

1 OPTIMALMP

1.1 Optimisation problem

$$\max_{pH_t, yH_t, i_t, pL_t, yL_t} U_t = -0.25 (piH - piCB + piH_t)^2 - 0.25 (-piCB + piL + piL_t)^2 + \beta E_t [U_{t+1}] - 0.25 \kappa \theta^{-1} yH_t^2 - 0.25 \kappa \theta^{-1} yL_t^2 \quad (1.1)$$

s.t. :

$$piH_{t-1} = \log etapi_{t-1} + \beta piH_t + \kappa yH_{t-1} + \beta (1 - pH) (-piH_t + piL_t) \quad \left(\lambda_t^{\text{OPTIMALMP}^1} \right) \quad (1.2)$$

$$yH_{t-1} = yH_t - \sigma (i_{t-1} - piH_t) + (1 - pH) (-yH_t + yL_t) + \sigma (1 - pH) (-piH_t + piL_t) \quad \left(\lambda_t^{\text{OPTIMALMP}^2} \right) \quad (1.3)$$

$$piL_{t-1} = \log etapi_{t-1} + \kappa yL_{t-1} + \beta pL piL_t + \beta (1 - pL) (piH_t - piL_t) \quad \left(\lambda_t^{\text{OPTIMALMP}^3} \right) \quad (1.4)$$

$$yL_{t-1} = yL_t - \sigma (i_{t-1} - piL_t) + (1 - pL) (yH_t - yL_t) + \sigma (1 - pL) (piH_t - piL_t) \quad \left(\lambda_t^{\text{OPTIMALMP}^4} \right) \quad (1.5)$$

1.2 First order conditions

$$-0.5piH + 0.5piCB - 0.5piH_t - \beta E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^1} \right] + \lambda_t^{\text{OPTIMALMP}^1} (\beta - \beta (1 - pH)) + \lambda_t^{\text{OPTIMALMP}^2} (\sigma - \sigma (1 - pH)) + \beta \lambda_t^{\text{OPTIMALMP}^3} (1 - pL) + \sigma \lambda_t^{\text{OPTIMALMP}^4} (1 - pL) = 0 \quad (piH_t) \quad (1.6)$$

$$\beta \left(\kappa E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^1} \right] - E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^2} \right] \right) + pH \lambda_t^{\text{OPTIMALMP}^2} + \lambda_t^{\text{OPTIMALMP}^4} (1 - pL) - 0.5 \kappa \theta^{-1} yH_t = 0 \quad (yH_t) \quad (1.7)$$

$$\beta \left(-\sigma E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^2} \right] - \sigma E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^4} \right] \right) = 0 \quad (i_t) \quad (1.8)$$

$$0.5piCB - 0.5piL - 0.5piL_t - \beta E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^3} \right] + \lambda_t^{\text{OPTIMALMP}^3} (\beta pL - \beta (1 - pL)) + \lambda_t^{\text{OPTIMALMP}^4} (\sigma - \sigma (1 - pL)) + \beta \lambda_t^{\text{OPTIMALMP}^1} (1 - pH) + \sigma \lambda_t^{\text{OPTIMALMP}^2} (1 - pH) = 0 \quad (piL_t) \quad (1.9)$$

$$\beta \left(\kappa E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^3} \right] - E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^4} \right] \right) + pL \lambda_t^{\text{OPTIMALMP}^4} + \lambda_t^{\text{OPTIMALMP}^2} (1 - pH) - 0.5 \kappa \theta^{-1} yL_t = 0 \quad (yL_t) \quad (1.10)$$

2 EXOG

2.1 Identities

$$etapi_t = e^{\epsilon_t^\pi + \phi \log etapi_{t-1}} \quad (2.1)$$

3 Equilibrium relationships (after reduction)

$$-d\pi i_t + e^{\epsilon_t + \phi \log d\pi i_{t-1}} = 0 \quad (3.1)$$

$$\beta \left(\kappa E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^1} \right] - E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^2} \right] \right) + pH \lambda_t^{\text{OPTIMALMP}^2} + \lambda_t^{\text{OPTIMALMP}^4} (1 - pL) - 0.5\kappa\theta^{-1}yH_t = 0 \quad (3.2)$$

$$\beta \left(\kappa E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^3} \right] - E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^4} \right] \right) + pL \lambda_t^{\text{OPTIMALMP}^4} + \lambda_t^{\text{OPTIMALMP}^2} (1 - pH) - 0.5\kappa\theta^{-1}yL_t = 0 \quad (3.3)$$

$$-piH_{t-1} + \log d\pi i_{t-1} + \beta piH_t + \kappa yH_{t-1} + \beta (1 - pH) (-piH_t + piL_t) = 0 \quad (3.4)$$

$$-piL_{t-1} + \log d\pi i_{t-1} + \kappa yL_{t-1} + \beta pL piL_t + \beta (1 - pL) (piH_t - piL_t) = 0 \quad (3.5)$$

$$-yH_{t-1} + yH_t - \sigma (i_{t-1} - piH_t) + (1 - pH) (-yH_t + yL_t) + \sigma (1 - pH) (-piH_t + piL_t) = 0 \quad (3.6)$$

$$-yL_{t-1} + yL_t - \sigma (i_{t-1} - piL_t) + (1 - pL) (yH_t - yL_t) + \sigma (1 - pL) (piH_t - piL_t) = 0 \quad (3.7)$$

$$U_t + 0.25 (piH - piCB + piH_t)^2 + 0.25 (-piCB + piL + piL_t)^2 - \beta E_t [U_{t+1}] + 0.25\kappa\theta^{-1}yH_t^2 + 0.25\kappa\theta^{-1}yL_t^2 = 0 \quad (3.8)$$

$$-0.5piH + 0.5piCB - 0.5piH_t - \beta E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^1} \right] + \lambda_t^{\text{OPTIMALMP}^1} (\beta - \beta (1 - pH)) + \lambda_t^{\text{OPTIMALMP}^2} (\sigma - \sigma (1 - pH)) + \beta \lambda_t^{\text{OPTIMALMP}^3} (1 - pL) + \sigma \lambda_t^{\text{OPTIMALMP}^4} (1 - pL) = 0 \quad (3.9)$$

$$0.5piCB - 0.5piL - 0.5piL_t - \beta E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^3} \right] + \lambda_t^{\text{OPTIMALMP}^3} (\beta pL - \beta (1 - pL)) + \lambda_t^{\text{OPTIMALMP}^4} (\sigma - \sigma (1 - pL)) + \beta \lambda_t^{\text{OPTIMALMP}^1} (1 - pH) + \sigma \lambda_t^{\text{OPTIMALMP}^2} (1 - pH) = 0 \quad (3.10)$$

$$\beta \left(-\sigma E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^2} \right] - \sigma E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^4} \right] \right) = 0 \quad (3.11)$$

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4 Steady state relationships (after reduction)

$$-d\pi i_{ss} + e^{\phi \log d\pi i_{ss}} = 0 \quad (4.1)$$

$$\beta \left(-\lambda_{ss}^{\text{OPTIMALMP}^2} + \kappa \lambda_{ss}^{\text{OPTIMALMP}^1} \right) + pH \lambda_{ss}^{\text{OPTIMALMP}^2} + \lambda_{ss}^{\text{OPTIMALMP}^4} (1 - pL) - 0.5\kappa\theta^{-1}yH_{ss} = 0 \quad (4.2)$$

$$\beta \left(-\lambda_{ss}^{\text{OPTIMALMP}^4} + \kappa \lambda_{ss}^{\text{OPTIMALMP}^3} \right) + pL \lambda_{ss}^{\text{OPTIMALMP}^4} + \lambda_{ss}^{\text{OPTIMALMP}^2} (1 - pH) - 0.5\kappa\theta^{-1}yL_{ss} = 0 \quad (4.3)$$

$$-piH_{ss} + \log d\pi i_{ss} + \beta piH_{ss} + \kappa yH_{ss} + \beta (1 - pH) (-piH_{ss} + piL_{ss}) = 0 \quad (4.4)$$

$$-piL_{ss} + \log d\pi i_{ss} + \kappa yL_{ss} + \beta pL piL_{ss} + \beta (1 - pL) (piH_{ss} - piL_{ss}) = 0 \quad (4.5)$$

$$(1 - pH) (-yH_{ss} + yL_{ss}) - \sigma (i_{ss} - piH_{ss}) + \sigma (1 - pH) (-piH_{ss} + piL_{ss}) = 0 \quad (4.6)$$

$$(1 - pL) (yH_{ss} - yL_{ss}) - \sigma (i_{ss} - piL_{ss}) + \sigma (1 - pL) (piH_{ss} - piL_{ss}) = 0 \quad (4.7)$$

$$U_{ss} + 0.25 (piH - piCB + piH_{ss})^2 + 0.25 (-piCB + piL + piL_{ss})^2 - \beta U_{ss} + 0.25\kappa\theta^{-1}yH_{ss}^2 + 0.25\kappa\theta^{-1}yL_{ss}^2 = 0 \quad (4.8)$$

$$-0.5piH + 0.5piCB - 0.5piH_{ss} - \beta \lambda_{ss}^{\text{OPTIMALMP}^1} + \lambda_{ss}^{\text{OPTIMALMP}^1} (\beta - \beta (1 - pH)) + \lambda_{ss}^{\text{OPTIMALMP}^2} (\sigma - \sigma (1 - pH)) + \beta \lambda_{ss}^{\text{OPTIMALMP}^3} (1 - pL) + \sigma \lambda_{ss}^{\text{OPTIMALMP}^4} (1 - pL) = 0 \quad (4.9)$$

$$0.5piCB - 0.5piL - 0.5piL_{ss} - \beta \lambda_{ss}^{\text{OPTIMALMP}^3} + \lambda_{ss}^{\text{OPTIMALMP}^3} (\beta pL - \beta (1 - pL)) + \lambda_{ss}^{\text{OPTIMALMP}^4} (\sigma - \sigma (1 - pL)) + \beta \lambda_{ss}^{\text{OPTIMALMP}^1} (1 - pH) + \sigma \lambda_{ss}^{\text{OPTIMALMP}^2} (1 - pH) = 0 \quad (4.10)$$

$$\beta \left(-\sigma \lambda_{ss}^{\text{OPTIMALMP}^2} - \sigma \lambda_{ss}^{\text{OPTIMALMP}^4} \right) = 0 \quad (4.11)$$

5 Parameter settings

$$\beta = 0.99 \tag{5.1}$$

$$\kappa = 0.2465 \tag{5.2}$$

$$\phi = 0.95 \tag{5.3}$$

$$pitH = 2 \tag{5.4}$$

$$pitCB = 2 \tag{5.5}$$

$$pitL = 4 \tag{5.6}$$

$$pH = 0.99 \tag{5.7}$$

$$pL = 0.99 \tag{5.8}$$

$$\sigma = 1 \tag{5.9}$$

$$\theta = 6 \tag{5.10}$$

6 Steady-state values

	Steady-state value
$etapi$	1
i	-1.0001
$\lambda^{\text{OPTIMALMP}^1}$	-0.0243
$\lambda^{\text{OPTIMALMP}^2}$	-0.5105
$\lambda^{\text{OPTIMALMP}^3}$	0.0141
$\lambda^{\text{OPTIMALMP}^4}$	0.5105
piH	-0.9997
piL	-1.0006
yH	-0.0405
yL	-0.0808
U	-49.9673

7 The solution of the 1st order perturbation

Matrix P

$$\begin{matrix}
 & etapi_{t-1} & i_{t-1} & piH_{t-1} & piL_{t-1} & yH_{t-1} & yL_{t-1} \\
 \begin{matrix} etapi_t \\ i_t \\ piH_t \\ piL_t \\ yH_t \\ yL_t \end{matrix} & \begin{pmatrix} 0.95 & 0 & 0 & 0 & 0 & 0 \\ -29.0023 & -3.0336 & -589.7847 & 607.7495 & 15.026 & -30.556 \\ -1.0103 & 0 & 1.0204 & -0.0104 & -0.0102 & 0.0002 \\ -1.0198 & 0 & -0.0104 & 1.0308 & 0.0001 & -0.0205 \\ 24.9235 & 24.6802 & -25.1736 & 0.2571 & 1.2617 & -0.0255 \\ 12.6276 & 12.3768 & 0.1288 & -12.7635 & -0.0064 & 1.2643 \end{pmatrix}
 \end{matrix}$$

Matrix Q

$$\begin{matrix}
 & \epsilon^\pi \\
 \begin{matrix} etapi \\ i \\ piH \\ piL \\ yH \\ yL \end{matrix} & \begin{pmatrix} 1 \\ -12.1391 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
 \end{matrix}$$

Matrix R

$$\begin{matrix}
 & etapi_{t-1} & i_{t-1} & piH_{t-1} & piL_{t-1} & yH_{t-1} & yL_{t-1} \\
 \begin{matrix} \lambda_t^{\text{OPTIMALMP}^1} \\ \lambda_t^{\text{OPTIMALMP}^2} \\ \lambda_t^{\text{OPTIMALMP}^3} \\ \lambda_t^{\text{OPTIMALMP}^4} \\ U_t \end{matrix} & \begin{pmatrix} 78354.8138 & 5351.9263 & 2931231.873 & -2990881.8819 & -74650.4938 & 150430.5551 \\ -1426.9835 & -97.4423 & -53335.8517 & 54422.0249 & 1358.3293 & -2737.2326 \\ -138117.8358 & -9440.8772 & -5150701.2185 & 5255767.5998 & 131175.5042 & -264343.799 \\ 1435.7948 & 98.1682 & 53590.8208 & -54683.1946 & -1364.8251 & 2750.3637 \\ -0.0034 & 0 & 0.0005 & -0.0003 & 0.0004 & -0.0008 \end{pmatrix}
 \end{matrix}$$

Matrix S

$$\begin{matrix}
 & \epsilon^\pi \\
 \begin{matrix} \lambda^{\text{OPTIMALMP}^1} \\ \lambda^{\text{OPTIMALMP}^2} \\ \lambda^{\text{OPTIMALMP}^3} \\ \lambda^{\text{OPTIMALMP}^4} \\ U \end{matrix} & \begin{pmatrix} 22259.7763 \\ -405.5204 \\ -39307.9945 \\ 408.4928 \\ -0.0034 \end{pmatrix}
 \end{matrix}$$

8 Model statistics

8.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
$etapi$	1	0.1303	0.017	Y
i	-1.0001	1.4658	2.1487	Y
$\lambda^{\text{OPTIMALMP}^1}$	-0.0243	2399.9295	5759661.7798	Y
$\lambda^{\text{OPTIMALMP}^2}$	-0.5105	43.7148	1910.9849	Y
$\lambda^{\text{OPTIMALMP}^3}$	0.0141	4228.1871	17877566.0584	Y
$\lambda^{\text{OPTIMALMP}^4}$	0.5105	43.9456	1931.2164	Y
pH	-0.9997	0.1683	0.0283	Y
pL	-1.0006	0.1688	0.0285	Y
yH	-0.0405	25.2506	637.5929	Y
yL	-0.0808	12.6478	159.9677	Y
U	-49.9673	0	0	Y

8.2 Correlation matrix

	$etapi$	i	$\lambda^{\text{OPTIMALMP}^1}$	$\lambda^{\text{OPTIMALMP}^2}$	$\lambda^{\text{OPTIMALMP}^3}$	$\lambda^{\text{OPTIMALMP}^4}$	pH	pL	yH
$etapi$	1	-0.099	0.753	-0.753	-0.751	0.751	0.156	0.153	-0.559
i		1	-0.398	0.398	0.402	-0.402	-0.206	-0.209	-0.656
$\lambda^{\text{OPTIMALMP}^1}$			1	-1	-1	1	-0.179	-0.182	-0.385
$\lambda^{\text{OPTIMALMP}^2}$				1	1	-1	0.18	0.183	0.386
$\lambda^{\text{OPTIMALMP}^3}$					1	-1	0.182	0.185	0.382
$\lambda^{\text{OPTIMALMP}^4}$						1	-0.18	-0.183	-0.382
pH							1	1	0.443
pL								1	0.447
yH									1
yL									

8.3 Cross correlations with the reference variable (i)

	$\sigma[\cdot]$ rel. to $\sigma[i]$	i_{t-5}	i_{t-4}	i_{t-3}	i_{t-2}	i_{t-1}	i_t	i_{t+1}	i_{t+2}	i_{t+3}	i_{t+4}	i_{t+5}
$etapi_t$	0.089	-0.021	-0.008	0.033	0.16	0.562	-0.099	-0.09	-0.08	-0.07	-0.06	-0.05
i_t	1	-0.004	-0.012	-0.035	-0.109	-0.343	1	-0.343	-0.109	-0.035	-0.012	0.004
$\lambda_t^{\text{OPTIMALMP}^1}$	1637.229	0	0.015	0.058	0.191	0.61	-0.398	-0.234	-0.135	-0.074	-0.036	0.004
$\lambda_t^{\text{OPTIMALMP}^2}$	29.822	0	-0.015	-0.058	-0.191	-0.61	0.398	0.235	0.135	0.073	0.036	-0.004
$\lambda_t^{\text{OPTIMALMP}^3}$	2884.463	0	-0.015	-0.058	-0.191	-0.611	0.402	0.236	0.134	0.072	0.035	-0.004
$\lambda_t^{\text{OPTIMALMP}^4}$	29.98	0	0.015	0.058	0.191	0.611	-0.402	-0.234	-0.134	-0.073	-0.036	0.004
pH_t	0.115	-0.007	-0.008	-0.011	-0.024	-0.067	-0.206	0.863	-0.019	-0.179	-0.157	-0.004
pL_t	0.115	-0.007	-0.008	-0.011	-0.025	-0.068	-0.209	0.863	-0.016	-0.177	-0.156	-0.004
yH_t	17.226	0.012	0.007	-0.008	-0.055	-0.199	-0.656	0.635	0.147	0.021	-0.004	0.004
yL_t	8.628	0.012	0.007	-0.008	-0.055	-0.199	-0.656	0.636	0.147	0.02	-0.004	0.004

8.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
$etapi$	0.713	0.471	0.271	0.11	-0.016
i	-0.343	-0.109	-0.035	-0.012	-0.004
$\lambda^{\text{OPTIMALMP}^1}$	0.467	0.156	-0.021	-0.115	-0.159
$\lambda^{\text{OPTIMALMP}^2}$	0.467	0.154	-0.022	-0.116	-0.159
$\lambda^{\text{OPTIMALMP}^3}$	0.463	0.151	-0.023	-0.116	-0.158
$\lambda^{\text{OPTIMALMP}^4}$	0.464	0.153	-0.022	-0.115	-0.158
pH	0.218	-0.026	-0.098	-0.113	-0.109
pL	0.217	-0.027	-0.098	-0.113	-0.109
yH	0.163	-0.029	-0.055	-0.05	-0.045
yL	0.162	-0.03	-0.056	-0.05	-0.045

9 Impulse response functions

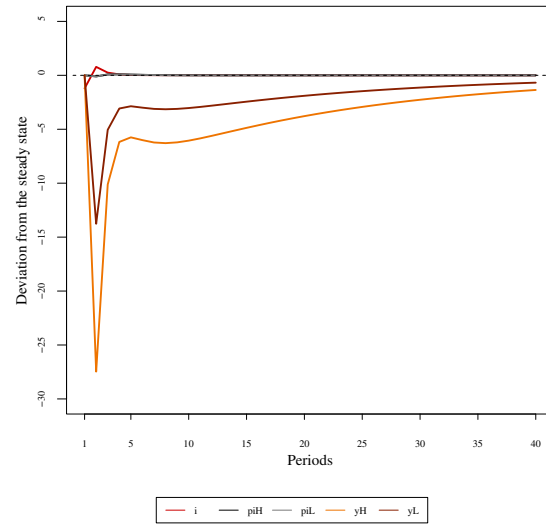


Figure 1: Impulse responses $(i, \pi H, \pi L, y H, y L)$ to ϵ^π shock