# STAT 222 Spring 2022 HW8

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```r
warpbreaks = read.table("http://www.stat.uchicago.edu/~yibi/s222/warpbreaks.txt", h=T)
warpbreaks$wool = as.factor(warpbreaks$wool)
warpbreaks$tension = factor(warpbreaks$tension, labels=c("L","M","H"))
lm1 = lm(breaks ~ wool*tension, data=warpbreaks)
```

## Q1 — 5 points

We can use Tukey's HSD to control the FWER, where the HSD is given as $\frac{q_{g,dfE,\alpha}}{\sqrt{2}} \times \sqrt{\text{MSE}(\frac{1}{r}+\frac{1}{r})}$

```r
anova(lm1)
## Analysis of Variance Table
##
## Response: breaks
##              Df Sum Sq Mean Sq F value   Pr(>F)
## wool          1    451   450.7   3.765 0.058213 .
## tension       2   2034  1017.1   8.498 0.000693 ***
## wool:tension  2   1003   501.4   4.189 0.021044 *
## Residuals    48   5745   119.7
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Calculating HSD:

```r
mse<-119.7
qval<-qtukey(1-0.02,6,48)/sqrt(2)
qval * sqrt(mse*(2/9))
## [1] 17.137
```

```r
sort(mean(breaks~wool+tension,data=warpbreaks))
##     2.H     1.M     1.H     2.L     2.M     1.L
## 18.7778 24.0000 24.5556 28.2222 28.7778 44.5556
```

```
    2.H      1.M      1.H      2.L      2.M      1.L
 18.7778  24.0000  24.5556  28.2222  28.7778  44.5556
 -------------------------------------------------
                              -----------------------------
```

## Q2 — 5 points

We can again use Tukey's HSD

```
anova(lm1)
## Analysis of Variance Table
##
## Response: breaks
##              Df Sum Sq Mean Sq F value    Pr(>F)
## wool          1    451   450.7   3.765 0.058213 .
## tension       2   2034  1017.1   8.498 0.000693 ***
## wool:tension  2   1003   501.4   4.189 0.021044 *
## Residuals    48   5745   119.7
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Calculating HSD:

```
mse<-119.7
qval<-qtukey(1-0.02,3,48)/sqrt(2)
qval * sqrt(mse*(2/(9*2)))
## [1] 10.192
```

```
sort(mean(breaks~tension,data=warpbreaks))
##       H       M       L
## 21.6667 26.3889 36.3889
```

```
      H         M        L
 21.6667   26.3889   36.3889
 -------------------
           -------------------
```

## Q3 — 5 points

We again use Tukey.

```
anova(lm1)
## Analysis of Variance Table
##
## Response: breaks
##              Df Sum Sq Mean Sq F value    Pr(>F)
## wool          1    451   450.7   3.765 0.058213 .
## tension       2   2034  1017.1   8.498 0.000693 ***
## wool:tension  2   1003   501.4   4.189 0.021044 *
## Residuals    48   5745   119.7
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
lm1emmean <- emmeans(lm1,~tension:wool)
summary(contrast(lm1emmean, method="pairwise", adjust="tukey"),
infer=c(T,F), level=0.98)
##  contrast  estimate   SE df lower.CL upper.CL
##  L 1 - M 1   20.556 5.16 48    3.419     37.7
##  L 1 - H 1   20.000 5.16 48    2.864     37.1
##  L 1 - L 2   16.333 5.16 48   -0.803     33.5
##  L 1 - M 2   15.778 5.16 48   -1.358     32.9
##  L 1 - H 2   25.778 5.16 48    8.642     42.9
##  M 1 - H 1   -0.556 5.16 48  -17.692     16.6
##  M 1 - L 2   -4.222 5.16 48  -21.358     12.9
##  M 1 - M 2   -4.778 5.16 48  -21.914     12.4
##  M 1 - H 2    5.222 5.16 48  -11.914     22.4
##  H 1 - L 2   -3.667 5.16 48  -20.803     13.5
##  H 1 - M 2   -4.222 5.16 48  -21.358     12.9
##  H 1 - H 2    5.778 5.16 48  -11.358     22.9
##  L 2 - M 2   -0.556 5.16 48  -17.692     16.6
##  L 2 - H 2    9.444 5.16 48   -7.692     26.6
##  M 2 - H 2   10.000 5.16 48   -7.136     27.1
##
## Confidence level used: 0.98
## Conf-level adjustment: tukey method for comparing a family of 6 estimates
```

$\mu_{1L} - \mu_{2L}$: (-0.803,33.5)

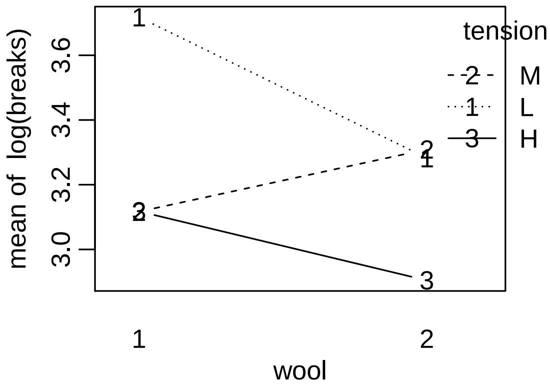
$\mu_{1M} - \mu_{2M}$: (-21.914,12.4)

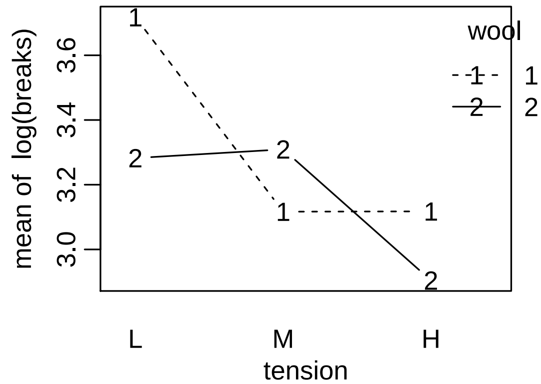
$\mu_{1H} - \mu_{2H}$: (-11.358,22.9)

We can conclude that there could be an interaction between wool type and level of tension since the CIs change with the level of tension. However, since the CIs all overlap, we cannot be sure that the interaction term is not insignificantly different than zero i.e. we cannot conclude that there is an interaction.

## Q4 — 6 points

```
par(mai=c(.6,.6,.1,.3),mgp=c(2,.6,0))
with(warpbreaks, interaction.plot(wool, tension, log(breaks), type="b"))
with(warpbreaks, interaction.plot(tension, wool, log(breaks), type="b"))
```




**i)** $C_1$: $\sum_i c_{ij}$

j=L: $(1)\mu_{1L} + (-1)\mu_{2L} = 1 + (-1) = 0$

j=M: $(-1)\mu_{1M} + (1)\mu_{2M} = (-1) + 1 = 0$

$\sum_j c_{ij}$

i=1: $(1)\mu_{1L} + (-1)\mu_{1M} = 1 + (-1) = 0$

i=2: $(-1)\mu_{2L} + (1)\mu_{2M} = (-1) + 1 = 0$

$C_2$: $\sum_i c_{ij}$

j=H: $(1)\mu_{1H} + (-1)\mu_{2H} = 1 + (-1) = 0$

j=M: $(-1)\mu_{1M} + (1)\mu_{2M} = (-1) + 1 = 0$

$\sum_j c_{ij}$

i=1: $(1)\mu_{1H} + (-1)\mu_{1M} = 1 + (-1) = 0$

i=2: $(-1)\mu_{2H} + (1)\mu_{2M} = (-1) + 1 = 0$

$C_3$: $\sum_i c_{ij}$

j=L: $(0.5)\mu_{1L} + (-0.5)\mu_{2L} = 0.5 + (-0.5) = 0$

j=M: $(-1)\mu_{1M} + (1)\mu_{2M} = (-1) + 1 = 0$

j=H: $(-0.5)\mu_{1H} + (0.5)\mu_{2H} = (-0.5) + 0.5 = 0$

$\sum_j c_{ij}$

i=1: $(0.5)\mu_{1L} + (0.5)\mu_{1H}(-1)\mu_{1M} = 0.5 + 0.5 + 1 = 0$

i=2: $(-0.5)\mu_{2L} + (-0.5)\mu_{2H} + (1)\mu_{2M} = (-0.5) + (-0.5) + 1 = 0$

**ii)** We use Scheffe since we are performing contrasts after observing the data/based off of prior analysis.

```
anova(lm1)
## Analysis of Variance Table
##
## Response: breaks
##              Df Sum Sq Mean Sq F value   Pr(>F)
```

```
## wool           1    451   450.7   3.765 0.058213 .
## tension        2   2034  1017.1   8.498 0.000693 ***
## wool:tension   2   1003   501.4   4.189 0.021044 *
## Residuals     48   5745   119.7
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
lm1emmean <- emmeans(lm1,~tension:wool)
summary(contrast(lm1emmean, method=list(C_1 = c(1,-1,0,-1,1,0), C_2 = c(0,-1,1,0,1,-1),C_3 =
infer=c(F,T), level=0.96)
##  contrast estimate   SE df t.ratio p.value
##  C_1          21.1 7.29 48   2.895  0.0210
##  C_2          10.6 7.29 48   1.447  0.3588
##  C_3          15.8 6.32 48   2.507  0.0522
##
## P value adjustment: scheffe method with rank 2
```

At FWER = 0.04, only the first contrast is significant. We can conclude that the change in effect of wool type on breaks is nonzero from low to medium.

### Q5 — 1 point

The upper bound of the FWER for the entire analysis is no more than the sum of the FWER for each family so it is $0.02 + 0.02 + 0.02 + 0.04 = 0.1$.

### Q6 — 3 points

Bonferroni.

```
anova(lm1)
## Analysis of Variance Table
##
## Response: breaks
##              Df Sum Sq Mean Sq F value   Pr(>F)
## wool           1    451   450.7   3.765 0.058213 .
## tension        2   2034  1017.1   8.498 0.000693 ***
## wool:tension   2   1003   501.4   4.189 0.021044 *
## Residuals     48   5745   119.7
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
lm1emmean <- emmeans(lm1,~tension:wool)
summary(contrast(lm1emmean, method=list(C_1 = c(1,-1,0,-1,1,0), C_2 = c(0,-1,1,0,1,-1),C_3 =
infer=c(F,T), level=0.96)
##  contrast estimate   SE df t.ratio p.value
##  C_1          21.1 7.29 48   2.895  0.0171
##  C_2          10.6 7.29 48   1.447  0.4630
##  C_3          15.8 6.32 48   2.507  0.0469
```

```
##
## P value adjustment: bonferroni method for 3 tests
```