

1 OPTIMALMP

1.1 Optimisation problem

$$\max_{\pi H_t, y H_t, \pi L_t, y L_t} U_t = -0.25 (\pi H_t - \pi CB + \pi H_t)^2 - 0.25 (-\pi CB + \pi L_t + \pi L_t)^2 + \beta E_t [U_{t+1}] - 0.25 \lambda y H_t^2 - 0.25 \lambda y L_t^2 \quad (1.1)$$

s.t. :

$$\pi H_{t-1} = \log \epsilon \pi i_{t-1} + \beta \pi H_t + \kappa y H_{t-1} + \beta (-\pi H_t + \pi L_t) \left(1 - p H_{ss} - \tau (-\pi CB + \pi H_t)^2 \right) \left(\lambda_t^{\text{OPTIMALMP}^1} \right) \quad (1.2)$$

$$\pi L_{t-1} = \log \epsilon \pi i_{t-1} + \beta \pi L_t + \kappa y L_{t-1} + \beta (\pi H_t - \pi L_t) \left(1 - p L_{ss} + \tau (-\pi CB + \pi L_t)^2 \right) \left(\lambda_t^{\text{OPTIMALMP}^2} \right) \quad (1.3)$$

1.2 First order conditions

$$-0.5 \pi H_t + 0.5 \pi CB - 0.5 \pi H_t - \beta E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^1} \right] + \lambda_t^{\text{OPTIMALMP}^1} \left(\beta - \beta \left(1 - p H_{ss} - \tau (-\pi CB + \pi H_t)^2 \right) - 2\beta \tau (-\pi CB + \pi H_t) (-\pi H_t + \pi L_t) \right) + \beta \lambda_t^{\text{OPTIMALMP}^2} \left(1 - p L_{ss} + \tau (-\pi CB + \pi L_t)^2 \right) = 0 \quad (y H_t) \quad (1.4)$$

$$-0.5 \lambda y H_t + \beta \kappa E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^1} \right] = 0 \quad (y H_t) \quad (1.5)$$

$$0.5 \pi CB - 0.5 \pi L_t - 0.5 \pi L_t - \beta E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^2} \right] + \lambda_t^{\text{OPTIMALMP}^2} \left(\beta - \beta \left(1 - p L_{ss} + \tau (-\pi CB + \pi L_t)^2 \right) + 2\beta \tau (-\pi CB + \pi L_t) (\pi H_t - \pi L_t) \right) + \beta \lambda_t^{\text{OPTIMALMP}^1} \left(1 - p H_{ss} - \tau (-\pi CB + \pi H_t)^2 \right) = 0 \quad (y L_t) \quad (1.6)$$

$$-0.5 \lambda y L_t + \beta \kappa E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^2} \right] = 0 \quad (y L_t) \quad (1.7)$$

2 EXOG

2.1 Identities

$$\epsilon \pi i_t = e^{\epsilon_t^\pi + \phi \log \epsilon \pi i_{t-1}} \quad (2.1)$$

3 Equilibrium relationships (after reduction)

$$-\epsilon \pi i_t + e^{\epsilon_t^\pi + \phi \log \epsilon \pi i_{t-1}} = 0 \quad (3.1)$$

$$-0.5 \lambda y H_t + \beta \kappa E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^1} \right] = 0 \quad (3.2)$$

$$-0.5 \lambda y L_t + \beta \kappa E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^2} \right] = 0 \quad (3.3)$$

$$-piH_{t-1} + \log \epsilon \pi i_{t-1} + \beta piH_t + \kappa y H_{t-1} + \beta (-piH_t + piL_t) \left(1 - pH_{ss} - \tau (-pitCB + piH_t)^2\right) = 0 \quad (3.4)$$

$$-piL_{t-1} + \log \epsilon \pi i_{t-1} + \beta piL_t + \kappa y L_{t-1} + \beta (piH_t - piL_t) \left(1 - pL_{ss} + \tau (-pitCB + piL_t)^2\right) = 0 \quad (3.5)$$

$$-0.5pitH + 0.5pitCB - 0.5piH_t - \beta E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^1} \right] + \lambda_t^{\text{OPTIMALMP}^1} \left(\beta - \beta \left(1 - pH_{ss} - \tau (-pitCB + piH_t)^2\right) - 2\beta\tau (-pitCB + piH_t) (-piH_t + piL_t) \right) + \beta \lambda_t^{\text{OPTIMALMP}^2} \left(1 - pL_{ss} + \tau (-pitCB + piL_t)^2\right) = 0 \quad (3.6)$$

$$0.5pitCB - 0.5pitL - 0.5piL_t - \beta E_t \left[\lambda_{t+1}^{\text{OPTIMALMP}^2} \right] + \lambda_t^{\text{OPTIMALMP}^2} \left(\beta - \beta \left(1 - pL_{ss} + \tau (-pitCB + piL_t)^2\right) + 2\beta\tau (-pitCB + piL_t) (piH_t - piL_t) \right) + \beta \lambda_t^{\text{OPTIMALMP}^1} \left(1 - pH_{ss} - \tau (-pitCB + piH_t)^2\right) = 0 \quad (3.7)$$

$$U_t + 0.25 (pitH - pitCB + piH_t)^2 + 0.25 (-pitCB + pitL + piL_t)^2 - \beta E_t [U_{t+1}] + 0.25 \lambda y H_t^2 + 0.25 \lambda y L_t^2 = 0 \quad (3.8)$$

4 Steady state relationships (after reduction)

$$-\epsilon \pi i_{ss} + e^{\phi \log \epsilon \pi i_{ss}} = 0 \quad (4.1)$$

$$-0.5 \lambda y H_{ss} + \beta \kappa \lambda_{ss}^{\text{OPTIMALMP}^1} = 0 \quad (4.2)$$

$$-0.5 \lambda y L_{ss} + \beta \kappa \lambda_{ss}^{\text{OPTIMALMP}^2} = 0 \quad (4.3)$$

$$-piH_{ss} + \log \epsilon \pi i_{ss} + \beta piH_{ss} + \kappa y H_{ss} + \beta (-piH_{ss} + piL_{ss}) \left(1 - pH_{ss} - \tau (-pitCB + piH_{ss})^2\right) = 0 \quad (4.4)$$

$$-piL_{ss} + \log \epsilon \pi i_{ss} + \beta piL_{ss} + \kappa y L_{ss} + \beta (piH_{ss} - piL_{ss}) \left(1 - pL_{ss} + \tau (-pitCB + piL_{ss})^2\right) = 0 \quad (4.5)$$

$$-0.5pitH + 0.5pitCB - 0.5piH_{ss} - \beta \lambda_{ss}^{\text{OPTIMALMP}^1} + \lambda_{ss}^{\text{OPTIMALMP}^1} \left(\beta - \beta \left(1 - pH_{ss} - \tau (-pitCB + piH_{ss})^2\right) - 2\beta\tau (-pitCB + piH_{ss}) (-piH_{ss} + piL_{ss}) \right) + \beta \lambda_{ss}^{\text{OPTIMALMP}^2} \left(1 - pL_{ss} + \tau (-pitCB + piL_{ss})^2\right) = 0 \quad (4.6)$$

$$0.5pitCB - 0.5pitL - 0.5piL_{ss} - \beta \lambda_{ss}^{\text{OPTIMALMP}^2} + \lambda_{ss}^{\text{OPTIMALMP}^2} \left(\beta - \beta \left(1 - pL_{ss} + \tau (-pitCB + piL_{ss})^2\right) + 2\beta\tau (-pitCB + piL_{ss}) (piH_{ss} - piL_{ss}) \right) + \beta \lambda_{ss}^{\text{OPTIMALMP}^1} \left(1 - pH_{ss} - \tau (-pitCB + piH_{ss})^2\right) = 0 \quad (4.7)$$

$$U_{ss} + 0.25 (pitH - pitCB + piH_{ss})^2 + 0.25 (-pitCB + pitL + piL_{ss})^2 - \beta U_{ss} + 0.25 \lambda y H_{ss}^2 + 0.25 \lambda y L_{ss}^2 = 0 \quad (4.8)$$

5 Parameter settings

$$\beta = 0.99 \quad (5.1)$$

$$\kappa = 0.2465 \quad (5.2)$$

$$\lambda = 0.04106 \quad (5.3)$$

$$\phi = 0.95 \quad (5.4)$$

$$pitH = 2 \quad (5.5)$$

$$pitCB = 2 \quad (5.6)$$

$$pitL = 4 \tag{5.7}$$

$$pHss = 0.99 \tag{5.8}$$

$$pLss = 0.99 \tag{5.9}$$

$$\sigma = 1 \tag{5.10}$$

$$\tau = 0.001 \tag{5.11}$$

$$\theta = 6 \tag{5.12}$$

6 Steady-state values

	Steady-state value
$\epsilon \pi$	1
$\lambda^{\text{OPTIMALMP}^1}$	0.004
$\lambda^{\text{OPTIMALMP}^2}$	-0.0243
pH	-0.0014
pL	-1.9979
yH	0.048
yL	-0.2894
U	-0.0885

7 The solution of the 1st order perturbation

Matrix P

$$\begin{matrix} & \epsilon \pi_{t-1} & pH_{t-1} & pL_{t-1} & yH_{t-1} & yL_{t-1} \\ \begin{matrix} \epsilon \pi_t \\ pH_t \\ pL_t \\ yH_t \\ yL_t \end{matrix} & \begin{pmatrix} 0.95 & 0 & 0 & 0 & 0 \\ -746.2354 & 1.0246 & -9.3875 & -8.8874 & 0.3352 \\ -0.5139 & 0 & 1.0545 & 0.0002 & -0.0377 \\ -137.0217 & 0.0819 & -2.1381 & -0.7104 & 0.0763 \\ -23.2817 & -0.0005 & 20.864 & 0.0046 & -0.745 \end{pmatrix} \end{matrix}$$

Matrix Q

$$\begin{matrix} \epsilon \pi \\ pH \\ pL \\ yH \\ yL \end{matrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ -82.1797 \\ -13.9222 \end{pmatrix}$$

Matrix R

$$\begin{matrix} \lambda^{\text{OPTIMALMP}^1}_t \\ \lambda^{\text{OPTIMALMP}^2}_t \\ U_t \end{matrix} \begin{pmatrix} \epsilon \pi_{t-1} & pH_{t-1} & pL_{t-1} & yH_{t-1} & yL_{t-1} \\ -260.6825 & 0.2606 & -14.6188 & -2.2605 & 0.522 \\ -46.2734 & -0.0017 & 67.4542 & 0.0144 & -2.4085 \\ -3.8568 & -0.0001 & 0.5497 & 0.0005 & -0.0196 \end{pmatrix}$$

Matrix S

$$\begin{matrix} \lambda^{\text{OPTIMALMP}^1} \\ \lambda^{\text{OPTIMALMP}^2} \\ U \end{matrix} \begin{pmatrix} \epsilon \pi \\ -81.0505 \\ -14.4476 \\ -3.8182 \end{pmatrix}$$

8 Model statistics

8.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
$\epsilon \pi$	1	0.1303	0.017	Y
$\lambda^{\text{OPTIMALMP}^1}$	0.004	11.186	125.126	Y
$\lambda^{\text{OPTIMALMP}^2}$	-0.0243	1.9061	3.6331	Y
pH	-0.0014	3.5273	12.4415	Y
pL	-1.9979	0	0	Y
yH	0.048	10.9038	118.8936	Y
yL	-0.2894	1.8217	3.3185	Y
U	-0.0885	0.4978	0.2478	Y

8.2 Correlation matrix

	$etpi$	$\lambda^{\text{OPTIMALMP}^1}$	$\lambda^{\text{OPTIMALMP}^2}$	pH	yH	yL	U
$etpi$	1	-0.998	-1	-0.677	-1	-1	-1
$\lambda^{\text{OPTIMALMP}^1}$		1	0.999	0.719	0.999	0.999	0.998
$\lambda^{\text{OPTIMALMP}^2}$			1	0.686	1	1	1
pH				1	0.691	0.68	0.677
yH					1	1	1
yL						1	1
U							1

8.3 Cross correlations with the reference variable (pH)

	$\sigma[\cdot]$ rel. to $\sigma[pH]$	pH_{t-5}	pH_{t-4}	pH_{t-3}	pH_{t-2}	pH_{t-1}	pH_t	pH_{t+1}	pH_{t+2}	pH_{t+3}	pH_{t+4}
$etpi_t$	0.037	-0.211	-0.403	-0.625	-0.848	-0.973	-0.677	-0.429	-0.226	-0.064	0.006
$\lambda_t^{\text{OPTIMALMP}^1}$	3.171	0.199	0.394	0.619	0.848	0.985	0.719	0.471	0.26	0.089	-0.006
$\lambda_t^{\text{OPTIMALMP}^2}$	0.54	0.208	0.401	0.624	0.849	0.976	0.686	0.438	0.234	0.07	-0.006
pH_t	1	0.005	0.161	0.357	0.587	0.829	1	0.829	0.587	0.357	0.161
yH_t	3.091	0.207	0.4	0.624	0.849	0.977	0.691	0.443	0.237	0.072	-0.006
yL_t	0.516	0.21	0.403	0.625	0.849	0.974	0.68	0.432	0.228	0.066	-0.006
U_t	0.141	0.211	0.403	0.625	0.848	0.973	0.677	0.429	0.226	0.064	-0.006

8.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
$etpi$	0.713	0.471	0.271	0.11	-0.016
$\lambda^{\text{OPTIMALMP}^1}$	0.747	0.505	0.296	0.125	-0.01
$\lambda^{\text{OPTIMALMP}^2}$	0.721	0.479	0.277	0.113	-0.015
pH	0.829	0.587	0.357	0.161	0.005
yH	0.724	0.482	0.279	0.115	-0.014
yL	0.716	0.474	0.273	0.111	-0.016
U	0.713	0.471	0.271	0.11	-0.016