№2

a) $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

$$\frac{1}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2} = \frac{A(n+2)+Bn}{n(n+2)} = \frac{1}{n(n+2)}$$

$$n(A+B) + 2A = 1$$

$$\begin{cases} A+B=0 \\ 2A=1 \end{cases} \qquad B=-\frac{1}{2} \qquad A=\frac{1}{2}$$

$$\sum_{n=1}^{\infty}\left(\frac{1}{2n} - \frac{1}{2(n+2)}\right)$$

$$S = \left(\frac{1}{2} - \cancel{\frac{1}{6}}\right) + \left(\frac{1}{4} - \cancel{\frac{1}{8}}\right) + \left(\cancel{\frac{1}{6}} - \cancel{\frac{1}{10}}\right) + \left(\cancel{\frac{1}{8}} - \frac{1}{12}\right) + \left(\cancel{\frac{1}{10}} - \frac{1}{14}\right)$$

$$S = \lim_{n\to\infty} S_n = \left(\frac{3}{4} + \left[\frac{1}{2n} - \frac{1}{2(n+2)}\right]\right) = \frac{3}{4}$$

b) $\sum_{n=1}^{\infty} \frac{1}{(3n-2)(3n+1)}$

$$\frac{A}{3n-2} + \frac{B}{3n+1} = \frac{A(3n+1)+B(3n-2)}{(3n-2)(3n+1)} = 1$$

$$\begin{cases} A+B=0 \\ A-2B=1 \end{cases} \qquad B=-A \qquad 3A=1 \qquad A=\frac{1}{3} \qquad B=-\frac{1}{3}$$

$$\sum_{n=1}^{\infty}\left[\frac{1}{3(3n-2)} - \frac{1}{3(3n+1)}\right]$$

$$S_n = \left(\frac{1}{3} - \cancel{\frac{1}{12}}\right) + \left(\cancel{\frac{1}{12}} - \cancel{\frac{1}{21}}\right) + \left(\cancel{\frac{1}{21}} - \frac{1}{30}\right) + \cdots = \frac{1}{3}$$

$$\lim_{n\to\infty} \frac{1}{3}+\frac{1}{3(3n-2)}-\frac{1}{3(3n+1)}=\frac{1}{3}$$

№3

а) $\lim\limits_{n\to\infty} \dfrac{5n+6}{100n-1}=\dfrac{1}{25}=0{,}04$ сход.

б) $\lim\limits_{n\to\infty} \dfrac{1+n}{1+n^2}=0$ расх.

с) $\lim\limits_{n\to\infty} \dfrac{\sqrt[3]{n^3}-25}{\sqrt{n}+50}=\lim\limits_{n\to\infty} \dfrac{n-25}{\sqrt{n}+50}=\lim\limits_{n\to\infty} \dfrac{\sqrt{n}\left(\sqrt{n}-\frac{25}{\sqrt{n}}\right)}{\sqrt{n}\left(1+\frac{50}{\sqrt{n}}\right)}=\infty$ расх.

№4

а) $\sum\limits_{n=1}^{\infty} \dfrac{1}{\sqrt{n(1+n^2)}}$ $\qquad \dfrac{1}{n^{\frac{3}{2}}+n^{\frac{1}{2}}}<\dfrac{1}{n^{\frac{3}{2}}}$

$\lim\limits_{n\to\infty} \dfrac{1}{n^{\frac{3}{2}}}=0 \to \dfrac{1}{n^{\frac{3}{2}}}$ сход. $\Rightarrow \sum\limits_{n=1}^{\infty} \dfrac{1}{\sqrt{n(1+n^2)}}$ сход.

б) $\sum\limits_{n=1}^{\infty} \dfrac{n+1}{\sqrt{n^3}}$ $\qquad \dfrac{n+1}{n^{\frac{3}{2}}}=\dfrac{n}{n^{\frac{3}{2}}}+\dfrac{1}{n^{\frac{3}{2}}}=\dfrac{1}{n^{\frac{1}{2}}}+\dfrac{1}{n^{\frac{3}{2}}}$

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}} \; ; \; \frac{1}{2} < 1 \to \text{расх.}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \; ; \; \frac{3}{2} > 1 \to \text{сход}$$

$$\frac{1}{\sqrt{n}} + \frac{1}{n^{3/2}} \geq \frac{1}{\sqrt{n}}$$

$\frac{1}{\sqrt{n}}$ – расх $\Rightarrow \frac{1}{\sqrt{n}} + \frac{1}{n^{3/2}}$ тоже расх.

c) $$\sum_{n=1}^{\infty} \frac{1}{(2n-1)\, 2^{2n-1}}$$

$$\frac{1}{(2n-1)\cdot 2^{2n-1}} < \frac{1}{2^{2n}} \qquad \left(\frac{1}{4}\right)^n \text{ сход. т.к } \frac{1}{4} < 1 \Rightarrow \frac{1}{(2n-1)\cdot 2^{2n-1}} \text{ сход.}$$

расх ← расх

d) $$\sum_{n=1}^{\infty} \frac{\cancel{n+1}}{(\cancel{n+1})(n-1)} \qquad \sum_{n=1}^{\infty} \frac{1}{n-1} \qquad \frac{1}{n-1} > \frac{1}{n} \Rightarrow \frac{1}{n-1} \text{ расх}$$

e) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+4)}$ $\qquad \frac{1}{(n+1)(n+4)} = \frac{A}{n+1} + \frac{B}{n+4} = \frac{A(n+4)+B(n+1)}{(n+1)(n+4)}$

$n(A+B) + 4A + B = 1$

$\begin{cases} A+B=0 & B=-A & B=-\frac{1}{3} \\ 4A+B=1 & 3A=1 & A=\frac{1}{3} \end{cases}$

$\frac{1}{(n+1)(n+4)} = \frac{1}{3(n+1)} - \frac{1}{3(n+4)}$

$\frac{1}{(n+1)} < \frac{1}{n}$

$\frac{1}{(n+4)} < \frac{1}{n}$

$\Rightarrow \frac{1}{(n+1)(n+4)} < \frac{1}{n^2}$ $\qquad \frac{1}{n^2}$ сход т.к $2>1 \Rightarrow$

$\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+4)}$ сход.

f) $\sum_{n=1}^{\infty} \frac{3n^2+2n}{n^3-5n-5}$

аппроксимация $\frac{3n^2}{n^3} \approx \frac{3}{n} > \frac{1}{n}$ расх

расх

$\Rightarrow$ ряд расх.

$$\lim_{n\to\infty} \frac{\frac{3n^2+2n}{n^3-5n-5}}{1/n} = 3 \quad 3 \text{ - конст} \Rightarrow \frac{3n^2+2n}{n^3-5n-5} \text{ ведет себя как } \frac{1}{n}$$

№5

а) $\sum_{n=1}^{\infty} \frac{n^2+n}{3^n}$ $\qquad U_{n+1} = \frac{(n+1)^2+n+1}{3\cdot 3^n}$

$\lim \frac{u_{n+1}}{u_n} = p$ $\quad p<1$ сход $\quad p>1$ расх

$$\lim_{n\to\infty} \frac{(n^2+2n+\cancel{1}+n+1)\cdot 3^n}{(3\cdot 3^n)(n^2+n)} = \frac{(n^2+3n+2)\cdot \cancel{3^n}}{(n^2+n)\cdot 3\cdot \cancel{3^n}} = \lim_{n\to\infty} \frac{1}{3}\,\frac{n^2+3n+2}{n^2+n} = \frac{1}{3} < 1$$

сход.

б) $\sum_{n=1}^{\infty} \frac{n!}{10^n}$ $\qquad u_{n+1} = \frac{(n+1)!}{10\cdot 10^n}$

$$\lim \frac{u_{n+1}}{u_n} = \frac{(n+1)!\cdot \cancel{10^n}}{10\cdot \cancel{10^n}\cdot n!} = \frac{\cancel{n!}(n+1)}{10\cdot \cancel{n!}} = \lim_{n\to\infty} \frac{n+1}{10} = \infty > 1 \text{ расх.}$$

c) $\sum_{n=1}^{\infty} \frac{n^3}{(n+1)!}$

$$u_{n+1} = \frac{(n+1)^3}{(n+1+1)!} = \frac{n^3+3n^2+3n+1}{(n+2)!}$$

$$\lim_{n\to\infty} \frac{(n+1)^3 \cancel{(n+1)!}}{\cancel{(n+1)!}(n+2)n^3} = \frac{n^3}{n^4} = 0<1 \text{ сх.}$$

$= 0 < 1$ сход.

d) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

$$u_{n+1} = \frac{(n+1)^{n+1}}{(n+1)!} =$$

$$\lim_{n\to\infty} \frac{(n+1)^n \cdot (n+1) \cdot n!}{(n+1)! \cdot n^n} =$$

$= \left(1+\frac{1}{n}\right)^n = e > 1$ расх.

e) $\sum_{n=1}^{\infty} \frac{1}{(2n+1)!}$

$u_{n+1} = \frac{1}{(2(n+1)+1)!} = \frac{1}{(2n+3)!}$

$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{1\cdot(2n+1)!}{(2n+3)!} = \frac{(2n+1)!}{(2n+1)!\,(2n+2)(2n+3)} = \lim_{n\to\infty} \frac{1}{(2n+2)(2n+3)} = 0 < 1$ сход.

f) $\sum_{n=1}^{\infty} \frac{n}{(n+1)!}$

$u_{n+1} = \frac{n+1}{(n+2)!}$

$\lim_{n\to\infty} \frac{(n+1)(n+1)!}{(n+2)!\cdot n} = \lim_{n\to\infty} \frac{(n+1)\cancel{(n+1)!}}{\cancel{(n+1)!}(n+2)\cdot n} = \lim_{n\to\infty} \frac{n+1}{n^2+2n} = 0 < 1$ сход.

№6

a) $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n$ $\lim_{n\to\infty} \sqrt[n]{\left(\frac{n}{2n+1}\right)^n} = \lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2} < 1$ сход.

b) $\sum_{n=1}^{\infty} \arcsin^n \frac{1}{n}$ $\quad \lim_{n\to\infty} \sqrt[n]{\arcsin^n \frac{1}{n}} = \lim_{n\to\infty} \arcsin \frac{1}{n} = 0 < 1$ схog.

c) $\sum_{n=1}^{\infty} \frac{\left(\frac{n+1}{n}\right)^{n^2}}{3^n}$ $\quad \lim_{n\to\infty} \sqrt[n]{\frac{\left(\frac{n+1}{n}\right)^{n^2}}{3^n}} = \lim_{n\to\infty} \frac{\left(\frac{n+1}{n}\right)^n}{3} = \lim_{n\to\infty} \frac{\left(1+\frac{1}{n}\right)^n}{3} = \frac{e}{3} < 1$

схog.

d) $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$ $\quad \int_1^{\infty} \frac{dn}{n \ln n} = \int_1^{\infty} \frac{dt \cdot n}{n \cdot t} = \int_1^{\infty} \frac{dt}{t} = \int_1^{\infty} \ln t =$

$\begin{cases} \ln n = t \\ \frac{1}{n} dn = dt \end{cases}$

$\ln|\ln n| \Big|_1^{\infty} =$

$\ln|\infty| - \ln|1| = \infty > 1$ расх.

e) $\sum_{n=1}^{\infty} \frac{1}{(n+1)\ln^2(n+1)}$ $\quad \int_1^{\infty} \frac{dn}{(n+1)\ln^2(n+1)} = \int_1^{\infty} \frac{dt(n+1)}{(n+1)\cdot t^2} = \int_1^{\infty} \frac{dt}{t^2} = \int_1^{\infty} \frac{t^{-1}}{-1} =$

$\begin{cases} \ln(n+1) = t \\ \frac{1 \cdot dn}{n+1} = dt \end{cases}$

$- \int_1^{\infty} \frac{1}{t} = - \int_1^{\infty} \frac{1}{\ln(n+1)}$

f) $\sum_{n=1}^{\infty} \frac{n}{1+n^2}$ $\int_1^{\infty} \frac{n \cdot dn}{1+n^2} = \left\{ \begin{array}{l} 1+n^2 = t \\ 2n\,dn = dt \end{array} \right.$ $= -\left(\frac{1}{\ln\infty} - \frac{1}{\ln 2}\right) = \frac{1}{\ln 2}$

cxog

$\int_1^{\infty} \frac{\cancel{n} \cdot dt}{2\cancel{n} \cdot t} = \frac{1}{2} \ln t \Big|_1^{\infty} = \frac{1}{2}(\ln|\infty| - \ln|1|) = \infty$ расх.

№7

a) $\sum_{n=1}^{\infty} \frac{4}{(2n-1)\cdot 2^{2n-1}}$

$u_{n+1} = \frac{4}{(2(n+1)-1)\cdot 2^{2(n+1)-1}} = \frac{4}{(2n+1)\cdot 2^{2n}\cdot 2}$

$\lim \frac{\cancel{4}\cdot(2n-1)\cdot \cancel{2^{n}}}{(2n+1)\cdot 2\cdot \cancel{2^{n}}\cdot \cancel{4}\cdot 2} = \lim_{n\to\infty} \frac{2n-1}{(2n+1)4} = \frac{1}{4} < 1$ cxog.

b) $\sum_{n=1}^{\infty} \frac{n^2+n-1}{n^3-n\,1}$

$$\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{n^2+n-1}{n^3-n+3}} = \frac{1\cdot(n^3-n+3)}{n(n^2+n-1)} = 1 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n} \text{ расх} \Leftrightarrow \text{расх}$$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\ln n+5}$

$$\lim_{n\to\infty} \frac{1}{\ln n+5} = \infty$$

d) $\displaystyle\sum_{1} \frac{1}{(5n-4)(4n-1)}$

$$\lim_{n\to\infty} \frac{1}{20n^2-21n+4} = 0 \quad \text{сход.}$$

e) $\sum \frac{3^n}{n2^n}$

$u_{n+1} = \frac{3\cdot 3^n}{(n+1)2^n\cdot 2}$

$\lim\limits_{n\to\infty} \frac{3\cdot \cancel{3^n}\cdot n\cdot \cancel{2^n}}{(n+1)\cancel{2^n}\cdot 2\cdot \cancel{3^n}} = \frac{3n}{2n+2} = \frac{3}{2} > 1$ расх

f) $\sum\limits_{n=1}^{\infty} \frac{2n-1}{3^n}$

$u_{n+1} = \frac{2(n+1)-1}{3\cdot 3^n}$

$\lim\limits_{n\to\infty} \frac{2n+1\cdot \cancel{3^n}}{3\cdot \cancel{3^n}\cdot(2n-1)} = \lim\limits_{n\to\infty} \frac{2n+1}{6n-3} = \frac{2}{6} = \frac{1}{3} < 1$ сход.

T 2

№ 1

a)