ECG denoising using Extended Kalman Filter

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***Abstract*—In this paper a combination of Extended Kalman Filter (EKF) and a dynamic model of a synthetic electrocardiogram (ECG) for ECG denoising is proposed. Experimental results show that the proposed algorithm is very efficient for the extraction of the ECG signals from noisy data measurements.**

Keywords-Extended; Kalman Filtre; Nonlinear dynamic model ;Electrocardiogram.

I. INTRODUCTION

An electrocardiogram describes the electrical activity in the heart, and can be decomposed in characteristic components, named P, Q, R, S and T waves [1]. The cardiac electrical activity is a convenient non-invasive tool for the detection, prediction and monitoring of rare cardiac events and related anomalies such as arrhythmias. ECG recording may include distortions due to noise contamination, artifacts and interference from other signals. Hence extraction of pure cardiological indices from noisy measurements has been one of the major concerns of biomedical signal processing and needs reliable techniques to preserve the diagnostic information of the recorded signal.

Electrocardiogram signals are time-varying signals, and their modeling is not an easy task. In [2], a model for generating synthetic ECG signals was proposed in order to give a standard realistic ECG signal. With this model we can generate ECG signals with different sampling frequencies and different noise levels.

The first attempt to apply Extended Kalman filter (KF) in Cartesian coordinates to ECG denoising was proposed in [3] where the authors combined the dynamic model of the synthetic ECG proposed in [1] with the extended Kalman filter, which is in our opinion not true because the dynamic model given in [1] is a continuous process and EKF is a discrete algorithm.

In this paper a discrete version in Cartesian coordinates of the dynamical model given in [1] is proposed, and then we will combine it with EKF in order to denoise ECG signals.

The outline of the paper is as follows. Section2 provides a brief review of EKF theory. In section 3 we summarize an already synthetic ECG model. In section 4 we propose a discrete model for the synthetic ECG signal. The linearization of the discrete synthetic model is presented in section 5. Simulation and experimental results are provided in section 6. Finally, conclusions are provided in section 7.

II. EXTENDED KALMAN FILTER (EKF)

The EKF is a nonlinear extension of conventional Kalman filter [4][5], that has been specifically developed for systems having nonlinear dynamic model. A discrete and nonlinear system $x_{k+1} = f(x_k, w_k)$ and its nonlinear observation $y_k = g(x_k, v_k)$ can be approximated linearly near reference point $(\hat{x}_k, \hat{w}_k, \hat{v}_k)$ as follows:

$$\begin{cases} x_{k+1} \approx f(\hat{x}_k, \hat{w}_k) + A_k(x_k - \hat{x}_k) + F_k(w_k - \hat{w}_k) \\ y_k \approx g(\hat{x}_k, \hat{v}_k) + C_k(x_k - \hat{x}_k) + G_k(v_k - \hat{v}_k) \end{cases} \quad (1)$$

where x_k the state vector, w_k the state noise with zero mean and covariance matrices $Q_k = E\{w_k w_k^T\}$, $f(.)$ the nonlinear dynamic, y_k observation vector, v_k the measurement noise with zero mean and covariance matrices $R_k = E\{v_k v_k^T\}$, $g(.)$ the nonlinear sensor model and A_k, F_k, C_k, G_k are the Jacobian matrices given by

$$\begin{aligned} A_k &= \left.\frac{\partial f(x, \hat{w}_k)}{\partial x}\right|_{x_k=\hat{x}_k} \qquad F_k = \left.\frac{\partial f(\hat{x}_k, w_k)}{\partial w}\right|_{w=\hat{w}_k} \\ C_k &= \left.\frac{\partial g(x, \hat{v}_k)}{\partial x}\right|_{x=\hat{x}_k} \qquad G_k = \left.\frac{\partial g(\hat{x}_k, v)}{\partial v}\right|_{v=\hat{v}_k} \end{aligned} \quad (2)$$

Then, extended Kalman equations can be given by:

$$\begin{aligned} &\hat{x}_{k/k-1} = f(\hat{x}_{k-1/k-1}, 0) \\ &P_{k/k-1} = A_k P_{k-1/k-1} A_k^T + F_k Q_k F_k^T \\ &\hat{x}_{k/k} = \hat{x}_{k/k-1} + K_k\left[y_k - g(\hat{x}_{k/k-1}, 0)\right] \\ &K_k = P_{k/k-1} C_k^T \left[C_k P_{k/k-1} C_k^T + G_k^T\right]^{-1} \\ &P_{k/k} = P_{k/k-1} - K_k C_k P_{k/k-1} \end{aligned} \quad (3)$$

where $\hat{x}_{k/k-1} = E\{x_k | y_{k-1}, y_{k-2}, \dots, y_1\}$ is estimate of the state vector x_k at instant k *given* observations y_1 to y_{k-1}, and $\hat{x}_{k/k} = E\{x_k | y_k, y_{k-1}, \dots, y_1\}$ is estimate of the state vector x_k at instant k *given* observations y_1 to y_k. $P_{k/k-1}$ and $P_{k/k}$ are defined in the same manner.

III. ECG DYNAMIC MODEL

In [2], the authors have proposed a dynamical model which can generates a synthetic ECG signal in the Cartesian space (x, y, z). This model is constituted by three coupled differential equations in the state space as follows:

$$\begin{aligned} \dot{x} &= \alpha x - \omega y \\ \dot{y} &= \alpha y + \alpha \omega \\ \dot{z} &= -\sum_{i \in \{P,Q,R,S,T\}} \frac{A_i \omega}{b_i^2} \Delta\theta_i \exp\left(-\frac{\Delta\theta_i^2}{2b_i^2}\right) - (z - z_0) \end{aligned} \quad (4)$$

where: $\alpha = 1 - \sqrt{x^2 + y^2}$, $\Delta\theta_i = (\theta - \theta_i) mod(2\pi), \theta = atan2(y, x)$ (the four quadrant arctangent of the real parts of the elements of x and y), with $-\pi \leq atan2(y, x) \leq \pi$, and ω is the angular velocity of the trajectory as it moves around the limit cycle. A_i, b_i and θ_i correspond to the amplitude, width and center parameters of the Gaussian. The baseline wander was introduced by coupling the baseline z_0 in (4) to the respiratory frequency f_2 as:

$$z_0 = A sin(2\pi f_2 t) \quad (5)$$

where $A = 0.15mV \quad f_2 = 0.25Hz$

Parameters values of (4) are listed in Table I.

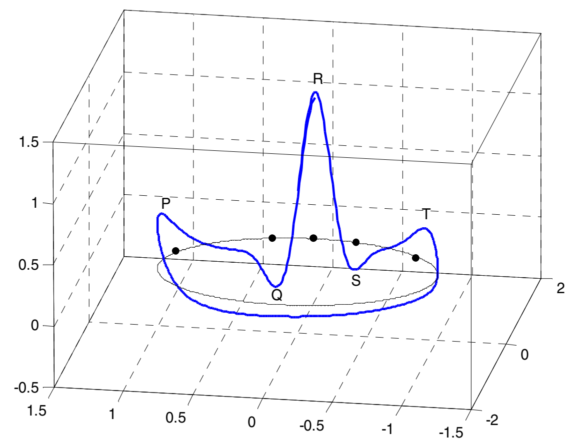

Figure 1. Output of the dynamial model (4)

TABLE I
PARAMETRES OF THE ECG MODEL (4)

Index (i)	**P**	**Q**	**R**	**S**	**T**
Time (sec)	-0.2	-0.05	0	0.05	0.3
θ_i (rads)	-π/3	-π/12	0	π/12	π/2
A_i	20	-50	300	-75	75
b_i	0.25	0.1	0.1	0.1	0.4

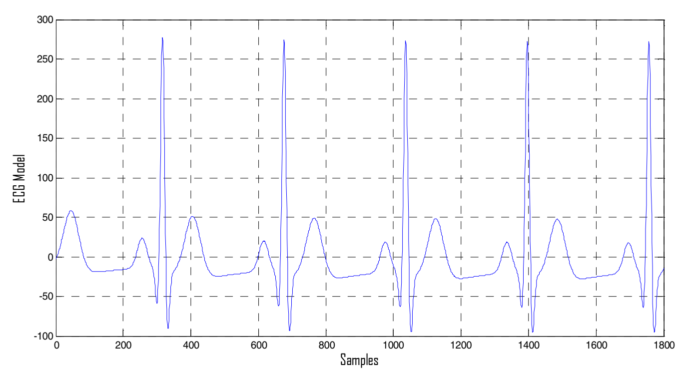

Figure 2. Synthetic ECG generated by (4)

IV. DISCRETISATION OF ECG MODEL

The nonlinear dynamic ECG model (4) is a continuous-time model, and since Kalman filter is a discrete-time algorithm, then a discretization of the continuous nonlinear dynamic ECG model (4) is necessary. We propose here to discretize (4) by using Euler method. Thus, the discrete form of (4) iwill be given by:

$$
\begin{aligned}
&x(k+1) = (1+\alpha h)x(k) - \omega h y(k) \\
&y(k+1) = (1+\alpha h)y(k) + \omega h x(k) \\
&z(k+1) = -\sum_{i\in\{P,Q,R,S,T\}} \frac{A_i\omega}{b_i^2} h\Delta\theta_i \exp\left(-\frac{\Delta\theta_i^2}{2b_i^2}\right) - ((h-1)z(k) - hz_0)
\end{aligned}
\tag{6}
$$

where h is the sampling time.

A. Quasi real ECG Model

The nonlinear discrete ECG model (6), can be rewritten in the following compact form:

$$X_{k+1} = f(X_k) \tag{7}$$

where X_k is the state vector given by $X_k = [x_k \quad y_k \quad z_k]^T$.

$$
\begin{pmatrix} x(k+1) \\ y(k+1) \\ z(k+1) \end{pmatrix} = \begin{pmatrix} (1+\alpha h)x(k) - \omega h y(k) \\ (1+\alpha h)y(k) + \omega h x(k) \\ -\sum_{i\in\{P,Q,R,S,T\}} \frac{A_i\omega}{b_i^2} h\Delta\theta_i \exp\left(-\frac{\Delta\theta_i^2}{2b_i^2}\right) - ((h-1)z(k) - hz_0) \end{pmatrix} \tag{8}
$$

The vector equation (8) represents the state equation without noise of the discrete EC G model. To represent a more realistic ECG signal, we need to introduce some random noises in (7) as follows:

$$X_{k+1} = f(X_k, w_k) \tag{9}$$

where $w_k = [w_1 \quad w_2 \quad w_3]^T$ is an additive random vector called state noise, which is normal and Gaussian with zero mean, then (8) becomes as follows:

$$
\begin{pmatrix} x(k+1) \\ y(k+1) \\ z(k+1) \end{pmatrix} = \begin{pmatrix} (1+\alpha h)x(k) - \omega h y(k) + w_1(k) \\ (1+\alpha h)y(k) + \omega h x(k) + w_2(k) \\ -\sum_{i\in\{P,Q,R,S,T\}} \frac{A_i\omega}{b_i^2} h\Delta\theta_i \exp\left(-\frac{\Delta\theta_i^2}{2b_i^2}\right) - ((h-1)z(k) - hz_0) + w_3(k) \end{pmatrix} \tag{10}
$$

The measurement equation corresponding to the state space representation (10) can be joined to the state vector $X_k = [x_k \quad y_k \quad z_k]^T$ by the following relation:

$$
\begin{aligned}
y_k &= g(X_k, v_k) \\
&= [0 \quad 0 \quad 1] X_k + v_k
\end{aligned}
\tag{11}
$$

with y_k is the considered measure, and v_k is a measurement noise which is also additive, normal and Gaussian with zero mean.

V. LINEARIZATION

In order to use EKF, a linear approximation of (10) is needed (see (1) and (2)). In order to compute the entries of

Jacobian matrices (see (2)), Let arrange (10) and (11) as follows:

$$
\begin{cases}
x(k+1) = X(x(k), y(k), z(k), w_1(k)) \\
y(k+1) = Y(x(k), y(k), z(k), w_2(k)) \\
z(k+1) = Z(x(k), y(k), z(k), w_3(k))
\end{cases}
\tag{12}
$$

$$y_k = g(X_k, v_k) \tag{13}$$

Then, the entries of Jacobian matrix A_k are

$$
\begin{aligned}
&\frac{\partial X}{\partial x} = 1 + h - \left(\frac{2hx(k)^2 + hy(k)^2}{\sqrt{x(k)^2 + y(k)^2}}\right) \\
&\frac{\partial X}{\partial y} = -\frac{hx(k)y(k)}{\sqrt{x(k)^2 + y(k)^2}} - \omega h \qquad \frac{\partial X}{\partial z} = 0 \\
&\frac{\partial Y}{\partial x} = -\frac{hx(k)y(k)}{\sqrt{x(k)^2 + y(k)^2}} + \omega h \\
&\frac{\partial Y}{\partial y} = 1 + h - \left(\frac{hx(k)^2 + 2hy(k)^2}{\sqrt{x(k)^2 + y(k)^2}}\right) \qquad \frac{\partial Y}{\partial z} = 0 \\
&\frac{\partial Z}{\partial x} = \sum_{i\in\{P,Q,R,S,T\}} \frac{\frac{A_i\omega}{b_i^2} hy(k)}{x(k)^2 + y(k)^2} \exp\left(-\frac{\Delta\theta_i^2}{2b_i^2}\right)\left[1 - \frac{\Delta\theta_i^2}{b_i^2}\right] \\
&\frac{\partial Z}{\partial y} = \sum_{i\in\{P,Q,R,S,T\}} \frac{-\frac{A_i\omega}{b_i^2} hx(k)}{x(k)^2 + y(k)^2} \exp\left(-\frac{\Delta\theta_i^2}{2b_i^2}\right)\left[1 - \frac{\Delta\theta_i^2}{b_i^2}\right] \\
&\frac{\partial Z}{\partial z} = 1 - h \\
&\frac{\partial X}{\partial w_1} = \frac{\partial Y}{\partial w_2} = \frac{\partial Z}{\partial w_3} = 1 \\
&\frac{\partial X}{\partial w_2} = \frac{\partial X}{\partial w_3} = \frac{\partial Y}{\partial w_1} = \frac{\partial Y}{\partial w_3} = \frac{\partial Z}{\partial w_1} = \frac{\partial Z}{\partial w_2} = 0 \\
&\frac{\partial g}{\partial x} = \frac{\partial g}{\partial y} = 0 \ ; \frac{\partial g}{\partial z} = 1 - h \ ; \frac{\partial g}{\partial v} = 1
\end{aligned}
\tag{14}
$$

VI. RESULTS

In this section, experimental studies of both synthetic and real ECGs are carried out to evaluate the performance of the proposed method. Let $s(t) = x(t) + n(t)$ the noisy signal to be denoised to obtain a smooth reconstructed version $\hat{x}(t)$ which means that the corrupted signal $s(t)$ is composed by an original clean signal $x(t)$ added to a noise realization $n(t)$.

For evaluating the performances, input SNR (signal to noise ratio), output SNR and MSE (mean squared error) are used. Input SNR will measure the amount of noise in $s(t)$, and output SNR will measure the amount of noise in $\hat{x}(t)$ and the denoising effectiveness will be the distance between these two SNRs (bigger distance, higher performances). Input, output SNRs and MSE are defined as follows:

$$SNR_{IN}\left[dB\right] = 10\log_{10}\left(\frac{\sum_i \left|x(i)\right|^2}{\sum_i \left|n(i)\right|^2}\right) \tag{15}$$

$$SNR_{OUT}\left[dB\right] = 10\log\left(\frac{\sum_i \left|x(i)\right|^2}{\sum_i \left|x(i) - \hat{x}(i)\right|^2}\right) \tag{16}$$

$$MSE = \frac{\sum_i \left(x(i) - \hat{x}(i)\right)^2}{N} \tag{17}$$

In what follows, we will test the proposed method for filtering electrocardiogram signals. In a first step, the signal considered for filtering purposes will be the synthetic ECG signal generated by the quasi real ECG model (10)-(11) (denoising of an artificial ECG), then in a second step, a real ECG signal will be considered.

A. Denoising of a sythetic ECG signal

The considered ECG signal is generated by the model (10) and its measure (11). The period $\omega/2\pi$ of this signal is chosen to be 1 sec. The parameters of this ECG signal are given in Table. 1. Fig. 3 represents the synthetic noised ECG to be denoised, corrupted by state noise of variance $var(w_k) = 10^{-8}mv^2$ and input SNR equal to 10 dB.

The simulation results are given in fig.4 and 5. In fig.4 we give the denoised ECG signal obtained by EKF with output SNR equal to *14.4069dB*. In fig.5 we display the corresponding errors (errors before and after denoising operation), where we note clearly that the error after denoising is smaller than error before denoising, which confirm the efficiency of EKF in synthetic ECG denoising. To see the quality of filtering numerically and evaluate the performances, the formulas (15), (16) and (17) are used, and their relative results are give in TABLE II where we approve that the proposed method can minimizes the effect of the noise on the generated ECG signal.

Figures 6 and 7 confirm this fact quantitatively (output SNR and output MSE) by using multiple realizations of Gaussian noise at different input SNRs ranged from 6 to 18 dB (Horizontal axes for both plots). Note that each output SNR (fig.6) and each output MSE (fig.7) is obtained from 100 Monte Carlo runs in order to represent averaged output SNR and MSE. We see clearly in figures 6 and that output SNR and output MSE are improved as input SNR is increased.

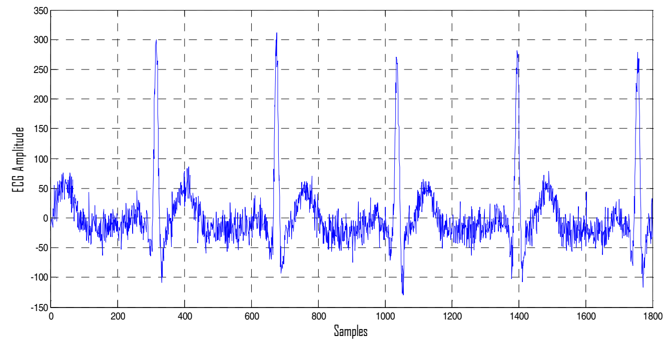

Figure 3. Noised synthetic ECG signal generated by (10) and (11) with white Gaussian noise having input SNR=10 dB .

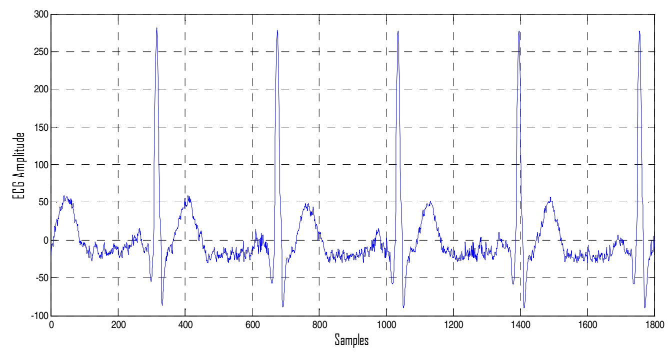

Figure 4. Denoised syntetic ECG signal with output SNR=*14.4069dB*

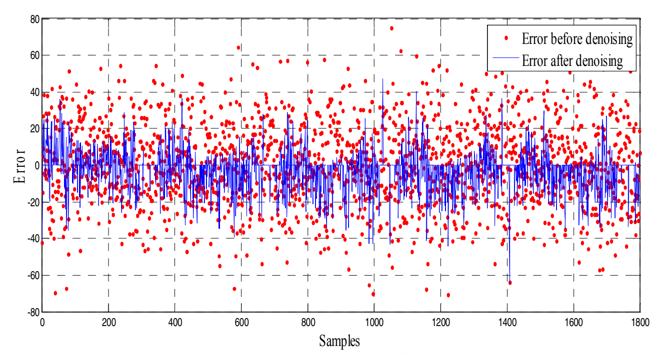

Figure.5 Errors before and after denoising

TABLE II
PERFORMANCE RESULTS FOR SYNTHETIC ECG DENOISING

Noisy synthetic ECG		**Denoised synthetic ECG**	
Input SNR [db]	**Input MSE**	**Output SNR [db]**	**Output MSE**
10	260.6487	14.4069	109.6316

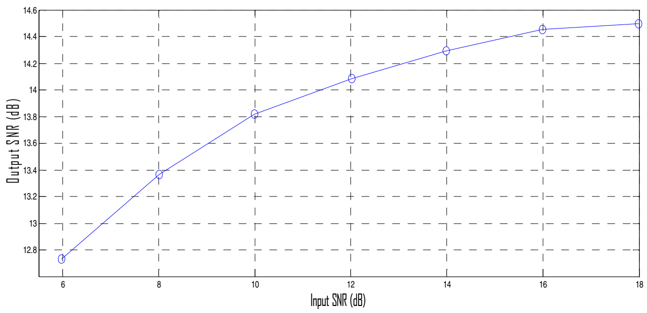

Figure 6. Output SNR vs Input SNR for syntetic ECG signal

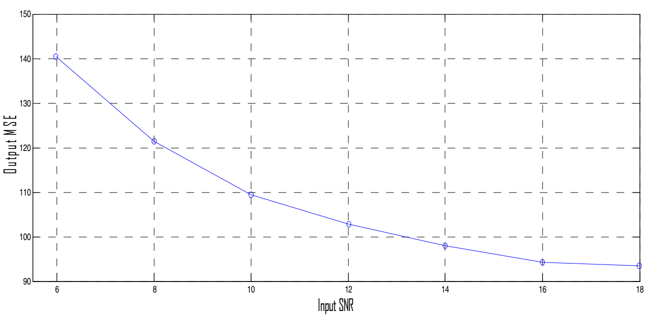

Figure 7. Output MSE vs Input SNR for syntetic ECG signal

B. *Denoising of a real ECG signal*

Real ECG signal considered here is taken from MIT-BIH database [6], referenced by 103.dat. The sampling frequency of this signal is 360 Hz. Parameters values in (4) proposed for this real ECG signal are listed in Table III.

TABLE III
PROPOSED PARAMETERS FOR ECG MODEL (4)
TO DENOISE REAL ECG SIGNAL 103.dat

Inde(i)	P	Q	R	S	T
θ_i (rads)	-π/3	-π/12	0	π/12	π/2
A_i	20	-50	450	-75	75
b_i	0.25	0.1	0.1	0.1	0.4

Fig. 8(a) represents the real clean ECG, Fig. 8(b) represents the corrupted ECG obtained by adding measurement noise with input SNR equal to 10 dB.

TABLE IV
PERFORMANCE RESULTS FOR REAL ECG DENOISING (103.dat)

Noisy synthetic ECG		Denoised synthetic ECG	
Input SNR [db]	Input MSE	Output SNR [db]	Output MSE
10	387.3437	13.6593	176.5962

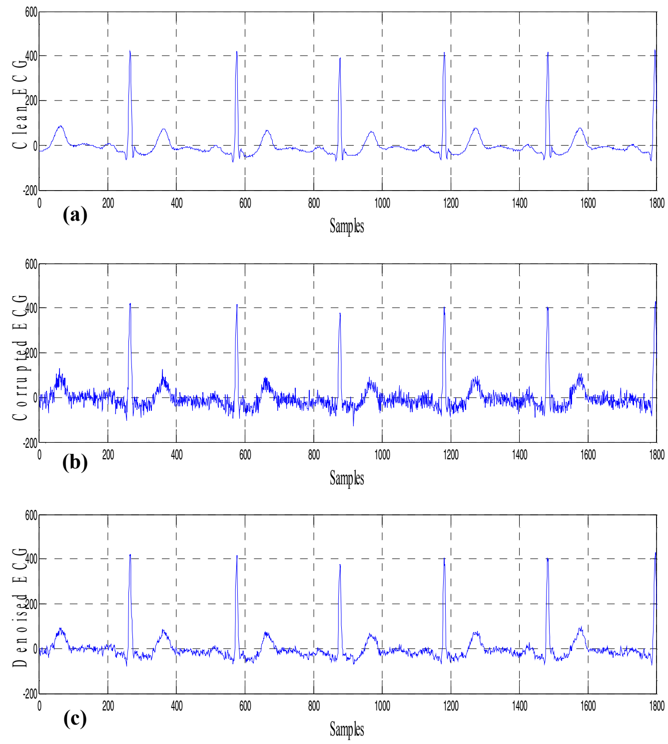

Figure 8. (a) Clean ECG signal 103.dat (b) Corrupted ECG signal 103 with input SNR 10dB (c) Denoised ECG signal 103.dat

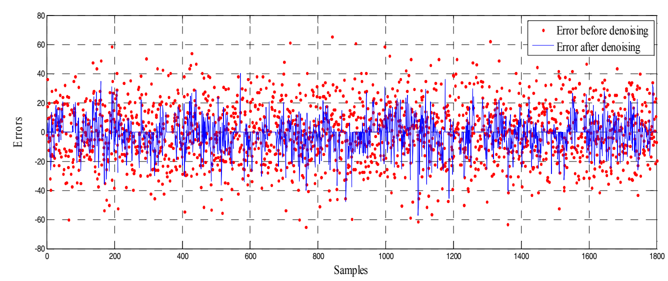

Figure.9 Errors before and after denoising fro ECG 103.dat

Denoising result is given in fig.8(c), where we give the denoised ECG signal 103.dat obtained by EKF with output SNR equal to *13.6593dB* relative to 10dB input SNR. In fig.9 we display the corresponding errors (errors before and after denoising), where we note clearly that the error after denoising is smaller than error before denoising except at R-peaks which constitute a difficult problem in ECG denoising. To see the quality of filtering numerically and evaluate the performances, the formulas (15), (16) and (17) are also used, and their relative results in real ECG denoising are give in TABLE IV where we approve also for real ECGs the effectiveness of EKF in ECG denoising.

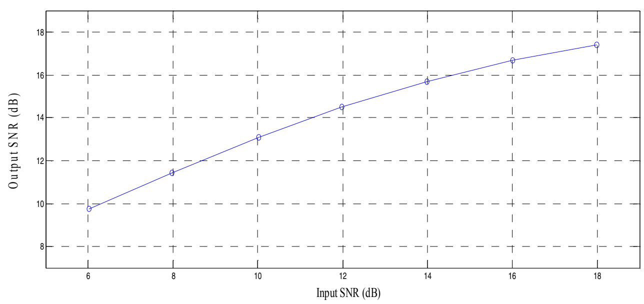

Figure 10. Output SNR vs Input SNR for ECG signal 103.dat

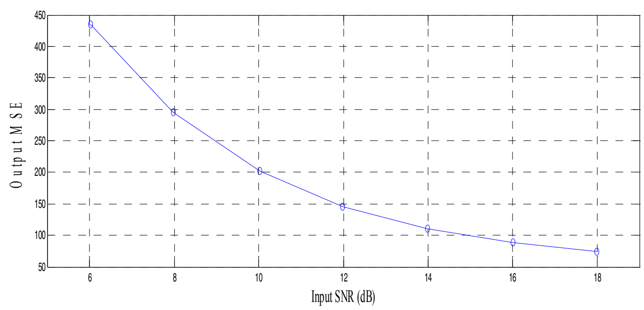

Figure 11. Output MSE vs Input SNR for ECG signal 103.dat

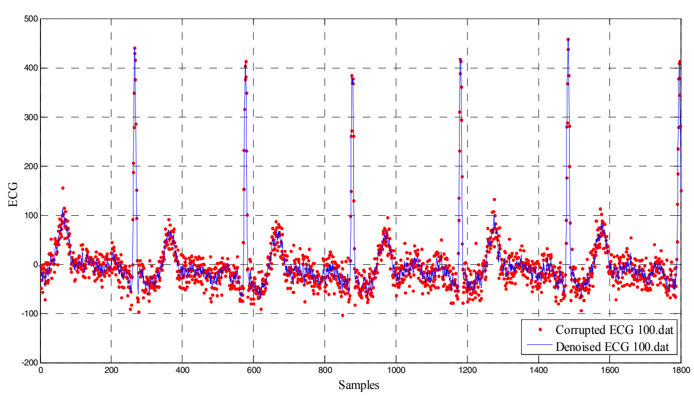

Figure 12. Output MSE vs Input SNR for ECG signal103.dat

Figures 10 and 11 confirm more generally the denoising performances quantitatively (output SNR and output MSE) by using multiple realizations of Gaussian noise at different input SNRs ranged from 6 to 18 dB and 100 Monte Carlo runs. In fig.12 we present a superposition of noisy and denoised ECG 103.dat which gives us a visual comparison of denoising performances.

VII. ONCLUSION

In this paper an EKF for synthetic and real ECGs denoising purposes was designed. Since EKF needs a discrete dynamical model, a previously proposed model was used. At this level, a discrete version of this model in Cartesian space was proposed in this paper and even its linearization. The results of this paper approve the applicability of the Extended Kalman Filter (EKF) in Cartesian space, for the denoising of corrupted ECG signals.

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