HW № 2

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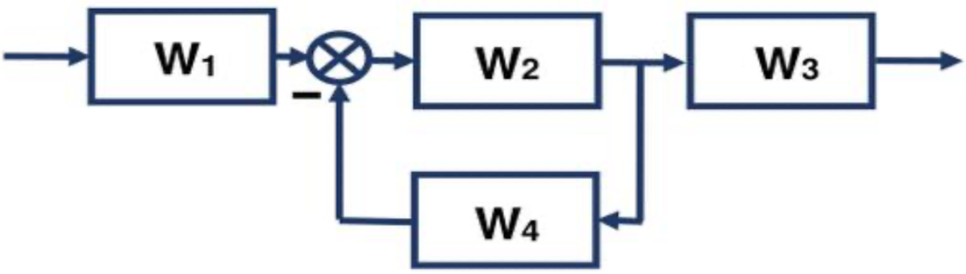
BS18-02
Variant (c)

Problem 1

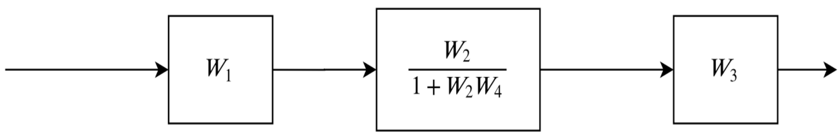


$$W_1 = \frac{2}{s^2+s-2}, W_2 = \frac{1}{3s+2}, W_3 = \frac{s+1}{s+0.3}, W_4 = \frac{1}{s+0.2}$$

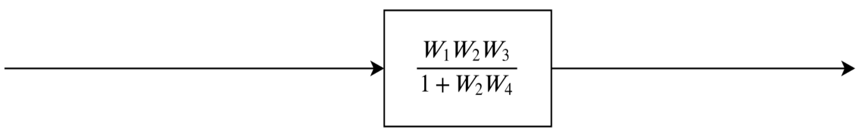
Part A

Description: Calculate the total Transfer Function of the system.

Solution: **Step I**. Reduce negative feedback loop:



Step II. Reduce series connections:



Step III. Substitute initial functions:

$$\frac{W_1W_2W_3}{1+W_2W_4} = \frac{\frac{2}{s^2+s-2} * \frac{1}{3s+2} * \frac{s+1}{s+0.3}}{1 + \frac{1}{3s+2} * \frac{1}{s+0.2}}$$

$$\frac{2(s+1)(s+0.2)}{(s+2)(s-1)(s+0.3)((3s+2)(s+0.2)+1)}$$

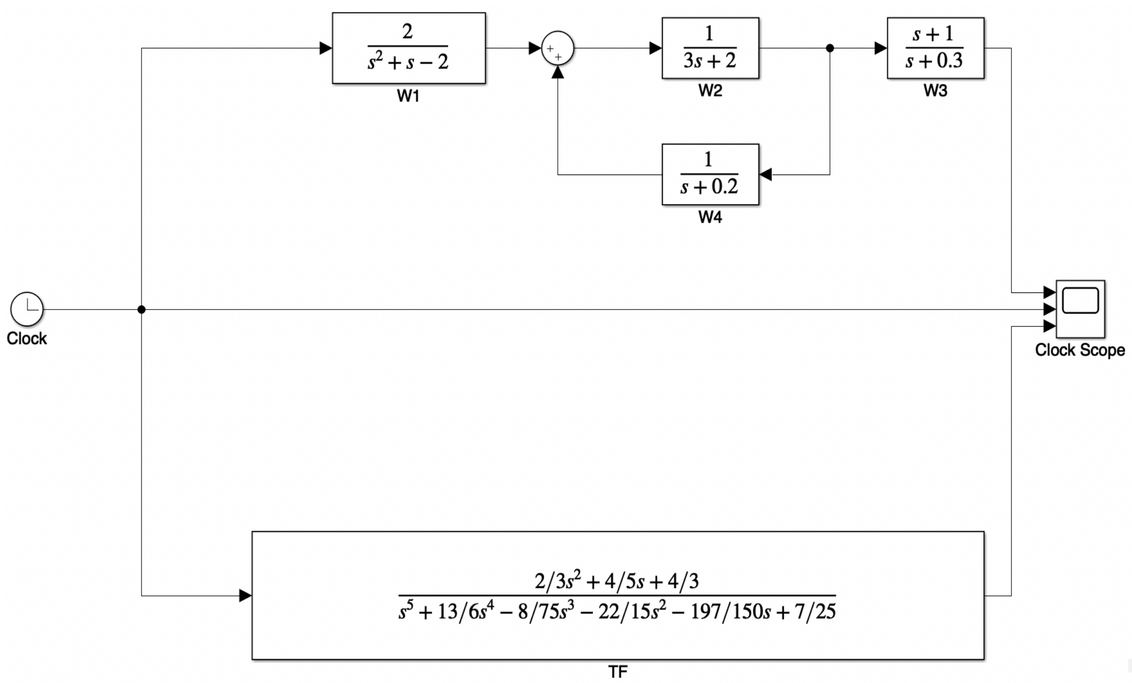
$$Answer : \frac{\frac{2}{3}s^2 + \frac{4}{5}s + \frac{4}{3}}{s^5 + \frac{13}{6}s^4 - \frac{8}{75}s^3 - \frac{22}{15}s^2 - \frac{197}{150}s + \frac{7}{25}}$$

Part B

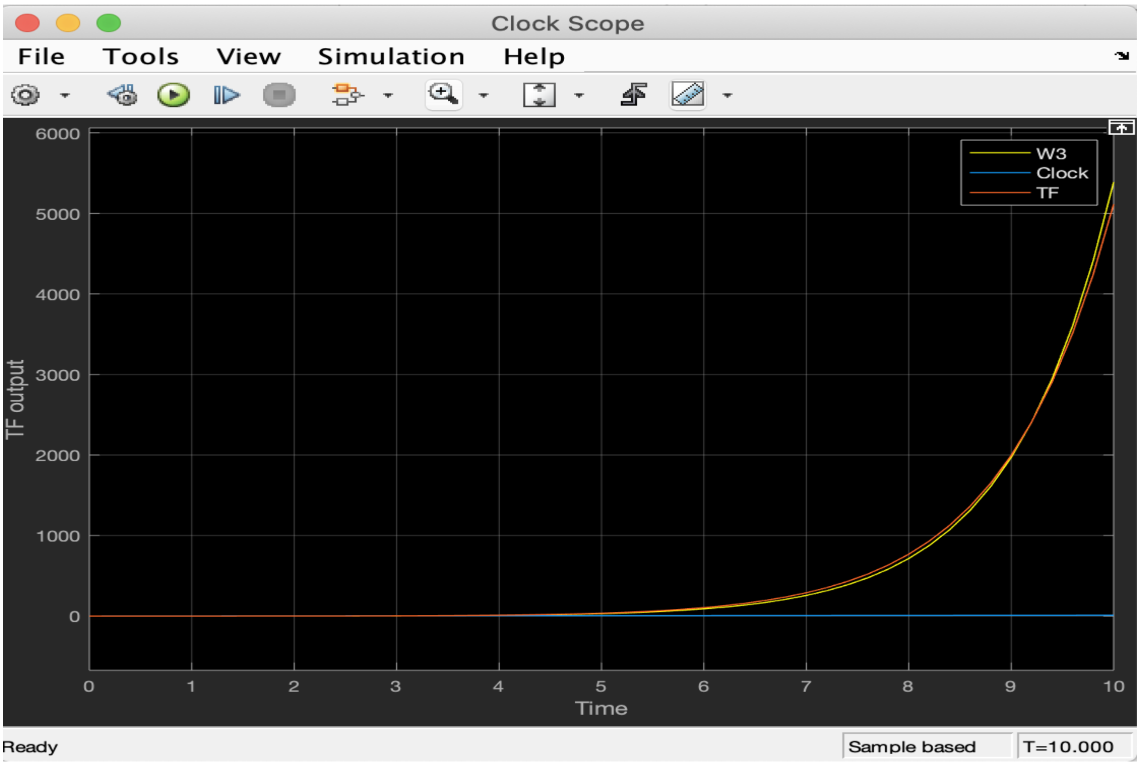
Description: Build initial system shown in the block diagram and simplified in one Simulink schema and analyze its Step, Impulse and Frequency responses. Results should have a schema with both systems and 3 Scope plots(for each input). Each plot should have 3 lines - input signal, and two outputs from each system.

Solution: In both plots Frequency and Step, inital TF and the total TF are close to each other. However, on Impulse there is a slight deviation.

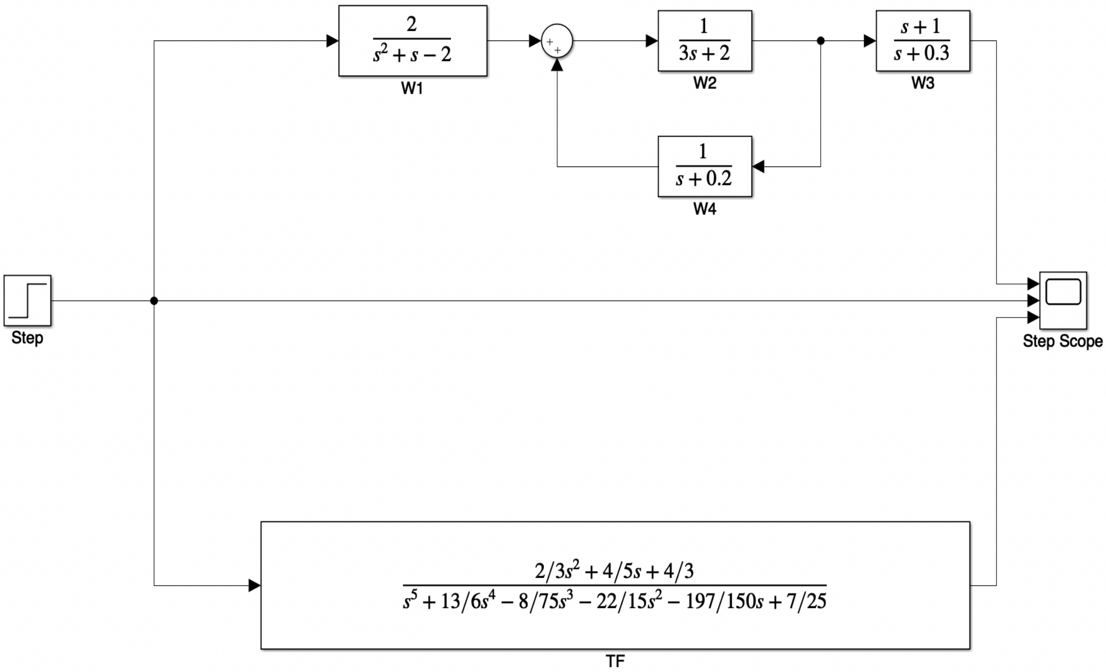
With Frequency Input



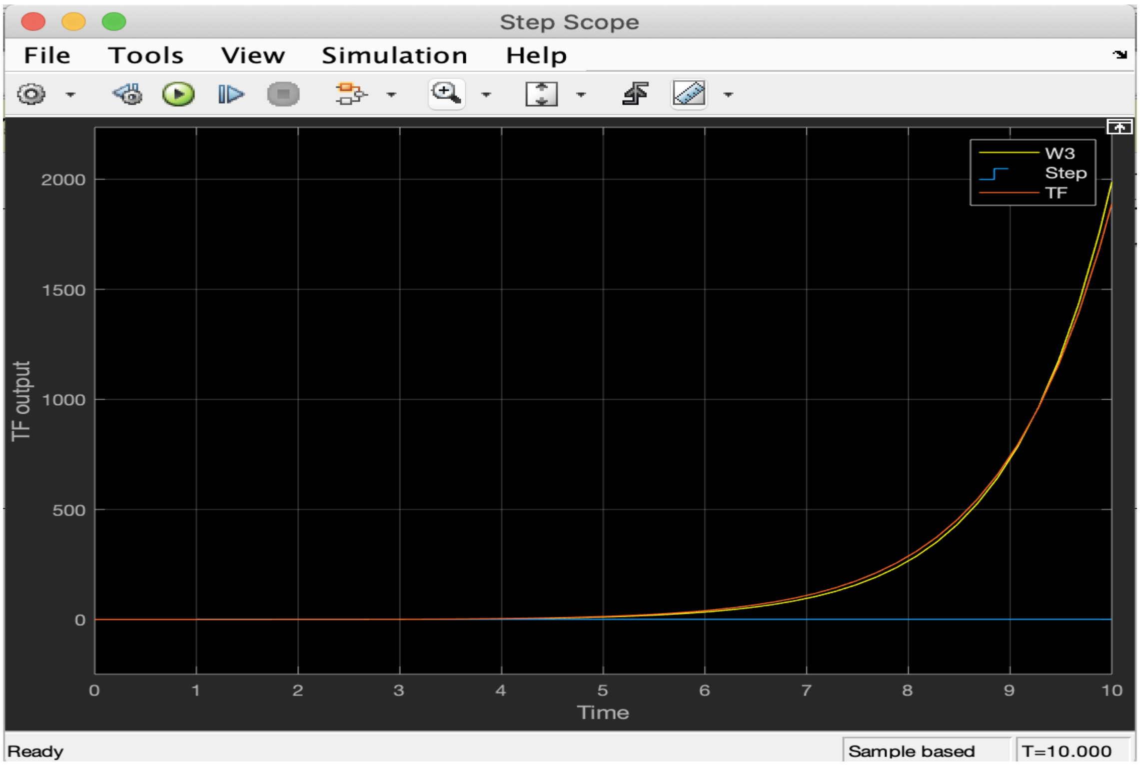
Frequency scope



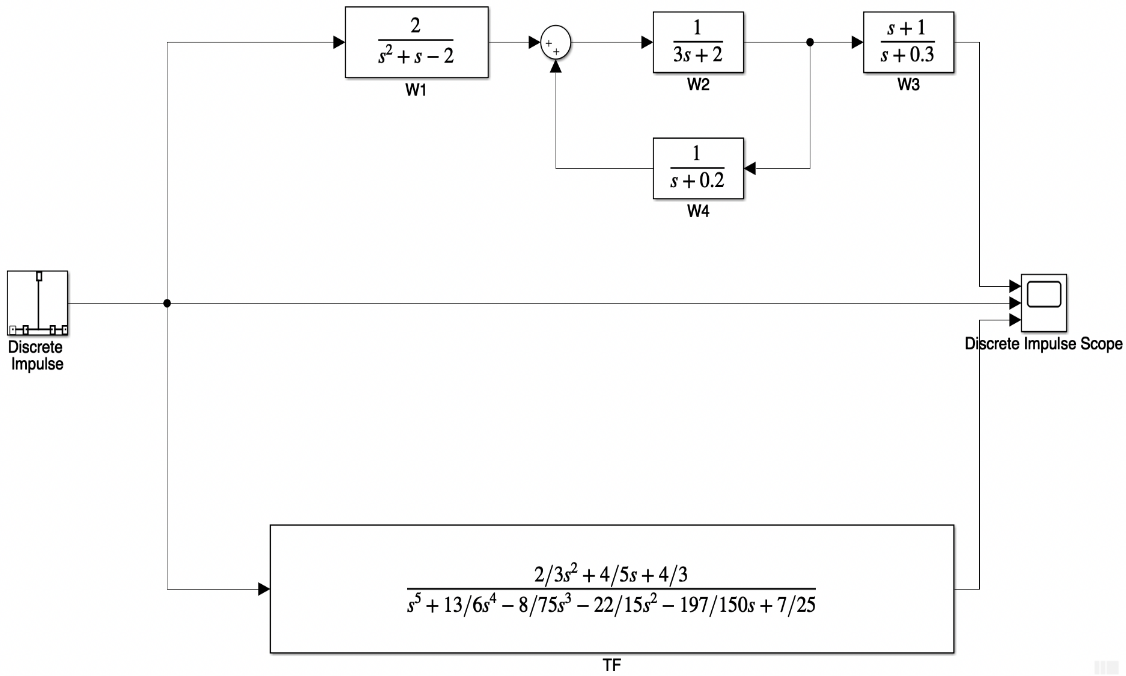
With step Input



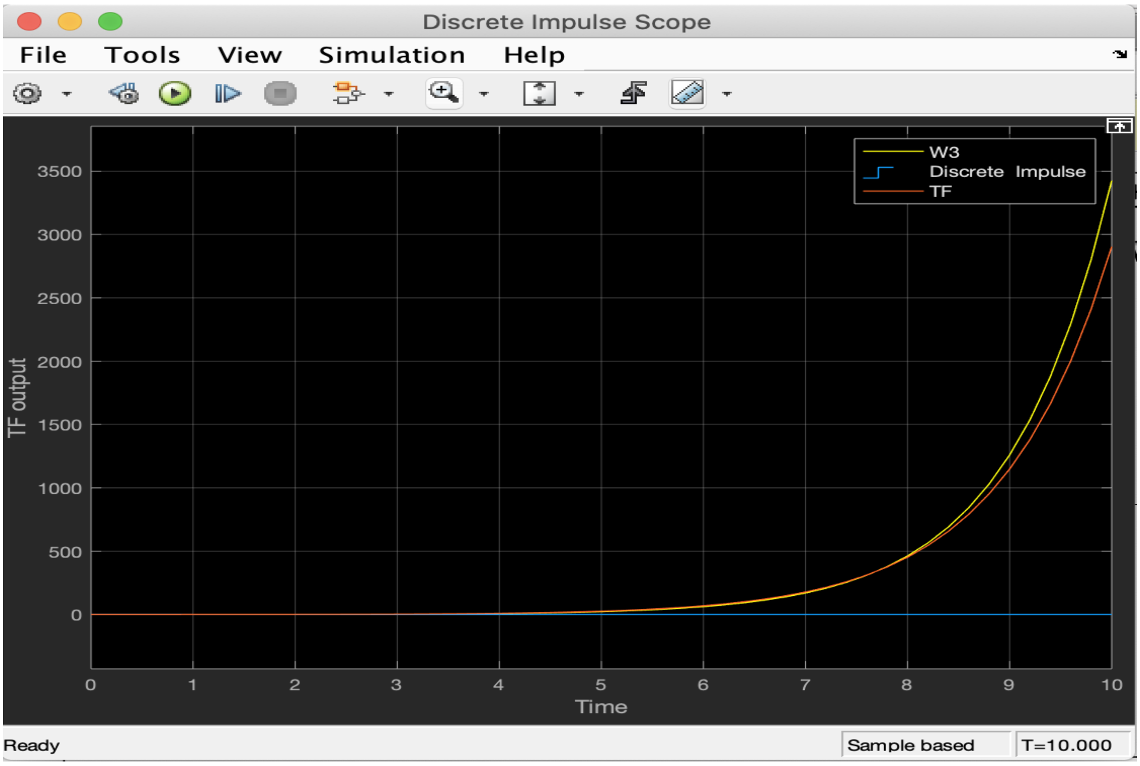
Step scope



With Impulse Input



Impulse scope



Part C

Description: For one of the inputs (write down what you choose) generate a Bode and Pole-Zero map plots. Put plots and result - stable or unstable is system and why - in the report.

Solution: Definition of stability for our problem:

$$Stable : f(t) \to 0, t \to \infty$$

$$f(t) = L^{-1}(tf)$$

L^{-1} is inverse Laplace transform.

From this rule, all roots of inverse denominator should be negative.

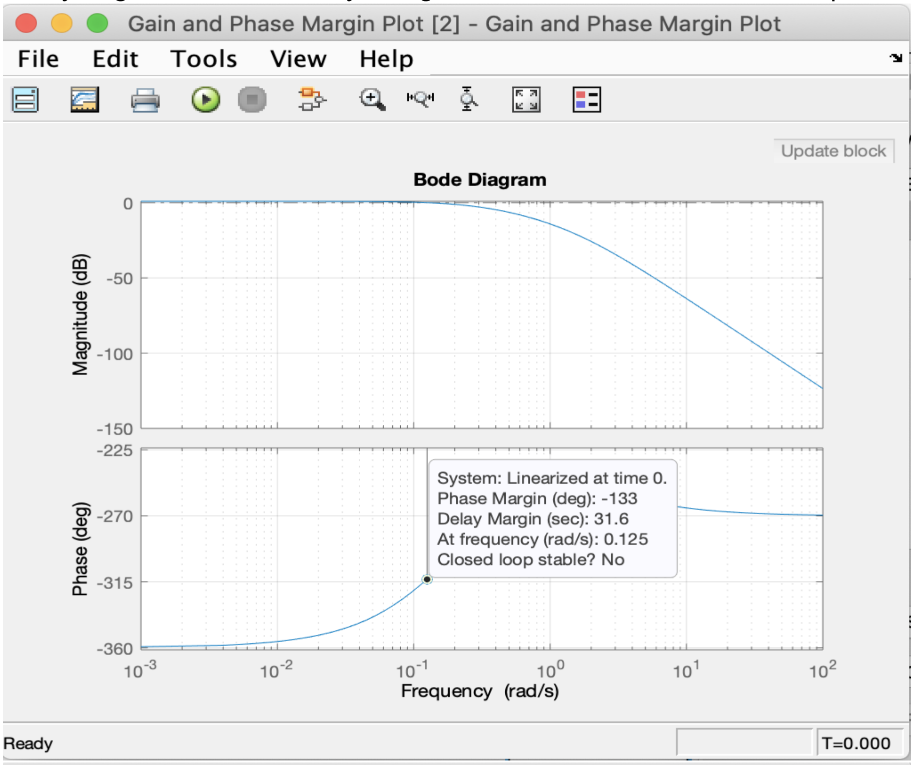
If we calculate roots of denominator of total TF using matlab:

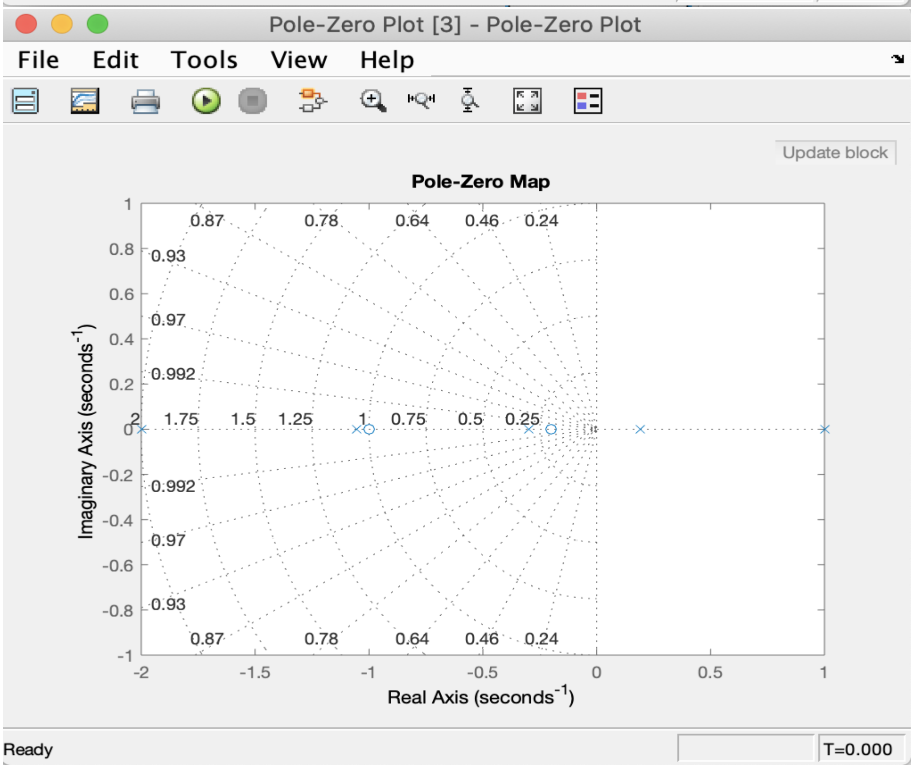
```
>> roots([1 13/6 -8/75 -22/15 -197/150 7/25])

ans =

  -2.0372 + 0.0000i
  -0.6175 + 0.6693i
  -0.6175 - 0.6693i
   0.9267 + 0.0000i
   0.1788 + 0.0000i
```

As we can see, the last 2 roots have positive signs. It means system is unstable.

Answer validation by BodePlot and Pole Zero Plot.

Analyzing minimum stability margins, we can see that closed loop is unstable.





Part D

Description: Analyze Bode plot - calculate asymptotes and frequence breaks and put calculations in report. Also calculate intersections of the plot with axes.

Solution: Asymptotes were calculated by general function for calculating asymptotes https://lpsa.swarthmore.edu/Bode/BodeHowGen.htmlusing Computations were implemented by MATLAB functions (code is below)
Frequence breaks are poles and zeros(roots of numerator and denominator) were calculated using MATLAB.

Roots of numerator

```
>> roots(num)

ans =

  -0.6000 + 1.2806i
  -0.6000 - 1.2806i
```

Roots of denominator

```
  >> roots(den)

ans =

  -2.0372 + 0.0000i
  -0.6175 + 0.6693i
  -0.6175 - 0.6693i
   0.9267 + 0.0000i
   0.1788 + 0.0000i
```

Frequency asymptotes pointwise graph generator

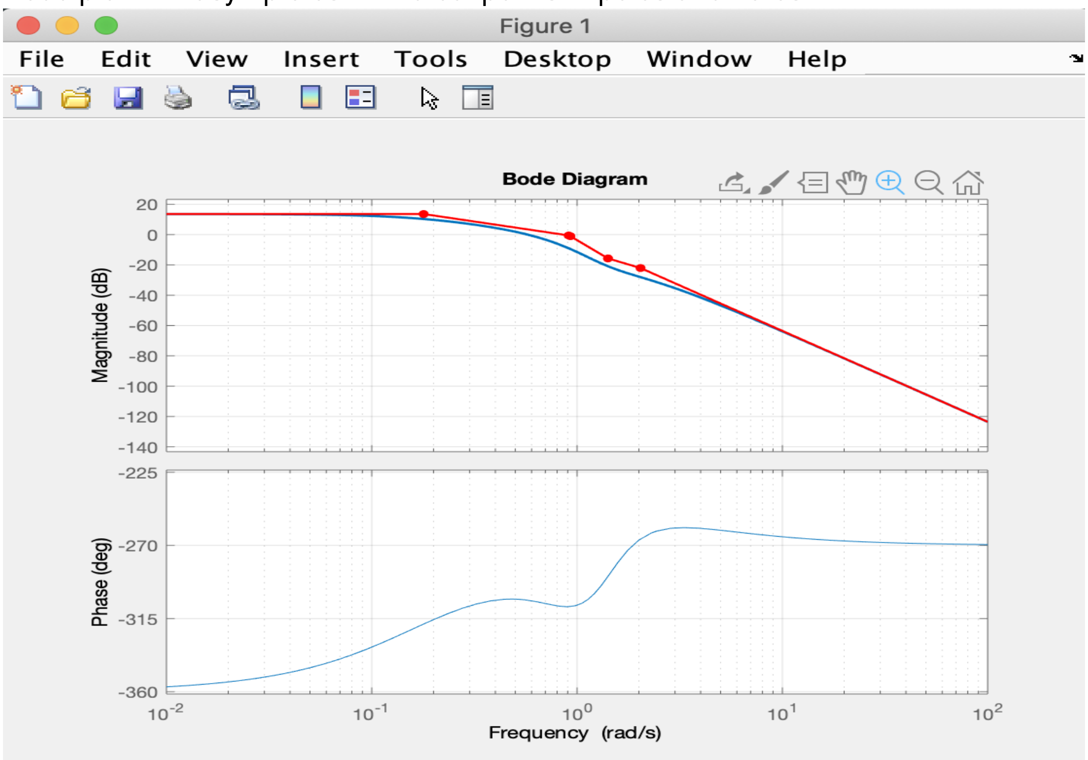
```
function res = asymptotes_graph(w, n_roots, d_roots)
c_prime = 2/3 * prod(n_roots)/prod(d_roots);
first_term = 0;
s = size(n_roots);
first_vector = reshape(n_roots, 1, s(1));
for term = first_vector
    first_term = first_term + 20 * log10(abs(1 + 1i*w/term));
end

second_term = 0;
s = size(d_roots);
second_vector = reshape(d_roots, 1, s(1));
```

```
for term = second_vector
    second_term = second_term + 20 * log10(abs(1 + 1i*w/term));
end


res = 20 * log10(abs(c_prime)) + first_term - second_term;
```

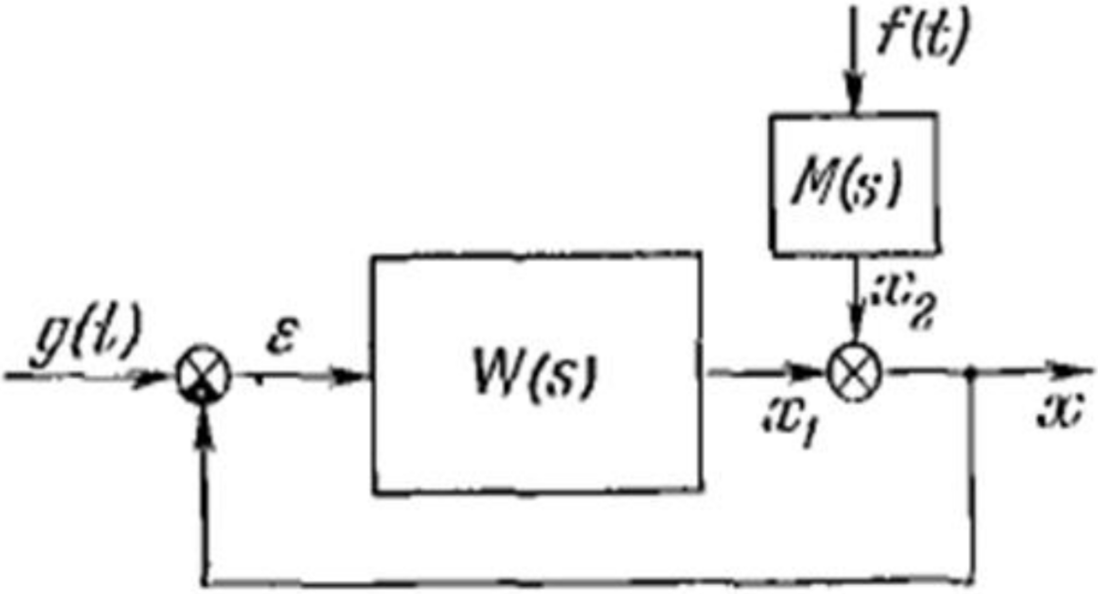
intersections of the plot with axes

```
>> asymptotes_graph(0, num_roots, den_roots)

ans =

   13.5556

>> asymptotes_graph(0.5733474, num_roots, den_roots)

ans =

    0.0273 (roundoff + plot error)
```

Bode plot with asymptotes with breakpoints in poles and zeros.



Problem 2



$$W(s) = \frac{s+4}{3s+2}$$

$$M(s) = \frac{1}{s+1}$$

Description: Find total transfer function for a closed-loop system.

Solution:

$$\Phi(s) = \frac{X}{G} = \frac{W(s)}{1+W(s)}$$

$$\Phi_f(s) = \frac{X}{F} = \frac{M(s)}{1+W(s)}$$

$$X = \Phi(s)G + \Phi_f(s)F = \frac{W(s)}{1+W(s)}G + \frac{M(s)}{1+W(s)}F$$

$$X = \frac{\frac{s+4}{3s+2}}{1+\frac{s+4}{3s+2}}G + \frac{\frac{1}{s+1}}{1+\frac{s+4}{3s+2}}F$$

$$X = \frac{s+4}{4s+6}G + \frac{3s+2}{(4s+6)(s+1)}F$$

$$Answer: X = \frac{s+4}{4s+6}G + \frac{3s+2}{4s^2+10s+6}F$$

Problem 3

$$A = \begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix}, B = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, C = \begin{pmatrix} -2 & 0 \end{pmatrix}, D = \begin{pmatrix} 2 \end{pmatrix}$$

Description: Find transfer function of the system.

Solution: Using formula for converting SS to TF:

$$Y(s) = \{C(sI - A)^{-1}B + D\}U(s)$$

$$(sI - A) = \begin{pmatrix} s-2 & 0 \\ 3 & s-1 \end{pmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s^2 - 3s + 2}\begin{pmatrix} s-1 & 0 \\ -3 & s-2 \end{pmatrix}$$

$$C(sI - A)^{-1} = \frac{1}{s^2 - 3s + 2}\begin{pmatrix} 2-2s & 0 \end{pmatrix}$$

$$C(sI - A)^{-1}B = \begin{pmatrix} \frac{2s-2}{s^2-3s+2} \end{pmatrix}$$

$$C(sI - A)^{-1}B + D = \begin{pmatrix} \frac{2s^2-4s+2}{s^2-3s+2} \end{pmatrix}$$

$$Answer: TF: \frac{2s^2 - 4s + 2}{s^2 - 3s + 2}$$

Answer validation in MATLAB

```
>> A = [2 0; -3 1];
>> B = [-1;1];
>> C = [-2, 0];
>> D = [2];
>> [b, a] = ss2tf(A, B, C, D)
b = 2     -4     2
a = 1     -3     2
```

Problem 4

$$A = \begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix}, B = \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} 1 & 3 \end{pmatrix}, D = \begin{pmatrix} 1 & 2 \end{pmatrix}$$

Description: Find transfer functions of the system.

Solution: Using formula for converting SS to TF:

$$Y(s) = \{C(sI - A)^{-1}B + D\}U(s)$$

$$(sI - A) = \begin{pmatrix} s-4 & -1 \\ 2 & s-1 \end{pmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s^2 - 5s + 6}\begin{pmatrix} s-1 & 1 \\ -2 & s-4 \end{pmatrix}$$

$$C(sI - A)^{-1} = \frac{1}{s^2 - 5s + 6}\begin{pmatrix} s-7 & 3s-11 \end{pmatrix}$$

$$C(sI - A)^{-1}B = \begin{pmatrix} \frac{11s-47}{s^2-5s+6} & \frac{s-7}{s^2-5s+6} \end{pmatrix}$$

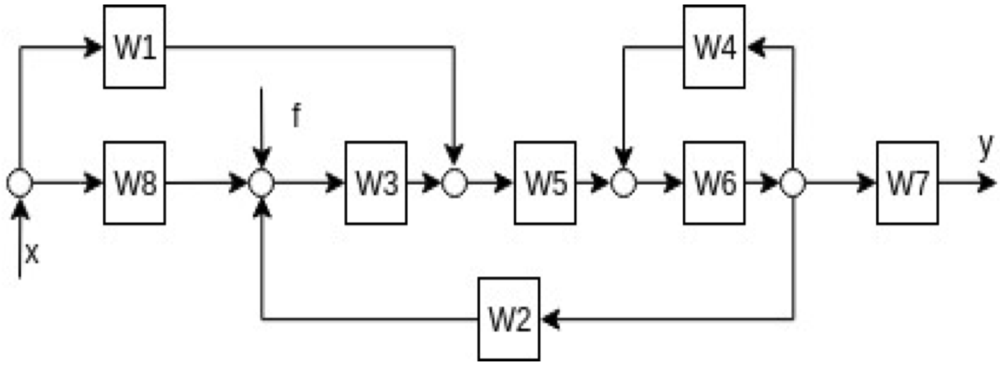
$$C(sI - A)^{-1}B + D = \begin{pmatrix} \frac{s^2+6s-41}{s^2-5s+6} & \frac{2s^2-9s+5}{s^2-5s+6} \end{pmatrix}$$

$$Answer : TF_1 : \frac{s^2 + 6s - 41}{s^2 - 5s + 6}, \quad TF_2 : \frac{2s^2 - 9s + 5}{s^2 - 5s + 6}$$

Answer validation in MATLAB

```
>> A = [4 1; -2 1];
>> B = [2;3];
>> C = [1 3];
>> D = [1];
>> [b, a] = ss2tf(A, B, C, D)
b = 1.0000     6.0000   -41.0000
a = 1     -5      6


>> A = [4 1; -2 1];
>> B = [1;0];
>> C = [1 3];
>> D = [2];
>> [b, a] = ss2tf(A, B, C, D)
b = 2     -9      5
a = 1     -5      6
```

Problem 5



(c)

Description: Find the total transfer function of the system.

Solution:

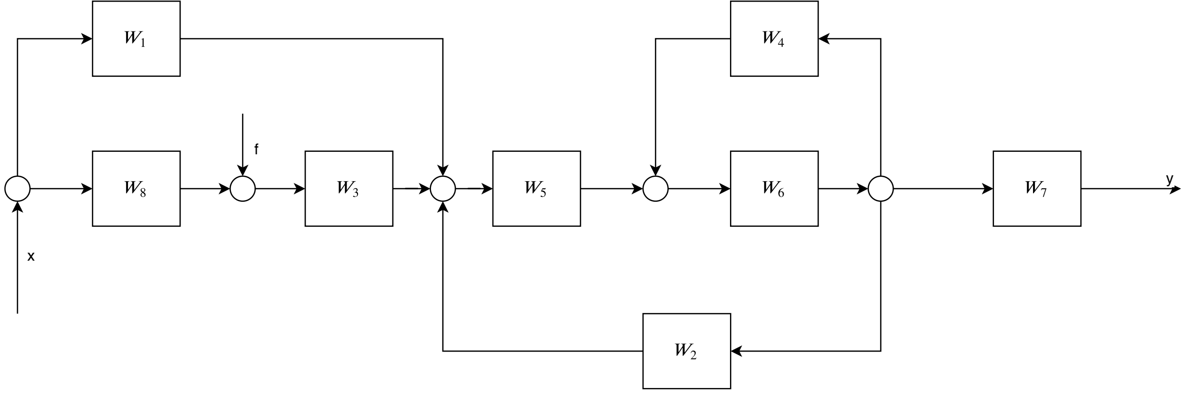
$$\Phi(s) = \frac{Y}{X} = (W_8 + \frac{W_1}{W_3}) * \frac{W_3W_5W_6W_7}{1 - W_6W_4 - W_2W_5W_6}$$

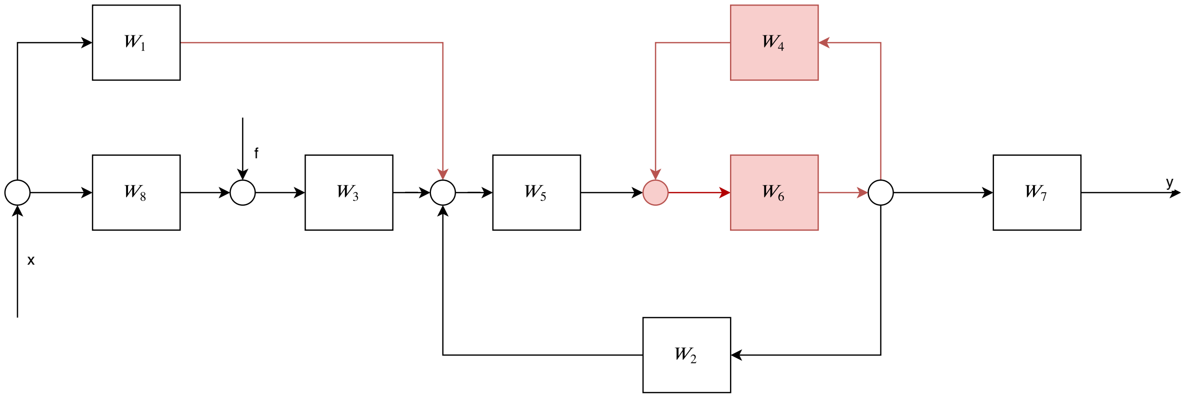
$$\Phi_f(s) = \frac{Y}{F} = \frac{W_3W_5W_6W_7}{1 - W_6W_4 - W_2W_5W_6}$$

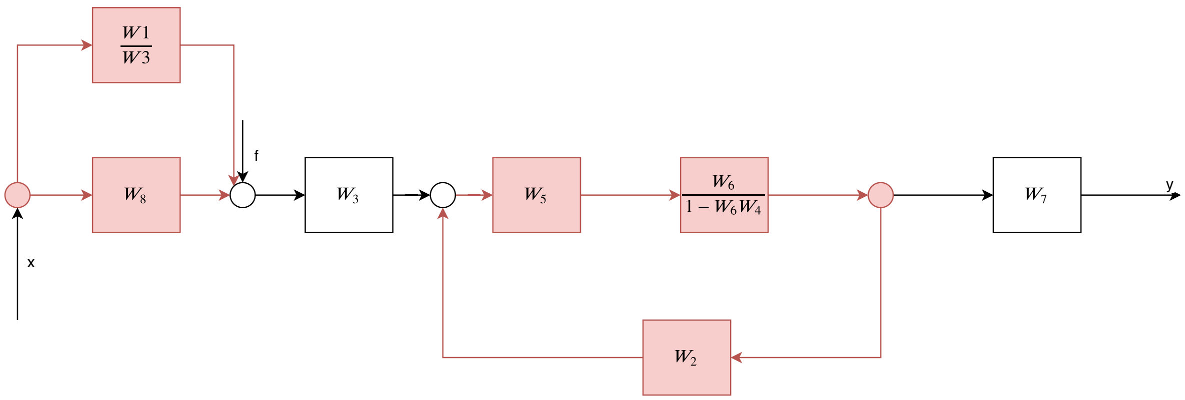
$$Y = \Phi(s)X + \Phi_f(s)F = \frac{W_3W_5W_6W_7}{1 - W_6W_4 - W_2W_5W_6} * ((W_8 + \frac{W_1}{W_3})X + F)$$

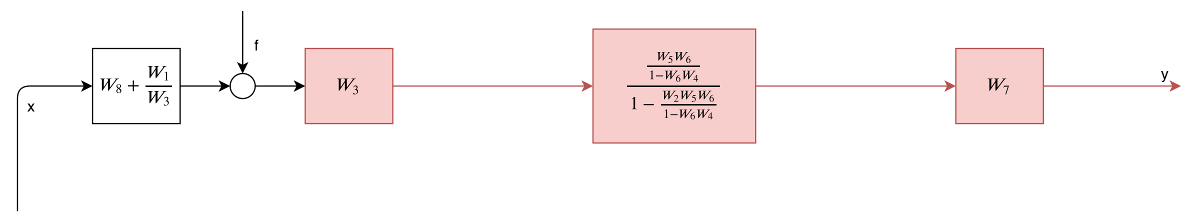
$$Answer : Y = \frac{W_3W_5W_6W_7}{1 - W_6W_4 - W_2W_5W_6} * ((W_8 + \frac{W_1}{W_3})X + F)$$

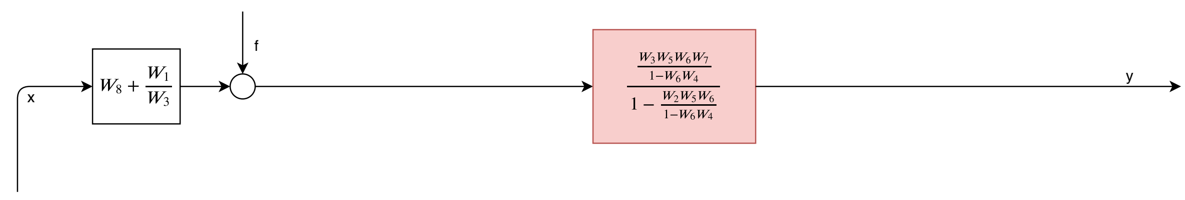
Reduction of Transfer function diagram is on the next page.

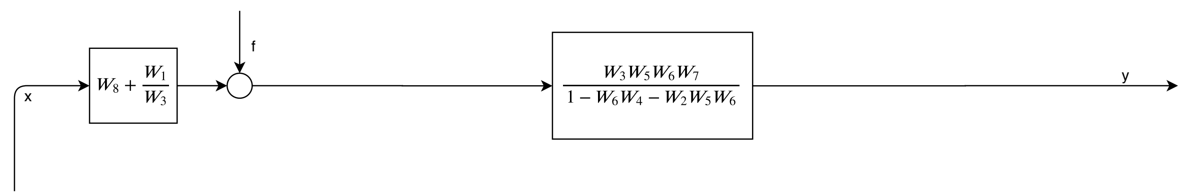

W_1
W_4
f
W_8
W_3
W_5
W_6
W_7
y
x
W_2


W_1
W_4
f
W_8
W_3
W_5
W_6
W_7
y
x
W_2


$\frac{W1}{W3}$
f
W_8
W_3
W_5
$\frac{W_6}{1 - W_6 W_4}$
W_7
y
x
W_2


f
$W_8 + \frac{W_1}{W_3}$
W_3
$\frac{\frac{W_5 W_6}{1 - W_6 W_4}}{1 - \frac{W_2 W_5 W_6}{1 - W_6 W_4}}$
W_7
y
x


f
$W_8 + \frac{W_1}{W_3}$
$\frac{\frac{W_3 W_5 W_6 W_7}{1 - W_6 W_4}}{1 - \frac{W_2 W_5 W_6}{1 - W_6 W_4}}$
y
x


f
$W_8 + \frac{W_1}{W_3}$
$\frac{W_3 W_5 W_6 W_7}{1 - W_6 W_4 - W_2 W_5 W_6}$
y
x