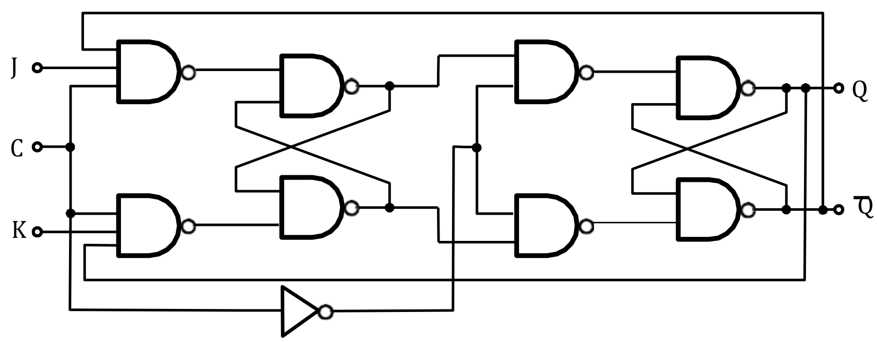
ICS 2022 Problem Sheet #9

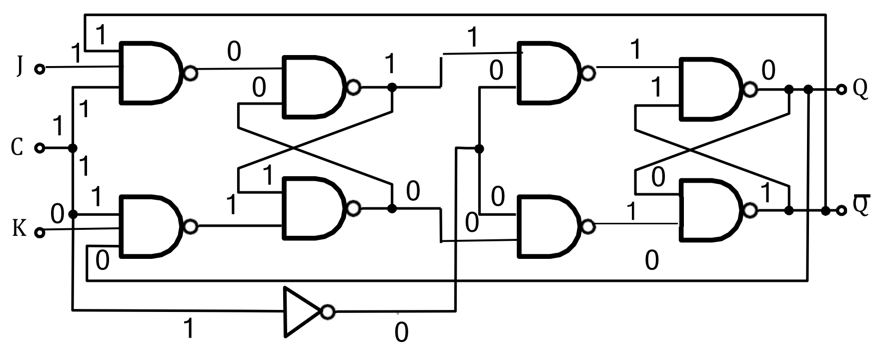
Problem 9.1: JK flip-flops

JK flip-flops, also colloquially known as jump/kill flip-flops, augment the behaviour of SR flip-flops. The letters J and K were presumably picked by Eldred Nelson in a patent application.

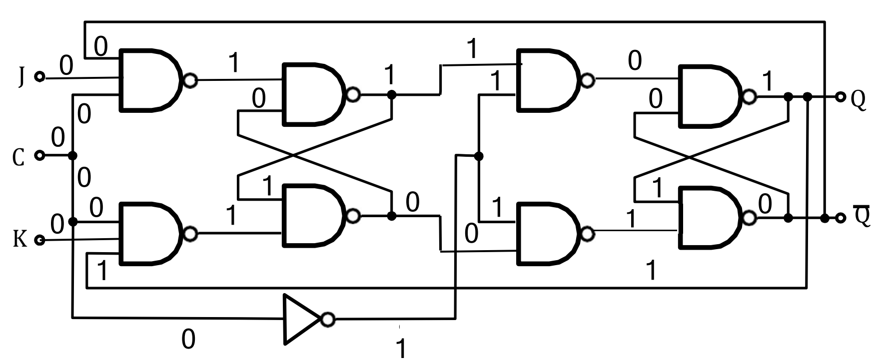
The sequential digital circuit shown below shows the design of a JK flip-flop based on two SR NAND latches. Assume the circuit's output is $Q = 0$ and that the inputs are $J = 0$ and $K = 0$, and that the clock input is $C = 0$. (You can make use of the fact that we already know how an SR NAND latch behaves.)



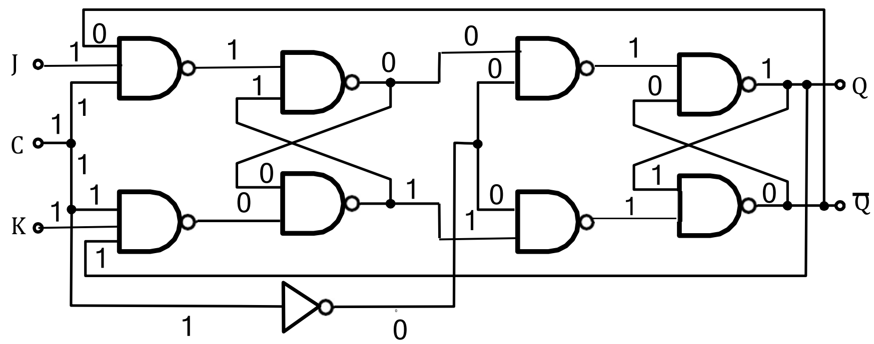
a) Suppose J transitions to 1 and C transitions to 1 soon after. Create a copy of the drawing and indicate for each line whether it carries a 0 or a 1.



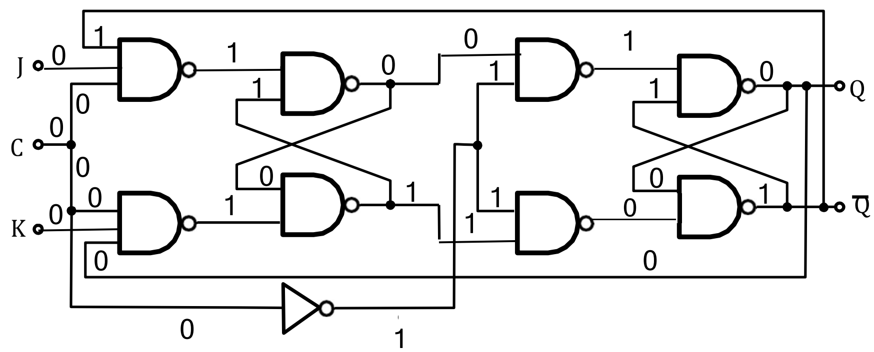
b) Some time later, C transitions back to 0 and soon after J transitions to 0 as well. Create another copy of the drawing and indicate for each line whether it carries a 0 or a 1



c) Some time later, J and K both transition to 1 and C transitions to 1 soon after. Create another copy of the drawing and indicate for each line whether it carries a 0 or a 1.



d) Finally, C transitions back to 0 and soon after J and K both transition to 0 as well. Create another copy of the drawing and indicate for each line whether it carries a 0 or a 1.



Problem 9.2: fold function duality theorems

The fold functions compute a value over a list (or some other type that is foldable) by applying an operator to the list elements and a neutral element. The foldl function assumes that the operator is left associative, the foldr function assumes that the operator is right associative. For example, the function application

foldl (+) 0 [3,5,2,1]

results in the computation of ((((0+3)+5)+2)+1) and the function application

foldr (+) 0 [3,5,2,1]

results in the computation of (3+(5+(2+(1+0)))). The value computed by the fold functions may be more complex than a simple scalar. It is very well possible to construct a new list as part of the fold. For example:

```haskell
map' :: (a -> b) -> [a] -> [b]
map' f xs = foldr ((:) . f) [] xs
```

The evaluation of map' succ [1,2,3]results in the list [2,3,4]. There are several duality theorems that can be stated for fold functions. Prove the following three duality theorems:

a) Let op be an associative operation with e as the neutral element:

op is associative: (x op y) op z = x op (y op z)

e is neutral element: e op x = x and x op e = x

Then the following holds for finite lists xs:

foldr op e xs = foldl op e xs   let xs = [1,2,3]

⇒ (1 op (2 op (3 op e))) = (((e op 1) op 2) op 3)

(1 op (2 op (e op 3))) = (((e op 1) op 2) op 3)   (e is neutral element)

((1 op (2 op e) op 3)) = (((e op 1) op 2) op 3)   (op is associative)

((1 op (e op 2) op 3)) = (((e op 1) op 2) op 3)   (e is neutral element)

(((1 op e) op 2) op 3) = (((e op 1) op 2) op 3)   (op is associative)

(((e op 1) op 2) op 3) = (((e op 1) op 2) op 3)   (e is neutral element)

⇒ foldr op e xs = foldl op e xs holds for finite lists xs

b) Let op1 and op2be two operations for which holds.

x `op1` (y `op2` z) = (x `op1` y) `op2` z
x `op1` e = e `op2` x

Then the following holds for finite lists xs:

foldr op1 e xs = foldl op2 e xs   let xs = [1,2,3]

⇒ (1 `op1` (2 `op1` (3 `op1` e))) = (((e `op2` 1) `op2` 2) `op2` 3)

(1 `op1` (2 `op1` (e `op2` 3))) = (((e `op2` 1) `op2` 2) `op2` 3)   (x `op1` e = e `op2` x)

((1 `op1` (2 `op1` e) `op2` 3)) = (((e `op2` 1) `op2` 2) `op2` 3)   (x `op1` (y `op2` z) = (x `op1` y) `op2` z)

((1 `op1` (e `op2` 2) `op2` 3)) = (((e `op2` 1) `op2` 2) `op2` 3)   (x `op1` e = e `op2` x)

(((1 `op1` e) `op2` 2) `op2` 3) = (((e `op2` 1) `op2` 2) `op2` 3)   (x `op1` (y `op2` z) = (x `op1` y) `op2` z)

(((e `op2` 1) `op2` 2) `op2` 3) = (((e `op2` 1) `op2` 2) `op2` 3)   (x `op1` e = e `op2` x)

⇒ foldr op1 e xs = foldl op2 e xs holds for finite lists xs

c) Let op be an associative operation and xs a finite list. (op is associative: (x op y) op z = x op (y op z))

foldr op a xs = foldl op' a (reverse xs)

Then holds with

<u>x op' y = y op x</u>

foldr op a xs = foldl op' a (reverse xs)   let xs = [1,2] ⇒

foldr op a [1,2] = foldl op' a [2,1]

⇒ (1 op (2 op a)) = ((a op' 2) op' 1)

((1 op 2) op a) = (a op' (2 op' 1))

Which is true ⇔ 1. 1 op 2 = 2 op' 1 = b ⇒ x op' y = y op x

2. b op a = a op' b ⇒ x op' y = y op x

Then this holds that if foldr op a xs = foldl op' a (reverse xs) then it follows that x op' y = y op x.