HW № 3

Matvey Plevako
m.plevako@innopolis.university
BS18-02
Variant (c)

Problem 1

HW3

March 30, 2020

[0]:
```
import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt

np.random.seed(42)

time = np.linspace(0, 1, 1000)     # interval from 0 to 1
```

1 Proportional-derivative (PD) control

Consider a second order linear ODE:

$$\ddot{x} + \mu\dot{x} + kx = u$$

We can use P control:

$$u = k_p(x^* - x)$$

or we can add a derivative term:

$$u = k_d(\dot{x}^* - \dot{x}) + k_p(x^* - x)$$

This is a **PD controller**.
Let us introduce $e = x^* - x$. Then we have:

$$u = k_d\dot{e} + k_p e$$

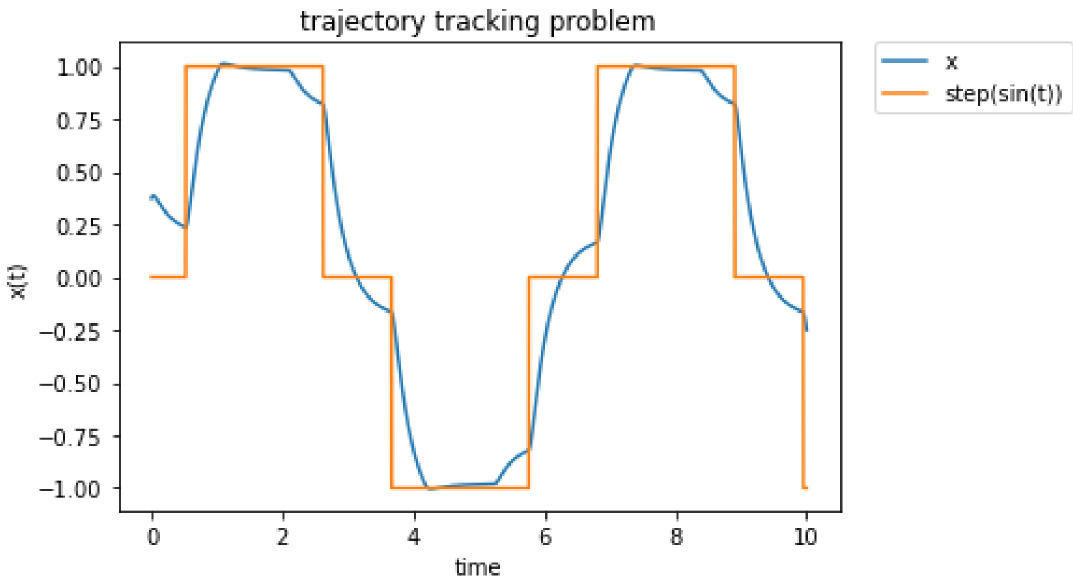
Variable e here is a **control error**.

2 Part A

Design PD-controller that tracks time varying reference states i.e. [x(t),x (t)] as closely as possible. Test your controller on different trajectories, at least two. System: x + x + kx = u, see variants below. ### Trajectory 1: [step(sin(t)), step(cos(t))] ### Trajectory 2: [step(cos(t)), step(sin(t))]

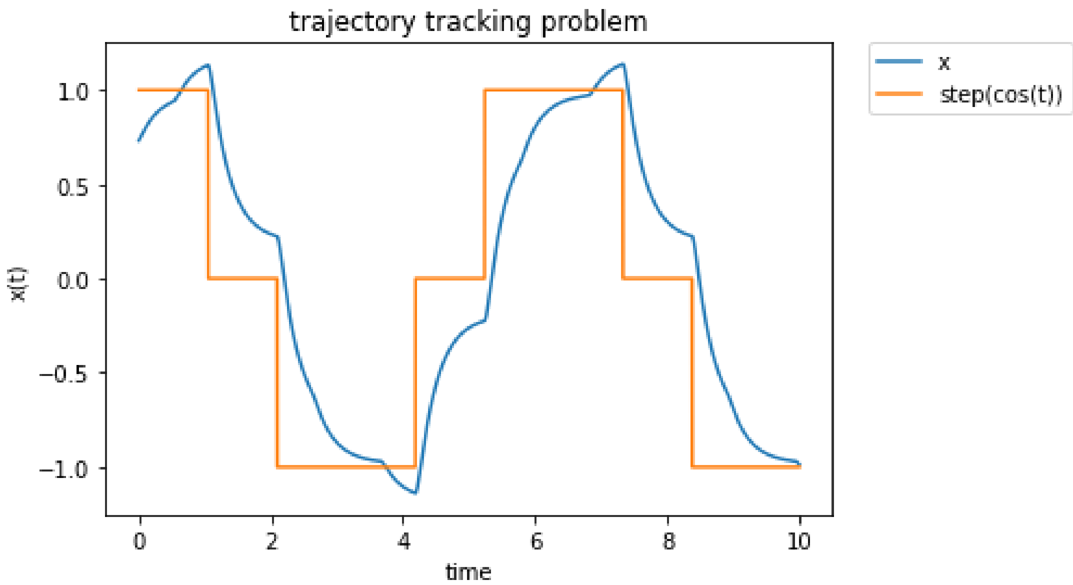
[2]:
```
mu = 13
k = 2

t_steps=10000
```

```
kp = 100
kd = 20


def trajectory(t):
  return np.round(np.sin(t))

def trajectory_dot(t):
  return np.round(np.cos(t))

def PD(x,t):
  x_desired = trajectory(t)
  x_dot_desired = trajectory_dot(t)
  error     = x_desired     - x[0]
  error_dot = x_dot_desired - x[1]
  u = kp*error + kd*error_dot
  return np.array([x[1], (u - mu*x[1] - k*x[0])])

x0 = np.random.rand(2)
time = np.linspace(0, 10, t_steps)

sol=odeint(PD, x0, time)
plt.plot(time, sol[:,0],label="x")
plt.plot(time, trajectory(time),label="step(sin(t))")
plt.legend(bbox_to_anchor=(1.05, 1), loc='upper left', borderaxespad=0.)
plt.xlabel('time')
plt.ylabel('x(t)')
plt.title("trajectory tracking problem")
plt.show()
```



[3]:
```
mu = 13
k = 2

t_steps=10000

kp = 100
kd = 20


def trajectory(t):
  return np.round(np.cos(t))

def trajectory_dot(t):
  return np.round(np.sin(t))

def PD(x,t):
  x_desired = trajectory(t)
  x_dot_desired = trajectory_dot(t)
  error     = x_desired     - x[0]
  error_dot = x_dot_desired - x[1]
  u = kp*error + kd*error_dot
  return np.array([x[1], (u - mu*x[1] - k*x[0])])

x0 = np.random.rand(2)
time = np.linspace(0, 10, t_steps)

sol=odeint(PD, x0, time)
```

```
plt.plot(time, sol[:,0],label="x")
plt.plot(time, trajectory(time),label="step(cos(t))")
plt.legend(bbox_to_anchor=(1.05, 1), loc='upper left', borderaxespad=0.)
plt.xlabel('time')
plt.ylabel('x(t)')
plt.title("trajectory tracking problem")
plt.show()
```



2.1 Part B and C

Tune controller gains kp and kd . Find gains that provide no ocsillations and no overshoot. Prove it with step input. Prove that controlled oscillator dynamics is stable for your choice of kp and kd.

[4]:
```
A=np.array([[-mu,-k],[1,0]])
B=np.array([[1],[0]])
ks=np.array([[kd,kp]])
np.linalg.eigvals(A-B.dot(ks))
```

[4]:
```
array([-29.54798835,  -3.45201165])
```

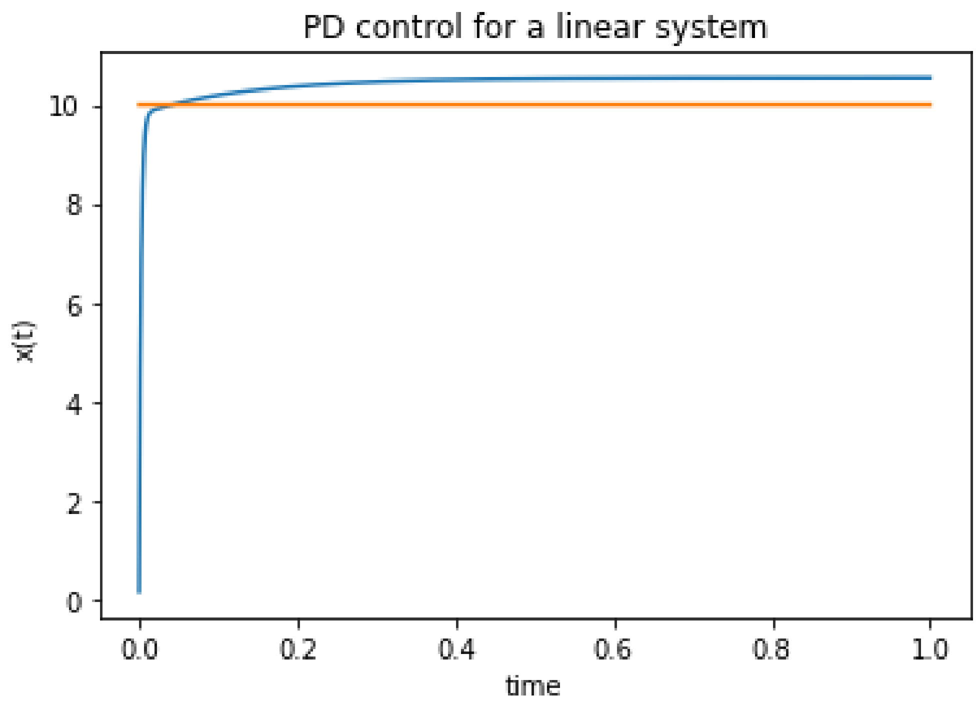
As we see Im(eigenvalues) = 0, that means we will have no **ocsillations**
And because Re(eigenvalues) < 0 for all eigenvalues this system is **stable**

2.2 Part D

2.2.1 Think of how you would implement PD control for a linear system:

[5]:
```
A = np.array([[10, 3], [5, -5]])
B=np.array([[1],[1]])
k=np.array([[400,20]])
print(np.linalg.eigvals(A-B.dot(k)))

t_steps=10000
time = np.linspace(0, 1, t_steps)
x0=np.random.rand(2)

def trajectory(t):
  return t*0+10

def trajectory_dot(t):
  return 0

def PD(x,t):
  x_desired = np.array([trajectory(t) , trajectory_dot(t)])
  error     = np.array([x_desired[0] - x[0] , x_desired[1] - x[1]])

  u=k.dot(error)
  return A.dot(x)+B.dot(u)


sol=odeint(PD, x0, time)


plt.plot(time, sol[:,0])
plt.xlabel('time')
plt.ylabel('x(t)')
plt.plot(time, trajectory(time),label="trajectory")
plt.title('PD control for a linear system')
plt.show()
```

```
[-407.55311795   -7.44688205]
```



3 Part E

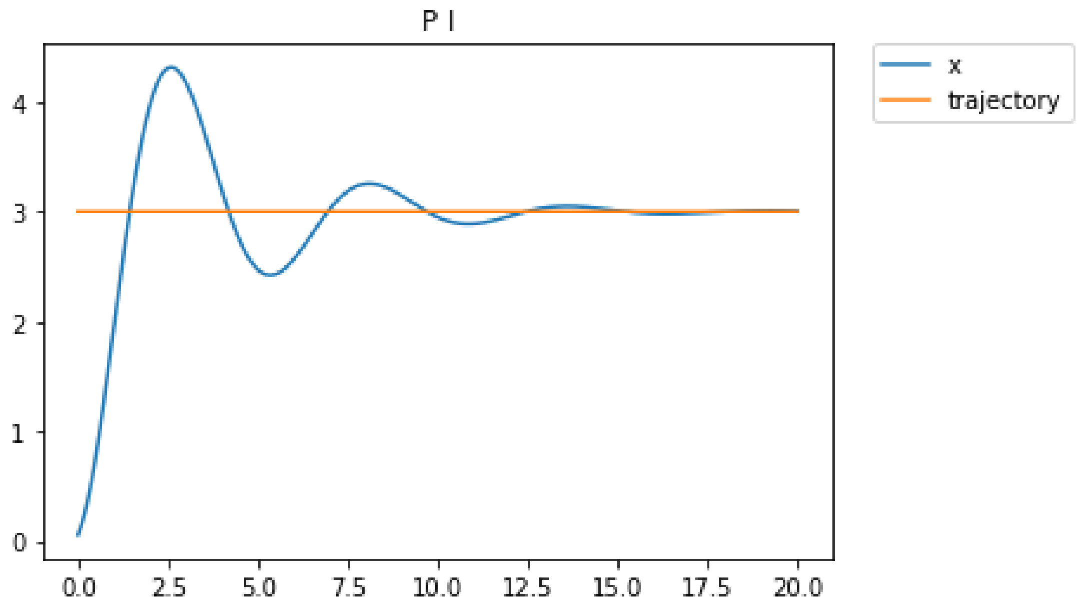
3.1 Implement a PI/PID controller for the system: $x + x + kx + 9.8 = u$ Test your controller on different trajectories, at least two.

3.1.1 Trajectory 1: PI with x_desired = 3

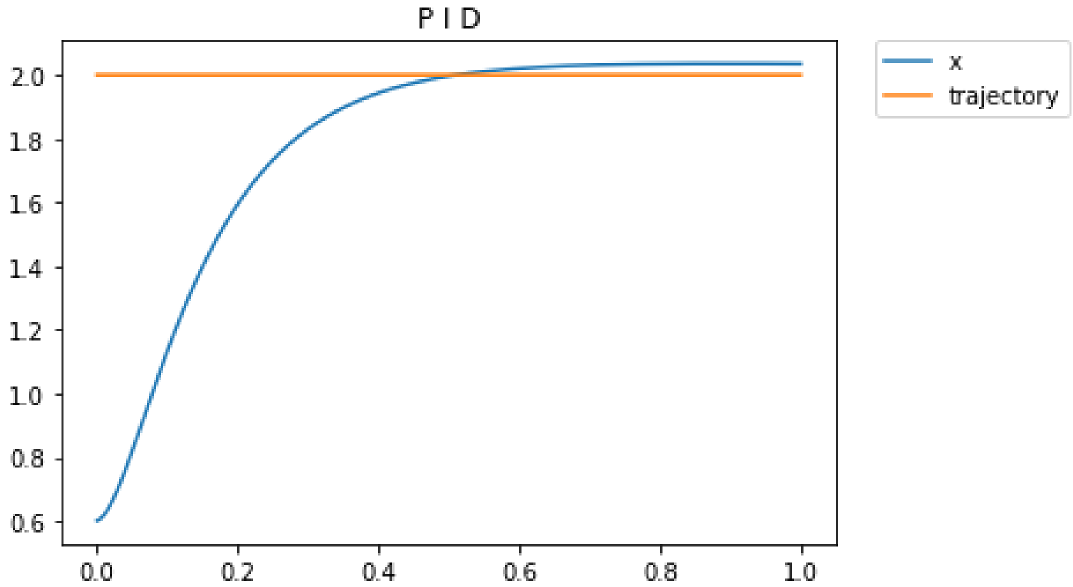
3.1.2 Trajectory 2: PID with x_desired = 2

[6]:
```
mu = 13
k = 2

kp=8
kd=2
ki=20

x=np.random.rand(2)

t_steps=10000
begin=0
end=20
step=(end-begin)/t_steps
graph=[]
```

```python
def trajectory(t):
  return t*0+3

def trajectory_dot(t):
  return (trajectory(t+step) - trajectory(t))/step


sum=0

time=np.linspace(begin,end,t_steps)


for t in time:
  graph.append(x[0])
  x_desired = trajectory(t)
  x_dot_desired = trajectory_dot(t)
  error      = x_desired      - x[0]
  error_dot = x_dot_desired - x[1]
  u = kp*error + kd*error_dot+ki*sum
  sum+=(error)*step
  x[0],x[1] = x[0]+x[1]*step,  x[1]+(u - mu*x[1] - k*x[0] - 9.8)*step

plt.plot(time, graph,label="x")
plt.plot(time, trajectory(time),label="trajectory")
plt.legend(bbox_to_anchor=(1.05, 1), loc='upper left', borderaxespad=0.)
plt.title("P I")
plt.show()
```



[7]:
```
mu = 13
k = 2
kp=200
kd=20
ki=100

x=np.random.rand(2)


t_steps=10000
begin=0
end=1
step=(end-begin)/t_steps
graph=[]

sum=0


time=np.linspace(begin,end,t_steps)

def trajectory(t):
  return t*0+2

def trajectory_dot(t):
  return (trajectory(t+step) - trajectory(t))/step


for t in time:
  graph.append(x[0])
  x_desired = trajectory(t)
  x_dot_desired = trajectory_dot(t)
  error     = x_desired     - x[0]
  error_dot = x_dot_desired - x[1]
  u = kp*error + kd*error_dot+ki*sum
  sum+=error*step
  x[0],x[1] = x[0]+x[1]*step, x[1]+(u - mu*x[1] - k*x[0] - 9.8)*step

plt.plot(time, graph,label="x")
plt.plot(time, trajectory(time),label="trajectory")
plt.legend(bbox_to_anchor=(1.05, 1), loc='upper left', borderaxespad=0.)
plt.title("P I D")
plt.show()
```


P I D
x
trajectory
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.0
0.2
0.4
0.6
0.8
1.0

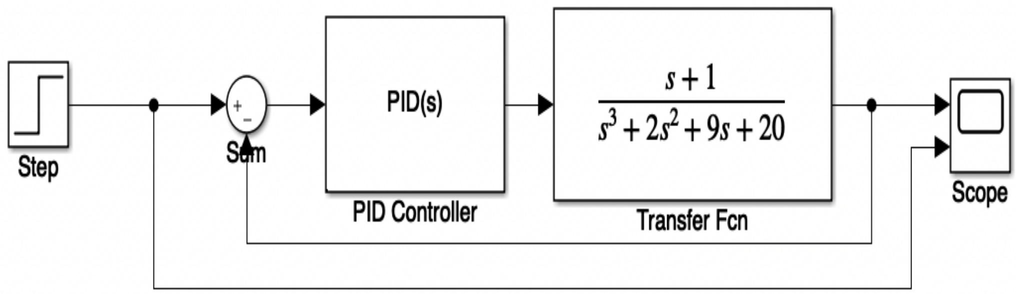
Problem 2

$$W(s) = \frac{s+1}{s^3 + 2s^2 + 9s + 20}$$

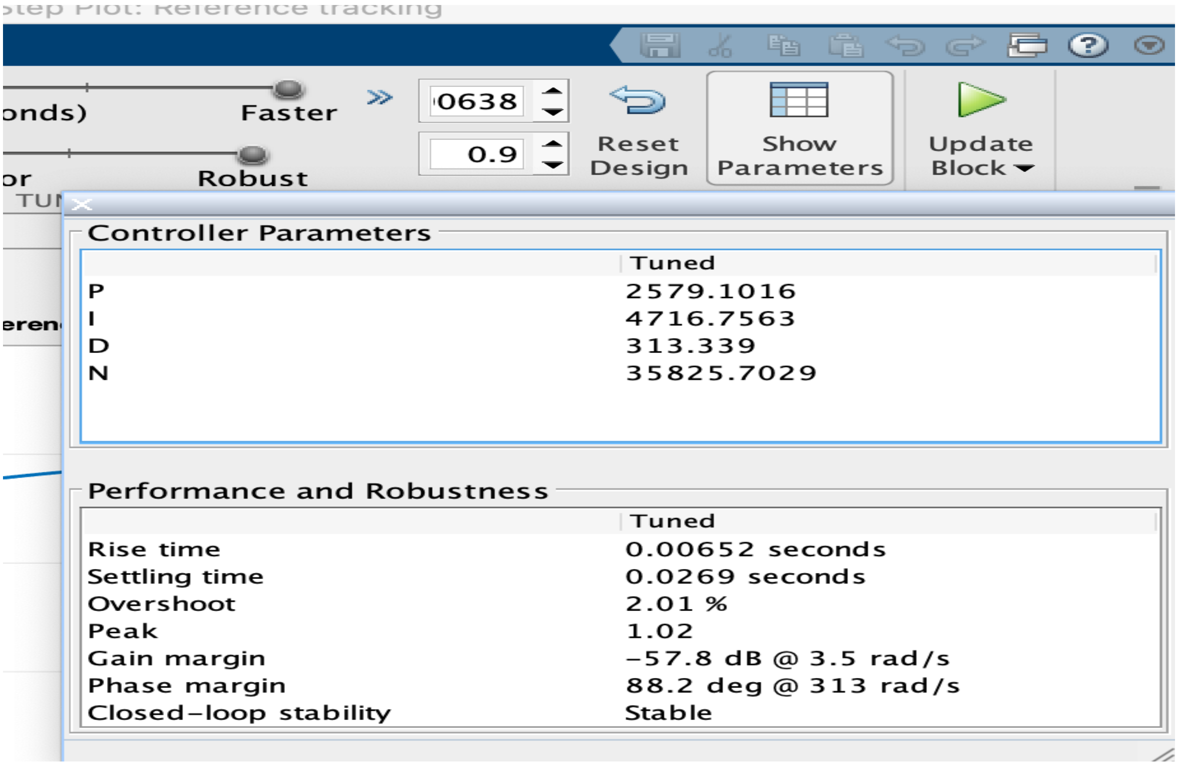
Description: Design a PID controller. Use step input function and try to improve rise time, overshoot and steady-state error, comparing with no controller system. Describe your actions. Use pidTuner in Matlab.

Solution:

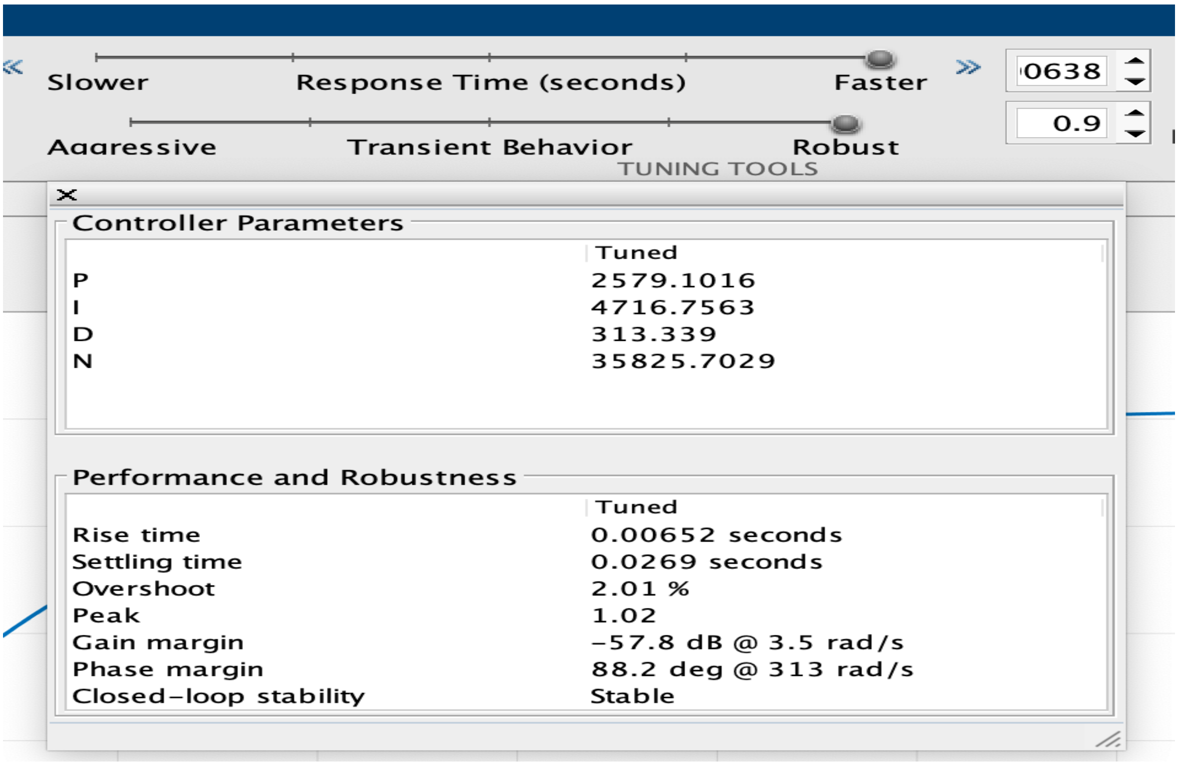
1. Design Simulink scheme with Transfer Function and PID controller



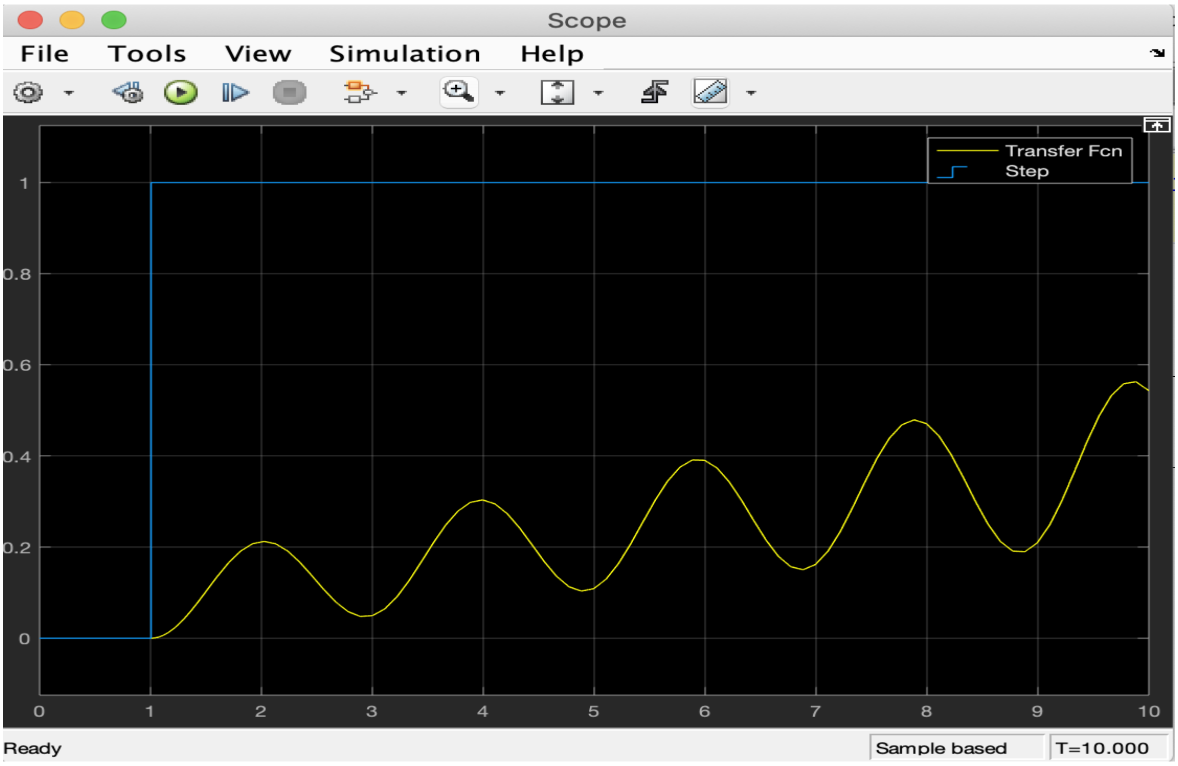
2. Open pidTuner in Matlab and show parametrs for tuning

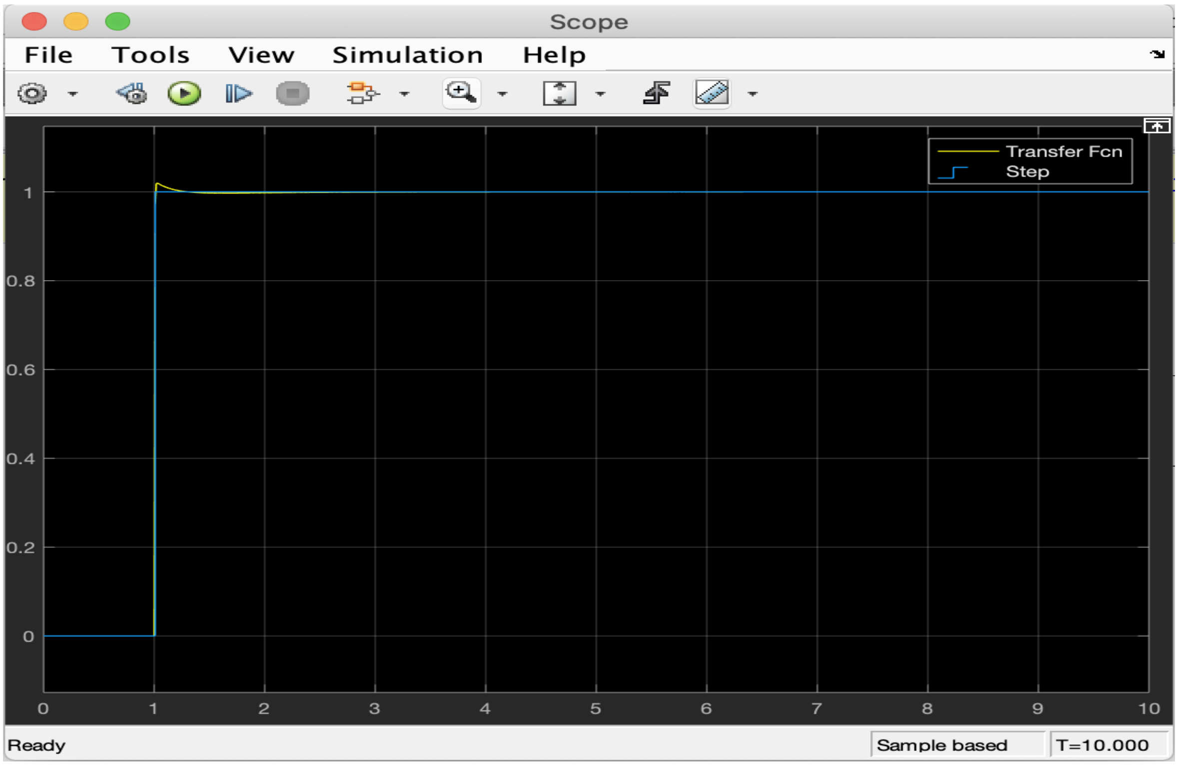


3. Tune Response time and Transient Behavior

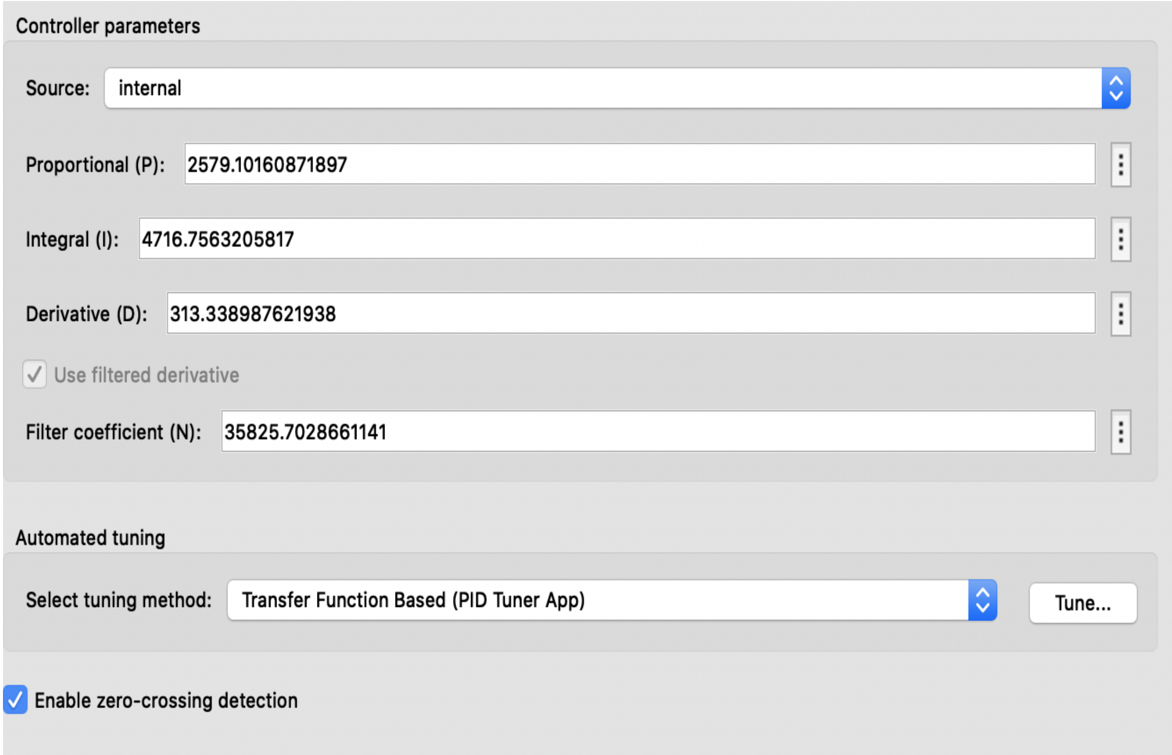


4. Compare results before tuning and after





5. PID parameters



Problem 3

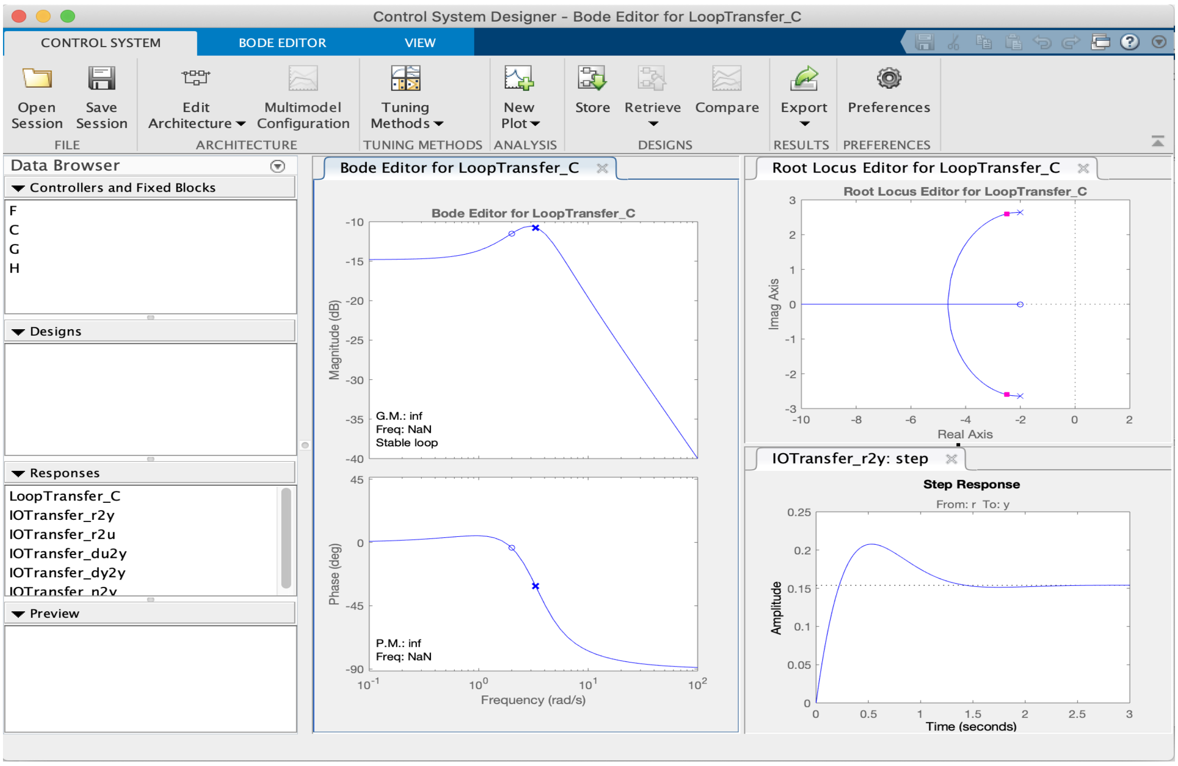
$$W(s) = \frac{s+2}{s^2+4s+11}$$

Description: Design a lag or lead compensator (if applicable), play with zero and pole to find optimal values (of overshoot, peak time, transient process time, stationary error, etc.) for transient process. Use editors in Matlab Control System Designer.

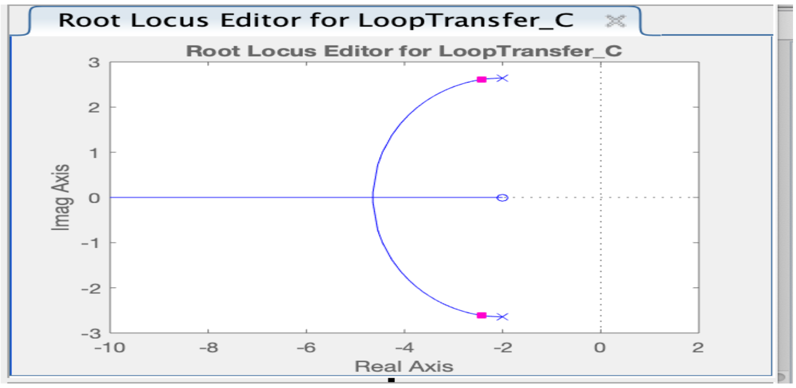
Solution:

1. Open Control System Designer in MATLAB

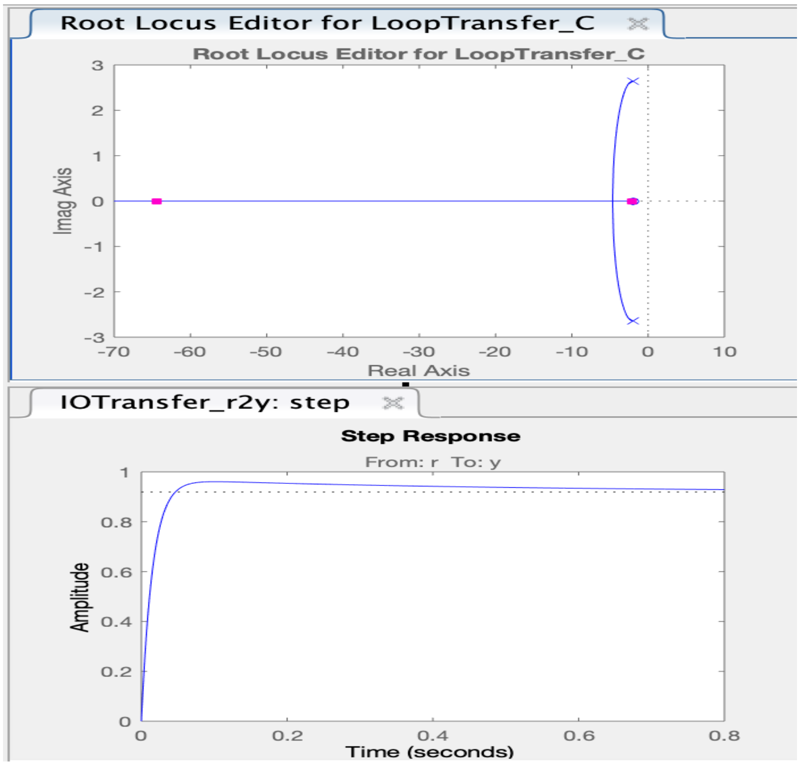
```
>> controlSystemDesigner(tf([1, 2],[1,4,11]))
```



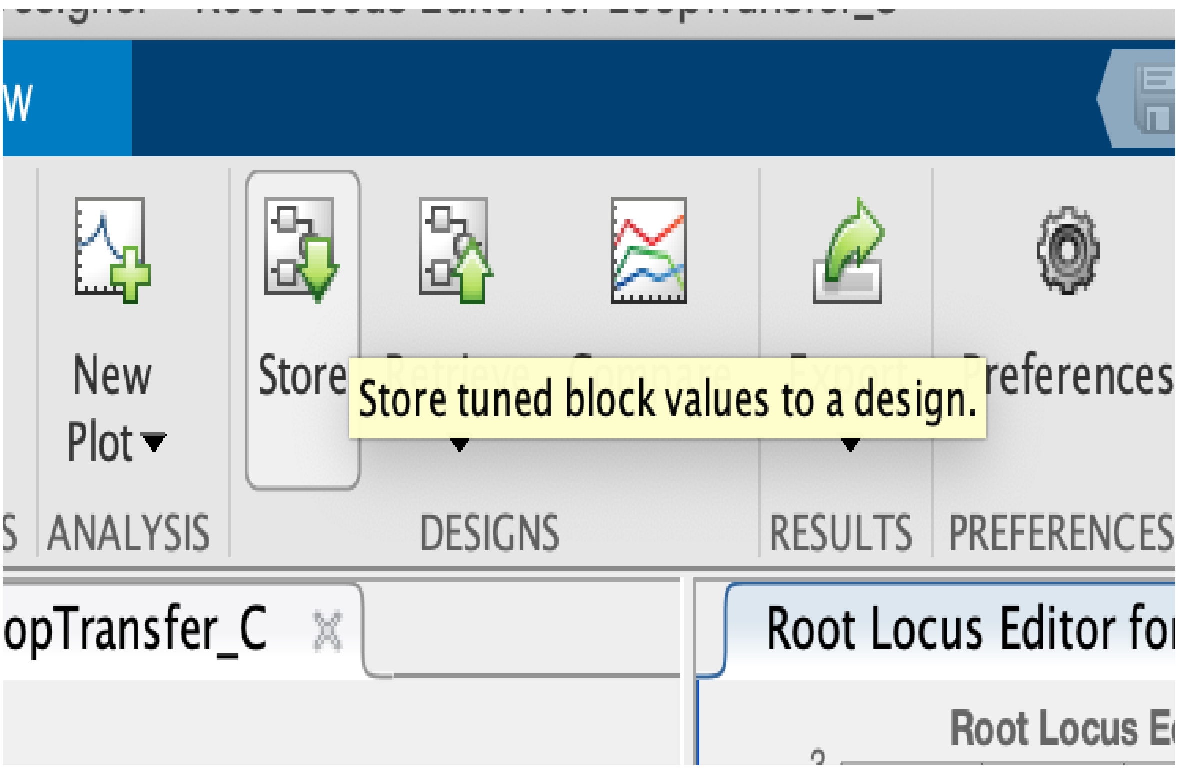
2. To tune poles and zeros we will use Root Locus plot



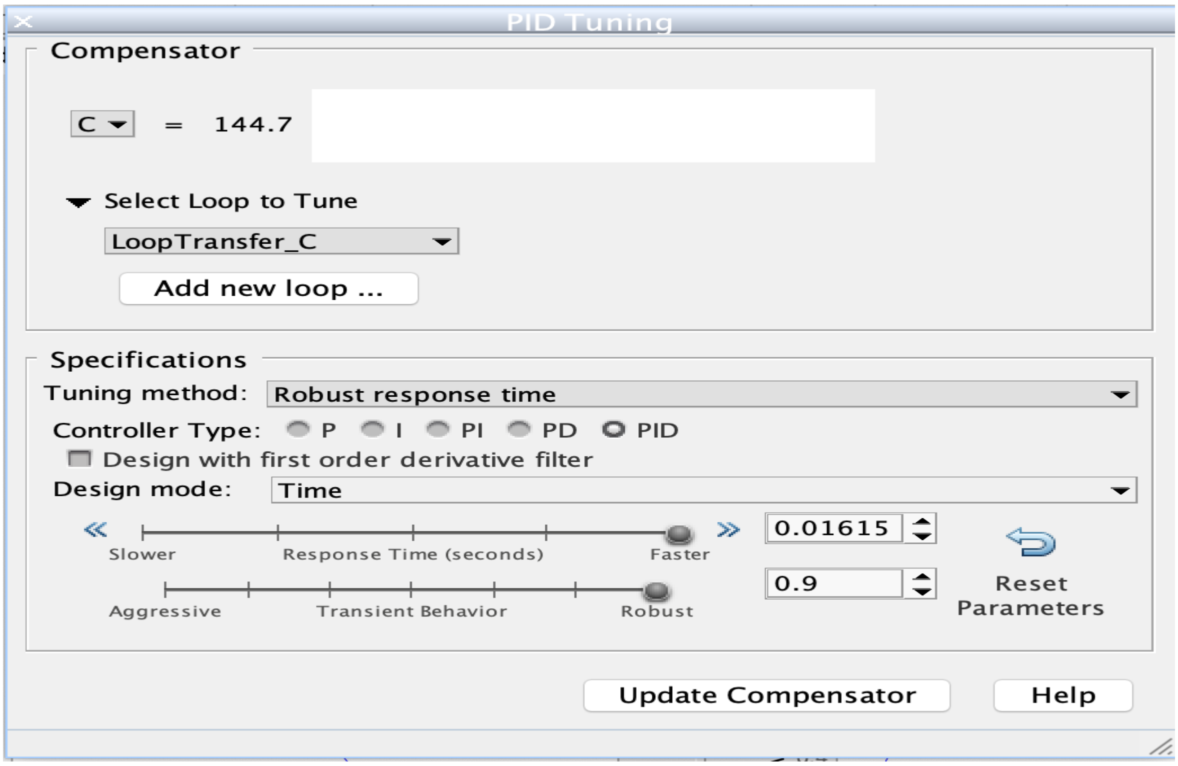
3. After tuning we will have following step response



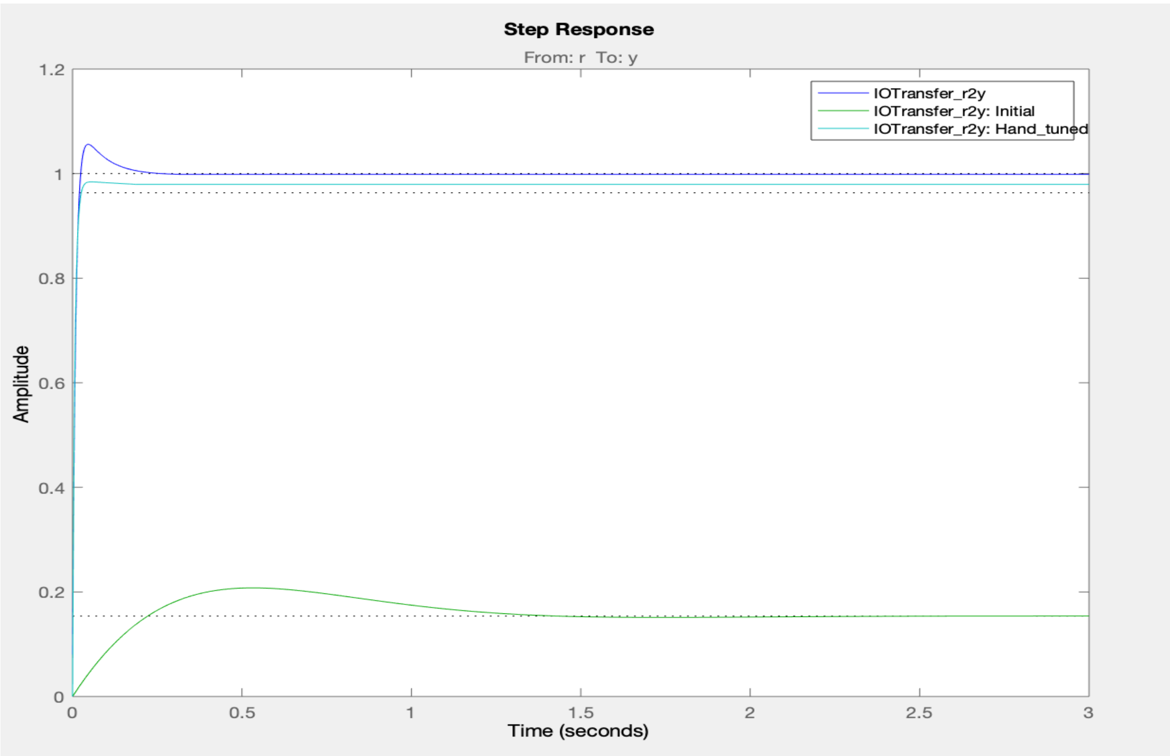
Save results for comparison



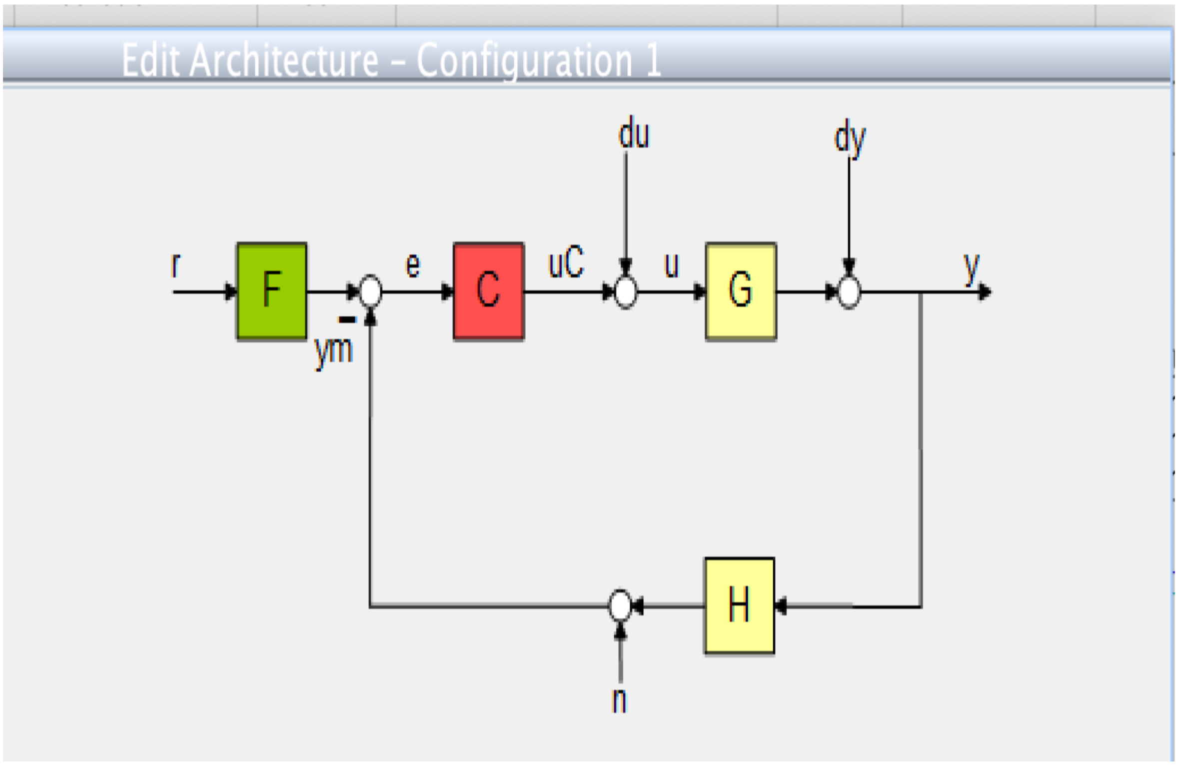
4. To compare results to optimal we will auto-tune with PID tuner



5. Compare PID tuned results with hand tuned and inital system



6. Get designed Lead–lag compensator from following architecute



$$Answer: C = \frac{0.086947(s+1411)(s+12.89)}{s}$$