Identification of the parameters in inverted pendulum model.

Stefan Brock
Poznań University of Technology
Institute of Industrial Electrical Engineering
ul. Piotrowo 3A, 60-965 Poznań, Poland
Stefan.Brock@put.poznan.pl

Abstract:
Inverted pendulum systems is excellent test beds for control theory. An identification method for a inverted pendulum based on simulation model is considered. This method is able to search dynamic parameters effectively even if pendulum has low resolution position encoders. Friction in the motor-cart system can be modeled with three components: viscous friction, and static (dry) friction terms Coulomb friction and stiction. Presented method is based on the pendulum time response, by using optimization tools. Experiments made both with Simulink based simulation model and the real electromechanical device have proven full functionality of the proposed model in the real life conditions.

1 Introduction.

For several decades, inverted pendulum systems have served as excellent test beds for control theory. Because they exhibit nonlinear, unstable, non-minimum phase dynamics, and because the full-state is not often fully measured, control objectives are always challenging.
Therefore, it is necessary to apply some advanced control algorithm to provide high quality control. There are two different methodologies. The first one is to design a robust controller using a minimum information about the dynamics, like sliding Mode Control. The second methodology - e.g. computed torque Control - is based on the use of the dynamic model of pendulum, that is as close as possible to the real one. The main drawback of the second approach is the need for an accurate plant model.
While the most of physical phenomena in the inverted pendulum can be described with well known laws of dynamics and parameters can be determined using rulers, micrometers, scales and timers, the modeling and parameterization of damping requires much more effort. Initially only viscous friction is taken into account in developing equations of motion. This is sufficient for the pendulum pivot joints but not adequate to describe friction forces in the motor, timing belt transmission and between the track and cart. Simple tests with the motor-cart system show that friction forces have a nonlinear nature.
For studying friction more precisely, the pendulum links can be removed from the cart. Thus the motor-cart can be handled mechanically as a one degree of freedom system.

In the industrial drives based on microprocessor control, the speed measure is done employing an optical, absolute or incremental encoder or an electromagnetic resolver. The speed is obtained by a derivative operation of the measured position. Then the speed is rather noisy because of the measured position discretization and, normally, a filtering operation becomes necessary, unless to employ an incremental encoder with a particularly high number of marks. The acceleration measurement results more noisy because it requires a further derivative operation.

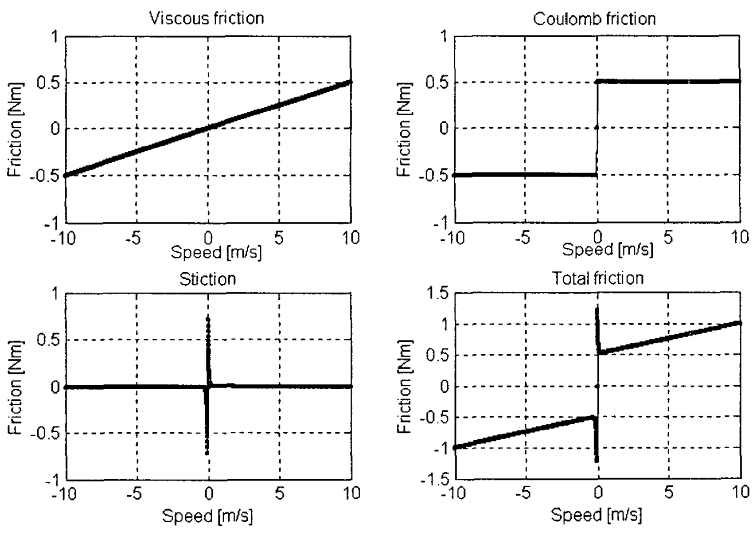


Fig. 1. Selected model of a friction: viscous, Coulomb, stick and mixed.

2 The model of friction

The main contributors to the friction forces are the DC motor and pulley-belt-cart system that transfers torque from motor to force on the cart. The motor has friction between the brushes and commutator as well as in the bearings. The friction in the pulley-timing belt-cart transmission is even more sophisticated. In both systems in addition to viscous friction, we encounter static friction terms like Coulomb friction and stiction that affect the control

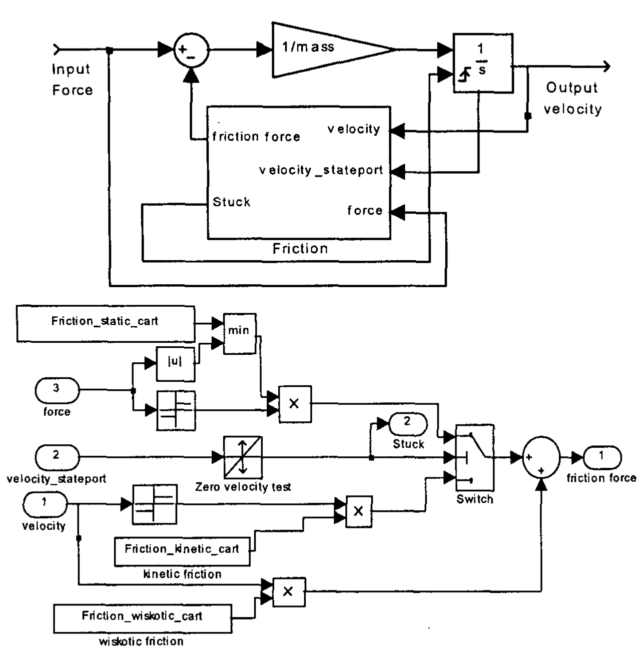


Fig 2. Model of friction

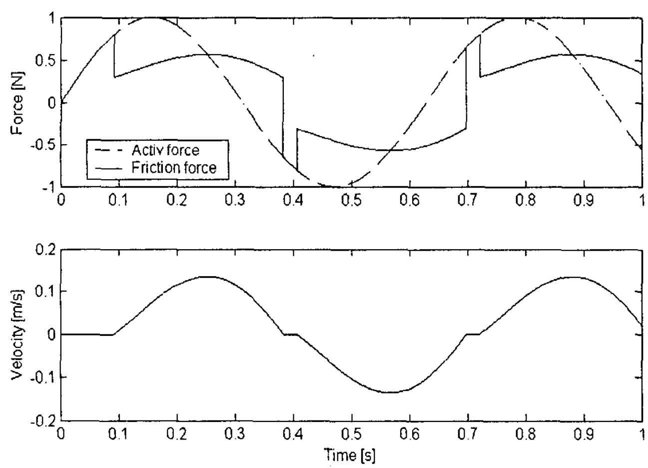


Fig. 3. Test of friction model.

performance. Friction in the motor-cart system can be modeled with three components: viscous friction, and static (dry) friction terms Coulomb friction and stiction. They are illustrated in Figure 1.

In inverted pendulum system friction occurs in cart and pendulum motion systems. The DC motor is the driven force for the cart at the base of leading screw and ball nut. The pendulum is located in the rolling bearings. In both systems viscous friction dependent on speed of rotation occurs. Moreover static friction terms like Coulomb friction and stiction should be encountered too. All three components are illustrated in Fig. 1. Relationship of viscous friction (Fig. 1a) and speed of rotation is linear:

$$F_w = k_{fw} * v$$

Where: F_w viscous friction force, v – motion speed of rubbing elements towards one another k_{fw} proportionality factor. Coulomb friction component depends on motion direction (Figure 1b). Coulomb force is dependent on rubbing material types and thrust force between them.

$$F_c = F_K * \mathrm{sgn}(v)$$

where: F_c Coulomb friction force, F_K constant characteristic for the system. In the case of rubbing of many materials stick – slip phenomena is present. It's connected to relaxation oscillation creation in the result of different values of stiction and kinetic friction. Stiction, illustrated in Fig 1c, compensates driven force during system rest. The motion occurs when driven force value is over stiction force – then stiction force disappears. The common result of static friction forces (Coulomb friction and stiction) can be determined the following:

$$F_{Stat} = k_{fwc} \cdot v + \begin{cases} F_K \cdot \mathrm{sgn}(v) & \text{for} \quad v \neq 0 \\ \min\left(F_S, |F_A|\right) \cdot \mathrm{sgn}(F_A) & \text{for} \quad v = 0 \end{cases}$$

where: F_{Stat} static friction force, F_S maximum stiction force, $F_{A\ to}$ active force of the system. Stick-slip phenomena can be present while $F_S > F_K$.

The total friction force, illustrated in Fig. 1c, can be obtained in the result of viscous friction and common friction force connection.

Simulation model of friction system is illustrated in Fig. 2. The model was created using Matlab/Simulink system. Speed and driven force are the input signals. Additionally state port of mass modeling integrator or system inertia can be introduced as input signals. The common friction force and additionally reset signal of mass modeling integrator or inertia are concerned as output signals. The factors of viscous friction, Coulomb friction and stiction are the model parameters.

In Figure 3 test results of the model are presented. We assumed mass 0.5 kg and input function of period force with amplitude 1 Nm and period 0.63 s. Moreover we assumed stic-

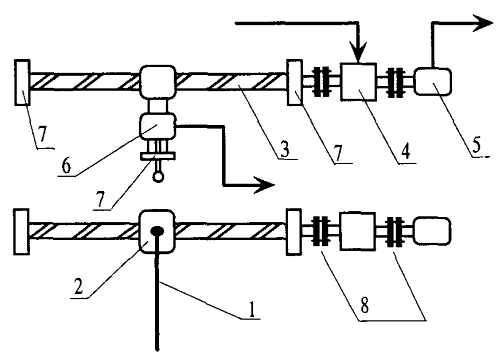


Fig 4. Scheme of the experimental plant.
1 - pendulum, 2 - cart, 3 - leading screw , 4 - DC motor,

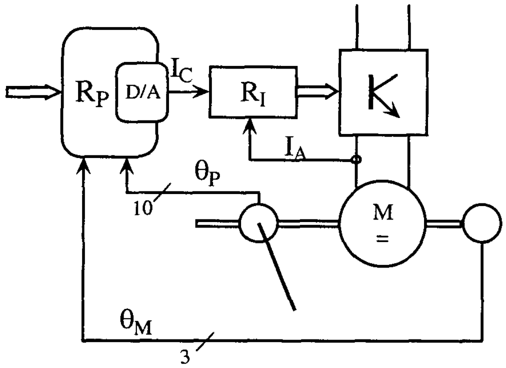


Fig 5. The functional scheme of pendulum control system

tiion value 0.8 Nm, Coulomb friction value 0.3 Nm, and viscous friction factor 2 Ns. The motion is started just at t=0.1 s, when input function force is over stiction value. The rest phase caused by stiction presence occurs during reverses too. The figure testifies to proper operation of friction model.

3 The inverted pendulum system.

Laboratory model of the inverted pendulum is illustrated in fig. 4. The pendulum (1) is fixed to the cart (2). The leading screw (3) and DC motor (4) are the driven force of the cart. The system has two measurement converters: increment rotary-impulse converter (5) (1024 impulses per rotation), connected to driven motor shaft and absolute code rotary converter (6) (10-bit) connected to pendulum rotary axle. The functional scheme of pendulum control system is illustrated in fig. 5. DC motor M is supplied from 4-quadrant transistor converter with current regulation loop. The measurement signals: θ_P (pendulum angle position) and θ_M (motor angle position change) are introduced to digital control system R_P. The current setting I_C is output signal of control system. After analog conversion (D/A) the signal is introduced to current controller input R_I.

Considered an inverted pendulum system illustrated in fig. 4. A cart (2) with a mass m_c can implement its linear motion on a leading screw (3) (with efficiency η), which is driven by the motor (4). A pendulum (1) with a mass m_p and length l is pivoted on the one point of M. Let x be the position of the cart in an internal frame, and θ the angle between the pendulum and the vertical. Then the equations of motions for the pendulum can be derived by the Lagrange equation:

$$\dot{I} = (\dot{I})_{\max} \cdot \mathrm{sat}\left(\frac{I_c - I/T_i}{(\dot{I})_{\max}}\right) \tag{1}$$

$$F_u = I \cdot k_M \cdot k_D \tag{2}$$

$$(m_c + m_p) \cdot \ddot{x} = F_{Ac} - F_{Fc} \tag{3}$$

$$F_{Ac} = \eta \cdot F_u + A(\eta) \cdot \left(\dot{\phi}^2 \cdot \sin(\phi) - \ddot{\phi} \cdot \cos(\phi)\right) \cdot m_p \cdot l \tag{4}$$

$$F_{Fc} = k_{fwc} \cdot \dot{x} + \begin{cases} F_{Kc} \cdot \mathrm{sgn}(\dot{x}) & \text{for} \quad \dot{x} \neq 0 \\ \min\left(F_{Sc}, |F_{Ac}|\right) \cdot \mathrm{sgn}(F_{Ac}) & \text{for} \quad \dot{x} = 0 \end{cases} \tag{5}$$

$$\frac{4}{3} m_p \cdot l^2 \cdot \ddot{\phi} = T_{Ap} - T_{Fp} \tag{6}$$

$$T_{Ap} = m_p \cdot l \cdot \left(g \cdot \sin(\phi) - \ddot{x} \cdot \cos(\phi)\right) \tag{7}$$

$$T_{Fp} = k_{fwp} \cdot \dot{\phi} + \begin{cases} T_{Kp} \cdot \mathrm{sgn}(\dot{\phi}) & \text{for} \quad \dot{\phi} \neq 0 \\ \min\left(T_{Sp}, |T_{Ap}|\right) \cdot \mathrm{sgn}(T_{Ap}) & \text{for} \quad \dot{\phi} = 0 \end{cases} \tag{8}$$

where:

I_c - command current of DC motor
I - actual current od DC motor
T_i - equivalent time constant for current control loop
$\dot{I}_{max}$ - the maximum slope of current
F_u - force driven the cart
k_M - torque gain of DC motor
k_D - translation gain of leading screw
F_{Ac} - active cart force
F_{Fc} - friction cart force
k_{fwc} - viscous friction gain of a cart
F_{Kc} - Coulomb kinetic friction of a cart
F_{Sc} - Coulomb static friction of a cart

T_{Ap} - active pendulum torque
T_{Fp} - friction pendulum torque

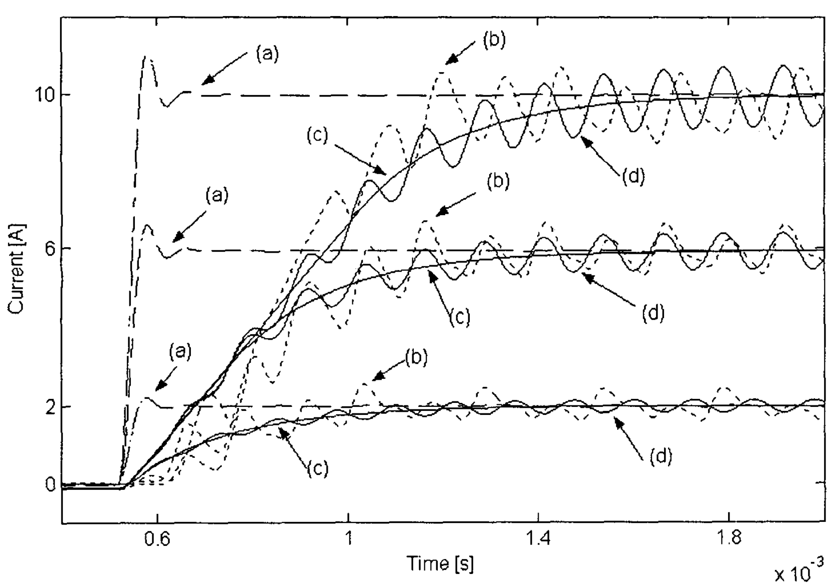


Fig 6. Current of DC motor: a) command current, b) real current, c) estimated current from simulation model, d) estimated current from simulation model with harmonic signal

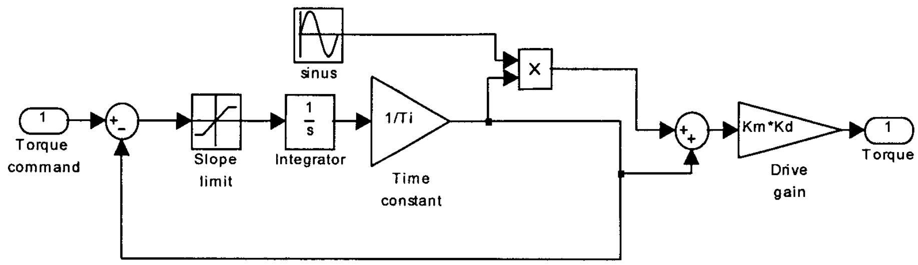


Fig. 7 Current control loop

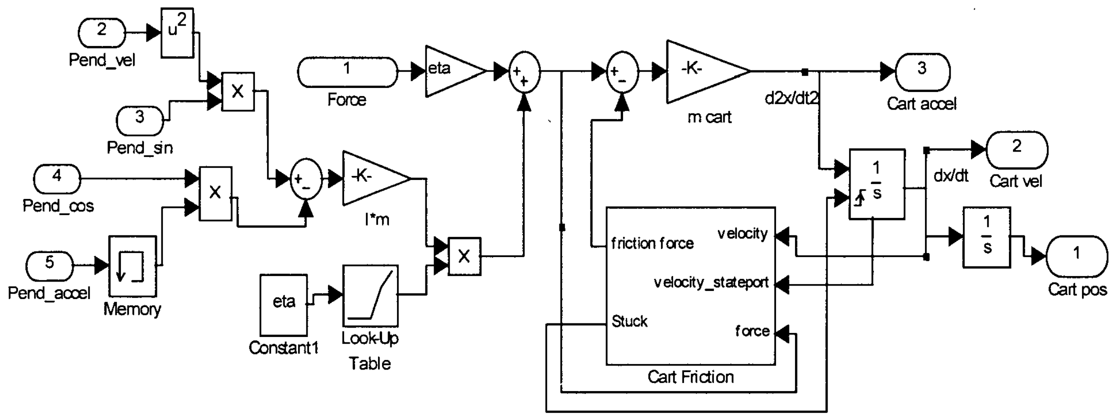


Fig. 8. Model of cart system

k_{fwp} - viscous friction gain of a cart
T_{Kc} - Coulomb kinetic friction torque of a pendulum
F_{Sc} - Coulomb static friction torque of a pendulum

$$\operatorname{sgn}(z) = \begin{cases} 1 & \text{for} \quad z \geq 0 \\ -1 & \text{for} \quad z < 0 \end{cases} \qquad \operatorname{sat}(z) = \begin{cases} z & \text{for} \quad |z| \leq 1 \\ \operatorname{sgn}(z) & \text{for} \quad |z| > 1 \end{cases}$$

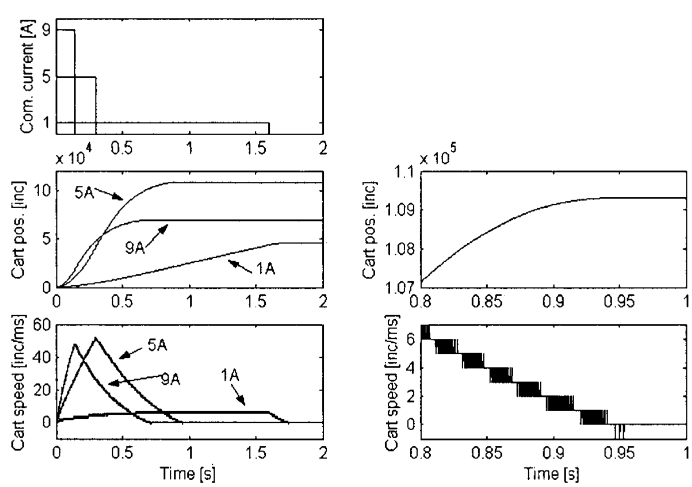


Fig. 9. Response of cart for step current signal a) - set of characteristic, b) - time zoom

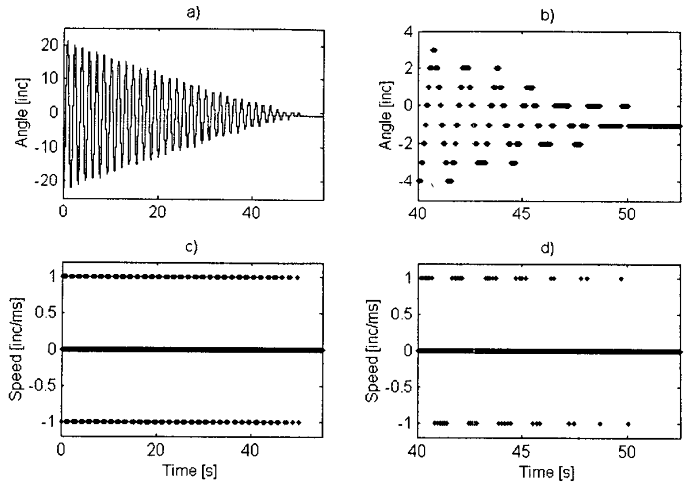
4 The current control loop.

The current control loop consists of current controller, transistor converter and DC motor. It can be determined using equivalent equation 1. The equation is based on limited current increase rate response to value setting step. Additionally equivalent time constant T_i, result of system electric parameters, is improved. Current step responses were used to parameter identification. Detected and actual results are shown in fig 6. The identification is based on current time response, by using optimization Toolbox library. Additional harmonic signal is improved, to model PWM inverter phenomena. The model of current control loop, according to Eq. 1 is presented on fig. 7

5 The cart system.

The cart model with friction is determined by Eq. 3-5. The tests of cart start-up without pendulum with current setting and cart bracking till its stop were realized to identification. The cart position was recorded. Thanks it rate an acceleration were calculated. The optimization let model parameters choice so that obtain compatibility of responses of simulation and actual models.

The model of cart with friction, according to



a) b)
Fig. 10. Response of pendulum for non zero initial a) - global, b) - time zoom

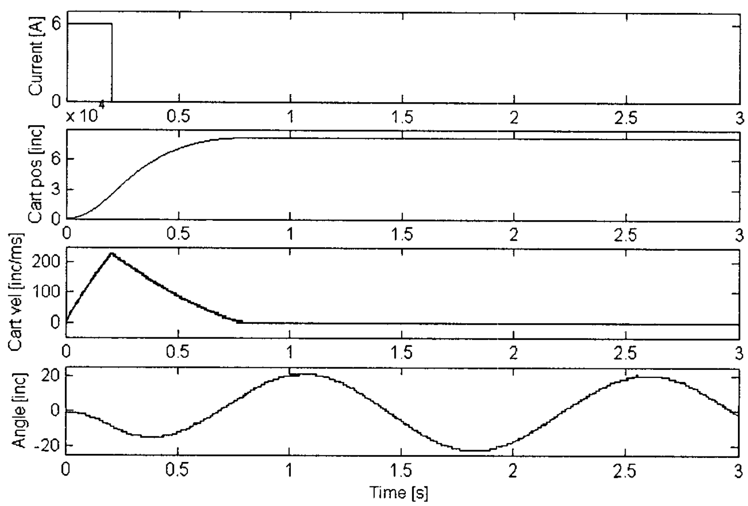


Fig. 11. Interaction between cart and pendulum.

Eq. 3-5 is presented on fig. 8.
The position and speed of cart are presented on fig. 9. Effect of nonlinear friction can be observed. Signal from simulating model is close to experimental results.

6 The pendulum system.

The cart model with friction is determined by Eq. 6-8. The tests of pendulum free damped vibration were recorded to identification. During pendulum vibration no cart motion was observed so efficiency of cart driven gear was less 0.5. This way expression A(eta) in Eq. 4 is 0, and cart motion can be considered pendulum motion independent. The limited resolution of pendulum position signal is the important difficulty occurring during recording. The used sensor resolution is 10 bit and measurements were realized every 0.2 ms.

Another test was concerned to observation of cart motion influence on pendulum. The start-up and breaking caused, according to equation 7, pendulum aberration in comparison to rest position and damped vibration later.

The model of pendulum system with friction, according to Eq. 6-8 is build like a cart system. On fig. 10. the experimental results are presented. The sampling period was adjusted to 0.2 ms, and the resolution of position encoder is equal 10 bits.
In last picture (Fig 11) the interaction between cart and pendulum is presented. Step change of current cause oscillation of pendulum.

7 Conclusion

Tests of system responses during optimal time control cart position were carried out to check the results. The tests were realized for some different pendulum start positions. Simulation results correspond to results recorded in actual system.

The offered test set makes possibility to identify pendulum model well. This model can be the beginning of effective control methods summary. The implementation of control system will be considered in next researches.

With presented set of experimental test one can to identify all parameters of model. Good identified model is a base for effective control of inverted pendulum.

References

1. Hubinski P., Ivanov I., Jurisica L.: Elimination of residual oscillation in electromechanical systems containing pendulum. Int Conf. on Electrical Drives and Power Electronics, Slovakia 3-5 October 2001.
2. Novikov A., Zakarov A., Halasz S.: Identification of robot dynamics model on the basis of genetic algorithms. Int Conf. on Electrical Drives and Power Electronics, Slovakia 3-5 October 2001.
3. Rong Y., Li C., Xiaodong Z.: Control of swing-up and stabilization for an inverted pendulum. 9th Int. Conf. on Power electronic and Motion Control, Slovakia 2000.
4. Kalev Sepp Friction Modeling in Linear Track Cart Pendulum System - *Internet*
5. Armando Bellini, Stefano Bifaretti, Stefano Costantini; Identification of the mechanical parameters in high performances drives, EPE 2001 – Graz
6. Ante Bozic, Danijel Pavkovic, Josko Deur and Nedjeljko Peric, Friction Compensation Based on Reset-Integrator Friction Model EPE 2001 – Graz