HW № 1

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BS18-02

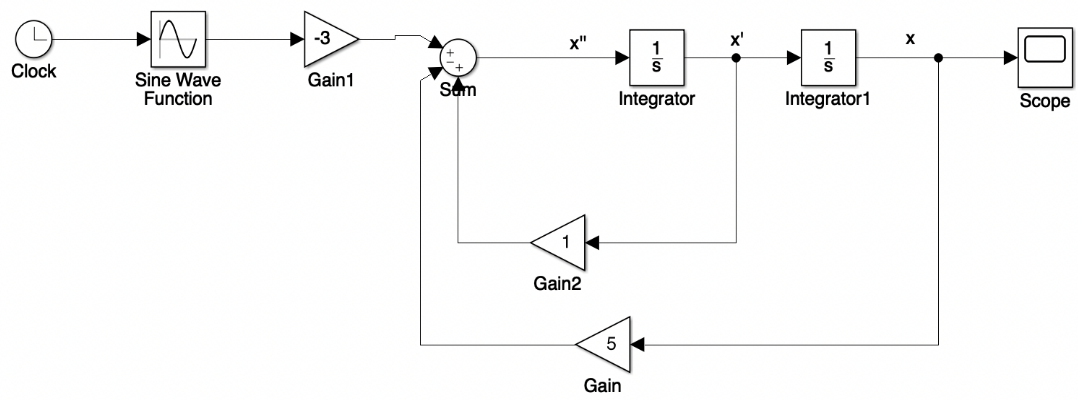
Variant (k)

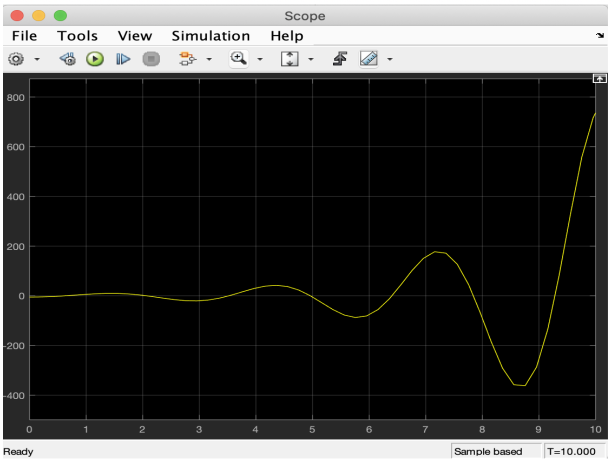
Problem 1

$x'' = x' - 5x - 3\sin t$

$x'(0) = 0, x(0) = -5$

Part A





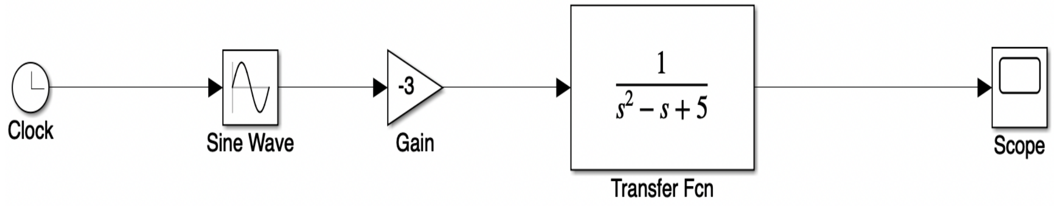
Part B

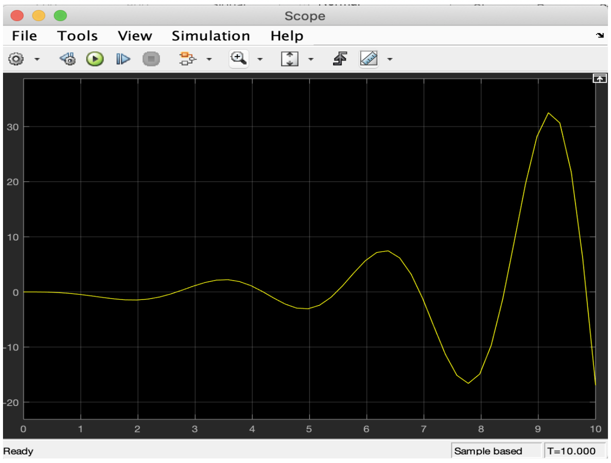
Calculations of transfer function.

$\frac{\partial}{\partial t} = p$

$p^2 x - px + 5x + 3\sin t = 0$

$$(p^2 - p + 5)x = -3\sin t$$
$$x = \frac{1}{p^2-p+5} * (-3\sin t)$$



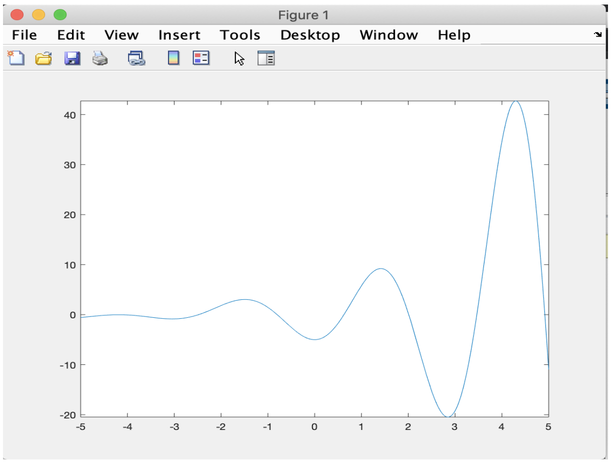


Part C

Code for solving differential equation

```
>> syms x(t)
>> Dx = diff(x,t);
>> eqn = diff(x,t,2) == Dx - 5 * x - 3 * sin(t);
>> cond = [x(0)==-5, Dx(0)==0];
>> xSol(t) = dsolve(eqn,cond)

xSol(t) =

- (82*exp(t/2)*cos((19^(1/2)*t)/2))/17 - cos((19^(1/2)*t)/2)*((3*cos(t - ↩
    (19^(1/2)*t)/2))/34 + (3*cos(t + (19^(1/2)*t)/2))/34 + (6*sin(t - ↩
    (19^(1/2)*t)/2))/17 + (6*sin(t + (19^(1/2)*t)/2))/17 + (9*19^(1/2)*cos↩
    (t - (19^(1/2)*t)/2))/323 - (9*19^(1/2)*cos(t + (19^(1/2)*t)/2))/323 +↩
     (21*19^(1/2)*sin(t - (19^(1/2)*t)/2))/646 - (21*19^(1/2)*sin(t + ↩
    (19^(1/2)*t)/2))/646) - (3*19^(1/2)*sin((19^(1/2)*t)/2)*((sin(t↩
    *(19^(1/2)/2 - 1))/2 + cos(t*(19^(1/2)/2 - 1))*(19^(1/2)/2 - 1))↩
    /((19^(1/2)/2 - 1)^2 + 1/4) - (sin(t*(19^(1/2)/2 + 1))/2 + cos(t↩
    *(19^(1/2)/2 + 1))*(19^(1/2)/2 + 1))/((19^(1/2)/2 + 1)^2 + 1/4)))/19 -↩
     (106*19^(1/2)*exp(t/2)*sin((19^(1/2)*t)/2))/(19*(19^(1/2) - 6)↩
    *(19^(1/2) + 6))
```

```
>>
>> fplot(xSol)
```

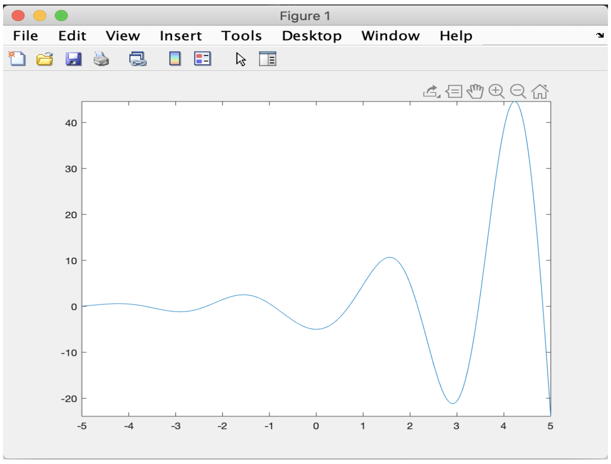


Part D

Code for solving differential equation with Laplace

```
>> syms x t s
>> f = -3 * sin(t)
f =
-3*sin(t)
>> F = laplace(f, t, s)
F =
-3/(s^2 + 1)
>> X1 = s * x + 5
X1 =
s*x + 5
>> X2 = s * X1
X2 =
s*(s*x + 5)
>> Sol = solve(X2 - X1 + 5 * x + 3 * sin(t), x)
```

```
Sol =

-(5*s + 3*sin(t) - 5)/(s^2 - s + 5)

>> sol = ilaplace(Sol,s,t)

sol =

-5*exp(t/2)*(cos((19^(1/2)*t)/2) + (2*19^(1/2)*sin((19^(1/2)*t)/2)*((3*sin(t))/5 - 1/2))/19)

>> fplot(sol)
```



Problem 2

$$\begin{cases} x'' - x' + 5 = t + 1 \\ y = 2x + x' \end{cases} \tag{1}$$

$x'' = x' + t - 4$

$$\begin{bmatrix} x' \\ x'' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ x' \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} * (t - 4)$$

$$\begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} 2 & 1 \end{bmatrix} * \begin{bmatrix} x \\ x' \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} * u$$

Problem 3

$$\begin{cases} x'''' - x''' - x'' + 3x' + x = 3u_1 + u_2 \\ y = x' + u_1 \end{cases} \tag{2}$$

$x'''' = x''' + x'' - 3x' - x + 3u_1 + u_2$

$$\begin{bmatrix} x' \\ x'' \\ x''' \\ x'''' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -3 & 1 & 1 \end{bmatrix} * \begin{bmatrix} x \\ x' \\ x'' \\ x''' \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} * \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} x \\ x' \\ x'' \\ x''' \end{bmatrix} + \begin{bmatrix} 1 & 0 \end{bmatrix} * \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Problem 4

```
import numpy as np

# return matrix A and b
# 1. divide by a_k
# 2. express y^(k) derivative
# 3. create matrix and replace the last row with reverse coefficients
def convert_ODE_to_SS(coefs, b0):
  b0 /= coefs[0]
  coefs = coefs / coefs[0]
  A = np.eye(len(coefs), k=1)
  A[-1] = coefs[::-1]
  B = np.zeros((3, 1))
  B[-1][0] = b0
  return A, B


coef = np.array([3, 2, 1])
convert_ODE_to_SS(coef, 3)

(array([[0.        , 1.        , 0.        ],
        [0.        , 0.        , 1.        ],
        [0.33333333, 0.66666667, 1.        ]]), array([[0.        ],
        [0.        ],
        [0.33333333]]))
```

Problem 5

```

def pend(y, t, coefs, u):
  '''
```

```
    y: np array of shape (N) in order x(0), x'(0), x''(0) ...
    t: range
    coefs: np array of shape (N) in order a1*x, a2*x', a3*x'' ...
    u: function of t
    '''
    dydt = np.append(y[1:], coefs.dot(y) + u(t))
    return dydt


from scipy.integrate import odeint
from math import sin

coefs = np.array([-5, 1])
y0 = np.array([-5, 0])
u = lambda t: -3 * sin(t)
t = np.linspace(0, 10, 101)
sol = odeint(pend, y0, t, args=(coefs, u))



import matplotlib.pyplot as plt

plt.plot(t, sol[:, 0], 'b', label='x(t)')
plt.legend(loc='best')
plt.xlabel('t')
plt.grid()
plt.show()
```