

## 1 OPTIMALMP

### 1.1 Optimisation problem

$$\max_{\pi H_t, y H_t, \pi L_t, y L_t} U_t = -0.25 (\pi H_t - \pi CB + \pi H_t)^2 - 0.25 (-\pi CB + \pi L_t + \pi L_t)^2 + \beta E_t [U_{t+1}] - 0.25 \lambda y H_t^2 - 0.25 \lambda y L_t^2 \quad (1.1)$$

s.t. :

$$\pi H_{t-1} = \log \epsilon \pi i_{t-1} + \beta \pi H_t + \kappa y H_{t-1} + \beta (1 - pH) (-\pi H_t + \pi L_t) \quad \left( \lambda_t^{\text{OPTIMALMP}^1} \right) \quad (1.2)$$

$$\pi L_{t-1} = \log \epsilon \pi i_{t-1} + \kappa y L_{t-1} + \beta \pi L_t + \beta (1 - pL) (\pi H_t - \pi L_t) \quad \left( \lambda_t^{\text{OPTIMALMP}^2} \right) \quad (1.3)$$

### 1.2 First order conditions

$$-0.5 \pi H_t + 0.5 \pi CB - 0.5 \pi H_t - \beta E_t \left[ \lambda_{t+1}^{\text{OPTIMALMP}^1} \right] + \lambda_t^{\text{OPTIMALMP}^1} (\beta - \beta (1 - pH)) + \beta \lambda_t^{\text{OPTIMALMP}^2} (1 - pL) = 0 \quad (\pi H_t) \quad (1.4)$$

$$-0.5 \lambda y H_t + \beta \kappa E_t \left[ \lambda_{t+1}^{\text{OPTIMALMP}^1} \right] = 0 \quad (y H_t) \quad (1.5)$$

$$0.5 \pi CB - 0.5 \pi L_t - 0.5 \pi L_t - \beta E_t \left[ \lambda_{t+1}^{\text{OPTIMALMP}^2} \right] + \lambda_t^{\text{OPTIMALMP}^2} (\beta pL - \beta (1 - pL)) + \beta \lambda_t^{\text{OPTIMALMP}^1} (1 - pH) = 0 \quad (\pi L_t) \quad (1.6)$$

$$-0.5 \lambda y L_t + \beta \kappa E_t \left[ \lambda_{t+1}^{\text{OPTIMALMP}^2} \right] = 0 \quad (y L_t) \quad (1.7)$$

## 2 EXOG

### 2.1 Identities

$$\epsilon \pi i_t = e^{\epsilon_t^\pi + \phi \log \epsilon \pi i_{t-1}} \quad (2.1)$$

## 3 Equilibrium relationships (after reduction)

$$-\epsilon \pi i_t + e^{\epsilon_t^\pi + \phi \log \epsilon \pi i_{t-1}} = 0 \quad (3.1)$$

$$-0.5 \lambda y H_t + \beta \kappa E_t \left[ \lambda_{t+1}^{\text{OPTIMALMP}^1} \right] = 0 \quad (3.2)$$

$$-0.5 \lambda y L_t + \beta \kappa E_t \left[ \lambda_{t+1}^{\text{OPTIMALMP}^2} \right] = 0 \quad (3.3)$$

$$-pH_{t-1} + \log \text{etapi}_{t-1} + \beta piH_t + \kappa yH_{t-1} + \beta (1 - pH) (-piH_t + piL_t) = 0 \quad (3.4)$$

$$-piL_{t-1} + \log \text{etapi}_{t-1} + \kappa yL_{t-1} + \beta pLpiL_t + \beta (1 - pL) (piH_t - piL_t) = 0 \quad (3.5)$$

$$-0.5piH + 0.5piCB - 0.5piH_t - \beta E_t \left[ \lambda_{t+1}^{\text{OPTIMALMP}^1} \right] + \lambda_t^{\text{OPTIMALMP}^1} (\beta - \beta (1 - pH)) + \beta \lambda_t^{\text{OPTIMALMP}^2} (1 - pL) = 0 \quad (3.6)$$

$$0.5piCB - 0.5piL - 0.5piL_t - \beta E_t \left[ \lambda_{t+1}^{\text{OPTIMALMP}^2} \right] + \lambda_t^{\text{OPTIMALMP}^2} (\beta pL - \beta (1 - pL)) + \beta \lambda_t^{\text{OPTIMALMP}^1} (1 - pH) = 0 \quad (3.7)$$

$$U_t + 0.25 (piH - piCB + piH_t)^2 + 0.25 (-piCB + piL + piL_t)^2 - \beta E_t [U_{t+1}] + 0.25 \lambda yH_t^2 + 0.25 \lambda yL_t^2 = 0 \quad (3.8)$$

## 4 Steady state relationships (after reduction)

$$-\text{etapi}_{ss} + e^{\phi \log \text{etapi}_{ss}} = 0 \quad (4.1)$$

$$-0.5 \lambda yH_{ss} + \beta \kappa \lambda_{ss}^{\text{OPTIMALMP}^1} = 0 \quad (4.2)$$

$$-0.5 \lambda yL_{ss} + \beta \kappa \lambda_{ss}^{\text{OPTIMALMP}^2} = 0 \quad (4.3)$$

$$-piH_{ss} + \log \text{etapi}_{ss} + \beta piH_{ss} + \kappa yH_{ss} + \beta (1 - pH) (-piH_{ss} + piL_{ss}) = 0 \quad (4.4)$$

$$-piL_{ss} + \log \text{etapi}_{ss} + \kappa yL_{ss} + \beta pLpiL_{ss} + \beta (1 - pL) (piH_{ss} - piL_{ss}) = 0 \quad (4.5)$$

$$-0.5piH + 0.5piCB - 0.5piH_{ss} - \beta \lambda_{ss}^{\text{OPTIMALMP}^1} + \lambda_{ss}^{\text{OPTIMALMP}^1} (\beta - \beta (1 - pH)) + \beta \lambda_{ss}^{\text{OPTIMALMP}^2} (1 - pL) = 0 \quad (4.6)$$

$$0.5piCB - 0.5piL - 0.5piL_{ss} - \beta \lambda_{ss}^{\text{OPTIMALMP}^2} + \lambda_{ss}^{\text{OPTIMALMP}^2} (\beta pL - \beta (1 - pL)) + \beta \lambda_{ss}^{\text{OPTIMALMP}^1} (1 - pH) = 0 \quad (4.7)$$

$$U_{ss} + 0.25 (piH - piCB + piH_{ss})^2 + 0.25 (-piCB + piL + piL_{ss})^2 - \beta U_{ss} + 0.25 \lambda yH_{ss}^2 + 0.25 \lambda yL_{ss}^2 = 0 \quad (4.8)$$

## 5 Parameter settings

$$\beta = 0.99 \quad (5.1)$$

$$\kappa = 0.2465 \quad (5.2)$$

$$\lambda = 0.04106 \quad (5.3)$$

$$\phi = 0.95 \quad (5.4)$$

$$piH = 2 \quad (5.5)$$

$$piCB = 2 \quad (5.6)$$

$$piL = 4 \quad (5.7)$$

$$pH = 0.99 \quad (5.8)$$

$$pL = 0.99 \quad (5.9)$$

$$\sigma = 1 \quad (5.10)$$

$$\theta = 6 \quad (5.11)$$

## 6 Steady-state values

	Steady-state value
$\epsilon \dot{a} p i$	1
$\lambda^{\text{OPTIMALMP}^1}$	0.0068
$\lambda^{\text{OPTIMALMP}^2}$	-0.0203
$pH$	-5e-04
$pL$	-1.9991
$yH$	0.0802
$yL$	-0.2416
$U$	-0.0666

## 7 The solution of the 1st order perturbation

Matrix  $P$

$$\begin{matrix} & \epsilon \dot{a} p i_{t-1} & pH_{t-1} & pL_{t-1} & yH_{t-1} & yL_{t-1} \\ \begin{matrix} \epsilon \dot{a} p i_t \\ pH_t \\ pL_t \\ yH_t \\ yL_t \end{matrix} & \begin{pmatrix} 0.95 & 0 & 0 & 0 & 0 \\ -1883.6741 & 1.0204 & -38.8204 & -37.6433 & 1.1567 \\ -0.5104 & 0 & 1.0308 & 1e-04 & -0.0307 \\ -81.9992 & 0.0191 & -1.2256 & -0.7053 & 0.0365 \\ -27.505 & -1e-04 & 24.0757 & 0.004 & -0.7174 \end{pmatrix} \end{matrix}$$

Matrix  $Q$

$$\begin{matrix} & \epsilon^\pi \\ \begin{matrix} \epsilon \dot{a} p i \\ pH \\ pL \\ yH \\ yL \end{matrix} & \begin{pmatrix} 1 \\ 0 \\ 0 \\ -49.4282 \\ -16.4896 \end{pmatrix} \end{matrix}$$

Matrix  $R$

$$\begin{matrix} & \epsilon \dot{a} p i_{t-1} & pH_{t-1} & pL_{t-1} & yH_{t-1} & yL_{t-1} \\ \begin{matrix} \lambda_t^{\text{OPTIMALMP}^1} \\ \lambda_t^{\text{OPTIMALMP}^2} \\ U_t \end{matrix} & \begin{pmatrix} -157.5291 & 0.0607 & -5.1479 & -2.2382 & 0.1534 \\ -53.4004 & -5e-04 & 76.8235 & 0.0169 & -2.2891 \\ -3.4276 & -1e-04 & 0.6104 & 0.002 & -0.0182 \end{pmatrix} \end{matrix}$$

Matrix  $S$

$$\begin{matrix} & \epsilon^\pi \\ \begin{matrix} \lambda^{\text{OPTIMALMP}^1} \\ \lambda^{\text{OPTIMALMP}^2} \\ U \end{matrix} & \begin{pmatrix} -49.4208 \\ -16.6586 \\ -3.3933 \end{pmatrix} \end{matrix}$$

## 8 Model statistics

### 8.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
$\epsilon \dot{a} p i$	1	0.1303	0.017	Y
$\lambda^{\text{OPTIMALMP}^1}$	0.0068	6.7372	45.3905	Y
$\lambda^{\text{OPTIMALMP}^2}$	-0.0203	2.2501	5.0628	Y
$pH$	-5e-04	7.2381	52.3895	Y
$pL$	-1.9991	0.0016	0	Y
$yH$	0.0802	6.5347	42.7018	Y
$yL$	-0.2416	2.1738	4.7255	Y
$U$	-0.0666	0.4427	0.196	Y

## 8.2 Correlation matrix

	$etpi$	$\lambda^{\text{OPTIMALMP}^1}$	$\lambda^{\text{OPTIMALMP}^2}$	$piH$	$piL$	$yH$	$yL$	$U$
$etpi$	1	-0.999	-0.999	-0.677	-0.677	-1	-1	-1
$\lambda^{\text{OPTIMALMP}^1}$		1	1	0.71	0.71	1	0.999	0.999
$\lambda^{\text{OPTIMALMP}^2}$			1	0.704	0.704	1	1	0.999
$piH$				1	1	0.688	0.686	0.677
$piL$					1	0.688	0.686	0.677
$yH$						1	1	1
$yL$							1	1
$U$								1

## 8.3 Cross correlations with the reference variable ( $piH$ )

	$\sigma[\cdot]$ rel. to $\sigma[piH]$	$piH_{t-5}$	$piH_{t-4}$	$piH_{t-3}$	$piH_{t-2}$	$piH_{t-1}$	$piH_t$	$piH_{t+1}$	$piH_{t+2}$	$piH_{t+3}$	$piH_{t+4}$
$etpi_t$	0.018	-0.211	-0.404	-0.626	-0.849	-0.972	-0.677	-0.429	-0.226	-0.064	0.000
$\lambda_t^{\text{OPTIMALMP}^1}$	0.931	0.202	0.396	0.621	0.849	0.982	0.71	0.462	0.253	0.084	-0.000
$\lambda_t^{\text{OPTIMALMP}^2}$	0.311	0.204	0.398	0.622	0.849	0.98	0.704	0.456	0.248	0.08	-0.000
$piH_t$	1	0.005	0.162	0.357	0.588	0.83	1	0.83	0.588	0.357	0.162
$piL_t$	0	0.005	0.162	0.357	0.588	0.83	1	0.829	0.587	0.357	0.162
$yH_t$	0.903	0.208	0.401	0.624	0.849	0.976	0.688	0.44	0.235	0.07	-0.000
$yL_t$	0.3	0.209	0.402	0.625	0.849	0.975	0.686	0.438	0.233	0.069	-0.000
$U_t$	0.061	0.211	0.404	0.626	0.849	0.973	0.677	0.429	0.226	0.064	-0.000

## 8.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
$etpi$	0.713	0.471	0.271	0.11	-0.016
$\lambda^{\text{OPTIMALMP}^1}$	0.74	0.498	0.291	0.122	-0.011
$\lambda^{\text{OPTIMALMP}^2}$	0.735	0.493	0.287	0.119	-0.012
$piH$	0.83	0.588	0.357	0.162	0.005
$piL$	0.829	0.587	0	0	0
$yH$	0.722	0.48	0.278	0.114	-0.015
$yL$	0.72	0.478	0.276	0.113	-0.015
$U$	0.714	0.472	0.271	0.11	-0.016

## 9 Impulse response functions

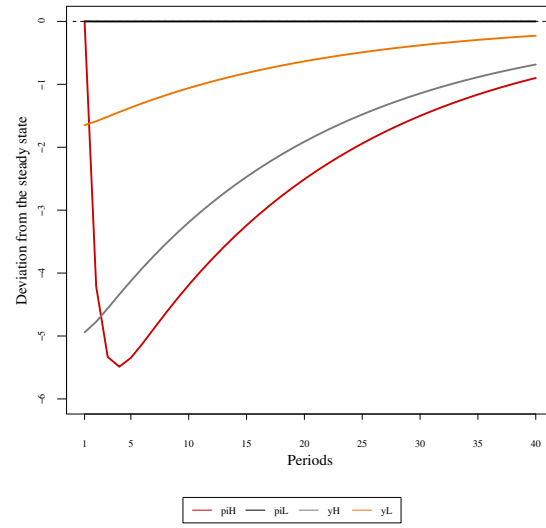


Figure 1: Impulse responses  $(\pi^H, \pi^L, y^H, y^L)$  to  $\epsilon^\pi$  shock