Interpolation

HES 505 Fall 2023: Session 19

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Objectives

By the end of today you should be able to:

- Distinguish deterministic and stochastic processes
- Define autocorrelation and describe its estimation
- Articulate the benefits and drawbacks of autocorrelation
- Leverage point patterns and autocorrelation to interpolate missing data

But first…

Patterns as realizations of spatial processes

- A **spatial process** is a description of how a spatial pattern might be *generated*
- **Generative models**
- An observed pattern as a *possible realization* of an hypothesized process

Deterministic vs. stochastic processes

- Deterministic processes: always produce the same outcome

$$z = 2x + 3y$$

- Results in a spatially continuous field

Deterministic vs. stochastic processes

```
x <- rast(nrows = 10, ncols=10, xmin = 0, xmax=10, ymin = 0, ymax=10)
values(x) <- 1
z <- x
values(z) <- 2 * crds(x)[,1] + 3*crds(x)[,2]
```

Deterministic vs. stochastic processes

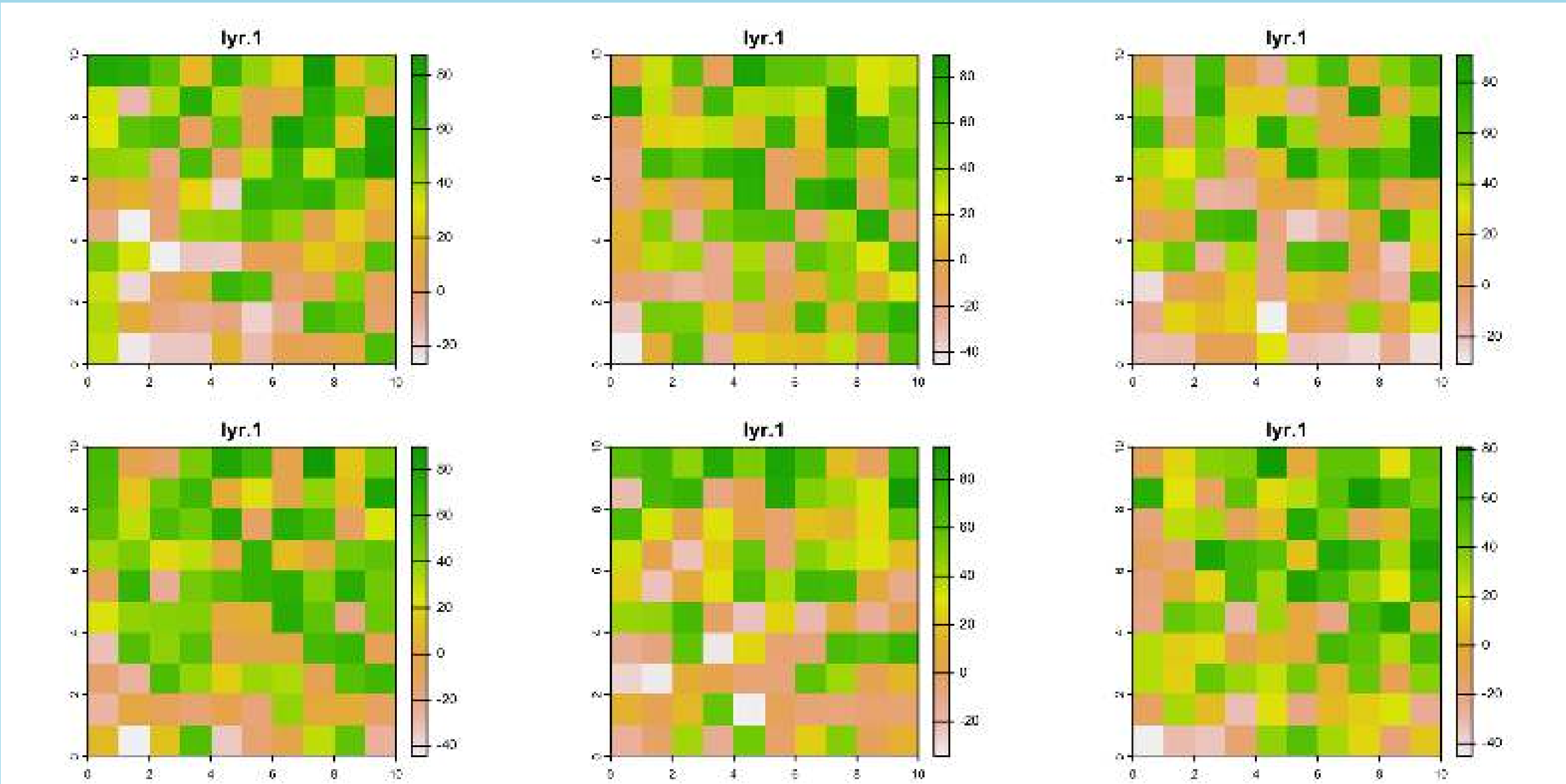
- Stochastic processes: variation makes each realization difficult to predict

$$z = 2x + 3y + d$$

- The *process* is random, not the result (!!)
- Measurement error makes deterministic processes appear stochastic

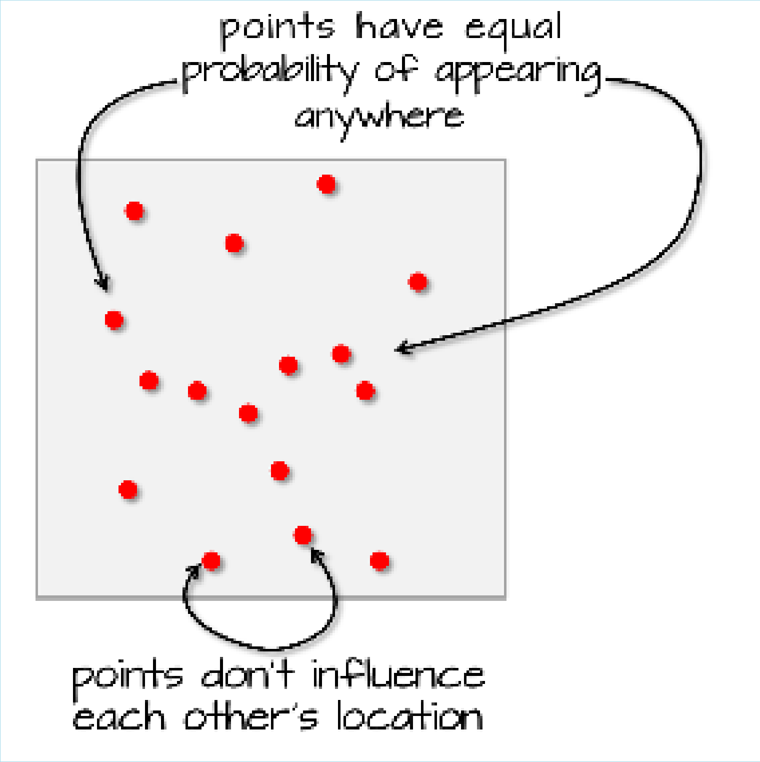
```
x <- rast(nrows = 10, ncols=10, xmin = 0,
values(x) <- 1
fun <- function(z){
a <- z
d <- runif(ncell(z), -50,50)
values(a) <- 2 * crds(x)[,1] + 3*crds(x)[,
return(a)
}

b <- replicate(n=6, fun(z=x), simplify=FAL
d <- do.call(c, b)
```

Deterministic vs. stochastic processes



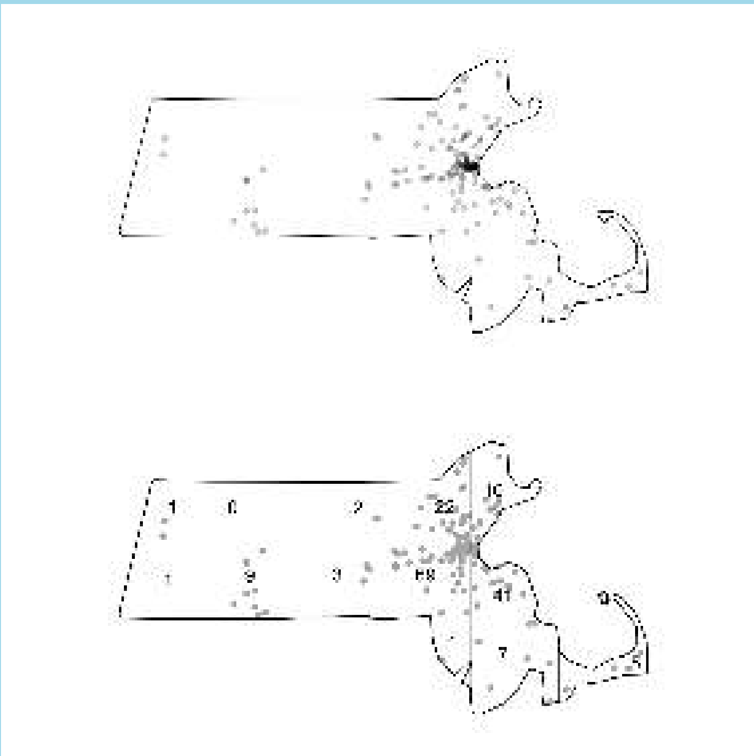
Expected values and hypothesis testing

- Considering each outcome as the realization of a process allows us to generate expected values
- The simplest spatial process is Completely Spatial Random (CSR) process
- **First Order** effects: any event has an equal probability of occurring in a location
- **Second Order** effects: the location of one event is independent of the other events



From Manuel Gimond

Generating expactations for CSR



- We can use quadrat counts to estimate the expected number of events in a given area
- The probability of each possible count is given by:

$$P(n,k) = \binom{n}{x} p^k (1-p)^{n-k}$$

- Given total coverage of quadrats, then $p = \frac{\frac{a}{x}}{a}$ and

$$P(k,n,x) = \binom{n}{k} \left(\frac{1}{x}\right)^k \left(\frac{x-1}{x}\right)^{n-k}$$

Revisiting Ripley's K

- Nearest neighbor methods throw away a lot of information
- If points have independent, fixed marginal densities, then they exhibit *complete, spatial randomness* (CSR)
- The *K* function is an alternative, based on a series of circles with increasing radius

$$K(d) = \lambda^{-1} E(N_d)$$

- We can test for clustering by comparing to the expectation:

$$K_{CSR}(d) = \pi d^2$$

- if $k(d) > K_{CSR}(d)$ then there is clustering at the scale defined by d

Ripley's K Function

- When working with a sample the distribution of K is unknown
- Estimate with

$$\hat{K}(d) = \hat{\lambda}^{-1} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{I(d_{ij} < d)}{n(n-1)}$$

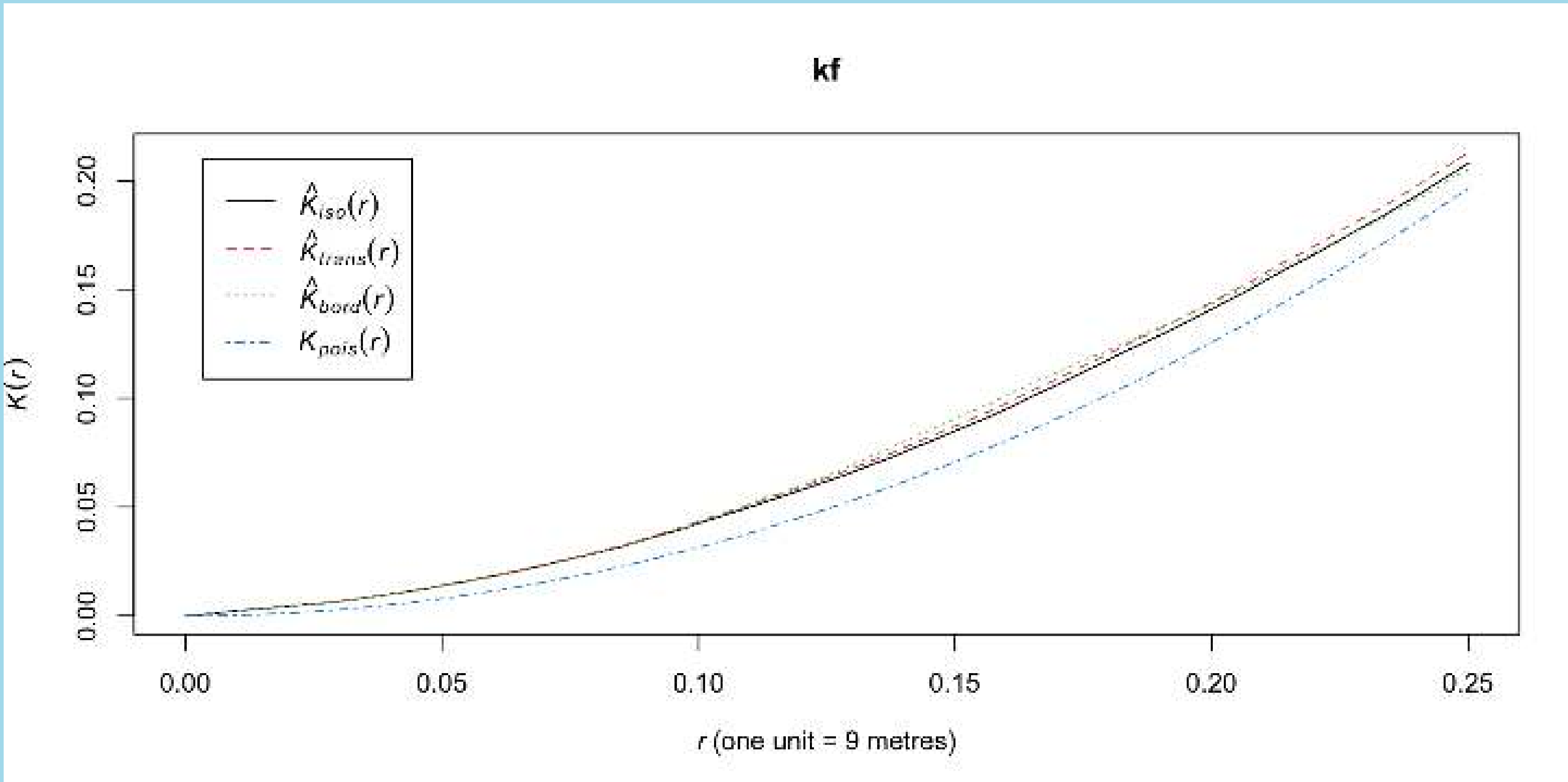
where:

$$\hat{\lambda} = \frac{n}{|A|}$$

Ripley's K Function

Ripley’s K Function

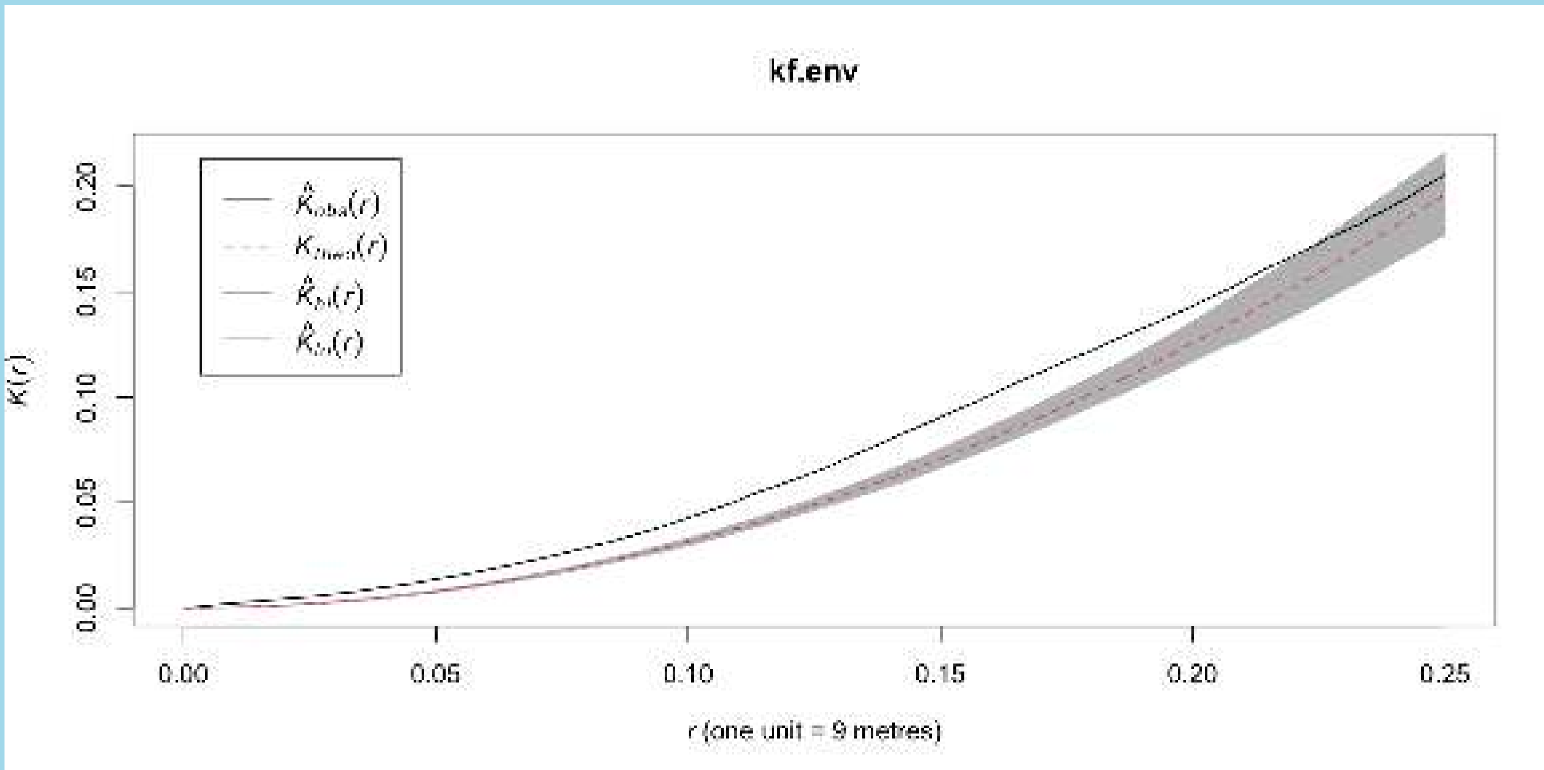
```r
kf <- Kest(bramblecanes, correction-"border")
plot(kf)
```



Ripley’s K Function

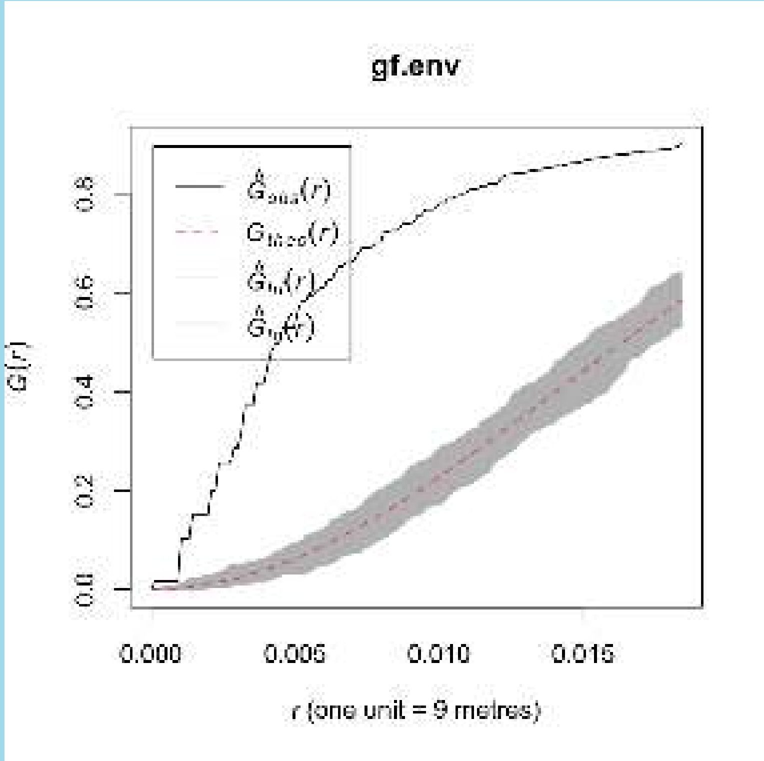
- accounting for variation in d

```
kf.env <- envelope(bramblecanes, correction="border", envelope = FALSE, ver
plot(kf.env)
```



Other functions

- L function: square root transformation of K
- G function: the cummulative frequency distribution of the nearest neighbor distances
- F function: similar to G but based on randomly located points



Tobler’s Law

> ‘everything is usually related to all else but those which are near to each other are more related when compared to those that are further away’.
> Waldo Tobler

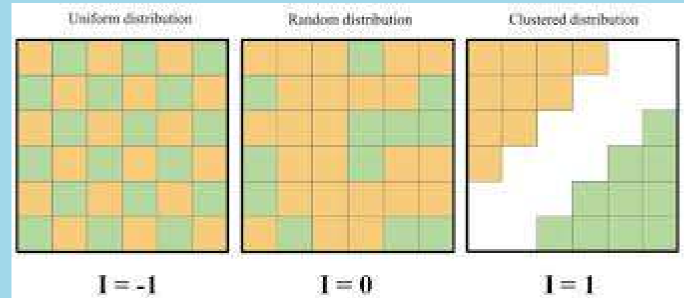
Spatial autocorrelation

From Manuel Gimond

(One) Measure of autocorrelation

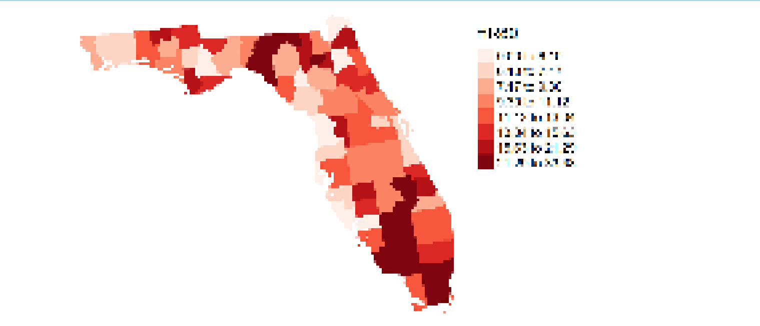
- Moran's I

$$I(d) = \frac{\sum_{i}^{n} \sum_{j \neq i}^{n} w_{ij} (x_i - \bar{x})(x_j - \bar{x})}{S^2 \sum_{i}^{n} \sum_{j \neq i}^{n} w_{ij}}$$



Moran’s I: An example

- Use `spdep` package
- Estimate neighbors
- Generate weighted average

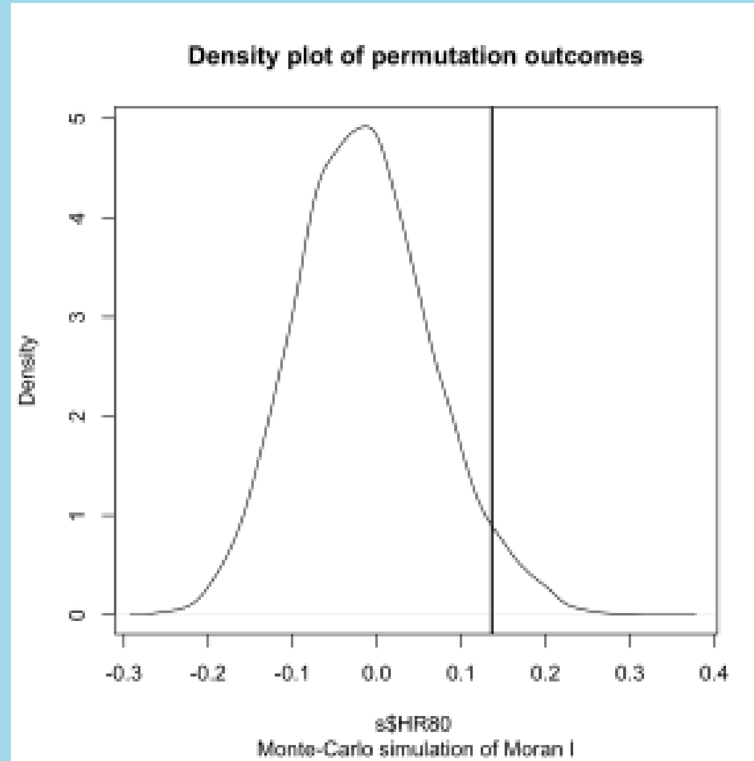


```r
set.seed(2354)
# Load the shapefile
s <- readRDS(url("https://github.com/mgimond/Data/raw/gh-pages

# Define the neighbors (use queen case)
nb <- poly2nb(s, queen=TRUE)

# Compute the neighboring average homicide rates
lw <- nb2listw(nb, style="W", zero.policy=TRUE)
#estimate Moran's I
moran.test(s$HR80,lw, alternative="greater")
```

```
	Moran I test under randomisation

data:  s$HR80  
weights: lw    

Moran I statistic standard deviate = 1.8891, p-value = 0.02944
alternative hypothesis: greater
sample estimates:
Moran I statistic       Expectation          Variance 
      0.136277593      -0.015151515       0.006425761 
```

Moran’s I: An example

```r
M1 <- moran.mc(s$HR80, lw, nsim=9999, alte



# Display the resulting statistics
M1
```




```
    Monte-Carlo simulation of Moran I

data:  s$HR80
weights: lw
number of simulations + 1: 10000

statistic = 0.13628, observed rank = 9575, p-
value = 0.0425
alternative hypothesis: greater
```

The challenge of areal data

- Spatial autocorrelation threatens *second order* randomness
- Areal data means an infinite number of potential distances
- Neighbor matrices, $\boldsymbol{W}$, allow different characterizations

Interpolation

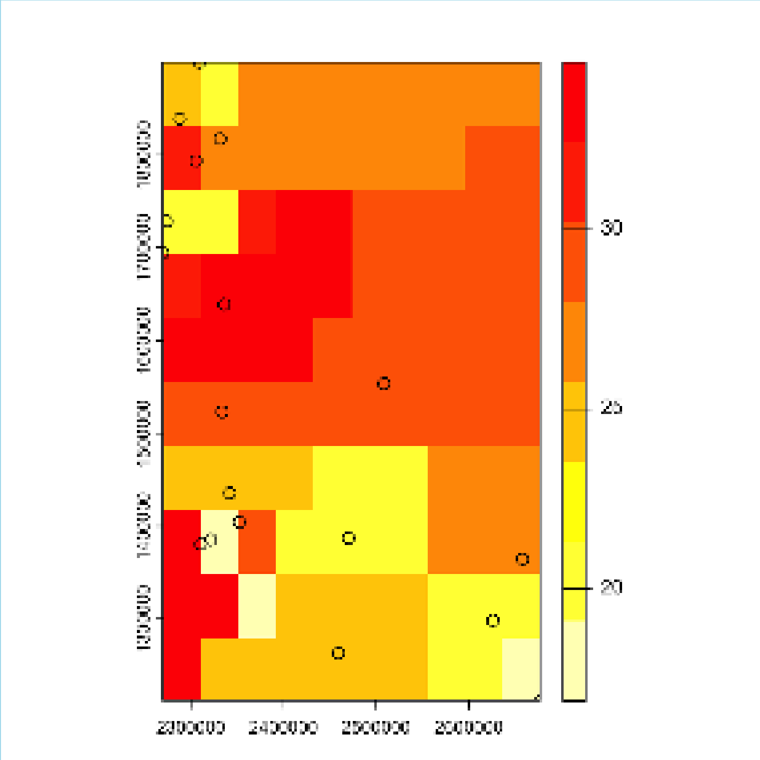
Interpolation

- Goal: estimate the value of z at new points in $\mathbf{x_i}$
- Most useful for continuous values
- Nearest-neighbor, Inverse Distance Weighting, Kriging

Nearest neighbor

- find i such that $|\mathbf{x_i} - \mathbf{x}|$ is minimized
- The estimate of z is z_i

```r
aq <- read_csv("data/ad_viz_plotval_data.csv") %>%
  st_as_sf(., coords = c("SITE_LONGITUDE", "SITE_LATITUDE"), c
  st_transform(., crs = "EPSG:8826") %>%
  mutate(date = as_date(parse_datetime(Date, "%m/%d/%Y"))) %>%
  filter(., date >= 2023-07-01) %>%
  filter(., date > "2023-07-01" & date < "2023-07-31")
aq.sum <- aq %>%
  group_by(., `Site Name`) %>%
  summarise(., meanpm25 = mean(DAILY_AQI_VALUE))

nodes <- st_make_grid(aq.sum,
                      what = "centers")

dist <- distance(vect(nodes), vect(aq.sum))
nearest <- apply(dist, 1, function(x) which(x == min(x)))
aq.nn <- aq.sum$meanpm25[nearest]
preds <- st_as_sf(nodes)
preds$aq <- aq.nn

preds <- as(preds, "Spatial")
sp::gridded(preds) <- TRUE
preds.rast <- rast(preds)
```



Inverse-Distance Weighting

- Weight closer observations more heavily

$$\hat{z}(\mathbf{x}) = \frac{\sum_{i=1} w_i z_i}{\sum_{i=1} w_i}$$

where

$$w_i = |\mathbf{x} - \mathbf{x}_i|^{-\alpha}$$

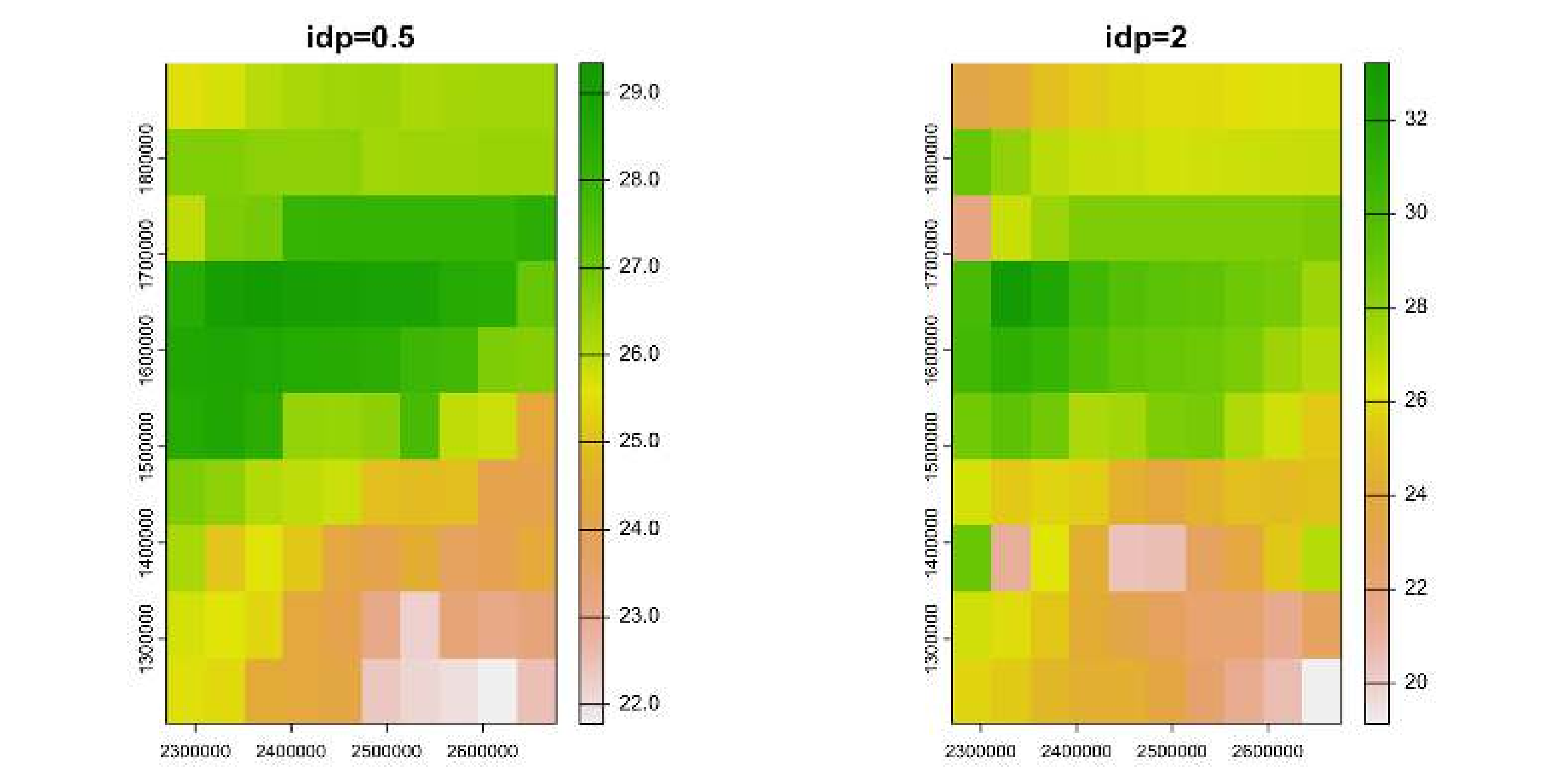
and $\alpha > 0$ ($\alpha = 1$ is inverse; $\alpha = 2$ is inverse square)

Inverse-Distance Weighting

- `terra::interpolate` provides flexible interpolation methods
- Use the `gstat` package to develop the formula

```
mgsf05 <- gstat(id = "meanpm25", formula = meanpm25~1, data=aq.sum,  nmax=7
mgsf2 <- gstat(id = "meanpm25", formula = meanpm25~1, data=aq.sum,  nmax=7,
interpolate_gstat <- function(model, x, crs, ...) {
    v <- st_as_sf(x, coords=c("x", "y"), crs=crs)
    p <- predict(model, v, ...)
    as.data.frame(p)[,1:2]
}
zsf05 <- interpolate(preds.rast, mgsf05, debug.level=0, fun=interpolate_gst
zsf2 <- interpolate(preds.rast, mgsf2, debug.level=0, fun=interpolate_gstat
```

Inverse-Distance Weighting



Inverse-Distance Weighting