

## 1 HIGHREGIME

### 1.1 Optimisation problem

$$\max_{pH_t, yH_t, iH_t} UH_t = -0.5 (piH - pitCB + piH_t)^2 + \beta (pHE_t [UH_{t+1}] + (1 - pH) E_t [UL_{t+1}]) - 0.5\kappa\theta^{-1}yH_t^2 \quad (1.1)$$

s.t. :

$$piH_{t-1} = \log etapi_{t-1} + \beta (pH piH_t + piL_t (1 - pH)) + \kappa yH_{t-1} \left( \lambda_t^{\text{HIGHREGIME}^1} \right) \quad (1.2)$$

$$yH_{t-1} = \log etag_{t-1} + pH yH_t - \sigma (iH_{t-1} - pH piH_t - piL_t (1 - pH)) + yL_t (1 - pH) \left( \lambda_t^{\text{HIGHREGIME}^2} \right) \quad (1.3)$$

### 1.2 First order conditions

$$-piH + pitCB - piH_t + \beta pH \lambda_t^{\text{HIGHREGIME}^1} - \beta pHE_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^1} \right] + pH \sigma \lambda_t^{\text{HIGHREGIME}^2} = 0 \quad (piH_t) \quad (1.4)$$

$$pH \lambda_t^{\text{HIGHREGIME}^2} + \beta pH \left( \kappa E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^1} \right] - E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^2} \right] \right) - \kappa \theta^{-1} yH_t = 0 \quad (yH_t) \quad (1.5)$$

$$-\beta pH \sigma E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^2} \right] = 0 \quad (iH_t) \quad (1.6)$$

## 2 LOWREGIME

### 2.1 Optimisation problem

$$\max_{piL_t, yL_t, iL_t} UL_t = -0.5 (-pitCB + piL + piL_t)^2 + \beta (pLE_t [UL_{t+1}] + (1 - pL) E_t [UH_{t+1}]) - 0.5\kappa\theta^{-1}yL_t^2 \quad (2.1)$$

s.t. :

$$piL_{t-1} = \log etapi_{t-1} + \beta (pL piL_t + piH_t (1 - pL)) + \kappa yL_{t-1} \left( \lambda_t^{\text{LOWREGIME}^1} \right) \quad (2.2)$$

$$yL_{t-1} = \log etag_{t-1} + pL yL_t - \sigma (iL_{t-1} - pL piL_t - piH_t (1 - pL)) + yH_t (1 - pL) \left( \lambda_t^{\text{LOWREGIME}^2} \right) \quad (2.3)$$

### 2.2 First order conditions

$$pitCB - piL - piL_t + \beta pL \lambda_t^{\text{LOWREGIME}^1} - \beta pLE_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^1} \right] + pL \sigma \lambda_t^{\text{LOWREGIME}^2} = 0 \quad (piL_t) \quad (2.4)$$

$$pL\lambda_t^{\text{LOWREGIME}^2} + \beta pL \left( \kappa E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^1} \right] - E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^2} \right] \right) - \kappa \theta^{-1} yL_t = 0 \quad (yL_t) \quad (2.5)$$

$$-\beta pL \sigma E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^2} \right] = 0 \quad (iL_t) \quad (2.6)$$

### 3 EXOG

#### 3.1 Identities

$$etapi_t = e^{\epsilon_t^\pi + \phi \log etapi_{t-1}} \quad (3.1)$$

$$etag_t = e^{\epsilon_t^g + \phi \log etag_{t-1}} \quad (3.2)$$

### 4 Equilibrium relationships (after reduction)

$$-etapi_t + e^{\epsilon_t^\pi + \phi \log etapi_{t-1}} = 0 \quad (4.1)$$

$$-etag_t + e^{\epsilon_t^g + \phi \log etag_{t-1}} = 0 \quad (4.2)$$

$$pH\lambda_t^{\text{HIGHREGIME}^2} + \beta pH \left( \kappa E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^1} \right] - E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^2} \right] \right) - \kappa \theta^{-1} yH_t = 0 \quad (4.3)$$

$$pL\lambda_t^{\text{LOWREGIME}^2} + \beta pL \left( \kappa E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^1} \right] - E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^2} \right] \right) - \kappa \theta^{-1} yL_t = 0 \quad (4.4)$$

$$-piH_{t-1} + \log etapi_{t-1} + \beta (pH piH_t + piL_t (1 - pH)) + \kappa yH_{t-1} = 0 \quad (4.5)$$

$$-piL_{t-1} + \log etapi_{t-1} + \beta (pL piL_t + piH_t (1 - pL)) + \kappa yL_{t-1} = 0 \quad (4.6)$$

$$UH_t + 0.5 (piH - pitCB + piH_t)^2 - \beta (pHE_t [UH_{t+1}] + (1 - pH) E_t [UL_{t+1}]) + 0.5 \kappa \theta^{-1} yH_t^2 = 0 \quad (4.7)$$

$$UL_t + 0.5 (-pitCB + piL + piL_t)^2 - \beta (pLE_t [UL_{t+1}] + (1 - pL) E_t [UH_{t+1}]) + 0.5 \kappa \theta^{-1} yL_t^2 = 0 \quad (4.8)$$

$$-yH_{t-1} + \log etag_{t-1} + pH yH_t - \sigma (iH_{t-1} - pH piH_t - piL_t (1 - pH)) + yL_t (1 - pH) = 0 \quad (4.9)$$

$$-yL_{t-1} + \log etag_{t-1} + pL yL_t - \sigma (iL_{t-1} - pL piL_t - piH_t (1 - pL)) + yH_t (1 - pL) = 0 \quad (4.10)$$

$$-pitH + pitCB - piH_t + \beta pH \lambda_t^{\text{HIGHREGIME}^1} - \beta pHE_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^1} \right] + pH \sigma \lambda_t^{\text{HIGHREGIME}^2} = 0 \quad (4.11)$$

$$pitCB - piL - piL_t + \beta pL \lambda_t^{\text{LOWREGIME}^1} - \beta pLE_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^1} \right] + pL \sigma \lambda_t^{\text{LOWREGIME}^2} = 0 \quad (4.12)$$

$$-\beta pH \sigma E_t \left[ \lambda_{t+1}^{\text{HIGHREGIME}^2} \right] = 0 \quad (4.13)$$

$$-\beta pL \sigma E_t \left[ \lambda_{t+1}^{\text{LOWREGIME}^2} \right] = 0 \quad (4.14)$$

## 5 Steady state relationships (after reduction)

$$-dapi_{ss} + e^{\phi \log dapi_{ss}} = 0 \quad (5.1)$$

$$-dag_{ss} + e^{phig \log dag_{ss}} = 0 \quad (5.2)$$

$$pH \lambda_{ss}^{\text{HIGHREGIME}^2} + \beta pH \left( -\lambda_{ss}^{\text{HIGHREGIME}^2} + \kappa \lambda_{ss}^{\text{HIGHREGIME}^1} \right) - \kappa \theta^{-1} yH_{ss} = 0 \quad (5.3)$$

$$pL \lambda_{ss}^{\text{LOWREGIME}^2} + \beta pL \left( -\lambda_{ss}^{\text{LOWREGIME}^2} + \kappa \lambda_{ss}^{\text{LOWREGIME}^1} \right) - \kappa \theta^{-1} yL_{ss} = 0 \quad (5.4)$$

$$-piH_{ss} + \log dapi_{ss} + \beta (pH piH_{ss} + piL_{ss} (1 - pH)) + \kappa yH_{ss} = 0 \quad (5.5)$$

$$-piL_{ss} + \log dapi_{ss} + \beta (pL piL_{ss} + piH_{ss} (1 - pL)) + \kappa yL_{ss} = 0 \quad (5.6)$$

$$UH_{ss} + 0.5 (piH - piCB + piH_{ss})^2 - \beta (pHUH_{ss} + UL_{ss} (1 - pH)) + 0.5 \kappa \theta^{-1} yH_{ss}^2 = 0 \quad (5.7)$$

$$UL_{ss} + 0.5 (-piCB + piL + piL_{ss})^2 - \beta (pLUL_{ss} + UH_{ss} (1 - pL)) + 0.5 \kappa \theta^{-1} yL_{ss}^2 = 0 \quad (5.8)$$

$$-yH_{ss} + \log dag_{ss} + pH yH_{ss} - \sigma (iH_{ss} - pH piH_{ss} - piL_{ss} (1 - pH)) + yL_{ss} (1 - pH) = 0 \quad (5.9)$$

$$-yL_{ss} + \log dag_{ss} + pL yL_{ss} - \sigma (iL_{ss} - pL piL_{ss} - piH_{ss} (1 - pL)) + yH_{ss} (1 - pL) = 0 \quad (5.10)$$

$$-piH + piCB - piH_{ss} + pH \sigma \lambda_{ss}^{\text{HIGHREGIME}^2} = 0 \quad (5.11)$$

$$piCB - piL - piL_{ss} + pL \sigma \lambda_{ss}^{\text{LOWREGIME}^2} = 0 \quad (5.12)$$

$$-\beta pH \sigma \lambda_{ss}^{\text{HIGHREGIME}^2} = 0 \quad (5.13)$$

$$-\beta pL \sigma \lambda_{ss}^{\text{LOWREGIME}^2} = 0 \quad (5.14)$$

3

## 6 Parameter settings

$$\beta = 0.99 \quad (6.1)$$

$$\kappa = 0.2465 \quad (6.2)$$

$$\phi = 0.95 \quad (6.3)$$

$$phig = 0.99 \quad (6.4)$$

$$piH = 0 \quad (6.5)$$

$$piCB = 0 \quad (6.6)$$

$$piL = 2 \quad (6.7)$$

$$pH = 0.99 \quad (6.8)$$

$$pL = 0.99 \quad (6.9)$$

$$\sigma = 1 \quad (6.10)$$

$$\theta = 6 \quad (6.11)$$

## 7 Steady-state values

	Steady-state value
$\epsilon \pi$	1
$\epsilon g$	1
$iH$	-0.0224
$iL$	-1.9776
$\lambda^{\text{HIGHREGIME}^1}$	0.0137
$\lambda^{\text{HIGHREGIME}^2}$	0
$\lambda^{\text{LOWREGIME}^1}$	-0.0275
$\lambda^{\text{LOWREGIME}^2}$	0
$\pi H$	0
$\pi L$	-2
$yH$	0.0803
$yL$	-0.1615
$UH$	-0.0266
$UL$	-0.0402

## 8 The solution of the 1st order perturbation

Matrix  $P$

$$\begin{matrix}
 & \epsilon \pi_{t-1} & \epsilon g_{t-1} & iH_{t-1} & iL_{t-1} & \pi H_{t-1} & \pi L_{t-1} & yH_{t-1} & yL_{t-1} \\
 \begin{matrix} \epsilon \pi_t \\ \epsilon g_t \\ iH_t \\ iL_t \\ \pi H_t \\ \pi L_t \\ yH_t \\ yL_t \end{matrix} & \begin{pmatrix}
 0.95 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0.99 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 -581.3663 & 130.7957 & -1.9645 & 1.9668 & 262.9418 & -6.4295 & -12.245 & 0.2885 & \\
 -6.5904 & 1.4827 & 3e-04 & -1.9645 & -0.0364 & 5.9614 & 0.0016 & -0.279 & \\
 -1.0101 & 0 & 0 & 0 & 1.0204 & -0.0206 & -0.0202 & 4e-04 & \\
 -0.5051 & 0 & 0 & 0 & -0.0052 & 1.0204 & 1e-04 & -0.0203 & \\
 12.5752 & -12.4495 & 0.2819 & -0.2512 & -12.7036 & 0.2566 & 1.2617 & -0.0256 & \\
 6.256 & -6.1935 & -0.0014 & 12.3731 & 0.0638 & -12.6397 & -0.0063 & 1.2617 & 
 \end{pmatrix}
 \end{pmatrix}$$

Matrix  $Q$

$$\begin{matrix}
 & \epsilon^\pi & \epsilon^g \\
 \begin{matrix} \epsilon \pi \\ \epsilon g \\ iH \\ iL \\ \pi H \\ \pi L \\ yH \\ yL \end{matrix} & \begin{pmatrix}
 1 & 0 \\
 0 & 1 \\
 -338.5676 & 44.6073 \\
 -3.838 & 0.5057 \\
 0 & 0 \\
 0 & 0 \\
 0 & 0 \\
 0 & 0
 \end{pmatrix}
 \end{pmatrix}$$

Matrix  $R$

$$\begin{matrix}
 & \epsilon \pi_{t-1} & \epsilon g_{t-1} & iH_{t-1} & iL_{t-1} & \pi H_{t-1} & \pi L_{t-1} & yH_{t-1} & yL_{t-1} \\
 \begin{matrix} \lambda_t^{\text{HIGHREGIME}^1} \\ \lambda_t^{\text{HIGHREGIME}^2} \\ \lambda_t^{\text{LOWREGIME}^1} \\ \lambda_t^{\text{LOWREGIME}^2} \\ UH_t \\ UL_t \end{matrix} & \begin{pmatrix}
 -438.2639 & 37.009 & -0.8469 & 1.5233 & 256.4894 & -9.1521 & -8.1131 & 0.3065 & \\
 1.0038 & -0.1323 & 0.003 & -0.0047 & -0.516 & 0.021 & 0.021 & -8e-04 & \\
 -218.0308 & 18.4115 & 0.0086 & -37.1681 & -2.2765 & 255.2005 & 0.0759 & -8.1131 & \\
 1.0038 & -0.1323 & -1e-04 & 0.2663 & 0.0105 & -1.0321 & -4e-04 & 0.0423 & \\
 5.5767 & 0.0054 & 0 & -0.0109 & -0.5135 & 0.0668 & 0.0102 & -0.0022 & \\
 -9.4149 & -0.0036 & 1e-04 & -2e-04 & -0.0256 & 1.3683 & 8e-04 & -0.0272 & 
 \end{pmatrix}
 \end{pmatrix}$$

## Matrix $S$

$$\begin{array}{c} \epsilon^\pi \quad \epsilon^g \\ \lambda^{\text{HIGHREGIME}^1} \\ \lambda^{\text{HIGHREGIME}^2} \\ \lambda^{\text{LOWREGIME}^1} \\ \lambda^{\text{LOWREGIME}^2} \\ UH \\ UL \end{array} \begin{pmatrix} -196.1584 & 0 \\ 0.5246 & 0 \\ -97.5864 & 0 \\ 0.5246 & 0 \\ 5.3648 & 0 \\ -9.2172 & 0 \end{pmatrix}$$

## 9 Model statistics

### 9.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
$\epsilon_{api}$	1	0.1303	0.017	Y
$\epsilon_{ag}$	1	0.1297	0.0168	Y
$iH$	-0.0224	34.9144	1219.0128	Y
$iL$	-1.9776	0.3958	0.1566	Y
$\lambda^{\text{HIGHREGIME}^1}$	0.0137	21.6778	469.9288	Y
$\lambda^{\text{HIGHREGIME}^2}$	0	0.0506	0.0026	N
$\lambda^{\text{LOWREGIME}^1}$	-0.0275	10.7845	116.3044	Y
$\lambda^{\text{LOWREGIME}^2}$	0	0.0506	0.0026	N
$pH$	0	0.0985	0.0097	N
$pL$	-2	0.0492	0.0024	Y
$yH$	0.0803	8.6232	74.3602	Y
$yL$	-0.1615	4.29	18.4037	Y
$UH$	-0.0266	0.7264	0.5277	Y
$UL$	-0.0402	1.2362	1.5281	Y

### 9.2 Correlation matrix

	$\epsilon_{api}$	$\epsilon_{ag}$	$iH$	$iL$	$\lambda^{\text{HIGHREGIME}^1}$	$\lambda^{\text{HIGHREGIME}^2}$	$\lambda^{\text{LOWREGIME}^1}$	$\lambda^{\text{LOWREGIME}^2}$	$pH$
$\epsilon_{api}$	1	0	-0.297	-0.297	-0.825	0.436	-0.825	0.436	-0.491
$\epsilon_{ag}$		1	0.166	0.166	0	0	0	0	0
$iH$			1	1	0.559	-0.964	0.559	-0.964	-0.318
$iL$				1	0.559	-0.964	0.559	-0.964	-0.318
$\lambda^{\text{HIGHREGIME}^1}$					1	-0.725	1	-0.725	0.567
$\lambda^{\text{HIGHREGIME}^2}$						1	-0.725	1	0.116
$\lambda^{\text{LOWREGIME}^1}$							1	-0.725	0.567
$\lambda^{\text{LOWREGIME}^2}$								1	0.116
$pH$									1
$pL$									
$yH$									
$yL$									
$UH$									
$UL$									

### 9.3 Cross correlations with the reference variable ( $iH$ )

	$\sigma[\cdot]$ rel. to $\sigma[iH]$	$iH_{t-5}$	$iH_{t-4}$	$iH_{t-3}$	$iH_{t-2}$	$iH_{t-1}$	$iH_t$	$iH_{t+1}$	$iH_{t+2}$	$iH_{t+3}$	$iH_{t+4}$
$\epsilon\alpha\pi_t$	0.004	0.097	0.129	0.174	0.255	0.438	-0.297	-0.25	-0.207	-0.166	-0.13
$\epsilon\alpha g_t$	0.004	0	0.021	0.047	0.08	0.119	0.166	0.119	0.08	0.047	0.021
$iH_t$	1	-0.027	-0.031	-0.041	-0.077	-0.193	1	-0.193	-0.077	-0.041	-0.031
$iL_t$	0.011	-0.027	-0.031	-0.041	-0.077	-0.193	1	-0.193	-0.077	-0.041	-0.031
$\lambda_t^{\text{HIGHREGIME}^1}$	0.621	-0.077	-0.099	-0.132	-0.201	-0.38	0.559	0.464	0.149	0.042	0.003
$\lambda_t^{\text{HIGHREGIME}^2}$	0.001	0.04	0.05	0.07	0.12	0.265	-0.964	0.043	0.042	0.04	0.038
$\lambda_t^{\text{LOWREGIME}^1}$	0.309	-0.077	-0.099	-0.132	-0.201	-0.38	0.559	0.464	0.149	0.042	0.003
$\lambda_t^{\text{LOWREGIME}^2}$	0.001	0.04	0.05	0.07	0.12	0.265	-0.964	0.043	0.042	0.04	0.038
$\pi H_t$	0.003	-0.045	-0.056	-0.07	-0.095	-0.154	-0.318	0.84	0.232	0.039	-0.022
$\pi L_t$	0.001	-0.045	-0.056	-0.07	-0.095	-0.154	-0.318	0.84	0.232	0.039	-0.022
$yH_t$	0.247	-0.068	-0.091	-0.119	-0.161	-0.241	-0.437	0.543	0.269	0.162	0.109
$yL_t$	0.123	-0.068	-0.091	-0.119	-0.161	-0.241	-0.437	0.543	0.269	0.162	0.109
$UH_t$	0.021	0.097	0.128	0.173	0.252	0.433	-0.263	-0.299	-0.215	-0.164	-0.124
$UL_t$	0.035	-0.097	-0.129	-0.173	-0.253	-0.434	0.271	0.288	0.214	0.164	0.125

### 9.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
$\epsilon\alpha\pi$	0.713	0.471	0.271	0.11	-0.016
$\epsilon\alpha g$	0.721	0.483	0.286	0.125	-0.003
$iH$	-0.193	-0.077	-0.041	-0.031	-0.027
$iL$	-0.193	-0.077	-0.041	-0.031	-0.027
$\lambda^{\text{HIGHREGIME}^1}$	0.51	0.081	-0.065	-0.117	-0.134
$\lambda^{\text{HIGHREGIME}^2}$	-0.074	-0.071	-0.066	-0.06	-0.054
$\lambda^{\text{LOWREGIME}^1}$	0.51	0.081	-0.065	-0.117	-0.134
$\lambda^{\text{LOWREGIME}^2}$	-0.074	-0.071	-0.066	-0.06	-0.054
$\pi H$	0.22	-0.024	-0.095	-0.11	-0.107
$\pi L$	0.22	-0.024	-0.095	-0.11	-0.107
$yH$	0.504	0.261	0.116	0.017	-0.055
$yL$	0.504	0.261	0.116	0.017	-0.055
$UH$	0.731	0.453	0.248	0.091	-0.029
$UL$	0.728	0.456	0.253	0.095	-0.026

## 10 Impulse response functions

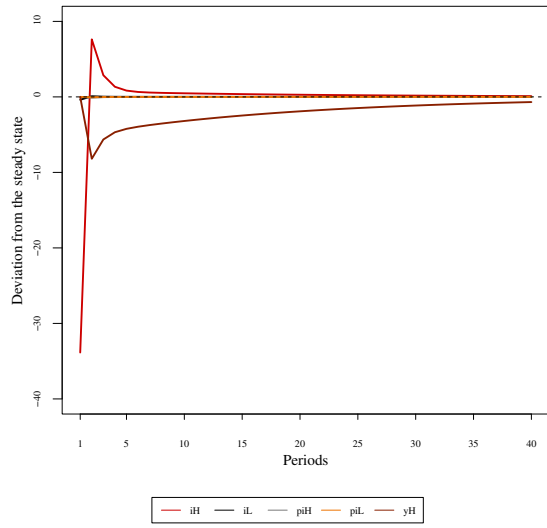


Figure 1: Impulse responses ( $iH, iL, \pi H, \pi L, yH$ ) to  $\epsilon^\pi$  shock

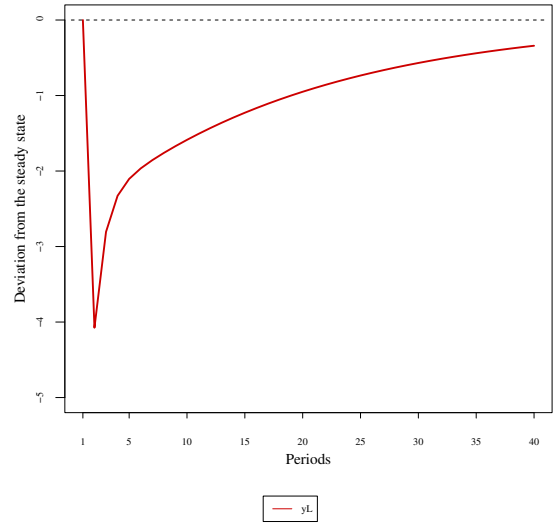


Figure 2: Impulse response ( $yL$ ) to  $\epsilon^\pi$  shock

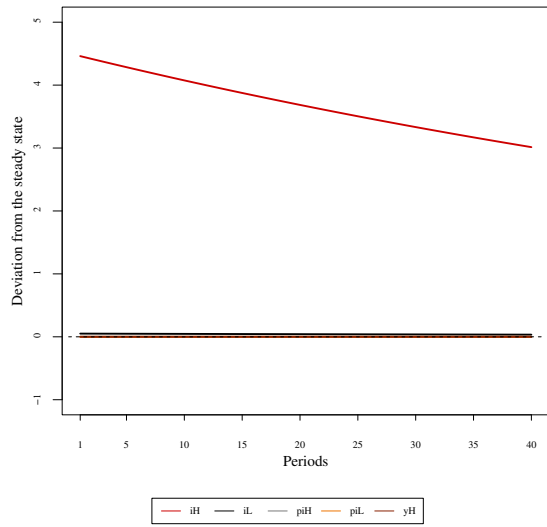


Figure 3: Impulse responses ( $iH, iL, \pi H, \pi L, yH$ ) to  $\epsilon^g$  shock

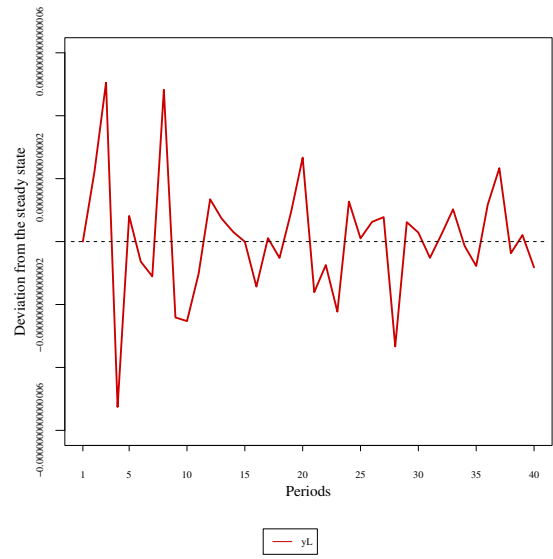


Figure 4: Impulse response ( $yL$ ) to  $\epsilon^g$  shock