ICS 2022 Problem Sheet #6

Problem 6.1: *brandy after dinner*

Leo, Mark and Nick often have dinner together, but we do not know who likes to have a brandy after the dinner. However, we know the following:

1. If Leo drinks a brandy, then so does Mark.
2. Mark and Nick never drink brandy together.
3. Leo or Nick drink a brandy (alone or together).
4. If Nick drinks a brandy, then so does Leo.

We introduce three boolean variables: The variable L is true if Leo enjoys a brandy, the variable M is true if Mark enjoys a brandy, and the variable N is true if Nick enjoys a brandy.

a) Provide a boolean formula for the function D(L, M, N) that captures the three rules.

   $D(L,M,N) := (L \rightarrow M) \land \neg(M \land N) \land (L \lor N) \land (N \rightarrow L)$

b) Construct a truth table that shows all interpretations of D(L, M, N). Break things into meaningful steps so that we can award partial points in case things go wrong somewhere.

| | L | M | N | D(L,M,N) |
|---|---|---|---|---|
| 1) | 0 | 0 | 0 | 0 |
| 2) | 0 | 0 | 1 | 0 |
| 3) | 0 | 1 | 0 | 0 |
| 4) | 1 | 0 | 0 | 0 |
| 5) | 0 | 1 | 1 | 0 |
| 6) | 1 | 1 | 0 | 1 |
| 7) | 1 | 0 | 1 | 0 |
| 8) | 1 | 1 | 1 | 0 |

1)

1. $(L \rightarrow M) = (0 \rightarrow 0) = 1$
2. $(M \land N) = (0 \land 0) = 0$
3. $\neg(M \land N) = \neg 0 = 1$
4. $(L \lor N) = (0 \lor 0) = 0$
5. $(N \rightarrow L) = (0 \rightarrow 0) = 1$
6. $(L \rightarrow M) \land \neg(M \land N) \land (L \lor N) \land (N \rightarrow L) = 1 \land 1 \land 0 \land 1 = 0$

2)

1. $(\mathrm{L} \rightarrow \mathrm{M}) = (0 \rightarrow 0) = 1$
2. $(\mathrm{M} \wedge \mathrm{N}) = (0 \wedge 1) = 0$
3. $\neg(\mathrm{M} \wedge \mathrm{N}) = \neg 0 = 1$
4. $(\mathrm{L} \vee \mathrm{N}) = (0 \vee 1) = 1$
5. $(\mathrm{N} \rightarrow \mathrm{L}) = (1 \rightarrow 0) = 0$
6. $(\mathrm{L} \rightarrow \mathrm{M}) \wedge \neg(M \wedge N) \wedge (\mathrm{L} \vee \mathrm{N}) \wedge (\mathrm{N} \rightarrow \mathrm{L}) = 1 \wedge 1 \wedge 1 \wedge 0 = 0$

3)

1. $(\mathrm{L} \rightarrow \mathrm{M}) = (0 \rightarrow 1) = 1$
2. $(\mathrm{M} \wedge \mathrm{N}) = (1 \wedge 0) = 0$
3. $\neg(\mathrm{M} \wedge \mathrm{N}) = \neg 0 = 1$
4. $(\mathrm{L} \vee \mathrm{N}) = (0 \vee 0) = 0$
5. $(\mathrm{N} \rightarrow \mathrm{L}) = (0 \rightarrow 0) = 1$
6. $(\mathrm{L} \rightarrow \mathrm{M}) \wedge \neg(M \wedge N) \wedge (\mathrm{L} \vee \mathrm{N}) \wedge (\mathrm{N} \rightarrow \mathrm{L}) = 1 \wedge 1 \wedge 0 \wedge 1 = 0$

4)

1. $(\mathrm{L} \rightarrow \mathrm{M}) = (1 \rightarrow 0) = 0$
2. $(\mathrm{M} \wedge \mathrm{N}) = (0 \wedge 0) = 0$
3. $\neg(\mathrm{M} \wedge \mathrm{N}) = \neg 0 = 1$
4. $(\mathrm{L} \vee \mathrm{N}) = (1 \vee 0) = 1$
5. $(\mathrm{N} \rightarrow \mathrm{L}) = (0 \rightarrow 1) = 1$
6. $(\mathrm{L} \rightarrow \mathrm{M}) \wedge \neg(M \wedge N) \wedge (\mathrm{L} \vee \mathrm{N}) \wedge (\mathrm{N} \rightarrow \mathrm{L}) = 0 \wedge 1 \wedge 1 \wedge 1 = 0$

5)

1. $(\mathrm{L} \rightarrow \mathrm{M}) = (0 \rightarrow 1) = 1$
2. $(\mathrm{M} \wedge \mathrm{N}) = (1 \wedge 1) = 1$
3. $\neg(\mathrm{M} \wedge \mathrm{N}) = \neg 1 = 0$
4. $(\mathrm{L} \vee \mathrm{N}) = (0 \vee 1) = 1$
5. $(\mathrm{N} \rightarrow \mathrm{L}) = (1 \rightarrow 0) = 0$
6. $(\mathrm{L} \rightarrow \mathrm{M}) \wedge \neg(M \wedge N) \wedge (\mathrm{L} \vee \mathrm{N}) \wedge (\mathrm{N} \rightarrow \mathrm{L}) = 1 \wedge 0 \wedge 1 \wedge 0 = 0$

6)

1. $(\mathrm{L} \rightarrow \mathrm{M}) = (1 \rightarrow 1) = 1$
2. $(\mathrm{M} \wedge \mathrm{N}) = (1 \wedge 0) = 0$
3. $\neg(\mathrm{M} \wedge \mathrm{N}) = \neg 0 = 1$
4. $(\mathrm{L} \vee \mathrm{N}) = (1 \vee 0) = 1$
5. $(\mathrm{N} \rightarrow \mathrm{L}) = (0 \rightarrow 1) = 1$

6. $(L \to M) \land \neg(M \land N) \land (L \lor N) \land (N \to L) = 1 \land 1 \land 1 \land 1 = 1$

7)

1. $(L \to M) = (1 \to 0) = 0$
2. $(M \land N) = (0 \land 1) = 0$
3. $\neg(M \land N) = \neg 0 = 1$
4. $(L \lor N) = (1 \lor 1) = 1$
5. $(N \to L) = (1 \to 1) = 1$
6. $(L \to M) \land \neg(M \land N) \land (L \lor N) \land (N \to L) = 0 \land 1 \land 1 \land 1 = 0$

8)

1. $(L \to M) = (1 \to 1) = 1$
2. $(M \land N) = (1 \land 1) = 1$
3. $\neg(M \land N) = \neg 1 = 0$
4. $(L \lor N) = (1 \lor 1) = 1$
5. $(N \to L) = (1 \to 1) = 1$
6. $(L \to M) \land \neg(M \land N) \land (L \lor N) \land (N \to L) = 1 \land 0 \land 1 \land 1 = 0$

c) Out of the truth table, derive a simpler boolean formula defining $D(L, M, N)$.

$D(L, M, N) = L \land M \land \neg N$

d) Take the boolean formula from a) and algebraically derive the simpler boolean formula from c). Annotate each step of your derivation with the equivalence law that you apply so that we can follow along.

$(L \to M) \land \neg(M \land N) \land (L \lor N) \land (N \to L) =$

$= (\neg L \lor M) \land \neg(M \land N) \land (L \lor N) \land (\neg N \lor L) =$ // the definition of $\to$

$= (\neg L \lor M) \land \neg(M \land N) \land (L \lor (N \land \neg N)) =$ // distributivity

$= (\neg L \lor M) \land \neg(M \land N) \land (L \lor 0) =$ // complementation

$= \neg(L \land \neg M) \land \neg(M \land N) \land L =$ // identity

**Problem 6.2:** *brandy after dinner*

a) Define a function

brandy :: Bool -> Bool -> Bool -> Bool

that implements the rules directly following the description given above and a function

brandy' :: Bool -> Bool -> Bool -> Bool

that implements a simplified boolean formula.

```
brandy :: Bool -> Bool -> Bool -> Bool
brandy  l m n = (not l || m) && not(m && n) && ( l || n) && (not n  && l )
brandy' :: Bool -> Bool -> Bool -> Bool
brandy'  l m n = l && m && (not n)
```