

1 OPTIMALMP

1.1 Optimisation problem

$$\max_{pH_t, pL_t, yH_t, yL_t, i_t} U_t = -0.25 (piH - piCB + piH_t)^2 - 0.25 (-piCB + piL + piL_t)^2 + \beta E_t [U_{t+1}] - 0.25 \kappa \theta^{-1} yH_t^2 - 0.25 \kappa \theta^{-1} yL_t^2 \quad (1.1)$$

s.t. :

$$piH_{t-1} = \log etapi_{t-1} + \beta (pH piH_t + pL_t (1 - pH)) + \kappa yH_{t-1} \quad \left(\lambda_t^{\text{OPTIMALMP}^1} \right) \quad (1.2)$$

$$piL_{t-1} = \log etapi_{t-1} + \beta (pL piL_t + piL_t (1 - pL)) + \kappa yL_{t-1} \quad \left(\lambda_t^{\text{OPTIMALMP}^2} \right) \quad (1.3)$$

$$yH_{t-1} = pH yH_t + yL_t (1 - pH) - \sigma (i_{t-1} - pH piH_t - piL_t (1 - pH)) \quad \left(\lambda_t^{\text{OPTIMALMP}^3} \right) \quad (1.4)$$

$$yL_{t-1} = pL yL_t + yH_t (1 - pL) - \sigma (i_{t-1} - pL piL_t - piH_t (1 - pL)) \quad \left(\lambda_t^{\text{OPTIMALMP}^4} \right) \quad (1.5)$$

1.2 First order conditions

$$-0.5 piH + 0.5 piCB - 0.5 piH_t - \beta E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^1} \right] + \beta pH \lambda_t^{\text{OPTIMALMP}^1} + pH \sigma \lambda_t^{\text{OPTIMALMP}^3} - \sigma \lambda_t^{\text{OPTIMALMP}^4} (-1 + pL) = 0 \quad (piH_t) \quad (1.6)$$

$$0.5 piCB - 0.5 piL - 0.5 piL_t + \beta \lambda_t^{\text{OPTIMALMP}^2} - \beta E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^2} \right] + \beta \lambda_t^{\text{OPTIMALMP}^1} (1 - pH) + pL \sigma \lambda_t^{\text{OPTIMALMP}^4} - \sigma \lambda_t^{\text{OPTIMALMP}^3} (-1 + pH) = 0 \quad (piL_t) \quad (1.7)$$

$$\beta \left(\kappa E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^1} \right] - E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^3} \right] \right) + pH \lambda_t^{\text{OPTIMALMP}^3} + \lambda_t^{\text{OPTIMALMP}^4} (1 - pL) - 0.5 \kappa \theta^{-1} yH_t = 0 \quad (yH_t) \quad (1.8)$$

$$\beta \left(\kappa E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^2} \right] - E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^4} \right] \right) + pL \lambda_t^{\text{OPTIMALMP}^4} + \lambda_t^{\text{OPTIMALMP}^3} (1 - pH) - 0.5 \kappa \theta^{-1} yL_t = 0 \quad (yL_t) \quad (1.9)$$

$$\beta \left(-\sigma E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^3} \right] - \sigma E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^4} \right] \right) = 0 \quad (i_t) \quad (1.10)$$

2 EXOG

2.1 Identities

$$etapi_t = e^{\epsilon_t^\pi + \phi \log etapi_{t-1}} \quad (2.1)$$

3 Equilibrium relationships (after reduction)

$$-d\pi i_t + e^{\epsilon_t + \phi \log d\pi i_{t-1}} = 0 \quad (3.1)$$

$$-pH_{t-1} + \log d\pi i_{t-1} + \beta (pH pH_t + pL_t (1 - pH)) + \kappa yH_{t-1} = 0 \quad (3.2)$$

$$-pL_{t-1} + \log d\pi i_{t-1} + \beta (pL pL_t + pL_t (1 - pL)) + \kappa yL_{t-1} = 0 \quad (3.3)$$

$$-yH_{t-1} + pH yH_t - \sigma (i_{t-1} - pH pH_t - pL_t (1 - pH)) + yL_t (1 - pH) = 0 \quad (3.4)$$

$$-yL_{t-1} + pL yL_t - \sigma (i_{t-1} - pL pL_t - pL_t (1 - pL)) + yH_t (1 - pL) = 0 \quad (3.5)$$

$$\beta \left(\kappa E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^1} \right] - E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^3} \right] \right) + pH \lambda_t^{\text{OPTIMALMP}^3} + \lambda_t^{\text{OPTIMALMP}^4} (1 - pL) - 0.5 \kappa \theta^{-1} yH_t = 0 \quad (3.6)$$

$$\beta \left(\kappa E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^2} \right] - E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^4} \right] \right) + pL \lambda_t^{\text{OPTIMALMP}^4} + \lambda_t^{\text{OPTIMALMP}^3} (1 - pH) - 0.5 \kappa \theta^{-1} yL_t = 0 \quad (3.7)$$

$$U_t + 0.25 (pH - pL_{CB} + pL_t)^2 + 0.25 (-pL_{CB} + pL + pL_t)^2 - \beta E_t [U_{t+1}] + 0.25 \kappa \theta^{-1} yH_t^2 + 0.25 \kappa \theta^{-1} yL_t^2 = 0 \quad (3.8)$$

$$-0.5 pL_{CB} + 0.5 pL_{CB} - 0.5 pH_t - \beta E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^1} \right] + \beta pH \lambda_t^{\text{OPTIMALMP}^1} + pH \sigma \lambda_t^{\text{OPTIMALMP}^3} - \sigma \lambda_t^{\text{OPTIMALMP}^4} (-1 + pL) = 0 \quad (3.9)$$

$$0.5 pL_{CB} - 0.5 pL - 0.5 pL_t + \beta \lambda_t^{\text{OPTIMALMP}^2} - \beta E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^2} \right] + \beta \lambda_t^{\text{OPTIMALMP}^1} (1 - pH) + pL \sigma \lambda_t^{\text{OPTIMALMP}^4} - \sigma \lambda_t^{\text{OPTIMALMP}^3} (-1 + pH) = 0 \quad (3.10)$$

$$\beta \left(-\sigma E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^3} \right] - \sigma E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^4} \right] \right) = 0 \quad (3.11)$$

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4 Steady state relationships (after reduction)

$$-d\pi i_{ss} + e^{\phi \log d\pi i_{ss}} = 0 \quad (4.1)$$

$$-pH_{ss} + \log d\pi i_{ss} + \beta (pH pH_{ss} + pL_{ss} (1 - pH)) + \kappa yH_{ss} = 0 \quad (4.2)$$

$$-pL_{ss} + \log d\pi i_{ss} + \beta (pL pL_{ss} + pL_{ss} (1 - pL)) + \kappa yL_{ss} = 0 \quad (4.3)$$

$$-yH_{ss} + pH yH_{ss} - \sigma (i_{ss} - pH pH_{ss} - pL_{ss} (1 - pH)) + yL_{ss} (1 - pH) = 0 \quad (4.4)$$

$$-yL_{ss} + pL yL_{ss} - \sigma (i_{ss} - pL pL_{ss} - pL_{ss} (1 - pL)) + yH_{ss} (1 - pL) = 0 \quad (4.5)$$

$$\beta \left(-\lambda_{ss}^{\text{OPTIMALMP}^3} + \kappa \lambda_{ss}^{\text{OPTIMALMP}^1} \right) + pH \lambda_{ss}^{\text{OPTIMALMP}^3} + \lambda_{ss}^{\text{OPTIMALMP}^4} (1 - pL) - 0.5 \kappa \theta^{-1} yH_{ss} = 0 \quad (4.6)$$

$$\beta \left(-\lambda_{ss}^{\text{OPTIMALMP}^4} + \kappa \lambda_{ss}^{\text{OPTIMALMP}^2} \right) + pL \lambda_{ss}^{\text{OPTIMALMP}^4} + \lambda_{ss}^{\text{OPTIMALMP}^3} (1 - pH) - 0.5 \kappa \theta^{-1} yL_{ss} = 0 \quad (4.7)$$

$$U_{ss} + 0.25 (pH - pL_{CB} + pL_{ss})^2 + 0.25 (-pL_{CB} + pL + pL_{ss})^2 - \beta U_{ss} + 0.25 \kappa \theta^{-1} yH_{ss}^2 + 0.25 \kappa \theta^{-1} yL_{ss}^2 = 0 \quad (4.8)$$

$$-0.5 pL_{CB} + 0.5 pL_{CB} - 0.5 pH_{ss} - \beta \lambda_{ss}^{\text{OPTIMALMP}^1} + \beta pH \lambda_{ss}^{\text{OPTIMALMP}^1} + pH \sigma \lambda_{ss}^{\text{OPTIMALMP}^3} - \sigma \lambda_{ss}^{\text{OPTIMALMP}^4} (-1 + pL) = 0 \quad (4.9)$$

$$0.5 pL_{CB} - 0.5 pL - 0.5 pL_{ss} + \beta \lambda_{ss}^{\text{OPTIMALMP}^1} (1 - pH) + pL \sigma \lambda_{ss}^{\text{OPTIMALMP}^4} - \sigma \lambda_{ss}^{\text{OPTIMALMP}^3} (-1 + pH) = 0 \quad (4.10)$$

$$\beta \left(-\sigma \lambda_{ss}^{\text{OPTIMALMP}^3} - \sigma \lambda_{ss}^{\text{OPTIMALMP}^4} \right) = 0 \quad (4.11)$$

5 Parameter settings

$$\beta = 0.99 \tag{5.1}$$

$$\kappa = 0.2465 \tag{5.2}$$

$$\phi = 0.95 \tag{5.3}$$

$$pitH = 2 \tag{5.4}$$

$$pitCB = 2 \tag{5.5}$$

$$pitL = 4 \tag{5.6}$$

$$pH = 0.99 \tag{5.7}$$

$$pL = 0.99 \tag{5.8}$$

$$\sigma = 1 \tag{5.9}$$

$$\theta = 6 \tag{5.10}$$