STAT 222 Spring 2022 HW4

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Question 1

Q1a — 4 points

Grand mean $\mu = \frac{\sum_{i=1}^{g} n_i \mu_i}{N} = \frac{45 * 3 + 33 * 3 + 60 * 4}{3 + 3 + 4} = 47.4$

```
(45 * 3 + 33* 3 + 60* 4)/(3 + 3 + 4)
## [1] 47.4
```

$\delta^2 = \frac{\sum_{i=1}^{g} n_i (\mu_i - \mu)^2}{\sigma^2} = 35.4$

```
((45-47.4)^2 * 3 + (33-47.4)^2* 3 + (60-47.4)^2* 4)/(36)
## [1] 35.4
```

The power can be calculated using the R code below:

```
pf(qf(1-0.01,3-1,10-3),3-1,10-3,ncp=35.4,lower.tail = F)
## [1] 0.882892
```

Q1b — 4 points

Grand mean $\mu = \frac{\sum_{i=1}^{g} n_i \mu_i}{N} = \frac{45 * n + 33 * n + 60 * n}{n + n + n} = \frac{138n}{3n} = 46$

$\delta^2 = \frac{\sum_{i=1}^{g} n_i (\mu_i - \mu)^2}{\sigma^2} = \frac{n(45-46)^2 + n(33-46)^2 + n(60-46)^2}{36} = \frac{n + 169n + 196n}{36} = \frac{366n}{36} = 10.167n$

```
# qf(alpha, g-1, N-g, lower.tail=F)
n=3
f_crit = qf(0.01, 3-1, 3*n-3, lower.tail=F)
pf(f_crit,3-1,3*n-3,ncp = 366*n/36,lower.tail=F)
## [1] 0.757129
```

```
n=4
f_crit = qf(0.01, 3-1, 3*n-3, lower.tail=F)
pf(f_crit,3-1,3*n-3,ncp = 366*n/36,lower.tail=F)
## [1] 0.967797
```

We only reach a power of at least 0.95 with 4 subjects per group, meaning that we need to use at least 4 subjects per group to get to a power of 0.95.

Question 2

```
insulate = read.table("http://www.stat.uchicago.edu/~yibi/s222/insulate.txt", h=T)
```

Q2a — 4 points

```
library(mosaic)
mean(failtime ~ as.factor(material), data=insulate)
##       1       2       3       4       5
##  159.75    6.25 2941.75 5723.00   10.75
anova(lm(failtime ~ as.factor(material), data=insulate))
## Analysis of Variance Table
##
## Response: failtime
##                      Df    Sum Sq  Mean Sq F value  Pr(>F)
## as.factor(material)   4 103191489 25797872   6.191 0.00379
## Residuals            15  62505657  4167044
```

$$HSD_{\text{tukey}} = \frac{q_\alpha(g, N-g)}{\sqrt{2}} \times \sqrt{\text{MSE}(\frac{1}{n}+\frac{1}{n})}$$

```
(qtukey(1-0.1,5,20-5)/sqrt(2))*(sqrt(4167044*((1/4)+(1/4))))
## [1] 3906.62
insl_means <- insulate %>% group_by(material) %>% summarise(group_mean = mean(failtime))
for (x in 1:(nrow(insl_means)-1)) {
  for (y in (x+1):nrow(insl_means)) {
    print(paste0(as.character(insl_means[x,1]),"-",
                 as.character(insl_means[y,1]), " ",
                 round(abs(insl_means$group_mean[x]-
                             insl_means$group_mean[y]),3),
                 "    ",
                 ifelse(abs(insl_means$group_mean[x]-
                              insl_means$group_mean[y]) < 3906.62,
                        "Not Significant", "Significant")))
  }
}
## [1] "1-2 153.5   Not Significant"
## [1] "1-3 2782   Not Significant"
## [1] "1-4 5563.25   Significant"
## [1] "1-5 149   Not Significant"
## [1] "2-3 2935.5   Not Significant"
## [1] "2-4 5716.75   Significant"
## [1] "2-5 4.5   Not Significant"
## [1] "3-4 2781.25   Not Significant"
## [1] "3-5 2931   Not Significant"
## [1] "4-5 5712.25   Significant"
```

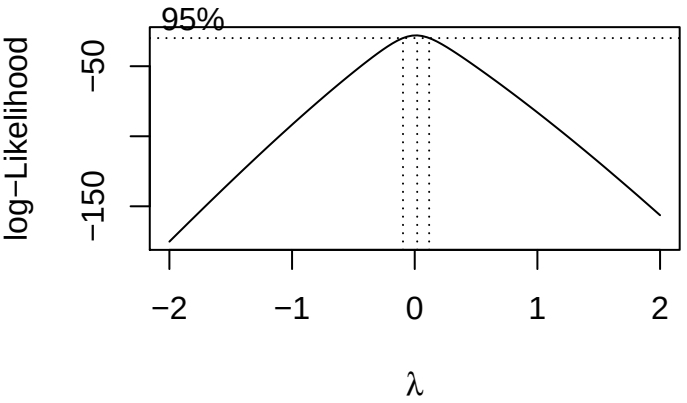
Underline Diagram

```
     2       5       1         3         4
  6.25   10.75  159.75    2941.75   5723.00
```

```
---------------------------------
                        ------------------
```

Q2b — 2 points

```
library(MASS)
boxcox(failtime ~ as.factor(material), data=insulate)
```



Since our interval is centered around 0, we should use the log transformation for the response.

Q2c — 4 points

```
mean(log10(failtime) ~ as.factor(material), data=insulate)
##        1        2        3        4        5
## 2.193879 0.539591 3.351191 3.567298 0.826874
anova(lm(log10(failtime) ~ as.factor(material), data=insulate))
## Analysis of Variance Table
##
## Response: log10(failtime)
##                     Df Sum Sq Mean Sq F value      Pr(>F)
## as.factor(material)  4  31.13   7.783   37.66 0.000000118
## Residuals           15   3.10   0.207
```

```
(qtukey(1-0.1,5,20-5)/sqrt(2))*(sqrt(0.207*((1/4)+(1/4))))
## [1] 0.870709
insl_means <- insulate %>%
  mutate(failtime = log10(failtime)) %>%
  group_by(material) %>%
  summarise(group_mean = mean(failtime))
for (x in 1:(nrow(insl_means)-1)) {
  for (y in (x+1):nrow(insl_means)) {
    print(paste0(as.character(insl_means[x,1]),"-",
                 as.character(insl_means[y,1]), " ",
                 round(abs(insl_means$group_mean[x]-
                             insl_means$group_mean[y]),3),
                 "    ",
                 ifelse(abs(insl_means$group_mean[x]-
                              insl_means$group_mean[y]) < 0.870709,
                        "Not Significant", "Significant")))
```

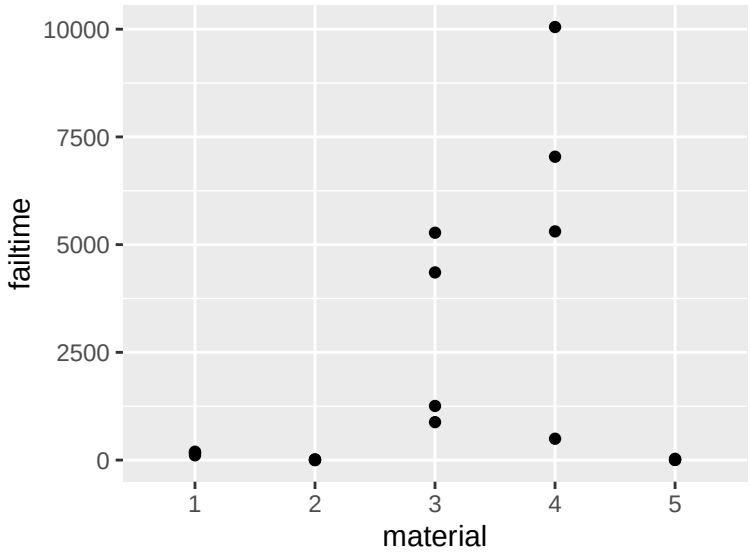
```
  }
}
## [1] "1-2 1.654   Significant"
## [1] "1-3 1.157   Significant"
## [1] "1-4 1.373   Significant"
## [1] "1-5 1.367   Significant"
## [1] "2-3 2.812   Significant"
## [1] "2-4 3.028   Significant"
## [1] "2-5 0.287   Not Significant"
## [1] "3-4 0.216   Not Significant"
## [1] "3-5 2.524   Significant"
## [1] "4-5 2.74   Significant"
```

Underline Diagram

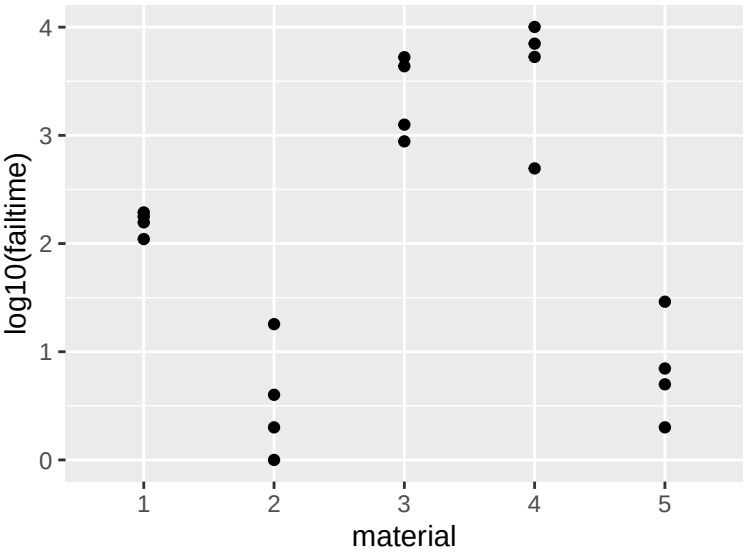
```
     2         5          1         3         4
  1.24245   1.90395    5.05159   7.71640   8.21401
  ------------------           -------------------
```

Q2d — 3 points

```
ggplot(data = insulate, aes(x=as.factor(material),
                            y=failtime)) +
  geom_point() +
  labs(x="material", y="failtime")
```




```
ggplot(data = insulate, aes(x=as.factor(material),
                            y=log10(failtime))) +
  geom_point() +
  labs(x="material", y="log10(failtime)")
```



After transforming the response, the standard deviation across groups appears to be more similar than before, with the standard deviations of the groups fairly close at a glance (besides group 1).

Q2e — 4 points

After the transformation, 1-2, 1-3, 1-5, 2-3, and 3-5 all become significant. All of the previously significant pairs are still significant. The discrepancies are due to the fact that prior to transformation, the variance among groups with high failure times was extremely high, resulting in a high MSE which caused the HSD to be high, making it difficult to reject the null. After transformation, the variances were stabilized and that resulted in a lower MSE and HSD, making previously not significant pairs significant. I trust the transformed results more since they allow for a valid comparison using the HSD with groups having more similar variances than before where the variation in variances made comparisons unatural.