

1 RW

1.1 Optimisation problem

$$\max_{\pi_t, y_t, i_t} U_t = -0.5 (-\textit{pistar} + \pi_t)^2 + \beta E_t [U_{t+1}] - 0.5 \kappa \theta^{-1} y_t^2 \quad (1.1)$$

s.t. :

$$\pi_{t-1} = \beta \pi_t + \kappa y_{t-1} \quad \left(\lambda_t^{\text{RW}^1} \right) \quad (1.2)$$

$$y_{t-1} = y_t - \sigma (i_{t-1} - m_{t-1} - \pi_t) \quad \left(\lambda_t^{\text{RW}^2} \right) \quad (1.3)$$

1.2 First order conditions

$$\textit{pistar} - \pi_t + \beta \lambda_t^{\text{RW}^1} - \beta E_t [\lambda_{t+1}^{\text{RW}^1}] + \sigma \lambda_t^{\text{RW}^2} = 0 \quad (\pi_t) \quad (1.4)$$

$$\lambda_t^{\text{RW}^2} + \beta \left(\kappa E_t [\lambda_{t+1}^{\text{RW}^1}] - E_t [\lambda_{t+1}^{\text{RW}^2}] \right) - \kappa \theta^{-1} y_t = 0 \quad (y_t) \quad (1.5)$$

$$-\beta \sigma E_t [\lambda_{t+1}^{\text{RW}^2}] = 0 \quad (i_t) \quad (1.6)$$

2 EXOG

2.1 Identities

$$m_t = e^{\epsilon_t^Z + \phi \log m_{t-1}} \quad (2.1)$$

3 Equilibrium relationships (after reduction)

$$-m_t + e^{\epsilon_t^Z + \phi \log m_{t-1}} = 0 \quad (3.1)$$

$$-\pi_{t-1} + \beta \pi_t + \kappa y_{t-1} = 0 \quad (3.2)$$

$$-y_{t-1} + y_t - \sigma (i_{t-1} - m_{t-1} - \pi_t) = 0 \quad (3.3)$$

$$\lambda_t^{\text{RW}^2} + \beta \left(\kappa E_t [\lambda_{t+1}^{\text{RW}^1}] - E_t [\lambda_{t+1}^{\text{RW}^2}] \right) - \kappa \theta^{-1} y_t = 0 \quad (3.4)$$

$$U_t + 0.5 (-\textit{pistar} + \pi_t)^2 - \beta E_t [U_{t+1}] + 0.5 \kappa \theta^{-1} y_t^2 = 0 \quad (3.5)$$

$$\bar{p}star - \pi_t + \beta \lambda_t^{\text{RW}^1} - \beta \mathbb{E}_t \left[\lambda_{t+1}^{\text{RW}^1} \right] + \sigma \lambda_t^{\text{RW}^2} = 0 \quad (3.6)$$

$$-\beta \sigma \mathbb{E}_t \left[\lambda_{t+1}^{\text{RW}^2} \right] = 0 \quad (3.7)$$

4 Steady state relationships (after reduction)

$$-m_{\text{ss}} + e^{\phi \log m_{\text{ss}}} = 0 \quad (4.1)$$

$$-\pi_{\text{ss}} + \beta \pi_{\text{ss}} + \kappa y_{\text{ss}} = 0 \quad (4.2)$$

$$-\sigma (i_{\text{ss}} - \pi_{\text{ss}} - m_{\text{ss}}) = 0 \quad (4.3)$$

$$\lambda_{\text{ss}}^{\text{RW}^2} + \beta \left(-\lambda_{\text{ss}}^{\text{RW}^2} + \kappa \lambda_{\text{ss}}^{\text{RW}^1} \right) - \kappa \theta^{-1} y_{\text{ss}} = 0 \quad (4.4)$$

$$U_{\text{ss}} + 0.5 (-\bar{p}star + \pi_{\text{ss}})^2 - \beta U_{\text{ss}} + 0.5 \kappa \theta^{-1} y_{\text{ss}}^2 = 0 \quad (4.5)$$

$$\bar{p}star - \pi_{\text{ss}} + \sigma \lambda_{\text{ss}}^{\text{RW}^2} = 0 \quad (4.6)$$

$$-\beta \sigma \lambda_{\text{ss}}^{\text{RW}^2} = 0 \quad (4.7)$$

5 Parameter settings

$$\beta = 0.99 \quad (5.1)$$

$$\kappa = 0.2465 \quad (5.2)$$

$$\phi = 0.95 \quad (5.3)$$

$$\bar{p}star = 0 \quad (5.4)$$

$$\sigma = 1 \quad (5.5)$$

$$\theta = 6 \quad (5.6)$$

6 Steady-state values

	Steady-state value
i	1
λ^{RW^1}	0
λ^{RW^2}	0
π	0
m	1
y	0
U	0

7 The solution of the 1st order perturbation

Matrix P

$$\begin{matrix} & i_{t-1} & \pi_{t-1} & m_{t-1} & y_{t-1} \\ \begin{matrix} i_t \\ \pi_t \\ m_t \\ y_t \end{matrix} & \begin{pmatrix} -1.9422 & 5.8224 & 2.8922 & -3.3774 \\ 0 & 1.0101 & 0 & -0.249 \\ 0 & 0 & 0.95 & 0 \\ 1 & -1.0101 & -1 & 1.249 \end{pmatrix} \end{matrix}$$

Matrix Q

$$\begin{matrix} & \epsilon^Z \\ \begin{matrix} i \\ \pi \\ m \\ y \end{matrix} & \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \end{matrix}$$

Matrix R

$$\begin{matrix} & i_{t-1} & \pi_{t-1} & m_{t-1} & y_{t-1} \\ \begin{matrix} \lambda_t^{\text{RW}^1} \\ \lambda_t^{\text{RW}^2} \\ U_t \end{matrix} & \begin{pmatrix} -0.5005 & 3.4065 & 0.5005 & -1.3402 \\ 0.1309 & -0.5005 & -0.1309 & 0.2543 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

Matrix S

$$\begin{matrix} & \epsilon^Z \\ \begin{matrix} \lambda^{\text{RW}^1} \\ \lambda^{\text{RW}^2} \\ U \end{matrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \end{matrix}$$

8 Model statistics

8.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
i	1	0.1303	0.017	Y
λ^{RW^1}	0	0	0	N
λ^{RW^2}	0	0	0	N
π	0	0	0	N
m	1	0.1303	0.017	Y
y	0	0	0	N
U	0	0	0	N

8.2 Correlation matrix

	i	m
i	1	1
m		1

8.3 Cross correlations with the reference variable (i)

	$\sigma[\cdot]$ rel. to $\sigma[i]$	i_{t-5}	i_{t-4}	i_{t-3}	i_{t-2}	i_{t-1}	i_t	i_{t+1}	i_{t+2}	i_{t+3}	i_{t+4}	i_{t+5}
i_t	1	-0.016	0.11	0.271	0.471	0.713	1	0.713	0.471	0.271	0.11	-0.016
m_t	1	-0.016	0.11	0.271	0.471	0.713	1	0.713	0.471	0.271	0.11	-0.016

8.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
i	0.713	0.471	0.271	0.11	-0.016
m	0.713	0.471	0.271	0.11	-0.016

9 Impulse response functions

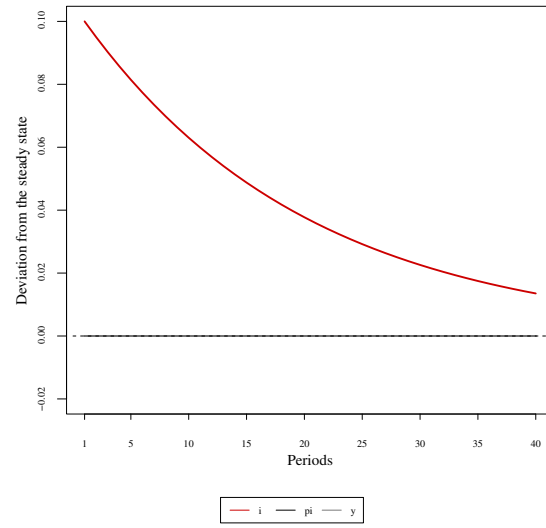


Figure 1: Impulse responses (i, π, y) to ϵ^Z shock