Tutorial: a bursting cartoon

A Morris-Lecar model,

Consider the Morris-Lecar model

$$\begin{aligned} C_m \dot{v} &= I_{app} - g_L(v - E_L) - g_K n(v - E_K) - g_{Ca} m_\infty(v)(v - E_{Ca}) \\ \dot{n} &= \phi(n_\infty(v) - n)/\tau_n(v) \end{aligned} \qquad (MLM)$$

where

$$\begin{aligned} m_\infty(v) &= \frac{1}{2}\left[1 + \tanh\left(\frac{v - V_1}{V_2}\right)\right] \\ n_\infty(v) &= \frac{1}{2}\left[1 + \tanh\left(\frac{v - V_3}{V_4}\right)\right], \qquad \tau_n(v) = \left[\cosh\left(\frac{v - V_3}{2V_4}\right)\right]^{-1}, \end{aligned}$$

In this exercise, all parameters except I_{app} are fixed as in the “Homoclinic” column of Table 3.1 in the book by Ermentrout and Terman, reported below for your reference.

Parameter	Hopf	SNLC	Homoclinic
ϕ	0.04	0.067	0.23
g_{Ca}	4.4	4	4
V_3	2	12	12
V_4	30	17.4	17.4
E_{Ca}	120	120	120
E_K	−84	−84	−84
E_L	−60	−60	−60
g_K	8	8	8
g_L	2	2	2
V_1	−1.2	−1.2	−1.2
V_2	18	18	18
C_M	20	20	20

SNLC saddle–node on a limit cycle

Table 3.1, from the book by Ermentrout and Terman

Question 1

Compute a bifurcation diagram of steady states and periodic orbits of (MLM) in the parameter I_app. If you want to progress without computing the bifurcation diagram you can it here (.fig) or here (.png). Classify all the bifurcations found in the bifurcation diagram.

Question 2

Let us time-step (MLM) for fixed values of I_{app}. Produce numerical evidence (together with exemplary values $I_1 < I_2 < I_3$) in support of the following statements:

S1: for $I_{app}(t) \equiv I_1$, the solution of (MLM) tends asymptotically to an equilibrium with $\lim_{t\to\infty} v(t) = v_* < 0$.

S2: for $I_{app}(t) \equiv I_2$, the solution of (MLM) tends asymptotically to an equilibrium with $\lim_{t\to\infty} v(t) = v_* < 0$, or to a periodic orbit, depending on initial conditions.

S1: for $I_{app}(t) \equiv I_3$, the solution of (MLM) tends asymptotically to an equilibrium with $\lim_{t\to\infty} v(t) = v_* > 0$.

Question 3

Provide numerical evidence that, if a harmonic, slow current $I_{app}(t)$ is applied to (MLM), then the model can produce bursting. In other words, find values A, I_0, ϵ such that the applied current

$$I_{app}(t) = I_0 + A\sin(\epsilon t)$$

produces bursting behaviour. This exercise requires some detective work to guess good parameters $I_0, A. \epsilon$. In your search, make use of the previous exercise, Q2, as well as the bifurcation diagram in Q1. It may be useful to plot solutions $v(t), n(t)$ together with the candidate $I_{app}(t)$. Also, it may help superimposing $v(t), I(t)$ on top of the bifurcation diagram of Q1 (this is why we provide you with a .fig file, which you can load, and modify by adding the orbit projected on the (v, I_{app})-plane.