Mathematical Neuroscience; Tutorial 4

H.G.E. Meijer

February 26, 2025

These are adaptations of exercises 2, 9 and 5 from Chapter 4 (in that order).

1. In this exercise, we explore the T-type current in more detail. The goal is to compute the orbits to obtain the sketches drawn in the lecture. The equations are given below, condensed for use in pplane9. A separate driver PlanarRebound is also provided.

```
Equations for V & h:
V'=-.05*(V-EL)-minf^2*h*cfedrive+I0;
h'=(hinf-h)/tauh;
Parameters & Expressions:
I0=0.0;
EL=-60;
cfedrive=2.2557*V*(1e-4-2*exp(-.0779*V))/(1-exp(-.0779*V));
minf=1/(1+exp(-(V+59)/6.2));
hinf=1/(1+exp((V+83)/4));
tauh=22.7+.27/(exp((V+48)/4)+exp(-(V+407)/50));
```

   - For $I_0 = 0$, decrease E_L from -60 mV to -85 mV in small steps and simulate. The goal is to observe oscillations for some values and to get a feeling for the phase plane. Also, check the position of the fixed point on the nullclines. Is it on the left branch of the V-nullcline or somewhere else?
   - Simulate rebound behaviour by starting from $I_0 = 0.0$, next inhibit the neuron during some time with $I_0 = -1.0$, and then release it setting $I_0 = 0.0$ again. Explain the orbit you observe by describing which nullcline the orbit is following or whether the orbit jumps to another part.
   - For $I_0 = 0$, Compute a one-parameter bifurcation diagram varying E_L. Identify two Hopf bifurcations and determine their criticality. Next, determine the range of E_L for which you find stable oscillations. Conclude that changing E_L may induce subthreshold oscillations, i.e., the potential hardly reaches the right branch of the V-nullcline.
   - (Extra) We may observe the same behaviour by changing I_0. In fact, as an extra, create a two-parameter diagram in the (E_L, I_0)-plane showing two disconnected Hopf bifurcation curves. These two curves are straight lines, which should be no surprise considering the differential equations.

2. Consider the following reduced neuronal model with an applied current I, an inward potassium current I_{Kir} and a chloride leak current I_L,

$$C\dot{V} = -g_{\text{Kir}} h_\infty(V)(V - E_K) - g_L(V - E_L) + I,$$

   with instantaneous sigmoidal gate $h_\infty(V) = \left(1 + \exp\left((V_{1/2} - V)/k\right)\right)^{-1}$. The parameter values are: $C = 1$, $g_L = 0.1$, $E_L = -60$, $g_{\text{Kir}} = 0.1$, $E_K = -85$, $V_{1/2} = -71$ and $k = -0.8$. The first goal is to draw a bifurcation diagram for this nonlinear one-dimensional model with I as a parameter.

   - You cannot simply solve for steady-state values of V given I, but the other way round works, i.e., compute $I(V)$ and plot $I(V)$ on the horizontal axis, and V on the vertical axis.
   - Determine the stability of the equilibria.

- Conclude that this model shows bistability.

Next, we add passive uptake of potassium

$$\tau \dot{K}_{\text{out}} = \alpha I_{\text{Kir}} + K_0 - K_{\text{out}},$$

with $\alpha = 0.2$, $K_0 = 0.1$ and $\tau = 600$ ms.

- Decrease $I_0 = 0$ to $I_0 = -0.4$ to observe oscillations. Characterize the corresponding bifurcations and cycles.
- The variation in K_{out} effectively modulates the applied current I_0 over time. Compute this modulation $I_{\text{eff}}(t)$ from the simulation, and plot the orbit in the (I, V)-plane you obtained above. Explain how the orbit follows the branch of equilibria.

3. (Adaptation of Exc 5) Consider the reduced Connor-Stevens model; the code is given online (ConnorSteves_Red.m).

   - Fix $g_A = 4$, and then increase the applied current I_{app}. Explain that the neuron is of Class II for $g_A = 4$ by looking at the (instantaneous) frequency for values of I_{app} near the switch from rest to spiking.
   - The neuron is still Class II for $g_A = 40$, but the onset of spiking involves a different mechanism.
   - Compute a bifurcation diagram for $g_A = 4$ and $g_A = 40$. Use this diagram to corroborate the findings above.
   - (Extra) Determine the phase plane for both values of g_A. This case is more involved as there are isolated branches of the V-nullcline.
   - (Extra) Modify the code by uncommenting and adding variables to compare the reduced and the full model.