

Model-Based ECG Fiducial Points Extraction Using a Modified Extended Kalman Filter Structure

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Abstract—This paper presents an efficient algorithm based on a nonlinear dynamical model for the precise extraction of the characteristic points of electrocardiogram (ECG), which facilitates the HRV analysis. Determining the precise position of the waveforms of an ECG signal is complicated due to the varying amplitudes of its waveforms, the ambiguous and changing form of the complex and morphological variations with unknown sources of drift. A model-based approach handles these complications; therefore a method based on the usage of this concept in an extended Kalman filter structure has been developed. The fiducial points are detected using both the parameters of Gaussian-functions of the model, and the state variable estimates. Several MIT-BIH ECG records are used for performance evaluation. Results show that the proposed method has an average sensitivity of 100% and a specificity of 99.93%. Simulation results illustrate that the method can contribute to and enhance the clinical ECG points extractions performance and exact tachogram representation.

Index Terms—Electrocardiogram, ECG dynamical model, Extended Kalman filter, Fiducial points, HRV.

I. INTRODUCTION

THE automatic analysis of the electrocardiogram (ECG) has been the subject of intense research during the last three decades and is well-known in the biomedical engineering field. The particular interest for ECG analysis comes from its role as an efficient noninvasive investigative method which provides useful information for the detection, diagnosis and treatment of cardiac diseases [1].

The ECG signal has a time periodicity allowing defining an elementary beat composed by specific waveforms, appearing pseudo-periodically in time. The study of the waveform amplitudes, timings and patterns constitutes the basis of the ECG analysis. For instance, one can easily show that the heart rate is estimated after detecting the QRS-complex from a beat sequence. In the same way, the time-distance between two consecutive QRS-complexes, known as RR-interval, is used to detect premature beats. We can extend this analysis to other conditions like the ST-segment deviation from a long period, necessary to early diagnosis of ischemia. As a result, reliable ECG analysis depends directly on the ECG beat segmentation results [1], [2].

The focus of this paper is to detect all of the fiducial points related to the five main waveforms in an electrocardiogram with baseline wander. Fiducial points (FPs) in an ECG signal are the onset and offset of P and T waveforms, and the locations of QRS complex. Efforts have

aimed to cope with the problem of characteristic points extraction in an ECG. Most works in this field employ heuristic rules to segment heartbeat automatically from the ECG signal after performing a suitable preprocessing technique [3]–[6], and many authors underline the advantages of the wavelet transform. The multiscale decomposition improves robustness, when the signal is corrupted by noise [6]. On the other hand, regarding the beat classification task, a large number of methods have already been proposed. In general, the classification approaches are heuristic [7]–[9], namely decision trees and fuzzy logic [10], and statistics, namely discriminant analysis [10], hidden Markov models (HMMs) [11], neural networks [12]–[14], and statistical ruled based systems [15].

On the other hand, a synthetic model has been proposed for generating artificial electrocardiograms, which has unified the morphology and pulse timing in a single nonlinear dynamic model [16]. Concerning the simplicity and flexibility of this model, it can be easily used as a base for ECG processing, as demonstrated by Clifford et al [17], where the use of the model to filter, compress and classify the ECG was first proposed. This approach was based on the Least Squares Error (LSE) optimization. Recently, we investigated the usage of LSE optimization based on the ECG Dynamical Model (EDM) for fiducial points extraction from wandered ECG recordings [18].

The model may be further used in dynamic adaptive filters, such as the *Kalman Filter* (KF). Sameni *et al* proposed the use of a KF framework to update the model on a beat-to-beat basis in order to filter noisy ECGs [19]–[22]. The polar form of the dynamical equations was also used for Kalman based ECG denoising [21]. In addition, a modified form of the model was used for simultaneous denoising and compression of ECG signals [23], [24]. In this paper, the modified KF framework has been used to estimate the system state variables, with which the point extraction task is accomplished. In fact, the modified KF structure is aimed at estimating new parameters, as well as the ECG signal, which are directly related to the locations of the ECG characteristic points. Meanwhile, the model is nonlinear and requires the nonlinear counterparts of the conventional Kalman filter. Although there are several Bayesian filters such as the Extended Kalman Smoother (EKS) and Unscented Kalman Filter (UKF), in this work we have chosen the Extended Kalman Filter (EKF) for its simplicity and more numerical stability. However, the overall filtering performance is expected to be better with EKS or UKF.

The paper is organized as follows. Section II summarizes the ECG artificial model. Section III provides backgrounds on the EKF theory, and the construction of EKF framework.

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In section IV our proposed algorithm for fiducial points determination and tachogram extraction is explained in details. Simulation results are provided in section V. Finally, discussion and conclusions are provided in section VI.

II. ECG DYNAMICAL MODEL (EDM)

McSharry *et al.* [16] proposed a realistic synthetic ECG generator using a set of three dimensional state equations that generates a trajectory in the Cartesian coordinates. Sameni *et al.* [21] transformed these dynamic equations into the polar form to obtain a simpler compact set, with the simplified discrete form shown as:

$$\begin{cases} \theta_{k+1} = (\theta_k + \omega\delta) \bmod (2\pi) \\ z_{k+1} = - \sum_{i \in \{P, Q, R, S, T\}} \delta \frac{\alpha_i \omega}{b_i^2} \Delta \theta_i \exp\left(-\frac{\Delta \theta_i^2}{2b_i^2}\right) + z_k + \eta \end{cases} \quad (1)$$

where δ is the sampling time, α_i, b_i, θ_i are the amplitude, angular spread and location of the Gaussian functions, and $\Delta \theta_i = (\theta - \theta_i) \bmod (2\pi)$ represents the RR interval variability. The ECG is then described by the set of discrete samples formed by z . The additive term η is a random white noise which represents the baseline wander effects and models other additive sources of process noise [21]. As it can be seen, the PQRST waves are modeled with a sum of five Gaussian functions, each of which located at a specific angular position θ_i . In fact, the three dimensional trajectory consists of a circular limit cycle in the polar plane which is pushed up and down as it approaches each of the θ_i , as shown in Fig. 1.

III. THE MODIFIED EKF STRUCTURE

The Extended Kalman Filter is a nonlinear extension of conventional Kalman Filter that has been specifically developed for systems having nonlinear dynamic models [25]. For a discrete nonlinear system with the state vector x_k and observation vector y_k , the dynamic model and its linear approximation near a desired reference point may be formulated as follows:

$$\begin{cases} \underline{x}_{k+1} = f(\underline{x}_k, \underline{w}_k, k) \\ \quad \approx f(\hat{\underline{x}}_k, \hat{\underline{w}}_k, k) + A_k(\underline{x}_k - \hat{\underline{x}}_k) + F_k(\underline{w}_k - \hat{\underline{w}}_k) \\ \underline{y}_k = g(\underline{x}_k, \underline{v}_k, k) \\ \quad \approx g(\hat{\underline{x}}_k, \hat{\underline{v}}_k, k) + C_k(\underline{x}_k - \hat{\underline{x}}_k) + G_k(\underline{v}_k - \hat{\underline{v}}_k) \end{cases} \quad (2)$$

where

$$\begin{aligned} A_k &= \left. \frac{\partial f(\underline{x}, \underline{w}, k)}{\partial \underline{x}} \right|_{\underline{x}=\hat{\underline{x}}_k} & F_k &= \left. \frac{\partial f(\hat{\underline{x}}_k, \underline{w}, k)}{\partial \underline{w}} \right|_{\underline{w}=\hat{\underline{w}}_k} \\ C_k &= \left. \frac{\partial g(\underline{x}, \underline{v}, k)}{\partial \underline{x}} \right|_{\underline{x}=\hat{\underline{x}}_k} & G_k &= \left. \frac{\partial g(\hat{\underline{x}}_k, \underline{v}, k)}{\partial \underline{v}} \right|_{\underline{v}=\hat{\underline{v}}_k} \end{aligned} \quad (3)$$

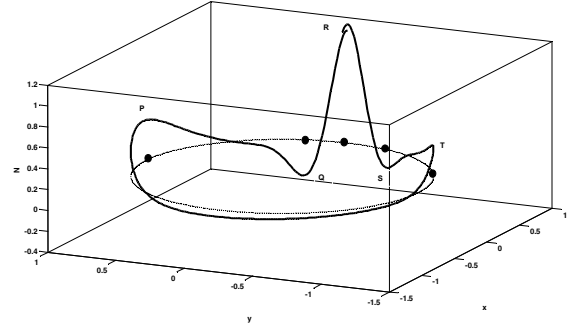


Fig. 1. Typical trajectory generated by the dynamical model (1) in the 3-D space given by (x, y, z) . The small circles show the positions of the P, Q, R, S, and T events on the limit cycle of unit radius.

Here w_k and v_k are the process and measurement noises respectively with covariance matrices $Q_k = E\{w_k w_k^T\}$ and $R_k = E\{v_k v_k^T\}$. In order to implement the EKF, the time propagation and the measurement propagation equations are summarized as follows:

$$\begin{cases} \hat{\underline{x}}_{k+1}^- = f(\hat{\underline{x}}_k^+, \underline{w}, k) \Big|_{\underline{w}=0} \\ P_{k+1}^- = A_k P_k^+ A_k^T + F_k Q_k F_k^T, \\ \hat{\underline{x}}_k^+ = \hat{\underline{x}}_k^- + K_k [y_k - g(\hat{\underline{x}}_k^-, \underline{v}, k)] \Big|_{\underline{v}=0} \\ K_k = P_k^- C_k^T [C_k P_k^- C_k^T + G_k]^{-1} \\ P_k^+ = P_k^- - K_k C_k P_k^- \end{cases} \quad (4)$$

where $\hat{\underline{x}}_k^- = \hat{E}\{\underline{x}_k | y_{k-1}, y_{k-2}, \dots, y_1\}$ is the *a priori* estimate of the state vector, x_k , at the k^{th} update, using the observations y_1 to y_{k-1} , and $\hat{\underline{x}}_k^+ = \hat{E}\{\underline{x}_k | y_k, y_{k-1}, \dots, y_1\}$ is the *a posteriori* estimate of the state vector after adding the k^{th} observations y_k . P_k^- and P_k^+ are defined in the same manner to be the estimations of the covariance matrices, in the k^{th} stage, before and after using the k^{th} observation, respectively.

The idea that long term ECGs can be described by a series of states described by a given power-spectral density with similar parameters over the short term, motivated us to propose autoregressive (AR) dynamics to obtain the modified EDM [23]. The state-space model is given by:

$$\begin{cases} \theta[k+1] = \theta[k] + \omega\delta = F_1(\theta_k, \omega, k) \\ z[k+1] = - \sum_{i \in \{P, Q, R, S, T\}} \delta \frac{\alpha_i[k]\omega}{b_i[k]^2} \Delta \theta_i[k] \exp\left(-\frac{\Delta \theta_i[k]^2}{2b_i[k]^2}\right) \\ \quad + z[k] + \eta = F_2(\theta_k, z_k, \alpha_{i_k}, b_{i_k}, \theta_{i_k}, N, k) \\ \alpha_P[k+1] = \alpha_P[k] + u_1[k] = F_3(\alpha_{P_k}, u_1, k) \\ \vdots \\ \theta_T[k+1] = \theta_T[k] + u_{15}[k] = F_{17}(\alpha_{T_k}, u_{15}, k) \end{cases} \quad (5)$$

in which the state and process noise vectors are as follows:

$$\begin{aligned} \underline{x}_k &= [\theta_k, z_k, \alpha_P, \dots, b_P, \dots, \theta_P, \dots, \theta_T]^T \\ \underline{w}_k &= [\omega, \eta, u_1, \dots, u_{15}]^T \end{aligned} \quad (6)$$

Equation (7) represents the linearized state-space model (5) at each time instant around the most recent state estimation, according to (3):

$$\begin{aligned}
\frac{\partial F_2}{\partial \theta_k} &= - \sum_{i \in \{P, Q, R, S, T\}} \delta \frac{\alpha_i \omega}{b_i^2} \left[1 - \frac{\Delta \theta_i^2}{b_i^2} \right] \exp\left(-\frac{\Delta \theta_i^2}{2b_i^2}\right), \\
\frac{\partial F_2}{\partial \alpha_{ik}} &= -\delta \frac{\omega}{b_i^2} \Delta \theta_i \exp\left(-\frac{\Delta \theta_i^2}{2b_i^2}\right), \\
\frac{\partial F_2}{\partial b_{ik}} &= -2\delta \frac{\alpha_i \omega}{b_i^3} \Delta \theta_i \left[1 - \frac{\Delta \theta_i^2}{2b_i^2} \right] \exp\left(-\frac{\Delta \theta_i^2}{2b_i^2}\right), \\
\frac{\partial F_2}{\partial \theta_{ik}} &= \delta \frac{\alpha_i \omega}{b_i^2} \left[1 - \frac{\Delta \theta_i^2}{2b_i^2} \right] \exp\left(-\frac{\Delta \theta_i^2}{2b_i^2}\right), \\
\frac{\partial F_3}{\partial \alpha_{Pk}} &= \frac{\partial F_4}{\partial \alpha_{Qk}} = \frac{\partial F_5}{\partial \alpha_{Rk}} = \frac{\partial F_6}{\partial \alpha_{Sk}} = \frac{\partial F_7}{\partial \alpha_{Tk}} = \frac{\partial F_1}{\partial \theta_k} = \frac{\partial F_2}{\partial z_k} = 1, \\
\frac{\partial F_8}{\partial b_{Pk}} &= \frac{\partial F_9}{\partial b_{Qk}} = \frac{\partial F_{10}}{\partial b_{Rk}} = \frac{\partial F_{11}}{\partial b_{Sk}} = \frac{\partial F_{12}}{\partial b_{Tk}} = 1, \\
\frac{\partial F_{13}}{\partial \theta_{Pk}} &= \frac{\partial F_{14}}{\partial \theta_{Qk}} = \frac{\partial F_{15}}{\partial \theta_{Rk}} = \frac{\partial F_{16}}{\partial \theta_{Sk}} = \frac{\partial F_{17}}{\partial \theta_{Tk}} = 1.
\end{aligned} \tag{7}$$

In the proposed EKF structure, we have only two noisy observations corresponding to the state variables θ and z [22] which are related to the state vector as follows:

$$\begin{bmatrix} \varphi_k \\ s_k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \end{bmatrix} \cdot \underline{x}_k + \begin{bmatrix} v_{1k} \\ v_{2k} \end{bmatrix} \tag{8}$$

where φ and s are the noisy observations corresponding to the phase and the ECG signal, respectively. v_1 and v_2 are the observation noises. Other state variables, for which we have no observations, are considered as the hidden states.

IV. FIDUCIAL POINTS AND RR-INTERVAL EXTRACTION

Once the EKF structure is constructed, we can perform selected processing applications on ECG signals. The proposed framework can estimate any of its states according to the dynamical equations and the observations. In fact, they enable us to build up beat-to-beat Gaussian kernels, a concept that is addressed next.

A. Fiducial Points Determination

The proposed nonlinear Bayesian framework estimates its variables using the state dynamical equations and its observations, noisy phase and noisy ECG. Since we have considered these variables as the states of EKF, we can easily estimate their values from $\hat{x}_{3:17}$. But the EKF updates its estimations when a new sample is observed. This means that the EKF estimates the α_i , b_i , and θ_i parameters time series, in a similar manner to θ and z . We expect a constant value for each of the 15 Gaussians' parameters, during each heart beat. This is especially true, because the amplitude, spread and angular location of the PQRST do not vary within a single ECG beat. In practice, since the estimated series of α_i , b_i , and θ_i are not a definitely constant

function, we use its average value over the non-fluctuating part, estimated during each heart beat. Fig. 2 shows an ECG signal, with 15 estimated Gaussians' parameters, i.e. $\hat{\alpha}_i$, \hat{b}_i , $\hat{\theta}_i$, shown as time series. Note that they remain nearly constant-valued during each normal beat. Using the parameters of the Gaussian functions of EDM, it is possible to locate the FPs. However, more accurate reconstruction is possible if we take advantage of the behavior of the estimated hidden variables. As depicted in Fig. 2, they show fluctuations corresponding to the waveform they describe and then they remain constant, giving the estimated value for their corresponding parameter. Hence, we may use the fluctuating part to obtain the onset and offset of each waveform. Specifically, when the EKF encounters a different rhythm, the Gaussian parameters change because the rhythm has been changed. This will cause their estimations not to remain constant anymore during that beat, but the fluctuating parts remain honest. Consequently, the locations of FPs may be derived from the estimated Gaussian parameters, and corrected according to the fluctuations in the same estimations provided by the EKF architecture. Accordingly, a set of combined rules are given in (9) which specify the locations of the FPs of each beat.

$$\begin{aligned}
P_{on} &= \min(\theta_P - 3b_P, f_P^{on}) \\
P_{off} &= \max(\theta_P + 3b_P, f_P^{off}) \\
QRS_{on} &= \min(\theta_Q - 3b_Q, f_Q^{on}) \\
QRS_{off} &= \max(\theta_S + 3b_S, f_S^{off}) \\
T_{on} &= \min(\theta_T - 3b_T, f_T^{on}) \\
T_{off} &= \max(\theta_T + 3b_T, f_T^{off})
\end{aligned} \tag{9}$$

where f_i^{on} and f_i^{off} stand for the onset and offset of the fluctuating part of waveform i .

B. Tachogram Extraction

Heart rate variability (HRV) is a widely used quantitative marker of autonomic nervous system activity. Various methods have been applied to extract tachograms, a graph representing the sequential RR-intervals. However, real HRV is usually non-stationary. Non-stationarities like slow linear or more complex trends in the HRV signal, can cause distortion to time- and frequency-domain analysis.

In contrast, when using adaptive dynamical filters, such as EKF, the estimation is truth worthy. Hence, similar to the fiducial points extraction problem, HRV may be obtained by locating the R peaks in an ECG signal, as follows:

$$R_{peak} = \theta_R \tag{10}$$

V. SIMULATION RESULTS

The proposed algorithm was implemented in MATLAB®. The MIT-BIH arrhythmia database [26] was used to study the performance of the proposed method. The manual detection was used to provide a known reference for the exploration, so these ECG's were first annotated completely

by experienced cardiologists from Tehran Heart Center (THC). Furthermore we have used the following parameters to evaluate our method: number of true positive detections (TP), number of false positive detections (FP), number of true negative detections (TN), and number of false negative detections (FN). Accordingly, sensitivity (Sn), specificity (Sp) and positive predictivity ($+P$) criteria are defined as:

$$Sn = \frac{TP}{TP + FN} \quad (11)$$

$$Sp = \frac{TN}{TN + FP} \quad (12)$$

$$+P = \frac{TP}{TP + FP} \quad (13)$$

Fig. 3 shows typical results of the algorithm. As it can be seen, the distinguished precise fiducial points indicated by circles are clearly at their exact locations. For quantitative evaluation, we have considered 10 different ECG records. Performance evaluation results are provided in Table I. It is worth noting that no false negative detections would occur, and consequently the sensitivity would be 100%, since we have restricted the EDM (5) to include only five Gaussians.

There are few works which focus on determining all the fiducial points of an ECG. The majority of these include feature extraction approaches for classification, or compression evaluation. Here we can obtain all of the features relating to locations (9) and (10) or time intervals. But to investigate the validity of our results, we have considered two applications of fiducial points extraction: QRS detection and T wave detection. The results have been compared to the best previously stated ones, the Filter Bank (FB) method [27], and the Wavelet Transform Modulus Maxima (WTMM) [28] in Table II. The results confirm the efficiency of the proposed method.

VI. DISCUSSION AND CONCLUSION

We have presented and validated a modified EKF algorithm which incorporates the parameters of the ECG dynamical model, and its applications to ECG points extraction. By introducing a simple AR model for each of the 15 dynamic parameters of the Gaussians, an EKF structure was constructed. The proposed set of equations aims at integrating into the ECG model a mechanism that estimates the new hidden state variables without having any corresponding observation, which was further used for FP extraction and R detection for obtaining tachograms. This presents the greatest potential of the presented mathematical model-based framework, with which the morphology is tracked efficiently. We also derived rules to locate the FPs based on the estimated state variables. In comparison to other proposed methods for point detection, the model-based approach has a superior performance, for there is no decision rules based on comparison against thresholds.

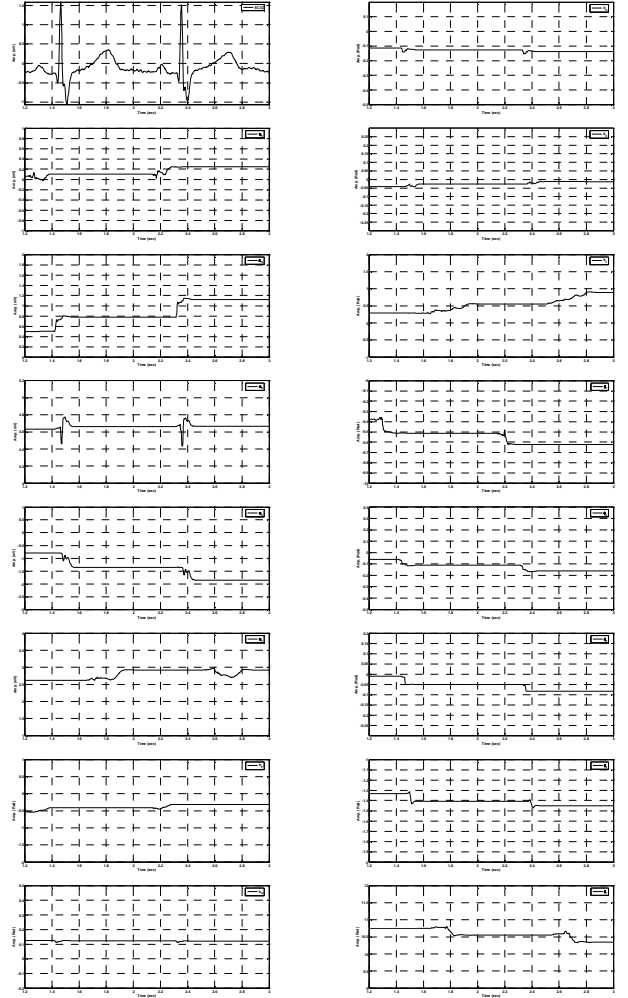


Fig. 2. Estimated Gaussians' parameters with EKF17 for record 231 (analyzed portion 1.2 sec:3 sec).

TABLE I
MODEL-BASED ECG FIDUCIAL POINTS EXTRACTION PERFORMANCE ON THE MIT-BIH ARRHYTHMIA DATABASE

Rec. No.	TP	TN	FP	FN	Sn(%)	Sp(%)	+P(%)
106	107	3490	3	0	100	99.91	97.27
111	118	3279	3	0	100	99.90	97.52
115	100	3489	10	0	100	99.71	90.91
116	140	3259	0	0	100	100	100
117	99	3454	0	0	100	100	100
121	108	3490	2	0	100	99.94	98.18
122	163	3434	2	0	100	99.94	98.79
123	85	3415	0	0	100	100	100
124	88	3427	0	0	100	100	100
215	195	3262	3	0	100	99.91	98.48
Average:					100	99.93	98.12

TABLE II
COMPARING SELECTED APPLICATIONS OF BENCHMARK TECHNIQUES USED TO DETECT ECG FIDUCIAL POINTS FOR THE TEN RECORDS USED IN TABLE I

Selected Approach	Algorithm	Sn(%)	Sp(%)	+P(%)
T-wave detection	WTMM	99.83±0.24	99.97±0.02	99.92±0.11
	Our method	100±0	99.96±0.01	99.92±0.03
QRS detection	FB	99.64±0.14	99.99±0.01	98.59±0.12
	Our method	100±0	100±0	99.01±0.08

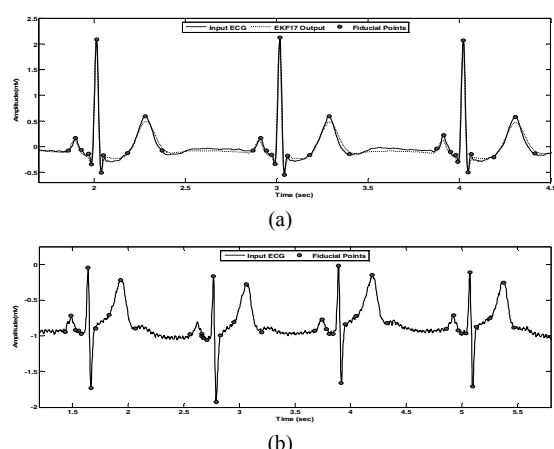


Fig. 3. Fiducial points extraction results for (a) record 106, (b) record 117.

The method has been validated using several ECG records from the MIT-BIH arrhythmia database. None of the more complex cases result in sensitivity less than 100% and specificity of 99.93%. These results show that the developed method provides a reliable and accurate detection of the fiducial points. It outperforms the other algorithms and has an average positive predictivity of 98.12% which is well within the acceptable range.

In addition, through simple modifications, it would be robust to PQRST variations, which incorporates several pathological conditions. Moreover, compared to our LSE approach [18], the proposed EKF based method does not need an initial estimation for beat-to-beat optimization, but is highly dependent to the selection of the initial covariance matrices. It worth noting that the LSE method [18] was developed for baseline wandered ECGs, while our current algorithm deals with ECG signals with no drifts.

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