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A Computer Model of Electrocardiogram Signals

by

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Chapter 1

Introduction

Computer modelling of Electrocardiogram Signals (ECGs) is a field of computer science involved with the generation of synthetic ECG signals. This is achieved using a dynamic mathematical model which describes the motion of an ECG. The usage of a dynamic computer model allows users to generate synthetic ECG patterns based on different criteria such as noise, sampling frequency and heart rate. This can be used to simulate ECGs of heart conditions.

Signal processing techniques are valuable for the clinical diagnosis of heart conditions such as arrhythmia. Automatic diagnosis of a patient through signal processing is motivated by conditions where constant monitoring of a patient's condition is infeasible. One such case includes Holter monitoring, which requires 24-hour monitoring of a patient's ECG.

The development of computer models for ECGs have facilitated the development and evaluation of signal processing techniques. Prior to computer models, testing of signal processing techniques such as R-peak detection [1], [2], QT-interval detection [3] and derivation of the heart rate and respiratory rate [4] [5] occurred on real ECGs obtained from patients or from the Physionet database [6]. However, access to patients and equipment to record ECGs can be difficult. Moreover, an ECG obtained from a patient or from a database will contain fixed noise levels and sampling frequencies.

Computer models facilitate the evaluation of signal processing methods by providing parameters to generate signals of varying noise and sampling frequencies. This provides a larger dataset of ECG signals that can be used to evaluate algorithms under different circumstances.

There have been several dynamic models proposed, each utilizing different backgrounds of mathematics to model ECG signals. Proposed dynamic models include [7], [8], [9] and [10].

The main focus in most computer models research is in introducing new models or extending existing models to incorporate features such as parameter fitting [11] or noise filtering [12] while retaining features of the ECG for clinical diagnosis of heart conditions. Implementations of computer models and extensions utilize proprietary software, which is hard to obtain for users to obtain due to licensing requirements. Open source implementations [13] exist, but lack features introduced in extensions over the last two decades. An implementation of a computer that provides the basic properties of an ECG and can generate and fit parameters to normal and abnormal ECG signals would be a good base model for an open source ECG generator.

This thesis aims to develop an open source alternative to a dynamic ECG model that can produce synthetic ECG signals through manual input or by fitting parameters to the closest ECG signal. Open source implementations can be used by any user and would serve as a good basic resource that is able to produce any sort of ECG signal. Chapter 2 will detail a summary of the background required to understand ECGs as well as several proposed dynamic models for ECGs. Chapter 3 states the proposal, timeline and initial results of a computer model for ECG signals.

Chapter 2

Background

2.1 ECG Signal Collection and Usage

An ECG is a record of electrical activity occurring through a heartbeat that is used in diagnosing heart conditions. ECG data is collected by an electrocardiograph, which displays the signal onto a screen or printed onto paper. Modern electrocardiographs typically have screens and have built-in interpretation algorithms for ECG intervals. ECG collection occurs by placing electrodes onto the surface of the skin at various positions across the heart and the limbs. The potential difference between electrodes is recorded and produces the ECG signal.

ECGs can be used to diagnose heart conditions such as arrhythmia, which occurs when the heartbeat is too slow, too fast or irregular. Some arrhythmia are harmless, although some may be life-threatening, such as ventricular fibrillation, which occurs when the heart beats too rapidly and quivers instead of pumping blood. Other conditions such as stroke motivate the need of signal processing techniques to discover heart conditions on ECGs.



Figure 2.1: Modern electrocardiograph machine

2.2 ECG Morphology

A single beat of the heart can be observed as a series of deflections from the baseline of an ECG. Deflections on the ECG represent the electrical activity produced by the heart as it initiates muscle contraction. A single cycle of the ECG is associated with a number of deflections that are traditionally labelled as P, Q, R, S and T.

The ECG may be divided into the following sections:

- P-wave: a small voltage deflection caused by the depolarization of the atria prior to atrial depolarization.
- PQ-interval: the time between the start of atrial depolarization and start of ventricular depolarization.
- QRS-complex: the largest amplitude portion of the ECG that represents atrial repolarization and ventricular depolarization. Atrial repolarization is not observed as ventricular repolarization has greater amplitude.

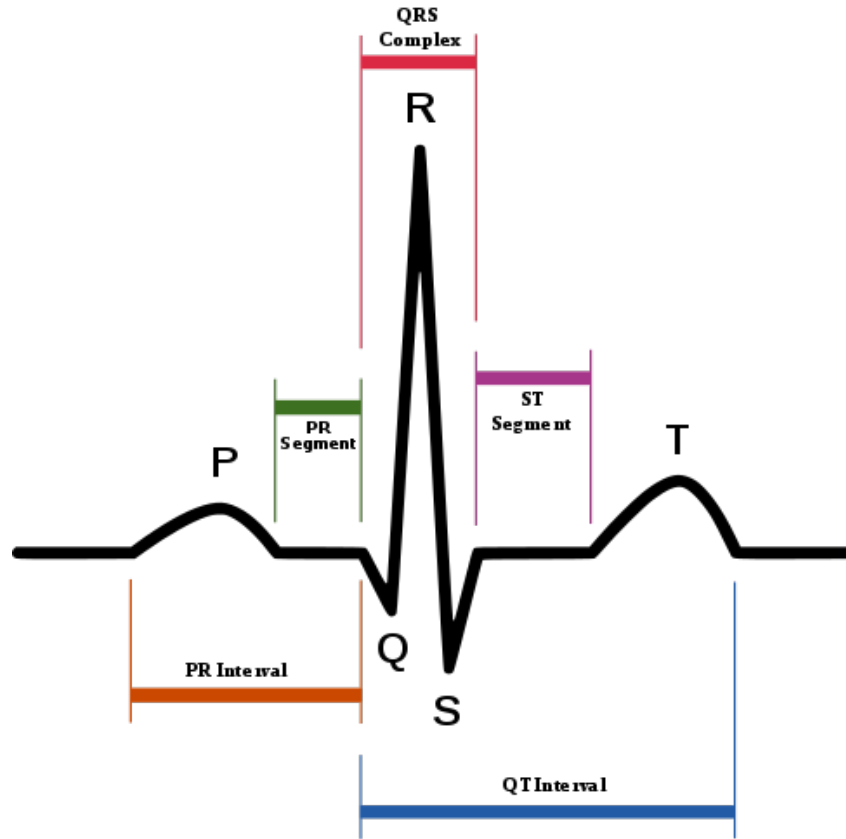


Figure 2.2: Morphology of one PQRST-complex of the ECG

- QT-interval: the time between the onset of ventricular depolarization and the end of ventricular repolarization.
- ST-interval: the time between the end of the S-wave and the beginning of the T-wave.
- T-wave: end of ventricular repolarization, where the heart prepares for the next heartbeat. The period between the T-wave and the P-wave is the same as the baseline for a healthy heart.

2.3 Dynamic Models of ECG Signals

Several ECG models utilizing different mathematical backgrounds have been proposed over time. Of the models, [7] is the most commonly cited and is often used in extensions for modelling ECGs.

2.3.1 ECG Model based on Ordinary Differential Equations

McSharry et al. proposed a synthetic ECG generator based on three dynamic equations of motion. The model generates a quasi-periodic trajectory in a three dimensional (3D) state space with coordinates (x, y, z) . The trajectory revolves around a unit radius around the (x, y) plane, which corresponds to a single heartbeat. The points P, Q, R, S and T are described by events placed at fixed angles along the unit circle labelled as $\theta_P, \theta_Q, \theta_R, \theta_S$ and θ_T .

The equations of motion are given by three ordinary differential equations:

$$\begin{aligned}\dot{x} &= \alpha x - \omega y \\ \dot{y} &= \alpha y + \omega x \\ \dot{z} &= - \sum_{i \in P, Q, R, S, T} a_i \Delta \theta_i \exp \left(- \frac{\Delta \theta_i^2}{2b_i^2} \right) - (z - z_0)\end{aligned}$$

where $\alpha = 1 - \sqrt{x^2 + y^2}$, $\Delta \theta_i = (\theta - \theta_i) \bmod 2\pi$, $\theta = \text{atan2}(y, x)$ and ω is the angular velocity of the trajectory as it moves around the x, y plane. The constants a_i and b_i for $i = 1..5$ are based on the morphology of a healthy ECG. Baseline wander of ECG signals has been modeled with z_0 and respiratory frequency f_2 :

$$z_0 = A \sin(2\pi f_2 t), \quad A = 0.15mV, \quad f_2 = 0.25Hz$$

The model introduced by McSharry has an open source implementation [13] that implements the model. Extensions to the model include model-based filtering, compression [14] and parameter fitting [11]. Other authors have also used this model in extensions, such as the simulation of fetal ECGs [15], hidden Markov model [16] to account for

beat-type changes as well as a wave-based model accompanied by Bayesian Filtering and Kalman smoother [17]. Implementations of these extensions are done in MATLAB.

The success of the McSharry model is attributed to its ease of use and its flexibility to be adapted to extensions. The quasi-periodic nature of the dynamic model, and the maturity of extensions make this model worth studying and implementing.

2.3.2 ECG Model based on Geometrical Features

P. Kovacs [8] proposed a model based on the geometrical features which gives strict mathematical control over the ECG signal.

The model utilizes 15 base points x_1, \dots, x_{15} to find a spline S that satisfies:

$$S^{(i)}(x_k) = f^{(i)}(x_k), \quad (k = 1, \dots, 15; i = 0, 1, 2)$$

where $f : \mathbb{R} \rightarrow \mathbb{R}$ is a time-varying function that represents the ECG curve.

There are 3 values associated with each base point, which makes the model require up to more than 50 parameters.

The base points can be classified into two different classes:

- Diagnostic: represents the base points required for PR, QRS, QT, ST.
- Geometric: represents the positions and amplitudes of P, Q, R, S, T.

Hermite interpolation is used to find the polynomial approximation of the curve. The model uses profiles of lower and upper bounds to create the ECG signal.

The usage of profiles can be used to generate characteristic waves of an ECG. When used with the many parameters the model can provide strict control over ECG signals. However, the number of control points makes this model difficult to implement. In addition to the existing parameters, additional parameters would need to be introduced in order to simulate ECGs of heart conditions. Furthermore, the implementation of extensions of interest to clinical diagnosis such as feature extraction, parameter fitting and noise filtering is uncertain for this model.

2.3.3 ECG Model based on Fourier Series

The work of J. Kubicek [9] aimed to elucidate the use of the Fourier series in producing ECG signals and arrhythmia's.

The model describes P and T waves as a composition of repeating sine waves and the QRS complex as ascending and descending triangle waves. Each part is further divided into 11 parts and described by mathematical functions using coefficients a, b, c, d, k, l, m, n to describe the shifts and slopes of the functions.

Model of P and T waves

P and T waves can be approximated by a parabolic function $f(t) = -t^2$.

$$\begin{aligned} a_0 &= \frac{1}{\pi} \int_c^{c+2\pi} -t^2 dt = \frac{1}{\pi} \left[-\frac{t^3}{3} \right]_c^{c+2\pi} = -\frac{2}{3}\pi^2 \\ a_k &= \frac{1}{\pi} \int_{c+2\pi}^c -t^2 \cos(kt) dt \\ b_k &= \frac{1}{\pi} \int_{c+2\pi}^c t \sin(tx) dx = 0 \\ f(t) &= 4 \sum_{k^2}^{\infty} \frac{(-1)^k}{k^2} \cos(kt) \end{aligned}$$

Model of QRS complex

Waves of QRS can be divided into falling and rising functions $f(t) = \pm t$.

$$\begin{aligned} a_0 &= \frac{1}{\pi} \int_c^{c+2\pi} \pm t dt = \frac{1}{\pi} \left[\frac{\pm t^2}{2} \right]_c^{c+2\pi} = 0 \\ a_k &= \frac{1}{\pi} \int_{c+2\pi}^c \pm t \cos(kt) dt = 0 \\ b_k &= \frac{1}{\pi} \int_{c+2\pi}^c -t \sin(kt) dx = \pm((-1)^{k+1} \frac{2}{k}) \\ f(t) &= \pm 2 \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \sin(kt) \end{aligned}$$

The model is able to simulate various arrhythmia (tachycardia, bradycardia, atrial flutter). The aim of the model is to be used as a basis for detecting pathological

phenomena in ECG and as a proof of theory. It is worthwhile to note that there are multiple studies involving the use of the Fourier series (and FFT) in ECG analysis. These include detection of cardiac arrhythmia [18], automatic feature extraction of ECGs [19] and noise filtering features [20]. The features included indicate that there is potential for development in this model and its extensions. Unlike other models however, the Fourier series is periodic. This is in contrast to the quasi-periodic nature of the McSharry model, which varies slightly per cycle and would be better for simulating other conditions of heart signals.

Chapter 3

Proposal and Plan

3.1 Proposal

This thesis will focus on developing an open source computer model of ECGs that allows for ready solving and visualisation of ECG signals through a graphical user interface that allows parameter input. In addition to solving and visualisation, the computer model will also be able to fit parameters to an ECG trace.

Implementing an open source model was chosen to introduce a basic model that can produce a variety of ECG signals by implementing a proposed extension which is not available in open source software.

3.2 Methodology

The computer model will utilise the McSharry model to generate ECG signals. The McSharry model involves a system of ordinary differential equations that require solving to generate an ECG signal. Components of the model such as baseline wander will not be implemented in this thesis. The McSharry model was chosen for its flexibility and environment of extensions that can be implemented for it.

A Kalman filter will be implemented for parameter fitting. A Kalman filter is a dynamic adaptive filter that can be used to estimate the state of a system with a linear model. As the model proposed by McSharry et al. is non-linear, a linearised version of the McSharry model will be implemented to utilise the concepts of an Extended Kalman Filter [15], which is a Kalman filter that works for non-linear models.

The implementation of the computer model will be written in Python, utilizing the third-party mathematics and science library SciPy (includes SciPy, NumPy, Matplotlib) to solve ordinary differential equations and perform Kalman filtering based on state-spaces.

A graphical user interface will also be written in tkinter, the standard GUI package included in Python distributions. While other modules for implementing a Python-based GUI exist (PyQt, PySide, wxpython), tkinter is native to Python and would be simpler for distribution.

Benefits of using Python include rapid development and support for cross-platform software. Python is also gaining momentum in scientific programming communities alongside SciPy.

3.2.1 Timeline

For this thesis, an agile development methodology utilising the Kanban style with GitHub issues will be implemented to manage the development of the computer model. This is beneficial for the thesis as there are uncertainties of development as well as a lack of members to fulfil other methodologies such as Scrum.

Weekly consultations with Dr. Socrates Dokos will be incorporated to track the progress of development, as well as relevant upskilling required to develop mathematical components required for the model. This includes understanding and implementing the mathematics behind the McSharry model and an Extended Kalman Filter.

Other upskilling requirements include the use of SciPy (and its other modules) as well

as learning to design an effective GUI using tkinter.

Timeline for Term 2

Weeks 1 - 2

- Organise weekly meetings for consultations and meetings to track progress.
- Reading on Kalman Filter, McSharry model mathematics.
- Research into improving tkinter graphical user interface.

Weeks 2 - 5

- Improve graphical user interface.
- Implement Kalman filtering and parameter fitting.

Weeks 6 - 7

- Implement Kalman filtering and parameter fitting.

Weeks 8 - 10

- Implement Kalman filtering and parameter fitting.
- Report writing.

Timeline for Term 3

Weeks 1 - 2

- Organise weekly meetings for consultations and meetings to track progress.
- Continued development.
- Decide evaluation method (testing various ECG signals).

Weeks 3 - 6

- Testing ECG generation and parameter fitting.

Weeks 7

- Packaging and distribution of software package.

Weeks 8 - 10

- Write final report.

3.3 Initial Results

With the variables and equations presented in [7], a basic computer model was implemented in Python. The model exists within an application that allows building of simple ECG waves with parameters, not including parameter fitting.

The implementation serves as a base for work to be completed in the future and acts as experience in developing software using SciPy and tkinter.

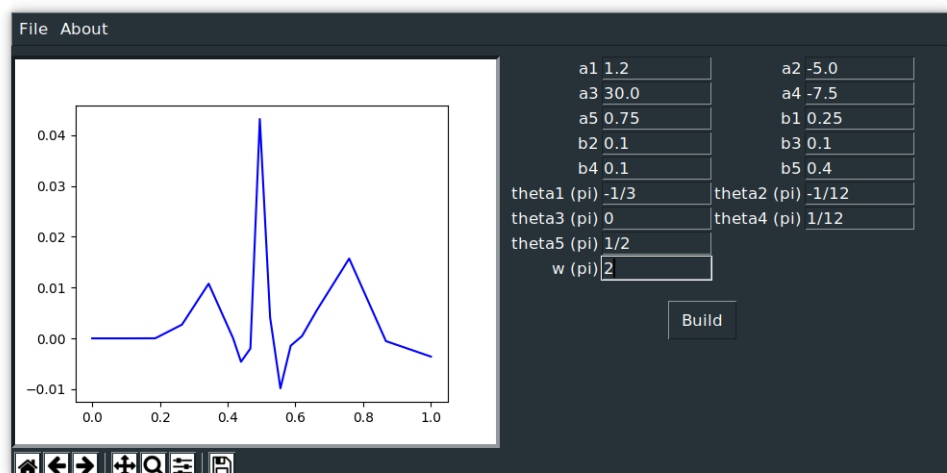


Figure 3.1: GUI application for simple McSharry model

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