

1 HIGHREGIME

1.1 Optimisation problem

$$\max_{\pi H_t, yH_t, iH_t} UH_t = -0.5 (\pi iH - \pi iCB + \pi iH_t)^2 + \beta (pHE_t [UH_{t+1}] + (1 - pH) E_t [UL_{t+1}]) - 0.5\kappa\theta^{-1}yH_t^2 \quad (1.1)$$

s.t. :

$$\pi H_{t-1} = \log etapi_{t-1} + \beta (pH\pi H_t + \pi L_t (1 - pH)) + \kappa yH_{t-1} \quad \left(\lambda_t^{\text{HIGHREGIME}^1} \right) \quad (1.2)$$

$$yH_{t-1} = pHyH_t + yL_t (1 - pH) - \sigma (iH_{t-1} - pH\pi H_t - \pi L_t (1 - pH)) \quad \left(\lambda_t^{\text{HIGHREGIME}^2} \right) \quad (1.3)$$

1.2 First order conditions

$$-\pi iH + \pi iCB - \pi iH_t + \beta pH\lambda_t^{\text{HIGHREGIME}^1} - \beta pHE_t \left[\lambda_{t+1}^{\text{HIGHREGIME}^1} \right] + pH\sigma\lambda_t^{\text{HIGHREGIME}^2} = 0 \quad (\pi H_t) \quad (1.4)$$

$$pH\lambda_t^{\text{HIGHREGIME}^2} + \beta pH \left(\kappa E_t \left[\lambda_{t+1}^{\text{HIGHREGIME}^1} \right] - E_t \left[\lambda_{t+1}^{\text{HIGHREGIME}^2} \right] \right) - \kappa\theta^{-1}yH_t = 0 \quad (yH_t) \quad (1.5)$$

$$-\beta pH\sigma E_t \left[\lambda_{t+1}^{\text{HIGHREGIME}^2} \right] = 0 \quad (iH_t) \quad (1.6)$$

2 LOWREGIME

2.1 Optimisation problem

$$\max_{\pi L_t, yL_t, iL_t} UL_t = -0.5 (-\pi iCB + \pi iL + \pi iL_t)^2 + \beta (pLE_t [UL_{t+1}] + (1 - pL) E_t [UH_{t+1}]) - 0.5\kappa\theta^{-1}yL_t^2 \quad (2.1)$$

s.t. :

$$\pi L_{t-1} = \log etapi_{t-1} + \beta (pL\pi L_t + \pi H_t (1 - pL)) + \kappa yL_{t-1} \quad \left(\lambda_t^{\text{LOWREGIME}^1} \right) \quad (2.2)$$

$$yL_{t-1} = pLyL_t + yH_t (1 - pL) - \sigma (iL_{t-1} - pL\pi L_t - \pi H_t (1 - pL)) \quad \left(\lambda_t^{\text{LOWREGIME}^2} \right) \quad (2.3)$$

2.2 First order conditions

$$\pi iCB - \pi iL - \pi iL_t + \beta pL\lambda_t^{\text{LOWREGIME}^1} - \beta pLE_t \left[\lambda_{t+1}^{\text{LOWREGIME}^1} \right] + pL\sigma\lambda_t^{\text{LOWREGIME}^2} = 0 \quad (\pi L_t) \quad (2.4)$$

$$pL\lambda_t^{\text{LOWREGIME}^2} + \beta pL \left(\kappa E_t \left[\lambda_{t+1}^{\text{LOWREGIME}^1} \right] - E_t \left[\lambda_{t+1}^{\text{LOWREGIME}^2} \right] \right) - \kappa \theta^{-1} yL_t = 0 \quad (yL_t) \quad (2.5)$$

$$-\beta pL\sigma E_t \left[\lambda_{t+1}^{\text{LOWREGIME}^2} \right] = 0 \quad (iL_t) \quad (2.6)$$

3 EXOG

3.1 Identities

$$etapi_t = e^{\epsilon_t^\pi + \phi \log etapi_{t-1}} \quad (3.1)$$

4 Equilibrium relationships (after reduction)

$$-etapi_t + e^{\epsilon_t^\pi + \phi \log etapi_{t-1}} = 0 \quad (4.1)$$

$$pH\lambda_t^{\text{HIGHREGIME}^2} + \beta pH \left(\kappa E_t \left[\lambda_{t+1}^{\text{HIGHREGIME}^1} \right] - E_t \left[\lambda_{t+1}^{\text{HIGHREGIME}^2} \right] \right) - \kappa \theta^{-1} yH_t = 0 \quad (4.2)$$

$$pL\lambda_t^{\text{LOWREGIME}^2} + \beta pL \left(\kappa E_t \left[\lambda_{t+1}^{\text{LOWREGIME}^1} \right] - E_t \left[\lambda_{t+1}^{\text{LOWREGIME}^2} \right] \right) - \kappa \theta^{-1} yL_t = 0 \quad (4.3)$$

$$-piH_{t-1} + \log etapi_{t-1} + \beta (pHpiH_t + piL_t(1 - pH)) + \kappa yH_{t-1} = 0 \quad (4.4)$$

$$-piL_{t-1} + \log etapi_{t-1} + \beta (pLpiL_t + piH_t(1 - pL)) + \kappa yL_{t-1} = 0 \quad (4.5)$$

$$-yH_{t-1} + pHyH_t - \sigma (iH_{t-1} - pHpiH_t - piL_t(1 - pH)) + yL_t(1 - pH) = 0 \quad (4.6)$$

$$-yL_{t-1} + pLyL_t - \sigma (iL_{t-1} - pLpiL_t - piH_t(1 - pL)) + yH_t(1 - pL) = 0 \quad (4.7)$$

$$UH_t + 0.5 (piH - piCB + piH_t)^2 - \beta (pHE_t[UH_{t+1}] + (1 - pH) E_t[UL_{t+1}]) + 0.5 \kappa \theta^{-1} yH_t^2 = 0 \quad (4.8)$$

$$UL_t + 0.5 (-piCB + piL + piL_t)^2 - \beta (pLE_t[UL_{t+1}] + (1 - pL) E_t[UH_{t+1}]) + 0.5 \kappa \theta^{-1} yL_t^2 = 0 \quad (4.9)$$

$$-piH + piCB - piH_t + \beta pH\lambda_t^{\text{HIGHREGIME}^1} - \beta pHE_t \left[\lambda_{t+1}^{\text{HIGHREGIME}^1} \right] + pH\sigma\lambda_t^{\text{HIGHREGIME}^2} = 0 \quad (4.10)$$

$$piCB - piL - piL_t + \beta pL\lambda_t^{\text{LOWREGIME}^1} - \beta pLE_t \left[\lambda_{t+1}^{\text{LOWREGIME}^1} \right] + pL\sigma\lambda_t^{\text{LOWREGIME}^2} = 0 \quad (4.11)$$

$$-\beta pH\sigma E_t \left[\lambda_{t+1}^{\text{HIGHREGIME}^2} \right] = 0 \quad (4.12)$$

$$-\beta pL\sigma E_t \left[\lambda_{t+1}^{\text{LOWREGIME}^2} \right] = 0 \quad (4.13)$$

5 Steady state relationships (after reduction)

$$-d\pi i_{ss} + e^{\phi \log d\pi i_{ss}} = 0 \quad (5.1)$$

$$pH \lambda_{ss}^{\text{HIGHREGIME}^2} + \beta pH \left(-\lambda_{ss}^{\text{HIGHREGIME}^2} + \kappa \lambda_{ss}^{\text{HIGHREGIME}^1} \right) - \kappa \theta^{-1} yH_{ss} = 0 \quad (5.2)$$

$$pL \lambda_{ss}^{\text{LOWREGIME}^2} + \beta pL \left(-\lambda_{ss}^{\text{LOWREGIME}^2} + \kappa \lambda_{ss}^{\text{LOWREGIME}^1} \right) - \kappa \theta^{-1} yL_{ss} = 0 \quad (5.3)$$

$$-piH_{ss} + \log d\pi i_{ss} + \beta (pH piH_{ss} + piL_{ss} (1 - pH)) + \kappa yH_{ss} = 0 \quad (5.4)$$

$$-piL_{ss} + \log d\pi i_{ss} + \beta (pL piL_{ss} + piH_{ss} (1 - pL)) + \kappa yL_{ss} = 0 \quad (5.5)$$

$$-yH_{ss} + pH yH_{ss} - \sigma (iH_{ss} - pH piH_{ss} - piL_{ss} (1 - pH)) + yL_{ss} (1 - pH) = 0 \quad (5.6)$$

$$-yL_{ss} + pL yL_{ss} - \sigma (iL_{ss} - pL piL_{ss} - piH_{ss} (1 - pL)) + yH_{ss} (1 - pL) = 0 \quad (5.7)$$

$$UH_{ss} + 0.5 (piH - piCB + piH_{ss})^2 - \beta (pH UH_{ss} + UL_{ss} (1 - pH)) + 0.5 \kappa \theta^{-1} yH_{ss}^2 = 0 \quad (5.8)$$

$$UL_{ss} + 0.5 (-piCB + piL + piL_{ss})^2 - \beta (pL UL_{ss} + UH_{ss} (1 - pL)) + 0.5 \kappa \theta^{-1} yL_{ss}^2 = 0 \quad (5.9)$$

$$-piH + piCB - piH_{ss} + pH \sigma \lambda_{ss}^{\text{HIGHREGIME}^2} = 0 \quad (5.10)$$

$$piCB - piL - piL_{ss} + pL \sigma \lambda_{ss}^{\text{LOWREGIME}^2} = 0 \quad (5.11)$$

$$-\beta pH \sigma \lambda_{ss}^{\text{HIGHREGIME}^2} = 0 \quad (5.12)$$

$$-\beta pL \sigma \lambda_{ss}^{\text{LOWREGIME}^2} = 0 \quad (5.13)$$

6 Parameter settings

$$\beta = 0.99 \quad (6.1)$$

$$\kappa = 0.2465 \quad (6.2)$$

$$\phi = 0.95 \quad (6.3)$$

$$piH = 2 \quad (6.4)$$

$$piCB = 0 \quad (6.5)$$

$$piL = -2 \quad (6.6)$$

$$pH = 0.99 \quad (6.7)$$

$$pL = 0.99 \quad (6.8)$$

$$\sigma = 1 \quad (6.9)$$

$$\theta = 6 \quad (6.10)$$

7 Steady-state values

	Steady-state value
$et\pi$	1
iH	-1.9552
iL	1.9552
$\lambda^{\text{HIGHREGIME}^1}$	-0.0411
$\lambda^{\text{HIGHREGIME}^2}$	0
$\lambda^{\text{LOWREGIME}^1}$	0.0411
$\lambda^{\text{LOWREGIME}^2}$	0
πH	-2
πL	2
yH	-0.2418
yL	0.2418
UH	-0.1201
UL	-0.1201

8 The solution of the 1st order perturbation

Matrix P

$$\begin{matrix}
 & et\pi_{t-1} & iH_{t-1} & iL_{t-1} & \pi H_{t-1} & \pi L_{t-1} & yH_{t-1} & yL_{t-1} \\
 \begin{matrix} et\pi_t \\ iH_t \\ iL_t \\ \pi H_t \\ \pi L_t \\ yH_t \\ yL_t \end{matrix} & \begin{pmatrix} 0.95 & 0 & 0 & 0 & 0 & 0 & 0 \\ -6.6659 & -1.9645 & 0.0223 & 6.0298 & -0.0737 & -0.4226 & 0.005 \\ -6.6659 & 0.0223 & -1.9645 & -0.0737 & 6.0298 & 0.005 & -0.4226 \\ -0.5051 & 0 & 0 & 1.0204 & -0.0103 & -0.0304 & 0.0003 \\ -0.5051 & 0 & 0 & -0.0103 & 1.0204 & 0.0003 & -0.0304 \\ 4.1777 & 8.1689 & -0.0825 & -8.4406 & 0.0853 & 1.2617 & -0.0127 \\ 4.1777 & -0.0825 & 8.1689 & 0.0853 & -8.4406 & -0.0127 & 1.2617 \end{pmatrix}
 \end{matrix}$$

Matrix Q

$$\begin{matrix}
 & \epsilon^\pi \\
 \begin{matrix} et\pi \\ iH \\ iL \\ \pi H \\ \pi L \\ yH \\ yL \end{matrix} & \begin{pmatrix} 1 \\ -3.882 \\ -3.882 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
 \end{matrix}$$

Matrix R

$$\begin{matrix}
 & et\pi_{t-1} & iH_{t-1} & iL_{t-1} & \pi H_{t-1} & \pi L_{t-1} & yH_{t-1} & yL_{t-1} \\
 \begin{matrix} \lambda_t^{\text{HIGHREGIME}^1} \\ \lambda_t^{\text{HIGHREGIME}^2} \\ \lambda_t^{\text{LOWREGIME}^1} \\ \lambda_t^{\text{LOWREGIME}^2} \\ UH_t \\ UL_t \end{matrix} & \begin{pmatrix} -145.5977 & -24.5389 & 0.5003 & 170.4191 & -3.0404 & -8.1131 & 0.1525 \\ 1.0038 & 0.2633 & -0.0047 & -1.0321 & 0.021 & 0.0633 & -0.0012 \\ -145.5977 & 0.5003 & -24.5389 & -3.0404 & 170.4191 & 0.1525 & -8.1131 \\ 1.0038 & -0.0047 & 0.2633 & 0.021 & -1.0321 & -0.0012 & 0.0633 \\ -4.386 & -0.0001 & 0.0048 & 0.6855 & -0.032 & -0.0204 & 0.0015 \\ 4.386 & -0.0048 & 0.0001 & 0.032 & -0.6855 & -0.0015 & 0.0204 \end{pmatrix}
 \end{matrix}$$

Matrix S

$$\begin{matrix} & \epsilon^\pi \\ \lambda^{\text{HIGHREGIME}^1} & -65.1667 \\ \lambda^{\text{HIGHREGIME}^2} & 0.5246 \\ \lambda^{\text{LOWREGIME}^1} & -65.1667 \\ \lambda^{\text{LOWREGIME}^2} & 0.5246 \\ UH & -4.2729 \\ UL & 4.2729 \end{matrix}$$

9 Model statistics

9.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
$\epsilon\pi$	1	0.1303	0.017	Y
iH	-1.9552	0.3948	0.1559	Y
iL	1.9552	0.3948	0.1559	Y
$\lambda^{\text{HIGHREGIME}^1}$	-0.0411	7.2017	51.8645	Y
$\lambda^{\text{HIGHREGIME}^2}$	0	0.0506	0.0026	N
$\lambda^{\text{LOWREGIME}^1}$	0.0411	7.2017	51.8645	Y
$\lambda^{\text{LOWREGIME}^2}$	0	0.0506	0.0026	N
pH	-2	0.0492	0.0024	Y
pL	2	0.0492	0.0024	Y
yH	-0.2418	2.8648	8.2069	Y
yL	0.2418	2.8648	8.2069	Y
UH	-0.1201	0.5746	0.3302	Y
UL	-0.1201	0.5746	0.3302	Y

9.2 Correlation matrix

	$\epsilon\pi$	iH	iL	$\lambda^{\text{HIGHREGIME}^1}$	$\lambda^{\text{HIGHREGIME}^2}$	$\lambda^{\text{LOWREGIME}^1}$	$\lambda^{\text{LOWREGIME}^2}$	pH	pL
$\epsilon\pi$	1	-0.301	-0.301	-0.825	0.436	-0.825	0.436	-0.491	-0.491
iH		1	1	0.567	-0.977	0.567	-0.977	-0.323	-0.323
iL			1	0.567	-0.977	0.567	-0.977	-0.323	-0.323
$\lambda^{\text{HIGHREGIME}^1}$				1	-0.725	1	-0.725	0.567	0.567
$\lambda^{\text{HIGHREGIME}^2}$					1	-0.725	1	0.116	0.116
$\lambda^{\text{LOWREGIME}^1}$						1	-0.725	0.567	0.567
$\lambda^{\text{LOWREGIME}^2}$							1	0.116	0.116
pH								1	1
pL									1
yH									
yL									
UH									
UL									

9.3 Cross correlations with the reference variable (iH)

	$\sigma[\cdot]$ rel. to $\sigma[iH]$	iH_{t-5}	iH_{t-4}	iH_{t-3}	iH_{t-2}	iH_{t-1}	iH_t	iH_{t+1}	iH_{t+2}	iH_{t+3}	iH_{t+4}
$\epsilon\pi i_t$	0.33	0.098	0.131	0.177	0.258	0.444	-0.301	-0.254	-0.21	-0.169	-0.132
iH_t	1	-0.028	-0.035	-0.051	-0.093	-0.219	1	-0.219	-0.093	-0.051	-0.035
iL_t	1	-0.028	-0.035	-0.051	-0.093	-0.219	1	-0.219	-0.093	-0.051	-0.035
$\lambda_t^{\text{HIGHREGIME}^1}$	18.242	-0.078	-0.1	-0.134	-0.204	-0.385	0.567	0.47	0.151	0.043	0.003
$\lambda_t^{\text{HIGHREGIME}^2}$	0.128	0.04	0.051	0.071	0.122	0.269	-0.977	0.043	0.043	0.041	0.038
$\lambda_t^{\text{LOWREGIME}^1}$	18.242	-0.078	-0.1	-0.134	-0.204	-0.385	0.567	0.47	0.151	0.043	0.003
$\lambda_t^{\text{LOWREGIME}^2}$	0.128	0.04	0.051	0.071	0.122	0.269	-0.977	0.043	0.043	0.041	0.038
πH_t	0.125	-0.046	-0.056	-0.071	-0.096	-0.156	-0.323	0.852	0.236	0.04	-0.022
πL_t	0.125	-0.046	-0.056	-0.071	-0.096	-0.156	-0.323	0.852	0.236	0.04	-0.022
yH_t	7.256	-0.069	-0.092	-0.121	-0.164	-0.245	-0.443	0.551	0.273	0.164	0.111
yL_t	7.256	-0.069	-0.092	-0.121	-0.164	-0.245	-0.443	0.551	0.273	0.164	0.111
UH_t	1.455	-0.098	-0.13	-0.176	-0.256	-0.44	0.272	0.295	0.217	0.166	0.127
UL_t	1.455	0.098	0.13	0.176	0.256	0.44	-0.272	-0.295	-0.217	-0.166	-0.127

9.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
$\epsilon\pi i$	0.713	0.471	0.271	0.11	-0.016
iH	-0.219	-0.093	-0.051	-0.035	-0.028
iL	-0.219	-0.093	-0.051	-0.035	-0.028
$\lambda^{\text{HIGHREGIME}^1}$	0.51	0.081	-0.065	-0.117	-0.134
$\lambda^{\text{HIGHREGIME}^2}$	-0.074	-0.071	-0.066	-0.06	-0.054
$\lambda^{\text{LOWREGIME}^1}$	0.51	0.081	-0.065	-0.117	-0.134
$\lambda^{\text{LOWREGIME}^2}$	-0.074	-0.071	-0.066	-0.06	-0.054
πH	0.22	-0.024	-0.095	-0.11	-0.107
πL	0.22	-0.024	-0.095	-0.11	-0.107
yH	0.504	0.261	0.116	0.017	-0.055
yL	0.504	0.261	0.116	0.017	-0.055
UH	0.729	0.455	0.251	0.094	-0.027
UL	0.729	0.455	0.251	0.094	-0.027

10 Impulse response functions

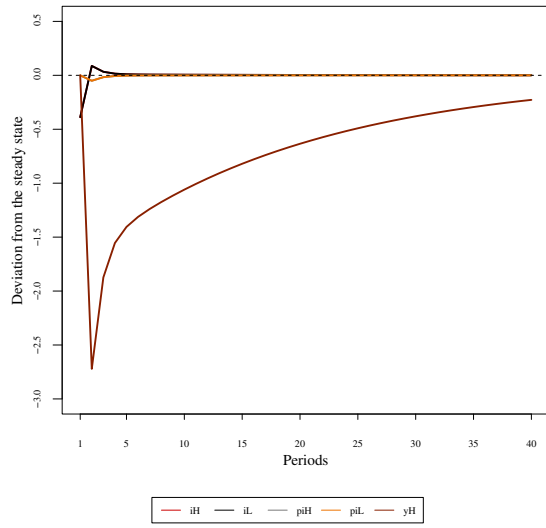


Figure 1: Impulse responses ($iH, iL, \pi H, \pi L, yH$) to ϵ^π shock

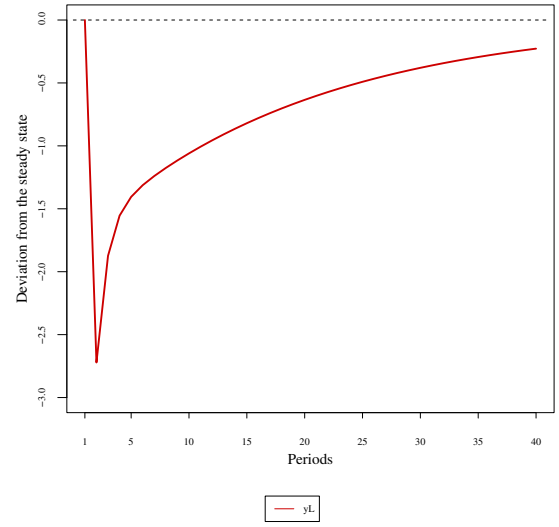


Figure 2: Impulse response (yL) to ϵ^π shock