the series is a component of an aggregate, in which case adjustments will probably need to be judgemental. In that case use external information (for example forecasts of sales patterns produced by the car industry ahead of registration change) or patterns in related series if possible

4. If 1-2 years of data are available after the break, consider not publishing the seasonally adjusted version of the series until more observations are available. X-13¬ARIMA-SEATS can provide adjustments, but they are generally of very poor quality and subject to large revisions as future observations become known. X-13ARIMA-SEATS estimates can therefore either replace, be used in combination with, or validate ad hoc attempts to adjust for the break

5. If 2-3 years of data are available after the break, consider forecasting 1 year of data beyond the end of the series using regARIMA. If the seasonal pattern after the break looks very regular (both the model and the SI ratios are stable), then publication of the seasonal adjustment can be reinstated at this point

6. If more than 3 years of data are available after the break, publication of the seasonal adjustment can generally be reinstated. But if the model is exceptionally poor, the series is exceptionally erratic or SI ratios are exceptionally inconsistent between years, do not reinstate publication at this point.

14.4.5 *Adjust for the seasonal break*

Use one of the three following methods to adjust for the seasonal break.

Use of regARIMA to derive permanent priors

It is possible to use the regARIMA modelling capabilities of X-13 ARIMA-SEATS to calculate the permanent prior adjustments. As discussed above, the effect of a change of regime can be reproduced by including appropriate user variables. These variables can be generated by formula or by saving the regression matrix in a run with a change of regime: the latter is probably the easier approach.

If both arguments **variables=(seasonal/1999.jan//)** and **save=(rmx)** are included in a spec file then the regressors for the change of regime variable will be saved in a file with the extension ".rmx". To then adjust for the seasonal break it is necessary to re-run the spec file removing the

variables=(seasonal/1999.jan//) and **save=(rmx)** arguments from the regression spec and including the variables previously saved in the ".rmx" file as user-defined regression variables. For a monthly series there should be 11 variables in the ".rmx" file, so the spec could be of the following form:

```
regression{
   user= (M1, M2, M3, M4, M5, M6, M7, M8, M9, M10, M11)
   file=name. rmx
   format="x13save"
   }

x11{
   mode=mult final=user
   }
```

where "name.rmx" is the ".rmx" file generated when you use the save=rmx option in the "name.spc" file. One important point is to ensure that the ".rmx" file only contains the 11 change of regime variables. If other regressors had been included when the ".rmx" file was saved then these regressors will also be included. To avoid this, either remove all other regressors when saving the ".rmx" file or manually remove the extra columns from the ".rmx" file.

There are two ways of ensuring that the change of regime regressors are reflected in the final seasonal adjustment. One way is shown above, with the inclusion of **final=user** in the **x11** spec. The alternative is to include the argument **usertype=seasonal** in the **user** argument of the regression spec. These should not be used together. The **usertype** method includes the prior adjustment as part of the seasonal component, while the **final** method treats it as an ordinary prior. As a result, although the two methods give the same final seasonally adjusted series they give somewhat different quality diagnostics (the M and Q statistics) because of the different breakdown of the original series.

The permanent priors derived by X-13ARIMA-SEATS are shown in table A9 (or A10 if the **usertype** argument is used in the regression spec). These prior adjustments can also be derived from the parameter estimates of the regression model used to parameterise the seasonal component. In other words the permanent priors are equal to the parameter estimates themselves if the series is additive and a seasonal variable has been included in the regression spec in conjunction with a change in regime option. The permanent priors, over any single year, should average out to approximately 100, for multiplicative cases.

The permanent priors derived by the regARIMA regression adjust for the leakage of the seasonal variation into the irregular component, and the estimated seasonally adjusted series and trend component are more robust as a result. Figure 14.4 shows the improvements in the result of seasonal adjustment. In fact, the SA series does not show any residual seasonality before or after the break.

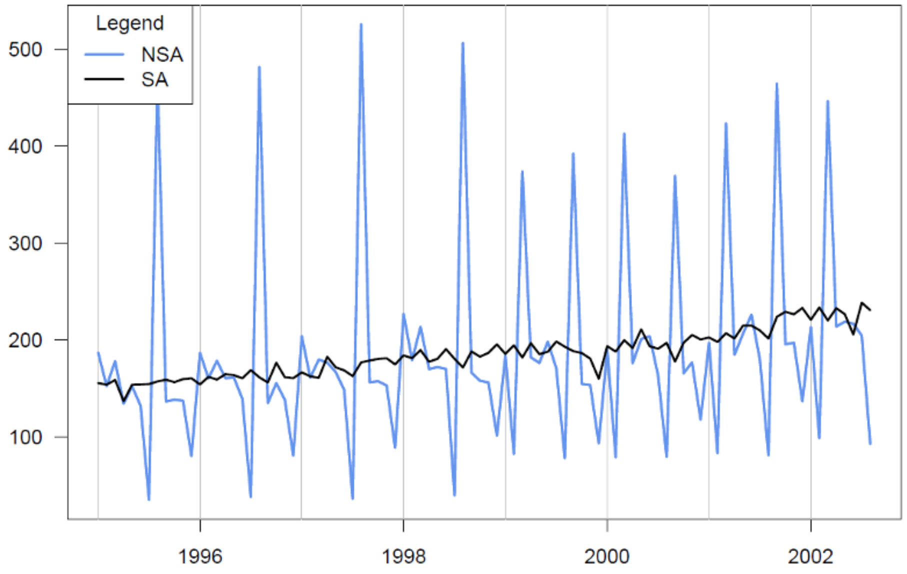

Figure 14.4: Seasonally adjusted car registrations prior adjusted for seasonal break

The improvement in the seasonal adjustment can also be seen in the SI ratios in Figure 14.5. The SI ratios for August after the seasonal break adjustments derived by regARIMA are much flatter, so an improved estimation of the seasonal factors can be calculated. In this example the priors have been treated as user defined and not part of the seasonal component. If the priors had been treated as part of the seasonal component, the SI ratios would still display the break. But the seasonal factor would contain what appears to be a level shift.

Problems:

- The interpretation of the seasonal diagnostics needs to be done with care if this approach is used. The usual Table D8a test for seasonality is based on the series input to the X-11 phase, which will have been prior adjusted for the seasonal break. Therefore, this test may not give a true picture of seasonality over the whole series; it will be dominated by the seasonal behaviour after the break. Additionally, the break adjustment applies a fixed seasonal effect to the series before the break; if the seasonal pattern was evolving before

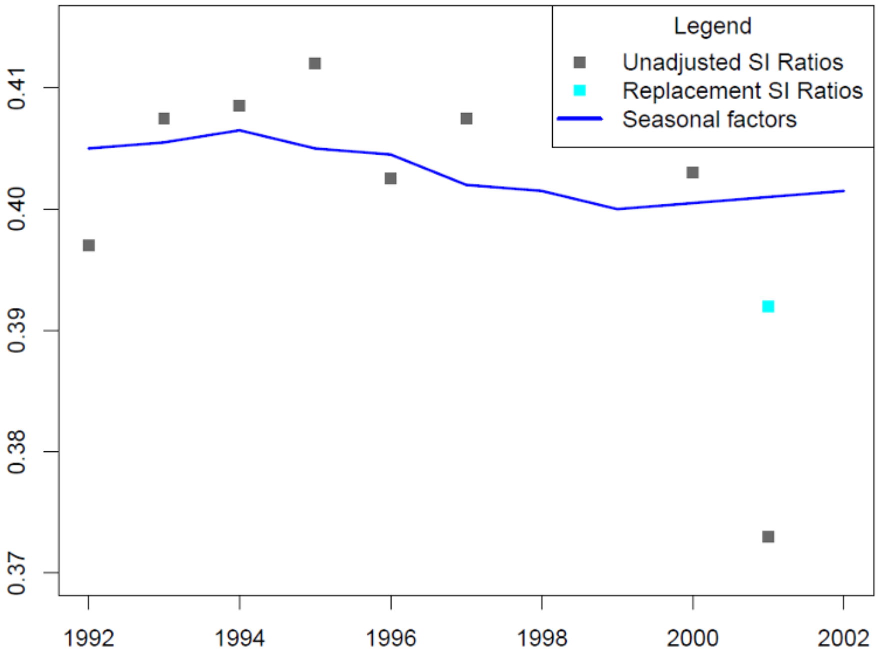

Figure 14.5: SI ratios with prior adjusted for a seasonal break

the break, the effect of the break adjustment may be to magnify the relative effect of the movement. In some circumstances, these two effects may move the diagnostic for identifiable seasonality towards "not present".

- If the break is non-compensating, the estimated permanent priors have to be modified so that any adjustment in one month can be balanced by a compensating adjustment in another month (or spread across several months). In this case the seasonal adjustment of the series should be run using the modified adjustments as permanent priors in the transform spec

- The analysis of seasonal breaks can be difficult in series with MCD = 1 or 2[a]. In fact, in those series the influence of the irregular component is very small, and the trend and seasonal components mainly drive the behaviour of the raw data. In this situation any small change in the seasonal factors is considered as a significant change in the seasonal pattern and produces a p-value from the chi-squared test of less than 0.05. Therefore, for these series the chi-squared test is less useful, meaning that a seasonal break analysis should rely more on the SI graphs analysis, with a seasonal break only being adjusted for if an economic reason has been confirmed

- The presence of other discontinuity factors (such as additive outliers or level shifts) not already adjusted for using regressors in the regARIMA may affect the performance of the chi-squared test in the analysis of seasonal breaks. For this reason, it is recommended to check the significance of the seasonal break and to use the estimated permanent priors calculated after the adjustment for outliers or level shifts has taken place

- Depending on the nature of the seasonal pattern and how it changes, it may be difficult to say from the data exactly when the break occurs. The chi-squared test statistics can be significant (p-value less than 0 .05) in a range between 7 months before and 9 months after the actual date of the seasonal break. Therefore, it is useful to check on the timing of the seasonal break in the C17 output table. In fact, if C17 presents, within this range, one or more zeros before or after the date indicated in the regression spec **variables=(seasonal/1999.jan//)**, it may be necessary to test alternative dates for the regressor. Of course, all this should be considered in conjunction with the views of the host branch on when the events causing the break may have occurred

[a] for more information on MCD, see Section 22.2

Adjust the two parts of the series separately

Although it is often easier to use the regressors, if there is more than five years of data before and after the break the two parts of the series can be seasonally adjusted separately.

This is probably most appropriate for historical seasonal breaks where the period of the break is not going to be revised in published data. If the movement of the seasonally adjusted series through the period of the break is important, treating it as one adjustment with a break is probably preferable. Furthermore, if it is a more recent break, adjusting the two parts of the series separately is not an option

Problems:

- Adjusting the two parts of the series separately can create discontinuities in the seasonally adjusted series
- It is possible to use this method only with long series that contain more than five years of data before and after the break

Use of output tables to manually calculate the permanent priors

It is possible to calculate manually permanent priors for seasonal break from table D8 of the output using the following procedure:

- Run the monthly/quarterly series in X-13ARIMA-SEATS using a default spec
- Use permanent, temporary priors and Easter and trading day adjustment if they have already been defined, otherwise, use default settings for all options
- This default run should include all the data. From Table D8 take the average of the values. within each month/quarter before the break, and similarly the average of the values after the break
- For each month/quarter divide the average before the break by the average after the break to give the permanent prior for that month/quarter
- Apply the permanent priors for the appropriate month/quarter to all data points before the break

Permanent priors may already be pre-defined for Easter or other reasons such as errors in the data. To incorporate these with the permanent priors for the seasonal break multiply them together and divide by 100 for multiplicative models or add them together for additive models.

Problems:

- This method can be used to validate the permanent priors derived by the regARIMA model, but it doesn't work well with fast-moving seasonality
- It may create additivity problems

Criteria for deciding which of the three methods should be used are as follows:

Length of the series before and after the break - The length of the series before and after the break influences the decision of which of the three methods should be used. In fact, if 1-3 years of data are available either side of the break, method 3.5.1 should be used in conjunction with method 3.5.3 to validate the quality of the derived permanent priors. If 3-5 years of data are available after the break, method 3.5.1 should be used. If more than 5 years of data are available after the break, method 3.5.1 and 3.5.2 can be used, although method 3.5.1 is preferable since it makes the process of updating the prior adjustments easier.

Ease of use/updating The use of regressors in the model (method 3.5.1) is easy to do and facilitates the update of the permanent priors. This is particularly useful when less than three years of data are available after the break and the parameters need to be re-estimated frequently until they become stable. Also, the method for adjusting the two parts of the series separately (3.5.2) is easy to use, but requires at least 5 years of data either side of the break. On the other hand, the manual calculation of the permanent priors (method 3.5.3) is more elaborate and makes the update more difficult.

Multiple seasonal breaks A particular strategy needs to be adopted in this case, since it is not possible to define two or more sets of change of regime variables in a regARIMA model to correct for seasonal discontinuities. In the case of multiple seasonal breaks, it is necessary to analyse the series in stages. Starting from the first seasonal break, test for significance using a change of regime specification, and if significant include user variables from the saved ".rmx" file in all later stages. Repeat the process for each of the subsequent breaks, including in the regARIMA model change of regime regressors only for the latest break. Finally, all identified breaks will have user-defined variables, which can be used for production running.

14.5 QUICK IMPLEMENTATION FOR SEASONAL BREAKS IN A SPEC FILE

Here are some quick instructions for implementing seasonal breaks, if the user has a good understanding of the seasonal break process.

14.5.1 *Seasonal → seasonal & non-seasonal → seasonal breaks*

```
regression{
   # variables = ()
   variables = (seasonal/2022.12//)
   save = rmx
   # user= (M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 M11) or (Q1 Q2 Q3)
      depending if monthly or quarterly
   # file="name.rmx"
   # format="x12save"
   # usertype = (seasonal)
   }
```

INSTRUCTIONS:

1. If a break is suspected, remove anything fixed, such as *arima {model=}*, *regression{variables=}* etc., and instead automate with *automdl{}*, *transform{function=auto}*. It is usually best not to put any regressors as this could distort the break, so remove *outlier{}*, *aictest* = etc. In addition, adding regressors at this stage would insert them into the upcoming RMX which makes the process more difficult. It is best to add the regressors conventionally after the break has been implemented

2. Insert *variables = (seasonal/xxxx.xx//)* and *save = rmx* into the *regression{}* spec

3. Run

4. If break is significant (User defined regressor p-value below 0.05), then comment out (#) the two lines in step 2 and insert user/file/format/usertype lines.

Note:

- Take caution with the output of the automdl{}. If the output is showing a non-seasonal ARIMA model, then it is best to manually overwrite with a safe alternative, such as the (0 1 1)(0 1 1) model
- If the ARIMA output is: ARIMA(p 0 q)(P D Q), ARIMA(p d q)(P 0 Q) or ARIMA(p 0 q)(P 0 Q) then this will produce a constant which will be embedded inside the RMX. As mentioned in step 1, this should be avoided. Either replace the 0 with 1 or manually remove the constant from the RMX and insert constant later into regression{variables=}

5. At this stage regressors can be tested but keeping everything automatic
6. Run
7. Now the break should be accounted for with good regressors. Check to make sure the output looks visually acceptable then fix the automatic options

Note:

- TSAB currently uses this method for both Seasonal → Seasonal and Non-Seasonal → Seasonal breaks. Before, TSAB only used this method for Seasonal → Seasonal and used a modelspan for Non-Seasonal → Seasonal breaks. This was changed as theoretically it is better to use an RMX for both
- It may be noticed visually on Win-X13's overlay graph that the average of the SA on one side of the break is too high. This is especially noticeable for big breaks. This is not an issue as the graph is showing the D11 (non-constrained seasonal adjustment). The SAA (seasonally adjusted values with constrained yearly totals) will not have the raised SA. The SAA is generally what is used by official statistics.

14.5.2 *Seasonal → non-seasonal (or any combination ending with non-seasonal) breaks*

```
transform{
   function=log
   file = "filename.ppp"
   format = "x12save"
   type = permanent
   mode = ratio
   }

x11{
```

```
type=summary
}
```

INSTRUCTIONS:

- If a break is suspected, remove anything fixed such as *arima{model=}*, *regressionvariables=* etc... and instead automate *automdl{}*, *transform-function=auto*. It is usually best not to put any regressors as this could distort the break, so remove outlier, no aictest= etc... It is best to add the regressors conventionally after the break has been implemented
- Insert variables = (seasonal/xxxx.xx//) into the regression{} spec
- Run
- If the break is significant (p-value < 0.05), then modelspan the seasonal part. This is done in the series{} spec: modelspan=(,2022.04). The adjustments within the modelspan need to be as good as possible. Therefore, now it is fine to use regressors. Ensure that save=(d10) is inserted in the x11{} spec
- Run
- Locate the D10 file that was produced, copy into Excel, and change any D10 values in the suspected non-seasonal part to 0s if additive or 1s if multiplicative. Optional is to extend the 0s/1s (and the dates) for a few years, especially if new data will be added in the future
- Put this into a new TXT file and save it as a PPP file with the series name
- Convert the SPC file to one that is used for non-seasonal adjustment: remove any regressors, ARIMA model, and modelspan
- In this non-seasonal adjustment SPC, read the PPP using the example above. Remember, if multiplicative, then 'function=log' and 'mode = ratio'; if additive, then 'function=none' and 'mode = diff'
- Run
- Now the break should be accounted for. Check to make sure the output looks visually acceptable.

15 THE EXISTENCE OF SEASONALITY

15.1 INTRODUCTION

The purpose of seasonal adjustment is to remove seasonality from a time series so that users are better able to interpret the series. In order to remove seasonality we must:

a) decide whether the series is seasonal

b) examine how accurately the seasonal component can be estimated

If there is no detectable seasonality or the seasonality is very weak and difficult to measure there will usually be little benefit from seasonal adjustment, so the series should not be adjusted. Seasonality is not a well defined concept. Roughly speaking, a seasonal pattern is a change in a time series that is repeated a whole number of times per year. This makes sense only for infinite series and exact repetition, which is impractical. A more practical approach is to allow the change to vary slightly in the short term, in both size and timing, and to drift systematically over the longer term. Sudden changes in pattern can also be modelled. The fact that seasonality is ill-defined implies the assessment of seasonality is subjective. Even within a narrow context, such as seasonal adjustment with X-13ARIMA-SEATS, different practitioners may disagree about whether a series is seasonal. This chapter provided suggestions for tests of seasonality. These tests have been found to work reasonably well for the majority of series so they are practical. However, there is a large minority of series for which more careful analysis is required, beyond the routine use of a single test. It is unwise to follow unthinkingly the rules below. In the more difficult cases the analyst must balance the evidence with their judgement, experience and wider knowledge.

15.2 CONTRADICTORY STATISTICS

Some statistics may suggest the series is seasonal and others may suggest it is not seasonal. This is often a warning sign that something unusual is happening and a simple decision based only on IDS would be unsound. When the statistics are contradictory a subjective judgement must be made about the presence of seasonality. Consideration of several statistics calculated by X-13ARIMA-SEATS and attempting to answer the following questions will help make the decision. A plot of the unadjusted with the seasonally adjusted series should be analysed: does the seasonal adjustment make interpretation of the time series easier? Does it seem that there is recently emerging seasonality? Do the results of the sliding spans analysis (Chapter 18) suggest that the seasonality is changing? Does examination of the time series plots, tables C17, E5 and E6, suggest one or more seasonal breaks (Chapter 14)? If the spectra are available, do they suggest seasonality may be present? What is the wider context? Are related series being classed as seasonal? For example, if the series is in current prices there will be an associated series in constant prices or chained volume measure. It is usual to judge both the series as seasonal or not.

15.3 WEAK SEASONALITY

The IDS may change after new data are added to a time series. Often, if the series is weakly seasonal the seasonality test alternates between not seasonal and seasonal. It is undesirable to make frequent changes to the policy of seasonally adjusting because this causes large revisions. The recommendation is to do the same as last time unless an unusually large change has occurred in the time series. The following guidelines have been found useful, but are not absolute rules.

a) If a series is being looked at for a first time then adjust it if the seasonality test after table D8 gives either "identifiable seasonality present" or "identifiable seasonality probably not present". Do not adjust if the test gives "identifiable seasonality not present"

b) If the series was adjusted in last year's re-analysis then continue adjusting it, unless the seasonality test shows absence of seasonality and the M7 is higher than 1.250 (monthly series) or 1.050 (quarterly series)

c) If the series was not adjusted in last year's re-analysis then continue not adjusting it, unless the seasonality test shows that there is seasonality present and the M7 is lower than 1.150 (monthly series) or 0.900 (quarterly series)

It must however be emphasised that these margins may not be appropriate for all series. It is therefore recommended that judgement is used, especially when the decision of whether to adjust a series or not has already changed at least once in the past. Finally, it should be mentioned that both the IDS test and the M7 are saved in the log file, if this is explicitly specified in the command file. This makes it easier to decide for many series at the same time, for example when using a data metafile.

15.4 COMPOSITE TIME SERIES

For sets of related time series it is quite common to perform the seasonal adjustment at a low level of aggregation and combine the seasonally adjusted series to get the seasonally adjusted version of a composite series. This process is covered in more detail in the chapter about the composite specification, Chapter 20 and Chapter 6 on aggregate time series. The problem with this approach is that there is often residual seasonality in the composite series. If we run the X-13ARIMA-SEATS program on the indirectly seasonally adjusted composite series the IDS test can show that seasonality is present.

The cause of this problem is that the more volatile the irregular component is, the more difficult for any seasonality in the series to be detected and estimated accurately. Usually the irregular component is larger for lower level series, and consequently some of the seasonal series are deemed non-seasonal, or have their seasonality measured incorrectly. When the lower level series are combined, the noise is reduced and the residual seasonal components become noticeable.

X-13ARIMA-SEATS provides assistance in investigating direct versus indirect adjustment in the form of the composite spec. In this case, a conventional spec file is provided for each component series, while for the composite the series spec is replaced by a composite spec. The link between the components and the composite is provided by an input metafile, with filename extension ".mta". The program will run a conventional adjustment for each component and for direct adjustment of the composite, while also calculating the indirectly adjusted composite. The full range of diagnostics is provided for each case, including the indirect composite; thus it is easy to compare the quality of the two adjustments, and in particular to see whether there is residual seasonality in the indirect adjustment. The components that are not adjusted are treated by setting type=summary in the x11 component of their spec files.

Ideally we would prefer to adjust each series individually, but it is often the case that certain constraints need to be satisfied. One option is to use a direct adjustment and apportion the differences between the direct and indirect adjustments to the component series. This is not easy to do, because of the amount of calculation required and the possibility that the constraining process will generate residual seasonality in some of the component series.

An alternative approach is to seasonally adjust more of the component series in order to exactly capture the part of the seasonality that is negligible for an individual component series but significant in the aggregate. To decide which component series should be seasonally adjusted an alternative to the IDS test can be used.

The IDS test result is based on the following tests:

1. Test for the presence of seasonality assuming stability
2. Nonparametric test for the presence of seasonality assuming stability
3. Moving seasonality test

An alternative test is to seasonally adjust a series if one of the first two tests passes.

If more of the component series are adjusted then the indirectly adjusted composite series is less likely to have any residual seasonality. If it does, then adjusting all component series in the data set regardless of seasonality tests may help, though the seasonality in these series is more difficult to estimate correctly. Some imagination may be required in the application of permanent priors for this method to be effective.

15.5 TESTING A COMPOSITE DATASET FOR SEASONALITY

The following steps should be followed in order to test a large composite data set for seasonality.

1. Specify the X-13ARIMA-SEATS command file as follows: x11{ savelog=(ids, M7)}
2. Run seasonal adjustment of all series in the data set, using a data metafile
3. Check the log file, to find the IDS test classifying each series as seasonal or not seasonal
4. Use the strategy in Section 15.3 to decide which of the series should be adjusted. The M7 diagnostic is given in the log file
5. Keep the seasonal adjustment of those series that in the previous stage it was decided to adjust. For the other series, the seasonal adjustment is the same as the original series, with any possible calendar effects removed
6. Derive the indirect seasonal adjustment for all aggregate series. Run it through X-13ARIMA-SEATS to check if there is any residual seasonality by IDS. If there is no residual seasonality in any of the indirectly adjusted series then the seasonal adjustment is completed

7. If there is residual seasonality in any of the indirectly adjusted series, then study those component series that were not adjusted in the first place, or were adjusted only for calendar effects. Adjust those that pass at least one of the two stable seasonality tests

8. Derive the indirect seasonal adjustment of the aggregate series again, also using the adjusted versions of the series that were adjusted in stage 7. Run the indirect seasonal adjustment through X-13ARIMA-SEATS again to check for residual seasonality

9. If the IDS is "no" then the seasonal adjustment is complete. If it is not, then try adjusting all component series regardless of the IDS. Some imagination may be required in the application of permanent priors to remove all residual seasonality.

16 X-13ARIMA-SEATS STANDARD OUTPUT

16.1 INTRODUCTION

This chapter will introduce the main output files generated by X-13ARIMA-SEATS. These include:

- An output ('.out') file for each series that is processed
- An error ('.err') file for each series that is processed
- A log ('.log') file for each series processed, where certain diagnostics requested by the user are stored

Of them, the most important is the output file, which gives the results of the seasonal adjustment as well as useful quality diagnostics. The output file consists of a fairly long list of tables, which are identified by a capital letter followed by a number, and they are organised as follows.

- **A-tables**: These tables include the original data and prior adjustment of the original data. Any prior adjustments are specified by the user in the transform spec, but also can be specified in the regression part of the regARIMA model. This should also include any automatically identified outliers or breaks
- **B-, C- and D-tables**: A seasonal adjustment run consists of 3 iterations of the X-11 method. The output of the first iteration is saved in the B-tables which are designed to estimate outliers and trading day effects in the RegARIMA model rather than in the x11regression spec. The output of the second iteration is saved on the C-tables and the output of the final iteration is saved in the D-tables which provide final estimates of all components. Of course, only the D-tables are final
- **E- and F-tables**: These tables provide diagnostics of the seasonal adjustment
- **G-tables**: These are graphics[1]
- **R-tables**: These tables show the revision histories[2]
- **S-tables**: These tables show the sliding spans diagnostics[3]

1 more in Chapter 17
2 more in Chapter 7 and Chapter 19
3 more in Chapter 18

D8	Final unmodified SI ratios
D9	Final replacement values for extreme SI ratios
D10	Final Seasonal factors
D11	Final Seasonally adjusted series
D11.A or SAA	Final Seasonally adjusted series with forced yearly totals (also adjusted for trading day)
D12	Final trend-cycle
D13	Final irregular series

Table 16.1: Frequently used D-tables

Some of these tables are more important than the others and in fact the majority of the tables are not even printed in the output unless specifically requested by the user. Therefore, it is deemed more appropriate to organise the presentation of the output tables not by their numbers but by the purpose they serve.

This is presented in Section 16.2 and Section 16.3, Section 16.4 and Section 16.5 provide new or improved diagnostics of X-13ARIMA-SEATS for SEATS estimates. Finally, the log and error files are described in Section 16.6.

16.2 OUTPUT DIAGNOSTICS

Before using the results of the seasonal adjustment, the users need to satisfy themselves that the adjustment is of a good quality, in other words, check the diagnostics. An output file from X-13ARIMA-SEATS starts by repeating the specification file used to generate these diagnostics. It is not a bad idea to check that the specifications are those intended by the user, especially if alternative specifications are tried.

Table A1 follows immediately after and shows the original series. Again, it is not a bad idea to have a quick look to make sure that it is the correct data, especially when it is imported from another file. Having a quick look at the first and last data points to ensure that all data points have been correctly imported is a good idea and after these basic checks, the output diagnostics should also be checked. Generally speaking, the output diagnostics can be classified into two types: diagnostics of the prior adjustment and diagnostics of the seasonal adjustment.

16.2.1 *Diagnostics of the prior adjustments*

Since good prior adjustment is a prerequisite for good seasonal adjustment, this implies that any external factors that affect the series are appropriately treated. These effects are captured either through:

1. the regression part of the regARIMA model, as it was described in Chapter 8, or

2. by putting in prior adjustments directly by means of the transform spec.

Table A2 shows the total prior adjustments specified by the user. As with Table A1, it is necessary to check that they are those intended by the user. However, if both permanent and temporary priors are used, Table A2 will show their combined effect, so it will be different from each of them taken individually.

For effects captured with the regression model, one should look at the regARIMA fit, following the guidance of Chapter 8. However, the prior adjustment might be inadequate even if the regARIMA diagnostics are good. This would be the case when significant regression variables are missing. For example, there might be a strong trading day effect that the user did not include or included incorrectly. But the most common cases where a significant variable is omitted are outliers, trend or seasonal breaks. These can be identified by means of the following diagnostics:

- **Automatic outlier identification**: Including the outlier spec in the spec file will enable the automatic identification of any unaccounted outliers or level shifts. Of course, only those among them that are valid should be included in the regression[4]. It should be noted that if many outliers are automatically detected in the same month or quarter, it could be evidence of a seasonal break

- **Table D8** (SI ratios): If there is a sudden increase or decrease for one or more months or quarters, this might imply a seasonal break. This is more likely if it occurs for many months or quarters and at the same time

Example:

In this example the fourth quarter is persistently higher than 110 up to 1995, falling to 107 in 1996, to 103 in 1997, and finishing below 100. This could be either because of evolving seasonality or a seasonal break. However, the speed of this evolution (completed in just 3 periods with stability before and after), implies that it is probably a seasonal break.

4 see Chapter 11

	1^{st}	2^{nd}	3^{rd}	4^{th}	AVGE
1991	83.7	100.6	104.1	111.9	100.1
1992	83.2	100.8	104.5	110.6	99.8
1993	84.7	100.0	104.0	112.1	100.2
1994	83.1	101.6	103.9	110.7	99.8
1995	80.9	101.1	104.9	112.9	99.9
1996	81.8	100.3	104.7	107.0	98.5
1997	89.1	100.1	104.9	103.2	99.3
1998	92.4	101.6	104.2	101.1	99.8
1999	94.1	102.8	103.6	100.1	100.1
2000	93.8	102.3	104.8	98.2	99.8
2001	94.1	104.5	102.4	98.6	99.9
2002	94.2	104.5	103.4	97.5	99.9

Table 16.2: D8 Final unmodified SI ratios from 1991.1 to 2002.4 with 48 observations

Inspection of the first quarter supports this suspicion; whilst for the period up to 1996 the SI ratio was between 80 and 85, in 1997 it jumps to 89 and thereafter it is persistently above 90. Noteworthy also is that this rapid evolution of the 1^{st} and 4^{th} quarters takes place at the same time, which again supports the suspicion of a seasonal break. Note finally that one does not need all quarters to be affected to argue that there is a seasonal break; indeed, in the example above the 2^{nd} and 3^{rd} quarters are fairly stable- the break basically consists of shifting activity between the 4^{th} and the 1^{st} quarter.

- **Table E5** (month-to-month or quarter-to-quarter changes in the unadjusted series): These are expected to be dominated by the seasonal component. Thus, it is expected that at least some months/quarters have the same sign all the time. If this pattern changes, this implies a seasonal break

Example:

The above example is from the same output as the previous example. One can immediately see that the sign for the 4^{th} quarter is positive till 1996 and negative afterwards. Further, although the sign in the other quarters does not change there are some significant changes in the size of the figures: The 1st quarter rises from below -20 to single digit, the 2^{nd} quarter falls from 20-30 to around 10, while the 3rd quarter is affected too. This is good evidence of a seasonal break, which probably occurred sometimes between 1996.3 and 1997.1.

	1st	2nd	3rd	4th	AVGE
1991		21.3	4.6	8.3	11.4
1992	-24.3	24.7	6.1	8.1	3.7
1993	-22.3	18.6	5.6	10.1	3.0
1994	-24.7	24.1	4.3	8.3	3.0
1995	-24.8	28.9	6.0	8.2	4.6
1996	-26.6	26.9	9.7	5.3	3.8
1997	-18.2	7.9	2.7	-0.5	-2.0
1998	-8.7	10.5	1.7	-1.9	0.4
1999	-4.4	11.7	1.7	-2.6	1.6
2000	-6.5	9.5	3.9	-4.6	0.6
2001	-2.6	11.9	-1.6	-1.9	1.5
2002	-2.1	12.5	-0.6	-5.7	1.0

Table 16.3: E 5 quarter-to-quarter percent change in the original series from 1991.2 to 2002.4 with 47 observations

- **Table E5** is also useful in identifying outliers or level shifts. Outliers manifest themselves with a large value which is followed by a large change of the opposite sign in the following month; level shifts manifest themselves with large changes which are not balanced with subsequent opposite changes

- **Table E6** (month-to-month or quarter-to-quarter changes in the adjusted series): As with Table E5, outliers show up as large numbers which are followed by a change back, while level shifts do not change back

- **Table C17** (final weights of the irregular component): The weight of each point in Table C17 is 100, unless this point has been picked up as an outlier in the irregular component during the X-11 algorithm iterations. Table C17 is very useful in identifying problems such as breaks or outliers. For example, if there is a concentration of outliers (values lower than 100) within a particular year or month this could be indicative of a seasonal break; if outliers dominate March and April this may indicate a need for Easter adjustments

In summary, good prior adjustment consists of the following:

- Appropriate fit of the regression model

- No significant variable is omitted

- All outliers, level shifts and seasonal breaks have been identified and adjusted for

Finally, it must be emphasised that the diagnostics listed above should be used to help identify a potential problem but should not be entirely relied upon to decide on whether the problem is significant or not. Instead, once a problem is identified it should be modelled in the regARIMA model, and it is through the regression diagnostics that it will be eventually decided whether it is insignificant, or it is significant and needs to be adjusted for. Once the user is satisfied with the quality of the prior adjustment the next step is to check the quality of the seasonal adjustment itself.

16.2.2 *Diagnostics of the seasonal adjustment*

The first thing to check is the seasonality tests in Table D8a and the M7 diagnostic. This will determine whether the series is seasonal or not, as described in Chapter 15. If the conclusion is that the series is not seasonal then the series should generally not be seasonally adjusted. Assuming that the series is seasonal, the next steps should be to check other diagnostics of the seasonal adjustment. These include the M-diagnostics in Table F3. These take values from 0 to 3, and a value higher than 1 indicates a source for potential problems for the seasonal adjustment. In particular:

- **M7** is the most important among the M-diagnostics, showing the amount of moving seasonality compared to stable seasonality, or in other words how regular the seasonal pattern is. Although M7 is used as a test for existence of seasonality, it is important to remember that it is not a binary (existent/non-existent) test but it takes continuous values

- Next most important is **M1**, which shows how large the irregular component is compared to the seasonal. Failure (that is a value higher than 1) of M1 implies that the irregular component is large and therefore it might be difficult to estimate the seasonal component accurately

- **M6** measures the irregular too but is valid only when a 3x5 seasonal filter is used. Failure of M6 means that a filter shorter than 3x5 should be used

- Next most important are the **M8** through **M11** diagnostics, which show the fluctuations in the seasonal component. These diagnostics are useful in showing potential problems in the seasonal component, such as seasonal breaks. M10 and M11 are the same as M8 and M9, but only for the end of the series. Thus, comparison of M10 and M11 with M8 and M9 can also help identify problems at the end of the series. It should also be noted that M10 and M11 might fail even if there are no problems with the seasonal adjustment, for example if the fit of the ARIMA model that was used to generate forecasts is poor. Finally, it must be mentioned that if M7 is high then M8-M11 are likely to be high as well

- **M4** is a measure of autocorrelation in the irregular component. It is a less important diagnostic, as good quality of seasonal adjustment does not require an uncorrelated irregular
- **M2** measures the amount of the irregular compared to the variance of the raw series made stationary. As a consequence, M2 may be misleading if the series has a trend that is not well-approximated by a straight line
- **M3** and M5 measure the irregular compared to the trend. They are not important diagnostics.

16.2.3 *Other diagnostics*

Other diagnostics include:

- Tests for residual seasonality, shown after **Table D11**. If the tests show that residual seasonality is present, it can be eliminated by one of the following means:
  - If the series is very long, one might consider shortening the series, as the cause of the problem might be that the seasonal pattern has changed with time. This may also be resolved by using seasonal breaks
  - An alternative is to change the lengths of the seasonal filters. This also includes the option of using different filters for different months/quarters, if appropriate
- **Table E6**, previously mentioned as an aid to check for outliers and breaks, can be also used to check for residual seasonality. In particular, the seasonally adjusted series is not supposed to contain any seasonal elements. Consequently, if the month-to-month change always has the same sign for some months, this might imply residual seasonality. However, if the same sign is present for most of the values of the table, this is probably caused by a steady trend, and not by residual seasonality

- **Table D9a** (moving seasonality ratio) provides the annual change of the (preliminary) seasonal and irregular component for each month/quarter. This is used by the program to automatically select the appropriate seasonal moving average (unless a particular filter was specified by the user). However, sometimes it might be appropriate to use different seasonal filters for some months or quarters. This is the case when in a particular month/quarter the seasonal component evolves too fast, or the irregular component fluctuates too much. In such cases one might wish to use a shorter filter for the month/quarter in question[5]. Table D9a can be used to identify a problem of this kind. The second line of the table provides the fluctuation of the seasonal component. If this is much higher for one month than the others, it may indicate the need for a longer filter

- Another criterion for heteroskedasticity is the SI ratios (from **Table D8** or from the corresponding graph). If they fluctuate too much for one month (yet not in the way that implies a seasonal break), this may also suggest that a longer filter might be needed

- Graphical output of the spectrum of some components is produced for monthly series after the F-tables. These graphs (labelled G.0 to G.2) are the only parts of the G outputs which are produced by default. These can be used to identify significant peaks at seasonal and trading day frequencies in the prior adjusted, seasonally adjusted and irregular series. A peak is defined as a value which exceeds the adjacent values by at least six stars on the plot. Any such peaks are mentioned in a brief note just above the G.0 plot. If regARIMA modelling has been carried out, a similar plot of the spectrum of the regARIMA residuals appears after the modelling output, with a similar note if any peaks are found. Any peaks found are also mentioned in the console output and the error file

- The **QS statistic** (in **Table F3**) can be used to test for the presence of seasonality within a series. The QS statistic is approximately distributed as a chi-squared distribution with two degrees of freedom and has a null hypothesis that there is no seasonality, so a p-value less than 0.05 would indicate seasonality is present. The QS statistics produces values for the non seasonally adjusted series, the seasonally adjusted series, the irregular series and the residuals

- Finally, useful diagnostics are X-13ARIMA-SEATS graphs, revisions history analysis and sliding spans which are described in Chapter 17, Chapter 19 and Chapter 18 respectively.

5 for more details see Section 14.4.2

16.3 THE RESULTS OF THE SEASONAL ADJUSTMENT

Once the users are satisfied that the seasonal adjustment is of good quality (or at least as good as possible) then they can proceed and use the results of the seasonal adjustment. Before describing the relevant output tables, it is useful to mention that there are different lengths of output available depending on how much detail is required. The user can specify the desired output length with the print argument in the x11 spec. The options, from the shortest to the longest output, include:

- **print=none**: With this option X-13ARIMA-SEATS gives only the results from the estimation of the regARIMA model. In particular, it gives the A-tables, regARIMA coefficients, residual checking (if requested) and forecasts. Nothing related to the seasonal adjustment is printed
- **print=brief**: With this option the X-13ARIMA-SEATS output includes everything that is printed with the "none" option, as well as basic seasonal adjustment tables and diagnostics
- **print=default**: Gives the same output as above, only with more seasonal adjustment tables and diagnostics
- With **print=alltables** all output tables are printed. Additionally, to the previous option, these include the intermediate tables from the first and second iteration of the X-11 method
- **Print=all** gives additionally all graphs that are produced. This option would not be recommended as the quality of the graphs is poor and there are other graphical tools available

Next, the output tables are described, with more emphasis given to the most important of them.

16.3.1 *Most important seasonal adjustment tables*

Tables A6, A7, A8 and **A9** respectively show the trading day, holiday, outlier and user-defined regression components estimated from the regARIMA model. These components include the effects of both program and user-specified variables, as long as the latter have been assigned the appropriate usertype. The effects given in Tables A6-A9 can be used not only for analysis, but also as prior adjustments for production running- as an alternative to re-estimating these effects every time a new data point becomes available.

Table B1 shows the original series, after all prior adjustments - including the regression model and automatically detected outliers. It is effectively this series B1 that goes through the X-11 algorithm. Table B1a shows the forecasts of the prior-adjusted original series.

Table D10 shows the final seasonal component. On the top of the table the seasonal moving average filter that was used is shown, which should be fixed and used until the next re-analysis.

Table D11 shows the seasonally adjusted series. However, if the user wants and has specified to constrain the annual totals then Table D11A, also known as SAA is the appropriate table.

Table D12 shows the final trend component. The Henderson moving average filter that was used is shown at the top of the table. This should be fixed and used for production running until the next re-analysis.

Table D18 shows the combined trading day and holiday factors that are used in the seasonal adjustment and it is equal to the sum (or product for multiplicative series) of Tables A6 and A7. As with those tables, it includes the effects of both program and user-defined calendar effect variables, and it can be used for analysis or as prior adjustment for production running- if the calendar effects are kept constant between seasonal adjustment reviews.

Also important are **Tables C17, D8, D8a, D9, D9a, E5, E6,** and **F3** which were mentioned in the previous section as useful diagnostics.

16.3.2 *Less important seasonal adjustment tables*

Theses tables are used infrequently, however a short summary is provided in case they are useful.

Table D13 provides the final irregular component.

Table D16 is the sum (or product) of **Tables D10** and **D18** and shows the total calendar and seasonal adjustment to the series.

Table E4 shows the annual totals of the original series divided by the annual totals of the seasonally adjusted series (In the case of additive seasonal adjustment it is subtraction rather than division). The second column of Table E4 shows the same ratio (or difference) but for the extreme-values-adjusted original and seasonally adjusted series. This table can be used as a quality diagnostic, especially when the annual totals are constrained; in that case, the more the ratio is away from 100 (or the more the difference is away from 0 in case of an additive model), the more the seasonal adjustment is impacted by annual constraining.

Table E7 shows the month-to-month change in the trend-cycle component (Table D12).

Tables F2A-F2I show certain diagnostics in more detail. These include changes, duration of run, or analysis of variance for certain components of the series. Most of these diagnostics are used to derive the single-value M-diagnostics of Table F3. Of the F2-tables it is deemed useful to mention

Table F2E which gives the Months for Cyclical Dominance (MCD), that is, the number of months it takes for the variation of the trend-cycle to become larger than the variation of the irregular component. This is very useful for presentation purposes, because if changes during spans shorter than the MCD are presented, they will be dominated by the irregular, thus they will be uninformative and perhaps misleading. Further, if the MCD is greater than 6 this indicates that the series is very volatile and the quality of the seasonal adjustment is not likely to be good[6].

16.3.3 *Tables not printed by default*

The user will only need tables in this subsection for an exceptionally thorough analysis. They are presented here briefly, more detail can be found in theX-13ARIMA-SEATS user manual (USCB 2017).

Tables C1 and **D1** print the series that go through the X-11 algorithm in the second and third iterations. These series are the original series adjusted for prior adjustments, regression effects, and extreme values identified in the previous X-11 iteration.

Tables D2, D4, D5, D6 and **D7** provide the preliminary estimates of the components of the series in the final X-11 iteration.

Tables B2-B13 and **C2-C13** are similar to the corresponding D-Tables, but for the first and second X-11 iteration.

Table B17 is the same as Table C17, but for the first X-11 iteration.

Table B20 and **Table C20** show the factors by which the extreme values identified in the first and second X-11 iteration are adjusted before the following (second or third) iteration.

Table C15 and **Table C16** are generated only when the irregular regression is run (x11regression). Table C15 gives the regression output while Table C16 gives the resulting trading day, holiday, etc., components.

Table D8B is same as Table D8, only it marks any extreme values according to whether they were identified during the regression/automatic outlier procedure, or during the X-11 iterations. This can be useful information for analysis purposes.

Table D12B and Table D13B show the trend and irregular components net of extreme values identified in the regression part of the model (the users are reminded that additive outliers and temporary changes are assigned to the irregular component, while level shifts and ramps are assigned to the trend-cycle).

Tables E1, E2 and **E3** show the original series, the seasonally adjusted series, and the irregular component, corrected for extreme values.

6 for more information, see Section 22.2

Table E11 shows a robust estimate of the final seasonally adjusted series. It is equivalent to Table E2, except for those points considered extreme, which are those which have been assigned zero weight in Table C17.

Table E16 shows the final adjustment ratios.

Table F1 is a smooth seasonal adjustment, derived by smoothing the original seasonal adjustment by means of a simple moving average, the length of which depends on the months for cyclical dominance diagnostic.

Table F4 is produced only if a trading day component has been included in the model of a monthly series. It shows the effect of the trading day component on the monthly adjusted figures, as a function of the length of month and the day of the week on which the month starts. It may be used as a second check on the plausibility of the trading day effects; the user should ask whether there is some known reason in the pattern of weekly activity which would explain why some figures are high and some low.

The example below shows an example of the F4 table.

Day of Week Component for regARIMA Trading Day Factors:

months starting on:	Mon	Tue	Wed	Thu	Fri	Sat	Sun
31-day months	101.54	101.54	101.54	99.75	97.99	97.99	99.75
30-day months	101.02	101.02	101.02	101.02	99.24	97.49	99.24
Leap year	100.51	*****	100.51	*****	*****	98.74	*****

Table 16.4: F4: multiplicative trading day component factors: day of week and leap year factors

16.4 DIAGNOSTICS OF X-13ARIMA-SEATS INHERITED FROM X-12ARIMA

As the successor to X-12ARIMA, X-13ARIMA-SEATS provides diagnostics inherited from X-12ARIMA.

The spectrum is a fundamental diagnostic for detecting the need for seasonal and trading day adjustments because these effects are periodic with known periods. The spectrum can also guide regARIMA modelling decisions in various ways, as it is illustrated in the log spectral plots of the (usually differenced and log-transformed) original and seasonally adjusted series and of the irregular component produced by X-13ARIMA-SEATS[7].

7 see Soukup and Findley (1998) and Soukup et al. (2001) for details concerning the spectrum estimator and trading day frequencies

X-13ARIMA-SEATS also includes model comparison diagnostics, such as the out-of-sample forecast error diagnostics of X-12ARIMA and diagnostics of the stability of seasonal adjustment and trend estimates, such as the revisions history diagnostics and sliding spans diagnostics. For the sliding spans diagnostics of SEATS estimates, a new criterion for choosing the span length is used. The span length is determined by the ARIMA model's seasonal moving average parameter, which generally determines the effective length of the seasonal adjustment filter. Because SEATS filters can have effective lengths much greater than X-11 filters, this modified criterion only permits standard sliding spans comparisons to be made when $\prec 0.685$ with monthly series of length thirteen years.

Summary statistics for the unstandardised residuals and the results of normality statistics for regARIMA model residuals are also produced by X-13ARIMA-SEATS for seasonal adjustment quality checks. Sample autocorrelations of the residuals with the Ljung-Box can be used for diagnostic checks to determine if the model selected is suitable for the data. The p-values approximate the probability of observing a Q-value at least this large when the model fitted is correct. Small p-values, customarily those below 0.05, indicate model inadequacy.

16.5 NEW OR IMPROVED DIAGNOSTICS OF X-13ARIMA-SEATS FOR SEATS ESTIMATE

For SEATS estimates, new or improved diagnostics of X-13ARIMA-SEATS are included in the ARIMA model-based signal extraction. This section covers:

- ARIMA estimation
- Derivation of the models for the components and estimators
- Error analysis
- Estimates of the components (levels)
- Rates of growth

ARIMA estimation: Used for detecting whether the model is fitted through test-statistics on extended residuals.

Derivation of the models for the components and estimators: Models for the components and decomposition through WIENER- KOLMOGOROV filters. Bias-correcting modifications for over- or underestimation of the irregular component and of the stationary transforms of the seasonal, trend and seasonally adjusted series from decomposition $Y_t = S_t + T_t + I_t, 1 \leqslant t \leqslant N$ a finite-sample-based diagnostic are provided. It also provides a set of associated test statistics.

The filter diagnostics - Weights for asymmetric trend concurrent estimator filter (semi-infinite realisation) and transfer function and phase delay of asymmetric trend filter (semi-infinite realisation) - for seasonal adjustment and trend estimates suggest the extent of the trade-off between smoothness and the delay or exaggeration of business cycle components in the estimates.

Error analysis: Significance of seasonality is assessed using the variances of the total estimation error, which includes the error in the preliminary estimator (the revision error) and the error in the final estimator. Because the S.E. of the seasonal component estimator varies (it reaches a minimum for historical estimation and a maximum for the most distant forecast), the significance of seasonality will be different for different periods. An extreme example would be a series showing significant seasonality for historical estimates that is poorly captured concurrently, and useless for forecasting.

Estimates of the components (levels): Various model-based tests are included in this section. These include estimates of the trend-cycle, seasonal and irregular components with standard errors. The final estimates of the trend-cycle, seasonal and irregular components also included. Model comparison diagnostics include diagnostic for difference between aggregate and aggregate of components, and diagnostics comparison of means in this Section.

Rates of growth: The rate-of-growth of series Z(t) over the period (t_1, t_2) is expressed in percent points as $(\frac{Z(t_2)}{Z(t_1)} - 1) \cdot 100$ although these results may be less important part of the whole output except for interested in the annual growth.

16.6 ERROR AND LOG FILES

One **error file** is generated for each series that is processed through X-13ARIMA-SEATS. This file stores the following information:

- Errors in the input files. Examples include syntax errors, internal inconsistencies of the input file, or problems with reading data files
- Problems encountered during processing, which halted the procedure. Singularity of the regression matrix and non-convergence are such examples
- Problems encountered during the processing, for which an automatic fix was put in place, of which the user must be warned. Examples include changes to certain procedures because there is insufficient data to run them with the options specified by the user (such as shorter spans for history analysis)

Properties of the series identified during the processing and for which no specific action has been taken. For example, trading day or seasonal peaks identified in the spectrum.

When running a seasonal adjustment of many series with a data or an input metafile, it is possible to save certain information for all series in one file, instead of checking a large number of output files. This is achieved using the **savelog** argument of the appropriate spec. The following is an example of a spec file and the log file that was generated when it was run for a data metafile:

```
series{start=1991.1
       period=12
      }
transform{
            function=AUTO
         }
automdl{
        MAXORDER=(4 1)
        MAXDIFF=(2 1)
        savelog=(AMD MU)
        }
REGRESSION{
            AICTEST=(EASTER TD)
            SAVELOG=AICTEST
          }
X11{
    SAVELOG=(IDS M7)
    }
```

This spec file has requested to save the automatic ARIMA model selected, the results of the AIC tests for Easter and trading day, and two diagnostics of the seasonal adjustment (identifiable seasonality and M7), as specified by the savelog argument in the appropriate specs. This information is summarised for all series in the log file, an extract of which is presented here:

```
M-ADD NI -------- -------- X-13ARIMA-SEATS run of NI
         automean: not significant.
 Automatic model chosen: (0 0 0)(0 1 1)
 AICtd: rejected
 AICeaster: rejected
 Identifiable seasonality: no
 M07 : 1.907

M-ADD SCOTLA -------- -------- X-13ARIMA-SEATS run of SCOTLAND
         automean: not significant.
```

```
 Automatic model chosen: (0 0 0)(0 1 1)
 AICtd: rejected
AICeaster: rejected
Identifiable seasonality: yes
M07: 0.676

M-ADD WALES -------- -------- X-13ARIMA-SEATS run of WALES
       automean: not significant.
Automatic model chosen: (0 0 0)(0 1 1)
 AICtd: rejected
AICeaster: rejected
Identifiable seasonality: yes
M07: 0.787

M-AUTO UK -------- -------- X-13ARIMA-SEATS run of UK
      automean: not significant.
Automatic model chosen: (0 1 1)(0 1 1)
      AICtd: rejected
      AICeaster: accepted
      Identifiable seasonality
      Identifiable seasonality: yes
      M07: 0.296
```

16.7 WIN X-13 DIAGNOSTICS TABLE

This section contains information about commonly used diagnostics in Win X-13. This is not a comprehensive list of diagnostics, rather the diagnostics that TSAB would frequently look at for determining quality of seasonal adjustment. All are illustrated from the Win X-13 diagnostics table.

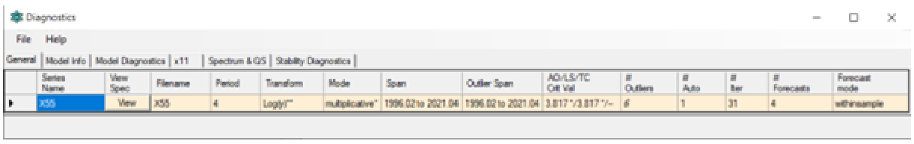

Figure 16.1: General tab

Period - the number of observations of the data per year (either 4 or 12 datapoints per year)

Transform – Whether the data are log transformed or not. The Mode column indicates whether the series uses multiplicative or additive decomposition. The ** indicates that this has been automatically selected, using the function=auto argument in the transform specification

Span – The length of time the series covers. This can be altered using the start or span argument in the series specification

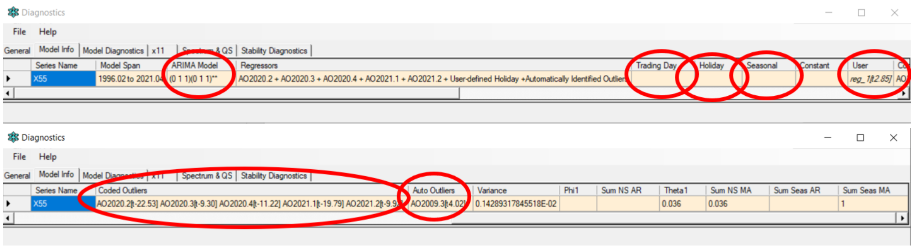

Figure 16.2: Model info tab

ARIMA Model – the ARIMA model used for the seasonal adjustment. The ** indicates that this has been selected using the automdl{} specification[8].

Trading day – regressors and t-values for regressors accounting for trading day effects[9].

Holiday – regressors and t-values for moving holiday regressors[10].

Seasonal – Date of break in the series and t-values for testing a seasonal change of regime[11].

User – User defined regressors and t-values, such as rmx regressors[12].

Coded Outliers – Fixed Additive outliers, Temporary changes, and Level shifts (AO, TC, LS) and their associated t-values. Dependent on the length of the series, t-values for new outliers should be greater than at least ±2 before these are implemented[13].

Auto Outliers – Suggested AO, TC, and LS's and their t-values, if the outlier specification is used. A manual check on these outliers should occur before they are implemented as Coded Outliers, as suggested outliers may not be the most suitable[14].

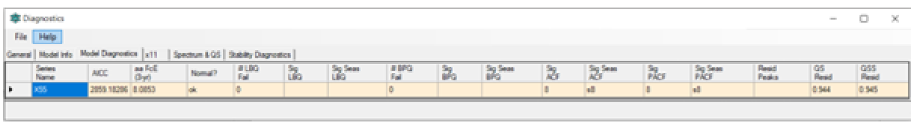

Figure 16.3: Model diagnostics tab

AICC – F-adjusted Akaike information criteria (corrected for sample size). The lower the value the better the model fit.

8 see Section 8.4 for further information
9 see Chapter 9 for further information
10 see Chapter 10 for further information
11 see Section 14.4 for further information
12 see Section 14.4 for further information
13 see Chapter 11 for further information
14 see Chapter 11 for further information

aa FcE (3yr) – absolute forecast error. The absolute error of a 3-year forecast compared to the latest data. The lower the value the better the model fit.

Normal – indicates whether residuals pass a normality test[15].

Sig (Seas) (P)ACF – Marks significant peaks in the AFC / PCAF of the residuals. These can also be identified visually in the ACF and PACF of the Residuals graphical output. Fewer entries in this indicates less residual seasonality[16].

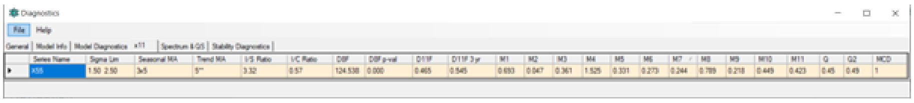

Figure 16.4: X11 tab

Seasonal MA – the length of moving average used in calculating the seasonal component[17].

Trend MA – the length of moving average used in calculating the trend component. The ** indicates that this has been automatically calculated, and can be fixed using the trendma=5 argument in the x11 specification. Increasing the Trend MA will result in a smoother trend[18].

D11F - a p-value for the D11 F statistic for residual seasonality. Values below 0.05 indicate strong evidence of residual seasonality, which could be resolved by use of additional regressors, or a different ARIMA model.

M1-M6 – values greater than 1 often indicate that the outlier treatments have not been fully resolved[19].

M7 – the ratio of moving to stable seasonality. Values less than 0.7 indicate the series is likely to be seasonal, values greater than 1.3 indicate the series in unlikely to be seasonal[20].

M8 – M11 – values greater than 1 often indicate that changes the Seasonal MA or Trend MA would be needed[21].

Q – A weighted average of the M1-M11 statistics, all of which are forms of seasonality test. Values less than 0.7 indicate the series is likely to be seasonal, values greater than 1.3 indicate the series is unlikely to be seasonal.

15 see Section 23.3 for further information
16 see Table 26 for further information
17 see Section 13.4 for further information
18 see Section 13.3 for further information
19 see Chapter 26 for further information
20 see Section 23.3 for further information
21 see Chapter 26 for further information

17 GRAPHS AND JAVA GRAPHS

17.1 INTRODUCTION

When analysing data one of the most basic but powerful tools is to graph the time series. There are several different methods for producing graphical output, depending on the software used. We primarily focus on the graphics used in Win X-13 software. This chapter will also show some graphs produced by X-13-Graph, which is a useful aid in the analysis of a time-series. X-13-Graph can be used to produce graphical output for users running the x11 method on MS-DOS, but users of Win X-13 will find it adds little extra. Full details on how to install and use X-13-Graph can be found from the website of the U.S. Census Bureau.

Graphics mode

X-13ARIMA-SEATS can be run in "graphics mode". This produces graphics metafiles (.gmt files) that contain data for creating a variety of charts. These files can be read by X-13-Graph (and X-13-Graph Java or other software as required). Users of Win X-13 need to tick the "Run in graphics mode" box and specify a graphical output directory.

Users working from the command line need to use the "-g" flag during the run of X-13ARIMA-SEATS and supply the name of an existing directory where X-13ARIMA-SEATS will store the graphics files. For either case, the full path of the directory needs to be used, remembering that it must be different from the directory where the output ".out" file is stored. An example of the command line to run X-13ARIMA-SEATS in graphics mode is given below:

x13as myspec -g c:13as

This command will create graphics output files for the specification file "myspec.spc" in the graphics subdirectory. The seasonal adjustment diagnostics file and the model diagnostics file produced using the "-g" flag store only essential information about the seasonal adjustment and model run needed for the X-13-Graph external graphics procedure.

A description of how to start X-13-Graph in SAS and of the capability of X-13-Graph can be found from the website of the U.S. Census Bureau.

17.2 THE RAW DATA

Before seasonally adjusting a series, looking at a graph of the original estimates will enable the user to identify possible problems. For example, look for:

- sudden changes in the general level of the series (trend breaks)
- sudden changes in the months in which peaks and troughs occur (seasonal breaks)
- large extreme values - outliers

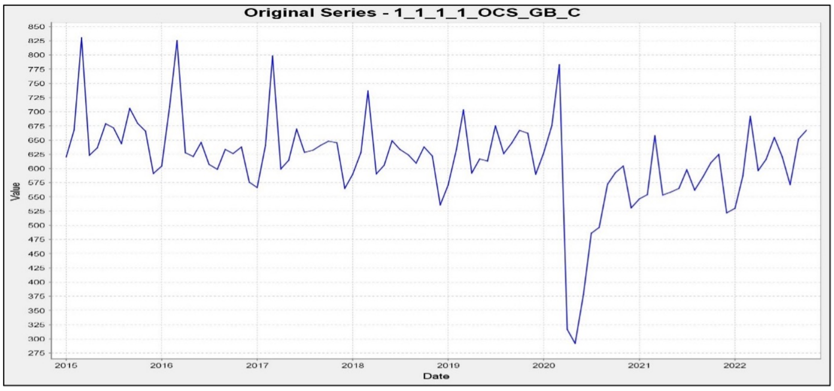

Figure 17.1: Graph of the original series

Figure 17.1 plots an example time series. On first viewing, it can be seen that the series is seasonal because of the regular seasonal peaks in March. There are several changes in the level of the series in 2020, indicating the possibility of a trend break (which could be corrected with a level shifts or ramps).

17.3 THE SEASONALLY ADJUSTED ESTIMATES

A graph of the original and seasonally adjusted estimates can show where breaks or outliers may have affected the series.

Figure 17.2 shows a steep fall and then gradual rise in the seasonal adjustment in 2020 (a likely trend break) and a possible outlier in August 2022. There is an option with Win X-13 output to highlight any span of data by clicking and dragging the desired span on the graph.

17.4 THE SEASONAL IRREGULAR RATIOS VS THE SEASONAL FACTORS

This type of chart plots the Seasonal Irregular (SI) ratios (de-trended series found in the D8 table of the output) and the seasonal factors (D10) over

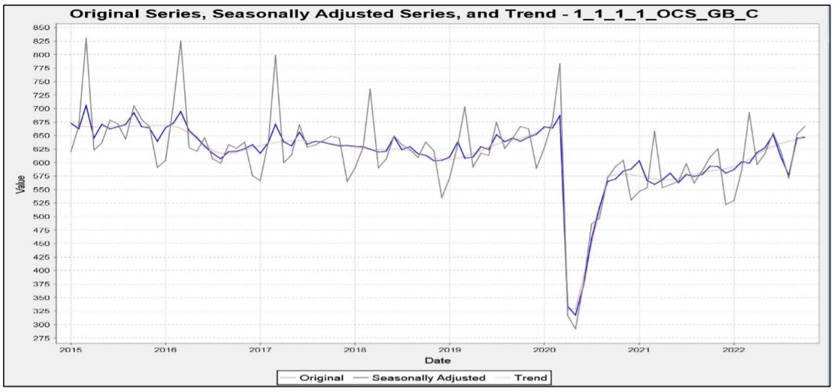

Figure 17.2: Graph of the seasonally adjusted series

years where the information has been grouped by different periods. See Figure 17.3 for an example.

A moving average is applied to the SI ratios to obtain the estimate of the seasonal component (which will be removed later by seasonal adjustment). Both the seasonally adjusted and non seasonally adjusted series are useful for helping decide whether there are any seasonal breaks within the time series, or whether there is any need to change the seasonal moving averages for any particular month or quarter. They can also indicate if there is any one month or quarter with more statistical variability than the other months or quarters. If the SI ratios are in an approximate straight line, then the seasonal component should follow this, so a short moving average is appropriate. If the SI ratios appear to be very erratic, the seasonal factors will try to follow too closely to the SI ratios, producing an erratic seasonal factor line. In this case a long moving average is appropriate. This will remove any unwanted variation in the seasonal without distorting important patterns in the SI ratios.

X-13ARIMA-SEATS replaces some outliers using an automatic process. Where there are lots of replaced SI values in a particular month (denoted by light blue points) this could be because the month is particularly volatile. This may indicate evidence of heteroskedasticity or non-constant variance.

SI ratio graphics can be used for quick visual understanding of the general seasonality of a series before proceeding with more thorough analysis. SI ratios also are a useful guide to the presence of seasonal breaks, shown as sudden changes to the level of the SI ratios. A seasonal break in the time series will distort the estimation of the seasonal component. Seasonal breaks may result in leakage in the variation of one component into the variation of another. Where the seasonal factor (D10) is much lower than the SI ratio, some of the seasonality could have been included as part of the irregular, this may result in residual seasonality in the seasonally adjusted series. Where the seasonal factor is much higher than the SI ratio,

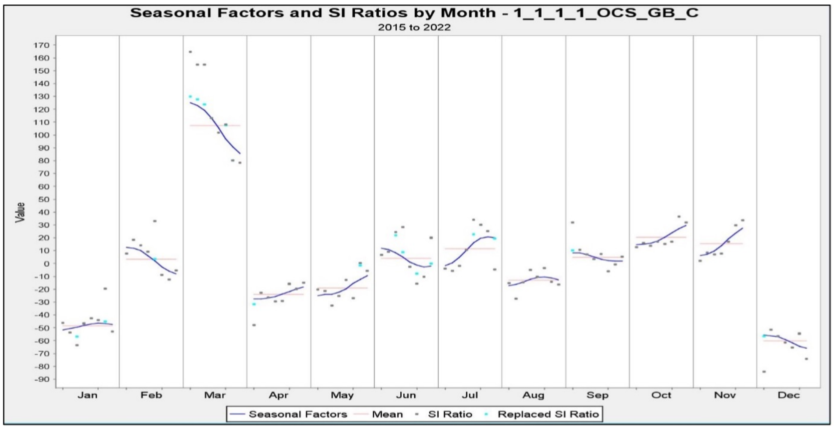

Figure 17.3: Graph of the replaced SI ratios

the seasonal factor could have included too much irregular variation. In this case, some variation may have been removed from the seasonally adjusted series that is not seasonal.

Another useful graph that can be produced in X-13-Graph Java is the seasonal factors (table D10 in the output) plotted against the mean seasonal factor for each month. Change in seasonality over time can be shown as a gradual movement in the seasonal factors. Seasonal moving averages are selected on the basis of the SI ratios, using information from the whole series.

Where one month or quarter is not being tracked well by the seasonal factors (it may be more volatile than other months or quarters) the moving average can be tailored to that month or quarter. For example, in Figure 17.4, the change in SI ratio in March could be used to show that a 3x7 moving average would be more suitable for capturing the changes in seasonality, when the program originally selected a 3x5 moving average to be applied to all months.

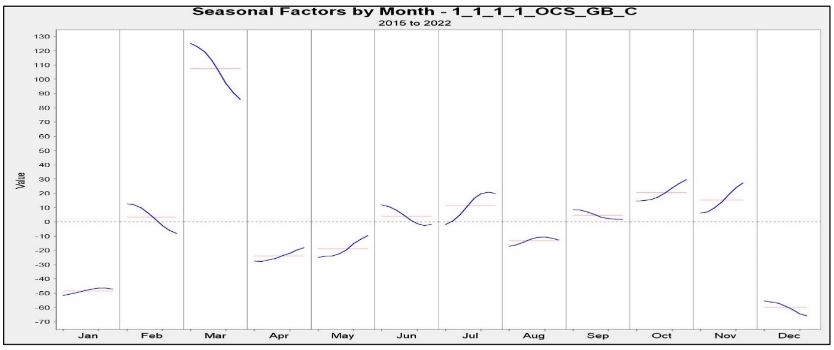

Figure 17.4: Graph of the seasonal factors

Since all the values are on the same scale it can be seen whether any change in any month is large or small compared with the overall pattern of the series. The overall pattern of the series shown above is one in which March is the highest month. The I/S ratios (in the D9A table) can be used in deciding whether distinct moving averages need to be applied to any one month or quarter.

17.5 THE SPECTRUM

The spectrum graph is a useful graphical tool. It can aid the user in establishing whether a series is seasonal or not. Figure 17.5 shows the spectrum of the prior adjusted data, produced by Win X-13, and the overlaid spectra of the original and seasonally adjusted series, produced by X-13-Graph. Spectral peaks, occurring at one or more of the seasonal frequencies provide evidence of seasonality. For example, in Figure 17.5 there are spectral peaks at four of the six seasonal frequencies which indicates strong seasonality (solid line). The seasonally adjusted series can be identified as non-seasonal because there are no spectral peaks evident (dashed line). Note for quarterly series, there will be two spectral frequencies ½ and ¼.

Marginally seasonal series can be identified from the graph as they will have spectral peaks at one or more frequencies (most probably at 1/12). However, the user should be aware that in some marginally seasonal cases, the peaks may not be as clearly defined as the ones shown in Figure 17.5.

The X-13ARIMA-SEATS program also produces several crude spectral graphs at the end of the output file (Table G), but using poor quality character graphs. Significant seasonal peaks are marked with an "S" and significant trading day peaks with a "T", with the colour of the letter matching the colour of the graph(s) which is significant. It is normally preferable to observe the Win X-13 plots, or X-13-Graph interface for SAS or the Java equivalent.

17.6 OTHER USEFUL GRAPHS

Two other graphical options of X-13-Graph should be used whilst checking the quality of the seasonal adjustment. Those options are the Component Graphs and the Comparison Graphs for Two Adjustments or Two Models.

17.6.1 *Component graphs*

This graphical option gives the possibility to select up to four different seasonal decomposition components to plot (Original Series, Trend, Seasonal Factors, Irregular). Seasonal Factors and Irregular components cannot be viewed in Win X-13 graphics, but X-13-Graph plots each component individually on a graphic output. If several components are selected, they

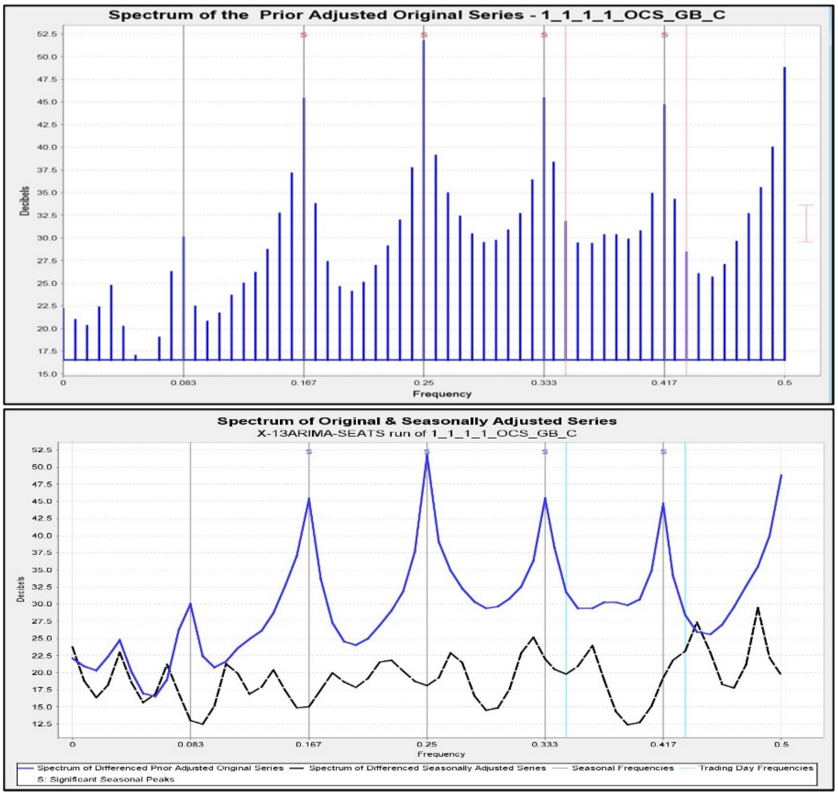

Figure 17.5: Graphs of the spectra of prior adjusted series from Win X-13 and X-13 graph Java

will be plotted in a reduced size on the same screen. An example of this is given in Figure 17.6.

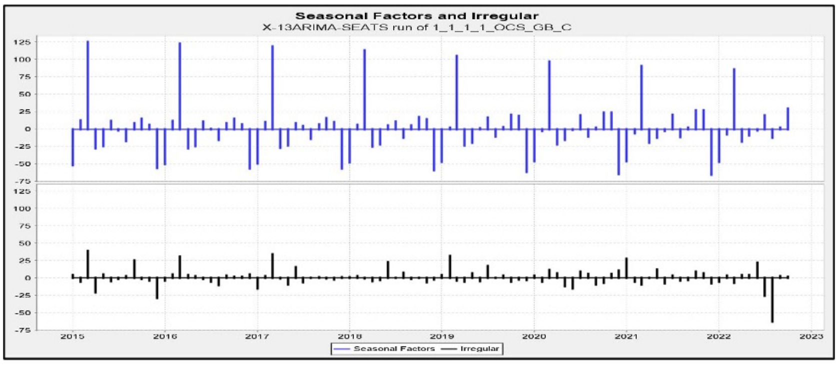

Figure 17.6: Graphs of the seasonal factors and irregular components

The first of the component graphs shows the seasonal factors (these data can be found in Table D10). This shows the seasonal pattern and how seasonality evolves. In the above example, it can be clearly seen that March is always high, December and January are always low, and the seasonal pattern is quite stable. A more useful representation is the SI chart (Section 14.5) The second of the component graphs shows the irregular component. The irregular shows the erratic, random part of the series (these data can be found in Table D13). It is important to use this graph on first analysis of the series to look for any residual pattern in the irregular which could harm the quality of seasonal adjustment. A concentration of outliers in one year may indicate a level shift or seasonal break at that point.

17.6.2 *Comparison graphs*

Comparison Graphs gives the possibility to compare the seasonal adjustment of two different series, or of two different adjustments of the same series. This cannot be done in Win X-13 graph but can be completed using the X-13-Graph Java software. Figure 17.7 compares two different seasonal adjustments of the same series (with and without the use of temporary priors to adjust for the level shifts in 2020 and additive outlier in August 2022). Using this graphing option, it is possible to compare the smoothness, the effect on the end of the series of two alternative seasonal adjustment methods or, as in this case, the effect of a temporary prior adjustment.

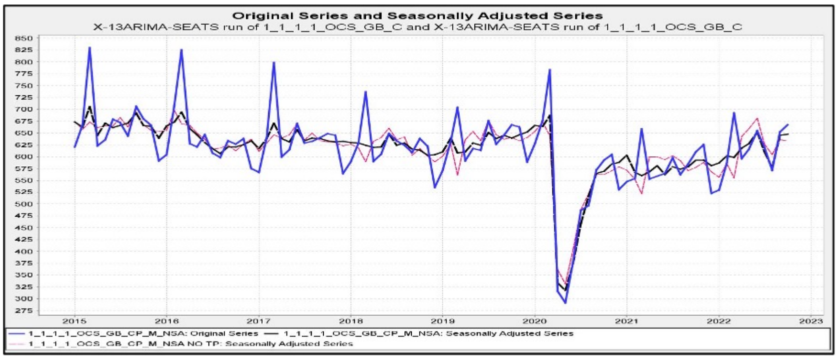

Figure 17.7: Graph of the original and comparison of seasonally adjusted series

17.6.3 *Forecast graph*

Forecast graphs show forecast data, and their 95% confidence interval the end of any series

17.6.4 *ACF and PACF*

Autocorrelation Function (ACF) is the correlation between a time series with a lagged version of itself. The ACF starts at a lag of 0, which is the correlation of the time series with itself and therefore results in a correlation of 1. This shows the correlation of the series with the lagged version of itself at, in Figure 17.9, up to 24 lags. Partial Autocorrelation Function (PACF) can be explained using a linear regression where we predict

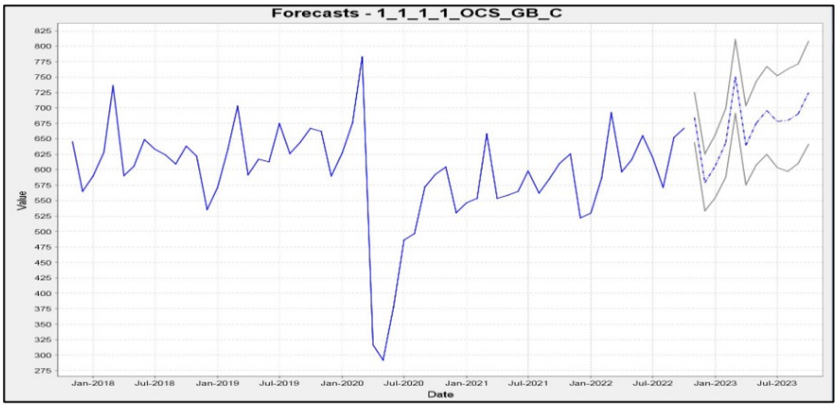

Figure 17.8: Graph of the forecast series and confidence intervals

$y(t)$ from $y(t-1)$, $y(t-2)$, and $y(t-3)$. Users of Win X-13 will normally look at ACF and PACF of the Residuals, and are looking for all lags to be below the line of statistical significance for a quality seasonal adjustment. If they are significant, this may indicate residual seasonality.

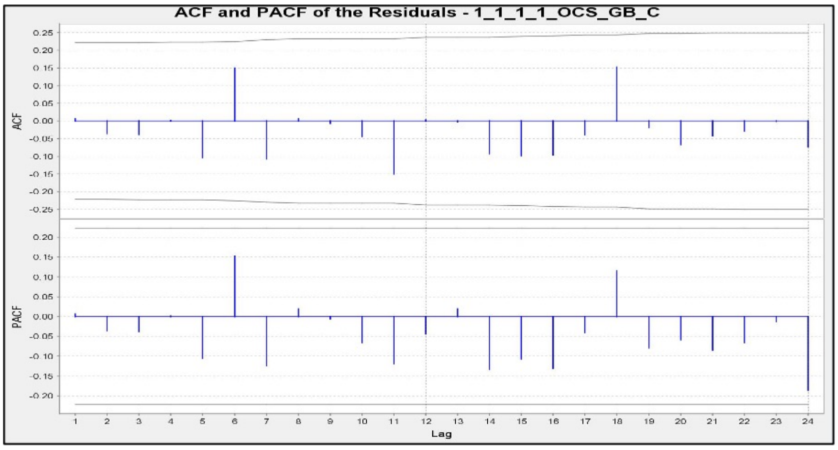

Figure 17.9: Graph of the ACF and PACF of residual component

18 SLIDING SPANS

18.1 INTRODUCTION

When a series is seasonally adjusted, an important property is stability. A series is defined to be stable if removing or adding data points at either end of the series does not overly affect the seasonal adjustment. If it does, any interpretation of the seasonally adjusted series would be unreliable. The sliding spans diagnostic is a way of deciding if a seasonal adjustment is stable.

Stability and quality are linked but are not the same. Most of the time, the stability of a seasonal adjustment is a good indicator of its quality. However, it will sometimes be the case that the best seasonal adjustment is an unreliable one. In this case, the needs of the user will dictate whether stability or quality of seasonal adjustment is more important.

Definition

A span is a range of data between two dates. A span length is the number of data points within that specific range. Sliding spans in X-13ARIMA-SEATS are a series of 2, 3 or 4 spans that overlap. For example, a span of data could be from January 2002 to December 2012. It has a span length of 132 months. An example of a span within this last series is March 2003 to November 2011, which has a span length of 105 months.

18.2 WHEN TO USE SLIDING SPANS

Sliding spans can be used when there is a need to know how well a seasonal adjustment is performing. Sliding spans analysis is particularly interesting if:

- Seasonal breaks, outliers or fast-moving seasonality in the series are suspected. Often, the Q statistic gives good results for series with clear seasonal breaks, but the sliding spans statistics will fail. Looking at the graph of original series and seasonally adjusted series will confirm this

- Two options for a seasonal adjustment have to be compared. Sliding spans can provide diagnostics on which adjustment would produce the most stable seasonal adjustment estimates. For example, comparing direct and indirect seasonal adjustment. When comparing direct and indirect seasonal adjustments, sliding spans statistics are produced for both methods. The series with the lower stability statistics is more stable, and therefore likely to be a better adjustment. However, it is important to make sure the lengths of the sliding spans of the component series are the same[1]

- When one or two months of a year are unstable, for example a sales series where November, December and January are particularly volatile or when there has been a period of instability in the data. One of the output tables (S3) can be useful for tracing particular months that are susceptible to producing unstable seasonal factors. For example, some series (typically in the USA) are very sensitive to winter temperatures. These series are still seasonal – but getting a stable seasonal adjustment is difficult

For many other different comparisons, such as with or without a trading day effect[2]. Figure 18.1 illlustration of 4 sliding spans, length 7 years. In all cases, the question to ask is if stability, or the best use of available information and options, is more important to users.

18.3 HOW SLIDING SPANS WORKS

The sliding spans diagnostic works by separately seasonally adjusting each of 2, 3 or 4 overlapping spans of a time series. Where two or more sliding spans overlap, the diagnostic compares the different seasonal adjustments. The full process is:

1. The program selects a span length, based on the seasonal moving averages used, the length of the series and whether the data are monthly or quarterly. The default span lengths are:
    - 6 years, if a 3x1 seasonal moving average is used
    - 7 years, if a 3x3 seasonal moving average is used
    - 8 years, if a 3x5 seasonal moving average is used
    - 11 years, if a 3x9 seasonal moving average is used

    If the data series is not a whole number of years long, the span length increases by the part year length. For example, a series running from January 2002 to March 2013 and using a 3x3 seasonal filter would have spans (7 years + 3 months) = 87 months long. Note that in this case:

1 see Section 6.3 for more information on direct and indirect seasonal adjustment
2 see Findley et al. (1990) for more examples

	1st span	2nd span	3rd span	4th span
Q1 1991	100			
Q2 1991	106			
Q3 1991	103			
Q4 1991	110			
Q1 1992	105	105		
Q2 1992	110	110		
Q3 1992	108	108		
Q4 1992	115	115		
Q1 1993	110	110	110	
Q2 1993	114	114	114	
Q3 1993	111	111	111	
Q4 1993	118	118	118	
Q1 1994	107	107	107	107
Q2 1994	116	116	116	116
Q3 1994	114	114	114	114
Q4 1994	121	121	121	121
Q1 1995	115	115	115	115
Q2 1995	124	124	124	124
Q3 1995	120	120	120	120
Q4 1995	125	125	125	125
Q1 1996	120	120	120	120
Q2 1996	128	128	128	128
Q3 1996	123	123	123	123
Q4 1996	129	129	129	129
Q1 1997	124	124	124	124
Q2 1997	133	133	133	133
Q3 1997	127	127	127	127
Q4 1997	135	135	135	135
Q1 1998		128	128	128
Q2 1998		139	139	139
Q3 1998		132	132	132
Q4 1998		138	138	138
Q1 1999			131	131
Q2 1999			143	143
Q3 1999			138	138
Q4 1999			144	144
Q1 2000				134
Q2 2000				149
Q3 2000				138
Q4 2000				150

Figure 18.1: Illustration of 4 sliding spans, length 7 years

- The first span would run between January 2003 and March 2010
- The second span would run between January 2004 and March 2011
- The third span would run between January 2005 and March 2012
- The final span would run between January 2006 and March 2013 (the most recent data point)

If different months or quarters have different seasonal filters, X-13ARIMA-SEATS uses the longest seasonal filter to set span length

2. The programme sets up a maximum of 4 spans. The spans start in 1-year intervals. So, if the first span starts in Q1 1991, the second span

would start in Q1 1992. If there is not enough data to create 4 spans, X-13ARIMA-SEATS will produce 2 or 3 sliding spans. In any case, all spans start at the beginning of a year (January or Q1) and the last span includes the most recent data point. If there is not enough data for X-13ARIMA-SEATS to produce at least 2 spans of the preferred length, the diagnostic is suppressed

3. The programme seasonally adjusts each span separately

4. Where the spans overlap, the programme compares the seasonal factors and the month-on-month (or quarter-on-quarter) and year-on-year percentage changes in each span for each data point. This is shown in Figure 18.1. Note that all the data for each corresponding date are the same; for example, the Q1 1997 value, 124, is the same in all four spans. However, as each span is seasonally adjusted individually, the seasonally adjusted Q1 1997 values will probably be different. This difference is the basis for sliding spans analysis

5. For each month (or quarter) that is covered by 2 or more spans, the programme works out as follows:

   - The percentage difference between the largest and smallest seasonal factor (S):

$$S_t^{max} = \left(\frac{\text{Largest S - Smallest S}}{\text{Smallest S}}\right) \cdot 100 \tag{12}$$

   - The difference between the greatest and smallest percentage change in the seasonally adjusted series since the previous month (or quarter)

$$\begin{aligned} MM_t^{max} = &(\text{Largest month-on-month change}) \\ &-(\text{Smallest month-on-month change}) \end{aligned} \tag{13}$$

$$\begin{aligned} QQ_t^{max} = &(\text{Largest quarter-on-quarter change}) \\ &-(\text{Smallest quarter-on-quarter change}) \end{aligned} \tag{14}$$

   - The difference between the greatest and smallest percentage change since the same month (or quarter) in the previous year

$$\begin{aligned} YY_t^{MAX} = &(\text{Largest year-on-year change}) \\ &-(\text{Smallest year-on-year change}) \end{aligned} \tag{15}$$

In all cases, a value of more than 3% is regarded as unstable.

	Description	Uses
S0	Summary of options	
S1	Period means of seasonal factors	
S2	Percentage of periods unstable	Main output table of sliding spans
S3	Breakdown of unstable periods	Highlights months and years that are particularly unstable
S7	Full Sliding Spans analysis	Gives an idea of the distribution of unstable months. Also contains SA spans for analysis.

Table 18.1: The sliding spans output table

6. The program then works out what proportion of data points in the series, where two or more spans overlap, qualify as unstable for each statistic given above. These proportions are called S (seasonal factors), MM (month-on-month change), QQ (quarter-on-quarter change) and YY (year-on-year change).

18.4 HOW TO USE SLIDING SPANS

Most of the time, just using the specification slidingspans{} is enough to use the diagnostic. Normally, the slidingspans spec is included at the bottom of the spec file. A typical example of part of an X-13ARIMA-SEATS spec file with a sliding spans analysis is:

```
series{
   title="Example of sliding spans spec" start=1996.1 period=4
       file="mydata.txt"
   }
arima{
   model=(0,1,1)(0,1,1)
   }
x11{
   mode=mult
   }
slidingspans{}
```

18.4.1 *The output*

Table 18.1 presents a short description, and some uses for each of the sliding spans output tables.

Tables S2, S3 and S7 are produced several times, labelled a) to e):

- Table a) represents seasonal factors
- Table b) covers trading days. This table is printed out only if fixmdl is set to no or clear
- Table c) is for the final seasonally adjusted series. This prints out only if one or more regression variables, such as trading days or an Easter effect, are included in the model and are not fixed
- Table d) is for the month-to-month or quarter-to-quarter changes
- Table e) is for year-on-year changes. In table S2 guideline percentages are produced only for seasonal factors, month-on-month (or quarter-on-quarter) and year-on-year changes. Looking at the values of, in particular, S and MM (or QQ if quarterly data) of table S2 gives an idea as to how good the adjustment is. The guidelines given in Findley et al. (1990) for interpreting S, MM (or QQ) and YY are:
  - For S, series with stable seasonal adjustments usually have S<15%. Series with S>25% almost never have good seasonal adjustments. Those for which S falls between 15% and 25% should cause concern but may give acceptable seasonal adjustments. Contact TSAB. for further guidance
  - Series for which MM>40% (QQ>40% for quarterly data) should not be seasonally adjusted. Those for which MM (QQ) falls between 35% and 40% should cause concern but may give acceptable seasonal adjustments. Contact TSAB for further guidance
  - YY<2% is common with good series. While there is no recommended upper limit for YY above which a series should not be seasonally adjusted, YY=10% is quite high. However, most series with high YY values also have poor results for S or MM, so the YY statistic is of less importance

These guidelines are summarised under table S2 in the X-13ARIMA-SEATS output, when sliding spans is run. The output of tables S3 can be particularly useful in tracking down areas of instability. It is not automatically printed with the print = brief option, so the default or print = all should be used when a single series is being analysed in more detail.

18.4.2 *Sliding spans options*

It is not necessary to specify any of the available options to use sliding spans. However, some of the options (such as length, print and outlier) can allow the user to be able to apply the sliding spans diagnostic in the widest range of situations.

Length of spans

The span length is of prime importance, since it can affect the performance of the sliding spans diagnostics, especially if fewer than 4 spans are created. The length argument allows the user to select the number of data points included in the spans and used to generate output for comparisons. The length argument is needed:

- If the length of the series is not a whole number of years. This will bring all the span lengths into line with the longest span used
- To run an indirect seasonal adjustment if different seasonal moving averages are used for the component series. In this case, the span lengths for each series will be incompatible. For example, a composite series of total employment is made up of monthly total male and female employment seasonally adjusted series. The adjustment for the male series uses a 3x5 seasonal moving average, while the adjustment for the female series uses a 3x3 seasonal filter. The two series will use different span lengths, so we need to change the span length. We would use the command length = 96 in the spec file for adjusting the series for female series to produce sliding spans diagnostics comparable to those from the male series

In these situations, the guide percentages given by Findley et al. (1990) (Findley, et al. 1990), become too high and cannot be reliably used. However, the results can still be used to say something useful about the adjustment.

When the slidingspans spec is included in the analysis of a series, users have to take into account that:

- Longer span lengths lead to a smaller proportion of unstable months. Therefore, using longer spans will, if anything, bias the stability statistics downwards. Therefore, any sliding spans statistics which are higher than the guideline percentages are still more unstable than is desirable
- When two or three spans are produced, the program will say that the guideline percentages are now too high and should be lowered. If the values of S and MM (or QQ) are too high when fewer than four spans are produced, the series is still unstable

In both these cases, there is a chance that a series may not be classified as unstable, when in fact it is. This is because the series would have been close to being classified as unstable under the guideline percentages, and either the length or number of spans has been changed. As it is not known how much to change the guideline percentages by, it cannot be said for sure whether the series is stable or unstable. Nevertheless, the benefits gained by using sliding spans in a large number of cases outweigh this disadvantage.

Other important options in the sliding spans analysis

There are two other important options that can be useful in the sliding spans analysis:

- The argument print = all allows you to view the full sliding spans output, which can be a useful additional source of information when analysing a series in more detail. However, the default output is usually enough
- If there are a lot of outliers in the data, it is recommended to use the command outlier = keep to allow the sliding spans analysis to use them. This may improve the stability statistics (see below) and provide further evidence for including them in the adjustment.

18.4.3 *When sliding spans will not work*

There are several situations in which Sliding Spans will either not work or produce output that is not useful in assessing the stability of the data. They are:

1. When an additive seasonal adjustment is selected. Sliding spans will not produce tables S1 and S2 in this case, and the remaining output should not be used in the same way as for multiplicative adjustments
2. If the series is too short for X-13ARIMA-SEATS to construct at least 2 spans, X-13ARIMA-SEATS will not carry out the sliding spans analysis and display an error message. However, the rest of the seasonal adjustment will be carried out
3. If X-13ARIMA-SEATS produces fewer than four spans, it will recommend that the threshold values of S, MM (or QQ) and YY be lowered[3]
4. When all the seasonal factors are close to 100. In this case, the threshold value of 3% is too large, and no S2 table will be produced. This is quite common, and unfortunately renders the sliding spans diagnostic useless when it happens. (Findley and Monsell 1986) have carried out a study as to how the threshold values should be lowered, but found no correlation between appropriate threshold values and the size of the seasonal movements[4]

3 see Section 18.4.2.1 for more details
4 see section 6.2 of the X-13ARIMA-SEATS manual (USCB 2017)

5. If adjusting a composite series indirectly, and the span lengths of the component series are different, a period in each span will have unrealistic seasonal factors. In this case, using the length argument as detailed earlier to even up the span lengths will produce more accurate seasonal factors. However, care must still be taken when interpreting the stability statistics[5].

18.5 summary

The sliding spans diagnostic is one of the more useful statistics in the X-13ARIMA-SEATS output to look at. The sliding spans diagnostic measures the stability of a seasonal adjustment, which is related to quality but not necessarily the same thing.

A series that fails sliding spans is likely to have a seasonal break or a period of instability. Sliding spans is particularly useful in these situations, as the M and Q statistics can misleadingly suggest the seasonal adjustment is good. Sliding spans can also be used to compare different methods of seasonal adjustment and give an indication of which option leads to a more stable adjusted series.

5 see Section 18.4.2 for more details

19 HISTORY DIAGNOSTICS

19.1 INTRODUCTION

Revisions history is one of the stability diagnostics available in X-13ARIMA-SEATS. It considers the revision of continuous seasonal adjustment over a period of years and is, therefore, a way of seeing how a time series is affected when new data points are introduced. The basic revision calculated by X-13ARIMA-SEATS, to enable visualisation of these effects, is the difference between the earliest seasonally adjusted estimate for given month (obtained when that month is the latest month in the series) and the most recent adjustment based on all future data available at the time of the diagnostic analysis.

When a new data point becomes available for a series, more is learnt about the behaviour of that series; this is especially true of seasonal series. With that new data point, more is found out about the seasonal pattern, as well as the underlying trend. However, when the series is updated with the extra information, the use of this extra information creates revisions to the seasonally adjusted series. Generally, users want to minimise the number and size of these revisions. The importance of this is a question for the user. Some users prefer to have the most up to date figures possible, regardless of the size of revisions. Other users prefer limited revisions, to make the data they work with more consistent. The choice of revision policy is also influenced by the nature of the data.

The revisions history diagnostic is a way of seeing how a time series is affected when new data points are introduced. Obviously, it is not possible to predict the future so instead, the diagnostic works by taking a period of existing data and adding the data points one by one as if they were new observations. This creates revisions in the data, which is what users are interested in.

19.2 WHEN TO USE REVISIONS HISTORY

The diagnostic is most often used when there are two competing methods for seasonally adjusting a series, both of which are acceptable in terms of other diagnostics, for example the Q statistics.

For example,

- Use of two different ARIMA models

- Use of different seasonal moving averages
- Choosing between direct and indirect seasonal adjustment of a composite series

To compare how prone two different adjustments are to revisions, the history diagnostic can be run on both series and then the revisions produced over the test period can be compared. In general, smaller revisions are better. An adjustment with small average annual revisions will be more reliable than an adjustment with larger revisions. Unlike some other diagnostics such as the Q statistic and sliding spans, there is no objective measure for accepting a revision.

There are eight different variables that X-13ARIMA-SEATS can produce a history for, and many of these tables can be used in more than one way. Any full explanation of the diagnostics and options available would be long and unwieldy. Therefore, this chapter will concentrate on three functions of the history diagnostic;

1. Using the revisions history to compare two competing adjustments
2. Comparing revisions in direct and indirect seasonal adjustments
3. Using AICC histories to choose between two adjustments

Therefore, the revisions history diagnostic is one of the later tests to apply to a series.

19.3 HOW THE REVISIONS HISTORY DIAGNOSTIC WORKS

The process by which the diagnostic works is:

1. The start date of the history diagnostics is chosen. This can either be the default X-13ARIMA-SEATS choice, or it can be specified by the user with the start argument
2. The program seasonally adjusts the series, up to and including the start point of the revisions history
3. X-13ARIMA-SEATS then seasonally adjusts the whole series again, but up to the observation after the revisions history start date. The program repeats this process, including one extra data point in each run until all the data have been added and the entire series has been adjusted
4. The final seasonal adjustment, which covers all the data, is the most recent adjustment available and represents the best estimate of the seasonal factors

5. The program calculates the percentage difference between the first seasonal adjustment of the starting point (calculated in step 2) and the final adjustment for the same month (as in step 4). The program repeats this for every data point up to, but not including, the final data point in the series. This is because the value in steps 2 and 4 will be identical. The program also calculates any other histories specified by the user

6. The program then produces the appropriate tables for the series.

19.4 HOW TO USE REVISIONS HISTORY?

The history specification can be used with different sets of arguments and the choice of the arguments depends on the scope of the revision analysis. An example of part of an X-13ARIMA-SEATS spec file with a revisions history analysis is:

```
series{
      title="Example of sliding spans spec"
      start=1996.1
      period=4
      file="NZJW.txt"
      }
arima{
      model=(0,1,1)(0,1,1)
      }
x11{
      mode=mult
      seasonalma=s3x3
      trendma=5
      }

history{
      start=1999.1
      sadjlags=(1,2)
      }
slidingspans{}
```

19.4.1 *Comparing two competing adjustments*

This is the simplest use of the revisions history diagnostic. It is used to produce just table R1, which gives the revision between the first and final estimates of the seasonally adjusted series. The idea of this diagnostic is to compare the stability of two competing adjustments when new data are introduced.

In the example spec file in Section 19.4, the program is being run on the series NZJW, which is from the Motor Vehicle Production Index data set. In this spec file, two options have been included within the history specification. The start argument determines the start date of the series. Users are most interested in how the seasonal adjustment performs with current data, rather than data that is several years old. It is recommended that the start argument be used to limit the length of the history diagnostic to the last couple of years. If the start argument is not used, X-13ARIMA-SEATS will select the start date – see section 7.8 of the X-13ARIMA-SEATS reference manual (USCB 2017) to see how the program does this.

The other argument used in the history specification above is sadjlags. This argument allows the user to choose up to 5 lags to be covered in the revisions history. These revisions compare the estimate of a given time point when n points are available after the point of interest to the final estimate (final by default – can be changed to concurrent). For example, a lag of n compares the estimate of time point t when data up to (t+n) are available with the final estimate when the full span is used. Table R1 from the output obtained when this spec file was run on the series NZJW is provided below.

```
R 1 Percent revisions of the concurrent seasonal adjustments

From         1999.1 to 2000.2
Observations   6

Date     Conc -   1 later- 2 later-
         Final    Final    Final
----     ------   ------   ------
1999
1st      0.89     -2.08    -2.27
2nd      -9.11    -2.88    -1.77
3rd      4.00     3.13      2.61
4th      1.08     -0.25    -0.52

2000
1st      -1.00    -1.03    *****
2nd      -2.55    *****    *****
```

In this case, the data are quarterly and the final adjustment (with all the data up to Q3 2000 available) is the target. The first column is the revisions history of the initial adjustment. It is the percentage difference between the first vintage (including all the data up to the point in question in the adjustment) and the final vintage (including all the data in the series). Although it is possible to do it with concurrent being the target, it is considered here from first to final vintage.

For example, the value 1.08 (shown in bold) has been calculated by working out the percentage difference between the seasonally adjusted value for Q4 1999 given all the data, and the value for Q4 1999 given the data up to Q4 1999. In other words, it is the percentage difference between the Q3 2000 vintage (the final vintage) and the Q4 1999 vintage (the first vintage) of Q4 1999. The next column is the revision at lag 1 (this is stated in the output in table R0). In this case, the value for Q4 1999 is -0.25. This has been calculated by working out the percentage difference between the seasonally adjusted value for Q4 1999 given all the data, and the value for Q4 1999 given the data up to and including Q1 2000. This is the percentage difference between the final vintage and the Q1 2000 vintage (the second vintage) of Q4 1999.

The final column is the revisions at lag 2. This is the percentage difference between the final vintage and the third vintage. For example, in table R1, the figure in italics (-0.52) has been calculated by working out the percentage difference between the seasonally adjusted value for Q4 1999 given all the data (up to Q3 2000), and the value for Q4 1999 given the data up to and including Q2 2000. In this example, the seasonally adjusted value for Q4 1999 given the data up to Q3 2000 is 0.52% smaller than the seasonally adjusted value for Q4 1999 given the data up to and including Q2 2000.

Why are the revisions at various lags of interest? They give some idea as to the size of revisions at each lag, which can say a lot about the data. For example, the relative sizes of the revisions at each lag give you some information as to how quickly adjusted data converge to its final values. This information can be used to inform decisions on a revision policy for the series. Up to five lags at a time can be specified, using the sadjlags argument. The diagnostic also prints out the summary table, shown below.

```
R 1.S Summary statistics: average absolute percent revisions of the
    seasonal adjustments

Quarters:
1st   0.95   1.55  2.27
2nd   5.83   2.88  1.77
3rd   4.00   3.13  2.61
4th   1.08   0.25  0.52
```

```
Years:
1999  3.77   2.08  1.79
2000  1.78   1.03 ****
Total:  3.11  1.87  1.79

Hinge Values:
Min   0.89   0.25  0.52
25%   1.00   1.03  1.14
Med   1.82   2.08  2.02
75%   4.00   2.88  2.44
Max   9.11   3.13  2.61
```

Of these statistics, the most useful value is the total revision. This is the average absolute percentage revision from the first (or second, third... etc, depending on the sadjlags option used) seasonal adjustment, and the adjustment using all the data available. This is a single figure that can be used to compare the quality of competing adjustments. These total revision values have been calculated by taking the mean of the absolute values of the entries of the corresponding column in table R1. For example, the Total Revisions Value in table R1S for the first column (two decimal places) is equal to:

$$3.11 = \frac{0.89 + 9.11 + 4.00 + 1.08 + 1.00 + 2.55}{6} \tag{16}$$

The other values can be useful too. Users will be more interested in recent data. Therefore, the size of revisions with recent data may be more important in deciding which adjustment to use than earlier years. This diagnostic could be used to compare revisions performance of competing adjustments. An example would be to check the revisions caused by different ARIMA models of the series NZJW. To do this, run the series repeatedly through X-13ARIMA-SEATS, once for each ARIMA model being compared. No other options should be changed. The total revisions values using different ARIMA models to adjust NZJW are presented in Table 19.1.

Model	Average absolute revision percentage		
	First estimate	At lag 1	At lag 2
Log(0 1 1)(0 1 1)	3.11	1.87	1.79
Log(0 1 2)(0 1 1)	3.11	1.92	1.90
Log(2 1 0)(0 1 1)	3.17	1.91	1.89
Log(0 2 2)(0 1 1)	3.11	1.86	1.79
Log(2 1 2)(0 1 1)	3.20	1.83	1.81

Table 19.1: Revisions history

Table 19.1 shows that changing the ARIMA model used does not have a very large effect on the size of revisions. The first and fourth model have the lowest revisions for the first estimate, and in practice a (0 1 1)(0 1 1) model would probably be selected as it uses the fewest parameters.

Other Options

Tables R2, R4 and R5 are all similar to table R1, but refer to slightly different target variables. The table below summarises the target variables available, and the commands needed to produce each variable.

Number	Command	Details
R1	Sadj	Final seasonally adjusted series (the default)
R2	Sadjchng	Period to period changes in final seasonally adjusted series
R3	Trend	Final Henderson trend component[1]
R4	Trendchng	Period to period changes in Henderson trend component

Table 19.2: Other options

The variables of interest can be specified in the estimates argument. The following history spec provides revisions to trend and monthly changes in the trend at lags 1, 2, 3 and 12:

```
history{
 start=1998.1
 estimates=(trend trendchng)
 trendlags=(1,2,3,12)
 }
```

Full details of all arguments can be found in section 7.8 of the X-13ARIMA-SEATS manual (USCB 2017). More complicated growth rates, such as the change of most recent 3 months on previous 3 months (for example, Apr-May-Jun compared with Jan-Feb-Mar) need a different approach. See Chapter 7 for information regarding revision triangles and details on how to construct revision histories manually for this purpose.

19.4.2 *Comparing direct and indirect seasonal adjustment*

Revisions history is useful in comparing the revisions performance of direct and indirect seasonal adjustments. Table R3 is produced when running a composite seasonal adjustment. This table provides revisions obtained by adjusting all the components individually, as if indirect seasonal adjustment had been performed. In the same output, Table R1 gives the revisions for the direct seasonally adjusted series. Other than setting up the composite adjustment (covered in Chapter 20) and including the history spec in the individual component series, no other changes to the spec files are needed.

As an example, here are the summary tables produced when the series GMAC, GMAD and GMAE, overseas visitors to the UK (from different locations), are run both directly and indirectly:

```
R 1.S Summary statistics: average absolute percent revisions of the
    seasonal adjustments
Years:
1999  0.73
2000  0.50
Total: 0.65

R 3.S Summary statistics: average absolute percent revisions of the
    concurrent indirect seasonal adjustments
Years:
1999  1.06
2000  0.67
Total: 0.92
```

The tables above show that since 1999, the indirect adjustment has produced greater revisions than the direct adjustment. This might not be a problem, and the indirect adjustment may have other virtues, for example, it is more stable under sliding spans. While the direct adjustment is better than the indirect in terms of the size of revisions, the table shows how big the difference is and allows it to be set against any perceived advantages of indirect adjustment.

19.4.3 *Using AICC history to choose between two adjustments*

Table R7 provides a history of the model's AIC values over the period of the history. Unlike the other tables discusses so far, this is not a revisions history. It is a history of the AICC values calculated when fitting the model. Lower AICC values indicate a better model fit. The AICC history is a useful and flexible way to choose between two different models for seasonal adjustment.

```
R 7. Likelihood statistics from estimating regARIMA model over spans
    with ending dates 1:1996 to 3:2000

Span End Log Likelihood    AICC
 -------- -------------    -------
 1999.1     -337.006      676.100
 1999.2     -345.626      693.338
 1999.3     -358.060      718.205
 1999.4     -364.566      731.214
 2000.1     -371.809      745.700
 2000.2     -378.743      759.565
 2000.3     -386.011      774.101
```

This is an example AICC history for the example series used above. The method is to produce two AICC histories, for two competing methods of seasonal adjustment and then calculate for each point in the history, the difference between the first and second model's AICC values, including any corrections you may need to apply. For example, when choosing between an additive or multiplicative decomposition model, X-13ARIMA-SEATS penalises the additive model by adding 2 to its AICC value, before comparing it against the multiplicative adjustment's AICC value. As stated earlier, a small AICC value is more desirable.

The following example uses a monthly series showing the number of men claiming Job Seekers' Allowance, as of the second Thursday in the month. There is an issue as to whether this series is best modelled with an additive or a multiplicative model. Therefore, looking at the AICC histories of both models might be useful. The history specification of the spec file required to run this diagnostic is:

```
history{
 start=1999.1
 estimates=aic
 save=lkh}
```

Span end date	AICC(mult. Model)	AICC (add. model)	Difference	Model choice
Jan 1999	2174	2166	8	Additive
Apr 1999	2241	2230	11	Additive
Jul 1999	2307	2292	15	Additive
Oct 1999	2372	2354	18	Additive
Jan 2000	2438	2421	17	Additive
Apr 2000	2504	2488	16	Additive
Jul 2000	2569	2549	20	Additive
Oct 2000	2635	2613	22	Additive
Nov 2000	2657	2633	24	Additive
Dec 2000	2678	2654	24	Additive

Table 19.3: Selected values from table R7

The option estimates = aic is used to produces the AICC history. The option save = lkh means that the AICC history will be saved as a separate file, with extension ".lkh".

Table 19.3 provides a selection of values from Table R7 when the series is adjusted using an additive and a multiplicative decomposition. Using the default bias of 2, the additive model is preferred if AICC (multiplicative) - AICC (additive) > 2. Table 19.3 shows that the choice of an additive model is stable. Throughout the last two years, X-13ARIMA-SEATS would have chosen an additive model in any month, even though the additive model is penalised in the model selection process[2].

19.5 WHEN REVISIONS HISTORY WILL NOT WORK

The Revisions history diagnostic needs a minimum of 5 years of data to work. This is because it needs 5 years of reference data to make a reasonable series of seasonal adjustments during the revisions history.

If there are less than 5 years of data, X-13ARIMA-SEATS will fail to run. If the start argument is used to specify a date that is less than 5 years from the beginning of the series, X-13ARIMA-SEATS will run. However, it will move the start date of the revisions history to 5 years after the series start date.

In some cases, the revisions history of an additive seasonal adjustment can be difficult to interpret. If an additive series has just positive or just negative seasonally adjusted values, the revisions in table R1 are calculated in just the same way as for a multiplicative seasonal adjustment. However, if a series has both positive and negative values, then the differ-

2 a further example can be found in Section 4.4 of Findley et al. (1998)

ence, rather than the percentage revision, is calculated. For example,

```
R 1 Revisions of the concurrent seasonal adjustments
From     2000.Jan to 2001.May
Observations 17
2001
Jan   -53.96
Feb    33.89
Mar    13.27
Apr    22.06
May    25.18
```

Most other variables are not affected by this effect. Details of which other variables are affected and how are given in (USCB, 2013).

19.6 SUMMARY

The revisions history diagnostic is most useful when comparing two methods for seasonal adjustment that are both acceptable in terms of M and Q statistics, sliding spans and other diagnostics. There is no absolute measure of what an acceptable level of revisions is, so the diagnostic is of limited use on a single series. There are several histories available, all of which can be useful in observing the performance of the seasonal adjustment. Of these, those in tables R1 and R3 are probably most useful. If month-to-month changes, the trend component, or the month-to-month trend changes are of interest. Tables R2, R4 and R5 respectively will also be worth looking at.

Table R7 is quite easy to use to compare two competing models for a series, such as two ARIMA models or two different prior adjustments. This chapter has introduced a basic use of the history spec; there are other tables that can be produced, details of which can be found in USCB (2013).

20 COMPOSITE SPEC

20.1 INTRODUCTION

The composite spec is used as part of the procedure for obtaining both direct and indirect adjustment of a composite series. See Chapter 6 for further details. This chapter describes the spec files of the composite series and of the components and how to run the seasonal adjustment.

Section 20.2 will describe how the composite spec can be used in X-13ARIMA-SEATS to compare the performance of direct and indirect seasonal adjustments. Section 20.3 will discuss the diagnostics that are produced and how these can be used to inform the best method of seasonally adjusting an aggregate series, also known as composite series. (An aggregate series is one which may be thought of as consisting of values at each point in time which can be obtained by summing the corresponding values of two or more series – for example unemployment in any month is the sum of male and female unemployment).

20.2 THE SPEC FILES

Running a composite seasonal adjustment (both direct and indirect) requires spec files for the component series (one spec for each series), a spec file for the aggregate series, and a metafile that contains the names of all the component series that form the aggregate series and the name of the aggregate series itself.

The individual spec files must define how they are combined to form the aggregate, using the comptype and the compwt arguments in the series spec. The comptype argument specifies whether that particular component is added, subtracted, multiplied or divided. The compwt argument specifies the size of the constant used to multiply that particular component before it is combined to form the aggregate series (this constant is the weight of the component series). If no compwt is specified, then the default weight is 1.

A composite adjustment run produces an indirect seasonal adjustment of the composite series as well as a direct seasonal adjustment. The indirect seasonal adjustment is the weighted aggregate of the seasonally adjusted component series specified by the comptype and comwt arguments. If one of the component series is not seasonal then specifying the summary measures option by setting type=summary in the x11 spec of that component will include the component in the indirect adjustment without seasonally adjusting that particular series. A sliding span or revisions history analysis[1] of the direct and indirect adjustments can be obtained but the options must be specified in each of the component series as well as the composite series.

When a composite series is adjusted using X-13ARIMA-SEATS the outputs are:

- A direct seasonal adjustment of the aggregate, with normal output
- An indirect adjustment of the aggregate found by combining the seasonal adjustment of the components, with an output that includes the D-tables onwards only
- Diagnostics comparing the smoothness of the direct and indirect adjustments of the aggregate series.

20.2.1 *Example*

The following example illustrates all the steps of a composite adjustment. The series with file names “enjq.txt” and “enjl.txt” are two component series that sum to an aggregate series called “trade”. For example, use of “comptype=add” will sum the component series into the composite series.

Step 1: Create a spec file for each of the component series, eg:

```
ENJQ.SPC
series{
   title="Food & Beverages Imports CP"
   file="enjq.txt"
   name="enjq"
   comptype=add
   }
x11{}
```

A spec file for a component series that is not seasonally adjusted is given below.

```
ENJL.SPC
series{
   title="Crude Oil Imports CP"
   file="enjl.txt"
   name="enjl"
   comptype=add
```

1 see Chapter 18 and Chapter 19

```
   }
x11{
   type=summary
   }
```

The two spec files specify that ENJQ and ENJL are summed to form an aggregate series. ENJQ will be seasonally adjusted. ENJL is not seasonally adjusted.

Step 2: Create a spec file for the composite series:

```
TRADE.SPC
Composite{
      title="Total Imports CP" name="Trade"
      }
x11{}
```

Step 3: Create a metafile with details of the component and composite series. The metafile saved as imports.mta is shown below:

```
Enjq
Enjl
Trade
```

NB the spec file for the composite series must be listed last.

20.3 COMPARING ADJUSTMENT USING X-13ARIMA-SEATS DIAGNOSTICS

For series with several layers within aggregation structures, the composite adjustment spec provides a powerful tool to decide what the optimal level of seasonal adjustment is. The following are some criteria to help choose between the direct and indirect approach and may be put in order of priority as follows:

20.3.1 *Residual seasonality in the seasonally adjusted series*

The estimated seasonally adjusted series should not have any statistically significant residual seasonality. X-13ARIMA-SEATS provides a set of spectral diagnostics which could be used as a quick check for the existence of significant seasonal peaks in one of the estimated spectra. Note that if a warning message for significant seasonal peaks is printed out while running the composite spec file, then it refers to the direct seasonal adjustment of the composite series. To check whether there are significant peaks in the indirect seasonally adjusted series[2], the spectral plots in the output should be examined. To be visually significant, the spectral peak at a seasonal frequency must exceed both of its neighbours by at least 6 stars. If there are any significant peaks in one of the two options, then the alternative approach should be chosen.

20.3.2 *Revision errors*

X-13ARIMA-SEATS produces a set of revisions history diagnostics[3]. In general, the preferred alternative is the one that minimises the average percentage of revisions in the seasonally adjusted series.

20.3.3 *Stability*

X-13ARIMA-SEATS produces a set of sliding span diagnostics[4] as a way of deciding if a seasonal adjustment is stable. In general, the preferred alternative is the one that produces a more stable seasonally adjusted series.

20.3.4 *Interpretability of seasonally adjusted series*

The M-statistics measure various areas in the quality of seasonal adjustment while the Q statistic is a weighted average of all M-statistics. If the Q statistic fails (is greater than 1), there might be problems interpreting short-term movements in the seasonally adjusted series. In general, the chosen approach should minimise the Q statistic. Note however that the US Census Bureau have found evidence that, in an indirect analysis, some of the statistics, namely, M8 to M11 are misleadingly high. If it appears that the indirect Q statistic is unduly high mainly because of these statistics, it may be justifiable to give less weight to this fact.

20.3.5 *Smoothess of seasonally adjusted series*

The choice between direct and indirect methods is based on the comparison between the roughness measures, computed for the two series derived under the two different approaches. In general, the method of adjustment which gives the smoother series should be used. The measure of roughness is given at the end of the output (for the aggregate series), or before

2 see an example in Figure 17.5
3 see Chapter 19
4 see Chapter 18

the sliding spans and history diagnostics if these have been activated. The following is an example of roughness measures.

MEASURES OF ROUGHNESS R1 AND R2 FOR SEASONALLY ADJUSTED SERIES

	DIRECT		INDIRECT		PERCENTAGE CHANGE	
	FULL SERIES	LAST THREE YEARS	FULL SERIES	LAST THREE YEARS	FULL SERIES	LAST THREE YEARS
R1-MEANSQUARE ERROR	1.453	2.692	1.663	3.366	-14.47%	-25.04%
R1-ROOTMEAN SQUARE ERROR	1.205	1.641	1.290	1.835	-6.99%	-11.82%
R2-MEANSQUARE ERROR	0.000	0.000	0.000	0.000	-25.77%	-34.11%
R2-ROOTMEAN SQUARE ERROR	0.008	0.011	0.008	0.013	-12.15%	15.81%

POSITIVE PERCENTAGE CHANGES INDICATE THAT THE INDIRECT SEASONALLY ADJUSTED COMPOSITE IS SMOOTHER THAN THE DIRECT SEASONALLY ADJUSTED COMPOSITE.

The measures of roughness describe the size of the deviations from a smooth trend of the adjusted series (with R1 and R2 using different methods of trend estimation). The test results reported above suggest that the direct seasonal adjustment is smoother than the indirect adjustment. However, it should be noted that smoothness is not necessarily the most desirable characteristic of a seasonally adjusted series.

21 FORECASTING

21.1 INTRODUCTION

Forecasting techniques are used extensively in National Accounts to extend series beyond the most recent period for which data are available . This document describes the options to use for forecasting in seasonal adjustment and how to use X-13ARIMA-SEATS to forecast series for a purpose different from seasonal adjustment. This chapter is intended to be a reference document to allow users to make sound decisions without needing to understand the forecasting procedure in depth, although a more technical section, which explains the regARIMA method in more detail, can be found in Chapter 9.

21.2 FORECASTING IN SEASONAL ADJUSTMENT

The regARIMA part of the X-13ARIMA-SEATS program enables time series to be modelled to extend the series forwards (adding forecasts) and backwards (adding backcasts), to estimate calendar effects and to enable the series to be adjusted for unusual and disruptive features such as outliers or breaks.

Chapter 9 described the regARIMA method used in X-13ARIMA-SEATS and explained the options available to fit a model before the seasonal adjustment itself is performed. This section will revisit regARIMA models and provide details on the method to use to forecast series for seasonal adjustment purposes. Section 21.3 will deal with the problem of model selection for forecasting series for a purpose different from seasonal adjustment.

When fitting ARIMA models using X-13ARIMA-SEATS, there are decisions that need to be made by the user, for example whether to specify an ARIMA model or use automatic model detection. Related to this, the forecast horizon should be taken into account for the forecasts and backcasts whose generation is one of the main purposes of modelling. The default horizon for forecasts, if none is specified, is one year (the default for backcasts is none). Thus, the standard approach envisaged by the designers of X-13ARIMA-SEATS is to use the model selected by automdl with a year of forecasts.

Extensive empirical analysis of ONS data has shown that performances of pickmdl and automdl are very similar. In fact, using pickmdlmethod=first, with the list of models (reported in Chapter 8, Section 8.4.2) and one year of forecasts will result in a substantial reduction of revisions compared with no forecasts at all – typically the mean absolute revision is reduced by 10-%. If the recommended method fails to select a model, the appropriate alternative forecasting method should be adopted, as highlighted in Table 21.1.

Series Length	Recommended Method
< 6 years	automdl maxorder = (2,1) no constant
6-10 years	automdl maxorder = (3,1) no constant
10-12 years	Monthly series: automdl maxorder = (4,1) no constant
	Quarterly series: automdl maxorder = (3,1) no constant
>12 years	automdl maxorder = (4,2) constant

Table 21.1: Appropriate modelling method for forecasting

The Table 21.1 shows the recommended method for forecasting monthly and quarterly series. The maxorder parameter specifies the maximum order of regular and seasonal ARMA polynomials that could be chosen by automdl.

For monthly series with more than 12 years of data available the alternative method allows us to test the significance of a constant term in the model. Particular attention should be paid when interpreting the results. In fact, a significant constant should be maintained only if the selected model does not present regular differencing (the model picked is of the type (p 0 q)(P D Q)). This is because when regular differencing ($d \neq 0$) is selected in the model, the constant term corresponds to an exogenous trend, which is not so common in real life series. On the contrary without regular differencing in the model the constant term would refer just to the mean of the series, useful to set the long-term behaviour of the series.

Once a model has been selected, it should be specified in the arima spec in the following way:

```
arima{
   model=(p d q)(P D Q)
   }
```

where p, d, q are the orders of the regular part of the model and P, D, Q are the orders of the seasonal part. The arima specification should be used for the production runs of the forthcoming year. The form of the model should be re-estimated once a year during the seasonal adjustment review process.

Finally, it must be emphasised that if the model used describes the series poorly, then the forecasts and backcasts that are generated are also of poor quality. However, for seasonal adjustment purposes, the gain in using the forecasts is greater than using asymmetric weights in the moving average calculation. Therefore if the automdl fails to select a model, a simple model (generally (0 1 1)(0 1 1)) should be fixed in the arima spec so that forecasts can be used in the calculation of the moving averages.

Once the model is decided, the forecast spec can be added:

```
forecast{
  maxlead=12
  }
```

where maxlead specifies the number of forecasts. Conventionally, maxlead=12 is used for monthly data, maxlead=4 for quarterly data, and maxlead=1 for annual data, all of which will produce one year of forecast. However, maxlead can be set to any value up to 120

21.3 FORECASTING FOR A PURPOSE DIFFERENT FROM SEASONAL ADJUSTMENT

When forecasting for purposes other than seasonal adjustment (for example some sections of a statistical bulletin are of insufficient timeliness, and must be forecasted), then less concern can be given to the stability of the series. The regARIMA part of the X-13ARIMA-SEATS program can also be used for this aim to extend the series forwards (adding forecasts) and backwards (adding backcasts). Depending on the nature of the forecasting problem, then forecast evaluation may require other considerations.

21.3.1 *Model selection to generate forecasts*

There are two criteria to select an ARIMA model with X-13ARIMA-SEATS to generate forecasts:

1. Manual model selection using autocorrelation and partial autocorrelation functions

2. Automatic model selection

The model selection criteria depend on the length of the series. In fact:

- If less than 3 years of data are available, other methods of forecasting should be used (for example simple extrapolation methods). At least 3 years of data are required for X-13ARIMA-SEATS to operate

- If 3-5 years of data are available, X-13ARIMA-SEATS can use ARIMA modelling or provide trading day and Easter adjustments, but they are generally of very poor quality and subject to large revisions as future observations become known. At least 5 years of data are required for any automatic model selection to operate. In this case, users could undertake manual model identification from scratch to identify the model to generate forecasts
- If more than 5 years of data are available, X-13ARIMA-SEATS can use automatic model selection and provide good estimates for trading day and Easter adjustments

Identifying a model manually to generate forecasts

The program can produce the necessary autocorrelation outputs, and there are textbooks that describe the procedure, but it requires considerable experience to produce reliable results. In fact, the process of manual refinement needs a degree of skill and care and advice should be sought if in any doubt. The program provides a number of diagnostics that are helpful in this process particularly the model identification statistics (AICC, BIC etc.). Details of the use of these may be found in the full X-13ARIMA-SEATS documentation.

Once the appropriate model has been found, it should be specified in the arima spec in the following way:

```
arima{
   model=(p d q)(P D Q)
   }
```

where p, d, q are the orders of the regular part of the model and P, D, Q are the orders of the seasonal part. In common with other X-13ARIMA-SEATS specs, the commas in the inner brackets may be omitted provided the figures are separated by spaces. The other possible arguments of arima are concerned with pre-specifying parameter values and should not generally be used.

Another approach to X-13ARIMA-SEATS can be used if only 3-5 years of data are available: the Holt-Winters method. Holt-Winters is an extension of exponential smoothing. This is a process in which a predicted value is updated each time new information becomes available at the end of a series. It takes its name from the use of moving averages with exponentially declining weights that ensure that the most recent and relevant data points in the series supply the most information to the predictions.

In the Holt-Winters method, predictions of the level, slope, and seasonality of a series are updated using exponential smoothing. These predictions are then combined to give a forecast of the next observation in the series. Holt-Winters has additive and multiplicative forms, which determine whether the seasonal component is added to, or multiplied by, the level and slope components.

For example, if simple forecasts of a large number of series are required and between 3-5 years of data are available then the most sensible approach would be to use Holt-Winters across the board. This would provide good quality forecasts of most series without the lengthy manual modelling of the ARIMA approach.

Although the Holt-Winters method cannot be run in X-13ARIMA-SEATS, it can be produced in other software.

Using regARIMA model to generate forecasts

Section 21.2 described the model selection criteria and the options that should be used to fit a model before the seasonal adjustment itself is performed. This section will revisit those suggestions and provide details on the method and forecast horizon to use to forecast series for a purpose other than seasonal adjustment.

Extensive analysis of the ONS data shows that the appropriate forecasting methods derived in the previous section for seasonal adjustment purposes are still appropriate and that forecast performances for the recommended method are not dissimilar from the performances of the alternative methods. This suggests that if the recommended method fails to select a model, users can equally rely on the alternative approaches in terms of the quality of forecasts. A major difference from the methods described in Section 21.2 is the forecast horizon. In fact, the forecast horizon can now be increased to the maximum reported in Table 21.2 if the fit of the selected model is satisfactory.

Series Length	Recommended method	Maximum recommended length
<6 years	automdl maxorder = (2,1), no constant	1 year
6-10 years	automdl maxorder = (3,1), no constant	2 years
10-12 years	Monthly series: automdl maxorder = (4,1), no constant	3 years
	Quarterly series: automdl maxlag = (3,1), no constant	
>12 years	automdl maxlag = (4,2), no constant	3 years

Table 21.2: Appropriate modelling method and forecast horizon

The first entry in each cell in Table 21.2 is the recommended method, and the second is the maximum number of forecasts that could be generated. The same attention mentioned in Section 21.2 on the constant term should be paid for series with more than 12 years of data available.

Another major difference between forecasting for seasonal adjustment purposes and forecasting for other purposes. For forecasting for seasonal adjustment, the focus is not on the quality of the forecast but on the quality of the moving average. For forecasting for other purposes, the accuracy of the forecast is very important. In some situations, for example, forecasting a one-step ahead forecast, it may be desirable not to fix a model, and use the automatic modelling procedures every time a new forecast is required. If the automatic model selection procedure fails to select a model, whether this automatic procedure is the automdl or the pickmdl one, no arima forecasts should be generated. In this situation users should not use X-13ARIMA-SEATS for forecasting and another forecasting technique should be used, such as the Holt-Winters one.

It is important to keep in mind that forecasts estimated beyond the maximum recommended length reported in Table 21.2 have a high forecast error and therefore are not reliable, and should not be used in any system. The reason why that series required a longer forecast horizon should be considered (for example, unavailability of recent data points) and a different solution should be implemented (for example, to investigate the issue with data compiler and agree, if possible, on an alternative way to get the recent data points).

If automatic identification has selected a model that, while satisfying the tests, still has some unsatisfactory features then the following refinements can be used to improve the quality of the model selected by X-13ARIMA-SEATS:

- If the X-13 output states that there is evidence of regular (non-seasonal) over-differencing, automdl approach should be used allowing for a constant term to be selected if significant. However, if the order of the non-seasonal difference is not reduced (for example from 1 to 0), it is better to use the initially selected model

- If the X-13 output states that there is evidence of seasonal over-differencing, automdl approach should be used with seasonal dummies in the regression part of the model. However, the seasonal dummies should be removed if they are not statistically significant. This could be the case when the series is not seasonal

The use of these refinements is a matter of judgement and should be used together with other manual refinements when the automatic model selected seems counterintuitive, when the series to be forecasted is important and when experienced resources are available to carry out the analysis.

21.3.2 *Model validation*

The forecasting methodology used by X-13ARIMA-SEATS carries with it certain hazards. In particular, since model selection is an iterative procedure, it is possible to end up with a model which describes the data fairly well which could be because of one of the two following reasons:

1. The model is a good mode
2. The model is bad model that just happened to fit their particular data reasonably well, but will not fit future data well

If the model is good, then it will produce good forecasts. But if the model used describes the series poorly then the forecasts and backcasts that are generated are probably also poor. Using the quality measures reported in Chapter 23, for model validation, can reduce the probability of selecting a bad model by chance. This means before using a model for forecasting users should verify the accuracy of the model and manually refine it where necessary.

For example, although the test on the serial correlations of the residuals may be passed (for example Ljung-Box Q statistic, ACF and PACF peaks below the threshold) there may still be some individual significant correlations at fairly low lags. It may then be justifiable to try manual refinement of the automatic model. In this example, it could be worthwhile adding an extra coefficient at the appropriate lag to the AR or MA component; if the extra coefficient is significant and the significant serial correlation has been removed, the extra term might be justified.

21.3.3 *Considerations*

- **How far ahead to forecast?** This depends on individual user requirements although it is often found that only one or two future points are estimated. Users should be aware that the further beyond the end of the real data a point being forecast is, the less reliable the estimate is and the less likely it is to be correct when the real data for that point becomes available. For this reason, it is recommended that forecasts of data points further than the recommended forecast horizon are not used as estimates
- **Are the data non-seasonal?** If the time series being forecast are non-seasonal or already seasonally adjusted then the automdl method should be used to select the model and check that no seasonal coefficients have been included in the model. The same applies for series for which it is known from the seasonal adjustment re-analysis that

are not seasonal (check the seasonal adjustment analysis output diagnostics).

- **If the data are seasonal, then is the time series additive or multiplicative?** This will involve using log or none function in the transform spec to invoke either the log transformation (multiplicative) model or no transformation (additive) model. This is determined by whether the seasonal component of the series is in the form of a factor or an additive quantity. The decision between additive and multiplicative is similar to that in seasonal adjustment:
  - Graph the series to check whether it has additive or multiplicative properties
  - Check the seasonal adjustment review report if available. This will state whether the series is seasonally adjusted on an additive or multiplicative basis
  - If a series contains any zero values, then the additive method could be used to prevent any division by zero or a multiplicative method with a constant argument in the series spec could be used
  - If it is still unclear, it is recommended that the multiplicative method be used
- **Is the series long enough?** ARIMA models require a minimum of five years to automatically select a model but a minimum of three years to fix a regARIMA model to carry out a basic forecast. Less than three years of data would not provide enough information about the seasonal pattern of the series. Even with three years though, the forecast would be of poor quality and ideally the series should be as long as possible. If three years of data are not available, it would not be possible to use the ARIMA specification and other methods should be used (such as Holt-Winters or extrapolation methods)
- **Is the series being forecast an interpolated series?** Interpolated data should not normally be forecast. Forecasting should be conducted prior to interpolation wherever possible
- **Outliers, Level Shifts and Seasonal Breaks.** These are all unusual features which can occur in time series data but which have the potential to undermine or distort the forecasts produced by the regARIMA model. This is particularly true where the feature is close to the end of the series. There are two possible ways of approaching this problem:

1. An easy solution is to ignore the series up to the point of the break/outlier and just use the subsequent part of the series for the forecast. The effectiveness of this will obviously depend on the length of series available after the event (see comment above on length of series) and the irregularity over that period

2. A more reliable way to treat outliers, level shifts or seasonal breaks is to treat them in the same manner as in seasonal adjustment procedures, that is, with regressors in the regARIMA model. This involves setting up a regressor variable as described in Chapter 11 for level shifts and outliers, or in Chapter 14 for a seasonal break.

22 TREND ESTIMATION

22.1 INTRODUCTION

A seasonally adjusted series contains irregular movements that can sometimes obscure the underlying behaviour in the data. Trends are seasonally adjusted series with some of the irregularity removed; this can often help to reveal the medium- to long-term behaviour of the process generating the time series.

Following the recommendations of a research project conducted by Kenny and Knowles (1997), ONS adopted a standard method for estimating trends for monthly time series; many of these trends were subsequently published in first releases. The standard method was used for some time until the ONS publication policy was altered to stop publishing trends. The current ONS policy is not to publish the trend, as the trend provides little additional information to most users.

Trends are calculated using X-13ARIMA-SEATS, and this is the only method ONS currently recommends for estimating trends. Trends are calculated in X-13ARIMA-SEATS by applying moving averages. The number of periods over which the average is taken determines the amount of irregularity removed and so the smoothness of the resulting trend. Usually, the more irregular the series is the larger the number of periods over which the moving averages are calculated. The amount of irregularity is measured using the I/C ratio and the months or quarter to cyclical dominance (MCD or QCD, respectively). The I/C ratio is the ratio of the average absolute period variations in the trend (C) and the same for the irregular (I).

22.2 THE I/C RATIO AND THE MCD

When estimating trends, there are two related characteristics of the seasonally adjusted series that are important:

- The noise to trend ratio, commonly referred to as the I/C ratio
- The months for cyclical dominance MCD

These can both be determined by performing a seasonal adjustment in X-13ARIMA-SEATS. The MCD can be found in table F2E of the analytic output, along with the I/C ratio for certain spans, as shown in the example output below:

F 2.E: I/C RATIO FOR MONTHS SPAN

1	2	3	4	5	6	7	8	9	10	11	12
0.79	0.39	0.23	0.17	0.14	0.12	0.10	0.09	0.08	0.07	0.06	0.05

MONTHS FOR CYCLICAL DOMINANCE: 1

The I/C ratio represents the average percentage change in the irregular (I) component in the series in comparison to the average percentage change in the trend (C) over a certain span. This gives a measure of the volatility of a series. The output has a number of different I/C ratios, all for different spans of data. The first I/C ratio, 0.79 (shown in red), is for a one-month span. It is equal to the average percentage change in the irregular from one month to the next divided by the average percentage change in the trend from one month to the next. The value is less than one, which means that the change in the trend dominates the movement of the series over a one-month span, indicating that the series is not particularly volatile.

The second I/C ratio, 0.39 (shown in blue), is for a two-month span. It is equal to the average percentage change in the irregular from one month to two months later divided by the average percentage change in the trend from one month to two months later. This value is typically less than that for a one-month span, because the longer the span over which we look, the more dominant the long-term trend of the series becomes, and the less influence the irregular has. The ratios are tabulated up to a 12-month span.

The MCD represents the time it takes the trend to dominate the irregular. It is the number of months it takes for the I/C ratio to fall below one. In X-13ARIMA-SEATS the MCD is rounded up to the nearest month. In this example, the I/C ratio is less than 1 for a one-month span, so the MCD is 1 – this implies that it is possible to get an indication of the underlying short-term trend of the series by looking at the movement from one month to the next.

The MCD is less useful for series in which the trend changes very slowly. For these series the average percentage change in the trend will be small even over longer spans. Since we divide by this value to calculate the I/C ratio, I/C could become large even if there is not much irregularity in the series. This could lead to a value for the MCD greater than 1.

F 2.E: I/C RATIO FOR MONTHS SPAN

1	2	3	4	5	6	7	8	9	10	11	12
2.47	1.28	0.81	0.59	0.44	0.42	0.35	0.31	0.27	0.23	0.21	0.20

MONTHS FOR CYCLICAL DOMINANCE: 3
In the second example the I/C ratio first falls below 1 when a three-month span is used, so the MCD is 3. This means that, on average, we will need to look across a three-month span of the seasonally adjusted series in order to discern the underlying short-term trend. A more formal explanation can be found in Economic Trends (1972). A copy of this can be obtained from TSAB.

22.3 THE STANDARD TREND ESTIMATION METHOD

Trend estimates are produced as part of the standard seasonal adjustment process. The trend estimates that are generated by X-13ARIMA-SEATS can be extracted and used. This is the easiest option to obtain trend estimates.

An alternative approach is to use the derived seasonally adjusted estimates and apply a user-defined smoother to the seasonally adjusted estimates. This can give greater flexibility in calculating a trend estimate as the user has more control of the smoothing functions that will be applied.

22.4 PRESENTATION OF TRENDS

The following quote is from the article (Compton, 1998) and contains some useful advice about the presentation of trends:

"Trend estimates should not be quoted as headline figures; they should always be given less emphasis than the seasonally adjusted series. They should be presented in a graphical form on the front page of a First Release and numbers should be made available on request. The graph on the front page should show the last 15 months seasonally adjusted data and trend. The trend should be represented by a solid line with a dashed end to reflect the relative uncertainty of the trend at the end of the series. The length of the dashed part of the line should be determined by the following: For a Months for Cyclical Dominance [. . .] of:

1 - use a dashed line for the most recent month
2 - use a dashed line for the most recent two months
3+ - use a dashed line for the most recent three months.

"All commentary should be written in the past tense. As an optional addition, "what-if" graphs can be shown [. . .] These give a clearer indication of the degree of uncertainty of trend estimates at the end of the series."

Figure 22.1 shows an example of the standard presentation of the headline seasonally adjusted data with the trend, while Figure 22.2 gives an example of a "what-if" or "trumpet" graph (note: the trumpet begins 4 points from the end of the series, and the confidence intervals depicted in the graph differ only in the most recent three points).

22.5 CONSIDERATIONS

When using X-13ARIMA-SEATS to estimate the trend component, the following issues need to be taken into account:

1. **What is the form of the input series?** The data that feed into the trend estimation method should be the published seasonally adjusted series from the first release

2. **Should the additive or multiplicative decomposition be used?** The two approaches to estimate the trend component depend on the decomposition method in the x11 spec of the specification file. The choice between additive and multiplicative should already have been made when the input series was seasonally adjusted. This information can be found through a check of the X-13ARIMA-SEATS spec file used to seasonally adjust the data or a look at the output file. If the input series was not seasonally adjusted directly then this information will obviously not be available. The choice should then be made using the following criteria:
   a) Does the series have negative values? If so, use add in the mode option of the x11 specification. If not, proceed to the next step
   b) When viewing a graph of the time series, does the irregularity increase as the level of the series increases? If so, use mult in the mode option of the x11 specification. If not, use add. If unsure, proceed to next step
   c) What is the nature of your data? If the series is an index, use mult in the mode option of the x11 specification. If the series is a difference (for example Balance of Payments = Exports - Imports), use add. If a choice has still not been made, run the function first with add, then with mult and compare the quality of the two resulting trends.

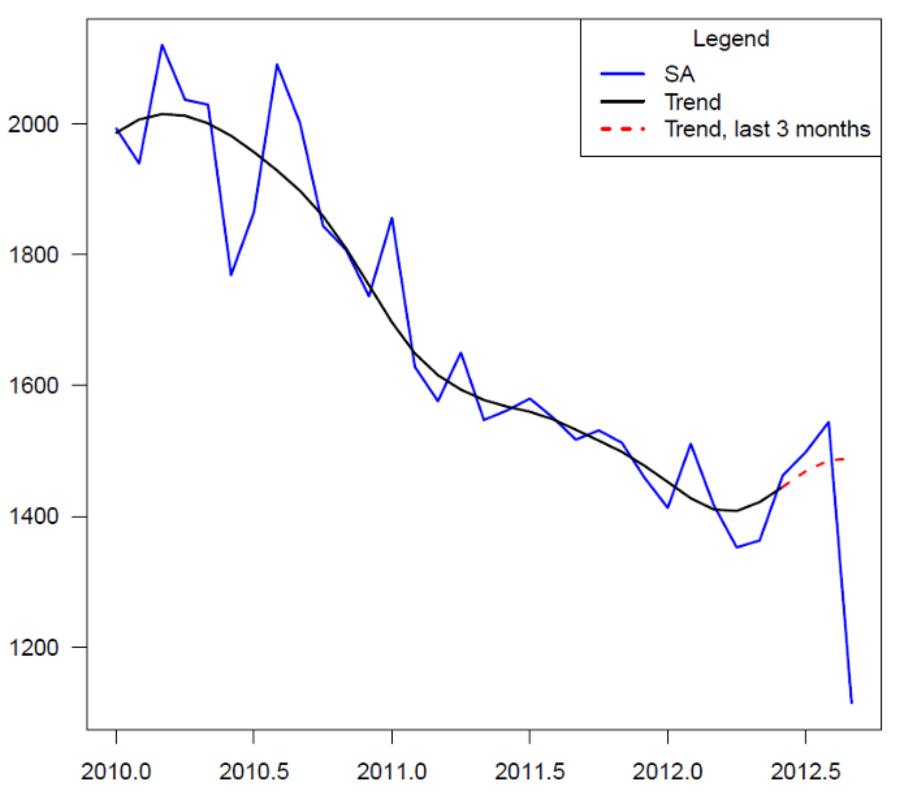

Figure 22.1: Front page, first release

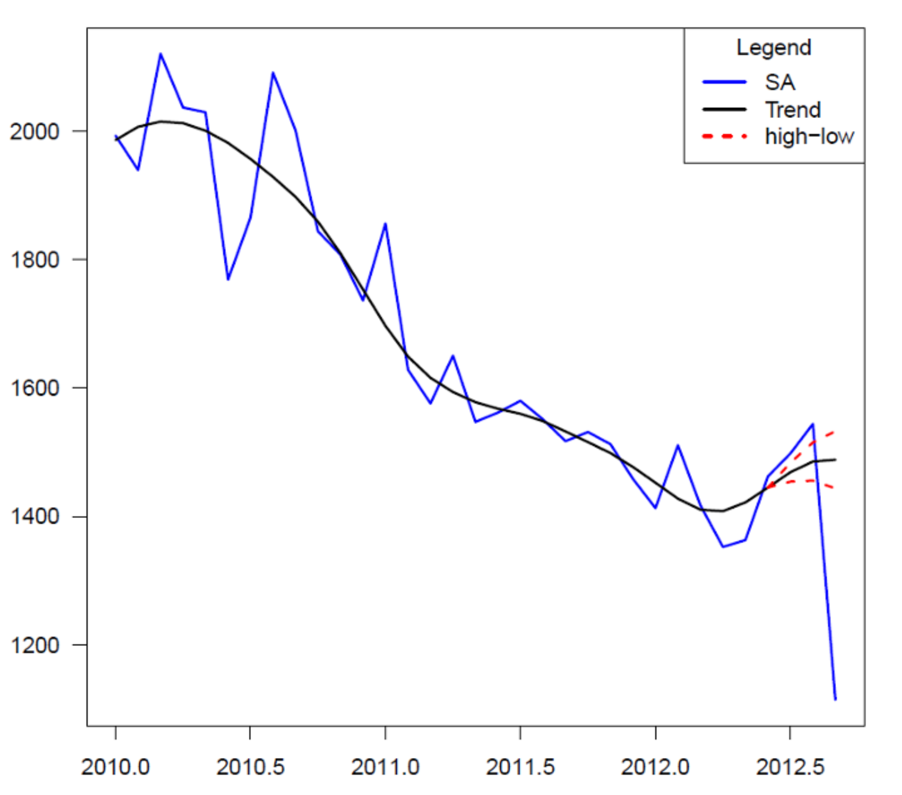

Figure 22.2: "What-If" graph, background notes

23 QUALITY MEASURES

23.1 INTRODUCTION

The Government Statistical Service (GSS) has long recognised the need to provide users with information about the quality of statistics and about the analytical techniques used to derive the figures. This chapter provides information about quality measures and indicators that can be used for considerations when measuring and reporting on the quality of time series outputs[1].

23.2 WHAT IS QUALITY

Quality, in terms of statistical outputs, can generally be thought of as a degree to which the data meet user needs, or simply put, the degree to which the data are fit for purpose.

Quality has often been associated with accuracy and timeliness. But even if statistics are accurate and timely, they cannot be deemed to be good quality if they are not based on concepts which are meaningful and relevant to the users. In addition, different users will have different needs. Quality measurement and reporting for statistical outputs is therefore concerned with providing the user with sufficient information to judge by themselves whether or not the data are of sufficient quality for their intended use.

The quality of a statistical output should be determined by its performance against a range of attributes that together can be used to assess whether an output meets users' quality criteria. The Office for National Statistics (ONS) has adopted the data quality attributes defined for the European Statistical System (ESS), which are shown in Table 23.1. This table has been superseded, but it is reproduced for brevity.

1 More information on quality measures can be found in Guidelines for Measuring Statistical Output Quality

Definition	Key components
1. RELEVANCE	
The degree to which the statistical product meets user needs for both coverage and content.	Any assessment of relevance needs to consider: • who are the users of the statistics? • what are their needs? • how well does the output meet these needs?
2. ACCURACY	
The closeness between an estimated result and the (unknown) true value.	Accuracy can be split into sampling error and non-sampling error, where non-sampling error includes: • coverage error • non-response error • measurement error • processing error; and • model assumption error
3. TIMELINESS AND PUNCTUALITY	
Timeliness refers to the lapse of time between publication and the period to which the data refer. Punctuality refers to the time lag between the actual and planned dates of publication.	An assessment of timeliness and punctuality should consider the following: • production time • frequency of release; and • punctuality of release
4. ACCESSIBILITY AND CLARITY	
Accessibility is the ease with which users are able to access the data, also reflecting the format(s) in which the data are available and the availability of supporting information. Clarity refers to the quality and sufficiency of the metadata, illustrations and accompanying advice.	Specific areas where accessibility and clarity may be addressed include: • Needs of analysts • Assistance to locate information • Clarity; and • Dissemination
5. COMPARABILITY	
The degree to which data can be compared over time and domain.	Comparability should be addressed in terms of comparability over: • Time • Spatial domains (sub-national, national and international); and • Domain or sub-population (industrial sector, household type)
6. COHERENCE	
The degree to which data that are derived from different sources or methods, but which refer to the same phenomenon are similar.	Coherence should be addressed in terms of coherence between: • Data produced at different frequencies • Other statistics in the same socio-economic domain • Sources and outputs

Table 23.1: ONS data quality attributes

Quality of data can rarely be explicitly measured. For example, in the case of accuracy, it is almost impossible to measure non-response bias as the characteristics of those who do not respond can be difficult to ascertain. Instead, certain information can be provided to help indicate quality. Quality indicators usually consist of information that is a by-product of the statistical process. They do not measure quality directly but can provide enough information to make inferences about the quality. Section 23.4 includes both quality measures and suitable quality indicators that can either supplement or act as substitute for the desired quality measure.

23.3 TIME SERIES QUALITY MEASURES

The following table reports the quality measures specific for time series analysis together with the output for which they are relevant and the European Statistical System to which they refer.

N.	Quality Measure	X13 In-built	Outputs applicable to	ESS Dimension
1	Original data visual check	✓	All time series	Accuracy
2	Comparison of the original and seasonally adjusted data	✓	All seasonal adjustments	Accuracy
3	Graph of Seasonal-Irregular (SI) ratios	✓	All time series	Comparability
4	Analysis of Variance (ANOVA)		All seasonal adjustments and trend estimations	Accuracy
5	Months (or Quarters) for Cyclical Dominance	✓	All seasonal adjustments and trend estimations	Comparability
6	The M7 statistic	✓	All seasonal adjustments	Accuracy
7	Contingency Table Q	✓	All seasonal adjustments and trend estimations	Comparability
8	Stability of Trend and Adjusted Series Rating (STAR)		All seasonal adjustments and trend estimations	Accuracy
9	Comparison of annual totals before and after seasonal adjustment	✓	All seasonal adjustments	Accuracy
10	Normality test	✓	All forecasts	Accuracy
11	p-values		All forecasts	Accuracy
12	Percentage standard forecast error		All forecasts	Accuracy
13	Graph of the confidence intervals		All forecasts	Accuracy
14	Percentage difference of unconstrained to constrained values		All constrained series	Accuracy

Table 23.2: Quality measures used in seasonal adjustment

Each quality measure is presented below with an example or formula describing their use and with notes providing more detail on the measure/indicator.

1. Original data visual check. The graph below shows the Airline Passengers original series. From the graph it is possible to see that the series has repeated peaks and troughs that occur around the same time each year. This implies seasonality. It is also possible to see that the trend is affecting the impact of the seasonality. In fact, the amplitude of the seasonal peaks and troughs change proportionally with the level of the trend. This suggests that a multiplicative decomposition model is appropriate for the seasonal adjustment. This series does not show any particular discontinuities (outliers, level shifts or seasonal breaks).

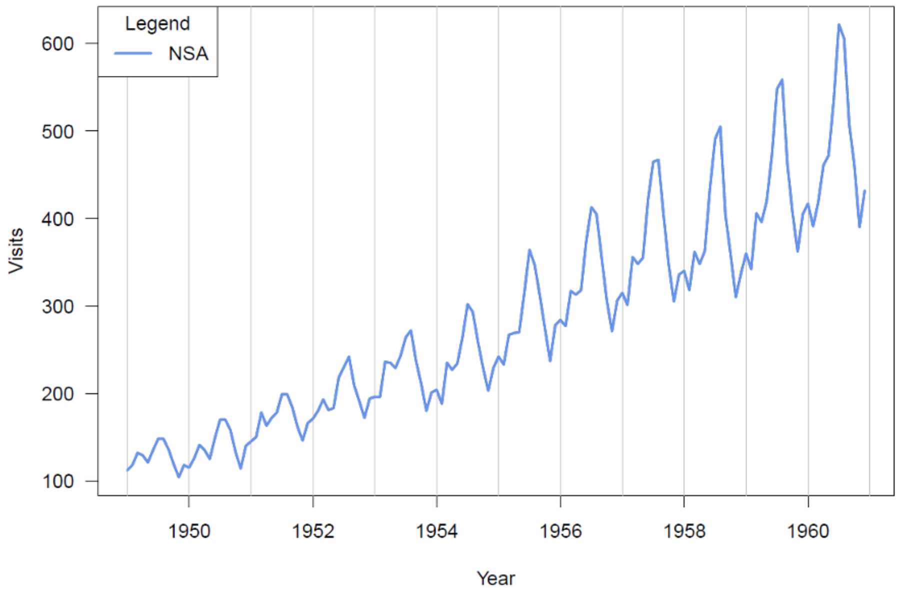

Figure 23.1: Airline passengers

Graphed data can be used in a visual check for the presence of seasonality, decomposition type model (multiplicative or additive), extreme values, trend breaks and seasonal breaks.

2. Comparison of the original and seasonally adjusted data. Figure 23.2 below contains a seasonal break that has not been accounted for. By looking at the seasonally adjusted series, there appears to be residual seasonality left over after seasonal adjustment has taken place (as can be seen every June and October up until 2003). This is because the change in seasonal pattern in January 2003 has not been accounted for. Information on how to deal with seasonal breaks can be found in Section 14.3.

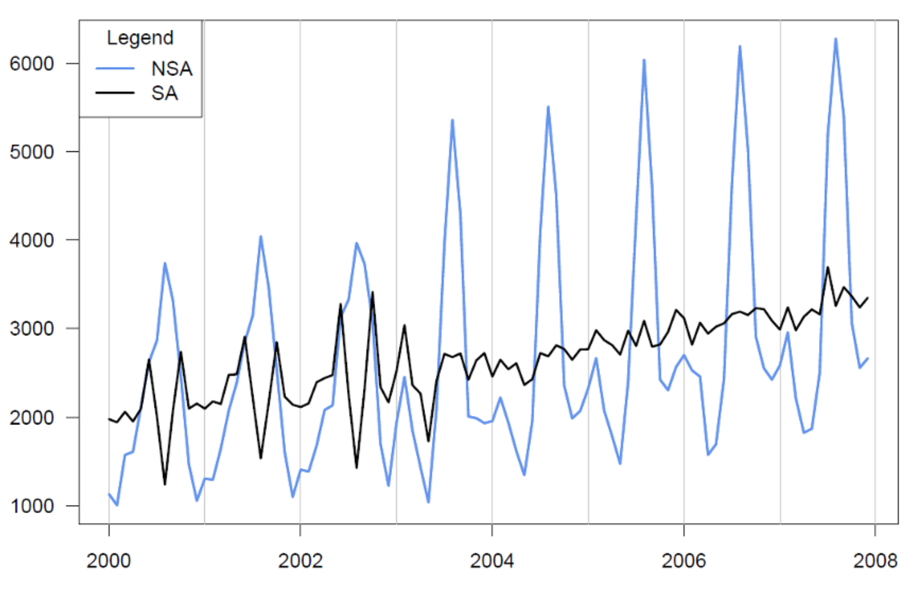

Figure 23.2: Comparison of seasonally adjusted and original series

By graphically comparing the original and seasonally adjusted series, it can be seen whether the quality of the seasonal adjustment is affected by any extreme values, trend breaks or seasonal breaks and whether there is any residual seasonality in the seasonally adjusted series.

3. Graph of Seasonal-Irregular (SI) ratios. In the example below, there has been a sudden drop in the level of the Seasonal-Irregular component (called unadjusted SI ratios) for August between 1998 and 1999. This is caused by a seasonal break in the Car Registration series which was because of the change in the car number plate registration legislation. Permanent prior adjustments should be estimated to correct for this break. If no action is taken to correct for this break, some of the seasonal variation will remain in the irregular component resulting in residual seasonality in the seasonally adjusted series. The result would be a higher level of volatility in the seasonally adjusted series and a greater likelihood of revisions.

It is possible to identify a seasonal break by a visual inspection of the seasonal irregular graph (graph of the SI ratios). Any change in the seasonal pattern indicates the presence of a seasonal break.

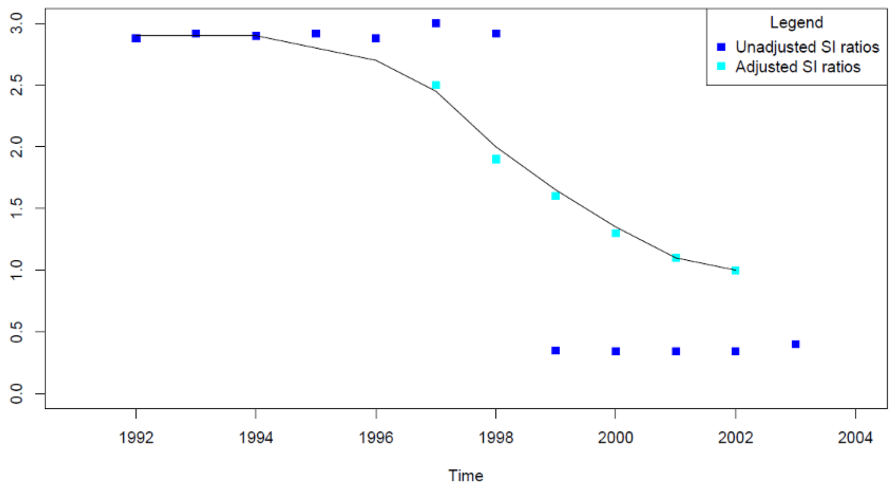

Figure 23.3: SI ratios for August

4. Analysis of Variance (ANOVA). The formula to calculate the Analysis of Variance statistic is as follows.

$$\text{ANOVA} = \frac{\sum_{t=2}^{n}(D12_t - D12_{t-1})^2}{\sum_{t=2}^{n}(D11_t - D11_{t-1})^2} \quad (17)$$

Where:
$D12_t$ = data point value for time t in table D12 (final trend-cycle) of the analytical output;
$D11_t$ = data point value for time t in table D11 (final seasonally adjusted data) of the output.
When calculating the ANOVA, the following considerations should be taken into account:

- If constraining to the annual totals is used and hence the D11A table is produced, D11A values are used in place of D11
- If the statistic is used as a quality indicator for the trend, table A1 should be used instead table D11. The D12t values should be taken

The Analysis of Variance (ANOVA) compares the variation in the trend component with the variation in the seasonally adjusted series. The variation of the seasonally adjusted series consists of variations of the trend and the irregular components. ANOVA indicates how much of the change of the seasonally adjusted series is attributable to changes primarily in the trend component. The statistic can take values between 0 and 1 and it can be interpreted as a percentage. For example, if ANOVA=0.716, this means that 71.6% of the movement in the seasonally adjusted series can be explained by the movement in the trend component and the remainder is attributable to the irregular component.

This indicator can also be used to measure the quality of the estimated trend.

5. **Months (or Quarters) for Cyclical Dominance.** The months for cyclical dominance (MCD) or quarters for cyclical dominance (QCD) are measures of volatility of a monthly or quarterly series respectively. The formula to derive the statistic is as follows.

$$\mathrm{MCD(or\ QCD)} = \min\left\{ d \in \mathbb{N} : \frac{I_d}{C_d} < 1 \right\} \tag{18}$$

where d is the number of months span considered so that the I/C ratio falls below 1, I_d is the final irregular component at d, and C_d is the final trend component at d.

This statistic measures the number of periods (months or quarters) that need to be spanned for the average absolute percentage change in the trend component of the series to be greater than the average absolute percentage change in the irregular component. For example, an MCD of 3 implies that the change in the trend component is greater than the change in the irregular component for a span at least 3 months long. The MCD (or QCD) can be used to decide the best measure of short-term change in the seasonally adjusted series; if MCD = 3, a three-month span will be a better estimate than a one-month span. The lower the MCD (or QCD), the less volatile the seasonally adjusted series is and the more appropriate the month-on-month growth rate is a measure of change. The MCD (or QCD) value is automatically calculated by X-13ARIMA-SEATS and is reported in table F2E of the analytical output. For monthly data the MCD takes values between 1 and 12, for quarterly data the QCD takes values between 1 and 4.

6. **The M7 statistic**. The formula behind M7 statistic is as follows.

$$M7 = \sqrt{\frac{1}{2}\left(\frac{7}{F_S} + \frac{3F_M}{F_S}\right)} \tag{19}$$

Where: $F_M = \frac{S_B^2(N-1)}{S_R^2(N-1)(k-1)}$

$F_S = \frac{S_A^2(K-1)}{S_R^2(n-k)}$

N = the number of observations

F_M = F-statistic capturing moving seasonality

S_B^2 = the inter-year sum of squares

S_R^2= the residual variance (the residual sum of squares)

F_S = F-statistic capturing stable seasonality

S_A^2 = the variance caused by the seasonality factor

M7 compares on the basis of F-test statistics the relative contribution of moving (statistic F_M) and stable (statistic F_S) seasonality.

This indicates whether the original series has a seasonal pattern or not. Furthermore, it shows the quality of the adjustment. M7 value lies between 0 and 3. Low values indicate clear and stable seasonality has been identified by X-13ARIMA-SEATS. Generally, values between 0 and 0.7 suggest seasonality is present; values between 0.7 and 1.3 show potential seasonality (but further tests are required to confirm); and values above 1.3 are considered to be non-seasonal. This is not an absolute rule and should be followed up with further checks. If the adjustment is not good, then the M7 value may output an inflated value, for example, the series might have a seasonal break not adjusted for. The M7 statistic is calculated by X-13ARIMA-SEATS and can be obtained from the F3 table of the analytical output.

7. Contingency Table Q (CTQ). The formula to analyse the Contingency Table Q is as follows:

$$CTQ = \frac{U_{11} + U_{22}}{U_{11} + U_{12} + U_{21} + U_{22}} \tag{20}$$

where U_{kl} is the value for contingency table cell with row k and column l and:

	$\Delta C > 0$	$\Delta C \leqslant 0$
$\Delta SA > 0$	U_{11}	U_{12}
$\Delta SA \leqslant 0$	U_{21}	U_{22}

where ΔSA is the change in the SA data, $\Delta SA = D11_t - D11_{t-1}$, and ΔC is the change in the trend $\Delta C = D12_t - D12_{t-1}$.

The CTQ shows how frequently the gradient of the trend and the seasonally adjusted series over a one period span have the same sign. CTQ can take values between 0 and 1. A value of 1 indicates that historically the trend component has always moved in the same direction as the seasonally adjusted series. A value of 0.5 suggests that the movement in the seasonally adjusted series is likely to be independent from the movement in the trend component, this can indicate that the series has a flat trend or that the series is very volatile. A value between 0 and 0.5 is unlikely but would indicate that there is a problem with the seasonal adjustment.

If constraining is used point values from table D11A are used in place of D11.

8. Stability of Trend and Adjusted Series Rating (STAR). The formula to calculate the Stability of Trend and Adjusted Series Rating (STAR) is as follows.

$$\text{STAR} = \frac{1}{N-1} \sum_{t=2}^{N} \left| \frac{D13_t - D13_{t-1}}{D13_{t-1}} \right| \tag{21}$$

Where $D13_t$ is the data point value for time t in table D13 (final irregular component) of the output, and N is the number of observations in Table D13.

This indicates the average absolute percentage change of the irregular component of the series. The **STAR statistic** is applicable to **multiplicative decompositions only**. The expected revision of the most recent estimate when a new data point is added is approximately half the value of the STAR value , for example, a STAR value of 7.8 suggests that the revision is expected to be around 3.9%.

9. Comparison of Annual Totals (CAT) before and after seasonal adjustment. The formula for multiplicative models is:

$$\frac{1}{n} \sum_{t=1}^{n} \left| E4^{D11}_{\mathtt{total(t)}} - 100 \right| \tag{22}$$

where $E4^{D11}_{\mathtt{total(t)}}$ is the unmodified ratio of annual totals for time t in table E4. E4 is the output table that shows the ratios of the annual totals of the original series to the annual totals of the seasonally adjusted series for all the n years in the series.

The formula for *additive models* is:

$$\frac{1}{n} \sum_{t=1}^{n} \left| \frac{E4^{D11}_{\mathtt{total(t)}}}{D11_{\mathtt{total(t)}}} \right| \tag{23}$$

$E4^{D11}_{\mathtt{total(t)}}$ is unmodified difference of annual totals for time t in table E4. For additive models E4 is the output table that calculates the difference between original annual total and seasonally adjusted annual totals for all the n years in the series.

This is a measure of the quality of the seasonal adjustment and of the distortion to the seasonally adjusted series brought about by constraining the seasonally adjusted annual totals to the annual totals of the original series. It is particularly useful to judge if it is appropriate for the seasonally adjusted series to be constrained to the annual totals of the original series.

DIAGNOSTIC CHECKING Sample Autocorrelation of Residuals												
Lag	1	2	3	4	5	6	7	8	9	10	11	12
ACF	-0.06	0.12	-0.1	-0.31	-0.25	-0.13	0.21	-0.09	0.21	-0.05	0.09	-0.15
SE	0.19	0.19	0.2	0.2	0.21	0.22	0.23	0.23	0.24	0.24	0.24	0.24
Q	0.09	0.56	0.86	4.04	6.2	6.8	8.86	8.86	10.8	10.89	11.31	12.44
DF	0	0	1	2	3	4	6	6	8	8	9	10
P	0	0	0.355	0.133	0.102	0.147	0.181	0.181	0.208	0.208	0.255	0.256

Model fitted is: ARIMA (0,1,1)(0,1,1). (Quarterly data so 12 lags will be extracted)

10. Normality test. The two statistics below (Geary's kurtosis (α) and sample kurtosis (b_2) test the regARIMA model residuals for deviations from normality.

$$\alpha = \frac{\frac{1}{n}\sum_{i=1}^{n}|X_i - \bar{X}|}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2}} \tag{24}$$

$$b_2 = \frac{m_4}{m_2^2}\frac{n\sum_{i=1}^{n}(X_i - \bar{X})^4}{(\sum_{i=1}^{n}(X_i - \bar{X})^2)^2} \tag{25}$$

The assumption of normality is used to calculate the confidence intervals from the standard errors. Consequently, if this assumption is rejected the estimated confidence intervals will be distorted even if the standard forecast errors are reliable. A significant value of one of these statistics (α or b_2) indicates that the standardised residuals do not follow a standard normal distribution, hence the reported confidence intervals might not represent the actual ones. (X-13ARIMA-SEATS tests for significance at one percent level.)

11. p-values. An example of the p-value quality measure is as follows.

The model will not be adequate if p-value $\leqslant 0.05$ for any of the 12 lags and will be adequate if p-value > 0.05 for all the lags. Because we are estimating 2 parameters in this model (one AR and one MA parameter), no p-values will be calculated for the first 2 lags as 2 degrees of freedom will be automatically lost, hence we will start at lag 3. Here all the p-values are greater than 0.05 at lag 12, Q=12.44, DF=10 and p-value = 0.256, therefore this model is adequate for forecasting.

This is found in the "Diagnostic checking Sample Autocorrelations of the Residuals" output table. The p-values show how good the fitted model is. They measure the probability of the ACF occurring under the hypothesis that the model has accounted for all serial correlation in the series up to the lag. Where p-values are greater than 0.05, up to lag 12 for quarterly data and lag 24 for monthly data, indicate that there is no significant residual auto-correlation and that, therefore, the model is adequate for forecasting.

12. Percentage forecast standard error. The percentage forecast standard error is given by:

$$\frac{\text{percentage forecast standard error}}{\text{forecast}} \text{x}100\% \qquad (26)$$

The percentage forecast standard error is required for each forecast produced and can be found in the forecast table. There will be one number for each period that has been forecasted. The percentage forecast standard error is applicable to multiplicative decompositions only.

13. Graph of the confidence intervals. An example of the graph of confidence intervals is reported below. Note that for seasonally adjusted series which are seasonally adjusted using X-11 based methods there are no variance estimates provided. To produce variance estimates and confidence intervals for the seasonally adjusted series see Chapter 24.

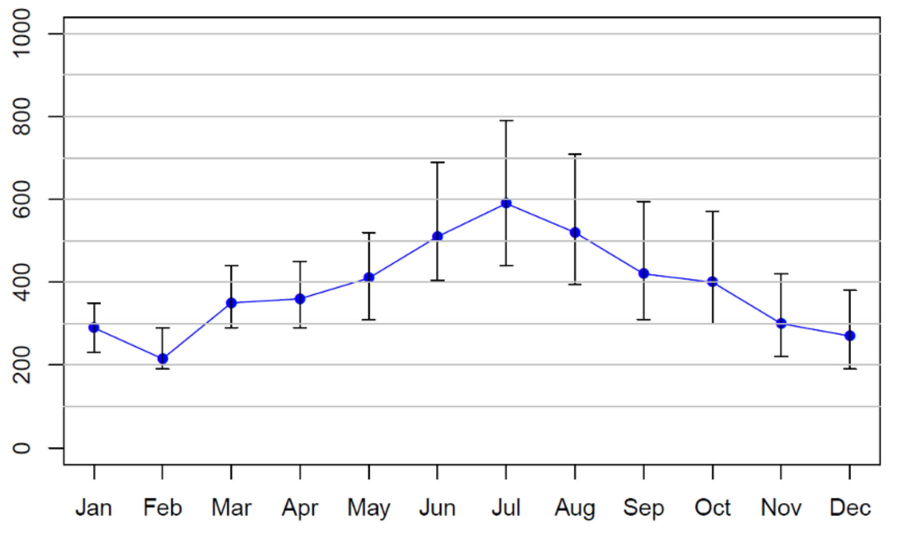

Figure 23.4: Graph of forecast estimates and their confidence intervals

14. Percentage difference of unconstrained to constrained values. The formula to compute the percentage difference of unconstrained to constrained values is as follows.

$$\left(\frac{\text{constrained data point}}{\text{unconstrained data point}} - 1\right) \text{x}100\% \qquad (27)$$

This indicates the percentage difference between the unconstrained and the constrained value. This will need to be calculated for each data point being constrained. The average, minimum and maximum percentage differences (among all data points of all the series) of the unconstrained should be calculated to give an overview of the effect of constraining.

23.4 HOW TO INTERPRET TIME SERIES QUALITY MEASURES

The following section provides examples to help users in interpreting the quality measures and the quality indicators when running seasonal adjustment, trend estimation, forecasting and constraining. When using a quality measure to interpret a time series it is important to keep in mind that none of the diagnostics should be used to explain the quality of a single component of a time series, but to give users information on the characteristics of the series. For example, the diagnostics should not be used to explain if the trend is a good or a bad estimate, but to relate the behaviour of the trend with that of the seasonal and irregular components.

23.4.1 *Quality measures for seasonal adjustment*

The quality indicators for seasonal adjustment should be used to give users information on the characteristics of the series (such as to relate the behaviour of the trend with that of the seasonal and irregular components). For example, the quality measure can be used to define:

- If the series has identifiable seasonality (by the size of M7, in the F3 table)
- How volatile the series is (by the size of STAR and MCD, in the F2 table), and
- How the trend relates to the seasonally adjusted series (by the size of the ANOVA and CTQ)

The visual inspection of the graph of the original series against the seasonally adjusted series can help users to identify residual seasonality in the seasonally adjusted series.

For example, Figure 23.5 shows the original and the seasonally adjusted series. The M7 statistic is high. This suggests that the series is non-seasonal, and it should not be seasonally adjusted. However, as can be seen in the graph, the series is not very irregular and most of the movement in the series is caused by the trend. This is confirmed by the high ANOVA and the Contingency Table Q (CTQ) statistics and by the very low MCD and STAR measures. The low Comparison of Annual Totals (CAT) also confirms that the irregular component does not affect much the series.

23.4.2 *Quality measures for forecasting*

The quality indicators for the forecast should be used to give users information on the reliability of the projections (for example, to relate the

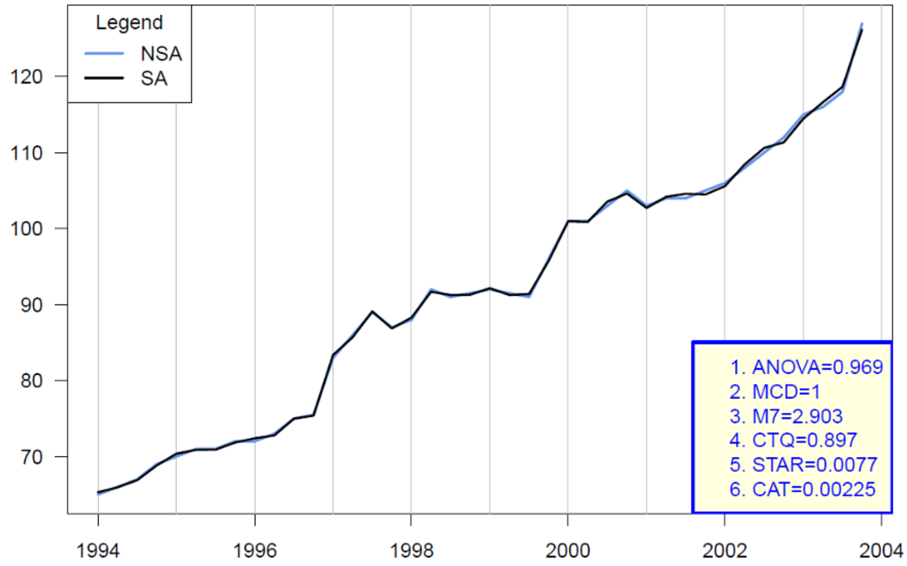

Figure 23.5: Quality measure for seasonal adjustment

goodness of the fitted model with goodness of the forecasts). For example, the quality measures can be used to define if the model is suitable for forecasting (by the size of the normality test and the p-value) and the dispersion of the forecasts themselves (by the size of the percentage standard forecast error and the width of the confidence intervals). The visual inspection of the confidence intervals can help users understanding how accurate the forecasts are (the wider the confidence intervals are, the less reliable the projections are).

Figure 23.6 shows the confidence intervals of the two-years-ahead forecast. The forecasts are shown by the blue line in the middle. The actual values are expected to lie somewhere between the lower (pink) and upper (green) lines, for 95% of the time.

An assumption of normality is used to calculate the confidence intervals. As the kurtosis statistic is significant, this assumption is rejected and consequently the reported confidence intervals might not represent the actual ones.

If the p-values presented are greater than 0.05 then we can reject the null hypothesis that there is no autocorrelation and therefore the model is fitting well. This implies that a forecast is more reliable though it should be noted that this does not guarantee a good forecast because of various factors such as unexpected events such as financial crises.

The Percentage Forecast Errors are a measure of how much the forecast is expected to differ from the actual value. For January 1998, the first forecasted value, we expect that the actual value will be different from the forecast by 25% on average. For December 1999, the final forecasted value,

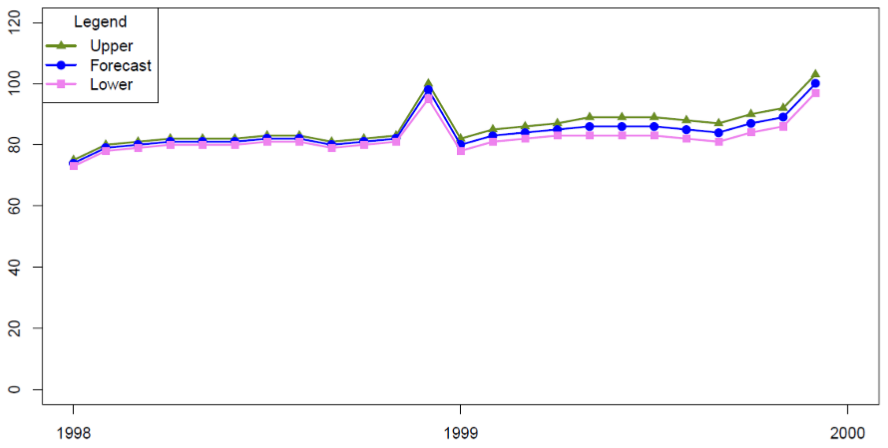

Figure 23.6: Confidence intervals with forecasts

we expect that the actual value will be different from the forecast by 75% on average.

23.4.3 *Quality measures for trend estimates*

The quality indicators for the estimated trend should be used to give users information on the characteristics of the series (such as relating the behaviour of the trend with that of the irregular component). For example, the quality measure can be used to define how volatile the series is (by the size of STAR and MCD) and how the trend relates to the seasonally adjusted series (by the size of the ANOVA and CTQ). The visual inspection of the graph of the seasonally adjusted series against the trend can help users to identify turning points in the behaviour of the series.

Figure 23.7 shows a seasonally adjusted series and the trend, which has been estimated for this series. The ANOVA statistic is low. This suggests, as can be seen in the graph, that most of the movement from one period to the next in the seasonally adjusted series is caused by the irregular component. Approximately one percent of the month-on-month change in the seasonally adjusted series is explained by movement in the trend. Also, the high MCD statistic confirms that the series is volatile. In fact, it takes on average 9 months for the trend component to explain more of the movement in the series than the irregular component. The CTQ statistic of 0.68, which is outside the range of 0 and 0.5, is a further indication that the series is volatile. The STAR value of 6.78 suggests that the expected revision of the most recent estimate when a new data point is added will be approximately 3.4%.

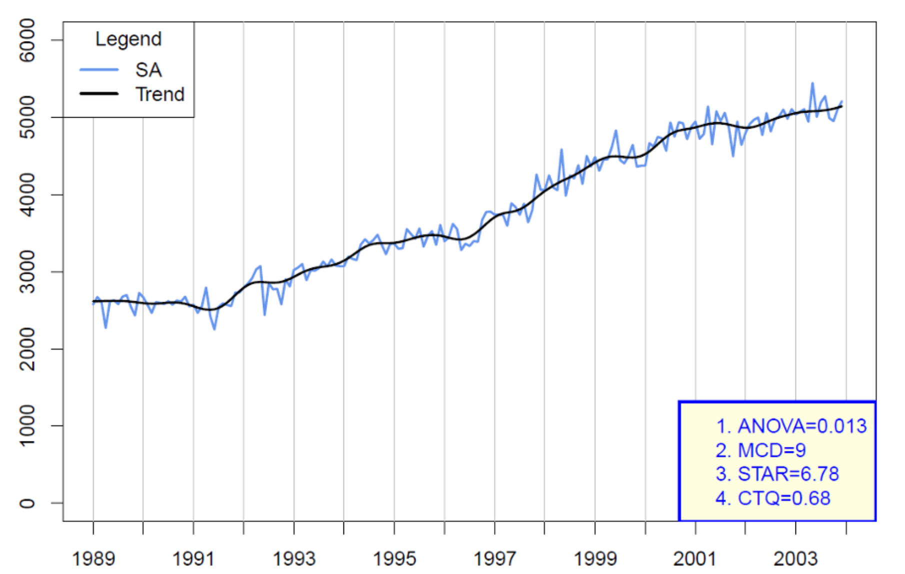

Figure 23.7: Quality measures for trend estimates, UK visits abroad

The quality indicators for the constrained series should be used to give users information on the effect of constraining. For example, the quality measures can be used to define how far the constrained series is from the original series. The summary statistics (minimum, average and maximum) give users an indication of the distribution of the percentage differences of unconstrained to constrained values. For example, after seasonal adjustment, Labour Force Survey series are constrained so that the property of additivity is fulfilled in every dimension - by age group, employment status etc. This constraining distorts the seasonal adjustment, so it is acceptable only if the distortion is small. Eight important high-level aggregation series of total numbers of active, inactive, employed and unemployed for both males and females were selected from the Labour Force Survey dataset. The average difference (among all data points of all series) was 0.11% of unconstrained. The maximum difference (among all data points of all series) was 0.12% of the unconstrained. For these high-level aggregation series, a Quality Measure of 0.11% suggests good constraining, so no problems have been detected with the constraining of these series. The acceptable threshold, for the distortion caused by constraining, should be considered for each individual series. In practice, the requirement to constrain may override any consideration of distortion.

24 VARIANCE ESTIMATION

24.1 INTRODUCTION

An important part of official statistics is the quantification of the uncertainty associated with the published statistics. Quantifying the uncertainty associated with a statistic allows users to make an assessment of the accuracy of the estimates and to take this into account when using the statistics in analysis and decision making. Uncertainty measures are usually published to quantify the size of the errors resulting from the sampling and imputation methods. This quantification of uncertainty most often takes the form of standard errors or confidence intervals associated with a particular estimate. Standard errors are routinely published by National Statistical Institutes (NSIs) for the non seasonally adjusted (NSA) series but not for the seasonally adjusted (SA) series, despite the fact that the SA series is of more interest to users.

24.2 WHY THE NSA STANDARD ERRORS CANNOT BE APPLIED TO THE SA SERIES

In X-13ARIMA-SEATS the outlier identification procedures used mean that the seasonal adjustment procedures are non-linear. The non-linear nature of the method means that simply applying the NSA standard errors to the SA series will not take account of the backcasts/forecasts implicit in seasonal adjustment at the ends of the series or the autocovariance structure of the series and the autocorrelation inherent in the series. For a series with constant standard error, applying the NSA variance estimates to the SA series will overestimate the variance by ten per cent in the centre of the series and underestimate it by 20 per cent at the end of the series[1].

24.3 APPROXIMATE METHODS

There are several methods available to estimate the variance of the seasonally adjusted estimators.

1 see Scott and Pfeffermann (2004)

24.3.1 *Wolter and Monsour*

The method Wolter and Monsour (1981) simply uses a linear approximation to the seasonal adjustment filter. The benefits of this method are that it is straightforward to explain and will account for the design variance of estimators. The disadvantages of this method are that it will not account for other sources of error, it requires the autocovariance of survey errors, and the variance estimates for a census will be zero.

24.3.2 *Pfeffermann*

The method Pfeffermann (1994) exploits a relationship between survey errors and the SA residual to obtain variance estimates. The benefits of this method are that it accounts for sampling and decomposition error, is largely model-free, can be applied to complex surveys, and does not require survey error autocovariances. The disadvantages of this method are that it assumes that the X-11 filter works correctly for the series, it requires a simplifying assumption for the survey error autocovariance, and there is a bias detected.

24.3.3 *Replication*

With the replication method, the variance estimates for the NSA series are used to replicate the NSA series. These replicates are then seasonally adjusted using the seasonal adjustment model for the original NSA series. The distribution of the SA estimates is then used as a proxy for the variance of the estimators. The benefits of this method are that it is straightforward to apply and can be applied to any series. The disadvantages are that it does not take into account error autocovariance or decomposition error.

25 SOFTWARE

25.1 DOWNLOADING AND INSTALLATION INSTRUCTION

Authorisation and advice should be sought from your Departmental IT and Security teams before downloading, installing or using any software.

The GSS recommended method is X-13ARIMA-SEATS. This is maintained by the U.S. Census Bureau (USCB). Advice on the installation of the software can be found on the USCB website or sought from email.

25.2 ALTERNATIVE SEASONAL ADJUSTMENT METHODS

There are several different software packages that can be used for seasonal adjustment:

X-13ARIMA-SEATS

The GSS recommended method is X-13ARIMA-SEATS. This is maintained by the U.S. Census Bureau. As well as using WinX-13 and JDemetra+, there are seasonal adjustment functions in R, SAS and Python, which are seas, PROC X13, and statsmodels.tsa.x13, respectively.

TRAMO-SEATS

TRAMO-SEATS software is available from the Bank of Spain. A version of the TRAMO-SEATS method is also available in WinX13 and in JDemetra+, where the most recent version of the method is maintained.

BV4.1

This freeware is developed by the Federal Statistical Office of Germany for the seasonal adjustment of economic time series. It is available free from the DESTATIS website.

25.3 GRAPHICAL USER INTERFACES FOR X-13ARIMA-SEATS

25.3.1 *Jdemetra+*

JDemerta+ is open source software for seasonal adjustment that has been developed by the National Bank of Belgium in co-operation with Eurostat.

It is available from the European Commission website. It allows for large numbers of series to be seasonally adjusted simultaneously, and parameters can easily be changed by clicking on the GUI.

For detailed instructions, see the documentation or the user guide.

Read in data in Jdemetra+

Brief user instructions follow:
To use JDemetra+, data needs to be read in from a spreadsheet, and include the dates to read in, rather than using a **dat** file. The **spc** file and regressor files can only be written, edited and saved within the JDemetra software:

- Open JD+
- From the Providers window → Right click in Spreadsheets → Open
- Click the ... button and select the input file of interest (e.g., from Documents folder). The date format should be automatically identified, but the spreadsheet needs to include the date, for example 01/01/2023 or 01-Jan-2023
- Select OK

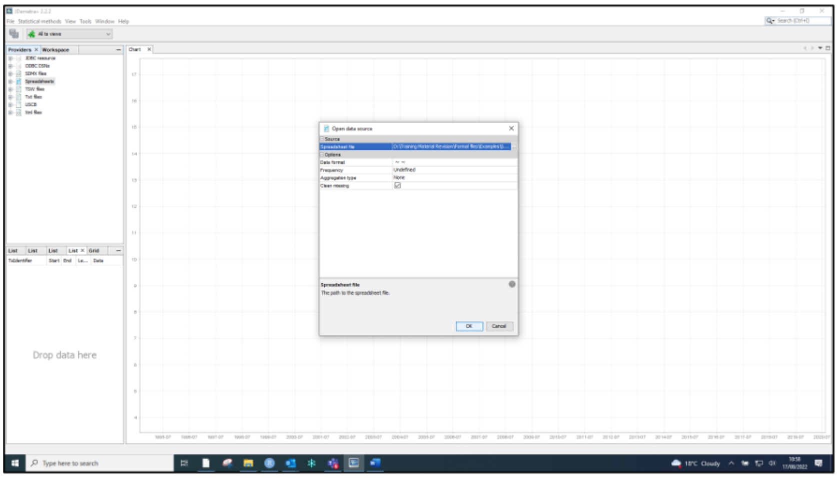

Figure 25.1: Loading data in JDemetra+

- Select the + button left from Spreadsheets in the Providers window to find the file you imported into JD+. Then right click in this file → Open with → Chart & grid

Seasonally adjust in Jdemetra+

To seasonally adjust a single series in JD+ (with a fixed spec)

- Select Statistical methods → Seasonal Adjustment → Single Analysis → X13

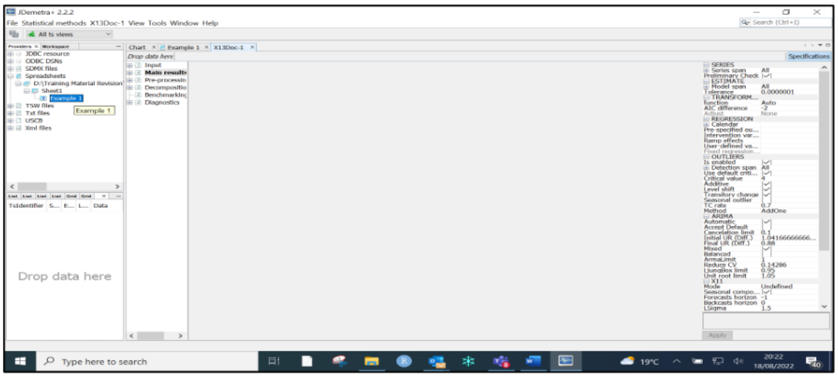

Figure 25.2: Performing seasonal adjustment using a single analysis in JDemetra+

- Drag and drop the imported file "Example 1" from Providers window into the grey area

To alter the specifications, equivalent of altering the spc file:

- From Specifications window (button to toggle this at the top right of the screen)
- Under TRANSFORMATION, change the function from Auto to None
- Under ARIMA, untick the Automatic ARIMA model and enter (manually) the parameters P, D, Q, BP, BD, and BQ as in Figure 25.3

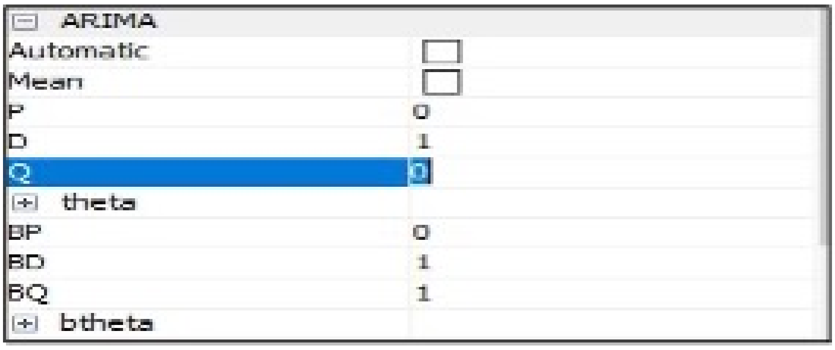

Figure 25.3: Changing ARIMA model under a single analysis in JDemetra+

- Under OUTLIERS, untick the Is Enabled box
- Under X11, unselect the Automatic Henderson filter, select S3X3 Seasonal filter, and enter 5 as Henderson filter
- Under REGRESSION, select the regressors, to adjust for Trading day and Easter effects

- To set AO, LS, TC and SO regressors, click next to the pre-specified outliers in the REGRESSION section. A window will open within this. Click in the boxes to add regressors
- The regressor diagnostics can be viewed under the pre-processing drop-down

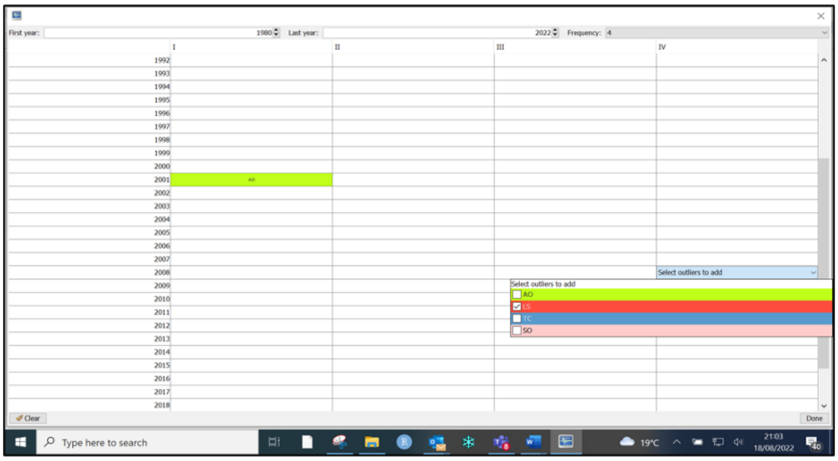

Figure 25.4: Adding AO and LS regressors under a single analysis in JDemetra+

25.3.2 *WinX13*

WinX13 is a Windows interface for running the X-13ARIMA-SEATS method. It allows for easy alterations of seasonal adjustment arguments, and allows for testing of different regressors and comparing diagnostics. Due to its lightweight nature, it is commonly used in TSAB operations. WinX13 is the preferred graphical user interface for X-13ARIMA-SEATS. See Chapter 4 for full instructions on initial use of WinX13. See also the USCB website for a fuller explanation of its usage.

25.3.3 *X13GraphJava*

This is also produced by the Census Bureau and is available from the USCB website. It does not run the X-13ARIMA-SEATS method itself, but uses the .gmt graphical files created by running the method to produce an extensive range of graphical output, which could further help with analysis.

25.4 *r* SEASON PACKAGE

For detailed instructions on seasonal adjustment using R, detailed instructions can be found at: http://www.seasonal.website/examples.html https://cran.r-project.org/web/packages/seasonal/vignettes/seas.pdf This requires the installation of the seasonal package, and other packages may be required for plotting outputs.

Read in data in R

In brief, you first need to read in the data, from either a csv file:

```
Data = read.csv (file=.. / input_data.csv)
```

Or a dat file with quarterly or monthly data in datevalue format:

```
Data = seasonal::import.ts(paste(directory, input_data.dat, sep=\\),
    format=datevalue, start = c(YYYY,Q), frequency = 4)
Data = seasonal::import.ts(paste(directory, input_data.dat, sep=\\),
    format=datevalue, start = c(YYYY,M), frequency = 12)
```

Where:

- Directory indicates the location of input file
- In c (YYYY, Q), YYYY indicates the year and Q indicates the quarter of the first data point ($1 \leqslant Q \leqslant 4$). For example, if the data start at first quarter of 2005, then start = c (2005, 1)
- In c (YYYY, M), M indicates the month of the first data point ($1 \leqslant M \leqslant 12$). For example, if the data start in July 2005, then start = c (2005, 7)

For monthly free input data, use one of the following two R scripts.

Option 1 Data = seasonal::import.ts(paste(directory, ”inputdata.dat”, sep=” ”), format=”free”, start=c(YYYY,M), frequency = 12)

Option 2 Data = read.csv(paste(directory, ”input$_d$ata.dat”, $sep = ”$ $”), header = FALSE)Data = ts(data, start = c(YYYY, M), frequency = 12)$

```
# Option 1
Data = seasonal::import.ts(paste(directory, input_data.dat, sep=\\),
    format=free, start=c(YYYY,M), frequency = 12)

# Option 2
Data = read.csv(paste(directory, input_data.dat, sep=\\),
    header=FALSE)
Data = ts(data, start = c(YYYY,M), frequency = 12)
```

R can span a subset of the series. For example, let the original time series data span from first quarter of 1995 to first quarter of 2020, but you are interested to analyse only the data from first quarter of 2001 to fourth quarter of 2009. To subset the time series data, use the following R script.

```
Data_interested = window(data, start = c(2001,1), end=c(2009,4))
```

Seasonally adjust in R

To perform seasonal adjustment in R, use the seas function. An example R script is illustrated below. Only the yellow-highlighted lines are necessary for this example, as other arguments will be used by default. The rest of the lines are presented for comparability reasons with Win-X13, but are not strictly necessary - more details can be seen in Table 25.1.

```
output <- seas(data_interested, transform.function = "none",
   arima.model = "(0 1 0)(0 1 1)",
   outlier = NULL,

# The reason for the outlier = NULL line above is to prevent the
    default automatic outlier detection
           regression.variables = c(AO2001.1, LS2008.4, Easter[1],
                td1coef ),
           regression.aictest = NULL,

# The reason for the regression.aictest = NULL line is to prevent the
    default AIC test for Easter and trading day effects
   estimate.maxiter = 5000,
   forecast.maxlead = 4,
   forecast.save = c("fct"),
   force.type = "denton",
   force.rho = 1,
   force.lambda = 1,
   force.round = "no",
   force.usefcst = "no",
   force.mode = "ratio",
   force.save = c("saa"),
   x11.seasonalma = "s3x3",
   x11.trendma = 5,
   x11.appendfcst = "yes",
   x11.save = c("d8","d9","d10","d11","d12","d13","d18")
)
```

To seasonally adjust in R without a pre-specified spec file:

X-13ARIMA-SEATS	R
estimate {maxiter = 5000}	estimate.maxiter = 5000

forecast{
maxlead = 4
}
	forecast.maxlead = 4,
	forecast.save = c("fct")

force{
type=denton
rho=1
lambda=1
round=no
usefcst=no
mode=ratio
save=saa
}
	force.type = "denton",
	force.rho = 1,
	force.lambda = 1,
	force.round = "no",
	force.usefcst = "no",
	force.mode = "ratio",
	force.save = c("saa")

x11{
appendfcst = “yes”
save =
c(“d8”,”d9”,”d10”,”d11”, ”d12”,”d13”,”d18”)
}
	x11.appendfcst = "yes",
	x11.save =
	c("d8","d9","d10","d11", "d12","d13","d18")

Table 25.1: List of specification arguments in X-13ARIMA-SEATS and R seasonal function equivalents

```
output <- seas(data_interested,transform.function = "auto",
          automdl =    ,
          regression.aictest =c (easter,  td ),
          outlier =    )
```

A large number of diagnostical statistics can be found with the udg function: https://www.rdocumentation.org/packages/seasonal/versions/1.9.0/topics/udg

25.4.1 *Plotting outputs in R*

To plot time series data in R, use:

- Any function among plot or ts.plot for standalone data visualization
- The package dygraphs for interactive data visualization

- It should be noted that many other options exist as well (e.g., ggplot2)

Example R scripts are displayed below:

```
plot(data, main=Title for the plot, ylab=Title for Y axis)
ts.plot(data, main=Title for the plot, ylab=Title for Y axis)
dygraphs::dygraph(data, main='Title for the plot', ylab = 'Title for
   Y axis') %>%
   dySeries("V1", label = "NSA") %>%
   dyOptions(colors = 'black')
```

Example time series plots are demonstrated in Figure 25.4 (output from plot/ts.plot function) and Figure 25.5 (output from dygraphs package).

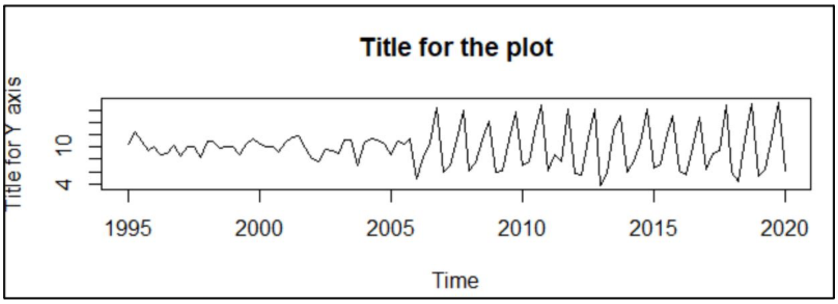

Figure 25.5: Graph of data produced using ts.plot

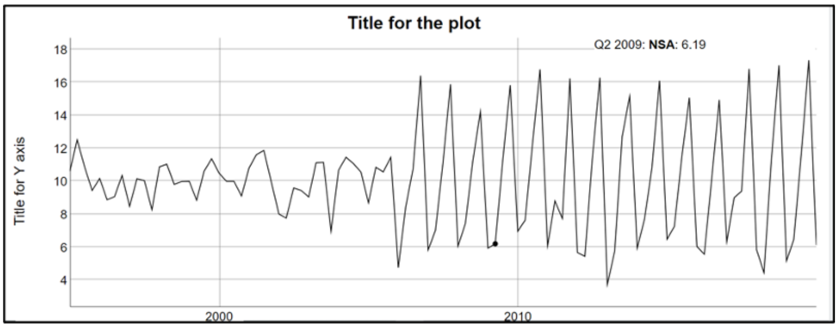

Figure 25.6: Graph of data produced using dygraphs

26

QUICK GUIDE TO SOLVING SEASONAL ADJUSTMENT PROBLEMS

This chapter aims to provide a quick guide for problems encountered while performing seasonal adjustment with the use of the X-13ARIMA-SEATS software. This may be useful as a quick reference for those new to seasonal adjustment.

What if the series is not long enough?

In order to seasonally adjust a series using X-13ARIMA-SEATS, at least 3 years of data are required. If the series is less than 3 years long, the seasonally adjusted series should not be published. However, it is recommended that a series be at least 5 years long before attempting seasonal adjustment. If a series is between 3 and 5 years long, you should consider not publishing it[1].

What if the series is very long?

If the series is longer than, say, 12 years, consider investigating seasonal breaks to account for evolving seasonality[2].

Should I use direct or indirect adjustment?

Indirect adjustment is only justifiable if it improves the seasonal adjustment of the aggregate or if there is a requirement for the component series to sum to the composite series[3]. You could also use the history spec to compare the results between direct adjustment and indirect adjustment of a particular series[4].

How do I check the quality of a seasonal adjustment?

To determine whether a seasonal adjustment is acceptable or not, the quality statistics produced in the output of X-13ARIMA-SEATS should be checked. If the Q statistic passes (less than 1) then, in most cases, the

1 see Chapter 4 for more information
2 see Chapter 4 for more information
3 see Chapter 6 for more information
4 see Chapter 19 for more information

seasonal adjustment will be acceptable. It is not necessary for all the M-statistics to pass for the seasonal adjustment to be acceptable, although it is ideal.

Other ways to test the suitability of an adjustment are to look at the seasonally adjusted series graphically and compare it to the non seasonally adjusted series. If the non seasonally adjusted looks smoother than the seasonally adjusted, then the adjustment is probably not suitable. Also, if the seasonally adjusted series appears to have a seasonal pattern, you need to check for residual seasonality[5].

How do I check for residual seasonality?

After running X-13ARIMA-SEATS with the seasonal adjustment parameters specified, the output produces a test for the presence of residual seasonality (found under table D11 - Final seasonally adjusted series) which will tell you if there is evidence of residual seasonality in whole series or just in the last three years of the series. You could also look at the diagnostics window in Win X-13 . In the Spectrum & QS tab, the Seasonal Peaks column tells you if there is residual seasonality present and in which series. If there is residual seasonality present, the current seasonal adjustment parameters are not suitable and need to be re-evaluated.

What should I do if the Q statistics fails (grater than 1)?

The Q statistic is just an average of the M statistics. Therefore, you need to look at the M statistics and consider which of them fail and how/if you should try to improve it.

What do the M statistics mean?

Table F3 of the X-13ARIMA-SEATS output shows the quality statistics and a number of different M statistics, each of which relate to a different aspect of the adjustment. These statistics each take a value between 0 and 3 with a value higher than 1 suggesting a potential problem. Table 26.1 is a rough guide of what each one means and options for how to proceed in the event of a failure in one of the statistics.

What if: "Identifiable seasonality not present"

The combined test for the presence of seasonality is an indicator of seasonality but there are other checks that can be done before considering a series non-seasonal. Something else to consider is the M7 statistic. If M7 is above

5 see Chapter 26 for more information)

Diagnostic:	What it does:	How to fix it in the event of a failure:
M1	Shows how large the irregular component is compared to the seasonal component	May need to consider suitability of prior adjustments (outliers, level shifts etc) or the need for such adjustments to be added
M2	Measures the contribution of the irregular component to the variance of the raw series (transformed to a stationary series) May need to consider suitability of prior adjustments (outliers, level shifts etc) or the need for such adjustments to be added	
M3	Measures the amount of period-to-period change in the irregular compared to that in the trend.	May need to consider suitability of prior adjustments (outliers, level shifts etc) or the need for such adjustments to be added
M4	A measure of autocorrelation in the irregular component.	Consider different moving averages
M5	Measures the irregular compared to the trend	May need to consider suitability of prior adjustments (outliers, level shifts etc) or the need for such adjustments to be added
M6	Also measures the irregular but is only valid when a 3x5 seasonal filter is used.	A seasonal filter shorter than 3x5 should be used
M7	Shows the amount of moving compared to stable seasonality (basically how regular the seasonal pattern is)	Suggests the series is not seasonal, or that seasonality cannot be identified.
M8	The size of the fluctuations in the seasonal component throughout the whole series	May indicate the presence of a seasonal break or the need for a change of moving average
M9	The average linear movement in the seasonal component throughout the whole series	May indicate the presence of a seasonal break or the need for a change of moving average
M10	The size of the fluctuations in the seasonal component for recent years only	May need to consider a change to the ARIMA model or consider the presence of a seasonal break within recent years
M11	The average linear movement in the seasonal component for recent years only	May need to consider a change to the ARIMA model or consider the presence of a seasonal break within recent years

Table 26.1: M-statistic meanings and suggestions

1.250 for monthly series or 1.050 for quarterly series, then this suggests that the series may not be seasonal. See Chapter 16 for more information.

Another way of checking if a series is seasonal is to use the sliding span diagnostic. This may indicate if the series is becoming non-seasonal/seasonal. See Chapter 18 for more information. It is also possible to compare the standard deviations given in the E5 and E6 outputs. E5 shows the month-to-month percent change in the original series and E6 shows the month-to-month percent change in the seasonally adjusted series. Both of which give the standard deviation of the respective series under the tables provided.

For a strongly seasonal series, it would be expected that the standard deviation in the month-to-month percent change would be much higher in the original series than the seasonally adjusted series. Therefore, similar standard deviations may suggest a series is not seasonal.

Other measures that could be used would be the combined test and M7 result in the sliding spans analysis. This may indicate emerging or decaying seasonality. Additionally, it might be worth looking at the graphs of the series and the SI ratios to check for seasonality.

All the above methods should be considered as a whole, as they sometimes contradict one another.

Which prior adjustment are permanent and which are temporary?

There are two types of prior adjustments; below are examples of both types of prior adjustments:

Temporary	**Permanent**
Level Shifts	Easter effects
Additive Outliers	Trading day effects
Ramps	Seasonal breaks

Table 26.2: Examples of temporary and permanent prior adjustments

What does it mean if Ljung-Box or Box-Pierce Q statistics fails?

The Ljung-Box and Box-Pierce statistics test if there is any evidence of autocorrelation in the residuals of the ARIMA model selected. In the Model Diagnostics tab, the #LBQ fail column tells you the number of lags from 1 to 24 where the Ljung-Box Q statistic indicates autocorrelation and the #BPQ fail column tells you the same for the Box-pierce Q statistic fails. Next look at the corresponding sig LBQ/sig BPQ column which will specify at which lags the autocorrelation occurs. Autocorrelation at lags 2, 4, 8 etc (for quarterly series) and 3, 4, 6, 12, 24 (for monthly series) suggests seasonality which is a problem. In this case, a change to the ARIMA model should be considered.

For example: if you are currently using the ARIMA model (011)(011), a change to (011)(012) may be considered more suitable and could be tested. For more information or advice, contact the Time Series Analysis Branch.

What does it mean if there are significant seasonal or trading day peaks in the spectrum of the seasonally adjusted or irregular series?

Seasonal peaks present in the seasonally adjusted series suggest that not all the seasonality has been removed during the seasonal adjustment process. In this case, see Section 27.5 above. You then need to alter the parameters selected for seasonal adjustment in order to remove the residual seasonality.

If there are significant trading day peaks in the spectrum of the seasonally adjusted series, this means that trading day effects have not been completely removed during the seasonal adjustment process. In the case that trading day peaks are present, first look at the diagnostics window in Win X-13. In the Spectrum & QS tab, the Nonsig TD Peaks column tells you if there are trading day peaks present and if so, in which series they are present. You can then look at the corresponding column to find out which type of peak is present; a t1 peak or a t2 peak. The occurrence of t1 peaks is more serious whereas t2 peaks are less important, therefore, in the event of a t1 peak being present, the following things could be considered. First you may want to consider removing any trading day regressors that have been added to the seasonal adjustment or, if no such regressors exist, try adding one to the adjustment parameters (see Chapter 9 for more information on the different types of trading day regressors). If neither of these options remove the t1 peaks, you could consider altering the seasonal filter[6].

What does it mean if there are significant seasonal or trading day peaks in the spectrum of the model residual series?

Significant trading day peaks in the spectrum of the model residual series means that trading day effects have not been accurately estimated and removed by the regARIMA model. If trading day peaks are present, as explained above, first look at the diagnostics window in Win X-13. Consider removing any trading day regressors that have been added to the regression spec or, if no such regressors exist, try adding one to the adjustment parameters[7].

If significant seasonal peaks are present in the spectrum of the model residual series, this may mean that seasonality has not been correctly estimated and removed by the regARIMA model. In this case, a change to the ARIMA model should be considered.

For example, if you are currently using the ARIMA model (011)(011), a change to (011)(012) or (011)(111) may be considered more suitable and could be tested. For more information or advice, contact the Time Series Analysis Branch.

6 see Section 13.4.1 for more information on the different types on seasonal filters
7 see Chapter 9 for more information on the different types of trading day regressors

What do I do if there are significant autocorrelation/partial autocorrelation peaks in the model residuals?

Autocorrelation shows how much a particular frequency contributes to a time series. If there is a significant peak in the Autocorrelation Function (ACF) or Partial Autocorrelation Function (PACF), this may indicate that the parameters used (such as trading day effects) should be checked for suitability. This can be checked by looking at the model diagnostics tab in the diagnostics window in Win X-13. The sig ACF and sig PACF columns will indicate at which lags the autocorrelation or partial autocorrelation occurs. Significant ACF/PACF peaks at lags 2, 4, 8 etc (for quarterly series) and 3, 4, 6, 12, 24 (for monthly series) are extremely important and peaks at these lags could indicate a problem with seasonality in the residuals. In this case, a change to the ARIMA model could be considered.

For example: if you are currently using the ARIMA model (011)(011) and there are seasonal ACF peaks, a change to (011)(012) may be considered more suitable and could be tested. If there are seasonal PACF peaks, a change to (011)(111) may be more suitable. For more information or advice, contact the Time Series Analysis Branch.

Why is my previously working seasonal adjustment now failing?

There are a few common reasons why a previously working seasonal adjustment spec file would stop working. Often, adding values which produce seasonally adjusted values at or below zero would cause the series to fail if the series in log transformed. A quick fix is to use transform-function=log constant=0.1, however, the revisions could be large. A more thorough fix would be to apply a ppp seasonal break before the start of any negative seasonal adjustment[8]. If a series is using a seasonal break, then the ppp or rmx files will be time limited. Once new data exceed this time limit, then the series will stop working. The ppp and rmx files will have to be extended beyond the latest data point. Often, these can be extended for years at a time, but will need review at a future date, to ensure they are correctly applied.

Why are my revisions so large?

Revisions to seasonally adjusted or forecast data can be caused by revisions to existing NSA data, or new unseasonal NSA data. Use of regressors such as Additive outliers can prevent the effect of un-seasonal NSA data from spreading to other periods, because the moving averages will be unaffected.

8 see Chapter 15

How do I adjust for COVID-19 effects?

COVID-19 has resulted in many non-seasonal outliers. In general, TSAB has had to make many interventions to prevent unseasonal changes to the data and large revisions to SA data in 2019. Copious use of level shifts in 2020 Q2, and additive outliers in 2020 mean that highly unseasonal effects of COVID-19 does not affect the moving averages and spread to other years. A selection of level shifts and ramps may be required through 2021 as series return to levels and seasonality similar to 2019. For further information, contact the Time Series Analysis Branch.

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