FACULTY OF INFORMATION TECHNOLOGY CTU IN PRAGUE

ASSIGNMENT OF MASTER'S THESIS

Title:	Summation polynomials and the discrete logarithm problem on elliptic curve
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Study Programme:	Informatics
Study Branch:	Computer Security
Department:	Department of Information Security
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Instructions

Discrete logarithm problem (DLP) is a fundamental problem arising in modern cryptography. While there exist subexponential algorithms that solve DLP in multiplicative groups of finite fields, no such algorithms are known for groups of points of elliptic curves (ECDLP). Attempts to develop index calculus methods for elliptic curves include so called summation polynomials that give algebraic relations whose solution may give a solution of ECDLP.

The goal of the thesis is to get acquainted with cryptography of elliptic curves, give thorough description of the state of the art of the summation polynomial algorithm, implement it in suitable language and test its performance. Student will focus on available methods of effective generation and solution (Groebner basis and other methods) of algebraic relations appearing in the algorithm.

References

[1] I. Semaev, New algorithm for the discrete logarithm problem on elliptic curves, Cryptology ePrint Archive, Report 2015/310
[2] S. D. Galbraith and S. W. Gebregiyorgis, Summation polynomial algorithms for elliptic curves in characteristic two, Cryptology ePrint Archive, Report 2014/806

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Master's thesis

Summation polynomials and the discrete logarithm problem on elliptic curve

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Department of Information Security
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April 4, 2019

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Czech Technical University in Prague
Faculty of Information Technology

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Abstrakt

V několika větách shrňte obsah a přínos této práce v českém jazyce.

Klíčová slova Replace with comma-separated list of keywords in Czech.

Abstract

Summarize the contents and contribution of your work in a few sentences in English language.

Keywords Replace with comma-separated list of keywords in English.

Contents

List of Figures

Introduction

Chapter 1

Mathematical background

V této kapitole definujeme základní matematické pojmy použité v této práce. Čerpali jsme především z přednášek [**?**] a skript [**?**] k předmětu BI-LIN, handoutů k předmětu MI-MPI [**?**], disertační práce naší vedoucí [**?**] a článků [**?**], [**?**]. Ostatní zdroje jsou uvedeny explicitně v místě, kde byly použity.

1.1 Introduction to general algebra

Definition 1.1.1. A **group** G is an ordered pair $(M, \circ)$, where M is any non-empty set and binary operation $\circ : M \times M \to M$ (sometimes called the group law of G) that satisfies three requirements known as group axioms:

- $\forall x, y, z \in M : x \circ (y \circ z) = (x \circ y) \circ z,$ (associativity)
- $\exists e \in M, \forall x \in M : e \circ x = x \circ e = x,$ (identity element)
- $\forall x \in M, \exists x^{-1} \in M : x \circ x^{-1} = x^{-1} \circ x = e.$ (inverse element)

Remark. M is closed under an operation $\circ$.

Notational Remark. When we are gonna talk about an element g of a group G ($g \in G$) we are actually gonna mean that g is an element of the underlying set M ($g \in M$).

Groups satisfying commutativity law:

- $\forall x, y \in M : x \circ y = y \circ x,$

are called **Abelian groups** (in honour of a famous Norwegian mathematician Niels Henrik Abel) `https://www.britannica.com/biography/Niels-Henrik-Abel`.

Chapter 2

Discrete logarithm problem on elliptic curves

Chapter 3

State-of-the-art

Chapter 4

Analysis and design

Chapter 5

Realisation

Conclusion

Appendix A

Acronyms

GUI Graphical user interface

XML Extensible markup language

Appendix B

Contents of enclosed CD

- `readme.txt` ..... the file with CD contents description
- `exe` ..... the directory with executables
- `src` ..... the directory of source codes
  - `wbdcm` ..... implementation sources
  - `thesis` ..... the directory of LaTeX source codes of the thesis
- `text` ..... the thesis text directory
  - `thesis.pdf` ..... the thesis text in PDF format
  - `thesis.ps` ..... the thesis text in PS format