PSET 3 ECON 21020

Matthew Zhao

Link to the GitHub Repository

Problem 1

a)

True.

$$\sqrt{n}(\frac{1}{n}\sum_{i=1}^{n} X_i - E(X)) = \frac{\sqrt{n}(\frac{1}{n}\sum_{i=1}^{n} X_i - E(X))}{\hat{\sigma}_x} \cdot \hat{\sigma}_x$$

Let $A_n = \frac{\sqrt{n}(\frac{1}{n}\sum_{i=1}^{n} X_i - E(X))}{\hat{\sigma}_x}$ and $B_n = \hat{\sigma}_x$. By CLT, A_n converges in distribution to $N(0,1)$, and by WLLN, $\hat{\sigma}_x$ converges in probability to σ_x. Therefore, by Slutsky, $\frac{\sqrt{n}(\frac{1}{n}\sum_{i=1}^{n} X_i - E(X))}{\hat{\sigma}_x} \cdot \hat{\sigma}_x$ converges in distribution to $\sigma_x N(0,1) = \sqrt{Var(X)}N(0,1) = N(0, Var(X))$.

b)

False. Suppose U is independent of W but U is not independent of X and U takes values 1 and -1 w.p. 0.5. As a result, $E(U|W) = E(U) = 0$ but $E(U|W,X) = E(U|X)$ which does not necessarily equal zero.

c)

$\beta = \frac{Cov(X,Y)}{Var(X)}$ $Cov(X,Y) = E(XY) - E(X)E(Y)$

$E(Y) = E(E(Y|X)) = E(X^2) = \frac{1}{3}$

$E(X) = \frac{1}{2}$

$E(XY) = E(E(XY|X)) = E(XE(Y|X)) = E(X^3) = \frac{1}{4}$

Problem 2

Problem 3

Problem 4

Problem 5

Problem 6

Loading data

```
# Load the ak91.csv data
df <- read.csv("data/ak91.csv")
n <- nrow(df)

# Store years of education and the weekly wage in separate variables
yrs_educ <- df$YRS_EDUC
wkly_wage <- df$WKLY_WAGE
```

a)

$$\hat{\beta_n} = \frac{\frac{1}{n}\sum_{i=1}^{n} Y_i X_i - (\frac{1}{n}\sum_{i=1}^{n} Y_i)(\frac{1}{n}\sum_{i=1}^{n} X_i)}{\frac{1}{n}\sum_{i=1}^{n} X_i^2 - (\frac{1}{n}\sum_{i=1}^{n} X_i)^2}$$

$$\hat{\alpha_n} = \frac{1}{n}\sum_{i=1}^{n} Y_i - (\frac{1}{n}\sum_{i=1}^{n} X_i)\hat{\beta_n}$$

b)

```
beta <- (mean(wkly_wage*yrs_educ) - mean(wkly_wage)*mean(yrs_educ))/(mean(yrs_educ^2)-mean(yrs_educ)^2)

alpha <- mean(wkly_wage) - mean(yrs_educ)*beta
cat(paste0("beta: ",round(beta,2),"\nalpha: ",round(alpha,2)))
```

```
## beta: 29.62
## alpha: 61.2
```

β captures the approximate change in weekly change associated with a unit change in years of education i.e. another year of education.

c)

```
alpha + 16*beta
```

```
## [1] 535.1538
```

The estimate here of $535.15 per week is lower than the solution in the previous pset. This is because in the previous pset, we took the sample of those with college education and calculated the estimator for weekly wage of those with a college education while here we use the entire sample to find the BLP coefficients relating weekly wage and years of education and use those to find an estimate for the weekly wage of those with college education.

d)

```
# sd <- sqrt(mean())
```

Problem 7