ICS 2022 Problem Sheet #7

Problem 7.1: quine-McCluskey algorithm

A Boolean function F is defined using the following sum of minterms:

F (A, B, C, D, E) $= m_0+m_2+m_4+m_6+m_9+m_{10}+m_{13}+m_{14}+m_{15}+m_{16}+m_{17}+m_{21}+m_{26}+m_{28}+m_{30}+m_{31}$

a) Calculate the prime implicants of F.

minterm	pattern	used	minterms	pattern	used	minterms	pattern	used
m_0	00000	*	$m_{0,2}$	000-0	*	$m_{0,2,4,6}$	00--0	
			$m_{0,4}$	00-00	*			
			$m_{0,16}$	-0000				
m_2	00010	*	$m_{2,6}$	00-10	*			
			$m_{2,10}$	0-010	*	$m_{2,10,6,14}$	0--10	
m_4	00100	*	$m_{4,6}$	001-0	*			
m_{16}	10000	*	$m_{16,17}$	1000-				
m_6	00110	*	$m_{6,14}$	0-110	*			
m_9	01001	*	$m_{9,13}$	01-01				
m_{10}	01010	*	$m_{10,26}$	-1010	*	$m_{10,14,26,30}$	-1-10	
			$m_{10,14}$	01-10	*			
m_{17}	10001	*	$m_{17,21}$	10-01				
m_{13}	01101	*	$m_{13,15}$	011-1				
m_{14}	01110	*	$m_{14,15}$	0111-	*	$m_{14,15,30,31}$	-111-	
			$m_{14,30}$	-1110	*			
m_{21}	10101	*						
m_{26}	11010	*	$m_{26,30}$	11-10	*			
m_{28}	11100	*	$m_{28,30}$	111-0				
m_{15}	01111	*	$m_{15,31}$	-1111	*			
m_{30}	11110	*	$m_{30,31}$	1111-	*			
m_{31}	11111	*						

Prime implicants: $m_{0,16}$ $m_{9,13}$ $m_{16,17}$ $m_{17,21}$ $m_{13,15}$ $m_{28,30}$ $m_{0,2,4,6}$ $m_{2,10,6,14}$ $m_{14,15,30,31}$ $m_{10,14,26,30}$

$F = m_{0,16} + m_{9,13} + m_{16,17} + m_{17,21} + m_{13,15} + m_{28,30} + m_{0,2,4,6} + m_{2,10,6,14} + m_{14,15,30,31} + m_{10,14,26,30}$

b) Construct the prime implicant chart and identify the essential prime implicants.

	m_0	m_2	m_4	m_6	m_9	m_{10}	m_{13}	m_{14}	m_{15}	m_{16}	m_{17}	m_{21}	m_{26}	m_{28}	m_{30}	m_{31}
$m_{0,16}$	*									*						
$m_{16,17}$										*	*					
$m_{9,13}$					+		+									
$m_{17,21}$											+	+				
$m_{13,15}$							*		*							
$m_{28,30}$														+	+	
$m_{0,2,4,6}$	+	+	+	+												
$m_{2,10,6,14}$		*		*		*		*								
$m_{10,14,26,30}$						+		+					+		+	
$m_{14,15,30,31}$								+	+						+	+
	+	+	+	+	+	+	+	+	+		+	+	+	+	+	+

Essential prime implicants: $m_{9,13}$ $m_{17,21}$ $m_{28,30}$ $m_{0,2,4,6}$ $m_{14,15,30,31}$ $m_{10,14,26,30}$

$F1 = m_{0,16} + m_{9,13} + m_{17,21} + m_{28,30} + m_{0,2,4,6} + m_{14,15,30,31} + m_{10,14,26,30}$ $\quad cost(F1) = 4 * 3 + 3 * 2 + 6 = 24$

$F2 = m_{16,17} + m_{9,13} + m_{17,21} + m_{28,30} + m_{0,2,4,6} + m_{14,15,30,31} + m_{10,14,26,30}$ $\quad cost(F2) = 4 * 3 + 3 * 2 + 6 = 24$

c) Write out all minimal boolean expressions defining F using the mathematical logic notation.

1. The minimal boolean expressions defining F using the mathematical logic notation with prime implicants:

$F = m_{0,16} + m_{9,13} + m_{16,17} + m_{17,21} + m_{13,15} + m_{28,30} + m_{0,2,4,6} + m_{2,10,6,14} + m_{14,15,30,31} + m_{10,14,26,30}$
$= B'C'D'E' + A'BC'D + AB'C'D' + AB'C'E + A'BCE + ABCE' + A'B'E' + BDE' + BCD + BDE' =$
$= (\neg B\neg C\neg D\neg E) \vee (\neg AB\neg CD) \vee (A\neg B\neg C\neg D) \vee (A\neg B\neg CE) \vee (\neg ABCE) \vee (ABC\neg E) \vee (A\neg B\neg E) \vee (BD\neg E) \vee (BCD) \vee (BD\neg E).$

2. The minimal boolean expressions defining F using the mathematical logic notation with essential prime implicants:

$F1 = m_{0,16} + m_{9,13} + m_{17,21} + m_{28,30} + m_{0,2,4,6} + m_{14,15,30,31} + m_{10,14,26,3}$
$= B'C'D'E' + A'BC'D + A'BC'E + ABCE' + A'B'E' + BCD + BDE' =$
$= (\neg B\neg C\neg D\neg E) \vee (\neg AB\neg CD) \vee (A\neg B\neg CE) \vee (ABC\neg E) \vee (A\neg B\neg E) \vee (BCD) \vee (BD\neg E).$

$F1 = m_{16,17} + m_{9,13} + m_{17,21} + m_{28,30} + m_{0,2,4,6} + m_{14,15,30,31} + m_{10,14,26,3}$
$= AB'C'D' + A'BC'D + A'BC'E + ABCE' + A'B'E' + BCD + BDE' =$
$= (A\neg B\neg C\neg D)) \vee (\neg AB\neg CD) \vee (A\neg B\neg CE) \vee (ABC\neg E) \vee (A\neg B\neg E) \vee (BCD) \vee (BD\neg E).$