

# Point Pattern Analysis

HES 505 Fall 2023: Session 18

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# Objectives

- Define a point process and their utility for ecological applications
- Define first and second-order Complete Spatial Randomness
- Use several common functions to explore point patterns
- Leverage point patterns to interpolate missing data

# What is a point pattern?

- *Point pattern*: A **set** of **events** within a study region (i.e., a *window*) generated by a random process
- **Set**: A collection of mathematical **events**
- **Events**: The existence of a point object of the type we are interested in at a particular location in the study region
- A *marked point pattern* refers to a point pattern where the events have additional descriptors

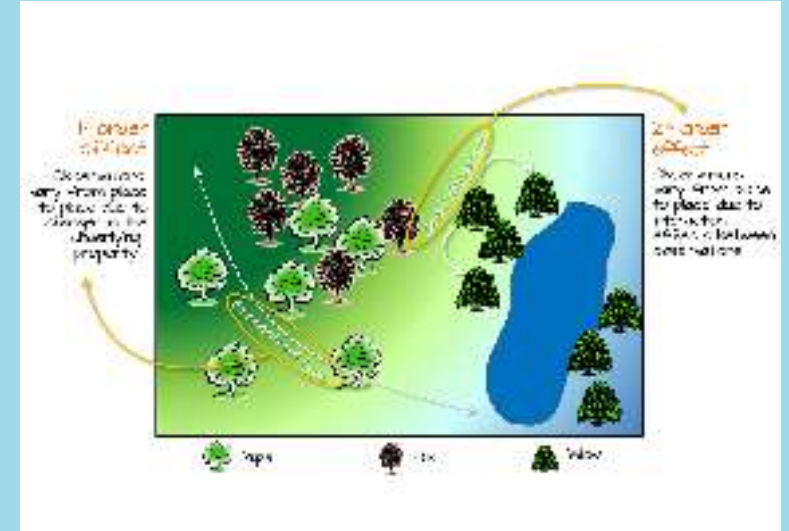
## Some notation:

- $S$ : refers to the entire set
- $s_i$  denotes the vector of data describing point  $s_i$  in set  $S$
- $\#(S \in A)$  refers to the number of points in  $S$  within study area  $A$

# Requirements for a set to be considered a point pattern

# Describing Point Patterns

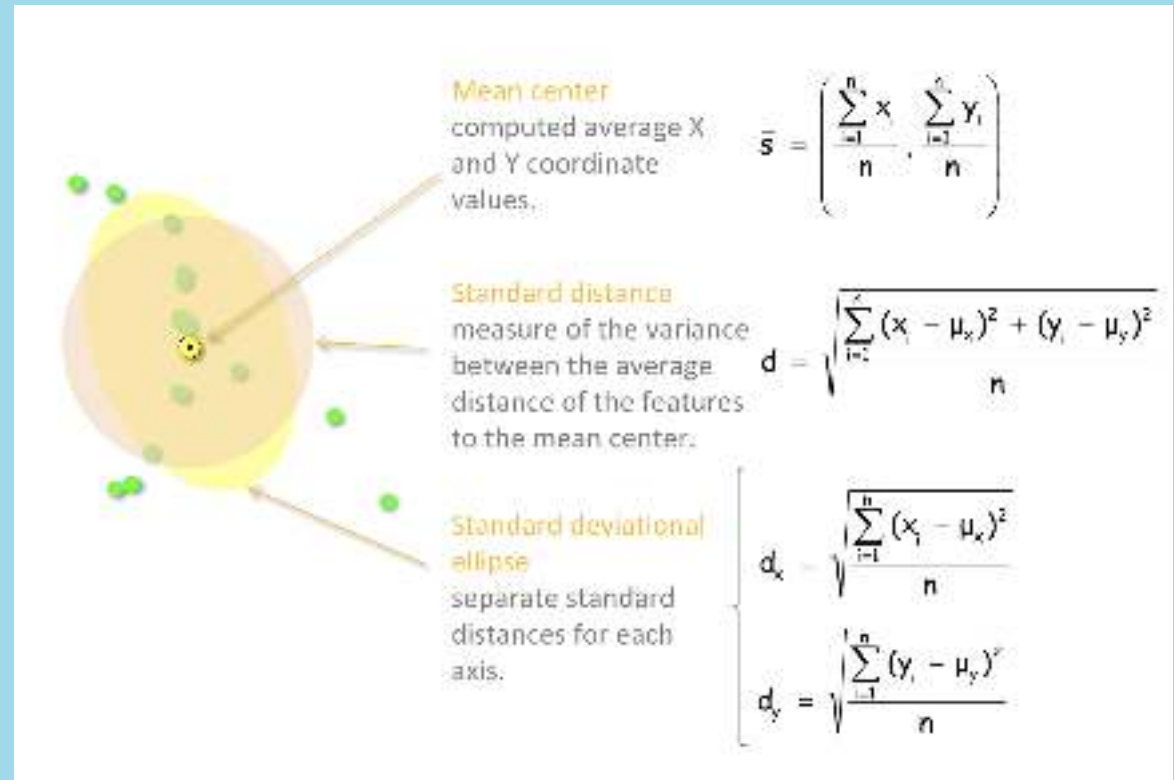
- *Density-based metrics*: the # of points within area,  $a$ , in study area  $A$
- *Distance-based metrics*: based on nearest neighbor distances or the distance matrix for all points
- *First order* effects reflect variation in **intensity** due to variation in the 'attractiveness' of locations
- *Second order* effects reflect variation in **intensity** due to the presence of points themselves



from Manuel Gimond

# Centrography

- *Mean center*: the point,  $\hat{s}$ , whose coordinates are the average of all events in the pattern
- *Standard distance*: a measure of the dispersion of points around the *mean center*
- *Standard ellipse*: dispersion in one dimension



From Manuel Gimond

# Analyzing Point Patterns

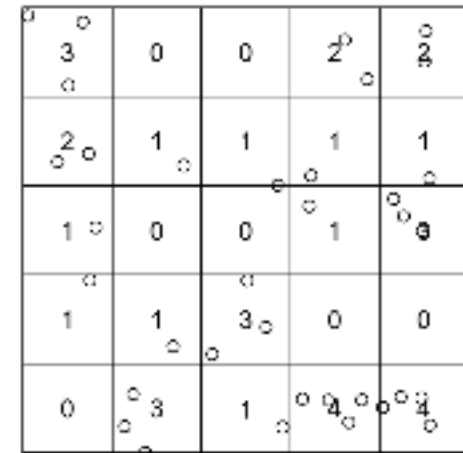
- Modeling random processes means we are interested in probability densities of the points (first-order; density)
- Also interested in how the presence of some events affects the probability of other events (second-order; distance)
- Finally interested in how the attributes of an event affect location (marked)
- Need to introduce a few new packages (**spatstat** and **gstat**)

# Density based methods

- The overall *intensity* of a point pattern is a crude density estimate

$$\hat{\lambda} = \frac{\#(S \in A)}{a}$$

\* Local density = quadrat counts





# Analyzing Point Patterns

# Kernel Density Estimates (KDE)

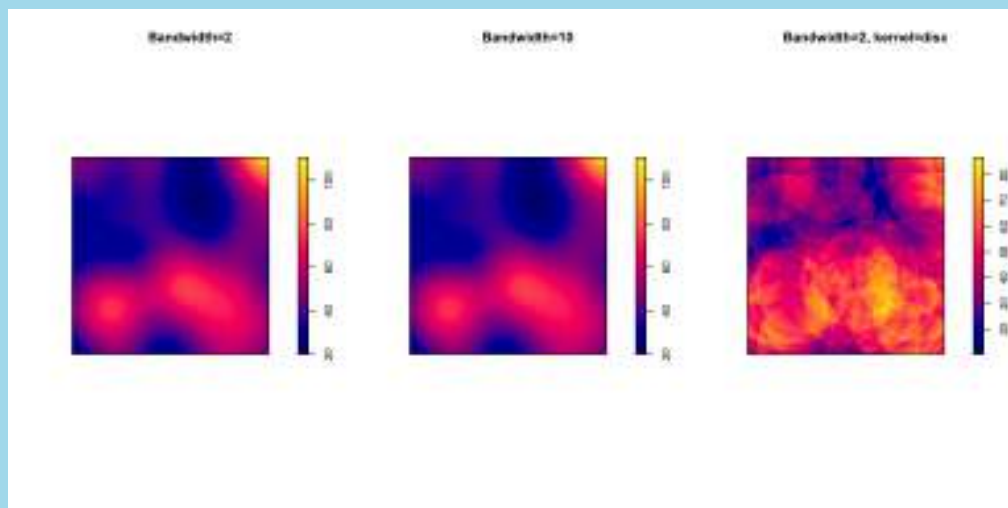
$$\hat{f}(\mathbf{x}) = \frac{1}{nh_x h_y} \sum_{i=1}^n k\left(\frac{x - x_i}{h_x}, \frac{y - y_i}{h_y}\right)$$

- Assume each location in  $\mathbf{s}_i$  drawn from unknown distribution
- Distribution has probability density  $f(\mathbf{x})$
- Estimate  $f(\mathbf{x})$  by averaging probability “bumps” around each location
- Need different object types for most operations in R (`as.ppp`)

# Kernel Density Estimates (KDE)

- $h$  is the bandwidth and  $k$  is the kernel
- We can use `stats::density` to explore
- **kernel**: defines the shape, size, and weight assigned to observations in the window
- **bandwidth** often assigned based on distance from the window center

```
1 x <- rpoispp(lambda =50)
2 K1 <- density(x, bw=2)
3 K2 <- density(x, bw=10)
4 K3 <- density(x, bw=2, kernel="disc")
```



# Choosing bandwidths and kernels

- Small values for  $h$  give 'spiky' densities
- Large values for  $h$  smooth much more
- Some kernels have optimal bandwidth detection
- **tmap** package provides additional functionality

# Second-Order Analysis

# Second-Order Analysis

- KDEs assume independence of points (first order randomness)
- Second-order methods allow dependence amongst points (second-order randomness)
- Several functions for assessing second order dependence (K, L, and G)

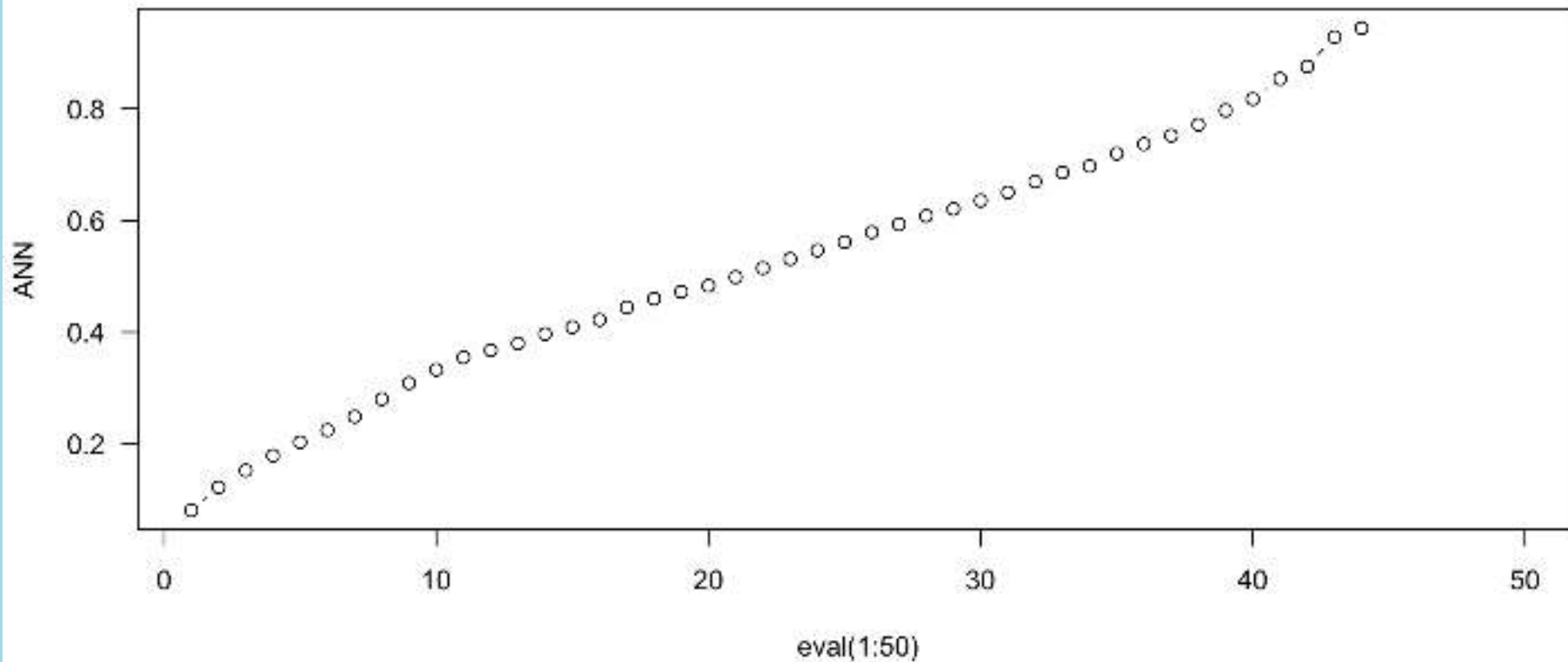
# Distance based metrics

- Provide an estimate of the *second order* effects
- *Mean nearest-neighbor distance*:

$$\hat{d}_{\min} = \frac{\sum_{i=1}^m d_{\min}(\mathbf{s}_i)}{n}$$

# Nearest-neighbor distance

```
1 ANN <- apply(nndist(x, k=1:50), 2, FUN=mean)
2 plot(ANN ~ eval(1:50), type="b", main=NULL, las=1)
```





# Ripley's K Function

- Nearest neighbor methods throw away a lot of information
- If points have independent, fixed marginal densities, then they exhibit *complete, spatial randomness* (CSR)
- The  $K$  function is an alternative, based on a series of circles with increasing radius

$$K(d) = \lambda^{-1} E(N_d)$$

- We can test for clustering by comparing to the expectation:

$$K_{CSR}(d) = \pi d^2$$

- if  $k(d) > K_{CSR}(d)$  then there is clustering at the scale defined by  $d$

# Ripley's K Function

- When working with a sample the distribution of K is unknown
- Estimate with

$$\hat{K}(d) = \hat{\lambda}^{-1} \sum_{i=1}^n \sum_{j=1}^n \frac{I(d_{ij} < d)}{n(n-1)}$$

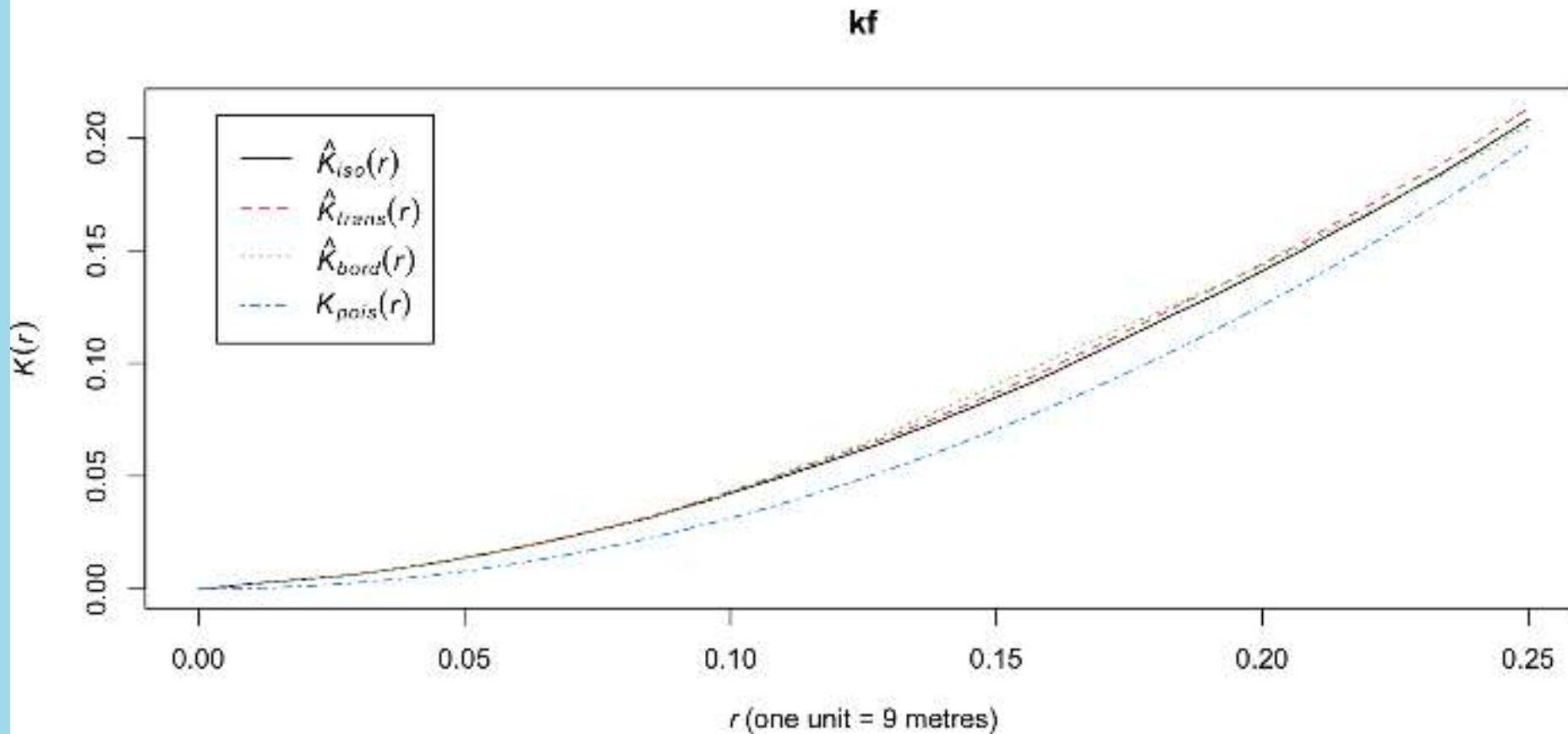
where:

$$\hat{\lambda} = \frac{n}{|A|}$$

# Ripley's K Function

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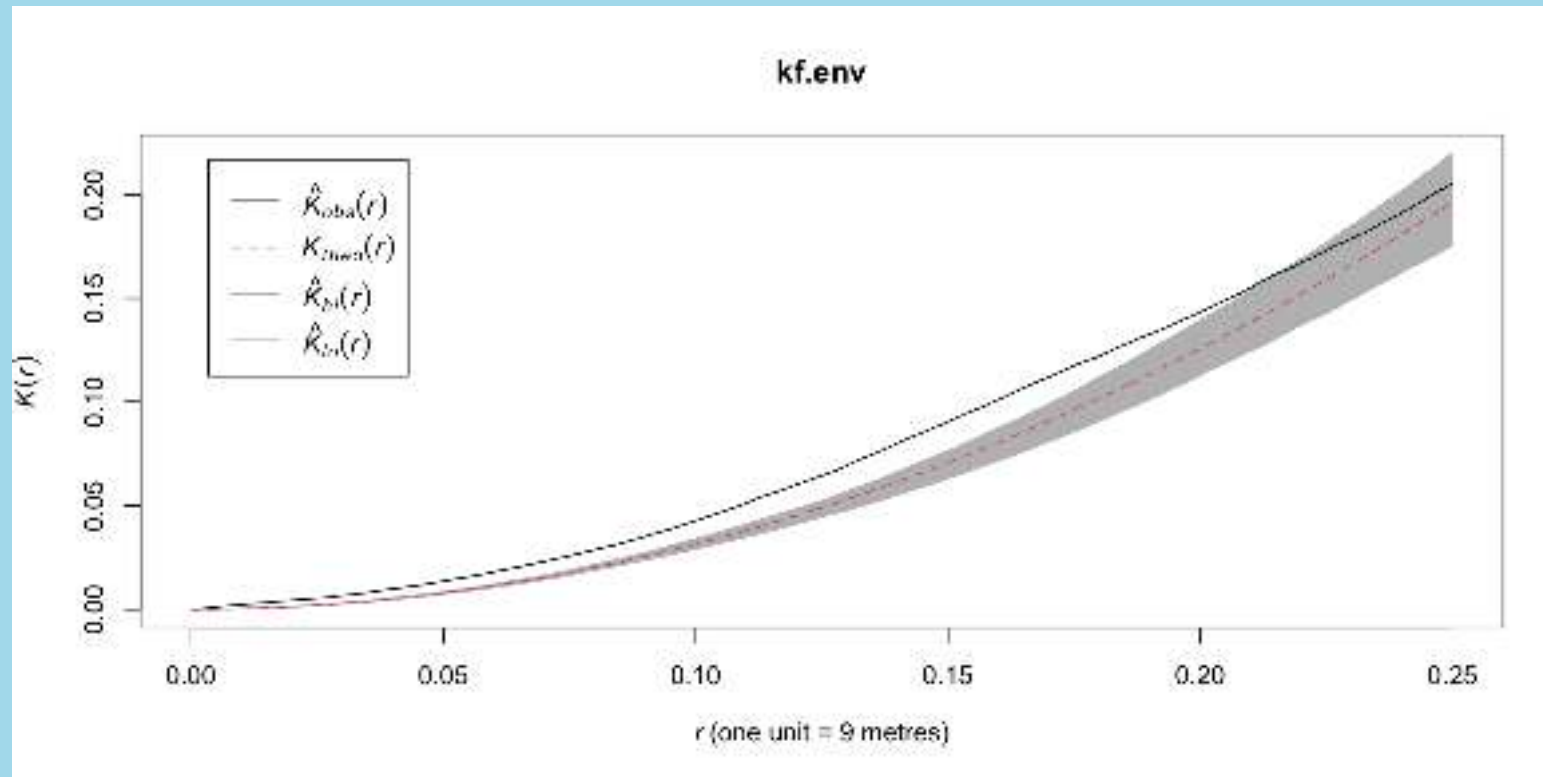
```
1 kf <- Kest(bramblecanes, correction="border")  
2 plot(kf)
```



# Ripley's K Function

- accounting for variation in d

```
1 kf.env <- envelope(bramblecanes, correction="border", envelope = FALSE, ver  
2 plot(kf.env)
```



# Other functions

- L function: square root transformation of K
- G function: the cumulative frequency distribution of the nearest neighbor distances
- F function: similar to G but based on randomly located points

