

pset 4 econ 21020

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5/22/2022

Problem 1

a)

False. Suppose W, U are iid on $\text{Uniform}[0, X]$ where X is a random variable that is equal to 1 with probability 0.5 and 2 with probability 0.5. In this case, W and U are independent by definition but W is not independent of $U|X$ since $U|X$ is a function of X and W is not independent of X .

b)

False.

c)

False. $E[\epsilon|X] = E[Y - \text{BLP}(Y|X)|X] = E[Y|X] - E[\text{BLP}(Y|X)|X] = E[Y|X] - \text{BLP}(Y|X)$. $E[\epsilon|X] = 0$ only when $E[Y|X] = \text{BLP}(Y|X)$ which is not always true.

Problem 2

$$\text{WTS } \frac{\hat{\text{ATE}}_n - \text{ATE}}{\text{se}(\hat{\text{ATE}}_n)} \xrightarrow{d} N(0, 1)$$

$$\text{where } \text{se}(\hat{\text{ATE}}_n) = \frac{1}{\sqrt{n}} \sqrt{\hat{\sigma}_{\text{ATE}}^2}$$

$$\frac{\hat{\text{ATE}}_n - \text{ATE}}{\text{se}(\hat{\text{ATE}}_n)} = \sqrt{n}(\hat{\text{ATE}}_n - \text{ATE}) \frac{1}{\sqrt{\hat{\sigma}_{\text{ATE}}^2}}$$

$$\text{Let } A_n = \sqrt{n}(\hat{\text{ATE}}_n - \text{ATE}), B_n = \hat{\sigma}_{\text{ATE}}^2$$

$$\text{By Lec 7 Theorem 4, } A_n \xrightarrow{d} N(0, \sigma_{\text{ATE}}^2)$$

$$B_n = \hat{\sigma}_{\text{CATE}}^2(1)\hat{p}_n(1)^2 + \hat{\sigma}_{\text{CATE}}^2(0)\hat{p}_n(0)^2 + (\hat{\text{CATE}}_n(1) - \hat{\text{CATE}}_n(0))\hat{p}_n(1)\hat{p}_n(0)$$

$$\text{where } \hat{p}_n(x) = \frac{1}{n} \sum_{i=1}^n 1(X_i = x)$$

$$\text{Let } C_n = \hat{\sigma}_{\text{CATE}}^2(1), D_n = \hat{\sigma}_{\text{CATE}}^2(0), E_n = \hat{p}_n(1), F_n = \hat{p}_n(0), G_n = \hat{\text{CATE}}_n(1), H_n = \hat{\text{CATE}}_n(0)$$

$$g(c, d, e, f, g, h) = ce^2 + df^2 + (g - h)ef$$

$$g(C_n, D_n, E_n, F_n, G_n, H_n) = B_n$$

$$\text{By WLLN, } C_n, D_n \xrightarrow{p} \sigma_{\text{CATE}}^2(x)$$

$$E_n, F_n \xrightarrow{p} p(x)$$

$$G_n, H_n \xrightarrow{p} \text{CATE}(x)$$

$$\text{By CMT, } B_n \xrightarrow{p} \sigma_{\text{ATE}}^2$$

$$\text{By Slutsky's Theorem, since } A_n \xrightarrow{d} N(0, \sigma_{\text{ATE}}^2) \text{ and } B_n \xrightarrow{p} \sigma_{\text{ATE}}^2,$$

$$\sqrt{n}(\hat{\text{ATE}}_n - \text{ATE}) \frac{1}{\sqrt{\hat{\sigma}_{\text{ATE}}^2}} \xrightarrow{d} \frac{1}{\sqrt{\sigma_{\text{ATE}}^2}} N(0, \sigma_{\text{ATE}}^2) \stackrel{d}{=} N(0, 1)$$

Problem 3

a)

b)

c)

d)

Problem 4

a)

$P(W = 1|X)$ is the likelihood of selecting into the policy given a set of observables/characteristics. It represents the probability of policy being selected as a function of the observables.

b)

An example would be winning big at the casino.

c)

A potential confounder would be family wealth. People who are wealthy are less likely to join the military and more likely to have higher lifetime earnings.

d)

The assumption states that the choice of serving the military is unrelated to unobserved determinants of lifetime income that the individual does not observe i.e. the choice of serving in the military is only based on factors the agent observes, here whether they are male or not.

This does not seem plausible here since it is likely that choosing to serve in the military is related to other factors besides your gender such as a family wealth or connections, which would allow you to dodge the draft, a frequent occurrence during the Vietnam War draft in particular.

e)

It does because it means that the event of serving in the military or not is completely random given gender, specifically that there is some random draft for males and none for females.

f)

$C\hat{ATE}(x) = E[g(1, U) - g(0, U)|X]$. The conditional average treatment effect here represents the expected lifetime earnings of an individual given their gender.

g)

Yes it is. Since we assume that W is independent of $U|X$, $C\hat{ATE}(1) = E[g(1, U) - g(0, U)|1] = E[g(1, U)|W = 1, x = 1] - E[g(0, U)|W = 0, x = 1] = E[g(W, U)|W = 1, x = 1] - E[g(W, U)|W = 0, x = 1] = E[Y|W = 1, x = 1] - E[Y|W = 0, x = 1]$

h)

Since we assume that W is independent of $U|X$, $C\hat{ATE}(0) = E[g(1, U) - g(0, U)|0] = E[g(1, U)|W = 1, x = 0] - E[g(0, U)|W = 0, x = 0] = E[g(W, U)|W = 1, x = 0] - E[g(W, U)|W = 0, x = 0] = E[Y|W = 1, x = 0] - E[Y|W = 0, x = 0]$. However, because we do not observe lifetime earnings for females who served in the military since we do not have $W = w, x = 0$, this is not point identified.

i)

$ATE = E[g(1, U) - g(0, U)]$. The average treatment effect is the expected difference in lifetime earnings for an individual if they had served in the military and if they had not.

j)

The ATE is point identified. Since we assume that W is independent of $U|X$, $ATE = E[g(1, U) - g(0, U)] = E[E[g(1, U) - g(0, U)|X]] = E[C\hat{ATE}(x)]$. Since the conditional average treatment effect is point identified, ATE is point identified.

Problem 5

$$\begin{aligned} & \arg \min_{\beta \in \mathbb{R}^{k+1}} E[(E[Y|X] - X^T \beta)^2] \\ & E[(E[Y|X]^2 - 2E[Y|X]X^T \beta - (X^T \beta)^2)] \\ & = E[E[Y|X]^2] - 2E[E[YX^T|X]]\beta - \beta^T E[X^T X]\beta \\ & = E[E[Y|X]^2] - 2E[YX^T]\beta - \beta^T E[XX^T]\beta \end{aligned}$$

FOC: $[\beta]$

$$\begin{aligned} & -2E[YX^T] - \beta^T(E[XX^T] + E[XX^T]) = 0_{k+1}^T \\ & \Rightarrow -2E[YX^T] + \beta^T E[XX^T] = 0_{k+1}^T \\ & \Rightarrow -2E[XY] + E[XX^T]\beta = 0 \\ & \Rightarrow E[XY] = E[XX^T]\beta \\ & \Rightarrow \beta = E[XX^T]^{-1}E[XY] \end{aligned}$$

$$\arg \min_{\beta \in \mathbb{R}^{k+1}} E[(Y - X^T \beta)^2]$$

$$E[(Y^2 - 2YX^T\beta - (X^T\beta)^2)]$$

$$= E[Y^2] - 2E[YX^T]\beta - \beta^T E[XX^T]\beta$$

FOC: $[\beta]$

$$-2E[YX^T] - \beta^T(E[X^TX]^T + E[XX^T]) = 0_{k+1}^T$$

$$\Rightarrow -2E[YX^T] + \beta^T E[XX^T] = 0_{k+1}^T$$

$$\Rightarrow -2E[XY] + E[XX^T]\beta = 0$$

$$\Rightarrow E[XY] = E[XX^T]\beta$$

$$\Rightarrow \beta = E[XX^T]^{-1}E[XY]$$

Problem 6

```
dat<-read.csv("data/bw06.csv")
dat<-as.matrix(dat)
```

```
y<-dat[, "birthweight"]
w<-dat[, "cigsdaily"]
x<-cbind(1,dat[,c("boy", "age", "highschool",
                  "somecollege", "college")])
x_tld<-dat[, "married"]
```

a)

```
ww_inv <- solve(t(w) %*% w)
wy <- t(w) %*% y
beta <- ww_inv %*% wy # 172
```

Beta here represents the change in birthweight associated with an additional cigarette smoked per day by the mother.

b)

```
x <- cbind(x,w)
xx_inv <- solve(t(x) %*% x )
xy <- t(x) %*% y
beta <- xx_inv %*% xy
```

```
beta[length(beta)]
```

```
## [1] -11.06251
```

Beta here represents the change in birthweight associated with an additional cigarette smoked per day by the mother when controlling for gender, age, and level of education.

c)

It does differ because we are controlling for other variables in addition to cigarettes smoked per day.

d)

e)

f)

Problem 7

a)

b)

c)

d)

e)