Point Pattern Analysis

HES 505 Fall 2022: Session 16

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Objectives

- Define a point process and their utility for ecological applications

- Define first and second-order Complete Spatial Randomness

- Use several common functions to explore point patterns

- Leverage point patterns to interpolate missing data

What is a point pattern?

- *Point pattern*: A **set** of **events** within a study region (i.e., a *window*) generated by a random process
- **Set**: A collection of mathematical **events**
- **Events**: The existence of a point object of the type we are interested in at a particular location in the study region
- A *marked point pattern* refers to a point pattern where the events have additional descriptors

Some notation:

- S: refers to the entire set
- s_i denotes the vector of data describing point s_i in set S
- $\#(S \in A)$ refers to the number of points in S within study area A

Requirements for a set to be considered a point pattern
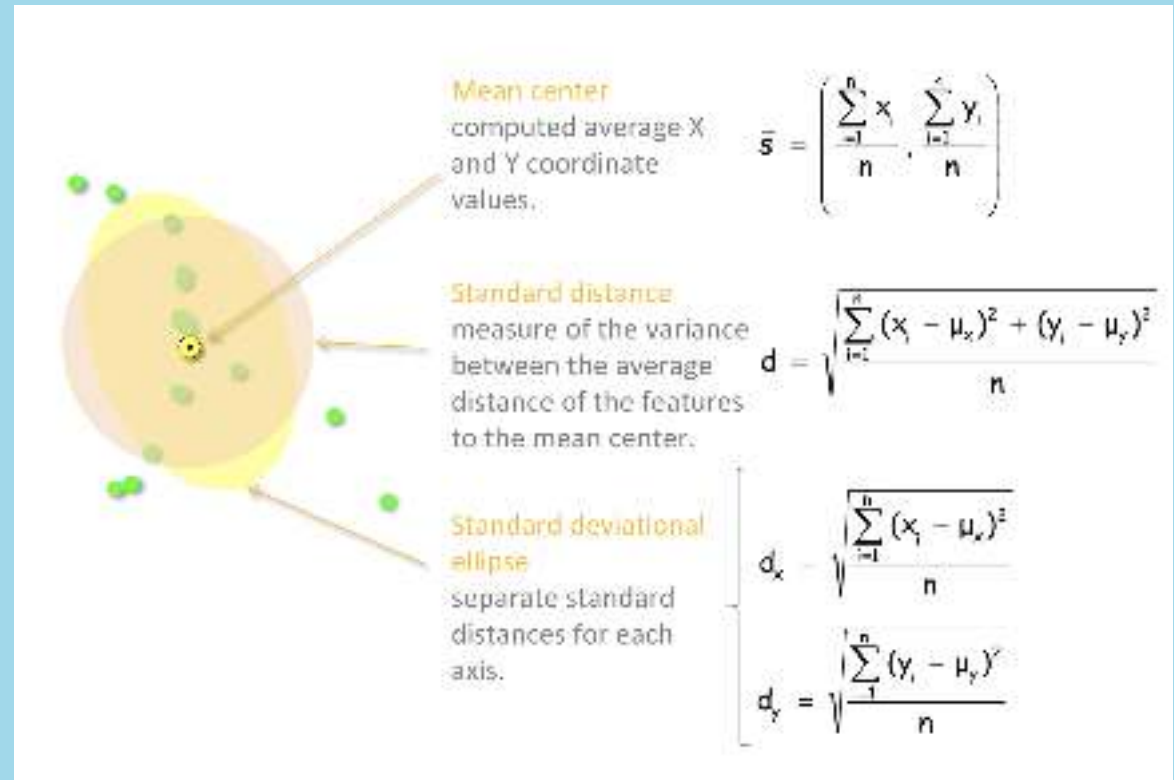
Describing Point Patterns

- *Density-based metrics*: the # of points within area, **a**, in study area A

- *Distance-based metrics*: based on nearest neighbor distances or the distance matrix for all points

- *First order* effects reflect variation in **intensity** due to variation in the 'attractiveness' of locations

- *Second order* effects reflect variation in **intensity** due to the presence of points themselves
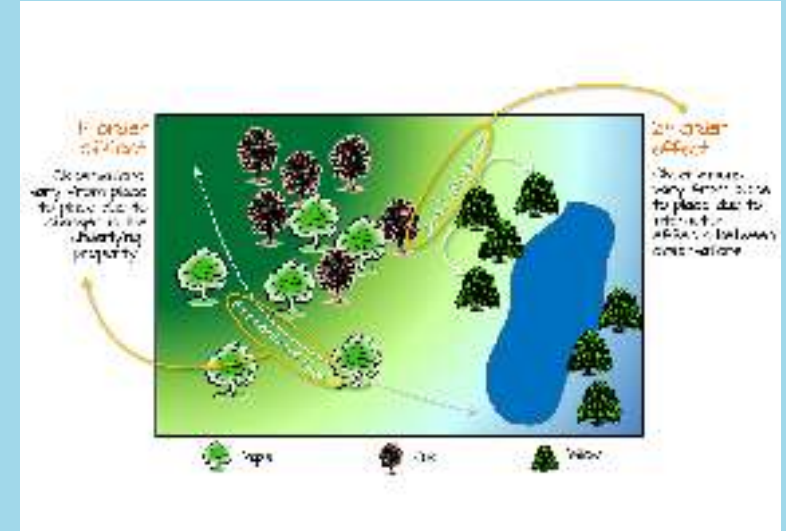
from Manuel Gimond

Centrography

- *Mean center*: the point, **ŝ**, whose coordinates are the average of all events in the pattern

- *Standard distance*: a measure of the dispersion of points around the *mean center*

- *Standard ellipse*: dispersion in one dimension

Mean center computed average X and Y coordinate values.

$$\bar{s} = \left(\frac{\sum_{i=1}^{n} x_i}{n}, \frac{\sum_{i=1}^{n} y_i}{n} \right)$$

Standard distance measure of the variance between the average distance of the features to the mean center.

$$d = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu_x)^2 + (y_i - \mu_y)^2}{n}}$$

Standard deviational ellipse separate standard distances for each axis.

$$d_x = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu_x)^2}{n}}$$

$$d_y = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \mu_y)^2}{n}}$$

From Manuel Gimond

Analyzing Point Patterns

- Modeling random processes means we are interested in probability densities of the points (first-order;density)

- Also interested in how the presence of some events affects the probability of other events (second-order;distance)

- Finally interested in how the attributes of an event affect location (marked)

- Need to introduce a few new packages (`spatstat` and `gstat`)

Density based methods

- The overall *intensity* of a point pattern is a crude density estimate

$$\hat{\lambda} = \frac{\#(S \in A)}{a}$$

- Local density = quadrat counts
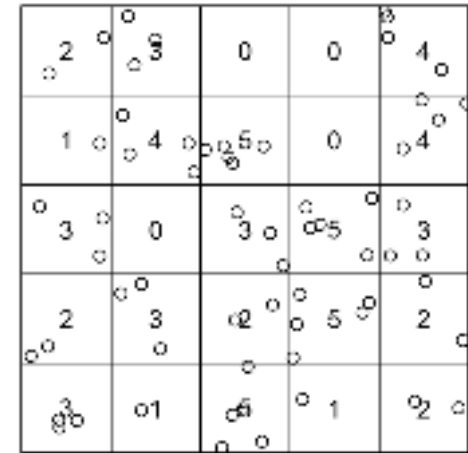
# Analyzing Point Patterns

Kernel Density Estimates (KDE)

$$\hat{f}(x) = \frac{1}{nh_x h_y} \sum_{i=1}^{n} k\left(\frac{x - x_i}{h_x}, \frac{y - y_i}{h_y} \right)$$

::: {style="font-size: 0.7em"} * Assume each location in s_i drawn from unknown distribution

Kernel Density Estimates (KDE)

- h is the bandwidth and k is the kernel

- We can use `stats::density` to explore

- **kernel**: defines the shape, size, and weight assigned to observations in the window

- **bandwidth** often assigned based on distance from the window center

```
1  x <- rpoispp(lambda =50)
2  K1 <- density(x, bw=2)
3  K2 <- density(x, bw=10)
4  K3 <- density(x, bw=2, kernel="disc")
```
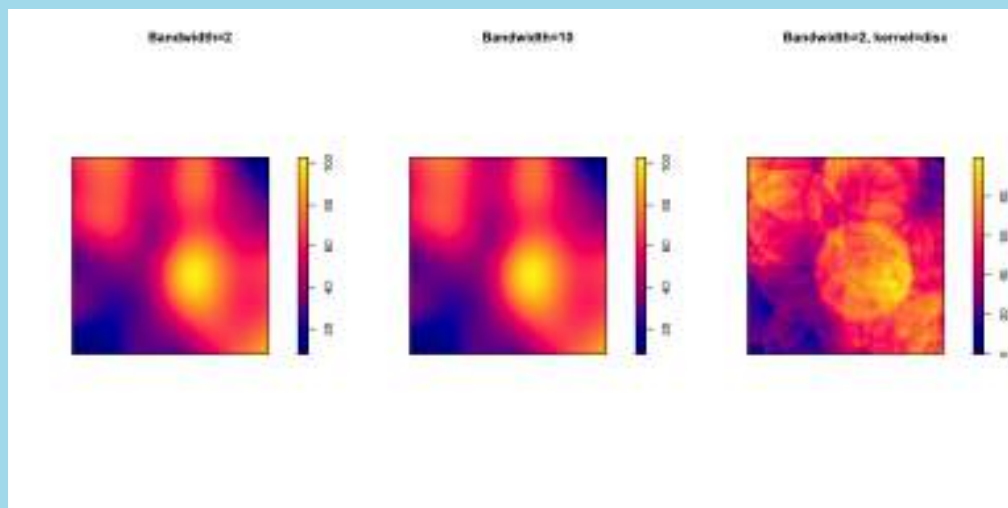
Choosing bandwidths and kernels

- Small values for h give 'spiky' densities

- Large values for h smooth much more

- Some kernels have optimal bandwidth detection

- `tmap` package (later) provides additional functionality

# Second-Order Analysis

Second-Order Analysis

- KDEs assume independence of points (first order randomness)

- Second-order methods allow dependence amongst points (second-order randomness)

- Several functions for assessing second order dependence (K, L, and G)

Distance based metrics

- Provide an estimate of the *second order* effects

- *Mean nearest-neighbor distance:*

$$\overset{\wedge}{d_{min}} = \frac{\sum_{i=1}^{m} d_{min}(s_i)}{n}$$

```
1  ANN <- apply(nndist(x, k=1
2  plot(ANN ~ eval(1:50), typ
```
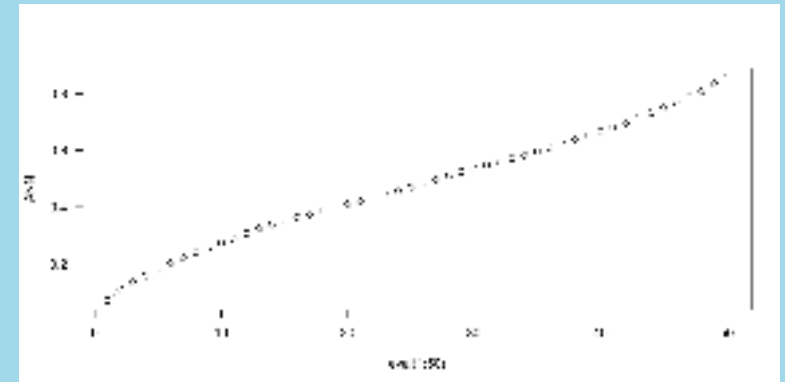
Ripley's K Function

- Nearest neighbor methods throw away a lot of information

- If points have independent, fixed marginal densities, then they exhibit *complete, spatial randomness* (CSR)

- The *K* function is an alternative, based on a series of circles with increasing radius

$$K(d) = \lambda^{-1} E(N_d)$$

- We can test for clustering by comparing to the expectation:

$$K_{CSR}(d) = \pi d^2$$

* if k(d) > $K_{CSR}(d)$ then there is clustering at the scale defined by d

Ripley's K Function

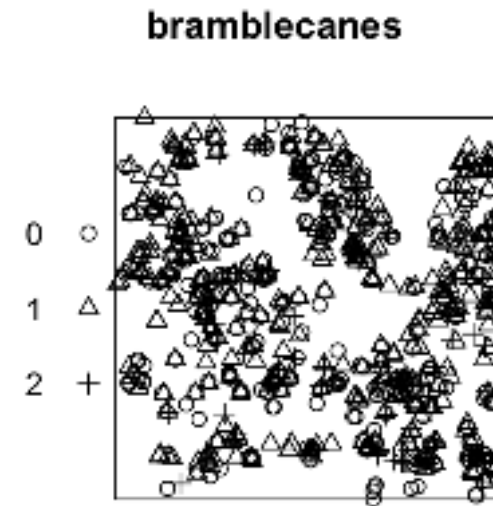- When working with a sample the distribution of K is unknown

- Estimate with

$$\hat{K}(d) = \hat{\lambda}^{-1} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{I(d_{ij} < d)}{n(n-1)}$$

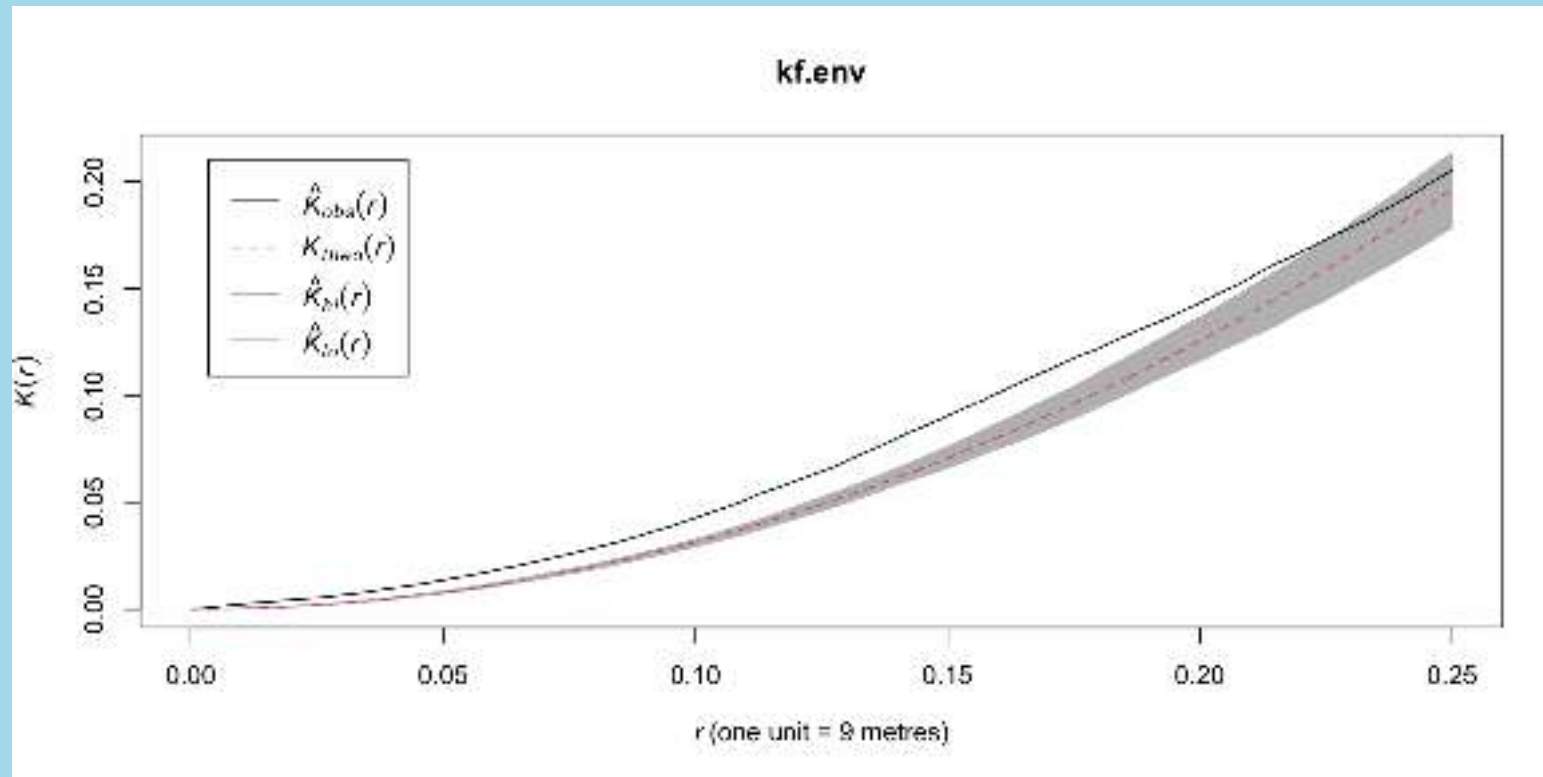where:

$$\hat{\lambda} = \frac{n}{|A|}$$

Ripley's K Function

- Using the `spatstat` package

bramblecanes

Ripley's K Function

- accounting for variation in d

```
1  kf.env <- envelope(bramblecanes, correction="border", envelope = FALSE, ver
2  plot(kf.env)
```

Other functions

- L function: square root transformation of K

- G function: the cummulative frequency distribution of the nearest neighbor distances

- F function: similar to G but based on randomly located points