PYU33A03 Stellar Structure Assignment 2

Prof. Kate Maguire (she/her), kate.maguire@tcd.ie

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Objective: The aim of this assignment is to understand some of the key properties of stars, investigate in more detail a ground-breaking mission in stellar astrophysics, and write some simple analysis code.
Marking: The assignment is worth 15% of the PYU33A03 module.
Submission and deadline: Your submission should be uploaded to Blackboard by the deadline of 23.59 on Wednesday 08 November 2023.
Format: Your submission should be a **single** PDF document and include your name and student number at the top of the first page. Your upload should include the answers to the questions (handwritten or typed) as well as your workings, e.g. computer code and any other handwritten/typed workings.
Plagiarism reminder: You are welcome to discuss the assignment but you must complete the questions and coding yourself. The use of AI for producing your answers is not permitted and detection of its use will be taken very seriously.

1. **Gaia space mission** **[15 marks]**

   (a) The European Space Agency Gaia mission (Gaia mission web page and Gaia description paper) is revolutionising our understanding of stars. Describe in less than 350 words, i) one astrophysical topic related to stars that Gaia is addressing and ii) one aspect of this topic that requires more data or theoretical modelling to explain fully. You must include one labelled figure and include a citation to at least one refereed journal paper (not including the one mentioned above). Hint: you could search through the citations for the Gaia description paper on the web page linked above for a topic of interest.

2. **Ionisation fractions in stars:** **[20 marks]**

   (a) The partition function, Z, is used to describe the number of available states in an atom, molecule, or ion and is given by,

$$Z = \sum_{n=1}^{\infty} g_n e^{-(E_n - E_1)/kT} \tag{1}$$

   where n represents the nth energy levels, g_n is the degeneracy of the nth energy level, E_n is the energy of the nth energy level, E_1 is the energy of the first energy level, k is the Boltzmann constant, and T is the temperature. Write down the first three terms of the partition function of H I. You may assume that $g_n = 2n^2$ for hydrogen and the standard values for its energy levels.

(b) Assuming that only the first three energy levels of H I contribute, plot using a computer programme of your choice, the fraction of ionised H in a stellar atmosphere in the temperature range of 2000 to 22000 K. You may assume a number density of free electrons of $X \times 10^{19}$ m^{-3}, where X is the last two digits of your student number. If the last two digits are '00', use '11'. Ensure to include comments in your code and explain any assumptions made.

(c) Estimate the temperature at which 90% of hydrogen atoms have become ionised.

(d) Assuming partition functions of $Z_I = 1.32$ and $Z_{II} = 2.30$ for Ca I and Ca II, respectively, plot the fraction of Ca II as a function of temperature. You may assume the same number density of free electrons as in the previous part, and an ionisation energy of Ca I of 6.11 eV.

(e) For a star with a photospheric temperature of 7000 K, would you expect to see visible-wavelength absorption lines produced from H I? Or what about visible wavelength absorption lines of singly-ionised Ca? What else does absorption line production depend on?

3. **Stellar interiors and main-sequence evolution** **[15 marks]**

(a) Write down the equation for the mean molecular weight of a star composed of completely ionised hydrogen and estimate its mean molecular weight.

(b) Show that for a completely ionised gas of hydrogen and helium the following relation holds,

$$\frac{1}{\mu_i} \simeq 2X + \frac{3}{4}Y \tag{2}$$

where μ_i is the mean molecular weight of a completely ionised gas, X is the mass fraction of hydrogen, and Y is the mass fraction of helium.

(c) When it arrives on the main sequence, a star has a hydrogen mass fraction of 0.65 and a helium mass fraction of 0.35. Estimate its mean molecular weight assuming the gas is fully ionised.

(d) Later in its stellar evolution, the star has fused Z% of its hydrogen atoms to helium, where Z is the third last digit of your student number+1, e.g. if the third last digit is '0', then the percentage is 0+1 = 1%. Estimate the change in its mean molecular weight.

(e) Determine how the luminosity of the star may change after undergoing this fusion.

Reminder: You can find an example Python notebook (.ipynb) for manipulating and plotting data here. There is also a .pdf ('PythonIntro_StephenPower.pdf') with instructions on various ways to access Python on Blackboard.