

## 1 HIGHREGIME

### 1.1 Optimisation problem

$$\max_{pH_t, yH_t, iH_t} UH_t = -0.5 (piH - pitCB + piH_t)^2 + \beta E_t [UH_{t+1}] + \beta (1 - pH) (-E_t [UH_{t+1}] + E_t [UL_{t+1}]) - 0.5 \kappa \theta^{-1} yH_t^2 \quad (1.1)$$

s.t. :

$$piH_{t-1} = \log etapi_{t-1} + \beta piH_t + \kappa yH_{t-1} + \beta (1 - pH) (-piH_t + piL_t) \quad \left( \lambda_t^{\text{HIGHREGIME}^1} \right) \quad (1.2)$$

$$yH_{t-1} = yH_t - \sigma (iH_{t-1} - piH_t) + (1 - pH) (-yH_t + yL_t) + \sigma (1 - pH) (-piH_t + piL_t) \quad \left( \lambda_t^{\text{HIGHREGIME}^2} \right) \quad (1.3)$$

### 1.2 First order conditions

$$-piH + pitCB - piH_t + \lambda_t^{\text{HIGHREGIME}^1} (\beta - \beta (1 - pH)) + \lambda_t^{\text{HIGHREGIME}^2} (\sigma - \sigma (1 - pH)) - (\beta - \beta (1 - pH)) E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^1} \right] = 0 \quad (piH_t) \quad (1.4)$$

$$pH \lambda_t^{\text{HIGHREGIME}^2} + (\beta - \beta (1 - pH)) \left( \kappa E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^1} \right] - E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^2} \right] \right) - \kappa \theta^{-1} yH_t = 0 \quad (yH_t) \quad (1.5)$$

$$-\sigma (\beta - \beta (1 - pH)) E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^2} \right] = 0 \quad (iH_t) \quad (1.6)$$

## 2 LOWREGIME

### 2.1 Optimisation problem

$$\max_{piL_t, yL_t, iL_t} UL_t = -0.5 (-pitCB + piL + piL_t)^2 + \beta E_t [UL_{t+1}] + \beta (1 - pL) (E_t [UH_{t+1}] - E_t [UL_{t+1}]) - 0.5 \kappa \theta^{-1} yL_t^2 \quad (2.1)$$

s.t. :

$$piL_{t-1} = \log etapi_{t-1} + \kappa yL_{t-1} + \beta pL piL_t + \beta (1 - pL) (piH_t - piL_t) \quad \left( \lambda_t^{\text{LOWREGIME}^1} \right) \quad (2.2)$$

$$yL_{t-1} = yL_t - \sigma (iL_{t-1} - piL_t) + (1 - pL) (yH_t - yL_t) + \sigma (1 - pL) (piH_t - piL_t) \quad \left( \lambda_t^{\text{LOWREGIME}^2} \right) \quad (2.3)$$

### 2.2 First order conditions

$$pitCB - piL - piL_t + \lambda_t^{\text{LOWREGIME}^1} (\beta pL - \beta (1 - pL)) + \lambda_t^{\text{LOWREGIME}^2} (\sigma - \sigma (1 - pL)) - (\beta - \beta (1 - pL)) E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^1} \right] = 0 \quad (piL_t) \quad (2.4)$$

$$pL\lambda_t^{\text{LOWREGIME}^2} + (\beta - \beta(1 - pL)) \left( \kappa E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^1} \right] - E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^2} \right] \right) - \kappa \theta^{-1} yL_t = 0 \quad (yL_t) \quad (2.5)$$

$$-\sigma (\beta - \beta(1 - pL)) E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^2} \right] = 0 \quad (iL_t) \quad (2.6)$$

### 3 EXOG

#### 3.1 Identities

$$\text{etapi}_t = e^{\epsilon_t^\pi + \phi \log \text{etapi}_{t-1}} \quad (3.1)$$

### 4 Equilibrium relationships (after reduction)

$$-\text{etapi}_t + e^{\epsilon_t^\pi + \phi \log \text{etapi}_{t-1}} = 0 \quad (4.1)$$

$$pH\lambda_t^{\text{HIGHREGIME}^2} + (\beta - \beta(1 - pH)) \left( \kappa E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^1} \right] - E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^2} \right] \right) - \kappa \theta^{-1} yH_t = 0 \quad (4.2)$$

$$pL\lambda_t^{\text{LOWREGIME}^2} + (\beta - \beta(1 - pL)) \left( \kappa E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^1} \right] - E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^2} \right] \right) - \kappa \theta^{-1} yL_t = 0 \quad (4.3)$$

$$-piH_{t-1} + \log \text{etapi}_{t-1} + \beta piH_t + \kappa yH_{t-1} + \beta(1 - pH)(-piH_t + piL_t) = 0 \quad (4.4)$$

$$-piL_{t-1} + \log \text{etapi}_{t-1} + \kappa yL_{t-1} + \beta pLpiL_t + \beta(1 - pL)(piH_t - piL_t) = 0 \quad (4.5)$$

$$-yH_{t-1} + yH_t - \sigma(iH_{t-1} - piH_t) + (1 - pH)(-yH_t + yL_t) + \sigma(1 - pH)(-piH_t + piL_t) = 0 \quad (4.6)$$

$$-yL_{t-1} + yL_t - \sigma(iL_{t-1} - piL_t) + (1 - pL)(yH_t - yL_t) + \sigma(1 - pL)(piH_t - piL_t) = 0 \quad (4.7)$$

$$UH_t + 0.5(piH - piCB + piH_t)^2 - \beta E_t[UH_{t+1}] - \beta(1 - pH)(-E_t[UH_{t+1}] + E_t[UL_{t+1}]) + 0.5\kappa\theta^{-1}yH_t^2 = 0 \quad (4.8)$$

$$UL_t + 0.5(-piCB + piL + piL_t)^2 - \beta E_t[UL_{t+1}] - \beta(1 - pL)(E_t[UH_{t+1}] - E_t[UL_{t+1}]) + 0.5\kappa\theta^{-1}yL_t^2 = 0 \quad (4.9)$$

$$-piH + piCB - piH_t + \lambda_t^{\text{HIGHREGIME}^1}(\beta - \beta(1 - pH)) + \lambda_t^{\text{HIGHREGIME}^2}(\sigma - \sigma(1 - pH)) - (\beta - \beta(1 - pH)) E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^1} \right] = 0 \quad (4.10)$$

$$piCB - piL - piL_t + \lambda_t^{\text{LOWREGIME}^1}(\beta pL - \beta(1 - pL)) + \lambda_t^{\text{LOWREGIME}^2}(\sigma - \sigma(1 - pL)) - (\beta - \beta(1 - pL)) E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^1} \right] = 0 \quad (4.11)$$

$$-\sigma(\beta - \beta(1 - pH)) E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^2} \right] = 0 \quad (4.12)$$

$$-\sigma(\beta - \beta(1 - pL)) E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^2} \right] = 0 \quad (4.13)$$

## 5 Steady state relationships (after reduction)

$$-d\pi_{ss} + e^{\phi \log d\pi_{ss}} = 0 \quad (5.1)$$

$$pH\lambda_{ss}^{\text{HIGHREGIME}^2} + (\beta - \beta(1 - pH)) \left( -\lambda_{ss}^{\text{HIGHREGIME}^2} + \kappa\lambda_{ss}^{\text{HIGHREGIME}^1} \right) - \kappa\theta^{-1}yH_{ss} = 0 \quad (5.2)$$

$$pL\lambda_{ss}^{\text{LOWREGIME}^2} + (\beta - \beta(1 - pL)) \left( -\lambda_{ss}^{\text{LOWREGIME}^2} + \kappa\lambda_{ss}^{\text{LOWREGIME}^1} \right) - \kappa\theta^{-1}yL_{ss} = 0 \quad (5.3)$$

$$-pH_{ss} + \log d\pi_{ss} + \beta pH_{ss} + \kappa yH_{ss} + \beta(1 - pH)(-pH_{ss} + pL_{ss}) = 0 \quad (5.4)$$

$$-pL_{ss} + \log d\pi_{ss} + \kappa yL_{ss} + \beta pL_{ss} + \beta(1 - pL)(pH_{ss} - pL_{ss}) = 0 \quad (5.5)$$

$$(1 - pH)(-yH_{ss} + yL_{ss}) - \sigma(iH_{ss} - pH_{ss}) + \sigma(1 - pH)(-pH_{ss} + pL_{ss}) = 0 \quad (5.6)$$

$$(1 - pL)(yH_{ss} - yL_{ss}) - \sigma(iL_{ss} - pL_{ss}) + \sigma(1 - pL)(pH_{ss} - pL_{ss}) = 0 \quad (5.7)$$

$$UH_{ss} + 0.5(pH - pCB + pH_{ss})^2 - \beta UH_{ss} - \beta(1 - pH)(-UH_{ss} + UL_{ss}) + 0.5\kappa\theta^{-1}yH_{ss}^2 = 0 \quad (5.8)$$

$$UL_{ss} + 0.5(-pCB + pL + pL_{ss})^2 - \beta UL_{ss} - \beta(1 - pL)(UH_{ss} - UL_{ss}) + 0.5\kappa\theta^{-1}yL_{ss}^2 = 0 \quad (5.9)$$

$$-pH + pCB - pH_{ss} + \lambda_{ss}^{\text{HIGHREGIME}^2}(\sigma - \sigma(1 - pH)) = 0 \quad (5.10)$$

$$pCB - pL - pL_{ss} - \lambda_{ss}^{\text{LOWREGIME}^1}(\beta - \beta(1 - pL)) + \lambda_{ss}^{\text{LOWREGIME}^1}(\beta pL - \beta(1 - pL)) + \lambda_{ss}^{\text{LOWREGIME}^2}(\sigma - \sigma(1 - pL)) = 0 \quad (5.11)$$

$$-\sigma\lambda_{ss}^{\text{HIGHREGIME}^2}(\beta - \beta(1 - pH)) = 0 \quad (5.12)$$

$$-\sigma\lambda_{ss}^{\text{LOWREGIME}^2}(\beta - \beta(1 - pL)) = 0 \quad (5.13)$$

## 6 Parameter settings

$$\beta = 0.99 \quad (6.1)$$

$$\kappa = 0.2465 \quad (6.2)$$

$$\phi = 0.95 \quad (6.3)$$

$$pH = 0 \quad (6.4)$$

$$pCB = 0 \quad (6.5)$$

$$pL = 2 \quad (6.6)$$

$$pH = 0.99 \quad (6.7)$$

$$pL = 0.99 \quad (6.8)$$

$$\sigma = 1 \quad (6.9)$$

$$\theta = 6 \quad (6.10)$$

## 7 Steady-state values

	Steady-state value
$etapi$	1
$iH$	-0.0232
$iL$	-1.9764
$\lambda^{\text{HIGHREGIME}^1}$	0.0137
$\lambda^{\text{HIGHREGIME}^2}$	0
$\lambda^{\text{LOWREGIME}^1}$	-0.0411
$\lambda^{\text{LOWREGIME}^2}$	0
$piH$	0
$piL$	-1.9996
$yH$	0.0803
$yL$	-0.2417
$UH$	-0.0487
$UL$	-0.0846

## 8 The solution of the 1st order perturbation

Matrix  $P$

$$\begin{matrix}
 & etapi_{t-1} & iH_{t-1} & iL_{t-1} & piH_{t-1} & piL_{t-1} & yH_{t-1} & yL_{t-1} \\
 \begin{matrix} etapi_t \\ iH_t \\ iL_t \\ piH_t \\ piL_t \\ yH_t \\ yL_t \end{matrix} & \begin{pmatrix} 0.95 & 0 & 0 & 0 & 0 & 0 & 0 \\ -561.3689 & -1.9645 & 1.8946 & 253.8981 & -6.2527 & -11.8214 & 0.4181 \\ -6.6986 & 0.0003 & -1.9791 & -0.0376 & 6.1014 & 0.0017 & -0.4239 \\ -1.01 & 0 & 0 & 1.0204 & -0.0208 & -0.0202 & 0.0006 \\ -0.5103 & 0 & 0 & -0.0052 & 1.0308 & 0.0001 & -0.0307 \\ 12.5765 & 0.292 & -0.2511 & -12.7062 & 0.2593 & 1.2617 & -0.0384 \\ 4.2212 & -0.001 & 8.2592 & 0.0431 & -8.5268 & -0.0042 & 1.2643 \end{pmatrix}
 \end{matrix}$$

Matrix  $Q$

$$\begin{matrix}
 & \epsilon^\pi \\
 \begin{matrix} etapi \\ iH \\ iL \\ piH \\ piL \\ yH \\ yL \end{matrix} & \begin{pmatrix} 1 \\ -326.9451 \\ -3.8787 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
 \end{matrix}$$

Matrix  $R$

$$\begin{matrix}
 & etapi_{t-1} & iH_{t-1} & iL_{t-1} & piH_{t-1} & piL_{t-1} & yH_{t-1} & yL_{t-1} \\
 \begin{matrix} \lambda_t^{\text{HIGHREGIME}^1} \\ \lambda_t^{\text{HIGHREGIME}^2} \\ \lambda_t^{\text{LOWREGIME}^1} \\ \lambda_t^{\text{LOWREGIME}^2} \\ UH_t \\ UL_t \end{matrix} & \begin{pmatrix} -438.2766 & -0.8773 & 1.53 & 256.5425 & -9.284 & -8.1131 & 0.4638 \\ 1.0037 & 0.0031 & -0.0048 & -0.516 & 0.0214 & 0.021 & -0.0012 \\ -150.6019 & 0.0061 & -25.5874 & -1.5735 & 176.6507 & 0.0524 & -8.3939 \\ 1.0299 & -0.0001 & 0.2703 & 0.0108 & -1.0642 & -0.0004 & 0.0648 \\ 2.5126 & 0 & -0.008 & -0.2808 & 0.0473 & 0.0056 & -0.0024 \\ -6.8483 & 0.0001 & -0.0001 & -0.0168 & 0.9727 & 0.0005 & -0.029 \end{pmatrix}
 \end{matrix}$$

## Matrix $S$

$$\begin{matrix} & \epsilon^\pi \\ \lambda^{\text{HIGHREGIME}^1} & \left( \begin{matrix} -196.1864 \\ 0.5245 \\ -67.1917 \\ 0.5353 \\ 2.3741 \\ -6.7143 \end{matrix} \right) \\ \lambda^{\text{HIGHREGIME}^2} & \\ \lambda^{\text{LOWREGIME}^1} & \\ \lambda^{\text{LOWREGIME}^2} & \\ UH & \\ UL & \end{matrix}$$

## 9 Model statistics

### 9.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
$etapi$	1	0.1303	0.017	Y
$iH$	-0.0232	33.249	1105.4967	Y
$iL$	-1.9764	0.3951	0.1561	Y
$\lambda^{\text{HIGHREGIME}^1}$	0.0137	21.6808	470.0577	Y
$\lambda^{\text{HIGHREGIME}^2}$	0	0.0506	0.0026	N
$\lambda^{\text{LOWREGIME}^1}$	-0.0411	7.4029	54.8026	Y
$\lambda^{\text{LOWREGIME}^2}$	0	0.0516	0.0027	N
$pH$	0	0.0985	0.0097	N
$pL$	-1.9996	0.0498	0.0025	Y
$yH$	0.0803	8.625	74.3904	Y
$yL$	-0.2417	2.8859	8.3285	Y
$UH$	-0.0487	0.3247	0.1055	Y
$UL$	-0.0846	0.8996	0.8093	Y

### 9.2 Correlation matrix

	$etapi$	$iH$	$iL$	$\lambda^{\text{HIGHREGIME}^1}$	$\lambda^{\text{HIGHREGIME}^2}$	$\lambda^{\text{LOWREGIME}^1}$	$\lambda^{\text{LOWREGIME}^2}$	$pH$	$pL$
$etapi$	1	-0.301	-0.298	-0.825	0.436	-0.821	0.436	-0.491	-0.491
$iH$		1	1	0.567	-0.977	0.573	-0.977	-0.323	-0.323
$iL$			1	0.564	-0.976	0.57	-0.976	-0.327	-0.327
$\lambda^{\text{HIGHREGIME}^1}$				1	-0.725	1	-0.725	0.567	0.567
$\lambda^{\text{HIGHREGIME}^2}$					1	-0.73	1	0.116	0.116
$\lambda^{\text{LOWREGIME}^1}$						1	-0.73	0.563	0.563
$\lambda^{\text{LOWREGIME}^2}$							1	0.116	0.116
$pH$								1	1
$pL$									1
$yH$									
$yL$									
$UH$									
$UL$									

### 9.3 Cross correlations with the reference variable ( $iH$ )

	$\sigma[\cdot]$ rel. to $\sigma[iH]$	$iH_{t-5}$	$iH_{t-4}$	$iH_{t-3}$	$iH_{t-2}$	$iH_{t-1}$	$iH_t$	$iH_{t+1}$	$iH_{t+2}$	$iH_{t+3}$	$iH_{t+4}$
$\epsilon\pi i_t$	0.004	0.098	0.131	0.177	0.258	0.444	-0.301	-0.254	-0.21	-0.169	-0.132
$iH_t$	1	-0.028	-0.035	-0.051	-0.093	-0.219	1	-0.219	-0.093	-0.051	-0.035
$iL_t$	0.012	-0.027	-0.035	-0.05	-0.092	-0.218	1	-0.222	-0.094	-0.051	-0.035
$\lambda_t^{\text{HIGHREGIME}^1}$	0.652	-0.078	-0.1	-0.134	-0.204	-0.385	0.567	0.47	0.151	0.043	0.003
$\lambda_t^{\text{HIGHREGIME}^2}$	0.002	0.04	0.051	0.071	0.122	0.269	-0.977	0.043	0.043	0.041	0.038
$\lambda_t^{\text{LOWREGIME}^1}$	0.223	-0.078	-0.099	-0.133	-0.203	-0.384	0.573	0.469	0.149	0.041	0.001
$\lambda_t^{\text{LOWREGIME}^2}$	0.002	0.04	0.051	0.071	0.122	0.269	-0.977	0.043	0.043	0.041	0.038
$\pi H_t$	0.003	-0.046	-0.056	-0.071	-0.096	-0.156	-0.323	0.852	0.236	0.04	-0.022
$\pi L_t$	0.001	-0.045	-0.056	-0.07	-0.095	-0.154	-0.32	0.855	0.233	0.037	-0.024
$yH_t$	0.259	-0.069	-0.092	-0.121	-0.164	-0.245	-0.443	0.551	0.273	0.164	0.111
$yL_t$	0.087	-0.069	-0.092	-0.121	-0.163	-0.244	-0.442	0.555	0.273	0.163	0.11
$UH_t$	0.01	0.098	0.13	0.175	0.255	0.438	-0.258	-0.313	-0.22	-0.165	-0.125
$UL_t$	0.027	-0.098	-0.13	-0.176	-0.256	-0.44	0.276	0.292	0.216	0.166	0.127

### 9.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
$\epsilon\pi i$	0.713	0.471	0.271	0.11	-0.016
$iH$	-0.219	-0.093	-0.051	-0.035	-0.028
$iL$	-0.221	-0.093	-0.05	-0.035	-0.027
$\lambda^{\text{HIGHREGIME}^1}$	0.51	0.081	-0.065	-0.117	-0.134
$\lambda^{\text{HIGHREGIME}^2}$	-0.074	-0.071	-0.066	-0.06	-0.054
$\lambda^{\text{LOWREGIME}^1}$	0.506	0.077	-0.068	-0.118	-0.134
$\lambda^{\text{LOWREGIME}^2}$	-0.074	-0.071	-0.066	-0.06	-0.054
$\pi H$	0.22	-0.024	-0.095	-0.11	-0.107
$\pi L$	0.217	-0.025	-0.096	-0.11	-0.107
$yH$	0.504	0.261	0.116	0.017	-0.055
$yL$	0.5	0.257	0.113	0.015	-0.055
$UH$	0.735	0.449	0.243	0.087	-0.031
$UL$	0.727	0.456	0.253	0.095	-0.026

## 10 Impulse response functions

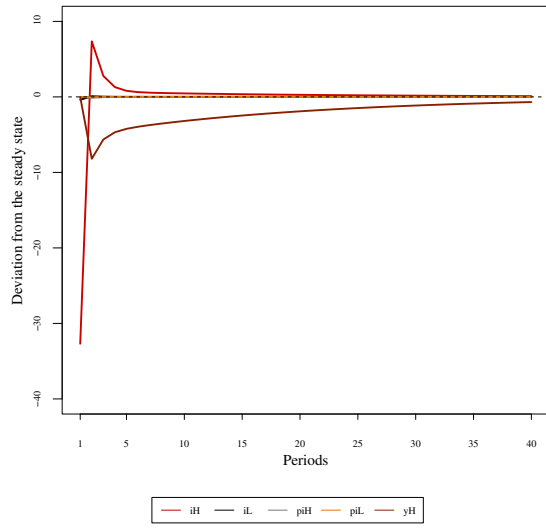


Figure 1: Impulse responses ( $iH, iL, \pi H, \pi L, yH$ ) to  $\epsilon^\pi$  shock

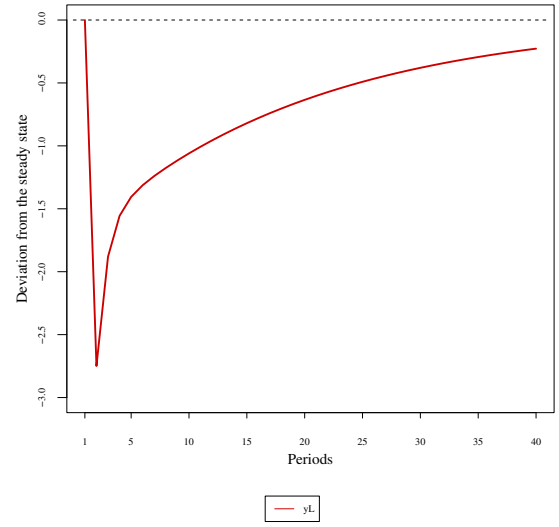


Figure 2: Impulse response ( $yL$ ) to  $\epsilon^\pi$  shock