

## 1 OPTIMALMP

### 1.1 Optimisation problem

$$\max_{\pi H_t, y H_t, \pi L_t, y L_t} U_t = -0.25 (\pi H_t - \pi CB + \pi H_t)^2 - 0.25 (-\pi CB + \pi L_t + \pi L_t)^2 + \beta E_t [U_{t+1}] - 0.25 \lambda y H_t^2 - 0.25 \lambda y L_t^2 \quad (1.1)$$

s.t. :

$$\pi H_{t-1} = \log \epsilon \pi i_{t-1} + \beta \pi H_t + \kappa y H_{t-1} + \beta (-\pi H_t + \pi L_t) \left( 1 - p H_{ss} - \tau (-\pi CB + \pi H_t)^2 \right) \left( \lambda_t^{\text{OPTIMALMP}^1} \right) \quad (1.2)$$

$$\pi L_{t-1} = \log \epsilon \pi i_{t-1} + \beta \pi L_t + \kappa y L_{t-1} + \beta (\pi H_t - \pi L_t) \left( 1 - p L_{ss} - \tau (-\pi CB + \pi L_t)^2 \right) \left( \lambda_t^{\text{OPTIMALMP}^2} \right) \quad (1.3)$$

### 1.2 First order conditions

$$-0.5 \pi H_t + 0.5 \pi CB - 0.5 \pi H_t - \beta E_t \left[ \lambda_{t+1}^{\text{OPTIMALMP}^1} \right] + \lambda_t^{\text{OPTIMALMP}^1} \left( \beta - \beta \left( 1 - p H_{ss} - \tau (-\pi CB + \pi H_t)^2 \right) - 2\beta \tau (-\pi CB + \pi H_t) (-\pi H_t + \pi L_t) \right) + \beta \lambda_t^{\text{OPTIMALMP}^2} \left( 1 - p L_{ss} - \tau (-\pi CB + \pi L_t)^2 \right) = 0 \quad (y H_t) \quad (1.4)$$

$$-0.5 \lambda y H_t + \beta \kappa E_t \left[ \lambda_{t+1}^{\text{OPTIMALMP}^1} \right] = 0 \quad (y H_t) \quad (1.5)$$

$$0.5 \pi CB - 0.5 \pi L_t - 0.5 \pi L_t - \beta E_t \left[ \lambda_{t+1}^{\text{OPTIMALMP}^2} \right] + \lambda_t^{\text{OPTIMALMP}^2} \left( \beta - \beta \left( 1 - p L_{ss} - \tau (-\pi CB + \pi L_t)^2 \right) - 2\beta \tau (-\pi CB + \pi L_t) (\pi H_t - \pi L_t) \right) + \beta \lambda_t^{\text{OPTIMALMP}^1} \left( 1 - p H_{ss} - \tau (-\pi CB + \pi H_t)^2 \right) = 0 \quad (y L_t) \quad (1.6)$$

$$-0.5 \lambda y L_t + \beta \kappa E_t \left[ \lambda_{t+1}^{\text{OPTIMALMP}^2} \right] = 0 \quad (y L_t) \quad (1.7)$$

## 2 EXOG

### 2.1 Identities

$$\epsilon \pi i_t = e^{\epsilon_t^\pi + \phi \log \epsilon \pi i_{t-1}} \quad (2.1)$$

## 3 Equilibrium relationships (after reduction)

$$-\epsilon \pi i_t + e^{\epsilon_t^\pi + \phi \log \epsilon \pi i_{t-1}} = 0 \quad (3.1)$$

$$-0.5 \lambda y H_t + \beta \kappa E_t \left[ \lambda_{t+1}^{\text{OPTIMALMP}^1} \right] = 0 \quad (3.2)$$

$$-0.5 \lambda y L_t + \beta \kappa E_t \left[ \lambda_{t+1}^{\text{OPTIMALMP}^2} \right] = 0 \quad (3.3)$$

$$-piH_{t-1} + \log \epsilon \pi i_{t-1} + \beta piH_t + \kappa y H_{t-1} + \beta (-piH_t + piL_t) \left(1 - pH_{ss} - \tau (-pitCB + piH_t)^2\right) = 0 \quad (3.4)$$

$$-piL_{t-1} + \log \epsilon \pi i_{t-1} + \beta piL_t + \kappa y L_{t-1} + \beta (piH_t - piL_t) \left(1 - pL_{ss} - \tau (-pitCB + piL_t)^2\right) = 0 \quad (3.5)$$

$$-0.5pitH + 0.5pitCB - 0.5piH_t - \beta E_t \left[ \lambda_{t+1}^{\text{OPTIMALMP}^1} \right] + \lambda_t^{\text{OPTIMALMP}^1} \left( \beta - \beta \left(1 - pH_{ss} - \tau (-pitCB + piH_t)^2\right) - 2\beta\tau (-pitCB + piH_t) (-piH_t + piL_t) \right) + \beta \lambda_t^{\text{OPTIMALMP}^2} \left(1 - pL_{ss} - \tau (-pitCB + piL_t)^2\right) = 0 \quad (3.6)$$

$$0.5pitCB - 0.5pitL - 0.5piL_t - \beta E_t \left[ \lambda_{t+1}^{\text{OPTIMALMP}^2} \right] + \lambda_t^{\text{OPTIMALMP}^2} \left( \beta - \beta \left(1 - pL_{ss} - \tau (-pitCB + piL_t)^2\right) - 2\beta\tau (-pitCB + piL_t) (piH_t - piL_t) \right) + \beta \lambda_t^{\text{OPTIMALMP}^1} \left(1 - pH_{ss} - \tau (-pitCB + piH_t)^2\right) = 0 \quad (3.7)$$

$$U_t + 0.25 (pitH - pitCB + piH_t)^2 + 0.25 (-pitCB + pitL + piL_t)^2 - \beta E_t [U_{t+1}] + 0.25 \lambda y H_t^2 + 0.25 \lambda y L_t^2 = 0 \quad (3.8)$$

## 4 Steady state relationships (after reduction)

$$-\epsilon \pi i_{ss} + e^{\phi \log \epsilon \pi i_{ss}} = 0 \quad (4.1)$$

$$-0.5 \lambda y H_{ss} + \beta \kappa \lambda_{ss}^{\text{OPTIMALMP}^1} = 0 \quad (4.2)$$

$$-0.5 \lambda y L_{ss} + \beta \kappa \lambda_{ss}^{\text{OPTIMALMP}^2} = 0 \quad (4.3)$$

$$-piH_{ss} + \log \epsilon \pi i_{ss} + \beta piH_{ss} + \kappa y H_{ss} + \beta (-piH_{ss} + piL_{ss}) \left(1 - pH_{ss} - \tau (-pitCB + piH_{ss})^2\right) = 0 \quad (4.4)$$

$$-piL_{ss} + \log \epsilon \pi i_{ss} + \beta piL_{ss} + \kappa y L_{ss} + \beta (piH_{ss} - piL_{ss}) \left(1 - pL_{ss} - \tau (-pitCB + piL_{ss})^2\right) = 0 \quad (4.5)$$

$$-0.5pitH + 0.5pitCB - 0.5piH_{ss} - \beta \lambda_{ss}^{\text{OPTIMALMP}^1} + \lambda_{ss}^{\text{OPTIMALMP}^1} \left( \beta - \beta \left(1 - pH_{ss} - \tau (-pitCB + piH_{ss})^2\right) - 2\beta\tau (-pitCB + piH_{ss}) (-piH_{ss} + piL_{ss}) \right) + \beta \lambda_{ss}^{\text{OPTIMALMP}^2} \left(1 - pL_{ss} - \tau (-pitCB + piL_{ss})^2\right) = 0 \quad (4.6)$$

$$0.5pitCB - 0.5pitL - 0.5piL_{ss} - \beta \lambda_{ss}^{\text{OPTIMALMP}^2} + \lambda_{ss}^{\text{OPTIMALMP}^2} \left( \beta - \beta \left(1 - pL_{ss} - \tau (-pitCB + piL_{ss})^2\right) - 2\beta\tau (-pitCB + piL_{ss}) (piH_{ss} - piL_{ss}) \right) + \beta \lambda_{ss}^{\text{OPTIMALMP}^1} \left(1 - pH_{ss} - \tau (-pitCB + piH_{ss})^2\right) = 0 \quad (4.7)$$

$$U_{ss} + 0.25 (pitH - pitCB + piH_{ss})^2 + 0.25 (-pitCB + pitL + piL_{ss})^2 - \beta U_{ss} + 0.25 \lambda y H_{ss}^2 + 0.25 \lambda y L_{ss}^2 = 0 \quad (4.8)$$

## 5 Parameter settings

$$\beta = 0.99 \quad (5.1)$$

$$\kappa = 0.2465 \quad (5.2)$$

$$\lambda = 0.04106 \quad (5.3)$$

$$\phi = 0.95 \quad (5.4)$$

$$pitH = 2 \quad (5.5)$$

$$pitCB = 2 \quad (5.6)$$

$$pitL = 4 \tag{5.7}$$

$$pHss = 0.99 \tag{5.8}$$

$$pLss = 0.99 \tag{5.9}$$

$$\sigma = 1 \tag{5.10}$$

$$\tau = 0.01 \tag{5.11}$$

$$\theta = 6 \tag{5.12}$$

## 6 Steady-state values

	Steady-state value
$\epsilon \Delta p_i$	1
$\lambda^{\text{OPTIMALMP}^1}$	-0.0202
$\lambda^{\text{OPTIMALMP}^2}$	0.0881
$p_i H$	-0.0236
$p_i L$	-1.9469
$y_i H$	-0.24
$y_i L$	1.0471
$U$	-1.269

## 7 The solution of the 1st order perturbation

Matrix  $P$

$$\begin{matrix} & \epsilon \Delta p_{i,t-1} & p_i H_{t-1} & p_i L_{t-1} & y_i H_{t-1} & y_i L_{t-1} \\ \begin{matrix} \epsilon \Delta p_i \\ p_i H_t \\ p_i L_t \\ y_i H_t \\ y_i L_t \end{matrix} & \begin{pmatrix} 0.95 & 0 & 0 & 0 & 0 \\ -46.2428 & 1.0637 & 2.0971 & -2.6721 & -0.278 \\ -0.4627 & 0.0014 & 0.7813 & -0.0036 & -0.1036 \\ -30.5425 & 0.3024 & 1.4628 & -0.7597 & -0.1939 \\ -5.1426 & 0.0093 & 3.2707 & -0.0234 & -0.4336 \end{pmatrix} \end{matrix}$$

Matrix  $Q$

$$\begin{matrix} & \epsilon^\pi \\ \begin{matrix} \epsilon \Delta p_i \\ p_i H \\ p_i L \\ y_i H \\ y_i L \end{matrix} & \begin{pmatrix} 1 \\ 0 \\ 0 \\ -17.8426 \\ -3.2281 \end{pmatrix} \end{matrix}$$

Matrix  $R$

$$\begin{matrix} & \epsilon \Delta p_{i,t-1} & p_i H_{t-1} & p_i L_{t-1} & y_i H_{t-1} & y_i L_{t-1} \\ \begin{matrix} \lambda_t^{\text{OPTIMALMP}^1} \\ \lambda_t^{\text{OPTIMALMP}^2} \\ U_t \end{matrix} & \begin{pmatrix} -65.8827 & 0.9888 & 8.6937 & -2.4839 & -1.1526 \\ -8.3814 & 0.0241 & 9.5166 & -0.0606 & -1.2616 \\ 0.8992 & 0.0004 & -0.1351 & -0.0009 & 0.0179 \end{pmatrix} \end{matrix}$$

Matrix  $S$

$$\begin{matrix} & \epsilon^\pi \\ \begin{matrix} \lambda^{\text{OPTIMALMP}^1} \\ \lambda^{\text{OPTIMALMP}^2} \\ U \end{matrix} & \begin{pmatrix} -20.455 \\ -2.5996 \\ 0.8902 \end{pmatrix} \end{matrix}$$

## 8 Model statistics

### 8.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
$\epsilon \Delta p_i$	1	0.1303	0.017	Y
$\lambda^{\text{OPTIMALMP}^1}$	-0.0202	2.4833	6.1668	Y
$\lambda^{\text{OPTIMALMP}^2}$	0.0881	0.44	0.1936	Y
$p_i H$	-0.0236	0.421	0.1772	Y
$p_i L$	-1.9469	0.0112	0.0001	Y
$y_i H$	-0.24	2.2561	5.0901	Y
$y_i L$	1.0471	0.4526	0.2048	Y
$U$	-1.269	0.1175	0.0138	Y

## 8.2 Correlation matrix

	$etpi$	$\lambda^{\text{OPTIMALMP}^1}$	$\lambda^{\text{OPTIMALMP}^2}$	$piH$	$piL$	$yH$	$yL$	$U$
$etpi$	1	-0.996	-0.974	0.666	-0.671	-0.999	-0.997	1
$\lambda^{\text{OPTIMALMP}^1}$		1	0.951	-0.599	0.604	0.999	0.987	-0.995
$\lambda^{\text{OPTIMALMP}^2}$			1	-0.817	0.821	0.966	0.988	-0.977
$piH$				1	-1	-0.64	-0.72	0.677
$piL$					1	0.645	0.725	-0.681
$yH$						1	0.994	-0.999
$yL$							1	-0.998
$U$								1

## 8.3 Cross correlations with the reference variable ( $piH$ )

	$\sigma[\cdot]$ rel. to $\sigma[piH]$	$piH_{t-5}$	$piH_{t-4}$	$piH_{t-3}$	$piH_{t-2}$	$piH_{t-1}$	$piH_t$	$piH_{t+1}$	$piH_{t+2}$	$piH_{t+3}$	$piH_{t+4}$
$etpi_t$	0.31	0.234	0.431	0.652	0.863	0.963	0.666	0.418	0.216	0.054	-0.001
$\lambda_t^{\text{OPTIMALMP}^1}$	5.899	-0.25	-0.442	-0.656	-0.856	-0.936	-0.599	-0.351	-0.16	-0.014	0.09
$\lambda_t^{\text{OPTIMALMP}^2}$	1.045	-0.185	-0.386	-0.617	-0.851	-0.999	-0.817	-0.575	-0.346	-0.152	0.00
$piH_t$	1	0.012	0.174	0.374	0.606	0.843	1	0.843	0.606	0.374	0.17
$piL_t$	0.027	-0.017	-0.179	-0.379	-0.611	-0.846	-1	-0.833	-0.593	-0.361	-0.17
$yH_t$	5.359	-0.241	-0.436	-0.654	-0.861	-0.953	-0.64	-0.391	-0.193	-0.038	0.00
$yL_t$	1.075	-0.219	-0.418	-0.644	-0.864	-0.98	-0.72	-0.473	-0.26	-0.087	0.00
$U_t$	0.279	0.232	0.429	0.651	0.864	0.967	0.677	0.429	0.224	0.06	-0.00

## 8.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
$etpi$	0.713	0.471	0.271	0.11	-0.016
$\lambda^{\text{OPTIMALMP}^1}$	0.661	0.416	0.229	0.084	-0.028
$\lambda^{\text{OPTIMALMP}^2}$	0.826	0.587	0.358	0.163	0.006
$piH$	0.843	0.606	0.374	0.174	0.012
$piL$	0.837	0.598	0.366	0.168	0.009
$yH$	0.693	0.449	0.254	0.099	-0.021
$yL$	0.755	0.514	0.303	0.13	-0.008
$U$	0.721	0.479	0.277	0.114	-0.015

## 9 Impulse response functions

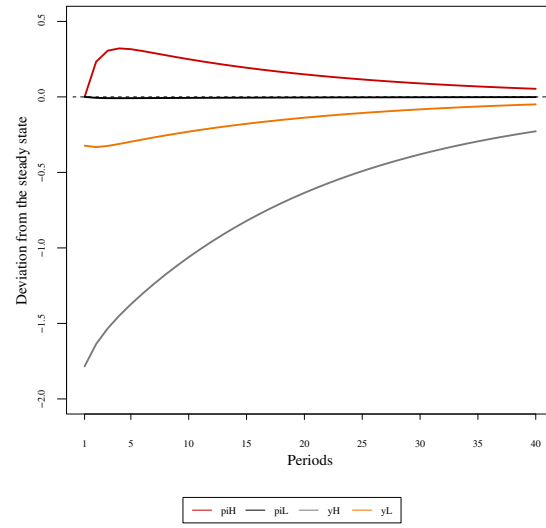


Figure 1: Impulse responses  $(\bar{p}H, \bar{p}L, yH, yL)$  to  $\epsilon^\pi$  shock