

1 OPTIMALMP

1.1 Optimisation problem

$$\max_{\pi H_t, y H_t, \pi L_t, y L_t} U_t = -0.25 (\pi H_t - \pi CB + \pi H_t)^2 - 0.25 (-\pi CB + \pi L_t + \pi L_t)^2 + \beta E_t [U_{t+1}] - 0.25 \lambda y H_t^2 - 0.25 \lambda y L_t^2 \quad (1.1)$$

s.t. :

$$\pi H_{t-1} = \log \epsilon \pi i_{t-1} + \beta \pi H_t + \kappa y H_{t-1} + \beta (1 - p H) (-\pi H_t + \pi L_t) \quad \left(\lambda_t^{\text{OPTIMALMP}^1} \right) \quad (1.2)$$

$$\pi L_{t-1} = \log \epsilon \pi i_{t-1} + \beta \pi L_t + \kappa y L_{t-1} + \beta (1 - p L) (\pi H_t - \pi L_t) \quad \left(\lambda_t^{\text{OPTIMALMP}^2} \right) \quad (1.3)$$

1.2 First order conditions

$$-0.5 \pi H_t + 0.5 \pi CB - 0.5 \pi H_t - \beta E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^1} \right] + \lambda_t^{\text{OPTIMALMP}^1} (\beta - \beta (1 - p H)) + \beta \lambda_t^{\text{OPTIMALMP}^2} (1 - p L) = 0 \quad (\pi H_t) \quad (1.4)$$

$$-0.5 \lambda y H_t + \beta \kappa E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^1} \right] = 0 \quad (y H_t) \quad (1.5)$$

$$0.5 \pi CB - 0.5 \pi L_t - 0.5 \pi L_t - \beta E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^2} \right] + \lambda_t^{\text{OPTIMALMP}^2} (\beta - \beta (1 - p L)) + \beta \lambda_t^{\text{OPTIMALMP}^1} (1 - p H) = 0 \quad (\pi L_t) \quad (1.6)$$

$$-0.5 \lambda y L_t + \beta \kappa E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^2} \right] = 0 \quad (y L_t) \quad (1.7)$$

2 EXOG

2.1 Identities

$$\epsilon \pi i_t = e^{\epsilon_t^\pi + \phi \log \epsilon \pi i_{t-1}} \quad (2.1)$$

3 Equilibrium relationships (after reduction)

$$-\epsilon \pi i_t + e^{\epsilon_t^\pi + \phi \log \epsilon \pi i_{t-1}} = 0 \quad (3.1)$$

$$-0.5 \lambda y H_t + \beta \kappa E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^1} \right] = 0 \quad (3.2)$$

$$-0.5 \lambda y L_t + \beta \kappa E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^2} \right] = 0 \quad (3.3)$$

$$-piH_{t-1} + \log etapi_{t-1} + \beta piH_t + \kappa yH_{t-1} + \beta (1 - pH) (-piH_t + piL_t) = 0 \quad (3.4)$$

$$-piL_{t-1} + \log etapi_{t-1} + \beta piL_t + \kappa yL_{t-1} + \beta (1 - pL) (piH_t - piL_t) = 0 \quad (3.5)$$

$$-0.5pitH + 0.5pitCB - 0.5piH_t - \beta E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^1} \right] + \lambda_t^{\text{OPTIMALMP}^1} (\beta - \beta (1 - pH)) + \beta \lambda_t^{\text{OPTIMALMP}^2} (1 - pL) = 0 \quad (3.6)$$

$$0.5pitCB - 0.5pitL - 0.5piL_t - \beta E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^2} \right] + \lambda_t^{\text{OPTIMALMP}^2} (\beta - \beta (1 - pL)) + \beta \lambda_t^{\text{OPTIMALMP}^1} (1 - pH) = 0 \quad (3.7)$$

$$U_t + 0.25 (pitH - pitCB + piH_t)^2 + 0.25 (-pitCB + pitL + piL_t)^2 - \beta E_t [U_{t+1}] + 0.25 \lambda yH_t^2 + 0.25 \lambda yL_t^2 = 0 \quad (3.8)$$

4 Steady state relationships (after reduction)

$$-etapi_{ss} + e^{\phi \log etapi_{ss}} = 0 \quad (4.1)$$

$$-0.5 \lambda yH_{ss} + \beta \kappa \lambda_{ss}^{\text{OPTIMALMP}^1} = 0 \quad (4.2)$$

$$-0.5 \lambda yL_{ss} + \beta \kappa \lambda_{ss}^{\text{OPTIMALMP}^2} = 0 \quad (4.3)$$

$$-piH_{ss} + \log etapi_{ss} + \beta piH_{ss} + \kappa yH_{ss} + \beta (1 - pH) (-piH_{ss} + piL_{ss}) = 0 \quad (4.4)$$

$$-piL_{ss} + \log etapi_{ss} + \beta piL_{ss} + \kappa yL_{ss} + \beta (1 - pL) (piH_{ss} - piL_{ss}) = 0 \quad (4.5)$$

$$-0.5pitH + 0.5pitCB - 0.5piH_{ss} - \beta \lambda_{ss}^{\text{OPTIMALMP}^1} + \lambda_{ss}^{\text{OPTIMALMP}^1} (\beta - \beta (1 - pH)) + \beta \lambda_{ss}^{\text{OPTIMALMP}^2} (1 - pL) = 0 \quad (4.6)$$

$$0.5pitCB - 0.5pitL - 0.5piL_{ss} - \beta \lambda_{ss}^{\text{OPTIMALMP}^2} + \lambda_{ss}^{\text{OPTIMALMP}^2} (\beta - \beta (1 - pL)) + \beta \lambda_{ss}^{\text{OPTIMALMP}^1} (1 - pH) = 0 \quad (4.7)$$

$$U_{ss} + 0.25 (pitH - pitCB + piH_{ss})^2 + 0.25 (-pitCB + pitL + piL_{ss})^2 - \beta U_{ss} + 0.25 \lambda yH_{ss}^2 + 0.25 \lambda yL_{ss}^2 = 0 \quad (4.8)$$

5 Parameter settings

$$\beta = 0.99 \quad (5.1)$$

$$\kappa = 0.2465 \quad (5.2)$$

$$\lambda = 0.04106 \quad (5.3)$$

$$\phi = 0.95 \quad (5.4)$$

$$pitH = 2 \quad (5.5)$$

$$pitCB = 2 \quad (5.6)$$

$$pitL = 4 \quad (5.7)$$

$$pH = 0.99 \quad (5.8)$$

$$pL = 0.99 \quad (5.9)$$

$$\sigma = 1 \quad (5.10)$$

$$\theta = 6 \quad (5.11)$$

6 Steady-state values

	Steady-state value
$\epsilon \pi$	1
$\lambda^{\text{OPTIMALMP}^1}$	0.0068
$\lambda^{\text{OPTIMALMP}^2}$	-0.0136
pH	-0.0004
pL	-1.9996
yH	0.0803
yL	-0.1614
U	-0.0334

7 The solution of the 1st order perturbation

Matrix P

$$\begin{matrix} \epsilon \pi_t \\ pH_t \\ pL_t \\ yH_t \\ yL_t \end{matrix} \begin{pmatrix} \epsilon \pi_{t-1} & pH_{t-1} & pL_{t-1} & yH_{t-1} & yL_{t-1} \\ 0.95 & 0 & 0 & 0 & 0 \\ -2509.0443 & 1.0204 & -51.1947 & -50.1556 & 1.0187 \\ -0.5052 & 0 & 1.0204 & 0.0001 & -0.0203 \\ -81.9707 & 0.0143 & -1.213 & -0.7053 & 0.0241 \\ -40.767 & -0.0001 & 35.444 & 0.006 & -0.7053 \end{pmatrix}$$

Matrix Q

$$\begin{matrix} \epsilon \pi \\ pH \\ pL \\ yH \\ yL \end{matrix} \begin{pmatrix} \epsilon \pi \\ 1 \\ 0 \\ 0 \\ -49.4067 \\ -24.5717 \end{pmatrix}$$

Matrix R

$$\begin{matrix} \lambda_t^{\text{OPTIMALMP}^1} \\ \lambda_t^{\text{OPTIMALMP}^2} \\ U_t \end{matrix} \begin{pmatrix} \epsilon \pi_{t-1} & pH_{t-1} & pL_{t-1} & yH_{t-1} & yL_{t-1} \\ -157.5109 & 0.0455 & -5.0667 & -2.2382 & 0.1008 \\ -78.3358 & -0.0005 & 112.4831 & 0.0249 & -2.2382 \\ -3.4381 & -0.0001 & 0.8138 & 0.004 & -0.0162 \end{pmatrix}$$

Matrix S

$$\begin{matrix} \lambda^{\text{OPTIMALMP}^1} \\ \lambda^{\text{OPTIMALMP}^2} \\ U \end{matrix} \begin{pmatrix} \epsilon \pi \\ -49.4067 \\ -24.5717 \\ -3.4037 \end{pmatrix}$$

8 Model statistics

8.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
$\epsilon \pi$	1	0.1303	0.017	Y
$\lambda^{\text{OPTIMALMP}^1}$	0.0068	6.7345	45.3534	Y
$\lambda^{\text{OPTIMALMP}^2}$	-0.0136	3.3493	11.2178	Y
pH	-0.0004	9.6383	92.8973	Y
pL	-1.9996	0.0019	0	Y
yH	0.0803	6.5317	42.6626	Y
yL	-0.1614	3.2484	10.5523	Y
U	-0.0334	0.4442	0.1973	Y

8.2 Correlation matrix

	$etpi$	$\lambda^{\text{OPTIMALMP}^1}$	$\lambda^{\text{OPTIMALMP}^2}$	piH	piL	yH	yL	U
$etpi$	1	-0.999	-0.999	-0.677	-0.677	-1	-1	-1
$\lambda^{\text{OPTIMALMP}^1}$		1	1	0.71	0.71	1	1	0.999
$\lambda^{\text{OPTIMALMP}^2}$			1	0.71	0.71	1	1	0.999
piH				1	1	0.688	0.688	0.677
piL					1	0.688	0.688	0.677
yH						1	1	1
yL							1	1
U								1

8.3 Cross correlations with the reference variable (piH)

	$\sigma[\cdot]$ rel. to $\sigma[piH]$	piH_{t-5}	piH_{t-4}	piH_{t-3}	piH_{t-2}	piH_{t-1}	piH_t	piH_{t+1}	piH_{t+2}	piH_{t+3}	piH_{t+4}
$etpi_t$	0.014	-0.211	-0.404	-0.626	-0.849	-0.972	-0.677	-0.429	-0.226	-0.064	0.000
$\lambda_t^{\text{OPTIMALMP}^1}$	0.699	0.202	0.396	0.621	0.849	0.982	0.71	0.462	0.253	0.084	-0.000
$\lambda_t^{\text{OPTIMALMP}^2}$	0.347	0.202	0.396	0.621	0.849	0.982	0.71	0.462	0.253	0.084	-0.000
piH_t	1	0.005	0.162	0.357	0.588	0.83	1	0.83	0.588	0.357	0.162
piL_t	0	0.005	0.162	0.357	0.588	0.83	1	0.83	0.588	0.357	0.162
yH_t	0.678	0.208	0.402	0.625	0.849	0.976	0.688	0.44	0.235	0.07	-0.000
yL_t	0.337	0.208	0.402	0.625	0.849	0.976	0.688	0.44	0.235	0.07	-0.000
U_t	0.046	0.211	0.404	0.626	0.849	0.973	0.677	0.43	0.227	0.064	-0.000

8.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
$etpi$	0.713	0.471	0.271	0.11	-0.016
$\lambda^{\text{OPTIMALMP}^1}$	0.74	0.498	0.291	0.122	-0.011
$\lambda^{\text{OPTIMALMP}^2}$	0.74	0.498	0.291	0.122	-0.011
piH	0.83	0.588	0.357	0.162	0.005
piL	0.83	0.588	0.357	0	0
yH	0.722	0.48	0.278	0.114	-0.015
yL	0.722	0.48	0.278	0.114	-0.015
U	0.714	0.472	0.272	0.11	-0.016

9 Impulse response functions

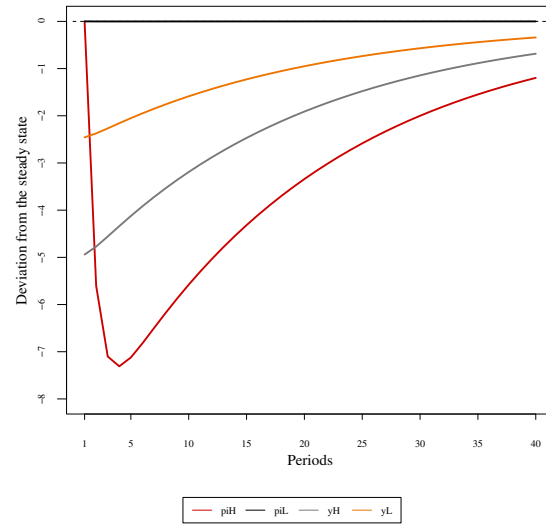


Figure 1: Impulse responses (π^H, π^L, y^H, y^L) to ϵ^π shock