

## 1 RW

### 1.1 Optimisation problem

$$\max_{\pi_t, y_t, i_t} U_t = -0.5(-\pi_{star} + \pi_t)^2 + \beta E_t[U_{t+1}] - 0.5\kappa\theta^{-1}y_t^2 \quad (1.1)$$

s.t. :

$$\pi_{t-1} = \eta\pi_{t-1} + \beta\pi_t + \kappa y_{t-1} \quad \left(\lambda_t^{RW^1}\right) \quad (1.2)$$

$$y_{t-1} = y_t - \sigma(i_{t-1} - \pi_t) \quad \left(\lambda_t^{RW^2}\right) \quad (1.3)$$

### 1.2 First order conditions

$$\pi_{star} - \pi_t + \beta\lambda_t^{RW^1} - \beta E_t[\lambda_{t+1}^{RW^1}] + \sigma\lambda_t^{RW^2} = 0 \quad (\pi_t) \quad (1.4)$$

$$\lambda_t^{RW^2} + \beta \left( \kappa E_t[\lambda_{t+1}^{RW^1}] - E_t[\lambda_{t+1}^{RW^2}] \right) - \kappa\theta^{-1}y_t = 0 \quad (y_t) \quad (1.5)$$

$$-\beta\sigma E_t[\lambda_{t+1}^{RW^2}] = 0 \quad (i_t) \quad (1.6)$$

## 2 EXOG

### 2.1 Identities

$$\eta\pi_t = e^{\epsilon_t^\pi + \phi \log \eta\pi_{t-1}} \quad (2.1)$$

## 3 Equilibrium relationships (after reduction)

$$-\eta\pi_t + e^{\epsilon_t^\pi + \phi \log \eta\pi_{t-1}} = 0 \quad (3.1)$$

$$-y_{t-1} + y_t - \sigma(i_{t-1} - \pi_t) = 0 \quad (3.2)$$

$$\lambda_t^{RW^2} + \beta \left( \kappa E_t[\lambda_{t+1}^{RW^1}] - E_t[\lambda_{t+1}^{RW^2}] \right) - \kappa\theta^{-1}y_t = 0 \quad (3.3)$$

$$\eta\pi_{t-1} - \pi_{t-1} + \beta\pi_t + \kappa y_{t-1} = 0 \quad (3.4)$$

$$U_t + 0.5(-\pi_{star} + \pi_t)^2 - \beta E_t[U_{t+1}] + 0.5\kappa\theta^{-1}y_t^2 = 0 \quad (3.5)$$

$$pistar - \pi_t + \beta \lambda_t^{RW1} - \beta E_t \left[ \lambda_{t+1}^{RW1} \right] + \sigma \lambda_t^{RW2} = 0 \quad (3.6)$$

$$-\beta \sigma E_t \left[ \lambda_{t+1}^{RW2} \right] = 0 \quad (3.7)$$

## 4 Steady state relationships (after reduction)

$$-etapi_{ss} + e^{\phi \log etapi_{ss}} = 0 \quad (4.1)$$

$$-\sigma (i_{ss} - \pi_{ss}) = 0 \quad (4.2)$$

$$\lambda_{ss}^{RW2} + \beta \left( -\lambda_{ss}^{RW2} + \kappa \lambda_{ss}^{RW1} \right) - \kappa \theta^{-1} y_{ss} = 0 \quad (4.3)$$

$$etapi_{ss} - \pi_{ss} + \beta \pi_{ss} + \kappa y_{ss} = 0 \quad (4.4)$$

$$U_{ss} + 0.5 (-pistar + \pi_{ss})^2 - \beta U_{ss} + 0.5 \kappa \theta^{-1} y_{ss}^2 = 0 \quad (4.5)$$

$$pistar - \pi_{ss} + \sigma \lambda_{ss}^{RW2} = 0 \quad (4.6)$$

$$-\beta \sigma \lambda_{ss}^{RW2} = 0 \quad (4.7)$$

## 5 Parameter settings

$$\beta = 0.99 \quad (5.1)$$

$$\kappa = 0.2465 \quad (5.2)$$

$$\phi = 0.95 \quad (5.3)$$

$$pistar = 0 \quad (5.4)$$

$$\sigma = 1 \quad (5.5)$$

$$\theta = 6 \quad (5.6)$$

## 6 Steady-state values

	Steady-state value
$\epsilon \pi i$	1
$i$	0
$\lambda^{\text{RW}1}$	-0.683
$\lambda^{\text{RW}2}$	0
$\pi$	0
$y$	-4.0568
$U$	-33.8066

## 7 The solution of the 1st order perturbation

Matrix  $P$

$$\begin{matrix} \epsilon \pi i_t \\ i_t \\ \pi_t \\ y_t \end{matrix} \begin{pmatrix} \epsilon \pi i_{t-1} & i_{t-1} & \pi_{t-1} & y_{t-1} \\ 0.95 & 0 & 0 & 0 \\ -13.033 & -1.9422 & 5.8225 & -13.7015 \\ -1.0101 & 0 & 1.0101 & -1.0101 \\ 0.249 & 0.2465 & -0.249 & 1.249 \end{pmatrix}$$

Matrix  $Q$

$$\begin{matrix} \epsilon \pi i \\ i \\ \pi \\ y \end{matrix} \begin{pmatrix} \epsilon^\pi \\ 1 \\ -7.59 \\ 0 \\ 0 \end{pmatrix}$$

Matrix  $R$

$$\begin{matrix} \lambda_t^{\text{RW}1} \\ \lambda_t^{\text{RW}2} \\ U_t \end{matrix} \begin{pmatrix} \epsilon \pi i_{t-1} & i_{t-1} & \pi_{t-1} & y_{t-1} \\ -8.6776 & -0.7328 & 4.9879 & -7.9606 \\ 0.9938 & 0.1309 & -0.5005 & 1.0316 \\ -0.3395 & 0 & 0.0202 & -0.0202 \end{pmatrix}$$

Matrix  $S$

$$\begin{matrix} \lambda^{\text{RW}1} \\ \lambda^{\text{RW}2} \\ U \end{matrix} \begin{pmatrix} \epsilon^\pi \\ -3.8839 \\ 0.5193 \\ -0.3361 \end{pmatrix}$$

## 8 Model statistics

### 8.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
$\epsilon \pi i$	1	0.1303	0.017	Y
$i$	0	0.7719	0.5958	N
$\lambda^{\text{RW}1}$	-0.683	0.4292	0.1842	Y
$\lambda^{\text{RW}2}$	0	0.0501	0.0025	N
$\pi$	0	0.0985	0.0097	N
$y$	-4.0568	0.1707	0.0292	Y
$U$	-33.8066	0.0448	0.002	Y

## 8.2 Correlation matrix

	$et\pi i$	$i$	$\lambda^{RW^1}$	$\lambda^{RW^2}$	$\pi$	$y$	$U$
$et\pi i$	1	-0.301	-0.825	0.436	-0.491	-0.7	-0.999
$i$		1	0.567	-0.977	-0.323	-0.443	0.28
$\lambda^{RW^1}$			1	-0.725	0.567	0.44	0.832
$\lambda^{RW^2}$				1	0.116	0.273	-0.421
$\pi$					1	0.826	0.524
$y$						1	0.721
$U$							1

## 8.3 Cross correlations with the reference variable ( $i$ )

	$\sigma[\cdot]$ rel. to $\sigma[i]$	$i_{t-5}$	$i_{t-4}$	$i_{t-3}$	$i_{t-2}$	$i_{t-1}$	$i_t$	$i_{t+1}$	$i_{t+2}$	$i_{t+3}$	$i_{t+4}$	$i_{t+5}$
$et\pi i_t$	0.169	0.098	0.131	0.177	0.258	0.444	-0.301	-0.254	-0.21	-0.169	-0.132	-0.099
$i_t$	1	-0.028	-0.035	-0.051	-0.093	-0.219	1	-0.219	-0.093	-0.051	-0.035	-0.028
$\lambda_t^{RW^1}$	0.556	-0.078	-0.1	-0.134	-0.204	-0.385	0.567	0.47	0.151	0.043	0.003	-0.013
$\lambda_t^{RW^2}$	0.065	0.04	0.051	0.071	0.122	0.269	-0.977	0.043	0.043	0.041	0.038	0.035
$\pi_t$	0.128	-0.046	-0.056	-0.071	-0.096	-0.156	-0.323	0.852	0.236	0.04	-0.022	-0.04
$y_t$	0.221	-0.069	-0.092	-0.121	-0.164	-0.245	-0.443	0.551	0.273	0.164	0.111	0.078
$U_t$	0.058	-0.098	-0.13	-0.176	-0.257	-0.441	0.28	0.286	0.215	0.167	0.128	0.095

## 8.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
$et\pi i$	0.713	0.471	0.271	0.11	-0.016
$i$	-0.219	-0.093	-0.051	-0.035	-0.028
$\lambda^{RW^1}$	0.51	0.081	-0.065	-0.117	-0.134
$\lambda^{RW^2}$	-0.074	-0.071	-0.066	-0.06	-0.054
$\pi$	0.22	-0.024	-0.095	-0.11	-0.107
$y$	0.504	0.261	0.116	0.017	-0.055
$U$	0.725	0.459	0.256	0.097	-0.025

## 9 Impulse response functions

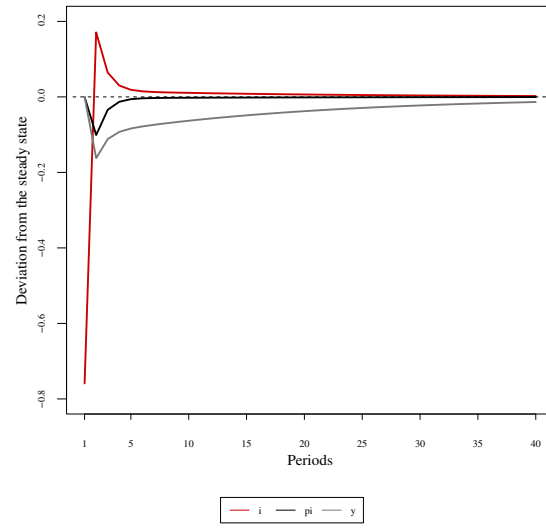


Figure 1: Impulse responses  $(i, \pi, y)$  to  $\epsilon^\pi$  shock