

Statistical Modelling I

HES 505 Fall 2023: Session 22

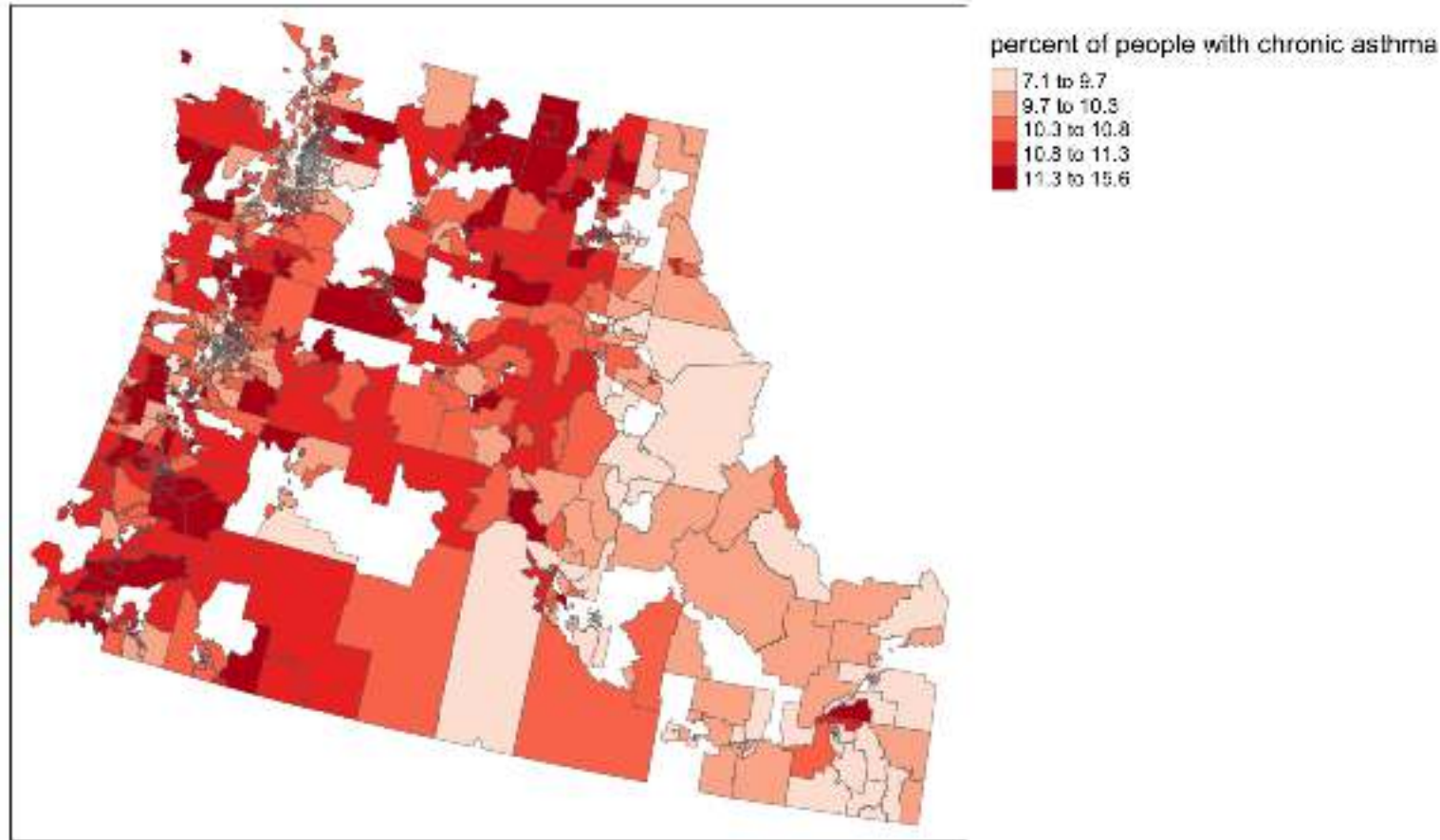
Matt Williamson

Objectives

By the end of today you should be able to:

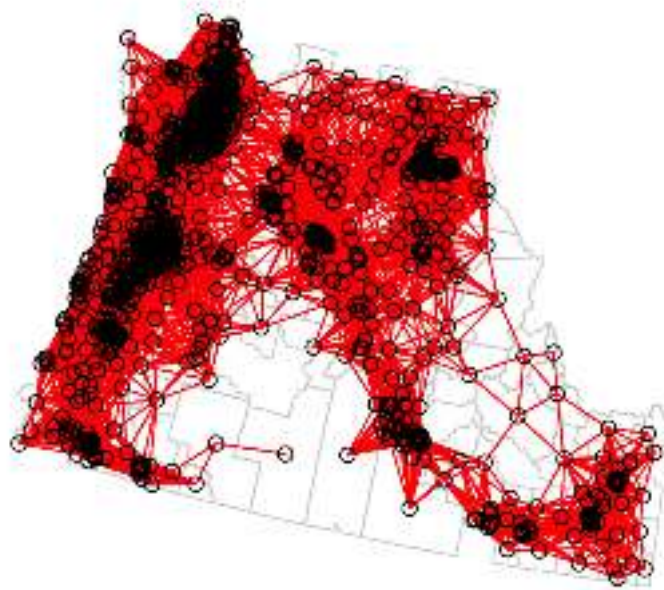
- Identify nearest neighbors based on distance
- Describe and implement overlay analyses
- Extend overlay analysis to statistical modeling
- Generate spatial predictions from statistical models

Revisiting neighbors and areal data



```
1 cdc.pt <- cdc %>% st_point_on_surface(.)
2 geog.earnb <- knn2nb(knearneigh(cdc.pt, k = 1), row.names = cdc.pt$GEOID,
3 nb.nearest <- dnearneigh(cdc.pt, 0, max(unlist(nbdists(geog.earnb, cdc.p
```

Getting Weights



```
1 lw.nearest <- nb2listw(nb.nearest, style="W")  
2 asthma.lag <- lag.listw(lw.nearest, cdc$casthma_cr)
```

Fit a model

- Moran's I coefficient is the slope of the regression of the *lagged* asthma percentage vs. the asthma percentage in the tract
- More generally it is the slope of the lagged average to the measurement

```
1 M <- lm(asthma.lag ~ cdc$casthma_cr)
```

```
cdc$casthma_cr  
0.1670774
```

Comparing observed to expected

- We can generate the expected distribution of Moran's I coefficients under a Null hypothesis of no spatial autocorrelation
- Using permutation and a loop to generate simulations of Moran's I

```
1  n <- 400L    # Define the number of simulations
2  I.r <- vector(length=n) # Create an empty vector
3
4  for (i in 1:n){
5    # Randomly shuffle income values
6    x <- sample(cdc$casthma_cr, replace=FALSE)
7    # Compute new set of lagged values
8    x.lag <- lag.listw(lw.nearest, x)
9    # Compute the regression slope and store its value
10   M.r    <- lm(x.lag ~ x)
```

```
11     I.r[i] <- coef(M.r)[2]  
12 }
```

Significance testing

- Pseudo p-value (based on permutations)
- Analytically (sensitive to deviations from assumptions)
- Using Monte Carlo

```
1 #Pseudo p-value
2 N.greater <- sum(coef(M)[2] > I.r)
3 (p <- min(N.greater + 1, n + 1 - N.greater) / (n + 1))
4
5 # Analytically
6 moran.test(cdc$casthma_cr, lw.nearest, zero.policy = TRUE)
7
8 # Monte Carlo
9 moran.mc(cdc$casthma_cr, lw.nearest, zero.policy = TRUE, nsim=400)
```


Significance testing

```
[1] 0.002493766
```

```
Moran I test under randomisation
```

```
data:  cdc$casthma_cr  
weights: lw.nearest
```

```
Moran I statistic standard deviate = 61.661, p-value < 2.2e-16
```

```
alternative hypothesis: greater
```

```
sample estimates:
```

Moran I statistic	Expectation	Variance
1.670774e-01	-4.990020e-04	7.385950e-06

```
Monte-Carlo simulation of Moran I
```

```
data:  cdc$casthma_cr  
weights: lw.nearest  
number of simulations + 1: 401
```

```
statistic = 0.16708, observed rank = 401, p-value = 0.002494
```

```
alternative hypothesis: greater
```

Overlay Analyses

Overlays

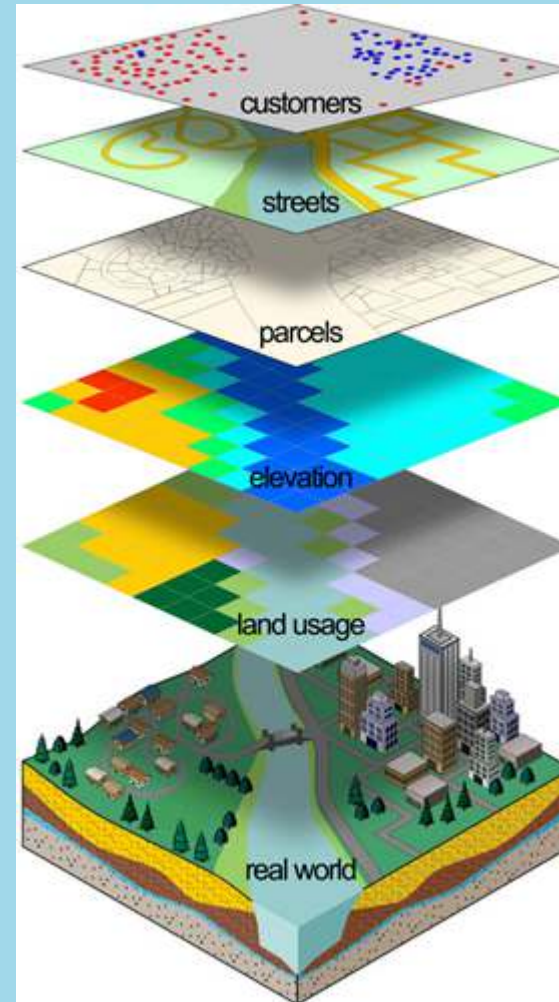
- Methods for identifying optimal site selection or suitability
- Apply a common scale to diverse or dissimilar outputs

Getting Started

1. Define the problem.
2. Break the problem into submodels.
3. Determine significant layers.
4. Reclassify or transform the data within a layer.
5. Add or combine the layers.
6. Verify

Boolean Overlays

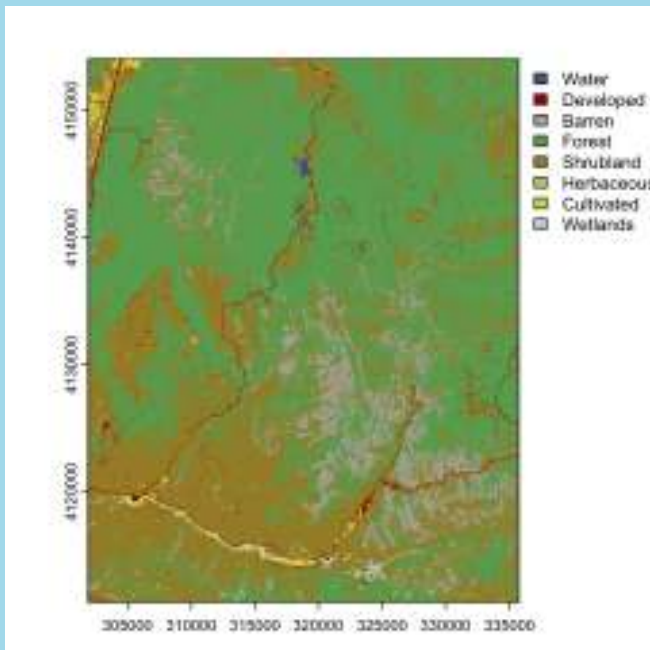
- Successive disqualification of areas
- Series of “yes / no” questions
- “Sieve” mapping



Boolean Overlays

- Reclassifying
- Which types of land are appropriate

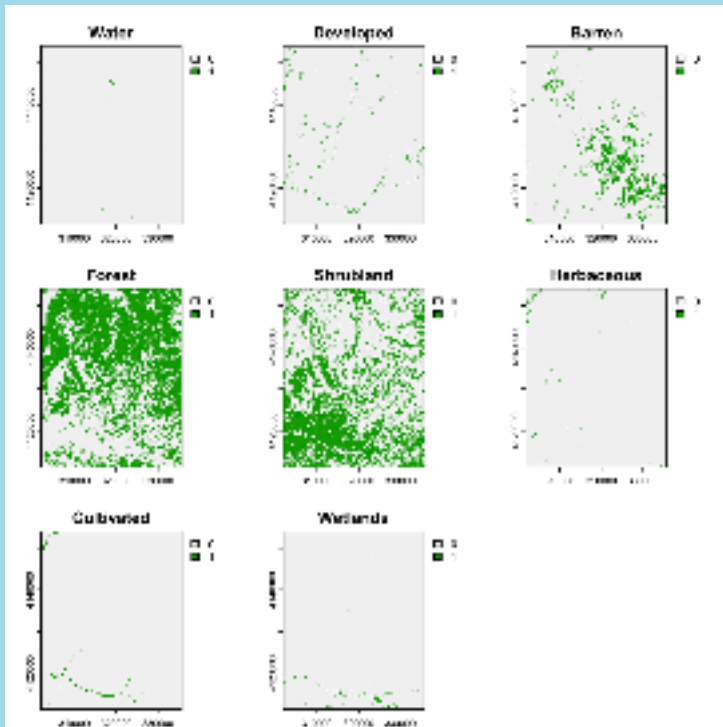
```
1 nlcd <- rast(system.file("raster/nlcd.tif", package = "spDataLarge"))  
2 plot(nlcd)
```



Boolean Overlays

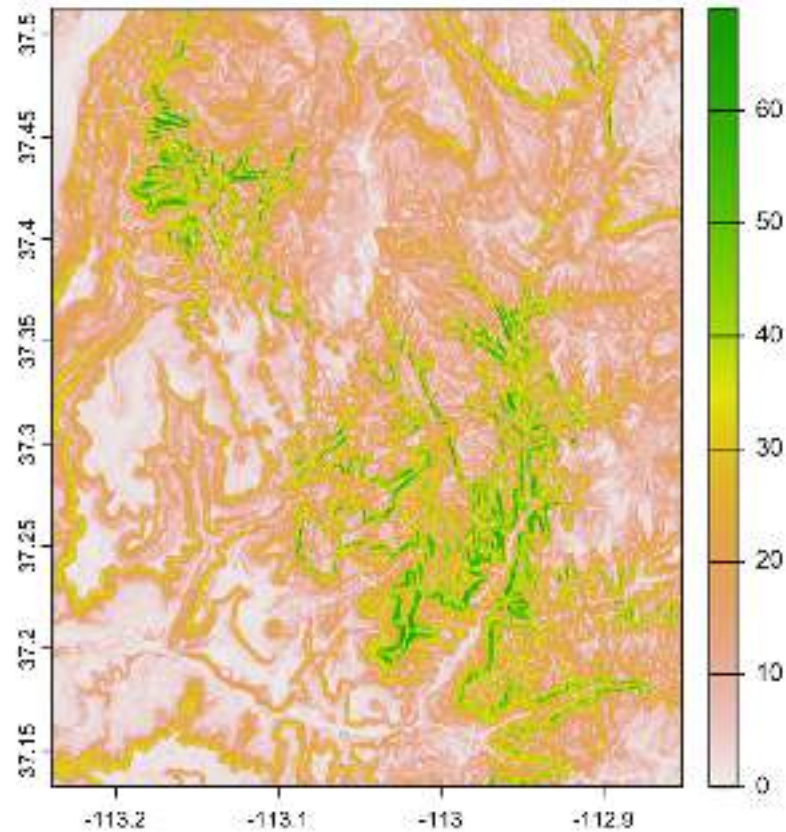
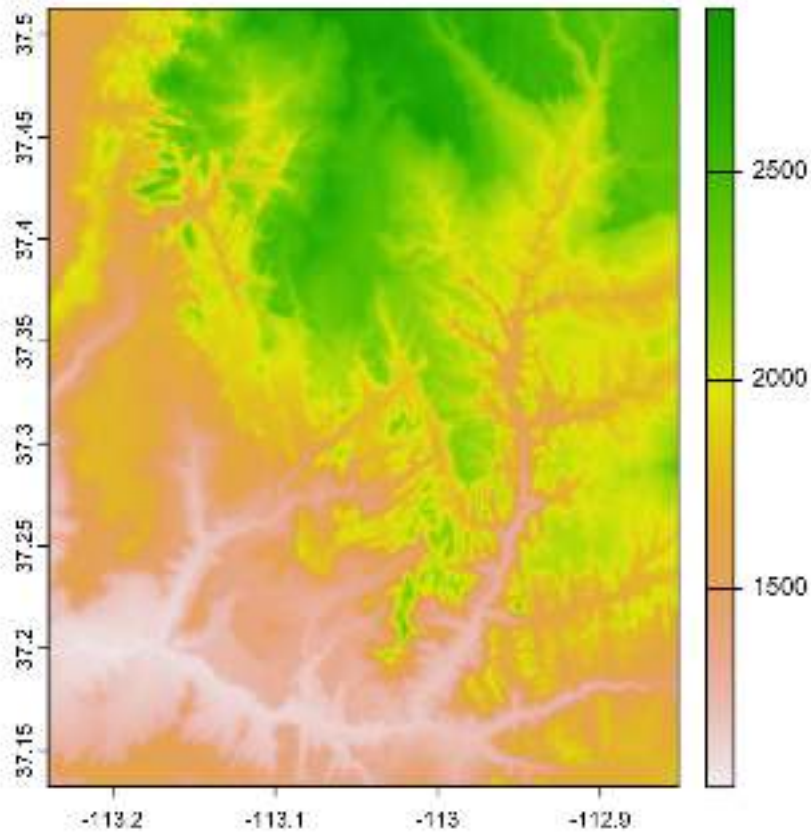
- Which types of land are appropriate?

```
1 nlcd.segments <- segregate(nlcd)
2 names(nlcd.segments) <- levels(nlcd)[[1]][-1,2]
3 plot(nlcd.segments)
```



Boolean Overlays

- Which types of land are appropriate?

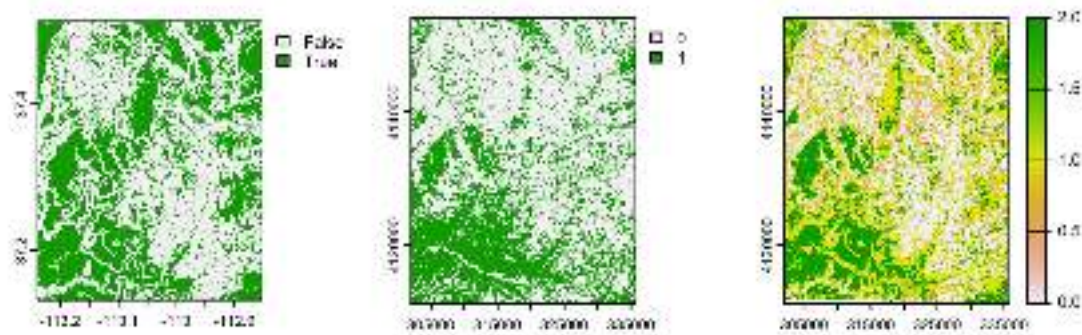


Boolean Overlays

- Make sure data is aligned!

```
1 suit.slope <- slope < 10
2 suit.landcov <- nlcd.segments["Shrubland"]
3 suit.slope.match <- project(suit.slope, suit.landcov)
4 suit <- suit.slope.match + suit.landcov
```

Boolean Overlays



Challenges with Boolean Overlays

1. Assume relationships are really Boolean
2. No measurement error
3. Categorical measurements are known exactly
4. Boundaries are well-represented

A more general approach

- Define a *favorability* metric

$$F(\mathbf{s}) = \prod_{M=1}^m X_m(\mathbf{s})$$

- Treat $F(\mathbf{s})$ as binary
- Then $F(\mathbf{s}) = 1$ if all inputs ($X_m(\mathbf{s})$) are suitable
- Then $F(\mathbf{s}) = 0$ if not

Estimating favorability

$$F(\mathbf{s}) = f(w_1 X_1(\mathbf{s}), w_2 X_2(\mathbf{s}), w_3 X_3(\mathbf{s}), \dots, w_m X_m(\mathbf{s}))$$

- $F(\mathbf{s})$ does not have to be binary (could be ordinal or continuous)
- $X_m(\mathbf{s})$ could also be extended beyond simply 'suitable / not suitable'
- Adding weights allows incorporation of relative importance
- Other functions for combining inputs ($X_m(\mathbf{s})$)

Weighted Linear Combinations

$$F(\mathbf{s}) = \frac{\sum_{i=1}^m w_i X_i(\mathbf{s})}{\sum_{i=1}^m w_i}$$

- $F(\mathbf{s})$ is now an index based on the values of $X_m(\mathbf{s})$
- w_i can incorporate weights of evidence, uncertainty, or different participant preferences
- Dividing by $\sum_{i=1}^m w_i$ normalizes by the sum of weights

Model-driven overlay

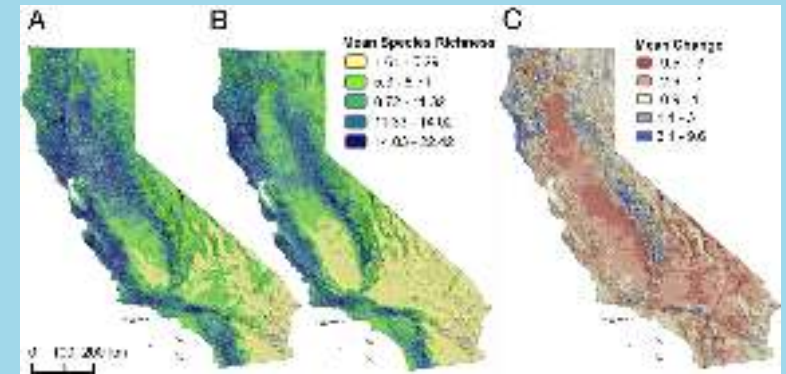
$$F(\mathbf{s}) = w_0 + \sum_{i=1}^m w_i X_i(\mathbf{s}) + \epsilon$$

- If we estimate w_i using data, we specify $F(\mathbf{s})$ as the outcome of regression
- When $F(\mathbf{s})$ is binary \rightarrow logistic regression
- When $F(\mathbf{s})$ is continuous \rightarrow linear (gamma) regression
- When $F(\mathbf{s})$ is discrete \rightarrow Poisson regression
- Assumptions about ϵ matter!!

Logistic Regression and Distribution Models

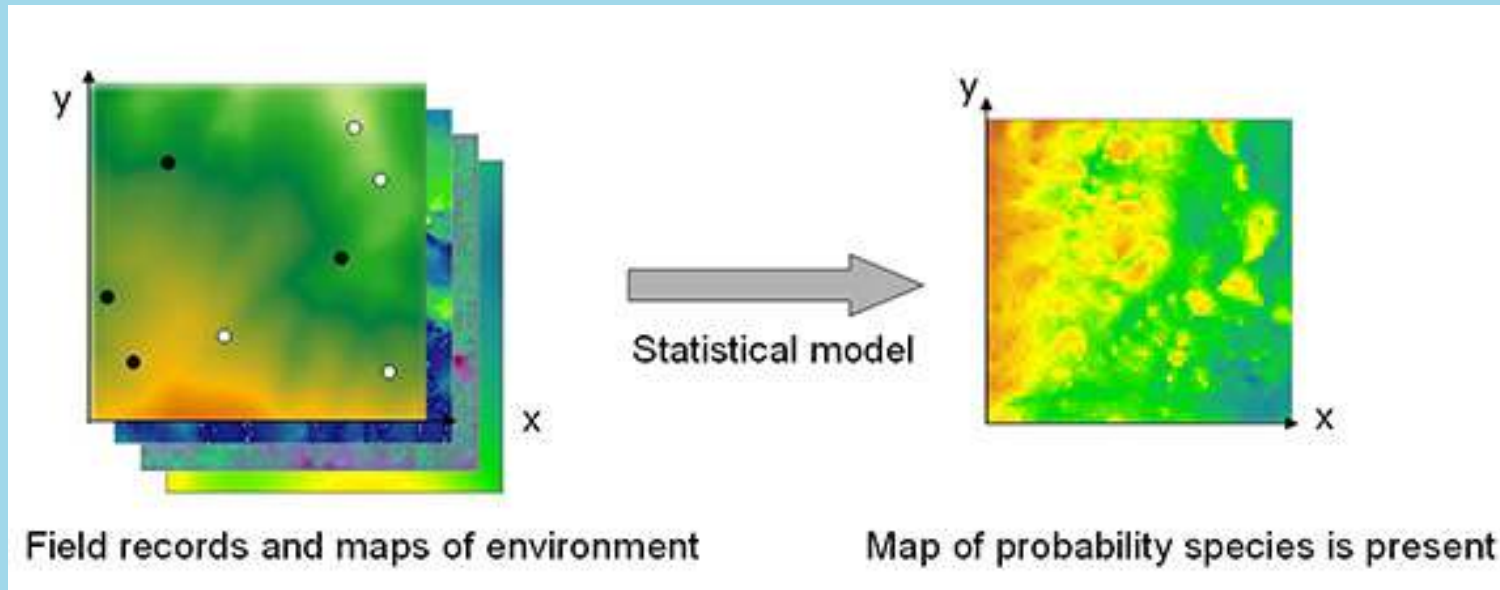
Why do we create distribution models?

- To identify important correlations between predictors and the occurrence of an event
- Generate maps of the 'range' or 'niche' of events
- Understand spatial patterns of event co-occurrence
- Forecast changes in event distributions



From Wiens et al. 2009

General analysis situation



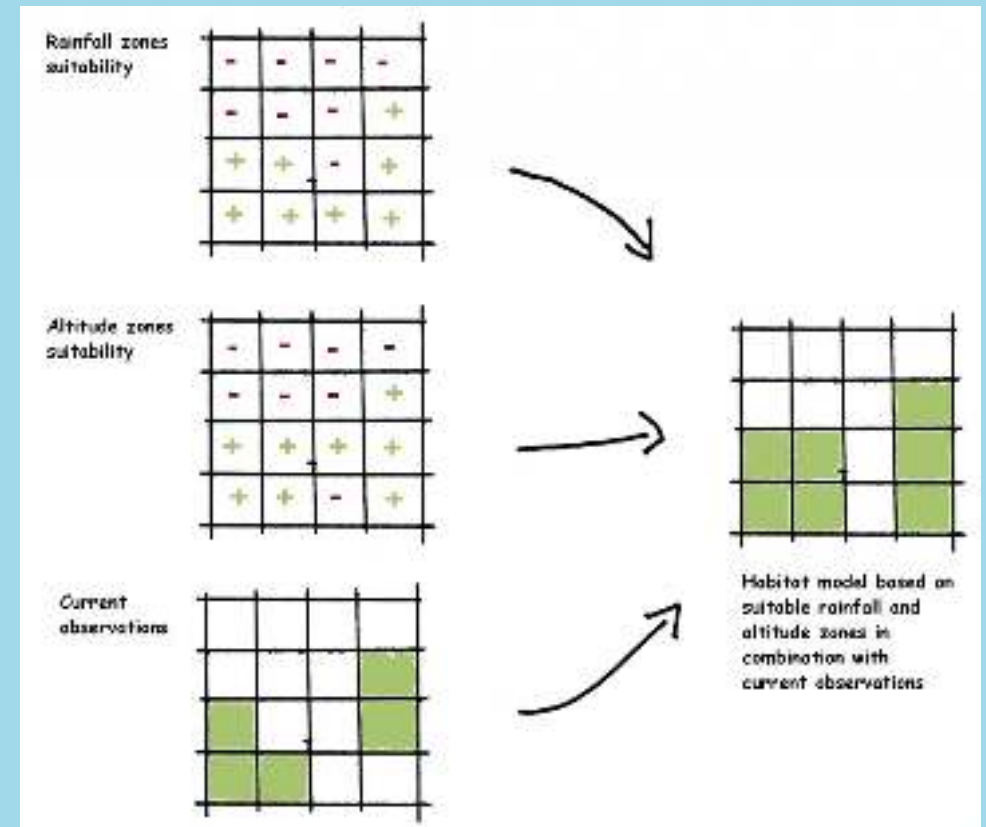
From Long

- Spatially referenced locations of events (\mathbf{y}) sampled from the study extent
- A matrix of predictors (\mathbf{X}) that can be assigned to each event based on spatial location

Goal: Estimate the probability of occurrence of events across unsampled regions of the study area based on correlations with predictors

Modeling Presence-Absence Data

- Random or systematic sample of the study region
- The presence (or absence) of the event is recorded for each point
- Hypothesized predictors of occurrence are measured (or extracted) at each point



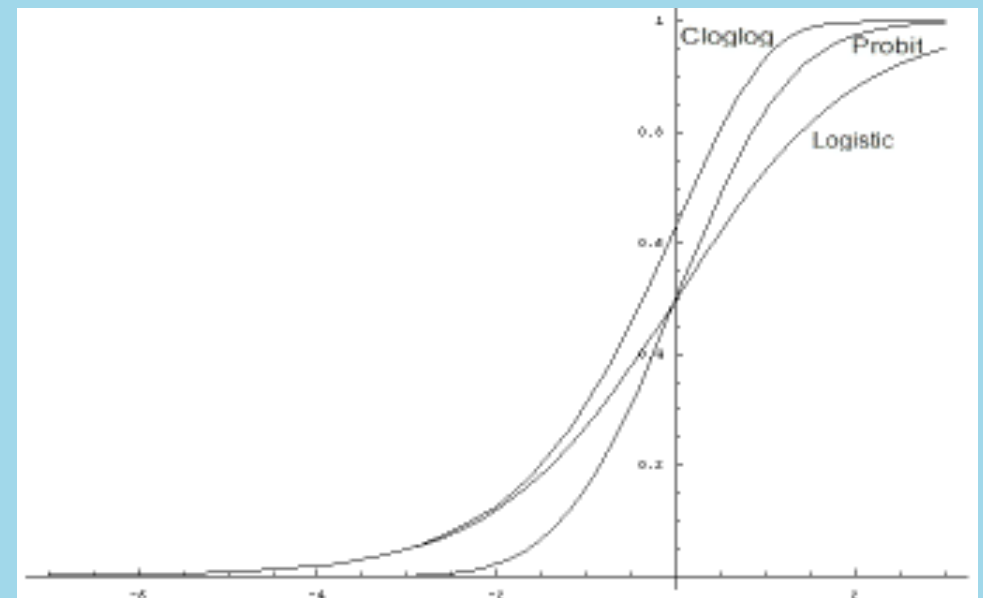
From By Ragnvald - Own work, CC BY-SA 3.0

Logistic regression

- We can model favorability as the **probability** of occurrence using a logistic regression
- A *link* function maps the linear predictor ($\mathbf{x}_i' \beta + \alpha$) onto the support (0-1) for probabilities
- Estimates of β can then be used to generate 'wall-to-wall' spatial predictions

$$y_i \sim \text{Bern}(p_i)$$

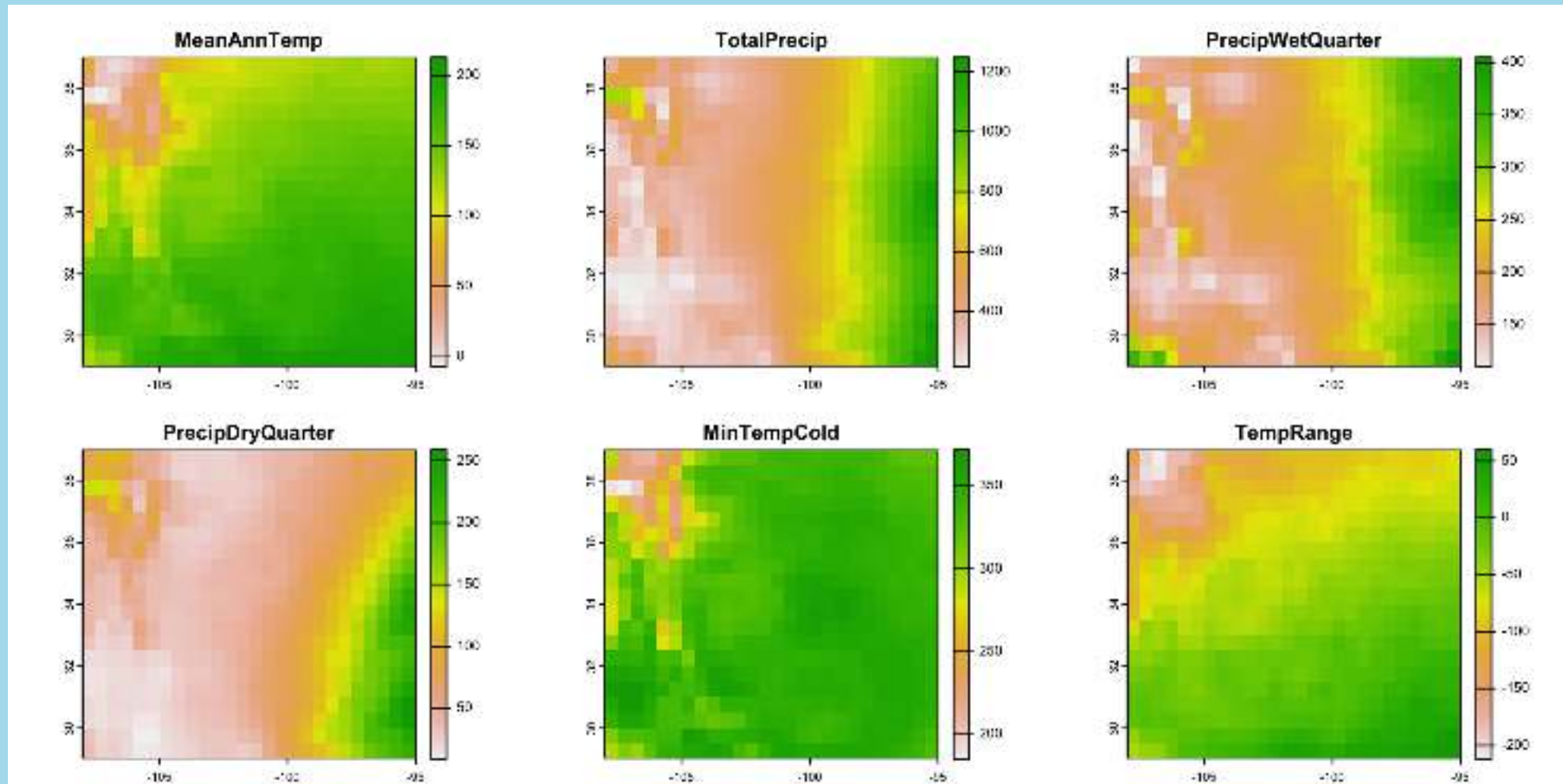
$$\text{link}(p_i) = \mathbf{x}_i' \beta + \alpha$$



From Mendoza

An Example

Inputs from the **dismo** package



An Example

The sample data

```
1 head(pres.abs)
```

Simple feature collection
with 6 features and 1 field

Geometry type: POINT

Dimension: XY

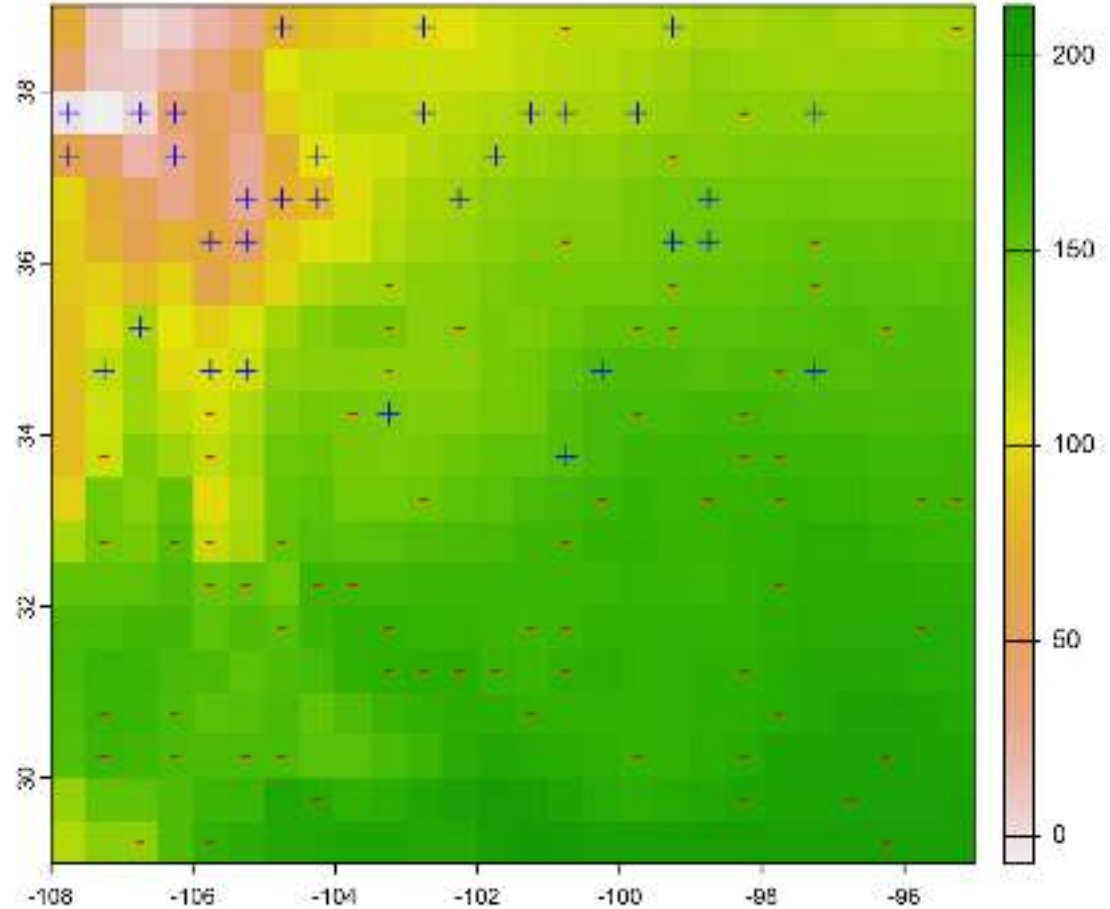
Bounding box: xmin: -106.75

ymin: 31.25 xmax: -98.75

ymax: 37.75

Geodetic CRS: GCS_unknown

	y	geometry
1	0	POINT (-99.25 35.25)
2	1	POINT (-98.75 36.25)
3	1	POINT (-106.75 35.25)
4	0	POINT (-100.75 31.25)
5	1	POINT (-99.75 37.75)
6	1	POINT (-104.25 36.75)



An Example

Building our dataframe

```
1 pts.df <- terra::extract(pred.stack, vect(pres.abs), df=TRUE)
2 head(pts.df)
```

	ID	MeanAnnTemp	TotalPrecip	PrecipWetQuarter	PrecipDryQuarter	MinTempCold
1	1	155	667	253	71	350
2	2	147	678	266	66	351
3	3	123	261	117	40	329
4	4	181	533	198	69	348
5	5	127	589	257	48	338
6	6	83	438	213	38	278

	TempRange
1	-45
2	-58
3	-64
4	-5
5	-81
6	-107

An Example

Building our dataframe

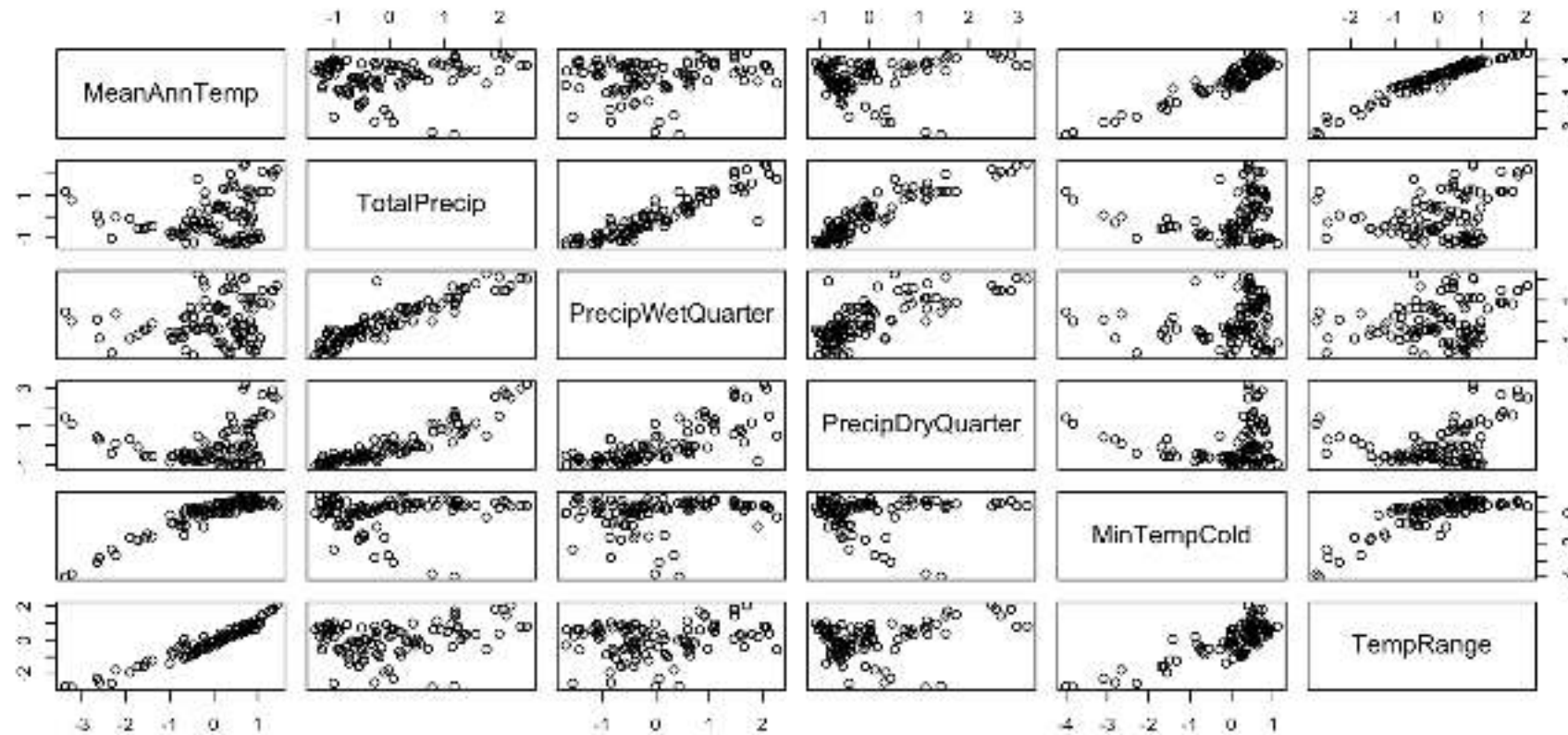
```
1 pts.df[,2:7] <- scale(pts.df[,2:7])
2 summary(pts.df)
```

ID	MeanAnnTemp	TotalPrecip	PrecipWetQuarter
Min. : 1.00	Min. : -3.3729	Min. : -1.3377	Min. : -1.6926
1st Qu.: 25.75	1st Qu.: -0.4594	1st Qu.: -0.7980	1st Qu.: -0.6895
Median : 50.50	Median : 0.2282	Median : -0.2373	Median : -0.2224
Mean : 50.50	Mean : 0.0000	Mean : 0.0000	Mean : 0.0000
3rd Qu.: 75.25	3rd Qu.: 0.7118	3rd Qu.: 0.7140	3rd Qu.: 0.6508
Max. : 100.00	Max. : 1.4285	Max. : 2.4843	Max. : 2.2713
PrecipDryQuarter	MinTempCold	TempRange	
Min. : -1.0828	Min. : -3.9919	Min. : -2.7924	
1st Qu.: -0.7013	1st Qu.: -0.0598	1st Qu.: -0.5216	
Median : -0.3770	Median : 0.3582	Median : 0.2075	
Mean : 0.0000	Mean : 0.0000	Mean : 0.0000	
3rd Qu.: 0.4290	3rd Qu.: 0.5495	3rd Qu.: 0.6450	
Max. : 3.1713	Max. : 1.1092	Max. : 2.0407	

An Example

Looking at correlations

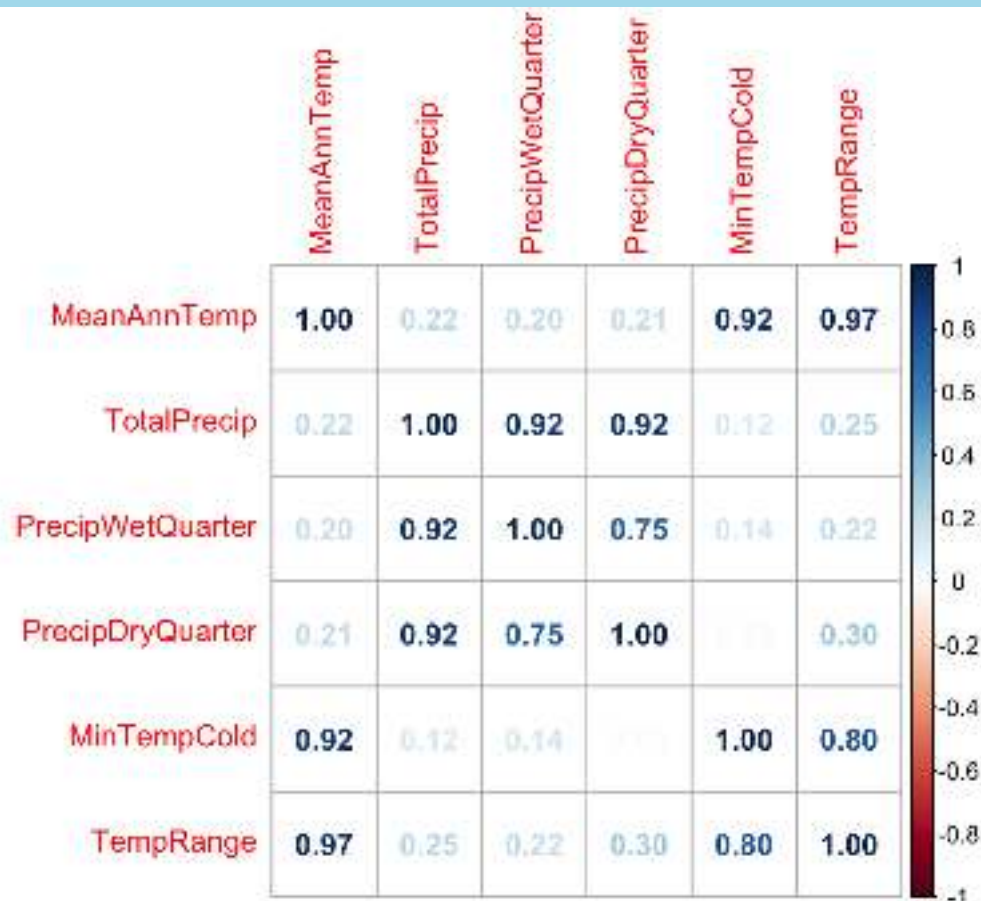
```
1 pairs(pts.df[,2:7])
```



An Example

Looking at correlations

```
1 corrplot(cor(pts.df[,2:7]), method = "number")
```



An Example

Fitting some models

```
1 pts.df <- cbind(pts.df, pres.abs$y)
2 colnames(pts.df)[8] <- "y"
3 logistic.global <- glm(y~., family=binomial(link="logit"), data=pts.df[,2:8])
4 logistic.simple <- glm(y ~ MeanAnnTemp + TotalPrecip, family=binomial(link="logit"), data=pts.df[,2:8])
5 logistic.rich <- glm(y ~ MeanAnnTemp + PrecipWetQuarter + PrecipDryQuarter, data=pts.df[,2:8])
```

An Example

Checking out the results

```
1 summary(logistic.global)
```

Call:

```
glm(formula = y ~ ., family = binomial(link = "logit"), data = pts.df[,  
  2:8])
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	-1.4461	0.5096	-2.837	0.00455	**
MeanAnnTemp	-6.3578	6.1645	-1.031	0.30237	
TotalPrecip	7.1453	4.5577	1.568	0.11694	
PrecipWetQuarter	-5.4207	3.0432	-1.781	0.07487	.
PrecipDryQuarter	-1.3110	2.2482	-0.583	0.55981	
MinTempCold	3.0890	2.6334	1.173	0.24080	
TempRange	-0.6213	4.5470	-0.137	0.89131	

```
chi.sq = 1.94, Df = 1, p-value = 0.061, likelihood ratio = 2.01, Wald = 2.05, Akaike = 1.94, AIC = 1.94
```

An Example

Checking out the results

```
1 summary(logistic.simple)
```

Call:

```
glm(formula = y ~ MeanAnnTemp + TotalPrecip, family = binomial(link =  
"logit"),  
     data = pts.df[, 2:8])
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	-0.9880	0.3145	-3.141	0.00168	**
MeanAnnTemp	-2.9990	0.6647	-4.512	6.42e-06	***
TotalPrecip	0.3924	0.3827	1.025	0.30517	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

An Example

Checking out the results

```
1 summary(logistic.rich)
```

Call:

```
glm(formula = y ~ MeanAnnTemp + PrecipWetQuarter + PrecipDryQuarter,  
     family = binomial(link = "logit"), data = pts.df[, 2:8])
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	-0.96504	0.35650	-2.707	0.00679	**
MeanAnnTemp	-2.85446	0.66142	-4.316	1.59e-05	***
PrecipWetQuarter	0.03212	0.43102	0.075	0.94060	
PrecipDryQuarter	0.16759	0.64935	0.258	0.79634	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

An Example

Comparing models

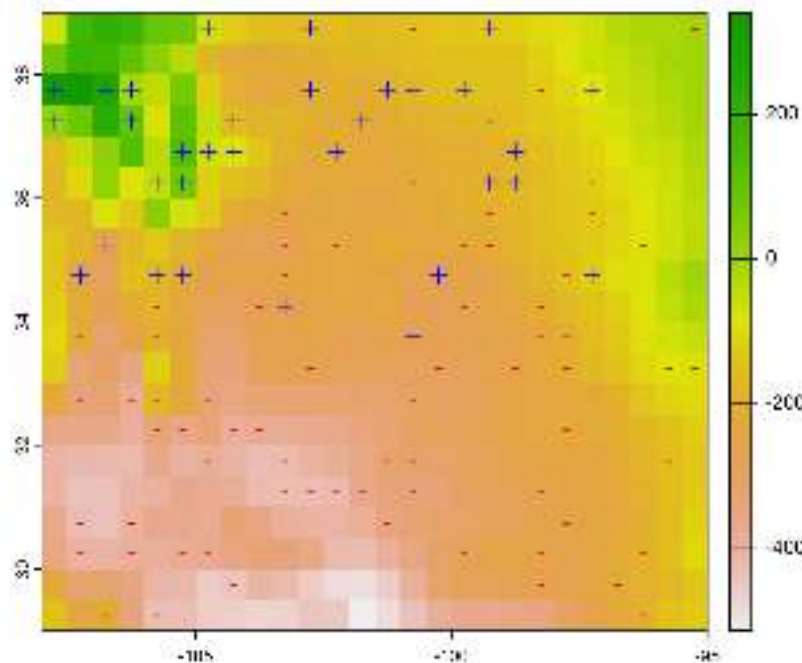
```
1 AIC(logistic.global, logistic.simple, logistic.rich)
```

	df	AIC
logistic.global	7	65.76394
logistic.simple	3	74.10760
logistic.rich	4	77.00622

An Example

Generating predictions

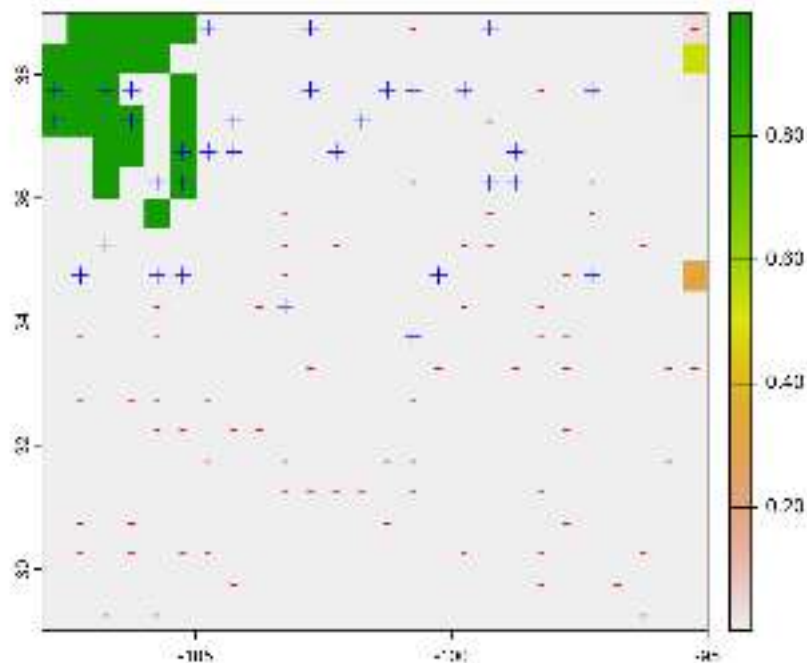
```
1 preds <- predict(object=pred.stack, model=logistic.simple)
2 plot(preds)
3 plot(pres.pts$geometry, add=TRUE, pch=3, col="blue")
4 plot(abs.pts$geometry, add=TRUE, pch = "-", col="red")
```



An Example

Generating predictions

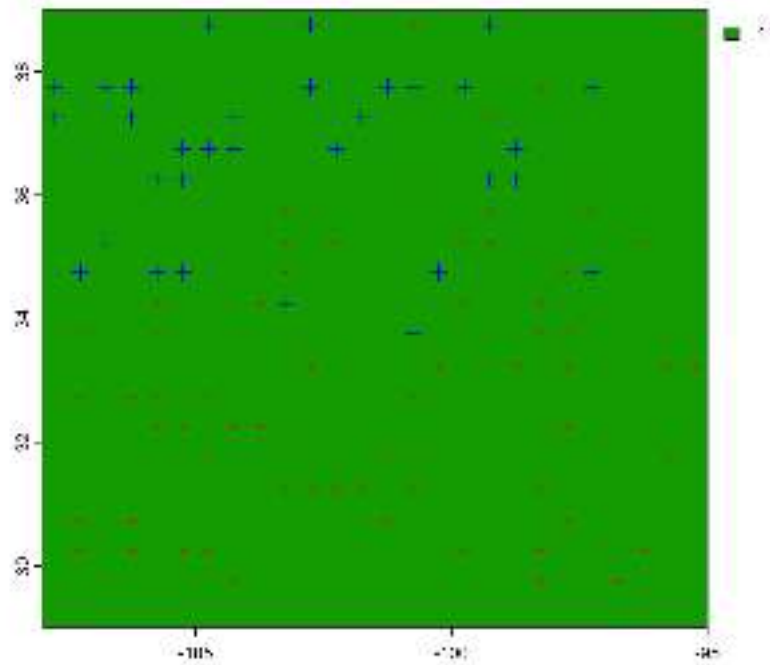
```
1 preds <- predict(object=pred.stack, model=logistic.simple, type="response")
2 plot(preds)
3 plot(pres.pts$geometry, add=TRUE, pch=3, col="blue")
4 plot(abs.pts$geometry, add=TRUE, pch = "-", col="red")
```



An Example

Generating predictions

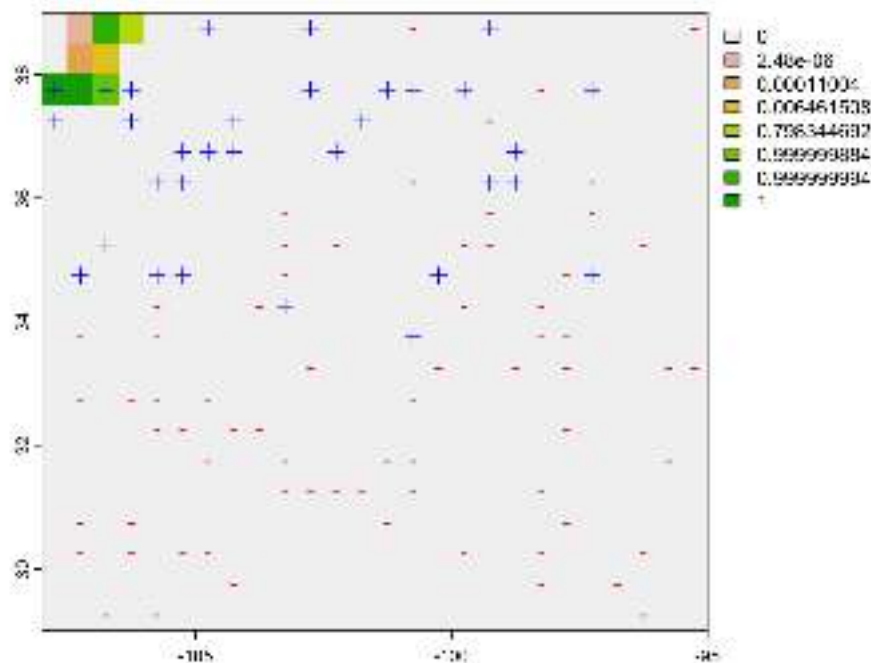
```
1 preds <- predict(object=pred.stack, model=logistic.global, type="response")
2 plot(preds)
3 plot(pres.pts$geometry, add=TRUE, pch=3, col="blue")
4 plot(abs.pts$geometry, add=TRUE, pch = "-", col="red")
```



An Example

Generating predictions

```
1 preds <- predict(object=pred.stack, model=logistic.rich, type="response")
2 plot(preds)
3 plot(pres.pts$geometry, add=TRUE, pch=3, col="blue")
4 plot(abs.pts$geometry, add=TRUE, pch = "-", col="red")
```

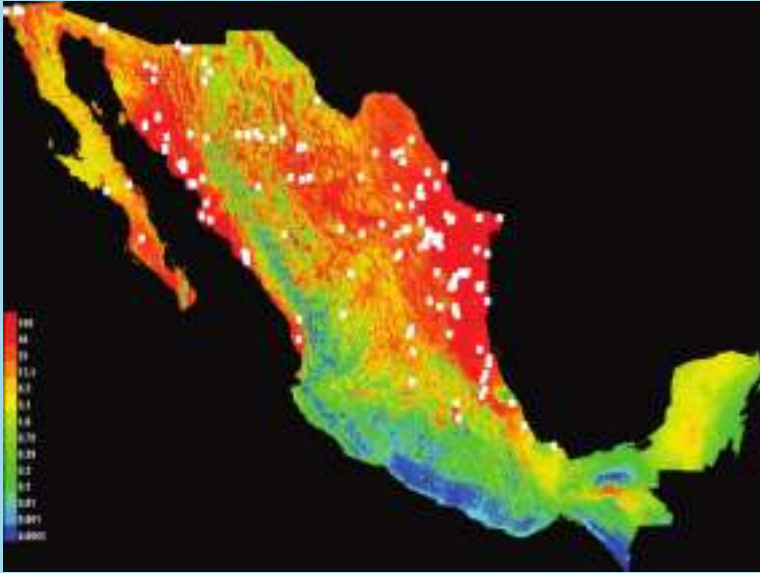


Key assumptions of logistic regression

- Dependent variable must be binary
- Observations must be independent (important for spatial analyses)
- Predictors should not be collinear
- Predictors should be linearly related to the log-odds
- **Sample Size**

Modelling Presence- Background Data

The sampling situation



From Lentz et al. 2008

- Opportunistic collection of presences only
- Hypothesized predictors of occurrence are measured (or extracted) at each presence
- Background points (or pseudoabsences) generated for comparison

The Challenge with Background Points

- What constitutes background?
- Not measuring *probability*, but relative likelihood of occurrence
- Sampling bias affects estimation
- The intercept

$$y_i \sim \text{Bern}(p_i)$$
$$\text{link}(p_i) = \mathbf{x}_i' \boldsymbol{\beta} + \alpha$$

