Let $F \in \mathbb{R}^{2\times2}$, $F = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Proove $tr(F^TF - I) = \|F\|_F^2 - 2$

Direct proof:

$$tr\left(\begin{bmatrix} a & c \\ b & d \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = tr\left(\begin{bmatrix} a^2+c^2-1 & ab+cd \\ ab+cd & b^2+d^2-1 \end{bmatrix}\right)$$

$$\Rightarrow a^2+b^2+c^2+d^2-2 = \|F\|_F^2 - 2 \quad \text{where } \|F\|_F = \sqrt{a^2+b^2+c^2+d^2}$$

$$\therefore tr(F^TF - I) = \|F\|_F^2 - 2$$

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