

## 1 HIGHREGIME

### 1.1 Optimisation problem

$$\max_{\mathbf{p}H_t, yH_t, iH_t} UH_t = -0.5(-\mathbf{p}iCB + \mathbf{p}H_t)^2 + \beta(pHE_t[UH_{t+1}] + (1 - pH)E_t[UL_{t+1}]) - 0.5\kappa\theta^{-1}yH_t^2 \quad (1.1)$$

s.t. :

$$\mathbf{p}H_{t-1} = \beta(pH\mathbf{p}H_t + \mathbf{p}L_t(1 - pH)) + \kappa yH_{t-1} \quad \left(\lambda_t^{\text{HIGHREGIME}^1}\right) \quad (1.2)$$

$$yH_{t-1} = pH yH_t + yL_t(1 - pH) - \sigma(iH_{t-1} - m_{t-1} - pH\mathbf{p}H_t - \mathbf{p}L_t(1 - pH)) \quad \left(\lambda_t^{\text{HIGHREGIME}^2}\right) \quad (1.3)$$

### 1.2 First order conditions

$$\mathbf{p}iCB - \mathbf{p}H_t + \beta pH \lambda_t^{\text{HIGHREGIME}^1} - \beta pH E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^1} \right] + pH \sigma \lambda_t^{\text{HIGHREGIME}^2} = 0 \quad (\mathbf{p}H_t) \quad (1.4)$$

$$pH \lambda_t^{\text{HIGHREGIME}^2} + \beta pH \left( \kappa E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^1} \right] - E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^2} \right] \right) - \kappa \theta^{-1} yH_t = 0 \quad (yH_t) \quad (1.5)$$

$$-\beta pH \sigma E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^2} \right] = 0 \quad (iH_t) \quad (1.6)$$

## 2 LOWREGIME

### 2.1 Optimisation problem

$$\max_{\mathbf{p}L_t, yL_t, iL_t} UL_t = -0.5(-\mathbf{p}iCB + \mathbf{p}L_t)^2 + \beta(pLE_t[UL_{t+1}] + (1 - pL)E_t[UH_{t+1}]) - 0.5\kappa\theta^{-1}yL_t^2 \quad (2.1)$$

s.t. :

$$\mathbf{p}L_{t-1} = \beta(pL\mathbf{p}L_t + \mathbf{p}H_t(1 - pL)) + \kappa yL_{t-1} \quad \left(\lambda_t^{\text{LOWREGIME}^1}\right) \quad (2.2)$$

$$yL_{t-1} = pL yL_t + yH_t(1 - pL) - \sigma(iL_{t-1} - m_{t-1} - pL\mathbf{p}L_t - \mathbf{p}H_t(1 - pL)) \quad \left(\lambda_t^{\text{LOWREGIME}^2}\right) \quad (2.3)$$

### 2.2 First order conditions

$$\mathbf{p}iCB - \mathbf{p}L_t + \beta pL \lambda_t^{\text{LOWREGIME}^1} - \beta pL E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^1} \right] + pL \sigma \lambda_t^{\text{LOWREGIME}^2} = 0 \quad (\mathbf{p}L_t) \quad (2.4)$$

$$pL\lambda_t^{\text{LOWREGIME}^2} + \beta pL \left( \kappa E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^1} \right] - E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^2} \right] \right) - \kappa \theta^{-1} yL_t = 0 \quad (yL_t) \quad (2.5)$$

$$-\beta pL\sigma E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^2} \right] = 0 \quad (iL_t) \quad (2.6)$$

### 3 EXOG

#### 3.1 Identities

$$m_t = e^{\epsilon_t^Z + \phi \log m_{t-1}} \quad (3.1)$$

### 4 Equilibrium relationships (after reduction)

$$-m_t + e^{\epsilon_t^Z + \phi \log m_{t-1}} = 0 \quad (4.1)$$

$$-piH_{t-1} + \beta (pH piH_t + piL_t (1 - pH)) + \kappa yH_{t-1} = 0 \quad (4.2)$$

$$-piL_{t-1} + \beta (pL piL_t + pH_t (1 - pL)) + \kappa yL_{t-1} = 0 \quad (4.3)$$

$$pH\lambda_t^{\text{HIGHREGIME}^2} + \beta pH \left( \kappa E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^1} \right] - E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^2} \right] \right) - \kappa \theta^{-1} yH_t = 0 \quad (4.4)$$

$$pL\lambda_t^{\text{LOWREGIME}^2} + \beta pL \left( \kappa E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^1} \right] - E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^2} \right] \right) - \kappa \theta^{-1} yL_t = 0 \quad (4.5)$$

$$-yH_{t-1} + pH yH_t - \sigma (iH_{t-1} - m_{t-1} - pH piH_t - piL_t (1 - pH)) + yL_t (1 - pH) = 0 \quad (4.6)$$

$$-yL_{t-1} + pL yL_t - \sigma (iL_{t-1} - m_{t-1} - pL piL_t - piH_t (1 - pL)) + yH_t (1 - pL) = 0 \quad (4.7)$$

$$UH_t + 0.5 (-piCB + piH_t)^2 - \beta (pHE_t [UH_{t+1}] + (1 - pH) E_t [UL_{t+1}]) + 0.5 \kappa \theta^{-1} yH_t^2 = 0 \quad (4.8)$$

$$UL_t + 0.5 (-piCB + piL_t)^2 - \beta (pLE_t [UL_{t+1}] + (1 - pL) E_t [UH_{t+1}]) + 0.5 \kappa \theta^{-1} yL_t^2 = 0 \quad (4.9)$$

$$piCB - piH_t + \beta pH\lambda_t^{\text{HIGHREGIME}^1} - \beta pHE_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^1} \right] + pH\sigma\lambda_t^{\text{HIGHREGIME}^2} = 0 \quad (4.10)$$

$$piCB - piL_t + \beta pL\lambda_t^{\text{LOWREGIME}^1} - \beta pLE_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^1} \right] + pL\sigma\lambda_t^{\text{LOWREGIME}^2} = 0 \quad (4.11)$$

$$-\beta pH\sigma E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^2} \right] = 0 \quad (4.12)$$

$$-\beta pL\sigma E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^2} \right] = 0 \quad (4.13)$$

## 5 Steady state relationships (after reduction)

$$-m_{ss} + e^{\phi \log m_{ss}} = 0 \quad (5.1)$$

$$-piH_{ss} + \beta (pH piH_{ss} + piL_{ss} (1 - pH)) + \kappa yH_{ss} = 0 \quad (5.2)$$

$$-piL_{ss} + \beta (pL piL_{ss} + piH_{ss} (1 - pL)) + \kappa yL_{ss} = 0 \quad (5.3)$$

$$pH \lambda_{ss}^{\text{HIGHREGIME}^2} + \beta pH \left( -\lambda_{ss}^{\text{HIGHREGIME}^2} + \kappa \lambda_{ss}^{\text{HIGHREGIME}^1} \right) - \kappa \theta^{-1} yH_{ss} = 0 \quad (5.4)$$

$$pL \lambda_{ss}^{\text{LOWREGIME}^2} + \beta pL \left( -\lambda_{ss}^{\text{LOWREGIME}^2} + \kappa \lambda_{ss}^{\text{LOWREGIME}^1} \right) - \kappa \theta^{-1} yL_{ss} = 0 \quad (5.5)$$

$$-yH_{ss} + pH yH_{ss} - \sigma (iH_{ss} - m_{ss} - pH piH_{ss} - piL_{ss} (1 - pH)) + yL_{ss} (1 - pH) = 0 \quad (5.6)$$

$$-yL_{ss} + pL yL_{ss} - \sigma (iL_{ss} - m_{ss} - pL piL_{ss} - piH_{ss} (1 - pL)) + yH_{ss} (1 - pL) = 0 \quad (5.7)$$

$$UH_{ss} + 0.5 (-piCB + piH_{ss})^2 - \beta (pHUH_{ss} + UL_{ss} (1 - pH)) + 0.5 \kappa \theta^{-1} yH_{ss}^2 = 0 \quad (5.8)$$

$$UL_{ss} + 0.5 (-piCB + piL_{ss})^2 - \beta (pLUL_{ss} + UH_{ss} (1 - pL)) + 0.5 \kappa \theta^{-1} yL_{ss}^2 = 0 \quad (5.9)$$

$$piCB - piH_{ss} + pH \sigma \lambda_{ss}^{\text{HIGHREGIME}^2} = 0 \quad (5.10)$$

$$piCB - piL_{ss} + pL \sigma \lambda_{ss}^{\text{LOWREGIME}^2} = 0 \quad (5.11)$$

$$-\beta pH \sigma \lambda_{ss}^{\text{HIGHREGIME}^2} = 0 \quad (5.12)$$

$$-\beta pL \sigma \lambda_{ss}^{\text{LOWREGIME}^2} = 0 \quad (5.13)$$

## 6 Parameter settings

$$\beta = 0.99 \quad (6.1)$$

$$\kappa = 0.2465 \quad (6.2)$$

$$\phi = 0.95 \quad (6.3)$$

$$piCB = 0 \quad (6.4)$$

$$pH = 0.99 \quad (6.5)$$

$$pL = 0.99 \quad (6.6)$$

$$\sigma = 1 \quad (6.7)$$

$$\theta = 6 \quad (6.8)$$

## 7 Steady-state values

	Steady-state value
$iH$	1
$iL$	1
$\lambda^{\text{HIGHREGIME}^1}$	0
$\lambda^{\text{HIGHREGIME}^2}$	0
$\lambda^{\text{LOWREGIME}^1}$	0
$\lambda^{\text{LOWREGIME}^2}$	0
$pH$	0
$pL$	0
$m$	1
$yH$	0
$yL$	0
$UH$	0
$UL$	0

## 8 The solution of the 1st order perturbation

Matrix  $P$

$$\begin{matrix}
 & iH_{t-1} & iL_{t-1} & pH_{t-1} & pL_{t-1} & m_{t-1} & yH_{t-1} & yL_{t-1} \\
 \begin{matrix} iH_t \\ iL_t \\ pH_t \\ pL_t \\ m_t \\ yH_t \\ yL_t \end{matrix} & \begin{pmatrix} -1.9645 & 0.0223 & 5.8945 & -0.0721 & 2.8922 & -3.4174 & 0.0401 \\ 0.0223 & -1.9645 & -0.0721 & 5.8945 & 2.8922 & 0.0401 & -3.4174 \\ 0 & 0 & 1.0204 & -0.0103 & 0 & -0.2515 & 0.0025 \\ 0 & 0 & -0.0103 & 1.0204 & 0 & 0.0025 & -0.2515 \\ 0 & 0 & 0 & 0 & 0.95 & 0 & 0 \\ 1.0102 & -0.0102 & -1.0204 & 0.0103 & -1 & 1.2617 & -0.0127 \\ -0.0102 & 1.0102 & 0.0103 & -1.0204 & -1 & -0.0127 & 1.2617 \end{pmatrix}
 \end{matrix}$$

Matrix  $Q$

$$\begin{matrix}
 & \epsilon^Z \\
 \begin{matrix} iH \\ iL \\ pH \\ pL \\ m \\ yH \\ yL \end{matrix} & \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}
 \end{matrix}$$

Matrix  $R$

$$\begin{matrix}
 & iH_{t-1} & iL_{t-1} & pH_{t-1} & pL_{t-1} & m_{t-1} & yH_{t-1} & yL_{t-1} \\
 \begin{matrix} \lambda_t^{\text{HIGHREGIME}^1} \\ \lambda_t^{\text{HIGHREGIME}^2} \\ \lambda_t^{\text{LOWREGIME}^1} \\ \lambda_t^{\text{LOWREGIME}^2} \\ UH_t \\ UL_t \end{matrix} & \begin{pmatrix} -0.516 & 0.0105 & 3.5035 & -0.0625 & 0.5055 & -1.3796 & 0.0259 \\ 0.1347 & -0.0024 & -0.516 & 0.0105 & -0.1323 & 0.2619 & -0.005 \\ 0.0105 & -0.516 & -0.0625 & 3.5035 & 0.5055 & 0.0259 & -1.3796 \\ -0.0024 & 0.1347 & 0.0105 & -0.516 & -0.1323 & -0.005 & 0.2619 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}
 \end{matrix}$$

## Matrix $S$

$$\begin{matrix} \lambda^{\text{HIGHREGIME}^1} \\ \lambda^{\text{HIGHREGIME}^2} \\ \lambda^{\text{LOWREGIME}^1} \\ \lambda^{\text{LOWREGIME}^2} \\ UH \\ UL \end{matrix} \epsilon^Z \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

## 9 Model statistics

### 9.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
$iH$	1	0.1303	0.017	Y
$iL$	1	0.1303	0.017	Y
$\lambda^{\text{HIGHREGIME}^1}$	0	0	0	N
$\lambda^{\text{HIGHREGIME}^2}$	0	0	0	N
$\lambda^{\text{LOWREGIME}^1}$	0	0	0	N
$\lambda^{\text{LOWREGIME}^2}$	0	0	0	N
$pH$	0	0	0	N
$pL$	0	0	0	N
$m$	1	0.1303	0.017	Y
$yH$	0	0	0	N
$yL$	0	0	0	N
$UH$	0	0	0	N
$UL$	0	0	0	N

### 9.2 Correlation matrix

	$iH$	$iL$	$m$
$iH$	1	1	1
$iL$		1	1
$m$			1

### 9.3 Cross correlations with the reference variable ( $iH$ )

	$\sigma[\cdot]$ rel. to $\sigma[iH]$	$iH_{t-5}$	$iH_{t-4}$	$iH_{t-3}$	$iH_{t-2}$	$iH_{t-1}$	$iH_t$	$iH_{t+1}$	$iH_{t+2}$	$iH_{t+3}$	$iH_{t+4}$	$iH_{t+5}$
$iH_t$	1	-0.016	0.11	0.271	0.471	0.713	1	0.713	0.471	0.271	0.11	-0.016
$iL_t$	1	-0.016	0.11	0.271	0.471	0.713	1	0.713	0.471	0.271	0.11	-0.016
$m_t$	1	-0.016	0.11	0.271	0.471	0.713	1	0.713	0.471	0.271	0.11	-0.016

### 9.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
$iH$	0.713	0.471	0.271	0.11	-0.016
$iL$	0.713	0.471	0.271	0.11	-0.016
$m$	0.713	0.471	0.271	0.11	-0.016

## 10 Impulse response functions

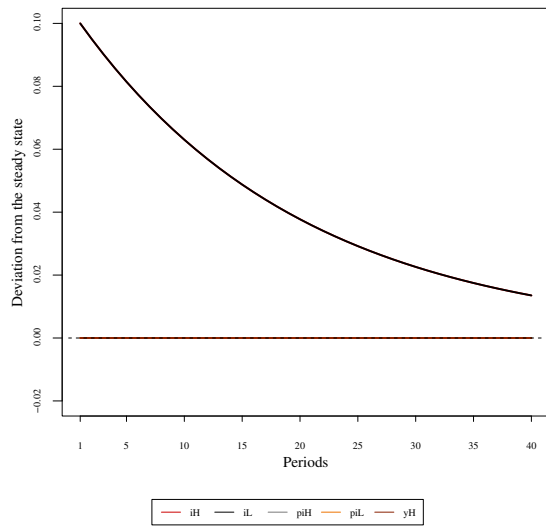


Figure 1: Impulse responses ( $iH, iL, pIH, pIL, yH$ ) to  $\epsilon^Z$  shock

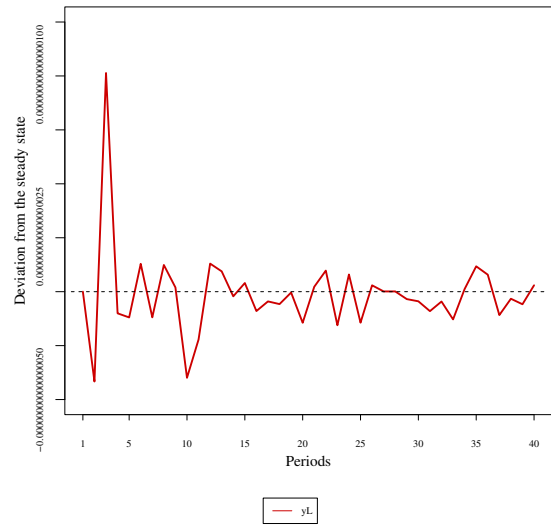


Figure 2: Impulse response ( $yL$ ) to  $\epsilon^Z$  shock