

- Write a Python class that defines a 2D Ising model on a square $L \times L$ lattice with periodic boundary conditions, including methods to perform a Metropolis Monte Carlo simulation.
- Plot the average magnetisation $\langle M \rangle$, energy $\langle E \rangle$, magnetic susceptibility χ , and heat capacity C as a function of the magnetic field h for system size $L = 10$ and two different temperatures: $k_B T/J = 1.0$ and $k_B T/J = 4.0$.
- Plot the same quantities as a function of temperature for $h = 0$. Find the critical temperature T_c of the phase transition between ferromagnetic and paramagnetic phases.
- Plot the magnetic susceptibility and the heat capacity as a function of system size at $T = T_c$.
- Discuss the effect of:
 - system size
 - initial state
 - number of Metropolis samples
 - sweeping through lattice sites sequentially or randomly
- Feel free to explore other features of the phase transition!