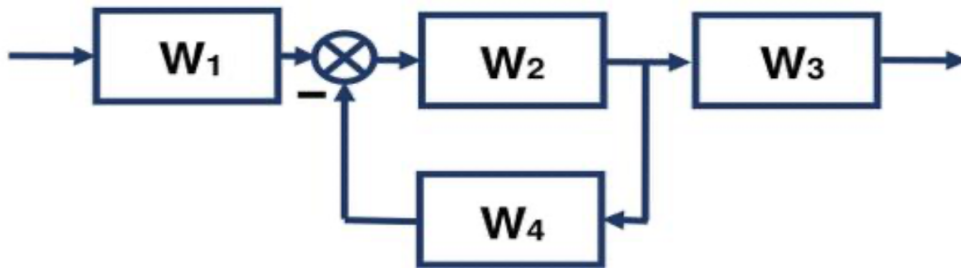


HW № 2

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BS18-02
Variant (c)

Problem 1

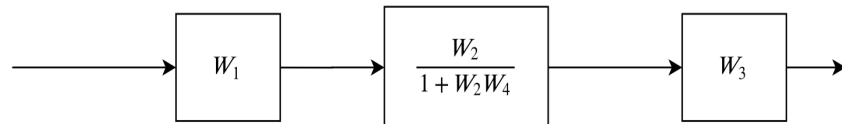


$$W_1 = \frac{2}{s^2 + s - 2}, W_2 = \frac{1}{3s + 2}, W_3 = \frac{s + 1}{s + 0.3}, W_4 = \frac{1}{s + 0.2}$$

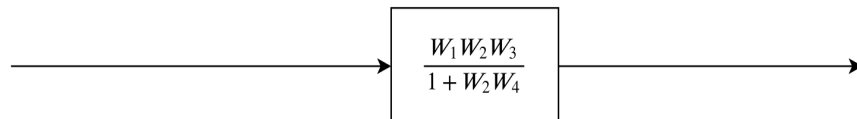
Part A

Description: Calculate the total Transfer Function of the system.

Solution: **Step I.** Reduce negative feedback loop:



Step II. Reduce series connections:



Step III. Substitute initial functions:

$$\frac{W_1 W_2 W_3}{1 + W_2 W_4} = \frac{\frac{2}{s^2 + s - 2} * \frac{1}{3s + 2} * \frac{s + 1}{s + 0.3}}{1 + \frac{1}{3s + 2} * \frac{1}{s + 0.2}}$$

$$\frac{20(s + 1)(5s + 1)}{(10s + 3)(s^2 + s - 2)(15s^2 + 13s + 7)}$$

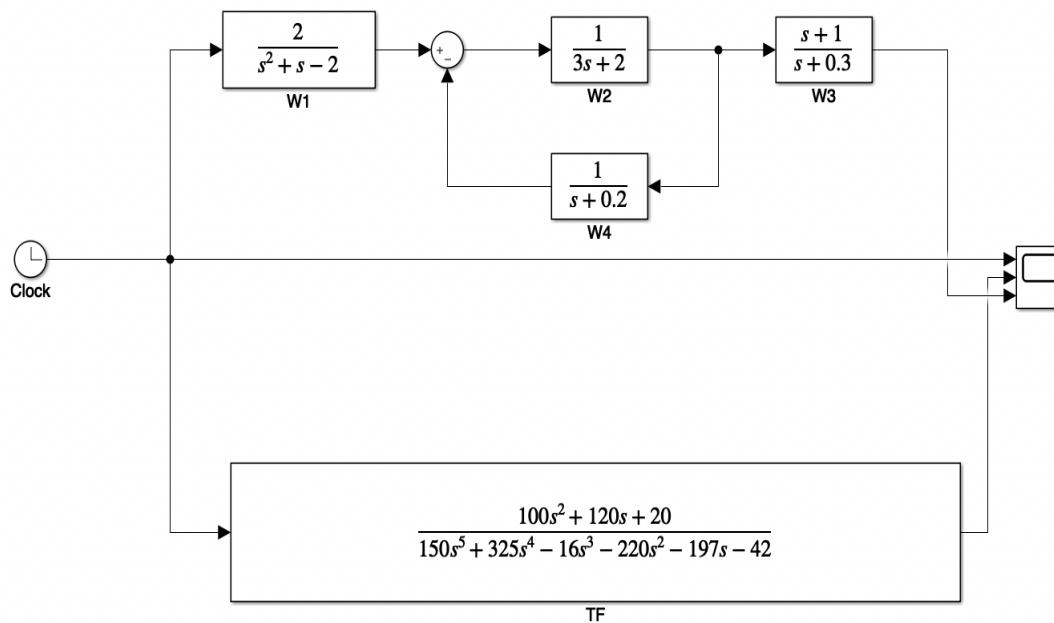
Answer : $\frac{100s^2 + 120s + 20}{150s^5 + 325s^4 - 16s^3 - 220s^2 - 197s - 42}$

Part B

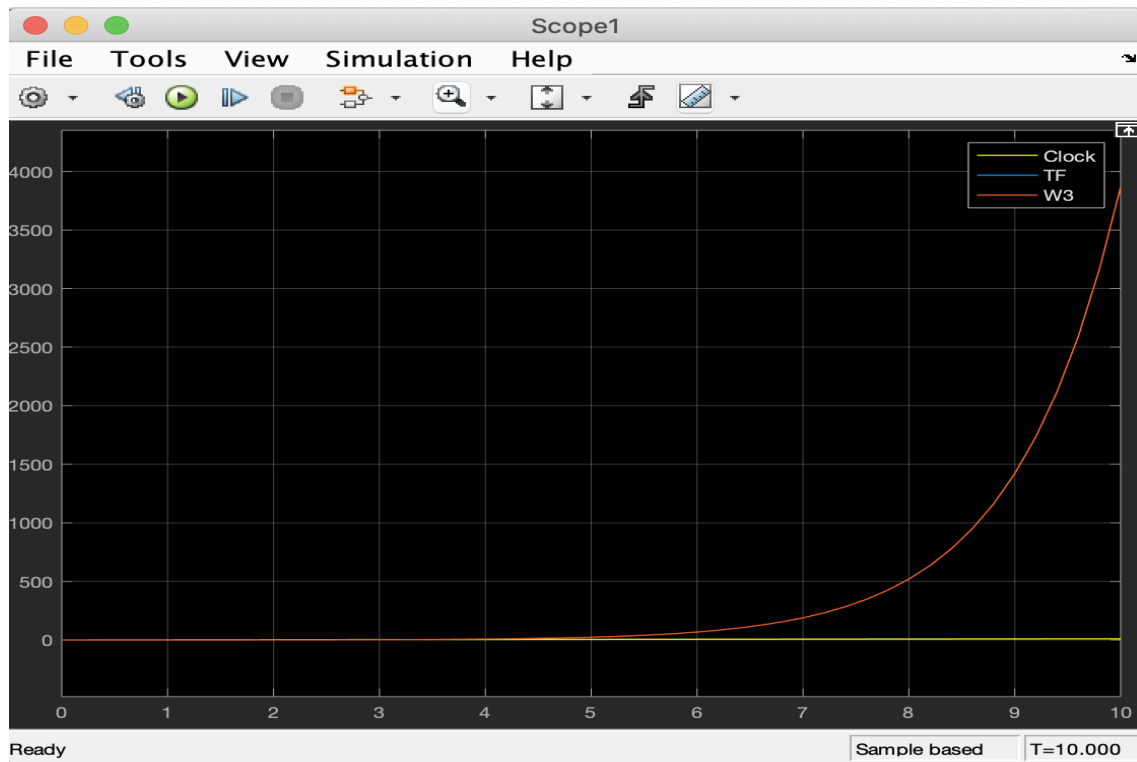
Description: Build initial system shown in the block diagram and simplified in one Simulink schema and analyze its Step, Impulse and Frequency responses. Results should have a schema with both systems and 3 Scope plots(for each input). Each plot should have 3 lines - input signal, and two outputs from each system.

Solution: In both plots Frequency and Step, initial TF and the total TF are close to each other. However, on Impulse there is a slight deviation.

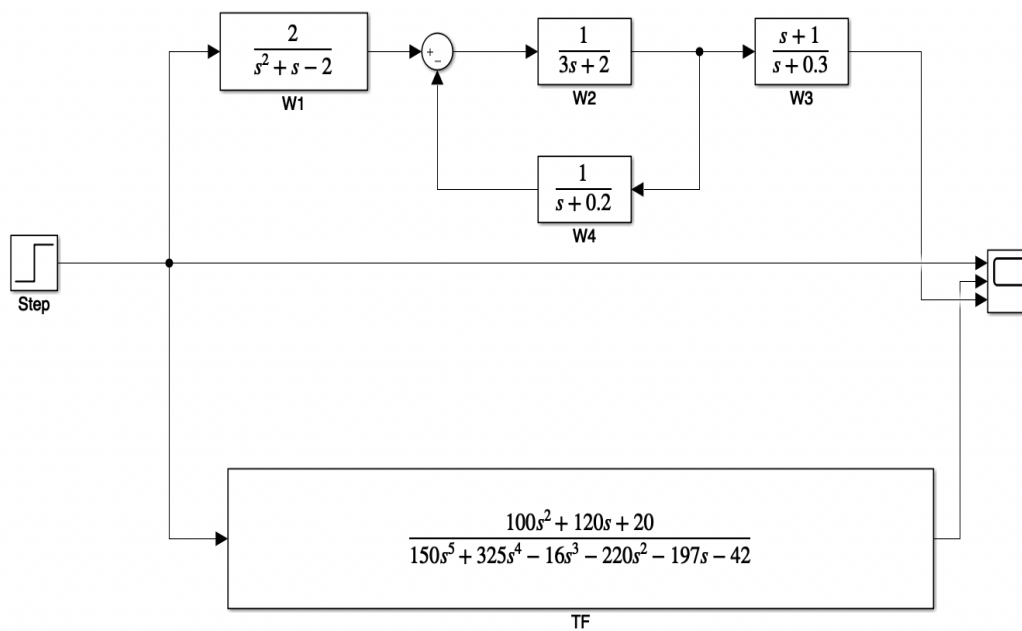
With Frequency Input



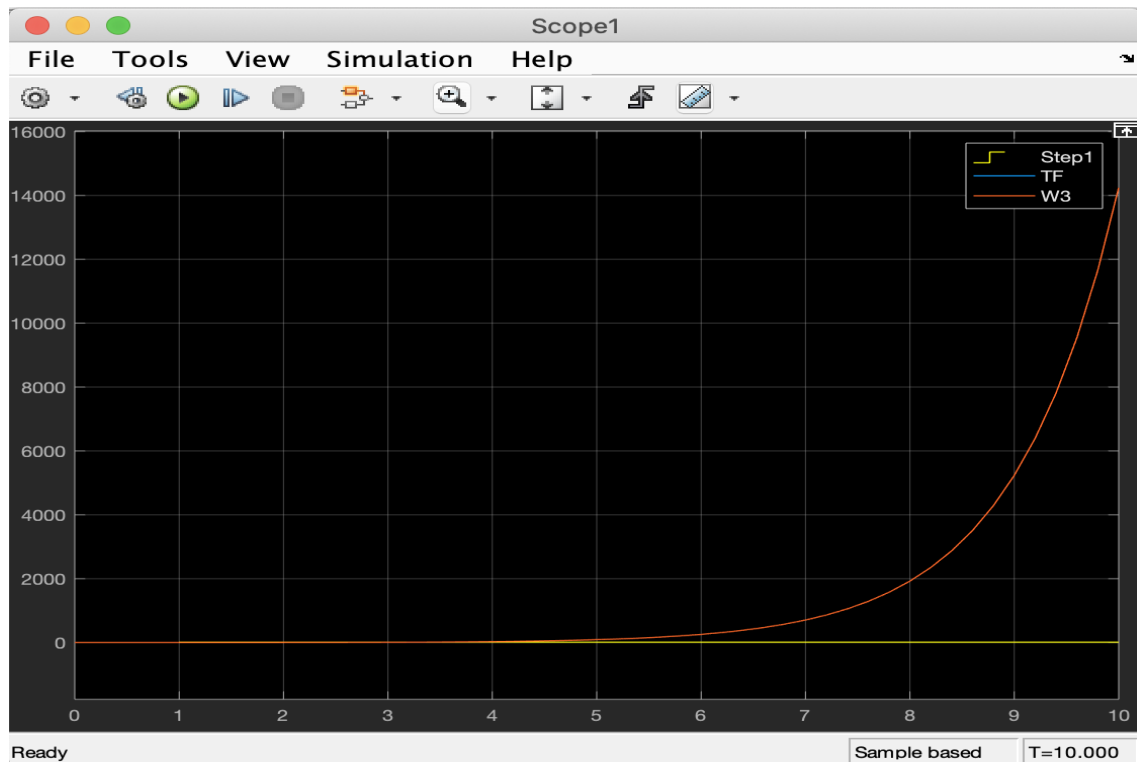
Frequency scope



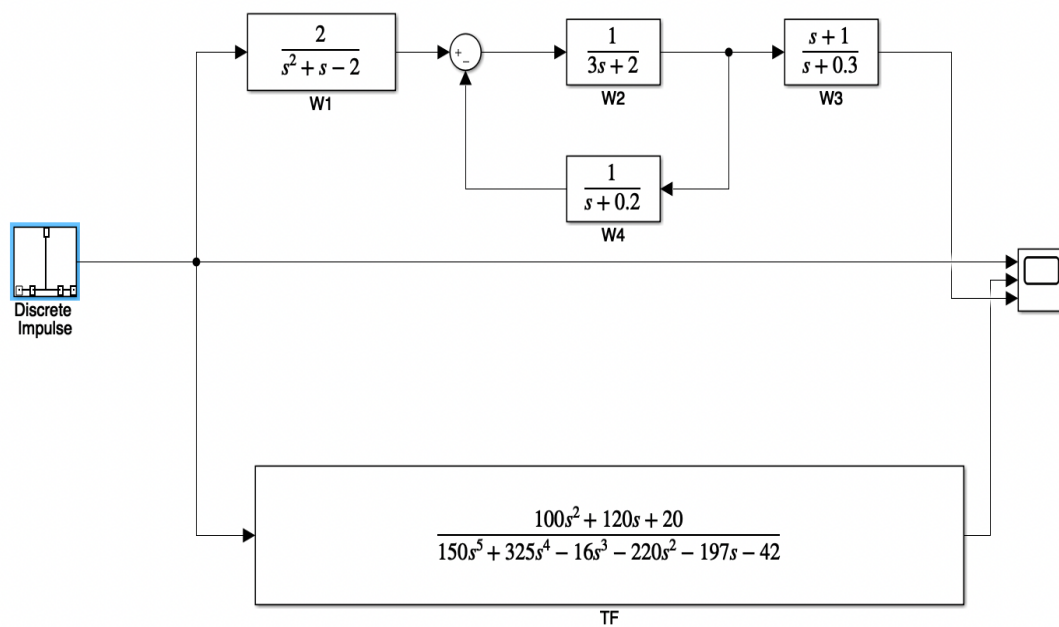
With step Input



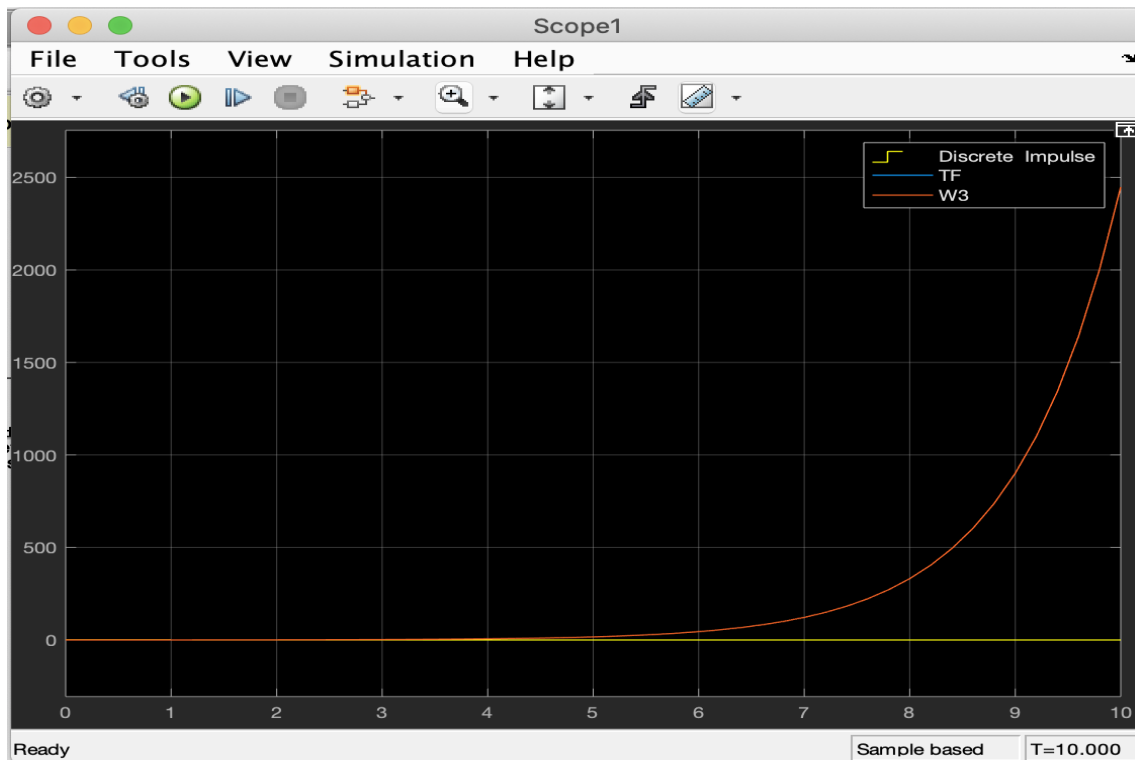
Step scope



With Impulse Input



Impulse scope



Part C

Description: For one of the inputs (write down what you choose) generate a Bode and Pole-Zero map plots. Put plots and result - stable or unstable is system and why - in the report.

Solution: Definition of stability for our problem:

$$\text{Stable} : f(t) \rightarrow 0, t \rightarrow \infty$$

$$f(t) = L^{-1}(tf)$$

L^{-1} is inverse Laplace transform.

From this rule, all roots of inverse denominator should be negative.

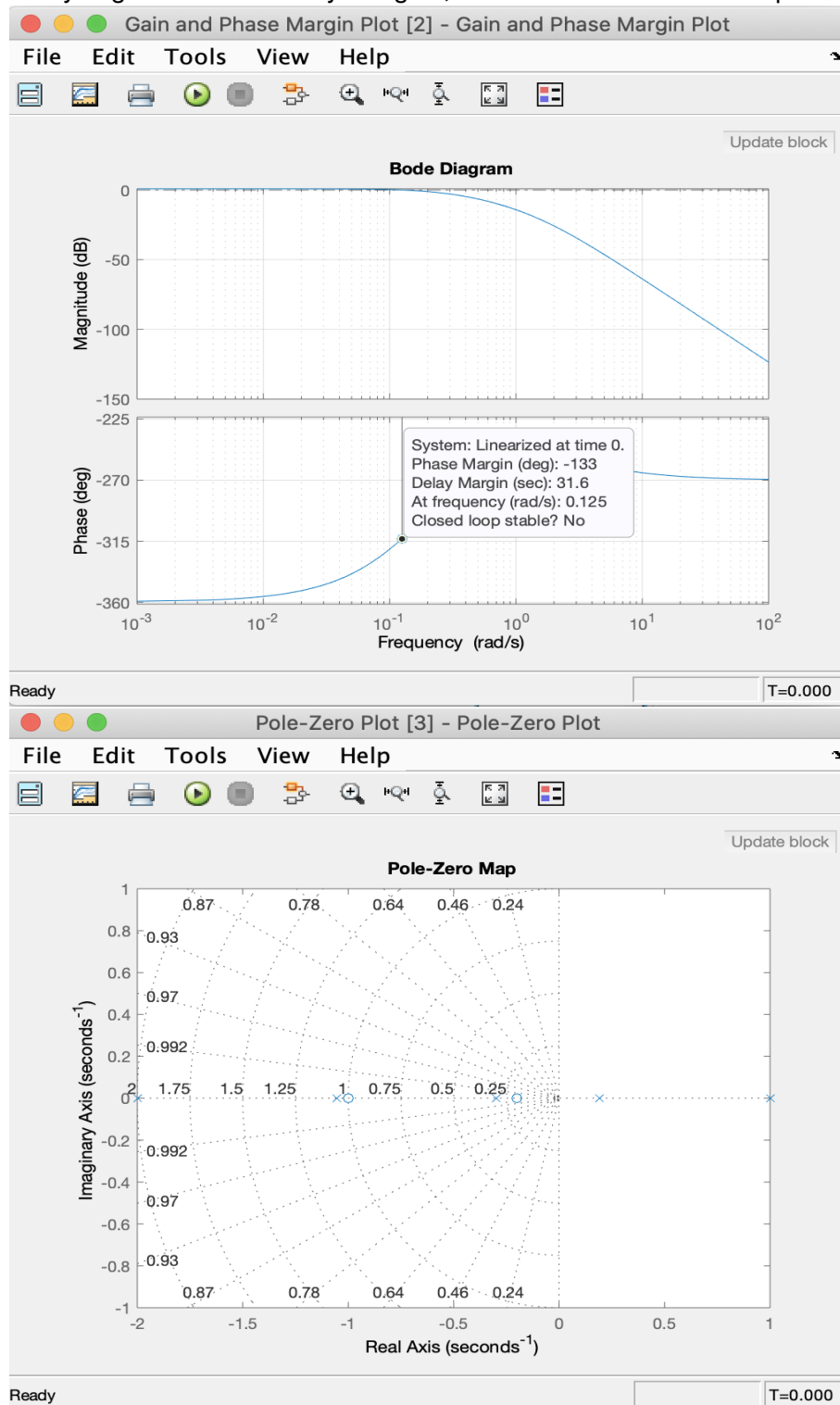
If we calculate roots of denominator of total TF using matlab:

```
1 >> roots([1 13/6 -8/75 -22/15 -197/150 7/25])
2
3 ans =
4
5 -2.0372 + 0.0000i
6 -0.6175 + 0.6693i
7 -0.6175 - 0.6693i
8 0.9267 + 0.0000i
9 0.1788 + 0.0000i
```

As we can see, the last 2 roots have positive signs. It means system is unstable.

Answer validation by BodePlot and Pole Zero Plot.

Analyzing minimum stability margins, we can see that closed loop is unstable.



Part D

Description: Analyze Bode plot - calculate asymptotes and frequency breaks and put calculations in report. Also calculate intersections of the plot with axes.

Solution: Asymptotes were calculated by general function for calculating asymptotes <https://lpsa.swarthmore.edu/Bode/BodeHowGen.html> using Computations were implemented by MATLAB functions (code is below)
Frequency breaks are poles and zeros (roots of numerator and denominator) were calculated using MATLAB.

Roots of numerator

```
1 >> roots(num)
2
3 ans =
4
5 -0.6000 + 1.2806i
6 -0.6000 - 1.2806i
```

Roots of denominator

```
1 >> roots(den)
2
3 ans =
4
5 -2.0372 + 0.0000i
6 -0.6175 + 0.6693i
7 -0.6175 - 0.6693i
8 0.9267 + 0.0000i
9 0.1788 + 0.0000i
```

Frequency asymptotes pointwise graph generator

```
1 function res = asymptotes_graph(w, n_roots, d_roots)
2 c_prime = 2/3 * prod(n_roots)/prod(d_roots);
3 first_term = 0;
4 s = size(n_roots);
5 first_vector = reshape(n_roots, 1, s(1));
6 for term = first_vector
7     first_term = first_term + 20 * log10(abs(1 + 1i*w/term));
8 end
9
10 second_term = 0;
11 s = size(d_roots);
12 second_vector = reshape(d_roots, 1, s(1));
```

```

13 for term = second_vector
14     second_term = second_term + 20 * log10(abs(1 + 1i*w/term));
15 end
16
17
18 res = 20 * log10(abs(c_prime)) + first_term - second_term;

```

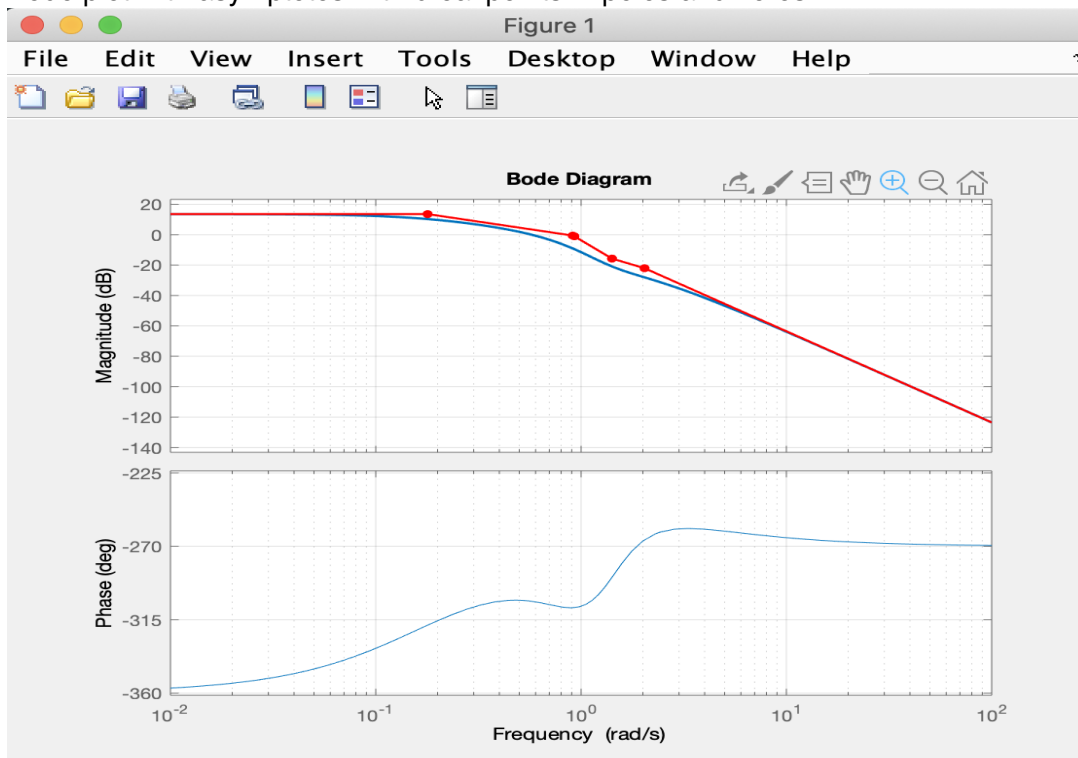
intersections of the plot with axes

```

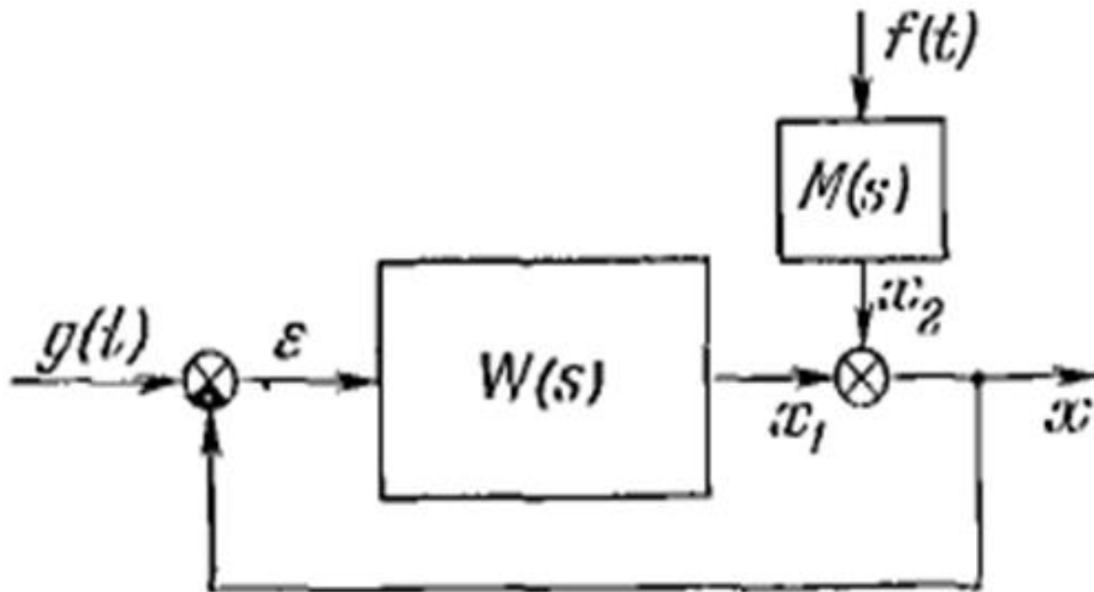
1 >> asymptotes_graph(0, num_roots, den_roots)
2
3 ans =
4
5     13.5556
6
7 >> asymptotes_graph(0.5733474, num_roots, den_roots)
8
9 ans =
10
11     0.0273 (roundoff + plot error)

```

Bode plot with asymptotes with breakpoints in poles and zeros.



Problem 2



$$W(s) = \frac{s+4}{3s+2}$$

$$M(s) = \frac{1}{s+1}$$

Description: Find total transfer function for a closed-loop system.

Solution:

$$\Phi(s) = \frac{X}{G} = \frac{W(s)}{1+W(s)}$$

$$\Phi_f(s) = \frac{X}{F} = \frac{M(s)}{1+W(s)}$$

$$X = \Phi(s)G + \Phi_f(s)F = \frac{W(s)}{1+W(s)}G + \frac{M(s)}{1+W(s)}F$$

$$X = \frac{\frac{s+4}{3s+2}}{1 + \frac{s+4}{3s+2}}G + \frac{\frac{1}{s+1}}{1 + \frac{s+4}{3s+2}}F$$

$$X = \frac{s+4}{4s+6}G + \frac{3s+2}{(4s+6)(s+1)}F$$

$$\text{Answer : } X = \frac{s+4}{4s+6}G + \frac{3s+2}{4s^2+10s+6}F$$

Problem 3

$$A = \begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix}, B = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, C = \begin{pmatrix} -2 & 0 \end{pmatrix}, D = \begin{pmatrix} 2 \end{pmatrix}$$

Description: Find transfer function of the system.

Solution: Using formula for converting SS to TF:

$$Y(s) = \{C(sI - A)^{-1}B + D\}U(s)$$

$$(sI - A) = \begin{pmatrix} s-2 & 0 \\ 3 & s-1 \end{pmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s^2 - 3s + 2} \begin{pmatrix} s-1 & 0 \\ -3 & s-2 \end{pmatrix}$$

$$C(sI - A)^{-1} = \frac{1}{s^2 - 3s + 2} (2 - 2s \quad 0)$$

$$C(sI - A)^{-1}B = \left(\frac{2s-2}{s^2-3s+2} \right)$$

$$C(sI - A)^{-1}B + D = \left(\frac{2s^2-4s+2}{s^2-3s+2} \right)$$

$$\text{Answer : TF : } \frac{2s^2 - 4s + 2}{s^2 - 3s + 2}$$

Answer validation in MATLAB

```
1 >> A = [2 0; -3 1];
2 >> B = [-1;1];
3 >> C = [-2, 0];
4 >> D = [2];
5 >> [b, a] = ss2tf(A, B, C, D)
6 b = 2      -4      2
7 a = 1      -3      2
```

Problem 4

$$A = \begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix}, B = \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}, C = (1 \quad 3), D = (1 \quad 2)$$

Description: Find transfer functions of the system.

Solution: Using formula for converting SS to TF:

$$Y(s) = \{C(sI - A)^{-1}B + D\}U(s)$$

$$(sI - A) = \begin{pmatrix} s - 4 & -1 \\ 2 & s - 1 \end{pmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s^2 - 5s + 6} \begin{pmatrix} s - 1 & 1 \\ -2 & s - 4 \end{pmatrix}$$

$$C(sI - A)^{-1} = \frac{1}{s^2 - 5s + 6} \begin{pmatrix} s - 7 & 3s - 11 \end{pmatrix}$$

$$C(sI - A)^{-1}B = \begin{pmatrix} \frac{11s-47}{s^2-5s+6} & \frac{s-7}{s^2-5s+6} \end{pmatrix}$$

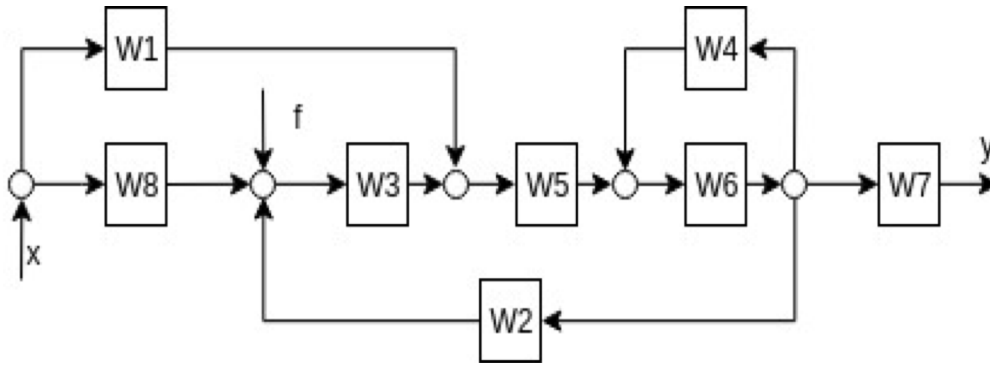
$$C(sI - A)^{-1}B + D = \begin{pmatrix} \frac{s^2+6s-41}{s^2-5s+6} & \frac{2s^2-9s+5}{s^2-5s+6} \end{pmatrix}$$

$$\text{Answer : } TF_1 : \frac{s^2 + 6s - 41}{s^2 - 5s + 6}, \quad TF_2 : \frac{2s^2 - 9s + 5}{s^2 - 5s + 6}$$

Answer validation in MATLAB

```
1 >> A = [4 1; -2 1];
2 >> B = [2;3];
3 >> C = [1 3];
4 >> D = [1];
5 >> [b, a] = ss2tf(A, B, C, D)
6 b = 1.0000    6.0000   -41.0000
7 a = 1    -5     6
8
9
10 >> A = [4 1; -2 1];
11 >> B = [1;0];
12 >> C = [1 3];
13 >> D = [2];
14 >> [b, a] = ss2tf(A, B, C, D)
15 b = 2    -9     5
16 a = 1    -5     6
```

Problem 5



(c)

Description: Find the total transfer function of the system.

Solution:

$$\Phi(s) = \frac{Y}{X} = \left(W_8 + \frac{W_1}{W_3}\right) * \frac{W_3 W_5 W_6 W_7}{1 - W_6 W_4 - W_2 W_5 W_6}$$

$$\Phi_f(s) = \frac{Y}{F} = \frac{W_3 W_5 W_6 W_7}{1 - W_6 W_4 - W_2 W_5 W_6}$$

$$Y = \Phi(s)X + \Phi_f(s)F = \frac{W_3 W_5 W_6 W_7}{1 - W_6 W_4 - W_2 W_5 W_6} * \left((W_8 + \frac{W_1}{W_3})X + F\right)$$

$$\text{Answer : } Y = \frac{W_3 W_5 W_6 W_7}{1 - W_6 W_4 - W_2 W_5 W_6} * \left((W_8 + \frac{W_1}{W_3})X + F\right)$$

Reduction of Transfer function diagram is on the next page.

