

# Statistical Modelling II

HES 505 Fall 2023: Session 23

Matt Williamson

# Objectives

By the end of today you should be able to:

- Articulate the differences between statistical learning classifiers and logistic regression
- Describe several classification trees and their relationship to Random Forests
- Describe MaxEnt models for presence-only data

# Revisiting Classification

# Favorability in General

$$F(\mathbf{s}) = f(w_1 X_1(\mathbf{s}), w_2 X_2(\mathbf{s}), w_3 X_3(\mathbf{s}), \dots, w_m X_m(\mathbf{s}))$$

- Logistic regression treats  $f(\mathbf{x})$  as a (generalized) linear function
- Allows for multiple qualitative classes
- Ensures that estimates of  $F(\mathbf{s})$  are  $[0,1]$

# Key assumptions of logistic regression

- Dependent variable must be binary
- Observations must be independent (important for spatial analyses)
- Predictors should not be collinear
- Predictors should be linearly related to the log-odds
- **Sample Size**

# Beyond Linearity

- Logistic (and other generalized linear models) are relatively interpretable
- Probability theory allows robust inference of effects
- Predictive power can be low
- Relaxing the linearity assumption can help

# Classification Trees

- Use decision rules to segment the predictor space
- Series of consecutive decision rules form a 'tree'
- Terminal nodes (leaves) are the outcome; internal nodes (branches) the splits

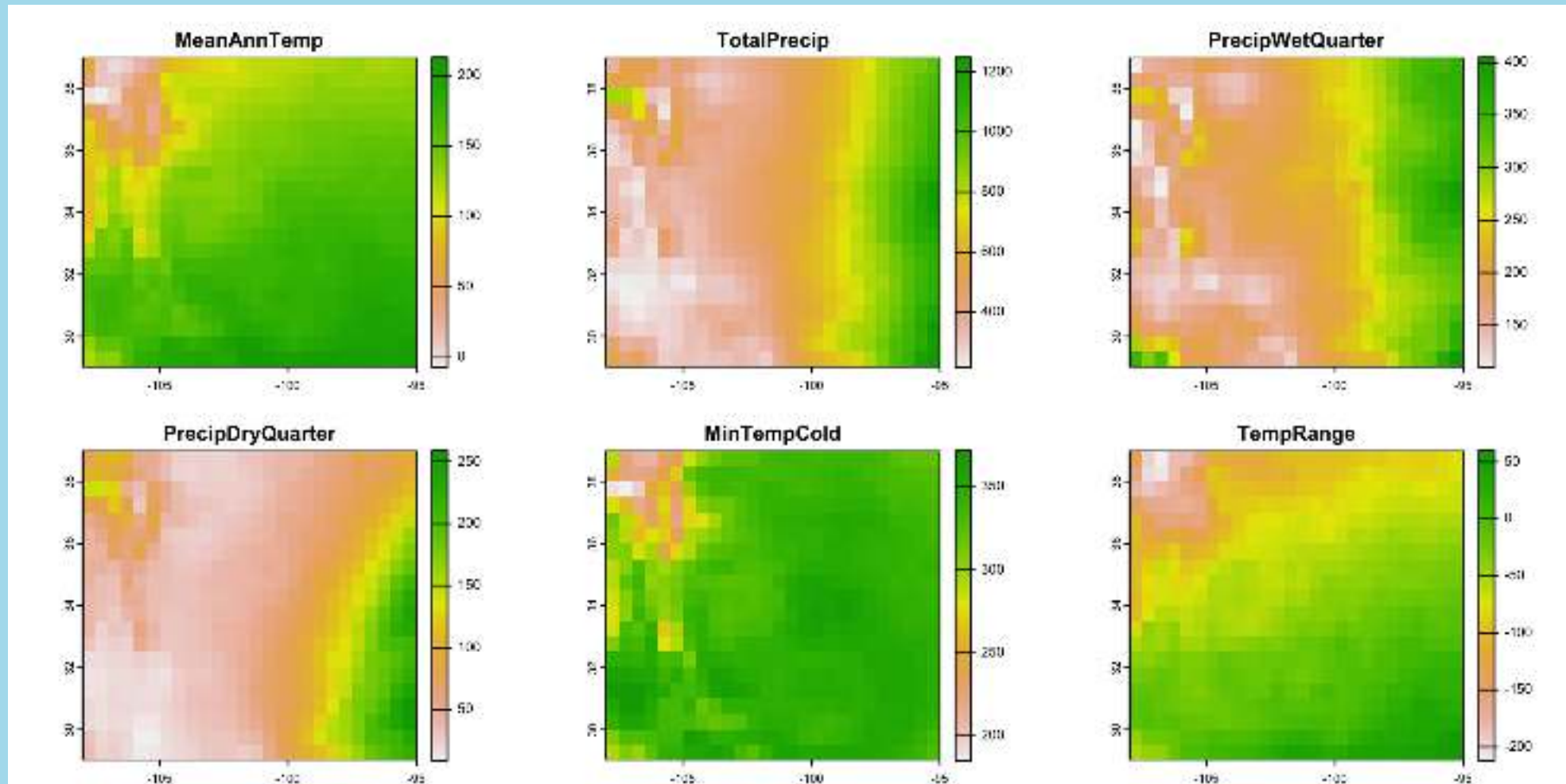
# Classification Trees

- Divide the predictor space ( $R$ ) into  $J$  non-overlapping regions
- Every observation in  $R_j$  gets the same prediction
- *Recursive binary splitting*
- Pruning and over-fitting



# An Example

Inputs from the **dismo** package



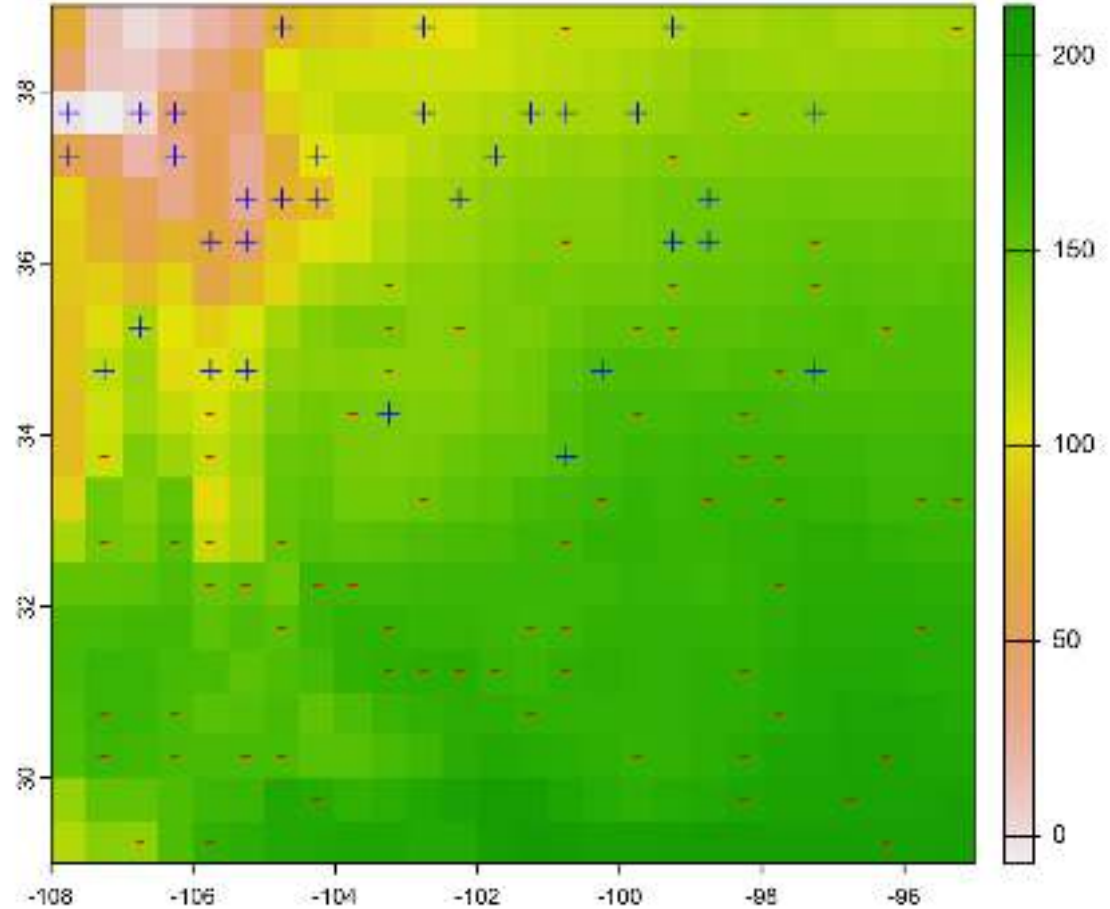
# An Example

The sample data

```
1 head(pres.abs)
```

Simple feature collection  
with 6 features and 1 field  
Geometry type: POINT  
Dimension: XY  
Bounding box: xmin: -106.75  
ymin: 31.25 xmax: -98.75  
ymax: 37.75  
Geodetic CRS: GCS\_unknown

	y	geometry
1	0	POINT (-99.25 35.25)
2	1	POINT (-98.75 36.25)
3	1	POINT (-106.75 35.25)
4	0	POINT (-100.75 31.25)
5	1	POINT (-99.75 37.75)
6	1	POINT (-104.25 36.75)



# An Example

## Building our dataframe

```
1 pts.df <- terra::extract(pred.stack, vect(pres.abs), df=TRUE)
2 head(pts.df)
```

	ID	MeanAnnTemp	TotalPrecip	PrecipWetQuarter	PrecipDryQuarter	MinTempCold
1	1	155	667	253	71	350
2	2	147	678	266	66	351
3	3	123	261	117	40	329
4	4	181	533	198	69	348
5	5	127	589	257	48	338
6	6	83	438	213	38	278

	TempRange
1	-45
2	-58
3	-64
4	-5
5	-81
6	-107

# An Example

## Building our dataframe

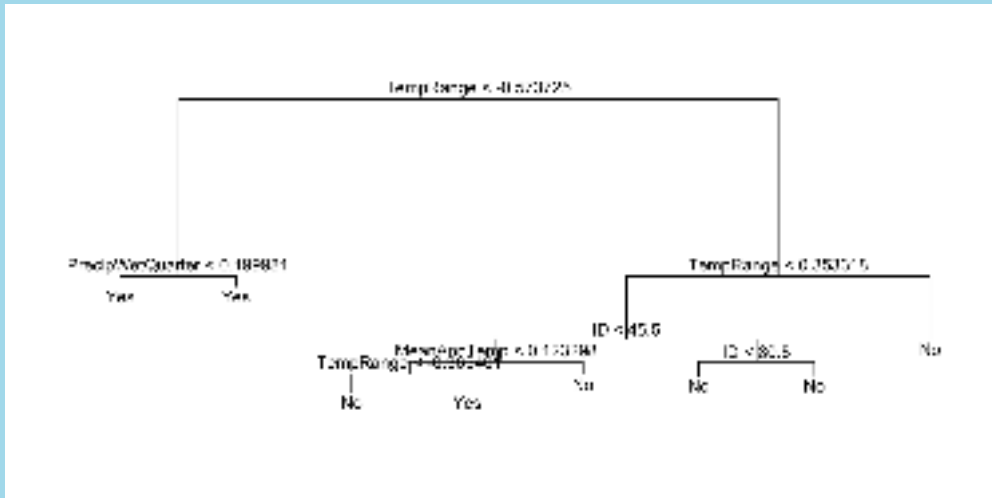
```
1 pts.df[,2:7] <- scale(pts.df[,2:7])
2 summary(pts.df)
```

ID	MeanAnnTemp	TotalPrecip	PrecipWetQuarter
Min. : 1.00	Min. : -3.3729	Min. : -1.3377	Min. : -1.6926
1st Qu.: 25.75	1st Qu.: -0.4594	1st Qu.: -0.7980	1st Qu.: -0.6895
Median : 50.50	Median : 0.2282	Median : -0.2373	Median : -0.2224
Mean : 50.50	Mean : 0.0000	Mean : 0.0000	Mean : 0.0000
3rd Qu.: 75.25	3rd Qu.: 0.7118	3rd Qu.: 0.7140	3rd Qu.: 0.6508
Max. : 100.00	Max. : 1.4285	Max. : 2.4843	Max. : 2.2713
PrecipDryQuarter	MinTempCold	TempRange	
Min. : -1.0828	Min. : -3.9919	Min. : -2.7924	
1st Qu.: -0.7013	1st Qu.: -0.0598	1st Qu.: -0.5216	
Median : -0.3770	Median : 0.3582	Median : 0.2075	
Mean : 0.0000	Mean : 0.0000	Mean : 0.0000	
3rd Qu.: 0.4290	3rd Qu.: 0.5495	3rd Qu.: 0.6450	
Max. : 3.1713	Max. : 1.1092	Max. : 2.0407	

# An example

- Fitting the classification tree

```
1 library(tree)
2 pts.df <- cbind(pts.df, pres.abs$y)
3 colnames(pts.df)[8] <- "y"
4 pts.df$y <- as.factor(ifelse(pts.df$y == 1, "Yes", "No"))
5 tree.model <- tree(y ~ . , pts.df)
6 plot(tree.model)
7 text(tree.model, pretty=0)
```



# An example

- Fitting the classification tree

```
1 summary(tree.model)
```

Classification tree:

```
tree(formula = y ~ ., data = pts.df)
```

Variables actually used in tree construction:

```
[1] "TempRange" "PrecipWetQuarter" "ID"
```

```
"MeanAnnTemp"
```

Number of terminal nodes: 8

Residual mean deviance: 0.3164 = 29.11 / 92

Misclassification error rate: 0.07 = 7 / 100

# Benefits and drawbacks

## Benefits

- Easy to explain
- Links to human decision-making
- Graphical displays
- Easy handling of qualitative predictors

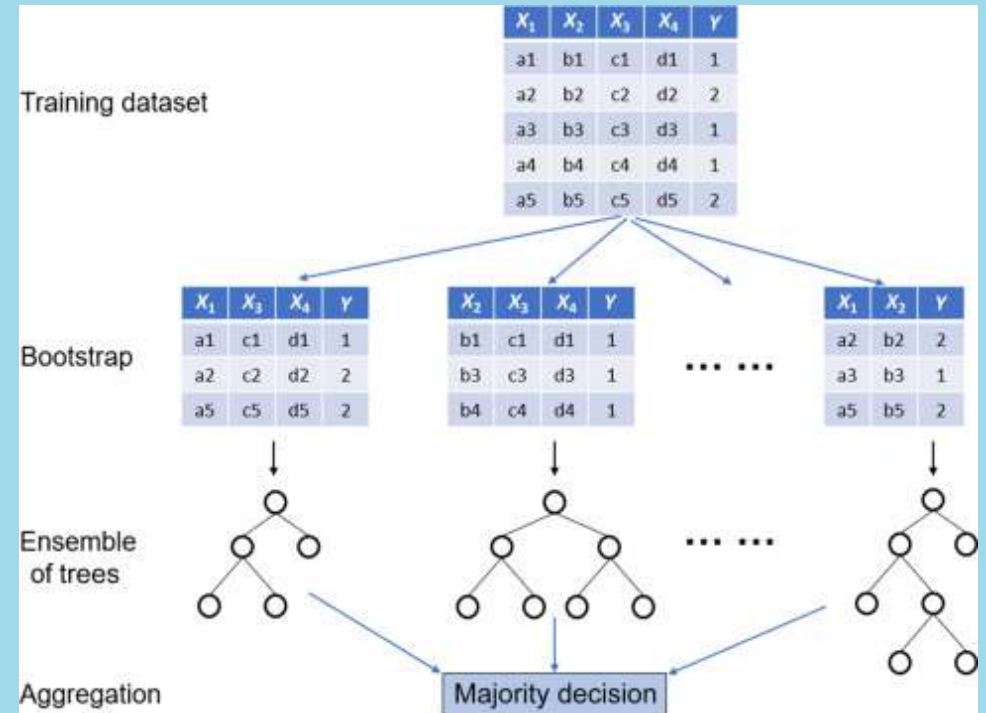
## Costs

- Lower predictive accuracy than other methods
- Not necessarily robust



# Random Forests

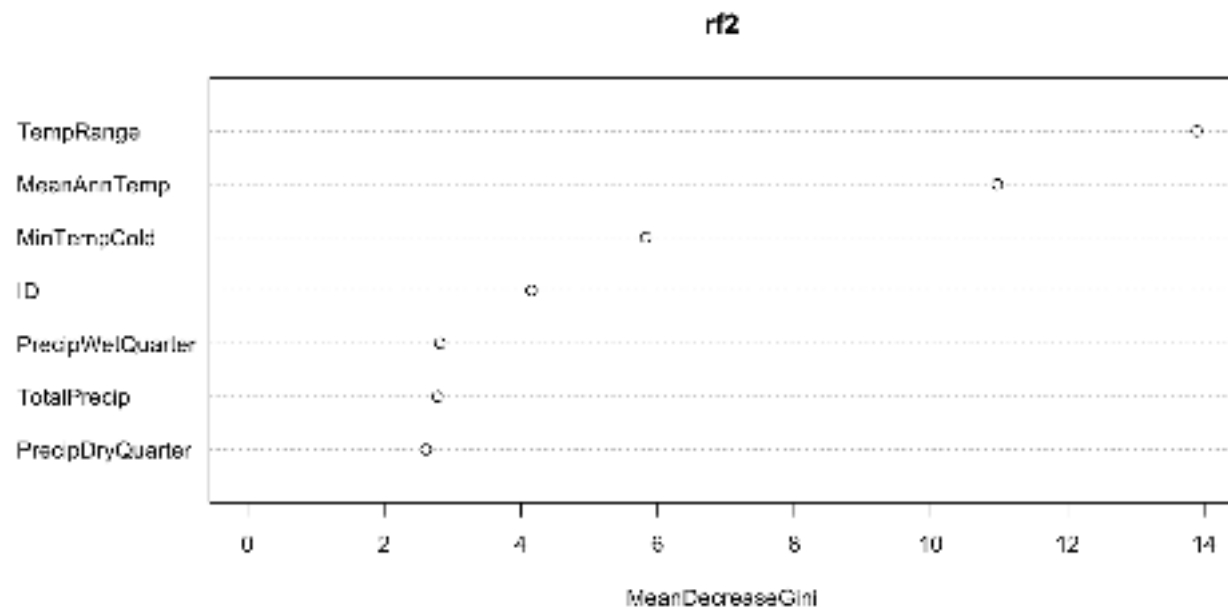
- Grow 100(000s) of trees using bootstrapping
- Random sample of predictors considered at each split
- Avoids correlation amongst multiple predictions
- Average of trees improves overall outcome (usually)
- Lots of extensions



# An example

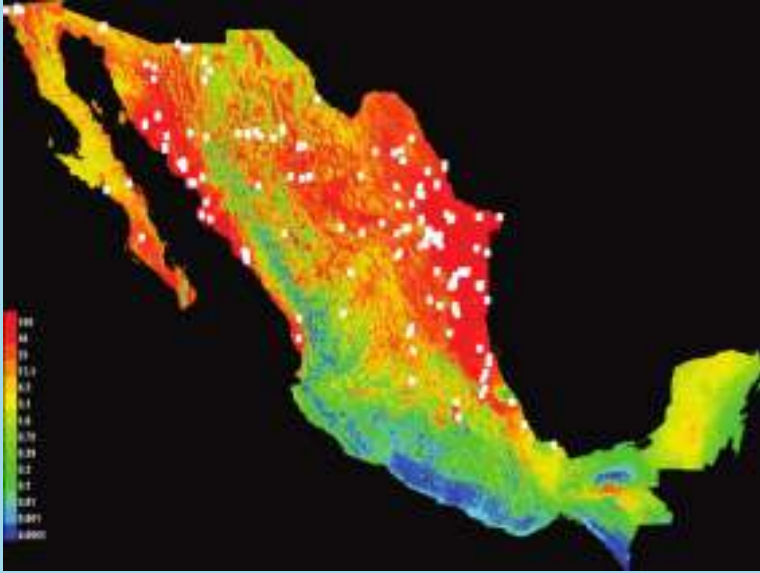
- Fitting the Random Forest

```
1 library(randomForest)
2 class.model <- y ~ .
3 rf2 <- randomForest(class.model, data=pts.df)
4 varImpPlot(rf2)
```



# Modelling Presence- Background Data

# The sampling situation



From Lentz et al. 2008

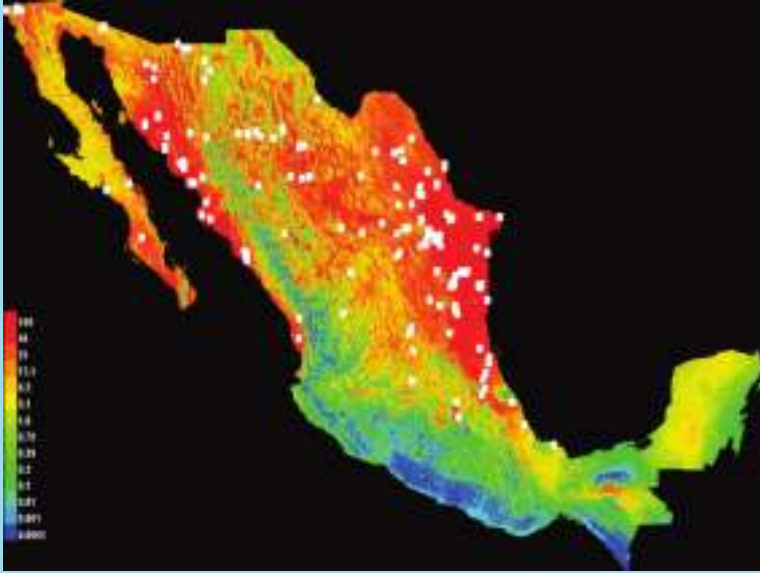
- Opportunistic collection of presences only
- Hypothesized predictors of occurrence are measured (or extracted) at each presence
- Background points (or pseudoabsences) generated for comparison

# The Challenge with Background Points

- What constitutes background?
- Not measuring *probability*, but relative likelihood of occurrence
- Sampling bias affects estimation
- The intercept

$$y_i \sim \text{Bern}(p_i)$$
$$\text{link}(p_i) = \mathbf{x}_i' \boldsymbol{\beta} + \alpha$$

# MaxEnt

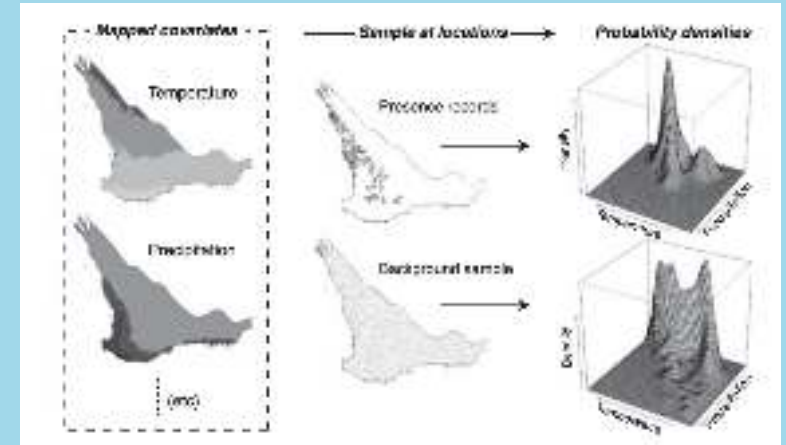


From Lentz et al. 2008

- Opportunistic collection of presences only
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# Maximum Entropy models

- MaxEnt (after the original software)
- Need *plausible* background points across the remainder of the study area
- Iterative fitting to maximize the distance between predictions generated by a spatially uniform model
- Tuning parameters to account for differences in sampling effort, placement of background points, etc
- Development of the model beyond the scope of this course, but see [Elith et al. 2010](#)



From [Elith et al. 2010](#)

# Challenges with MaxEnt

- Not measuring *probability*, but relative likelihood of occurrence
- Sampling bias affects estimation (but can be mitigated using tuning parameters)
- Theoretical issues with background points and the intercept
- Recent developments relate MaxEnt (with cloglog links) to Inhomogenous Point Process models



# Extensions

- Polynomial, splines, piece-wise regression
- Neural nets, Support Vector Machines, many many more

