Network Calculus Tests – Feed Forward (FF) Networks

Version 2.0 beta2 (2017-Jun-25)



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General Information

- The network calculus analyses presented in this document were created for the purpose of testing the Disco Deterministic Network Calculator (DiscoDNC)¹ an open-source deterministic network calculus tool developed by the Distributed Computer Systems (DISCO) Lab at the University of Kaiserslautern.
- Naming of the individual network settings depicts the name of the according functional test for the DiscoDNC.
- The naming scheme used in this document is detailed in NetworkCalculus NamingScheme.pdf.
- Arrival bounds for PmooArrivalBound.java and analyses using them are listed only if results differ from PbooArrivalBound_Concatenation.java.

Changelog:

Version 1.1 (2014-Dec-30):

- Streamlined the PMOO left-over latency $T_{\mathrm{e}2\mathrm{e}}^{\mathrm{l.o.}f}$ computation.
- Adaption to naming scheme version 1.1.

Version 2.0 beta (2015-Jul-11):

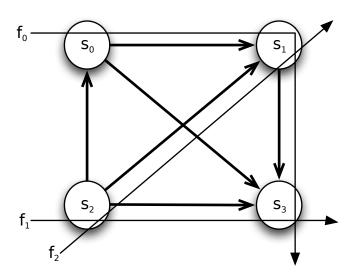
- Rework of Arrival Bounds documentation
 - Parameters: see DiscoDNC's computeArrivalBounds(Server server, Set<Flow> flows_to_bound, Flow flow_of_interest).
 - Bounding arrivals moved to the analysis requiring the specific bounds if they differ between flows of interest (may cause duplication).
 - The algebraic derivations is included within many tabular bounding procedures. They are adapted to PbooArrivalBound_Concatenation.java, yet, in contrast to the current DiscoDNC code, they may reuse known results.
- The naming scheme was slightly updated to include sets of servers S and sets of Flows F.
- Minor consistency fixes for variable names.

Version 2.0 beta2 (2017-Jun-25):

- Rework of the documentation according to code changes
 - New, more complete naming.
 - Separation of network and test.

¹http://disco.cs.uni-kl.de/index.php/projects/disco-dnc

$FF_4S_1SC_3F_1AC_3P_Network$



$$S = \{s_0, s_1, s_2, s_3\}$$
 with

$$\beta_{s_0} = \beta_{s_1} = \beta_{s_2} = \beta_{s_3} = \beta_{R_{s_i},T_{s_i}} = \beta_{20,20}, \ i \in \{0,1,2\}$$

$$\mathbb{F} = \{f_0, f_1, f_2\} \text{ with }$$

$$\alpha^{f_0} = \alpha^{f_1} = \alpha^{f_2} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{5,25}, \ n \in \{0, 1, 2\}$$

$FF_4S_1SC_3F_1AC_3P_Test$

Flow f_0

Total Flow Analysis

Arrival Bounds

$(s_1, \{f_0\}, \emptyset) \eqqcolon \alpha_{s_1}^{f_0}$		FIFO Multiplexing	Arbitrary Multiplexing		
	$\alpha_{s_1}^{f_0} = \alpha^{f_0} \oslash \beta_{s_0}^{\text{l.o.}f_0}$				
$\alpha_{s_0}^{x(f_0)}$		=	$=\gamma_{0,0}$		
$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^x$	(f_0)	_ 8	_ 0		
$= \beta_{R_{s_0}^{\text{l.o.}f_0}, T_{s_0}^{\text{l.}}}$	$= \beta_{R_{s_0}^{1.0.f_0}, T_{s_0}^{1.0.f_0}}$		$=\beta_{s_0}=\beta_{20,20}$		
$\alpha_{s_1}^{f_0} = \alpha^{f_0} \oslash \beta_{s_0}^{\text{l.o.} f_0}$			= 5		
$= \gamma_{f_0,f_0}$		= 0	$\alpha^{f_0}\left(T^{\text{l.o.}}\right)$		
$r_{s_1}^{r_{s_1}, b_{s_1}^{r_{s_1}}}$	$= \qquad \gamma_{r_{s_1}^{f_0}, b_{s_1}^{f_0}} \left \begin{array}{c} b_{s_1}^{f_0} \end{array} \right $		$5 \cdot 20 + 25$		
			125		
	=	=	$\gamma_{5,125}$		

$(s_3, \{f_1\}, \emptyset) =: \alpha_s^{f_1}$ $(s_1, \{f_2\}, \emptyset) =: \alpha_s^{f_n}$ $=: \alpha_{s_i}^{f_n} \text{ with } (n, i) \in \{(1, 3)\}$	f ₂ 1	FIFO Multiplexing	Arbitrary Multiplexing	
	α	$\beta_{s_i}^{f_n} = \alpha^{f_n} \oslash \beta_{s_2}^{\text{l.o.}f_n}$		
$\alpha_{s_2}^{x(f_n)}$		=	$\gamma_{5,25}$	
$\beta_{s_2}^{\text{l.o.}f_n} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_n)}$	$R_{s_2}^{\mathrm{l.o.}f_n}$			
$= \beta_{R_{s_2}^{1.0.f_n}, T_{s_2}^{1.0.f_n}}$		$\beta_{s_2} = b_{s_2}^{x(f_n)}$	$= 15$ $\beta_{s_2} = \alpha_{s_2}^{x(f_n)}$	
	$T_{s_2}^{\mathrm{l.o.}f_n}$	$20 \cdot \left[t - 20\right]^+ = 25$	$20 \cdot [t - 20]^+ = 5 \cdot t + 25$	
	- 2	$t = 21\frac{1}{4}$)	
	=	$=\beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$	
$\alpha_{s_i}^{f_n} = \alpha^{f_n} \oslash \beta_{s_2}^{\text{l.o.} f_n}$	$r_{s_i}^{f_n}$	I .	= 5	
$= \gamma_{r_{s_i}^{f_n}, b_{s_i}^{f_n}}$		$= \alpha^{f_n}(T_{s_2}^{\text{l.o.}f_n})$	$= \alpha^{f_n}(T_{s_2}^{\mathbf{l.o.}f_n})$	
	$b_{s_i}^{f_n}$	$= 5 \cdot 21\frac{1}{4} + 25$	$= 5 \cdot 28 \frac{1}{3} + 25$	
		$= 131\frac{1}{4}$	$=$ $166\frac{2}{3}$	
	=	$=\gamma_{5,131\frac{1}{4}}$	$=\gamma_{5,166\frac{2}{3}}$	

$(s_3, \{f_0\}, \emptyset) =: \alpha_{s_3}^{f_0}$		FIFO Multiplexing	Arbitrary Multiplexing	
	$\alpha_{s_3}^{f_0} =$	$lpha^{f_0} \oslash \left(eta_{s_0}^{\mathrm{l.o.}f_0} \otimes eta_{s_1}^{\mathrm{l.o.}f_0} ight)$		
		(reuse of previous resu	lt)	
	=	$\alpha^{f_0} \oslash \beta_{s_0}^{\mathrm{l.o.}f_0} \oslash \beta_{s_1}^{\mathrm{l.o}}$	$.f_0$	
	=	$lpha_{s_1}^{f_0}\oslasheta_{s_1}^{ ext{l.o}}$.f ₀	
$lpha_{s_1}^{x(f_0)}$		$=\alpha_{s_1}^{f_2} = \gamma_{5,131\frac{1}{4}}$	$=\alpha_{s_1}^{f_2} = \gamma_{5,166\frac{2}{3}}$	
alo f_2 $x(f_2)$	$R_{s_1}^{\mathrm{l.o.}f_0}$	$R_{s_1} - r_{s_1}^{x_0}$	(f_0) $\Big ^+ = 20 - 5$	
$\beta_{s_1}^{\text{l.o.}f_0} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_0)}$	r_{s_1}	_	= 15	
$= \beta_{R_{s_1}^{\mathrm{l.o.}f_0}, T_{s_1}^{\mathrm{l.o.}f_0}}$		$\beta_{s_1} = b_{s_1}^{x(f_0)}$	$\beta_{s_1} = \alpha_{s_1}^{x(f_0)}$	
	$T_{s_1}^{\mathrm{l.o.}f_0}$	$20 \cdot [t - 20]^+ = 131\frac{1}{4}$	$20 \cdot [t - 20]^{+} = 5 \cdot t + 166 \frac{2}{3}$	
			$t = 37\frac{7}{8}$	
	=		$=\beta_{15,37\frac{7}{8}}$	
$lpha_{s_1}^{f_0}$		$=\gamma_{5,125}$		
$\alpha_{s_3}^{f_0} = \alpha^{f_0} \oslash \beta_{s_1}^{\text{l.o.} f_0}$	$r_{s_3}^{f_0}$	f (10 f)	=5	
$= \gamma_{r_{s_3}^{f_0}, b_{s_3}^{f_0}}$		$= \alpha_{s_1}^{f_0} \left(T_{s_1}^{\text{l.o.} f_0} \right)$	$= \alpha_{s_1}^{f_0} \left(T_{s_1}^{\text{l.o.}f_0} \right)$	
/ \$3,0\$3	$b_{s_3}^{f_0}$	$= 5 \cdot 26 \frac{9}{16} + 125$	$= 5 \cdot 37\frac{7}{8} + 125$	
		$=$ $257\frac{13}{16}$	$=$ $313\frac{8}{9}$	
	=	$=\gamma_{5,257\frac{13}{16}}$	$=\gamma_{5,313rac{8}{9}}$	

Analysis

TFA		FIFO Multiplexing	Arbitrary Multiplexing
	α_{s_0}	$= \alpha_{s_0}^{f_0} = \gamma_{5,25}$	
s_0	30		FIFO per micro flow
		$\beta_{s_0} = b_{s_0}$	$eta_{s_0} = b_{s_0}$
	$D_{s_0}^{f_0}$	$20 \cdot \left[t - 20\right]^+ = 25$	$20 \cdot [t - 20]^+ = 25$
		$t = 21\frac{1}{4}$	$t = 21\frac{1}{4}$
		4	
	$B_{s_0}^{f_0}$	$\alpha_{s_0}\left(T_{s_0}\right) =$	
	1	=	$\frac{125}{f_0 \dots f_0}$
		$= \alpha_{s_1}^{f_0} + \alpha_{s_1}^{f_2}$	$= \alpha_{s_1}^{f_0} + \alpha_{s_1}^{f_2}$
	α_{s_1}	$= \gamma_{5,125} + \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,125} + \gamma_{5,166\frac{2}{3}}$
s_1		$= \gamma_{10,256\frac{1}{4}} \\ \beta_{s_1} = b_{s_1}$	$= \gamma_{10,291\frac{2}{3}}$ $\beta_{s_1} = \alpha_{s_1}$
	$D_{s_1}^{f_0}$	$20 \cdot [t - 20]^+ = 256 \frac{1}{4}$	$20 \cdot [t - 20]^{+} = 10 \cdot t + 291\frac{2}{3}$
	81	, 13	1
		$t = 32\frac{16}{16}$	6
	- f	$t = 32\frac{13}{16}$ $\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 256\frac{1}{4}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 291\frac{2}{3}$
	$B_{s_1}^{f_0}$		2
		$=$ $456\frac{1}{4}$	9
		$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1}$	$= \qquad \qquad \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1}$
	α_{s_3}	$= \gamma_{5,257\frac{13}{16}} + \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,313\frac{8}{9}} + \gamma_{5,166\frac{2}{3}}$
s_3		$= \gamma_{10,389\frac{1}{16}} \beta_{s_3} = b_{s_3}$	$= \gamma_{10,480\frac{5}{9}}$
			$\beta_{s_3} = \alpha_{s_3}$
	$D_{s_3}^{f_0}$	$20 \cdot [t - 20]^+ = 389 \frac{1}{16}$	$20 \cdot [t - 20]^{+} = 10 \cdot t + 480 \frac{5}{9}$
	S_3		_
		$t = 39\frac{29}{64}$	$t = 88\frac{5}{90}$
		$\alpha_{s_3}(T_{s_3}) = 10 \cdot 20 + 389 \frac{1}{16}$	$\alpha_{s_3}(T_{s_3}) = 10 \cdot 20 + 480\frac{5}{9}$
	$B_{s_3}^{f_0}$		5
		$=$ $589\frac{1}{16}$	
		$= \sum D_{s_i}^{f_0}$	$= \sum D_{s_i}^{f_0}$
	D^{f_0}	$i=\{0,1,3\}$	$i=\{0,1,3\}$
		$=$ 93 $\frac{33}{64}$	$=$ $178\frac{17}{36}$
		$= \max_{i=\{0,1,3\}} B_{s_i}^{f_0}$	$= \max_{i=\{0,1,3\}} B_{s_i}^{f_0}$
	B^{j_0}		* ' ' '
		$=$ $589\frac{1}{16}$	$=$ $680\frac{5}{9}$

Separate Flow Analysis and PMOO Analysis

Arrival Bounds

$(s_3, \{f_1\}, f_0) =: \alpha_{s_3}^{f_1} (s_1, \{f_2\}, f_0) =: \alpha_{s_1}^{f_2} =: \alpha_{s_i}^{f_n} \text{ with } (n, i) \in \{(1, 3), (2, 1)\}$		FIFO Multiplexing	Arbitrary Multiplexing
	α	$\frac{f_n}{s_i} = \alpha^{f_n} \oslash \beta_{s_2}^{\text{l.o.}f_n}$	
$\alpha_{s_2}^{x(f_n)}$		=	= 75,25
$\beta_{s_2}^{\text{l.o.}f_n} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_n)}$	$R_{s_2}^{\mathrm{l.o.}f_n}$	$\left[R_{s_2} - r_{s_2}^{x(f_n)} \right]^+ = 20 - 5$	
$= \beta_{R_{s_2}^{1.0.f_n}, T_{s_2}^{1.0.f_n}}$		$\beta_{s_2} = b_{s_2}^{x(f_n)} \qquad \qquad \beta_{s_2} = \alpha_s^x$	
	$T_{s_2}^{\mathrm{l.o.}f_n}$	$20 \cdot [t - 20]^+ = 25$	$20 \cdot [t - 20]^+ = 5 \cdot t + 25$
	2	$t = 21\frac{1}{4}$	$t = 28\frac{1}{3}$
	=		$=\beta_{15,28\frac{1}{3}}$
$\alpha_{s}^{f_n} = \alpha^{f_n} \oslash \beta_{s_0}^{\text{l.o.} f_n}$	$r_{s_i}^{f_n}$		= 5
$\alpha_{s_i}^{f_n} = \alpha^{f_n} \oslash \beta_{s_2}^{\text{l.o.}f_n}$ $= \gamma_{r_{s_i}^{f_n}, b_{s_i}^{f_n}}$		$= \alpha^{f_n} \left(T_{s_2}^{\text{l.o.}f_n} \right)$	$= \alpha^{f_n} \left(T_{s_2}^{\text{l.o.} f_n} \right)$
	$b_{s_i}^{f_n}$	$= 5 \cdot 21 \frac{1}{4} + 25$	$= 5 \cdot 28\frac{1}{3} + 25$
		$=$ $131\frac{1}{4}$	$=$ $166\frac{2}{3}$
Dl	=	$=\gamma_{5,131\frac{1}{4}}$	$=\gamma_{5,166\frac{2}{3}}$

Remark

In this network setting, we have $(s_3, \{f_1\}, f_0) = (s_3, \{f_1\}, \emptyset)$ and $(s_1, \{f_2\}, f_0) = (s_1, \{f_2\}, \emptyset)$ because neither (cross-)flow f_1 nor f_2 interferes with the flow of interest f_0 on multiple consecutive hops.

Analyses

SFA		FIFO Multiplexing Arbitrary Multiplexing		
0.0	s_0 a_{s_0} a_{s_0}		$=\gamma_{0,0}$	
30	$eta_{s_0}^{ ext{l.o.}f_0}=eta_{s_0}\ominuslpha_{s_0}^{x(f_0)}$		$=\beta_{20,20}$	
	$\alpha_{s_1}^{x(f_0)}$		$=\alpha_{s_1}^{f_2} = \gamma_{5,131\frac{1}{4}}$	$=\alpha_{s_1}^{f_2} = \gamma_{5,166\frac{2}{3}}$
s_1	$\beta_{s_1}^{\text{l.o.}f_0} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_0)}$	$R_{s_1}^{\mathrm{l.o.}f_0}$	$\left[R_{s_1} - r_{s_1}^{x(f_0)} \right]$	$\begin{bmatrix} = \alpha_{s_1}^2 = \gamma_{5,166\frac{2}{3}} \\ = 20 - 5 \end{bmatrix}$
	$= \beta_{R_{s_1}^{\text{l.o.}f_0}, T_{s_1}^{\text{l.o.}f_0}}$			$= 15$ $\beta_{s_1} = \alpha_{s_1}^{x(f_0)}$
	$R_{s_1}^{norj_0}, T_{s_1}^{norj_0}$		$\beta_{s_1} = b_{s_1}^{x(f_0)}$	- 1
		$T_{s_1}^{\mathrm{l.o.}f_0}$	$20 \cdot [t - 20]^+ = 131\frac{1}{4}$	$20 \cdot [t - 20]^{+} = 5 \cdot t + 166 \frac{2}{3}$
			$t = 26\frac{9}{16}$ $= \beta_{15,26\frac{9}{16}}$	$t = 37\frac{7}{9}$
		=		$=\beta_{15,37\frac{7}{9}}$
	$\alpha_{s_3}^{x(f_0)}$		$=\alpha_{s_3}^{f_1} = \gamma_{5,131\frac{1}{4}}$	$=\alpha_{s_3}^{f_1} = \gamma_{5,166\frac{2}{3}}$
s_3			$\left[R_{s_3} - r_{s_3}^{x(f_0)}\right]$	$\begin{bmatrix} + & 20 - 5 \end{bmatrix}$
	$= \beta_{R_{s_3}^{1.0.f_0}, T_{s_3}^{1.0.f_0}}$		$a = a x(f_0)$	= 15
	R_{s_3} $, T_{s_3}$		$\beta_{s_3} = b_{s_3}^{x(f_0)}$	$\beta_{s_3} = \alpha_{s_3}^{x(f_0)}$
		$T_{s_3}^{\mathrm{l.o.}f_0}$	$20 \cdot [t - 20]^+ = 131\frac{1}{4}$	$20 \cdot [t - 20]^{+} = 5 \cdot t + 166 \frac{2}{3}$
			$t = 26\frac{9}{16}$	$t = 37\frac{7}{9}$
		=	$=\beta_{15,26\frac{9}{16}}$	$=\beta_{15,37\frac{7}{9}}$
			$= \bigotimes \beta_{s_i}^{\text{l.o.}f_0}$	$=$ \bigotimes $\beta_{s_i}^{\text{l.o.}f_0}$
ß	$\beta_{\langle s_0, s_1, s_3 \rangle}^{\text{l.o.} f_0} = \beta_{R_{\langle s_0, s_1, s_3 \rangle}^{\text{l.o.} f_0}, T_{\langle s_0 \rangle}^{\text{l.o.}}}$	f_0	$i=\{0,1,3\}$	$i=\{0,1,3\}$
	$\langle s_0, s_1, s_3 \rangle' \langle s_0, s_1, s_3 \rangle$		$= \beta_{15,73\frac{1}{8}} \beta_{\langle s_0, s_1, s_3 \rangle}^{\text{l.o.}f_0} = b^{f_0}$	$= \beta_{15,95\frac{5}{9}}$ $\beta_{\langle s_0, s_1, s_3 \rangle}^{\text{l.o.} f_0} = b^{f_0}$
			1	
	D^{f_0}		$15 \cdot \left[t - 73\frac{1}{8}\right]^{+} = 25$	$15 \cdot \left[t - 95 \frac{5}{9} \right]^+ = 25$
			$t = 74\frac{19}{24}$	$t = 97\frac{2}{9}$
	B^{f_0}		$t = 74 \frac{19}{24}$ $\alpha^{f_0} \left(T_{\langle s_0, s_1, s_3 \rangle}^{\text{l.o.} f_0} \right) = 5 \cdot 73 \frac{1}{8} + 25$	$t = 97\frac{2}{9}$ $\alpha^{f_0}\left(T_{\langle s_0, s_1, s_3\rangle}^{\text{l.o.}f_0}\right) = 5 \cdot 95\frac{5}{9} + 25$
			$= 390\frac{5}{8}$	$= 502\frac{7}{9}$

PMOO		Arbitrary Multiplexing
s_0	$\frac{\alpha_{s_0}^{x(f_0)}}{\alpha_{s_0}^{\bar{x}(f_0)}}$	$=\gamma_{0,0}$
s_1	$\alpha_{s_1}^{x(f_0)}$ $\alpha_{s_1}^{\bar{x}(f_0)}$ $\alpha_{s_1}^{\bar{x}(f_0)}$	$=\alpha_{s_1}^{f_2}=\gamma_{5,166\frac{2}{3}}$
s_3	$\begin{array}{c} \alpha_{s_3}^{x(f_0)} \\ \alpha_{s_3}^{\bar{x}(f_0)} \end{array}$	$=\alpha_{s_3}^{f_1}=\gamma_{5,166\frac{2}{3}}$
$\beta^{\text{l.o.}f_0}_{\langle s_0,s_1,s_3\rangle} = \beta_{R^{\text{l.o.}f_0}_{\langle s_0,s_1,s_3\rangle}, T^{\text{l.o.}f_0}_{\langle s_0,s_1,s_3\rangle}}$	$R^{\mathrm{l.o.}f_0}_{\langle s_0,s_1,s_3\rangle}$	$= \bigwedge_{i \in \{0,1,3\}} \left(R_{s_i} - r_{s_i}^{x(f_0)} \right)$ $= (20 - 5) \wedge (20 - 5) \wedge (20 - 5)$ $= 15$
	g 0, fo	$= \sum_{i \in \{0,1,3\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_0)} + r_{s_i}^{x(f_0)} \cdot T_{s_i}}{R_{\text{e}2e}^{1.0 \cdot f_0}} \right)$ $= 20 + \frac{0 + 0 \cdot 20}{15} + 20 + \frac{166\frac{2}{3} + 5 \cdot 20}{15} + 20 + \frac{166\frac{2}{3} + 5 \cdot 20}{15}$
$\left egin{array}{c} T_{\langle s_0,s_1,s_3 angle}^{ ext{1.0.}J_0} \end{array} ight $		=
	=	$=\beta_{15,95\frac{5}{9}}$
D^{f_0}		$=\beta_{15,95\frac{5}{9}}$ $\beta_{\langle s_0,s_1,s_3\rangle}^{\text{l.o.}f_0} = b^{f_0}$ $15 \cdot \left[t - 95\frac{5}{9}\right]^+ = 25$ $t = 97\frac{2}{9}$ $\alpha^{f_0}\left(T_{\langle s_0,s_1,s_3\rangle}^{\text{l.o.}f_0}\right) = 5 \cdot 95\frac{5}{9} + 25$
B^{f_0}		$\alpha^{f_0} \left(T_{\langle s_0, s_1, s_3 \rangle}^{\text{l.o.} f_0} \right) = 5 \cdot 95 \frac{5}{9} + 25$ $= 502 \frac{7}{9}$

Flow f_1

Total Flow Analysis

Arrival Bounds

$(s_3, \{f_1\}, \emptyset) =: \alpha_{s_3}^{f_1} (s_1, \{f_2\}, \emptyset) =: \alpha_{s_1}^{f_2} =: \alpha_{s_i}^{f_n} \text{ with } (n, i) \in \{(1, 3), (2, 1)\}$		FIFO Multiplexing	Arbitrary Multiplexing	
S_i		$\frac{f_n}{s_i} = \alpha^{f_n} \oslash \beta_{s_2}^{\text{l.o.} f_n}$		
$\alpha_{s_2}^{x(f_n)}$		=	$\gamma_{5,25}$	
$\beta_{s_2}^{\text{l.o.}f_n} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_n)}$	$\beta_{s_2}^{\text{l.o.}f_n} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_n)} \qquad R_{s_2}^{\text{l.o.}f_n}$		$ \left[R_{s_2} - r_{s_2}^{x(f_n)} \right]^+ = 20 - 5 $ $= 15 $	
$= \beta_{R_{s_2}^{1.\circ.f_n}, T_{s_2}^{1.\circ.f_n}}$		$\beta_{s_2} = b_{s_2}^{x(f_n)}$		
	$T_{s_2}^{\mathrm{l.o.}f_n}$	$20 \cdot \left[t - 20\right]^+ = 25$	$20 \cdot [t - 20]^+ = 5 \cdot t + 25$	
	02	$t = 21\frac{1}{4}$		
=		$=\beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$	
$\alpha_{s_i}^{f_n} = \alpha^{f_n} \oslash \beta_{s_2}^{\text{l.o.} f_n}$	$r_{s_i}^{f_n}$		=5	
$= \gamma_{r_{s_i}^{f_n}, b_{s_i}^{f_n}} = \gamma_{r_{s_i}^{f_n}, b_{s_i}^{f_n}}$		$= \alpha^{f_n} \left(T_{s_2}^{\text{l.o.} f_n} \right)$	$= \alpha^{f_n} \left(T_{s_2}^{\text{l.o.} f_n} \right)$	
	$b_{s_i}^{f_n}$	$= 5 \cdot 21\frac{1}{4} + 25$	$= 5 \cdot 28\frac{1}{3} + 25$	
		$= 131\frac{1}{4}$	$=$ $166\frac{2}{3}$	
	=	$=\gamma_{5,131\frac{1}{4}}$	$=\gamma_{5,166\frac{2}{3}}$	

$(s_3, \{f_0\}, \emptyset) =: \alpha_{s_3}^{f_0}$		FIFO Multiplexing	Arbitrary Multiplexing
	$\alpha_{s_3}^{f_0} =$	$\alpha^{f_0} \oslash \left(\beta_{s_0}^{\mathrm{l.o.}f_0} \otimes \beta_{s_1}^{\mathrm{l.o.}f_0}\right)$	
$lpha_{s_0}^{x(f_0)}$			$=\gamma_{0,0}$
$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)}$		_ 2	$s_0 = \beta_{20,20}$
$= \beta_{R_{s_0}^{1.0.f_0}, T_{s_0}^{1.0.f_0}}$		$-\rho$	$s_0 - \rho_{20,20}$
$lpha_{s_1}^{x(f_0)}$		$=\alpha_{s_1}^{f_2}=\gamma_{5,131\frac{1}{4}}$	$=\alpha_{s_1}^{f_2} = \gamma_{5,166\frac{2}{3}}$
$\beta_{s_1}^{\text{l.o.}f_0} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_0)}$	$R_{s_1}^{\mathrm{l.o.}f_0}$	$\left[R_{s_1} - r_{s_1}^{x_0}\right]$	(f_0) $\Big]^+ = 20 - 5$
$= \beta_{R_{s_1}^{\text{l.o.}}f_0, T_{s_1}^{\text{l.o.}}f_0}$		(f)	= 15
$R_{s_1}^{n.o.j_0}, T_{s_1}^{n.o.j_0}$		$\beta_{s_1} = b_{s_1}^{x(f_0)}$	
	$T_{s_1}^{\mathrm{l.o.}f_0}$	$20 \cdot [t - 20]^+ = 131\frac{1}{4}$	$20 \cdot [t - 20]^{+} = 5 \cdot t + 166 \frac{2}{3}$
		$t = 26\frac{9}{16}$	$t = 37\frac{7}{8}$
	=	$= \beta_{15,26\frac{9}{16}}$	$=\beta_{15,37\frac{7}{8}}$
$\beta_{s_0}^{\mathrm{l.o.}f_0} \otimes \beta_{s_1}^{\mathrm{l.o.}f_0} = \beta_{\langle s_0, s_1}^{\mathrm{l.o.}f_0}$		$= \beta_{20,20} \otimes \beta_{15,26\frac{9}{16}}$	$= \beta_{20,20} \otimes \beta_{15,37\frac{7}{8}}$
$\rho_{s_0} \circ \otimes \rho_{s_1} \circ - \rho_{\langle s_0, s_1 \rangle}$,	$=$ $\beta_{15,46\frac{9}{16}}$	$=$ $\beta_{15,57\frac{7}{8}}$
	$r_{s_3}^{f_0}$		=5
$\alpha_{s_3}^{f_0} = \alpha^{f_0} \oslash \left(\beta_{s_0}^{\text{l.o.}f_0} \otimes \beta_{s_1}^{\text{l.o.}f_0}\right)$		$= \alpha_{s_1}^{f_0} \left(T_{\langle s_0, s_1 \rangle}^{\text{l.o.} f_0} \right)$	$= \alpha_{s_1}^{f_0} \left(T_{\langle s_0, s_1 \rangle}^{\text{l.o.} f_0} \right)$
	$b_{s_3}^{f_0}$	$= 5 \cdot 46 \frac{9}{16} + 25$	$= 5 \cdot 57\frac{7}{8} + 25$
		$=$ $257\frac{13}{16}$	$=$ $313\frac{8}{9}$
	=	$= \gamma_{5,257\frac{13}{16}}$	$=\gamma_{5,313\frac{8}{9}}$

Remark:

 ${\tt PmooArrivalBound.java}$ will have the same result as ${\tt PbooArrivalBound_Concatenation.java}$ because f_0 does not have cross-traffic interfering on multiple consecutive hops.

Analysis

TFA		FIFO Multiplexing	Arbitrary Multiplexing	
		= 0	$\alpha_{s_2}^{f_1} + \alpha_{s_2}^{f_2}$	
	α_{s_2}	$= \gamma_{5,:}$	$_{25} + \gamma_{5,25}$	
s_2		=	$\gamma_{10,50}$	
		$\beta_{s_2} = b_{s_2}$	$\beta_{s_2} = \alpha_{s_2}$	
	$D_{s_2}^{f_1}$	$20 \cdot [t - 20]^+ = 50$	$20 \cdot [t - 20]^+ = 10 \cdot t + 50$	
	32	$t = 22\frac{1}{2}$ $\alpha_{s_2}(T_{s_2}) =$	t = 45	
	Df_1	$\alpha_{s_2}\left(\tilde{T}_{s_2}\right) =$	$20 \cdot 10 + 50$	
	$B_{s_2}^{f_1}$	=	250	
		$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1}$	$= \qquad \qquad \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1}$	
	α_{s_3}	$= \gamma_{5,257\frac{13}{16}} + \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,313\frac{8}{9}} + \gamma_{5,166\frac{2}{3}}$	
s_3		$= \gamma_{10,389\frac{1}{16}} \\ \beta_{s_3} = b_{s_3}$	$= \gamma_{10,480\frac{5}{9}}$	
			$= \gamma_{10,480\frac{5}{9}}$ $\beta_{s_3} = \alpha_{s_3}$ 5	
	$D_{s_3}^{f_1}$	$20 \cdot [t - 20]^+ = 389 \frac{1}{16}$	$20 \cdot [t - 20]^{+} = 10 \cdot t + 480 \frac{5}{9}$	
		$t = 39\frac{29}{64}$	F	
	$B_{s_3}^{f_1}$	$\alpha_{s_3}(T_{s_3}) = 10 \cdot 20 + 389 \frac{1}{16}$	9	
	D_{s_3}	$=$ $589\frac{1}{16}$	$=$ $680\frac{5}{9}$	
		$\sum_{i=1}^{3} D_{i} f_{1}$	$\sum_{i=1}^{3} D_{i} f_{1}$	
D^{f_1}		$= \sum_{i=2} D_{s_i}^{f_1}$	$= \sum_{i=2} D_{s_i}^{f_1}$	
		$= 61\frac{61}{64}$	$= 185\frac{5}{9}$	
1	3^{f_1}	$= \max_{i=\{2,3\}} B_{s_i}^{f_1}$	$= \max_{i=\{2,3\}} B_{s_i}^{f_1}$	
	- ×ر 	$= 589\frac{1}{16}$	$=$ $680\frac{5}{9}$	

Separate Flow Analysis and PMOO Analysis

Arrival Bounds

$(s_1, \{f_2\}, \emptyset) \eqqcolon \alpha_{s_1}^{f_2}$		FIFO Multiplexing	Arbitrary Multiplexing			
	$lpha_{s_1}^{f_2} = ~ lpha^{f_2} \oslash eta_{s_2}^{ ext{l.o.}f_2}$					
$\alpha_{s_2}^{x(f_2)}$		=	$\gamma_{5,25}$			
$\beta_{s_2}^{\text{l.o.}f_2} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_2)}$	$R_{s_2}^{\mathrm{l.o.}f_n}$	$\left[R_{s_2} - r_{s_2}^{x(f)}\right]$	$\left[\frac{f_{2}}{f_{2}}\right]^{+} = 20 - 5$			
			= 15			
$= \beta_{R_{s_2}^{\text{l.o.}f_2}, T_{s_2}^{\text{l.o.}f_2}}$		$\beta_{s_2} = b_{s_2}^{x(f_2)}$	$\beta_{s_2} = \alpha_{s_2}^{x(f_2)}$			
	$T_{s_2}^{\mathrm{l.o.}f_n}$	$20 \cdot \left[t - 20\right]^+ = 25$	$20 \cdot [t - 20]^{+} = 5 \cdot t + 25$			
		$t = 21\frac{1}{4}$	$t = 28\frac{1}{3}$			
	=	$=\beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$			
$\alpha_{s_1}^{f_2} = \alpha^{f_2} \oslash \beta_{s_2}^{\text{l.o.} f_2}$	$r_{s_1}^{f_2}$		=5			
$= \gamma_{r_{s_1}^{f_2}, b_{s_1}^{f_2}} = \gamma_{r_{s_1}^{f_2}, b_{s_1}^{f_2}}$		$= \alpha^{f_2} \left(T_{s_2}^{\text{l.o.} f_2} \right)$	$= \alpha^{f_2} \left(T_{s_2}^{\text{l.o.} f_2} \right)$			
$\tau_{s_1, \sigma_{s_1}}$	$b_{s_1}^{f_2}$	$= 5 \cdot 21\frac{1}{4} + 25$	$= 5 \cdot 28 \frac{1}{3} + 25$			
		$=$ $131\frac{1}{4}$	$=$ $166\frac{2}{3}$			
	=	$=\gamma_{5,131\frac{1}{4}}$	$=\gamma_{5,166\frac{2}{3}}$			

$(s_3, \{f_0\}, \emptyset) \eqqcolon \alpha_{s_3}^{f_0}$		FIFO Multiplexing	Arbitrary Multiplexing
	$\alpha_{s_3}^{f_0} =$	$\alpha^{f_0} \oslash \left(\beta_{s_0}^{\mathrm{l.o.}f_0} \otimes \beta_{s_1}^{\mathrm{l.o.}f_0} \right)$	
$lpha_{s_0}^{x(f_0)}$			$=\gamma_{0,0}$
$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)}$		a	
$= \beta_{R_{s_0}^{1.o.f_0}, T_{s_0}^{1.o.f_0}}$ $\alpha_{s_1}^{x(f_0)}$		$=\rho$	$\beta_{s_0} = \beta_{20,20}$
$lpha_{s_1}^{x(f_0)}$		$=\alpha_{s_1}^{f_2} = \gamma_{5,131\frac{1}{4}}$	$=\alpha_{s_1}^{f_2} = \gamma_{5,166\frac{2}{3}}$
$\beta_{s_1}^{\text{l.o.}f_0} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_0)}$	$R_{s_1}^{\mathrm{l.o.}f_0}$	$\left[R_{s_1} - r_{s_1}^{x_0}\right]$	$\binom{f_0}{f}^+ = 20 - 5$
_	- "81	_	= 15
$= \beta_{R_{s_1}^{\text{l.o.}f_0}, T_{s_1}^{\text{l.o.}f_0}}$		$\beta_{s_1} = b_{s_1}^{x(f_0)}$	
	$T_{s_1}^{\mathrm{l.o.}f_0}$	$20 \cdot [t - 20]^+ = 131\frac{1}{4}$	$20 \cdot [t - 20]^{+} = 5 \cdot t + 166 \frac{2}{3}$
		$t = 26\frac{9}{16}$	$t = 37\frac{7}{8}$
	=	$= \beta_{15,26\frac{9}{16}}$	$=\beta_{15,37\frac{7}{8}}$
$\beta_{s_0}^{\text{l.o.}f_0} \otimes \beta_{s_1}^{\text{l.o.}f_0} = \beta_{\langle s_0, s_1}^{\text{l.o.}f_0}$		$= \beta_{20,20} \otimes \beta_{15,26\frac{9}{16}}$	$= \beta_{20,20} \otimes \beta_{15,37\frac{7}{8}}$
$\beta_{s_0} = \beta_{s_1} = \beta_{\langle s_0, s_1 \rangle}$,	$=$ $\beta_{15,46\frac{9}{16}}$	$=$ $\beta_{15,57\frac{7}{8}}$
	$r_{s_3}^{f_0}$		=5
$\alpha_{s_3}^{f_0} = \alpha^{f_0} \oslash \left(\beta_{s_0}^{\text{l.o.}f_0} \otimes \beta_{s_1}^{\text{l.o.}f_0}\right)$		$= \alpha_{s_1}^{f_0} \left(T_{\langle s_0, s_1 \rangle}^{\text{l.o.} f_0} \right)$	$= \alpha_{s_1}^{f_0} \left(T_{\langle s_0, s_1 \rangle}^{\text{l.o.} f_0} \right)$
	$b_{s_3}^{f_0}$	$= 5 \cdot 46 \frac{9}{16} + 25$	$= 5 \cdot 57\frac{7}{8} + 25$
		$= 257 \frac{13}{16}$	$=$ $313\frac{8}{9}$
Pamark:	=	$=\gamma_{5,257\frac{13}{16}}$	$=\gamma_{5,313\frac{8}{9}}$

Remark

 $\label{lower_possible} {\tt PmooArrivalBound_Concatenation.java} \ because \ f_0 \ does \ not \ have \ cross-traffic \ interfering \ on \ multiple \ consecutive \ hops.$

SFA			FIFO Multiplexing Arbitrary Multiplexing	
0.	$\alpha_{s_2}^{x(f_1)}$			
s_2	$\beta_{s_2}^{\text{l.o.}f_1} = \beta_{s_2} \ominus \alpha_{s_2}^x$	(f_1)	_ 8	_ ß
	$= \beta_{R_{s_2}^{1.o.f_1}, T_{s_2}^{1.}}$	o.f ₁	$=\beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$
	$\alpha_{s_3}^{x(f_1)} = \alpha_{s_3}^{f_0}$		$=\gamma_{5,257\frac{13}{16}}$	$=\gamma_{5,313\frac{8}{9}}$
s_3	$\beta_{s_3}^{\text{l.o.}f_1} = \beta_{s_3} \ominus \alpha_{s_3}^{x(f_1)}$	$R_{s_3}^{\mathrm{l.o.}f_1}$	$\left[R_{s_3} - r_{s_3}^{x(f_1)}\right]$	= 20 - 5
	$= \beta_{R_{s_3}^{\text{l.o.}f_1}, T_{s_3}^{\text{l.o.}f_1}}$		$\beta_{s_3} = b_{s_3}^{x(f_1)}$	$= 15$ $\beta_{s_3} = \alpha_{s_3}^{x(f_1)}$
		$T_{s_3}^{\mathrm{l.o.}f_1}$	$20 \cdot [t - 20]^+ = 257 \frac{13}{16}$	$20 \cdot [t - 20]^{+} = 5 \cdot t + 313 \frac{8}{9}$
			$t = 32\frac{57}{64}$	$t = 47\frac{16}{27}$
		=	$=\beta_{15,32\frac{57}{64}}$	$=\beta_{15,47\frac{16}{27}}$
	$\beta_{\langle s_2, s_3 \rangle}^{\text{l.o.}f_1} = \beta_{R_{\langle s_2, s_3 \rangle}^{\text{l.o.}f_1}, T_{\langle s_2, s_3 \rangle}^{\text{l.o.}f_1}}$		$= \bigotimes_{i=2}^{3} \beta_{s_i}^{\text{l.o.} f_1}$	$= \bigotimes_{i=2}^{3} \beta_{s_i}^{\text{l.o.} f_1}$
			$=$ $\beta_{15,54\frac{9}{64}}$	$=$ $\beta_{15,75\frac{25}{27}}$
			$= \beta_{15,54\frac{9}{64}} \beta_{\langle s_2, s_3 \rangle}^{\text{l.o.}f_1} = b^{f_1}$	$= \beta_{15,75\frac{25}{27}} \beta_{\langle s_2, s_3 \rangle}^{\text{l.o.}f_1} = b^{f_1}$
D^{f_1}			$15 \cdot \left[t - 54\frac{9}{64}\right]^+ = 25$	$15 \cdot \left[t - 75 \frac{25}{27} \right]^+ = 25$
			$t = 55\frac{155}{192}$	$t = 77\frac{16}{27}$
B^{f_1}			$t = 55\frac{155}{192}$ $\alpha^{f_1} \left(T_{\langle s_2, s_3 \rangle}^{\text{l.o.} f_1} \right) = 5 \cdot 54\frac{9}{64} + 25$ $- 295\frac{45}{64}$	$t = 77 \frac{16}{27}$ $\alpha^{f_1} \left(T_{\langle s_2, s_3 \rangle}^{\text{l.o.} f_1} \right) = 5 \cdot 75 \frac{25}{27} + 25$
			$= 295\frac{45}{64}$	$=$ $404\frac{17}{27}$

PMOO		Arbitrary Multiplexing
s_2	$\frac{\alpha_{s_2}^{x(f_1)}}{\alpha_{s_2}^{\bar{x}(f_1)}}$	$=lpha_{s_{2}}^{f_{2}}=\gamma_{5,25}$
s_3	$\begin{array}{c} \alpha_{s_3}^{x(f_1)} \\ \alpha_{s_3}^{\bar{x}(f_1)} \end{array}$	$=\alpha_{s_3}^{f_0}=\gamma_{5,313\frac{8}{9}}$
$\beta_{\langle s_2, s_3 \rangle}^{\text{l.o.}f_1} = \beta_{R_{\langle s_2, s_3 \rangle}^{\text{l.o.}f_1}, T_{\langle s_2, s_3 \rangle}^{\text{l.o.}f_1}}$	$R^{\mathrm{l.o.}f_1}_{\langle s_2,s_3 angle}$	$= \bigwedge_{i \in \{2,3\}} \left(R_{s_i} - r_{s_i}^{x(f_1)} \right)$ $= (20 - 5) \wedge (20 - 5)$ $= 15$
		$= \sum_{i \in \{2,3\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_1)} + r_{s_i}^{x(f_1)} \cdot T_{s_i}}{R_{\text{e}2e}^{\text{l.o.} f_1}} \right)$
	$T_{\langle s_2, s_3 \rangle}^{\text{l.o.} f_1}$	$= 20 + \frac{25 + 5 \cdot 20}{15} + 20 + \frac{313\frac{8}{9} + 5 \cdot 20}{15}$
		$= 40 + \frac{538\frac{8}{9}}{15} \\ = 75\frac{25}{27}$
	=	
D^{f_1}		$= \beta_{15,75\frac{25}{27}}$ $\beta_{\langle s_2, s_3 \rangle}^{\text{l.o.} f_1} = b^{f_1}$ $15 \cdot \left[t - 75\frac{25}{27} \right]^+ = 25$
		$t = 77\frac{16}{27}$ $\alpha^{f_1}\left(T_{\langle s_2, s_3 \rangle}^{1.0.f_1}\right) = 5 \cdot 75\frac{25}{27} + 25$
B^{f_1}		$\alpha^{f_1} \left(T_{\langle s_2, s_3 \rangle}^{1.0.f_1} \right) = 5 \cdot 75 \frac{25}{27} + 25$ $= 404 \frac{17}{27}$

Flow f_2

Total Flow Analysis

Arrival Bounds

$(s_1, \{f_0\}, \emptyset) =: \alpha_s^f$	0 1	FIFO Multiplexing	Arbitrary Multiplexing		
$lpha_{s_1}^{f_0} = lpha^{f_0} \oslash eta_{s_0}^{\mathrm{l.o.}f_0}$					
$\alpha_{s_0}^{x(f_0)}$		$=\gamma_{0,0}$			
$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x_0}$	(f_0)	$=\beta_{s_0}=\beta_{20,20}$			
$\alpha_{s_1}^{f_0} = \alpha^{f_0} \oslash \beta_{s_0}^{\text{l.o.} f_0}$	$r_{s_1}^{f_0}$		= 5		
$= \gamma_{r_{s_1}^{f_0}, b_{s_1}^{f_0}}$	$=$ γ_{f_0,f_0}		$\alpha^{f_0}\left(T_{s_0}^{\text{l.o.}}\right)$		
$b_{s_1}^{f_0}$		= 5	$5 \cdot 20 + 25$		
		=	125		
	=	=	$\gamma_{5,125}$		

$(s_1, \{f_2\}, \emptyset) =: \alpha_s^f$	2 1	FIFO Multiplexing	Arbitrary Multiplexing			
	$lpha_{s_1}^{f_2} = lpha^{f_2} \oslash eta_{s_2}^{ ext{l.o.}f_2}$					
$\alpha_{s_2}^{x(f_2)}$		α^{f_1}	$=\gamma_{5,25}$			
$\beta_{s_2}^{\text{l.o.}f_2} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_2)}$	$R_{s_2}^{\mathrm{l.o.}f_2}$	$\left[R_{s_2} - r_{s_2}^{x(f)}\right]$	$\left[R_{s_2} - r_{s_2}^{x(f_2)}\right]^+ = 20 - 5$			
			= 15			
$= \beta_{R_{s_2}^{1.o.f_2}, T_{s_2}^{1.o.f_2}}$			$\beta_{s_2} = \alpha_{s_2}^{x(f_2)}$			
	$T_{s_2}^{\mathrm{l.o.}f_2}$	$20 \cdot [t - 20]^+ = 25$	$20 \cdot [t - 20]^+ = 5 \cdot t + 25$			
	_	$t = 21\frac{1}{4}$	$t = 28\frac{1}{3}$			
	=	$=\beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$			
$\alpha_{s_1}^{f_2} = \alpha^{f_2} \oslash \beta_{s_2}^{\text{l.o.} f_2}$	$r_{s_1}^{f_2}$	= 5				
$= \gamma_{r_{s_1}^{f_2}, b_{s_1}^{f_2}} = \gamma_{r_{s_1}^{f_2}, b_{s_1}^{f_2}}$		$= \alpha^{f_2} \left(T_{s_2}^{\text{l.o.} f_2} \right)$	$= \alpha^{f_2} \left(T_{s_2}^{\text{l.o.} f_2} \right)$			
$ ^{1}r_{s_{1}}^{j_{2}},b_{s_{1}}^{j_{2}}$	$b_{s_1}^{f_2}$	$= 5 \cdot 21\frac{1}{4} + 25$	$= 5 \cdot 28\frac{1}{3} + 25$			
		$=$ $131\frac{1}{4}$	$=$ $166\frac{2}{3}$			
	=	$=\gamma_{5,131\frac{1}{4}}$	$=\gamma_{5,166\frac{2}{3}}$			

Analysis

TFA		FIFO Multiplexing	Arbitrary Multiplexing	
		=	$\alpha^{f_1} + \alpha^{f_2}$	
	α_{s_2}	$= \gamma_5$	$_{,25} + \gamma_{5,25}$	
s_2		=	$\gamma_{10,50}$	
		$\beta_{s_2} = b_{s_2}$	$\beta_{s_2} = \alpha_{s_2}$	
	$D_{s_2}^{f_2}$	$20 \cdot [t - 20]^+ = 50$	$20 \cdot [t - 20]^+ = 10 \cdot t + 50$	
	32	$t = 22\frac{1}{2}$		
		$t = 22\frac{1}{2} \\ \alpha_{s_2}(T_{s_2}) =$	$20 \cdot 10 + 50$	
	$B_{s_2}^{f_2}$	=	250	
		$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1}$	$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1}$	
	α_{s_3}	$= \gamma_{5,125} + \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,125} + \gamma_{5,166\frac{2}{3}}$	
s_1		$= \gamma_{10,256\frac{1}{4}} \\ \beta_{s_1} = b_{s_1}$	$= \gamma_{10,291\frac{2}{3}}$ $\beta_{s_1} = \alpha_{s_1}$	
		$\beta_{s_1} = b_{s_1}$		
	$D_{s_1}^{f_2}$	$20 \cdot [t - 20]^+ = 256 \frac{1}{4}$	$20 \cdot [t - 20]^{+} = 10 \cdot t + 291\frac{2}{3}$	
		$t = 32\frac{13}{16}$	$t = 69\frac{1}{6}$ $\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 291\frac{2}{3}$ $= 491\frac{2}{3}$	
	$B_{s_1}^{f_2}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 256\frac{1}{4}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 291\frac{2}{3}$	
	$D_{s_1}^{s_2}$	$=$ $456\frac{1}{4}$	$=$ $491\frac{2}{3}$	
		$\sum_{r=1}^{2} p_r f_2$	$\sum_{r=1}^{2} D_{r} f_{2}$	
D^{f_2}		$= \sum_{i=1}^{n} D_{s_i}^{f_2}$	$=\sum_{i=1}^{n}D_{s_{i}}^{f_{2}}$	
		$= 55\frac{5}{16}$	$= 114\frac{1}{6}$	
		$= \max_{i=\{1,2\}} B_{s_i}^{f_2}$	$= \max_{i=\{1,2\}} B_{s_i}^{f_2}$	
	\mathbf{B}^{f_2}	$= 456\frac{1}{4}$	$= 491\frac{2}{2}$	

Separate Flow Analysis and PMOO Analysis

Arrival Bounds

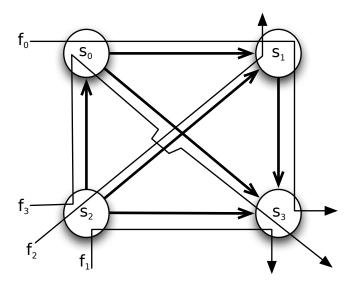
$(s_1, \{f_0\}, f_2) =: \alpha_{s_1}^{f_0}$		FIFO Multiplexing	Arbitrary Multiplexing			
	$lpha_{s_1}^{f_0}=lpha^{f_0}\oslasheta_{s_0}^{\mathrm{l.o.}f_0}$					
$lpha_{s_0}^{x(f_0)}$		$=\gamma_{0,0}$				
$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)}$		$=\beta_{s_0}=\beta_{20,20}$				
f f alof	$r_{s_1}^{f_0}$		= 5			
$\alpha_{s_1}^{f_0} = \alpha^{f_0} \oslash \beta_{s_0}^{\text{l.o.}f_0} = \gamma_{r_{s_1}^{f_0}, b_{s_1}^{f_0}}$		= 0	$\alpha^{f_0}\left(T_{s_0}^{\mathrm{l.o.}}\right)$			
	$b_{s_1}^{f_0}$	= 5	$5 \cdot 20 + 25$			
		=	125			
	=	=	$\gamma_{5,125}$			

Analyses

	SFA		FIFO Multiplexing Arbitrary Multiplexing	
6.0	$lpha_{s_2}^{x(f_2)}$		$=\alpha^{f_1}=\gamma_{5,25}$	
s_2	$\beta_{s_2}^{\text{l.o.}f_2} = \beta_{s_2} \ominus \alpha_{s_2}^{x(\zeta)}$	f 1)	$= \beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$
	$\alpha_{s_1}^{x(f_2)}$		$=\alpha_{s_1}^{f_0}$	$= \gamma_{5,125}$
s_1	$\beta_{s_1}^{\text{l.o.}f_2} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_2)}$	$R_{s_1}^{\mathrm{l.o.}f_2}$	$\left[R_{s_1} - r_{s_1}^{x(f_2)}\right]$	$\Big]^+ = 20 - 5$
	$= \beta_{R_{s_1}^{\text{l.o.}f_2}, T_{s_1}^{\text{l.o.}f_2}}$		$\beta_{s_1} = b_{s_1}$	= 15
	r_{s_1} , r_{s_1}	1 6	$20 \cdot [t - 20]^{+} = 125$	$\beta_{s_1} = \alpha_{s_1}^{x(f_2)}$
		$T_{s_1}^{\mathrm{l.o.}f_2}$	1	$20 \cdot [t - 20]^{+} = 5 \cdot t + 125$
			$t = 26\frac{1}{4}$	t = 35
		=	$=\beta_{15,26\frac{1}{4}}$	$=\beta_{15,35}$
	$\beta_{\langle s_2, s_1 \rangle}^{\text{l.o.} f_2} = \beta_{R_{\langle s_2, s_1 \rangle}^{\text{l.o.} f_2}, T_{\langle s_2, s_1 \rangle}^{\text{l.o.} f_2}}$		$= \bigotimes_{i=1}^{2} \beta_{s_i}^{\text{l.o.} f_2}$	$= \bigotimes_{i=1}^{2} \beta_{s_i}^{\text{l.o.} f_2}$
	(\$2,\$1), (\$2,\$	1/	$=$ $\beta_{15,47\frac{1}{2}}$	$= \beta_{15,63\frac{1}{3}}$
				$eta_{ ext{e2e}}^{ ext{l.o.}f_2} = b^{f_2}$
	D^{f_2}		$15 \cdot \left[t - 47\frac{1}{2}\right]^+ = 25$	$15 \cdot \left[t - 63\frac{1}{3} \right]^+ = 25$
			$t = 49\frac{1}{6}$	t = 65
	B^{f_2}		_	$\alpha^{f_2} \left(T_{\text{e2e}}^{\text{l.o.}f_2} \right) = 5 \cdot 63 \frac{1}{3} + 25$
	D		$=$ $262\frac{1}{2}$	$= 341\frac{2}{3}$

PMOO		Arbitrary Multiplexing
s_2	$\begin{array}{c c} \alpha_{s_2}^{x(f_2)} \\ \hline \alpha_{s_2}^{\bar{x}(f_2)} \end{array}$	$=\alpha^{f_1}=\gamma_{5,25}$
s_1	$\alpha_{s_1}^{x(f_2)}$ $\alpha_{s_1}^{\bar{x}(f_2)}$ $\alpha_{s_1}^{\bar{x}(f_2)}$	$=lpha_{s_{1}}^{f_{0}}=\gamma_{5,125}$
$\beta_{\langle s_2, s_1 \rangle}^{\text{l.o.}f_2} = \beta_{R_{\langle s_2, s_1 \rangle}^{\text{l.o.}f_2}, T_{\langle s_2, s_1 \rangle}^{\text{l.o.}f_2}}$	$R^{\mathrm{l.o.}f_2}_{\langle s_2,s_1 angle}$	$= \bigwedge_{i \in \{2,1\}} \left(R_{s_i} - r_{s_i}^{x(f_2)} \right)$ $= (20 - 5) \wedge (20 - 5)$ $= 15$
	$T_{\langle s_2, s_1 \rangle}^{\mathrm{l.o.}f_2}$	$= \sum_{i \in \{2,1\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_2)} + r_{s_i}^{\bar{x}(f_2)} \cdot T_{s_i}}{R_{\text{e2e}}^{\text{l.o.}f_2}} \right)$ $= 20 + \frac{25 + 5 \cdot 20}{15} + 20 + \frac{125 + 5 \cdot 20}{15}$ $= 40 + \frac{350}{15}$ $= 63\frac{1}{3}$
	=	$=\beta_{15,63\frac{1}{3}}$
D^{f_2}		$= \beta_{15,63\frac{1}{3}}$ $\beta_{\langle s_2,s_1\rangle}^{\text{l.o.}f_2} = b^{f_2}$ $15 \cdot \left[t - 63\frac{1}{3}\right]^+ = 25$ $t = 65$
B^{f_2}		$\alpha^{f_2} \left(T_{\langle s_2, s_1 \rangle}^{\text{l.o.} f_2} \right) = 5 \cdot 63 \frac{1}{3} + 25$ $= 341 \frac{2}{3}$

$FF_4S_1SC_4F_1AC_4P_Network$



$$S = \{s_0, s_1, s_2, s_3\}$$
 with

$$\beta_{s_0} = \beta_{s_1} = \beta_{s_2} = \beta_{s_3} = \beta_{R_{s_i},T_{s_i}} = \beta_{20,20}, \ i \in \{0,1,2,3\}$$

$$\mathbb{F} = \{f_0, f_1, f_2, f_3\}$$
 with

$$\alpha^{f_n} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{5,25}, \ n \in \{0, 1, 2, 3\}$$

$FF_4S_1SC_4F_1AC_4P_Test$

Arrival Bounds

$(s_0, \{f_3\}, \emptyset) =$ $(s_1, \{f_2\}, \emptyset) =$ $(s_3, \{f_1\}, \emptyset) =$ $=: \alpha_{s_i}^{f_n} \text{ with } (n, i) \in \{(3, 0)\}$	$\begin{array}{l} : \alpha_{s_0}^{f_3} \\ : \alpha_{s_1}^{f_2} \\ : \alpha_{s_3}^{f_1} \\ : \alpha_{s_3}^{f_1} \\)) , (2,1) , (1,3) \} \end{array}$	FIFO Multiplexing	Arbitrary Multiplexing
		$\alpha_{s_i}^{f_n} = \alpha^{f_n} \oslash \beta_{s_2}^{\text{l.o.}f_n}$	
$\alpha_{s_2}^{x(f_n)}$		$=\gamma_{5,25}+\gamma_{5,5}$	$q_{25} = \gamma_{10,50}$
$\beta_{s_2}^{\text{l.o.}f_n} = \beta_{s_2} \ominus \alpha^{x(f_n)}$	$R_{s_2}^{\mathrm{l.o.}f_n}$	$ = \gamma_{5,25} + \gamma_{5,25} = \gamma_{10,50} $ $ \left[R_{s_2} - r_{s_2}^{x(f_n)} \right]^+ = 20 - 5 $ $ = 15 $	
$= \beta_{R_{s_2}^{\text{l.o.}f_n}, T_{s_2}^{\text{l.o.}f_n}}$	$T_{s_2}^{\mathrm{l.o.}f_n}$	$\beta_{s_2} = b_{s_2}^{x(f_n)}$ $20 \cdot [t - 20]^+ = 50$ $t = 22\frac{1}{2}$	$\beta_{s_2} = \alpha_{s_2}^{x(f_n)}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 50$ $t = 45$
	=	$=\beta_{10,22\frac{1}{2}}$	$=\beta_{10,45}$
$\alpha_{s_i}^{f_n} = \alpha^{f_n} \oslash \beta_{s_2}^{\text{l.o.}f_n} \qquad \frac{r_{s_i}^{f_n}}{b_{s_i}^{f_n}}$		$= 5$ $\alpha^{f_n} \left(T_{s_2}^{\text{I.o.}f_n} \right) = 5 \cdot 22\frac{1}{2} + 25 = 137\frac{1}{2} \mid \alpha^{f_n} \left(T_{s_2}^{\text{I.o.}f_n} \right) = 5 \cdot 45 + 25 = 25$	
$= \gamma_{r_{s_i}^{f_n}, b_{s_i}^{f_n}}$	=	$= \gamma_{\text{E},127} $	$= \gamma_{5,250}$

$(s_3, \{f_0\}, \emptyset) = \alpha_s^f$	0	FIFO Multiplexing	Arbitrary Multiplexing				
	$lpha_{s_1}^{f_0}=lpha^{f_0}\oslasheta_{s_0}^{\mathrm{l.o.}f_0}$						
$\frac{\alpha_{s_0}^{f_0}}{\alpha_{s_0}^{x(f_0)}}$		$= \alpha^{f_0}$:	$=\gamma_{5,25}$				
$\alpha_{s_0}^{x(f_0)}$		$=\alpha_{s_0}^{f_3}=\gamma_{5,137\frac{1}{2}}$	$=lpha_{s_0}^{f_3}=\gamma_{5,250}$				
$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)} R_{s_0}^{\text{l.o.}f_0}$		$ \left[R_{s_0} - r_{s_0}^{x(f_0)} \right]^+ = 20 - 5 $					
		= 15					
$= \beta_{R_{s_0}^{1.0.f_0}, T_{s_0}^{1.0.f_0}}$		$\beta_{s_0} = b_{s_0}^{x(f_0)}$	$\beta_{s_0} = \alpha_{s_0}^{x(f_0)}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 250$				
	$T_{s_0}^{\mathrm{l.o.}f_0}$	$20 \cdot [t - 20]^+ = 137\frac{1}{2}$	$20 \cdot [t - 20]^+ = 5 \cdot t + 250$				
		$t = 26\frac{7}{8}$	$t = 43\frac{1}{3}$				
	=	$=\beta_{15,26\frac{7}{8}}$	$=\beta_{15,43\frac{1}{3}}$				
$\alpha_{s_1}^{f_0} = \alpha^{f_0} \oslash \beta_{s_0}^{\text{l.o.}f_0}$	$r_{s_1}^{f_0}$	= 5					
	$b_{s_1}^{f_0}$	$\alpha^{f_0}\left(T_{s_0}^{\text{l.o.}f_0}\right) = 5 \cdot 26\frac{7}{8} + 25 = 159\frac{3}{8}$	$\alpha^{f_0} \left(T_{s_0}^{\text{l.o.}f_0} \right) = 5 \cdot 43\frac{1}{3} + 25 = 241\frac{2}{3}$				
$= \gamma_{r_{s_1}^{f_0},b_{s_1}^{f_0}}$	=	$=\gamma_{5,159\frac{3}{8}}$	$=\gamma_{5,241\frac{2}{3}}$				

$(s_3, \{f_0\}, \emptyset) = \alpha_s^f$	0	FIFO Multiplexing	Arbitrary Multiplexing	
(*3, (30), **) ***	3	$\alpha_{s_3}^{f_0} = \alpha^{f_0} \oslash \left(\beta_{s_0}^{\text{l.o.}f_0} \otimes \beta_{s_1}^{\text{l.o.}f_0}\right)$	The state of the s	
		(reuse of previous result)		
		$= \qquad \alpha^{f_0} \oslash \beta_{s_0}^{\text{l.o.}f_0} \oslash \beta_{s_1}^{\text{l.o.}f_0}$		
		$= \qquad \qquad \alpha_{s_1}^{f_0} \oslash \beta_{s_1}^{\text{l.o.}f_0}$		
$lpha_{s_1}^{f_0}$		$=\gamma_{5,159\frac{3}{8}}$	$=\gamma_{5,241\frac{2}{3}}$	
$lpha_{s_1}^{x(f_0)}$		$=\alpha_{s_1}^{f_2}=\gamma_{5,137\frac{1}{2}}$	$=\alpha_{s_1}^{f_2} = \gamma_{5,250}$	
$\beta_{s_1}^{\text{l.o.}f_0} = \beta_{s_1} \otimes \alpha_{s_1}^{x(f_0)}$	$R_{s_1}^{\mathrm{l.o.}f_0}$	$\left[R_{s_1} - r_{s_1}^{x(f_0)}\right]^+ = 20 - 5$		
	81	= 15		
$= \beta_{R_{s_1}^{1.0.f_0}, T_{s_1}^{1.0.f_0}}$		$\beta_{s_1} = b_{s_1}^{x(f_0)}$	$\beta_{s_1} = \alpha_{s_1}^{x(f_0)}$	
	$T_{s_1}^{\mathrm{l.o.}f_0}$	$20 \cdot [t - 20]^+ = 137\frac{1}{2}$	$20 \cdot [t - 20]^+ = 5 \cdot t + 250$	
		$t = 26\frac{7}{8}$	$t = 43\frac{1}{3}$	
	=	$= \beta_{15,26\frac{7}{8}}$	$=\beta_{15,43\frac{1}{3}}$	
$\alpha_{s_3}^{f_0} = \alpha_{s_1}^{f_0} \oslash \beta_{s_1}^{\text{l.o.}f_0}$	$\begin{array}{c c} r_{s_3}^{f_0} \\ \hline b_{s_3}^{f_0} \end{array}$	=	5	
	$b_{s_3}^{f_0}$	$\alpha_{s_1}^{f_0} \left(T_{s_1}^{\text{l.o.}f_0} \right) = 5 \cdot 26\frac{7}{8} + 159\frac{3}{8} = 293\frac{3}{4}$	$\alpha_{s_1}^{f_0} \left(T_{s_1}^{\text{l.o.}f_0} \right) = 5 \cdot 43\frac{1}{3} + 241\frac{2}{3} = 458\frac{1}{3}$	
$= \gamma_{r_{s_3}^{f_0}, b_{s_3}^{f_0}}$	=	$=\gamma_{5,293\frac{3}{4}}$	$=\gamma_{5,458\frac{1}{3}}$	

$(s_3, \{f_3\}, \emptyset) = \alpha_{s_3}^{f_3}$		FIFO Multiplexing	Arbitrary Multiplexing	
	PbooA	rrivalBound_Concatena	ation.java	
	α	$ \frac{f_3}{s_3} = \alpha^{f_3} \oslash \left(\beta_{s_2}^{\text{l.o.}f_3} \otimes \right) $	$\beta_{s_0}^{\text{l.o.}f_3}$	
		(reuse of previous	result)	
		$= \alpha^{f_3} \oslash \beta_{s_2}^{\text{l.o.} f_3} \oslash$	$\beta_{s_0}^{\text{l.o.}f_3}$	
		$= \qquad \qquad \alpha_{s_0}^{f_3} \oslash$	$eta_{s_0}^{ ext{l.o.}f_3}$	
$\alpha_{s_0}^{x(f_3)}$				
$\beta_{s_0}^{\text{l.o.}f_3} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_3)}$	$R_{s_0}^{\mathrm{l.o.}f_3}$	R_s	$= \alpha^{f_0} = \gamma_{5,25}$	
	30	= 15		
$= \beta_{R_{s_0}^{1.0.f_3}, T_{s_0}^{1.0.f_3}}$		$\beta_{s_0} = b^{f_0}$	$\beta_{s_0} = \alpha^{f_0}$	
	$T_{s_0}^{\mathrm{l.o.}f_3}$	$20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	$20 \cdot [t - 20]^+ = 5 \cdot t + 25$	
	30		$t = 28\frac{1}{3}$	
	=	$=\beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$	
$\alpha_{s_0}^{f_3}$		$=\gamma_{5,137\frac{1}{2}}$	$=\gamma_{5,250}$	
$\alpha_{s_3}^{f_3} = \alpha_{s_0}^{f_3} \oslash \beta_{s_0}^{\text{l.o.}f_3}$	$r_{s_3}^{f_3}$	= 5		
$= \gamma_{r_{s_2}^{f_3}, b_{s_2}^{f_3}} $		$= \alpha_{s_0}^{f_3} \left(T_{s_0}^{\text{l.o.}f_3} \right)$	$= \alpha_{s_0}^{f_3} \left(T_{s_0}^{\text{l.o.} f_3} \right)$	
783,633	$b_{s_3}^{f_3}$	$= 5 \cdot 21\frac{1}{4} + 137\frac{1}{2}$	$= 5 \cdot 28 \frac{1}{3} + 250$	
		$=$ $243\frac{3}{4}$	$=$ $391\frac{2}{3}$	
	=	$=\gamma_{5,243\frac{3}{4}}$	$=\gamma_{5,391\frac{2}{3}}$	

PmooArrivalBound.java					
	$lpha_{s_3}^{f_3} = lpha^{f_3} \oslash eta_{\langle s_2, s_0 angle}^{\mathrm{l.o.}f_3}$				
s_2	$\begin{array}{c c} \alpha_{s_2}^{x(f_3)} \\ \hline \alpha_{s_2}^{\bar{x}(f_3)} \end{array}$		$= \alpha^{f_1} + \alpha^{f_2} = \gamma_{5,25} + \gamma_{5,25}$		
s_0	$\begin{array}{c} \alpha_{s_0}^{x(f_3)} \\ \overline{\alpha_{s_0}^{\overline{x}(f_3)}} \end{array}$		$= \alpha^{f_0} = \gamma_{5,25}^{\gamma_{105}}$		
$\beta_{\langle s_2, s_0 \rangle}^{\text{l.o.}f_3} = \beta_{R_{\langle s_2, s_0 \rangle}^{\text{l.o.}f_3}, T_{\langle s_2, s_0 \rangle}^{\text{l.o.}f_3}}$	$R^{\mathrm{l.o.}f_3}_{\langle s_2,s_0 angle}$		$= \bigwedge_{i \in \{2,0\}} \left(R_{s_i} - r_{s_i}^{x(f_3)} \right)$ $= (20 - 10) \wedge (20 - 5)$ $= 10$		
	$T_{\langle s_2, s_0 angle}^{\mathrm{l.o.}f_3}$		$= \sum_{i \in \{2,0\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_3)} + r_{s_i}^{\bar{x}(f_3)} \cdot T_{s_i}}{R_{\langle s_2, s_0 \rangle}^{\text{l.o.} f_3}} \right)$ $= 20 + \frac{50 + 10 \cdot 20}{10} + 20 + \frac{25 + 5 \cdot 20}{10}$		
	=		$= 77\frac{1}{2} = \beta_{10,77\frac{1}{2}}$		
$\alpha_{s_3}^{f_3} = \alpha^{f_3} \oslash \beta_{\langle s_2, s_0 \rangle}^{\text{l.o.} f_3}$	$r_{s_3}^{f_3}$	-	$= 5$ $= \alpha^{f_3}(T_{\langle s_2, s_0 \rangle}^{\text{l.o.} f_3})$		
	$b_{s_3}^{f_3}$		$= 5 \cdot 77\frac{1}{2} + 25$		
	=		$= 412\frac{1}{2}$		
			$=\gamma_{5,412\frac{1}{2}}$		

Flow f_0

Total Flow Analysis

Analysis

TFA FIFO Multiplexing Arbitrary Multiplexing		Multiplexing		
		PbooArrivalBound_C		PmooArrivalBound.java
		$= \qquad \qquad lpha_{s_0}^{f_0} + lpha_{s_0}^{f_3}$	=	$\alpha_{s_0}^{f_0} + \alpha_{s_0}^{f_3}$
	α_{s_0}	$= \gamma_{5,25} + \gamma_{5,137\frac{1}{2}}$	$= \gamma_{5,5}$	$_{25} + \gamma_{5,250}$
s_0		$= \gamma_{10,162\frac{1}{2}}$	=	$\gamma_{10,275}$
		$= \gamma_{10,162\frac{1}{2}} \\ \beta_{s_0} = b_{s_0}$	β_{s_0}	$=$ α_{s_0}
	$D_{s_0}^{f_0}$	$20 \cdot [t-20]^+ = 162\frac{1}{2}$	$20 \cdot [t-20]^{+}$	$= 10 \cdot t + 275$
	D_{s_0}	<u></u>	,	$=$ $67\frac{1}{2}$
		$t = 28\frac{1}{8}$ $\alpha_{s_0}(T_{s_0}) = 10 \cdot 20 + 162\frac{1}{2}$	L	$=$ $07\frac{1}{2}$
	- f	$\alpha_{s_0}(T_{s_0}) = 10 \cdot 20 + 162\frac{1}{2}$	$\alpha_{s_0}\left(T_{s_0}\right) =$	$10 \cdot 20 + 275$
	$B_{s_0}^{f_0}$	- 2621	=	475
		$= 262\frac{1}{2}$ $= \alpha_{s_1}^{f_0} + \alpha_{s_1}^{f_2}$		for fo
				$\alpha_{s_1}^{f_0} + \alpha_{s_1}^{f_2}$
s_1	α_{s_1}	$= \gamma_{5,159\frac{3}{8}} + \gamma_{5,137\frac{1}{2}}$		$_{11\frac{2}{3}} + \gamma_{5,250}$
91		$= \gamma_{10,296\frac{7}{8}} \\ \beta_{s_1} = b_{s_1}$	= β -	$=\frac{\gamma_{10,491\frac{2}{3}}}{\alpha_{s_1}}$
		 		
	$D_{s_1}^{f_0}$	$20 \cdot [t - 20]^+ = 296 \frac{7}{8}$	$20 \cdot [t - 20]^{+} =$	$= 10 \cdot t + 491\frac{2}{3}$
		$t = 34\frac{27}{32}$	t =	$= 89\frac{1}{c}$
		$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 296\frac{7}{9}$	$\alpha_{s_1}(T_{s_1}) =$	$= \frac{89\frac{1}{6}}{10 \cdot 20 + 491\frac{2}{3}}$
	$B_{s_1}^{f_0}$	2		
		$=$ $496\frac{7}{8}$	=	$691\frac{2}{3} = \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} + \alpha_{s_3}^{f_3}$
		$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} + \alpha_{s_3}^{f_3}$	$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} + \alpha_{s_3}^{f_3}$	$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} + \alpha_{s_3}^{f_3}$
	α_{s_3}	$= \gamma_{5,293\frac{3}{4}} + \gamma_{5,137\frac{1}{2}} + \gamma_{5,243\frac{3}{4}}$	$= \gamma_{5,458\frac{1}{3}} + \gamma_{5,250} + \gamma_{5,391\frac{2}{3}}$	$= \gamma_{5,458\frac{1}{3}} + \gamma_{5,250} + \gamma_{5,412\frac{1}{2}}$
s_3		$= \gamma_{15,675}$	$= \qquad \qquad \gamma_{15,1100}$	$= \gamma_{15,1120\frac{5}{6}}$ $\beta_{s_3} = \alpha_{s_3}$
		$\beta_{s_3} = b_{s_3}$	$\beta_{s_3} = \alpha_{s_3}$	
	$D_{s_3}^{f_0}$	$20 \cdot [t - 20]^+ = 675$	$20 \cdot [t - 20]^{+} = 15 \cdot t + 1100$	$20 \cdot [t - 20]^{+} = 15 \cdot t + 1120 \frac{5}{6}$
	-3	$t = 53\frac{3}{4}$	t = 300	$t = 304\frac{1}{c}$
		$\alpha_{s_3}(T_{s_3}) = 15 \cdot 20 + 675$	$\alpha_{s_3}(T_{s_3}) = 15 \cdot 20 + 1100$	$\alpha_{s_3}(T_{s_3}) = 15 \cdot 20 + 1120 \frac{5}{6}$
	$B_{s_3}^{f_0}$	= 975	= 1400	$=$ $1420\frac{5}{6}$
I	\mathbf{f}_0	$D^{f_0} + D^{f_0} + D^{f_0} = 116\frac{23}{23}$	$D^{f_0} + D^{f_0} + D^{f_0} = 456\frac{2}{5}$	$D^{f_0} + D^{f_0} + D^{f_0} = 460\frac{5}{5}$
	$3f_0$	$D_{s_0}^{f_0} + D_{s_1}^{f_0} + D_{s_3}^{f_0} = 116\frac{23}{32}$ $\max \left\{ B_{s_0}^{f_0}, B_{s_1}^{f_0}, B_{s_3}^{f_0} \right\} = 975$	$D_{s_0}^{f_0} + D_{s_1}^{f_0} + D_{s_3}^{f_0} = 456\frac{2}{3}$ $\max \left\{ B_{s_0}^{f_0}, B_{s_1}^{f_0}, B_{s_3}^{f_0} \right\} = 1400$	

Separate Flow Analysis and PMOO Analysis

Analyses

PbooArrivalBound_Concatenation.java

	SFA		FIFO Multiplexing	Arbitrary Multiplexing
	$lpha_{s_0}^{x(f_0)}$		$=\alpha_{s_0}^{f_3} = \gamma_{5,137\frac{1}{2}}$	$= \alpha_{s_0}^{f_3} = \gamma_{5,250}$
s_0	$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_3)}$	$R_{s_0}^{\mathrm{l.o.}f_0}$	$\begin{bmatrix} R_{s_0} - r_{s_0}^x \\ \beta_{s_0} = b_{s_0}^{x(f_0)} \end{bmatrix}$	$ f(f_0) ^+ = 15$
	$= \beta_{R_{s_0}^{1.0.f_0}, T_{s_0}^{1.0.f_0}}$		$\beta_{s_0} = b_{s_0}^{x(f_0)}$	$\beta_{s_0} = \alpha_{s_0}^{x(f_0)}$
	50 / 50	$T_{s_0}^{\mathrm{l.o.}f_0}$	$20 \cdot [t - 20]^+ = 137\frac{1}{2}$	$20 \cdot [t - 20]^+ = 5 \cdot t + 250$
			$t = 26\frac{7}{8}$	$t = 43\frac{1}{3}$
		=	$=\beta_{15,26\frac{7}{8}}$	$=\beta_{15,43\frac{1}{3}}$
	$lpha_{s_1}^{x(f_0)}$		$=\alpha_{s_1}^{f_1} = \gamma_{5,137\frac{1}{2}}$	$= \alpha_{s_1}^{f_1} = \gamma_{5,250}$
s_1	$\beta_{s_1}^{\text{l.o.}f_0} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_0)}$	$R_{s_1}^{\mathrm{l.o.}f_0}$	$\left[R_{s_1} - r_{s_1}^{x_1}\right]$	$\left[f_{0}\right]^{+}=15$
	$= \beta_{R_{s_1}^{\text{l.o.}f_0}, T_{s_1}^{\text{l.o.}f_0}}$		$\beta_{s_1} = b_{s_1}^{x(f_0)}$	$\beta_{s_1} = \alpha_{s_1}^{x(f_0)}$
	1	$T_{s_1}^{\mathrm{l.o.}f_0}$	$20 \cdot [t - 20]^+ = 137\frac{1}{2}$	$20 \cdot [t - 20]^+ = 5 \cdot t + 250$
			$t = 26\frac{7}{8}$ $= \beta_{15,26\frac{7}{8}}$ $= \alpha_{s_3}^{f_2} + \alpha_{s_3}^{f_3}$	$t = 43\frac{1}{3}$
		=	$=\beta_{15,26\frac{7}{8}}$	$= \beta_{15,43\frac{1}{3}}$ $= \alpha_{s_3}^{f_2} + \alpha_{s_3}^{f_3}$
			$= \alpha_{s_3}^{f_2} + \alpha_{s_3}^{f_3}$	$= \alpha_{s_3}^{f_2} + \alpha_{s_3}^{f_3}$
	$lpha_{s_3}^{x(f_0)}$		$= \gamma_{5,137\frac{1}{2}} + \gamma_{5,243\frac{3}{4}}$	$= \gamma_{5,250} + \gamma_{5,391\frac{2}{3}}$
s_3			$= \gamma_{10,381\frac{1}{4}}$	$= \gamma_{10,641\frac{2}{3}}$
	$\beta_{s_3}^{\text{l.o.}f_0} = \beta_{s_3} \ominus \alpha_{s_3}^{x(f_0)}$	$R_{s_3}^{\mathrm{l.o.}f_0}$	$= \gamma_{10,381\frac{1}{4}}$ $\begin{bmatrix} R_{s_3} - r_{s_3}^{x_1} \\ \beta_{s_3} = b_{s_3}^{x_1(f_0)} \end{bmatrix}$	$\begin{bmatrix} f_0 \end{bmatrix}$ = 10
	$= \beta_{R_{s_3}^{1.o.f_0}, T_{s_3}^{1.o.f_0}}$		$\beta_{s_3} = b_{s_3}^{x(f_0)}$	$\beta_{s_3} = \alpha_{s_3}^{x(f_0)}$
		$T_{s_3}^{\mathrm{l.o.}f_0}$	$20 \cdot [t - 20]^+ = 381\frac{1}{4}$	$20 \cdot [t - 20]^{+} = 10 \cdot t + 641\frac{2}{3}$
			$t = 39 \frac{1}{16}$ $= \beta_{10,39 \frac{1}{16}}$ $\bigotimes_{i=\{0,1,3\}} \beta_{s_i}^{\text{l.o.} f_0} = \beta_{10,92 \frac{13}{16}}$ $\beta_{\text{e2e}}^{\text{l.o.} f_0} = b^{f_0}$	$t = 104\frac{1}{6}$
		=	$=\beta_{10,39\frac{1}{16}}$	$=\beta_{10,104\frac{1}{6}}$
	$\beta_{\text{e2e}}^{\text{l.o.}f_0} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_0}, T_{\text{e2e}}^{\text{l.o.}f_0}}$		$\bigotimes_{i=\{0,1,3\}} \beta_{s_i}^{\text{l.o.} f_0} = \beta_{10,92\frac{13}{16}}$	
	***		$\beta_{\text{e2e}}^{\text{l.o.}f_0} = b^{f_0}$	$eta_{\mathrm{e}2\mathrm{e}}^{\mathrm{l.o.}f_0} = b^{f_0}$
	D^{f_0}		$10 \cdot \left[t - 92\frac{13}{16}\right]^+ = 25$	$10 \cdot \left[t - 190\frac{5}{6}\right]^+ = 25$
			$t = 95\frac{5}{16}$	4
	B^{f_0}		$t = 95\frac{5}{16}$ $\alpha^{f_0} \left(T_{\text{e2e}}^{\text{l.o.}f_0} \right) = 5 \cdot 92\frac{13}{16} + 25$	$t = 193\frac{1}{3}$ $\alpha^{f_0} \left(T_{\text{e2e}}^{\text{l.o.}f_0} \right) = 5 \cdot 190\frac{5}{6} + 25$
	B_{10}		$=$ $489\frac{1}{16}$	$=$ $979\frac{1}{6}$

PmooArrivalBound.java

PI	${\tt PmooArrivalBound.java}$				
SFA		Arbitrary Multiplexing			
	$\alpha_{s_0}^{x(f_0)}$		$= \alpha_{s_0}^{f_3} = \gamma_{5,250}$		
s_0	$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_3)}$	$R_{s_0}^{\mathrm{l.o.}f_0}$	$= \alpha_{s_0}^{f_3} = \gamma_{5,250}$ $\begin{bmatrix} R_{s_0} - r_{s_0}^{x(f_0)} \end{bmatrix}^+ = 15$ $\beta_{s_0} = \alpha_{s_0}^{x(f_0)}$		
	$= \beta_{R_{s_0}^{1.0.f_0}, T_{s_0}^{1.0.f_0}}$		$\beta_{s_0} = \alpha_{s_0}^{x(f_0)}$		
	κ_{s_0} , r_{s_0}	$T_{s_0}^{\mathrm{l.o.}f_0}$	$20 \cdot [t - 20]^+ = 5 \cdot t + 250$		
			$t = 43\frac{1}{3}$		
		=	$=\beta_{15,43\frac{1}{3}}$		
	$\alpha_{s_1}^{x(f_0)} = \alpha_{s_1}^{f_1}$		$= \gamma_{5,250}$		
s_1	$\beta_{s_1}^{\text{l.o.}f_0} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_0)}$	$R_{s_1}^{\mathrm{l.o.}f_0}$	$ = \gamma_{5,250} $ $ \left[R_{s_1} - r_{s_1}^{x(f_0)} \right]^+ = 15 $ $ \beta_{s_1} = \alpha_{s_1}^{x(f_0)} $		
	$= \beta_{R_{s_1}^{\text{l.o.}f_0}, T_{s_1}^{\text{l.o.}f_0}}$		$\beta_{s_1} = \alpha_{s_1}^{x(f_0)}$		
		$T_{s_1}^{\mathrm{l.o.}f_0}$	$20 \cdot [t - 20]^+ = 5 \cdot t + 250$		
			$t = 43\frac{1}{3}$		
		=	$=\beta_{15,43\frac{1}{3}}$		
	$\alpha_{s_3}^{x(f_0)} = \alpha_{s_3}^{f_2} + \alpha_{s_3}^{f_2}$	3	$= \gamma_{5,250} + \gamma_{5,412\frac{1}{2}} = \gamma_{10,662\frac{1}{2}}$		
s_3	$\beta_{s_3}^{\text{l.o.}f_0} = \beta_{s_3} \ominus \alpha_{s_3}^{x(f_0)}$	$R_{s_3}^{\mathrm{l.o.}f_0}$	$ \begin{bmatrix} R_{s_3} - r_{s_3}^{x(f_0)} \end{bmatrix}^+ = 10 $ $ \beta_{s_3} = \alpha_{s_3}^{x(f_0)} $		
	$= \beta_{R_{s_3}^{1.0.f_0}, T_{s_3}^{1.0.f_0}}$		$\beta_{s_3} = \alpha_{s_3}^{x(f_0)}$		
	1683 ,183	$T_{s_3}^{\mathrm{l.o.}f_0}$	$20 \cdot [t - 20]^{+} = 10 \cdot t + 662 \frac{1}{2}$		
			$t = 106\frac{1}{4}$		
		=	$=\beta_{10,106\frac{1}{4}}$		
	$\beta_{\text{e2e}}^{\text{l.o.}f_0} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_0}, T_{\text{e2e}}^{\text{l.o.}f_0}}$		$= \beta_{10,106\frac{1}{4}}$ $\bigotimes_{i=\{0,1,3\}} \beta_{s_i}^{\text{l.o.}f_0} = \beta_{10,192\frac{11}{12}}$ $\beta_{e2e}^{\text{l.o.}f_0} = b^{f_0}$		
			$\beta_{\text{e2e}}^{\text{l.o.}f_0} = b^{f_0}$		
	D^{f_0}		$10 \cdot \left[t - 192 \frac{11}{12} \right]^+ = 25$		
			$t = 195\frac{5}{12}$		
	B^{f_0}		$t = 195 \frac{5}{12}$ $\alpha^{f_0} \left(T_{\text{e2e}}^{\text{l.o.}f_0} \right) = 5 \cdot 192 \frac{11}{12} + 25$		
	D.		$=$ $989\frac{7}{12}$		

PbooArrivalBound_Concatenation.java

PbooArrivalBound_	Concater	
PMOO		Arbitrary Multiplexing
s_0	$\frac{\alpha_{s_0}^{x(f_0)}}{\alpha_{s_0}^{\bar{x}(f_0)}}$	$=\alpha_{s_0}^{f_3} = \gamma_{5,250}$
s_1	$\alpha_{s_1}^{x(f_0)}$ $\alpha_{s_1}^{\bar{x}(f_0)}$	$=lpha_{s_1}^{f_2}=\gamma_{5,250}$
s_3	$lpha_{s_3}^{x(f_0)}$	$= \alpha_{s_3}^{f_2} + \alpha_{s_3}^{f_3}$ $= \gamma_{5,250} + \gamma_{5,391\frac{2}{3}}$ $= \gamma_{5,641\frac{2}{3}}$
	$\alpha_{s_3}^{\bar{x}(f_0)}$	5,53
$\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_0} = \beta_{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}, T_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}}$	$R_{ m e2e}^{ m l.o.}f_0$	$= \bigwedge_{i \in \{0,1,3\}} \left(R_{s_i} - r_{s_i}^{x(f_0)} \right)$ $= (20 - 5) \wedge (20 - 5) \wedge (20 - 10)$ $= 10$
	$T^{1.0.f_0}$	$= \frac{10}{\sum_{i \in \{0,1,3\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_0)} + r_{s_i}^{x(f_0)} \cdot T_{s_i}}{R_{e2e}^{1.0.f_0}} \right)}$ $= 20 + \frac{250 + 5 \cdot 20}{10} + 20 + \frac{250 + 5 \cdot 20}{10} + 20 + \frac{641\frac{2}{3} + 10 \cdot 20}{10}$
	r e2e	$= 60 + \frac{1541\frac{2}{3}}{10}$ $= 214\frac{1}{6}$
	=	$=\beta_{10,214\frac{1}{6}}$
D^{f_0}		$=\beta_{10,214\frac{1}{6}}$ $\beta_{\text{e2e}}^{\text{l.o.}f_0} = b^{f_0}$ $10 \cdot \left[t - 214\frac{1}{6}\right]^+ = 25$ $t = 216\frac{2}{3}$
B^{f_0}		$t = 216\frac{2}{3}$ $\alpha^{f_0} \left(T_{\text{e2e}}^{\text{l.o.}f_0} \right) = 5 \cdot 214\frac{1}{6} + 25$ $= 1095\frac{5}{6}$

PmooArrivalBound.java

PMOOATTIVALBOUNG. Java PMOO		Arbitrary Multiplexing
1 1/100	/ 6	montary munipicang
	$\alpha_{s_0}^{x(f_0)}$	f_2
s_0	$\alpha_{s_0}^{\bar{x}(f_0)}$	$=lpha_{s_0}^{f_3}=\gamma_{5,250}$
	$x(f_0)$	
s_1	$\alpha_{s_1}^{x(f_0)}$	$=lpha_{s_1}^{f_2}=\gamma_{5,250}$
31	$\alpha_{s_1}^{\bar{x}(f_0)}$	$-\alpha_{s_1} - \gamma_{5,250}$
	1.01	$= \alpha_{s_3}^{f_2} + \alpha_{s_3}^{f_3}$
	(()	
	$\alpha_{s_3}^{x(f_0)}$	$= \gamma_{5,250} + \gamma_{5,412\frac{1}{2}}$
s_3		-
	$\bar{\pi}(f)$	$= \qquad \qquad \gamma_{10,662\frac{1}{2}}$
	$\alpha_{s_3}^{\bar{x}(f_0)}$	
		$\bigwedge \left(D x(f_0) \right)$
		$= \qquad \qquad \bigwedge \qquad \left(R_{s_i} - r_{s_i}^{x(f_0)} \right)$
al.o.fo	$R_{ m e2e}^{{ m l.o.}f_0}$	$i \in \{0,1,3\}$
	$n_{ m e2e}$	$= (20-5) \wedge (20-5) \wedge (20-10)$
$\beta_{\text{e2e}}^{\text{l.o.}f_0} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_0}, T_{\text{e2e}}^{\text{l.o.}f_0}}$		
		$= \frac{10}{\sum_{i \in \{0,1,3\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_0)} + r_{s_i}^{x(f_0)} \cdot T_{s_i}}{R_{\text{e}2e}^{\text{l.o.}f_0}} \right)}$
		$\sum \int_{T_i} b_{s_i}^{x(J_0)} + r_{s_i}^{x(J_0)} \cdot T_{s_i}$
		$ = \sum_{\mathbf{p} \text{l.o.} f_0} \left(T_{s_i} + \frac{1}{p \text{l.o.} f_0} \right) $
		$i \in \{0,1,3\} \setminus I_{e2e}$
		$250 + 5 \cdot 20$ $250 + 5 \cdot 20$ $662\frac{1}{9} + 10 \cdot 20$
	$T^{\text{l.o.}f_0}$	$= 20 + \frac{250 + 5 \cdot 20}{10} + 20 + \frac{250 + 5 \cdot 20}{10} + 20 + \frac{662\frac{1}{2} + 10 \cdot 20}{10}$
	- e2e	
		$= 60 + \frac{1462\frac{1}{2}}{10}$
		10
		$=$ 216 $\frac{1}{4}$
		$-\frac{210-}{4}$
	=	$=\beta_{10,216\frac{1}{4}}$
	•	$= \beta_{10,216\frac{1}{4}}$ $\beta_{e^{2}e}^{\text{l.o.}f_{0}} = b^{f_{0}}$
		. 626
7.6		$10 \cdot \left[t - 216 \frac{1}{4} \right]^+ = 25$
D^{f_0}		$10 \cdot \left t - 210 \frac{1}{4} \right = 25$
		L -J
		$t = 218\frac{3}{4}$
		$t = 218 \frac{3}{4}$ $\alpha^{f_0} \left(T_{\text{e2e}}^{\text{l.o.}f_0} \right) = 5 \cdot 216 \frac{1}{4} + 25$
B^{f_0}		$\alpha^{f_0} \left(T_{\text{e2e}}^{1.0.f_0} \right) = 5 \cdot 216 \frac{1}{4} + 25$
		\
		$= 1106\frac{1}{4}$
		4

Flow f_1

Total Flow Analysis

Analysis

PbooArrivalBound_Concatenation.java

FDOO	PbooArrivalBound_Concatenation.java						
Γ	ΓFA	FIFO Multiplexing	Arbitrary Multiplexing				
		$lpha_{s_3}^{f_3} = lpha_{s.}^f$	$\alpha_{s_2}^{1} + \alpha_{s_2}^{f_2} + \alpha_{s_2}^{f_3}$				
	α_{s_2}	$= \gamma_{5,25}$ $+$	$+ \gamma_{5,25} + \gamma_{5,25}$				
s_2		=	$\gamma_{15,75}$				
		$\beta_{s_2} = b_{s_2}$	$\beta_{s_2} = \alpha_{s_2}$				
	$D_{s_2}^{f_1}$	$20 \cdot \left[t - 20\right]^+ = 75$	$20 \cdot [t - 20]^+ = 15 \cdot t + 75$				
	- 2	$t = 23\frac{3}{4}$ $\alpha_{s_2}\left(T_{s_2}\right) =$	t = 95				
	$B_{s_2}^{f_1}$	$\alpha_{s_2}\left(T_{s_2}\right) =$	$15 \cdot 20 + 75$				
	$D_{s_2}^{r_1}$	=	375				
		$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} + \alpha_{s_3}^{f_3}$	$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} + \alpha_{s_3}^{f_3}$				
	α_{s_3}	$= \gamma_{5,137\frac{1}{2}} + \gamma_{5,293\frac{3}{4}} + \gamma_{5,243\frac{3}{4}}$	$= \gamma_{5,250} + \gamma_{5,458\frac{1}{3}} + \gamma_{5,391\frac{2}{3}}$				
s_3		$= \gamma_{15,675}$	$= \gamma_{15,1100}$				
		$\beta_{s_3} = b_{s_3}$	$\beta_{s_3} = \alpha_{s_3}$				
	$D_{s_3}^{f_1}$	$20 \cdot [t - 20]^+ = 675$	$20 \cdot [t - 20]^+ = 15 \cdot t + 1100$				
		$t = 53\frac{3}{4}$	t = 300				
	$B_{s_3}^{f_1}$	$\alpha_{s_3}(T_{s_3}) = 15 \cdot 20 + 675$	$\alpha_{s_3}(T_{s_3}) = 15 \cdot 20 + 1100$				
	$D_{s_3}^{r}$	= 975	= 1400				
	\mathcal{I}^{f_1}	$D_{s_2}^{f_1} + D_{s_3}^{f_1} = 77\frac{1}{2}$ $\max \left\{ B_{s_2}^{f_1}, B_{s_3}^{f_1} \right\} = 975$	$D_{s_2}^{f_1} + D_{s_3}^{f_1} = 395$ $\max \left\{ B_{s_2}^{f_1}, B_{s_3}^{f_1} \right\} = 1400$				
I	3^{f_1}	$\max\left\{B_{s_2}^{f_1}, B_{s_3}^{f_1}\right\} = 975$	$\max\left\{B_{s_2}^{f_1}, B_{s_3}^{f_1}\right\} = 1400$				

PmooArrivalBound.iava

Pı	${ t PmooArrivalBound.java}$				
Τ	FΑ	Arbitrary Multiplexing			
	_	$\alpha_{s_3}^{f_3} = \alpha_{s_2}^{f_1} + \alpha_{s_2}^{f_2} + \alpha_{s_2}^{f_3}$			
	α_{s_2}	$= \gamma_{5,25} + \gamma_{5,25} + \gamma_{5,25}$			
s_2		$=$ $\gamma_{15,75}$			
		$\beta_{s_2} = \alpha_{s_2}$			
	$D_{s_2}^{f_1}$	$20 \cdot [t - 20]^+ = 15 \cdot t + 75$			
		t = 95			
	$B_{s_2}^{f_1}$	$\alpha_{s_2}(T_{s_2}) = 15 \cdot 20 + 75$			
	$-s_2$	= 375			
		$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} + \alpha_{s_3}^{f_3}$			
	α_{s_3}	$= \gamma_{5,250} + \gamma_{5,458\frac{1}{3}} + \gamma_{5,412\frac{1}{2}}$			
s_3		$= \gamma_{15,1120\frac{5}{6}}$			
		$\beta_{s_3} = \alpha_{s_3}$			
	$D_{s_3}^{f_1}$	$20 \cdot [t - 20]^{+} = 15 \cdot t + 1120 \frac{5}{6}$			
		$t = 304\frac{1}{6}$			
	$B_{s_3}^{f_1}$	$\alpha_{s_3}(T_{s_3}) = 15 \cdot 20 + 1120 \frac{5}{6}$			
	\mathcal{L}_{s_3}	$=$ $1420\frac{5}{6}$			
1	$)^{f_1}$	$D_{s_2}^{f_1} + D_{s_2}^{f_1} = 399\frac{1}{6}$			
B^{f_1}		$D_{s_2}^{f_1} + D_{s_3}^{f_1} = 399\frac{1}{6}$ $\max \left\{ B_{s_2}^{f_1}, B_{s_3}^{f_1} \right\} = 1420\frac{5}{6}$			

Separate Flow Analysis and PMOO Analysis

Analyses

PbooArrivalBound_Concatenation.java

SFA			FIFO Multiplexing	Arbitrary Multiplexing
			$=$ α	$\frac{f_2}{f_{22}} + \alpha_{s_2}^{f_3}$
s_2	$\alpha_{s_2}^{x(f_1)}$		$= \gamma_{5,2}$	$_{5}+\gamma_{5,25}$
02			=	$\gamma_{10,50}$
	$\beta_{s_2}^{\text{l.o.}f_1} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f)}$	1)	$=\beta_{10,22\frac{1}{2}}$	$=\beta_{10,45}$
			$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_3}$	$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_3}$
	$\alpha_{s_3}^{x(f_1)}$		$= \gamma_{5,293\frac{3}{4}} + \gamma_{5,243\frac{3}{4}}$	$= \gamma_{5,458\frac{1}{3}} + \gamma_{5,391\frac{2}{3}}$
s_3			$= \gamma_{10,537\frac{1}{3}}$	$= \gamma_{10,850}$
	$\beta_{s_3}^{\text{l.o.}f_1} = \beta_{s_3} \ominus \alpha_{s_3}^{x(f_1)}$	$R_{s_3}^{\mathrm{l.o.}f_1}$	$R_{s_3} - r_{s_3}^{x(\cdot)}$	f_1 $= 10$
	$= \beta_{R_{s_3}^{1.0.f_1}, T_{s_3}^{1.0.f_1}}$		$\beta_{s_3} = b_{s_3}^{x(f_1)}$	$\beta_{s_3} = \alpha_{s_3}^{x(f_1)}$
	3 / 3	$T_{s_3}^{\mathrm{l.o.}f_1}$	$20 \cdot [t - 20]^+ = 537\frac{1}{2}$	$20 \cdot [t - 20]^{+} = 10 \cdot t + 850$
			$t = 46\frac{7}{8}$	t = 125
=		$=\beta_{10,46\frac{7}{8}}$	$=\beta_{10,125}$	
	$\beta_{\text{e2e}}^{\text{l.o.}f_1} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_1}, T_{\text{e2e}}^{\text{l.o.}f_1}}$		$\bigotimes_{i=2}^{3} \beta_{s_{i}}^{\text{l.o.}f_{1}} = \beta_{10,69\frac{3}{8}}$ $\beta_{\text{e2e}}^{\text{l.o.}f_{1}} = b^{f_{1}}$	$\bigotimes_{i=2}^{3} \beta_{s_i}^{\text{l.o.} f_1} = \beta_{10,170}$
				$\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_1} = b^{f_1}$
	D^{f_1}		$10 \cdot \left[t - 69 \frac{3}{8} \right]^+ = 25$	$10 \cdot [t - 170]^+ = 25$
			$t = 71\frac{7}{8}$ $\alpha^{f_1}\left(T_{\text{e2e}}^{\text{l.o.}f_1}\right) = 5 \cdot 69\frac{3}{8} + 25$ $= 371\frac{7}{2}$	$t = 172\frac{1}{2}$
	B^{f_1}		$\alpha^{f_1} \left(T_{\text{e2e}}^{\text{l.o.}f_1} \right) = 5 \cdot 69\frac{3}{8} + 25$	$\alpha^{f_1} \left(T_{\text{e2e}}^{\text{l.o.}f_1} \right) = 5 \cdot 170 + 25$
	B_{ij}		$=$ $371\frac{7}{8}$	= 875

PmooArrivalBound.java

	SFA		Arbitrary Multiplexing
	SFA		
			$= \alpha_{s_2}^{f_2} + \alpha_{s_2}^{f_3}$
	$\alpha_{s_2}^{x(f_1)}$		$= \gamma_{5,25} + \gamma_{5,25}$
s_2	~52		
		P \	$= \gamma_{10,50}$
	$\beta_{s_2}^{\mathbf{l.o.}f_1} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f)}$	1)	$= \beta_{10,45}$
			$= \alpha_{s_2}^{f_0} + \alpha_{s_2}^{f_3}$
	$lpha_{s_3}^{x(f_1)}$		3 3
	α_{s_3}		$= \gamma_{5,458\frac{1}{3}} + \gamma_{5,412\frac{1}{2}}$
s_3			$= \gamma_{10,870\frac{5}{6}}$
		$R_{s_3}^{\mathbf{l.o.}f_1}$	$\begin{bmatrix} R & -r^{x(f_1)} \end{bmatrix}^+ - 10$
	$\beta_{s_3}^{\mathbf{l.o.}f_1} = \beta_{s_3} \ominus \alpha_{s_3}^{x(f_1)}$	r_{s_3}	$\begin{bmatrix} R_{s_3} & R_{s_3} \end{bmatrix} = 10$
	$= \beta_{R_{s_3}^{\mathbf{l.o.}f_1}, T_{s_3}^{\mathbf{l.o.}f_1}}$		$= \gamma_{10,870\frac{5}{6}}$ $\left[R_{s_3} - r_{s_3}^{x(f_1)}\right]^+ = 10$ $\beta_{s_3} = \alpha_{s_3}^{x(f_1)}$
	1083 ,183	rel o f	$20 \cdot [t - 20]^{+} = 10 \cdot t + 870 \frac{5}{6}$
		$T_{s_3}^{\mathbf{l.o.}f_1}$	$\begin{bmatrix} 20 \cdot [t - 20] & = & 10 \cdot t + 870 \frac{1}{6} \end{bmatrix}$
			t _ 197 ¹
			$t = 127\frac{1}{12}$
		=	$=\beta_{10,127\frac{1}{12}}$
	$\beta_{\mathbf{e}2\mathbf{e}}^{\mathbf{l.o.}f_1} = \beta_{R_{\mathbf{e}2\mathbf{e}}^{\mathbf{l.o.}f_1}, T_{\mathbf{e}2\mathbf{e}}^{\mathbf{l.o.}f_1}}$		$\bigotimes_{i=2}^{3} \beta_{s_i}^{\text{l.o.}f_1} = \beta_{10,172\frac{1}{12}}$ $\beta_{\mathbf{e}2\mathbf{e}}^{\text{l.o.}f_1} = b^{f_1}$
	$R_{\mathbf{e}2\mathbf{e}}^{\mathbf{re}}$, $T_{\mathbf{e}2\mathbf{e}}^{\mathbf{re}}$		$\beta_{i=2}^{\text{l.o.}f_1} = b^{f_1}$
			$\beta_{\mathbf{e}2\mathbf{e}}^{\mathbf{n}\mathbf{e}\mathbf{j}_{1}}=$ $b^{\mathbf{j}_{1}}$
			1, [, ,,, 1]+
	D^{f_1}		$10 \cdot \left[t - 172 \frac{1}{12} \right]^+ = 25$
			$t = 174 \frac{7}{12}$ $\alpha^{f_1} \left(T_{\mathbf{e}2\mathbf{e}}^{\mathbf{l.o.}f_1} \right) = 5 \cdot 172 \frac{1}{12} + 25$
			f (-1 o fr) 1
	D f		$\alpha^{J_1} \left(T_{\mathbf{e}2\mathbf{e}}^{\mathbf{n}.\mathbf{o}.J_1} \right) = 5 \cdot 172 \frac{1}{12} + 25$
	B^{f_1}		
			$=$ $885\frac{5}{12}$
			1Z

PbooArrivalBound_Concatenation.java

PbooArrivalBound_Concatenation.java			
PMOO		Arbitrary Multiplexing	
		$= \alpha_{s_2}^{f_2} + \alpha_{s_2}^{f_3}$	
	$\alpha_{s_2}^{x(f_1)}$	$= \gamma_{5,25} + \gamma_{5,25}$	
s_2	3.32		
	$\bar{x}(f_1)$	$=$ $\gamma_{10,50}$	
	$\alpha_{s_2}^{\bar{x}(f_1)}$	I I	
		$= \qquad \qquad \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_3}$	
	$\alpha_{s_3}^{x(f_1)}$	$= \gamma_{5,458\frac{1}{3}} + \gamma_{5,391\frac{2}{3}}$	
s_3			
	$\bar{x}(f_1)$	$= \gamma_{10,850}$	
	$\alpha_{s_3}^{\bar{x}(f_1)}$		
		$= \bigwedge \left(R_{s_i} - r_{s_i}^{x(f_1)}\right)$	
	1 o f	$i \in \{2,3\}$	
alo f	$R_{\mathrm{e2e}}^{\mathrm{l.o.}f_{1}}$	$= (20 - 10) \wedge (20 - 10)$	
$\beta_{\text{e2e}}^{\text{l.o.}f_1} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_1}, T_{\text{e2e}}^{\text{l.o.}f_1}}$			
626 / 626		= 10	
		$\sum \left(\int_{T_i} b_{s_i}^{x(f_1)} + r_{s_i}^{x(f_1)} \cdot T_{s_i} \right)$	
		$= \frac{10}{\sum_{i \in \{2,3\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_1)} + r_{s_i}^{x(f_1)} \cdot T_{s_i}}{R_{\text{e2e}}^{\text{l.o.}f_1}} \right)}$	
		$i \in \{2,3\} \setminus \frac{10}{10} = 10$	
	$T_{\circ}^{\mathrm{l.o.}f_1}$	$= 20 + \frac{50 + 10 \cdot 20}{10} + 20 + \frac{850 + 10 \cdot 20}{10}$	
	eze		
		$= 40 + \frac{1300}{10}$	
		= 170	
	=	$=\beta_{10,170}$	
		$eta_{ ext{e}2 ext{e}}^{ ext{l.o.}f_1} = b^{f_1}$	
D^{f_1}		$10 \cdot [t - 170]^+ = 25$	
		$t = 172\frac{1}{2}$	
B^{f_1}			
		$\alpha^{f_1}\left(T_{\text{e2e}}^{\text{l.o.}f_1}\right) = 5 \cdot 170 + 25$	
		= 875	
		I .	

PmooArrivalBound.java

PmooArrivalBound.java			
PMOO		Arbitrary Multiplexing	
		$= \alpha_{s_2}^{f_2} + \alpha_{s_2}^{f_3}$	
	$\alpha_{s_2}^{x(f_1)}$	$= \gamma_{5,25} + \gamma_{5,25}$	
s_2			
	$\bar{x}(f_1)$	$=$ $\gamma_{10,50}$	
	$\alpha_{s_2}^{x(j_1)}$	$= \qquad \qquad \alpha_{s_2}^{f_0} + \alpha_{s_2}^{f_3}$	
	$\alpha_{s_3}^{x(f_1)}$		
s_3	α_{s_3}	$= \gamma_{5,458\frac{1}{3}} + \gamma_{5,412\frac{1}{2}}$	
	= (r)	$= \gamma_{10,870\frac{5}{6}}$	
	$\alpha_{s_3}^{\bar{x}(f_1)}$		
		$= \bigwedge \left(R_{s_i} - r_{s_i}^{x(f_1)} \right)$	
	$pl.o.f_1$	$i \in \{2,3\}$	
$\beta^{\text{l.o.}f_1} - \beta$	$R_{\mathrm{e2e}}^{\mathrm{l.o.}f_{1}}$	$= (20 - 10) \wedge (20 - 10)$	
$\beta_{\text{e2e}}^{\text{l.o.}f_1} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_1}, T_{\text{e2e}}^{\text{l.o.}f_1}}$		= 10	
		$b_{s}^{\bar{x}(f_1)} + r_{s}^{x(f_1)} \cdot T_{s}$	
		$= \sum_{i \in \{2,3\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_1)} + r_{s_i}^{x(f_1)} \cdot T_{s_i}}{R_{e2e}^{\text{l.o.}f_1}} \right)$	
	ml.o. f1	$= 20 + \frac{50 + 10 \cdot 20}{10} + 20 + \frac{870\frac{5}{6} + 10 \cdot 20}{10}$	
	$T_{ m e2e}$		
		$= 40 + \frac{1320\frac{5}{6}}{10}$	
		10	
		$=$ 172 $\frac{1}{12}$	
	=	$=\beta_{10,172\frac{1}{12}}$	
		$= \beta_{10,172\frac{1}{12}}$ $\beta_{e^{2}e}^{\text{l.o.}f_{1}} = b^{f_{1}}$	
		. 626	
D^{f_1}		$10 \cdot \left[t - 172 \frac{1}{12} \right]^+ = 25$	
		$t = 174 \frac{7}{12}$ $\alpha^{f_1} \left(T_{\text{e2e}}^{\text{l.o.}f_1} \right) = 5 \cdot 172 \frac{1}{12} + 25$	
B^{f_1}		$\alpha^{f_1}\left(T_{22}^{\text{l.o.}f_1}\right) = 5 \cdot 172 \frac{1}{2} + 25$	
		$=$ $885\frac{5}{12}$	
		12	

Flow f_2

Total Flow Analysis

Analysis

Γ	ΓFA	FIFO Multiplexing	Arbitrary Multiplexing				
		$= \qquad \alpha_{s_2}^{f_2} + \alpha_{s_2}^{f_1} + \alpha_{s_2}^{f_3}$					
	α_{s_2}	$= \gamma_{5,25} + \gamma_{5,25} + \gamma_{5,25}$					
s_2		=	$\gamma_{15,75}$				
		$\beta_{s_2} = b_{s_2}$	$\beta_{s_2} = \alpha_{s_2}$				
	$D_{s_2}^{f_2}$	$20 \cdot [t - 20]^+ = 75$	$20 \cdot [t - 20]^+ = 15 \cdot t + 75$				
	_	$t = 23\frac{3}{4}$	t = 95				
	$\mathbf{p}f_2$	$\alpha_{s_2}\left(\overset{4}{T}_{s_2}\right) =$	$15 \cdot 20 + 75$				
	$B_{s_2}^{f_2}$	= 375					
		$= \alpha_{s_1}^{f_2} + \alpha_{s_1}^{f_0}$	$= \alpha_{s_1}^{f_2} + \alpha_{s_1}^{f_0}$				
	α_{s_1}	$= \gamma_{5,137\frac{1}{2}} + \gamma_{5,159\frac{3}{8}}$	$= \gamma_{5,250} + \gamma_{5,241\frac{2}{3}}$				
s_1		$= \gamma_{10,296\frac{7}{8}}$	$= \gamma_{10,491\frac{2}{3}}$				
		$\beta_{s_1} = b_{s_1}$	$\beta_{s_1} = \alpha_{s_1}$				
	$D_{s_1}^{f_2}$	$20 \cdot [t - 20]^+ = 296\frac{7}{8}$	$20 \cdot [t - 20]^{+} = 10 \cdot t + 491\frac{2}{3}$				
		$t = 34\frac{27}{32}$	$t = 89\frac{1}{6}$				
	$\mathbf{D}f_2$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 296\frac{7}{8}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 491 \frac{2}{3}$				
	$B_{s_1}^{f_2}$	$=$ 496 $\frac{7}{8}$	$t = 89\frac{1}{6}$ $\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 491\frac{2}{3}$ $= 691\frac{2}{3}$				
	$)^{f_2}$	Ü					
1	3^{f_2}	$D_{s_2}^{f_2} + D_{s_1}^{f_2} = 58\frac{19}{32}$ $\max \left\{ B_{s_2}^{f_2}, B_{s_1}^{f_2} \right\} = 496\frac{7}{8}$	$D_{s_2}^{f_2} + D_{s_1}^{f_2} = 184\frac{1}{6}$ $\max \left\{ B_{s_2}^{f_2}, B_{s_1}^{f_2} \right\} = 691\frac{2}{3}$				

Separate Flow Analysis and PMOO Analysis

Analyses

		SFA		FIFO Multiplexing	Arbitrary Multiplexing
				$= \alpha$	$\frac{f_1}{s_2} + \alpha_{s_2}^{f_3}$
s_2		$\alpha_{s_2}^{x(f_2)}$		$= \gamma_{5,2}$	$_{5}+\gamma_{5,25}$
52				=	$\gamma_{10,50}$
	$\beta_{s_2}^{\mathrm{l.o.}}$	$f_2 = \beta_{s_2} \ominus \alpha_{s_2}^{x(j)}$	f ₂)	$=\beta_{10,22\frac{1}{2}}$	$=\beta_{10,45}$
		$\alpha_{s_1}^{x(f_2)}$		$=\alpha_{s_1}^{f_0} = \gamma_{5,159\frac{3}{8}}$	$=\alpha_{s_1}^{f_0} = \gamma_{5,241\frac{2}{3}}$
s_1	$\beta_{s_1}^{\text{l.o.}f_2} =$	$\beta_{s_1} \ominus \alpha_{s_1}^{x(f_2)}$	$R_{s_1}^{\mathrm{l.o.}f_2}$	$\left[R_{s_2} - r_{s_2}^{x(\cdot)}\right]$	
		$\beta_{R_{s_1}^{\text{l.o.}f_2}, T_{s_1}^{\text{l.o.}f_2}}$		$\beta_{s_1} = b_{s_1}^{x(f_2)}$	$\beta_{s_1} = \alpha_{s_1}^{x(f_2)}$
			$T_{s_1}^{\mathrm{l.o.}f_2}$	$20 \cdot [t - 20]^+ = 159\frac{3}{8}$	$20 \cdot [t - 20]^{+} = 5 \cdot t + 241 \frac{2}{3}$
				$t = 27\frac{31}{32}$	$t = 42\frac{7}{9}$
			=	$= \beta_{10,27\frac{31}{32}}$	$=\beta_{10,42\frac{7}{9}}$
	$eta_{\mathrm{e2e}}^{\mathrm{l.o.}f_2}$ =	$= \beta_{R_{\text{e2e}}^{\text{l.o.}f_2}, T_{\text{e2e}}^{\text{l.o.}f_2}}$		$\bigotimes_{i=1}^{2} \beta_{s_{i}}^{\text{l.o.}f_{2}} = \beta_{10,50\frac{15}{32}}$ $\beta_{\text{e2e}}^{\text{l.o.}f_{2}} = b^{f_{2}}$	$\bigotimes_{i=1}^{2} \beta_{s_{i}}^{\text{l.o.}f_{2}} = \beta_{10,87\frac{7}{9}}$ $\beta_{\text{e2e}}^{\text{l.o.}f_{2}} = b^{f_{2}}$
		Cac · Cac			$\beta_{\text{e2e}}^{\text{l.o.}f_2} = b^{f_2}$
		D^{f_2}		$10 \cdot \left[t - 50 \frac{15}{32} \right]^+ = 25$	$10 \cdot \left[t - 87\frac{7}{9}\right]^+ = 25$
				$t = 52\frac{31}{32}$ $\alpha^{f_2} \left(T_{\text{e2e}}^{\text{l.o.}f_2} \right) = 5 \cdot 50\frac{15}{32} + 25$	$t = 90\frac{5}{18}$
B^{f_2}				$\alpha^{f_2} \left(T_{\text{e2e}}^{\text{l.o.} f_2} \right) = 5 \cdot 50 \frac{15}{32} + 25$	$\alpha^{f_2} \left(T_{\text{e2e}}^{\text{l.o.} f_2} \right) = 5 \cdot 87 \frac{7}{9} + 25$
				$= 277\frac{11}{32}$	$=$ $463\frac{8}{9}$

PMOO		Arbitrary Multiplexing
		$= \alpha_{s_2}^{f_1} + \alpha_{s_2}^{f_3}$
	$\alpha_{s_2}^{x(f_2)}$	$= \gamma_{5,25} + \gamma_{5,25}$
s_2	- 2	$= \gamma_{10,50}$
	$\alpha_{s_2}^{\bar{x}(f_2)}$	710,50
	$\alpha_{s_1}^{x(f_2)}$	fo
s_1	$\alpha_{s_1}^{\bar{x}(f_2)}$	$=lpha_{s_1}^{f_0}=\gamma_{5,241rac{2}{3}}$
		$= \bigwedge \left(R_{s_i} - r_{s_i}^{x(f_2)} \right)$
	\mathbf{p} l o f_2	$ \begin{array}{c c} & \uparrow & \downarrow \\ & i \in \{2,1\} \\ \end{array} $
$\beta^{\text{l.o.}f_2} - \beta$	$R_{\mathrm{e2e}}^{\mathrm{l.o.}f_2}$	$= (20-10) \wedge (20-5)$
$\beta_{\text{e2e}}^{\text{l.o.}f_2} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_2}, T_{\text{e2e}}^{\text{l.o.}f_2}}$		= 10
		$= \frac{10}{\sum_{i \in \{2,1\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_2)} + r_{s_i}^{x(f_2)} \cdot T_{s_i}}{R_{e2e}^{\text{l.o.}f_2}} \right)}$
	$T_{ m e2e}^{{ m l.o.}f_2}$	$= 20 + \frac{50 + 10 \cdot 20}{10} + 20 + \frac{241\frac{2}{3} + 5 \cdot 20}{10}$
		$= 40 + \frac{591\frac{2}{3}}{10}$
		$=$ $99\frac{1}{6}$
	=	$=\beta_{10,99\frac{1}{6}}$
		$egin{align*} &=eta_{10,99rac{1}{6}} \ η_{e2e}^{\mathrm{l.o.}f_2} = &b^{f_2} \ & & & & & & & & & & & & & & & & & & $
D^{f_2}		$10 \cdot \left[t - 99\frac{1}{6} \right] = 25$
		$t = 101\frac{2}{3}$ $\alpha^{f_2} \left(T_{\text{e2e}}^{\text{l.o.} f_2} \right) = 5 \cdot 99\frac{1}{6} + 25$
B^{f_2}		
_		$=$ $520\frac{5}{6}$

Flow f_3

Total Flow Analysis

Analysis

 ${\tt PbooArrivalBound_Concatenation.java}$

TFA		FIFO Multiplexing	Arbitrary	Multiplexing
		PbooArrivalBound_C		PmooArrivalBound.java
			$= \alpha_{s_2}^{f_3} + \alpha_{s_2}^{f_1} + \alpha_{s_2}^{f_2}$	
	α_{s_2}		$= \gamma_{5,25} + \gamma_{5,25} + \gamma_{5,25}$	
s_2	-2		$= \gamma_{15,75}$	
		$\beta_{s_2} = b_{s_2}$		$=$ α_{s_2}
	$D_{s_2}^{f_3}$	$20 \cdot [t - 20]^+ = 75$	I -	$= 15 \cdot t + 75$
	D_{s_2}	$t = 23\frac{3}{4}$	1	= 10 t + 70 $=$ 95
		$t - 25\frac{1}{4}$		
	$B_{s_2}^{f_3}$		$\alpha_{s_2}(T_{s_2}) = 15 \cdot 20 + 75$	
		f_0 , f_2	= 375	fo fo
		$= \alpha_{s_0}^{f_0} + \alpha_{s_0}^{f_3}$		$\alpha_{s_0}^{f_0} + \alpha_{s_0}^{f_3}$
	α_{s_0}	$= \gamma_{5,25} + \gamma_{5,137\frac{1}{2}}$	$= \gamma_{5,:}$	$_{25}+\gamma_{5,250}$
s_0		$= \gamma_{10,162\frac{1}{2}} \\ \beta_{s_0} = b_{s_0}$	=	$\gamma_{10,275}$
		1		$=$ α_{s_0}
	$D_{s_0}^{f_3}$	$20 \cdot [t-20]^+ = 162\frac{1}{2}$	$20 \cdot [t-20]^+$	$= 10 \cdot t + 275$
	30	1 201	t	$=$ $67\frac{1}{2}$
		$t = 28\frac{1}{8}$ $\alpha_{s_0}(T_{s_0}) = 10 \cdot 20 + 162\frac{1}{2}$	-	2
	Df_0	$\alpha_{s_0}(T_{s_0}) = 10 \cdot 20 + 162\frac{1}{2}$	$\alpha_{s_0}\left(T_{s_0}\right) =$	$10 \cdot 20 + 275$
	$B_{s_0}^{f_3}$	$=$ $262\frac{1}{2}$	=	475
			2 2 2	f_ f_ f_
		$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} + \alpha_{s_3}^{f_3}$	$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} + \alpha_{s_3}^{f_3}$	
	α_{s_3}	$= \gamma_{5,293\frac{3}{4}} + \gamma_{5,137\frac{1}{2}} + \gamma_{5,243\frac{3}{4}}$	$= \gamma_{5,458\frac{1}{3}} + \gamma_{5,250} + \gamma_{5,391\frac{2}{3}}$	
s_3		$= \gamma_{15,675}$	$= \qquad \qquad \gamma_{15,1100}$	$= \gamma_{15,1120\frac{5}{6}}$ $\beta_{s_3} = \alpha_{s_3}$
		$\beta_{s_3} = b_{s_3}$	$\beta_{s_3} = \alpha_{s_3}$	
	$D_{s_3}^{f_3}$	$20 \cdot [t - 20]^+ = 675$	$20 \cdot [t - 20]^+ = 15 \cdot t + 1100$	$20 \cdot [t - 20]^{+} = 15 \cdot t + 1120 \frac{5}{6}$
	33	$t = 53\frac{3}{4}$	t = 300	2041
		4		$\iota = 304\frac{1}{5}$
	Df_2	$\alpha_{s_3}(T_{s_3}) = 15 \cdot 20 + 675$	$\alpha_{s_3}(T_{s_3}) = 15 \cdot 20 + 1100$	$t = 304\frac{1}{6}$ $\alpha_{s_3}(T_{s_3}) = 15 \cdot 20 + 1120\frac{5}{6}$ $= 1420\frac{5}{6}$
	$B_{s_3}^{f_3}$	= 975	= 1400	= 1420 -
7) fo	Dfo + Dfo + Dfo + 40x5	Dfo + Dfo + Dfo + 4001	$\frac{1420}{6}$
	9^{f_3}	$D_{s_2}^{f_3} + D_{s_0}^{f_3} + D_{s_3}^{f_3} = 105\frac{5}{8}$ $\max \{B_{s_0}^{f_3}, B_{s_0}^{f_3}, B_{s_3}^{f_3}\} = 975$	$D_{s_2}^{f_3} + D_{s_0}^{f_3} + D_{s_3}^{f_3} = 462\frac{1}{2}$ $\max\{B_{s_3}^{f_3}, B_{s_0}^{f_3}, B_{s_3}^{f_3}\} = 1400$	$D_{s_2}^{f_3} + D_{s_0}^{f_3} + D_{s_3}^{f_3} = 466\frac{2}{3}$ $\max \left\{ B_{s_2}^{f_3}, B_{s_0}^{f_3}, B_{s_3}^{f_3} \right\} = 1420\frac{5}{6}$
	· · ·	$D_{s_2}, D_{s_0}, D_{s_3} = 975$	$\max \left\{D_{\tilde{s}_{2}}, D_{\tilde{s}_{0}}, D_{\tilde{s}_{3}}\right\} = 1400$	$\max \{D_{s_2}^{s_2}, D_{s_0}^{s_0}, D_{s_3}^{s_3}\} = 1420\frac{1}{6}$

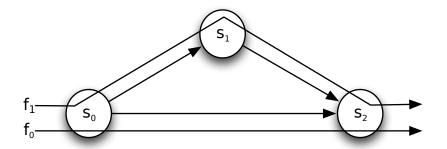
Separate Flow Analysis and PMOO Analysis

Analyses

		SFA		FIFO Multiplexing	Arbitrary Multiplexing
		$r(f_0)$			$\alpha_{s_2}^{f_1} + \alpha_{s_2}^{f_2}$
s_2		$\alpha_{s_2}^{x(f_3)}$		$=$ $\gamma_{5,2}$	$_{25} + \gamma_{5,25}$
	ol a	<i>t</i> 2 <i>r</i> (1	5)	=	$\gamma_{10,50}$
	$\beta_{s_2}^{\text{i.o.}}$	$f_3 = \beta_{s_2} \ominus \alpha_{s_2}^{x(j)}$		$= \beta_{10,22\frac{1}{2}}$	$=\beta_{10,45}$
$ s_0 $		$\alpha_{s_0}^{x(f_3)}$		$=lpha_{s_0}^{f_0}$	$=\gamma_{5,25}$
00	$\beta_{s_0}^{\mathrm{l.o.}}$	$f_3 = \beta_{s_0} \ominus \alpha_{s_0}^{x(j)}$	3)	$=\beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$
				$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1}$	$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1}$
		$lpha_{s_3}^{x(f_3)}$		$= \gamma_{5,293\frac{3}{4}} + \gamma_{5,137\frac{1}{2}}$	$= \gamma_{5,458\frac{1}{3}} + \gamma_{5,250}$
s_3				$= \gamma_{10,431\frac{1}{4}}$	$= \gamma_{10,708\frac{1}{3}}$
	$\beta_{s_3}^{\text{l.o.}f_3} =$	$\beta_{s_3} \ominus \alpha_{s_3}^{x(f_3)}$	$R_{s_3}^{\mathrm{l.o.}f_3}$	$R_{s_3} - r_{s_3}^{x_0}$	
	_	$\beta_{R_{s_3}^{1.0.f_3}, T_{s_3}^{1.0.f_3}}$		$\beta_{s_3} = b_{s_3}^{x(f_3)}$	$\beta_{s_3} = \alpha_{s_3}^{x(f_3)}$
		03 / 03	$T_{s_3}^{\mathrm{l.o.}f_3}$	$20 \cdot [t - 20]^+ = 431\frac{1}{4}$	$20 \cdot [t - 20]^{+} = 10 \cdot t + 708 \frac{1}{3}$
				$t = 41 \frac{9}{16}$ $= \beta_{10,41 \frac{9}{16}}$	$t = 110\frac{5}{6}$
			=	$=\beta_{10,41\frac{9}{16}}$	$=\beta_{10,110\frac{5}{6}}$
	$\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_3}$	$= \beta_{R_{\text{e2e}}^{\text{l.o.}f_3}, T_{\text{e2e}}^{\text{l.o.}f_3}}$		$\bigotimes_{i=\{2,0,3\}} \beta_{s_i}^{\text{l.o.}f_3} = \beta_{10,85\frac{5}{16}}$ $\beta_{e2e}^{\text{l.o.}f_3} = b^{f_3}$	$\bigotimes_{i=\{2,0,3\}} \beta_{s_i}^{\text{l.o.}f_3} = \beta_{10,184\frac{1}{6}}$ $\beta_{e,2e}^{\text{l.o.}f_3} = b^{f_3}$
		CAC - CAC			020
D^{f_3}				$10 \cdot \left[t - 85 \frac{5}{16} \right]^+ = 25$	$10 \cdot \left[t - 184\frac{1}{6}\right]^+ = 25$
				$t = 87\frac{13}{16}$	$t = 186\frac{2}{3}$ $\alpha^{f_3} \left(T_{\text{e2e}}^{\text{l.o.}f_3} \right) = 5 \cdot 184\frac{1}{6} + 25$
	B^{f_3}			$\alpha^{f_3} \left(T_{\text{e2e}}^{\text{l.o.}f_3} \right) = 5 \cdot 85 \frac{5}{16} + 25$	$\alpha^{f_3} \left(T_{\text{e2e}}^{\text{l.o.}f_3} \right) = 5 \cdot 184 \frac{1}{6} + 25$
		D.		$=$ $451\frac{9}{16}$	$=$ 945 $\frac{5}{6}$

PMOO		Arbitrary Multiplexing
s_2	$\alpha_{s_2}^{x(f_3)}$	$= \alpha_{s_2}^{f_1} + \alpha_{s_2}^{f_2}$ $= \gamma_{5,25} + \gamma_{5,25}$ $= \gamma_{10,50}$
s_0	$\begin{array}{c} \alpha_{s_2}^{\bar{x}(f_3)} \\ \alpha_{s_2}^{x(f_3)} \\ \alpha_{s_0}^{\bar{x}(f_3)} \end{array}$	$=lpha_{s_0}^{f_0}=\gamma_{5,25}$
s_3	$\alpha_{s_3}^{x(f_3)}$	$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1}$ $= \gamma_{5,458\frac{1}{3}} + \gamma_{5,250}$ $= \gamma_{10,708\frac{1}{3}}$
	$\alpha_{s_3}^{\bar{x}(f_3)}$	
$eta_{ ext{e2e}}^{ ext{l.o.}f_3} = eta_{R_{ ext{e2e}}^{ ext{l.o.}f_3}, T_{ ext{e2e}}^{ ext{l.o.}f_3}}$	$R_{ m e2e}^{ m l.o.}f_3$	$= \bigwedge_{i \in \{2,0,3\}} \left(R_{s_i} - r_{s_i}^{x(f_3)} \right)$ $= (20 - 10) \wedge (20 - 5) \wedge (20 - 10)$ $= 10$
	$T_{ m e2e}^{{ m l.o.}f_3}$	$= \sum_{i \in \{2,0,3\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_3)} + r_{s_i}^{\bar{x}(f_3)} \cdot T_{s_i}}{R_{e2e}^{\text{l.o.}f_3}} \right)$ $= 20 + \frac{50 + 10 \cdot 20}{10} + 20 + \frac{25 + 5 \cdot 20}{10} + 20 + \frac{708\frac{1}{3} + 10 \cdot 20}{10}$
	=	$= 60 + \frac{1283\frac{1}{3}}{10}$ $= 188\frac{1}{3}$ $= \beta_{10.188\frac{1}{3}}$
	<u> </u>	$= \beta_{10,188\frac{1}{3}}$ $\beta_{e^{2}e}^{\text{l.o.}f_{3}} = b^{f_{3}}$
D^{f_3}		$10 \cdot \left[t - 188\frac{1}{3}\right]^+ = 25$
		$t = 190\frac{5}{6}$ $\alpha^{f_3} \left(T_{\text{e2e}}^{\text{l.o.}f_3} \right) = 5 \cdot 188\frac{1}{3} + 25$
B^{f_3}		$\alpha^{f_3} \left(T_{\text{e2e}}^{\text{l.o.}f_3} \right) = 5 \cdot 188 \frac{1}{3} + 25$ $= 966 \frac{2}{3}$

$FF_3S_1SC_2F_1AC_2P_Network$



$$\begin{split} \mathbb{S} &= \{s_0, s_1, s_2\} \ \text{with} \\ & \beta_{s_0} = \beta_{s_1} = \beta_{s_2} = \beta_{R_{s_i}, T_{s_i}} = \beta_{20, 20}, \ i \in \{0, 1, 2\} \end{split}$$

$$\mathbb{F} &= \{f_0, f_1\} \ \text{with} \\ & \alpha^{f_0} = \alpha^{f_1} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{5, 25}, \ n \in \{0, 1\} \end{split}$$

$FF_3S_1SC_2F_1AC_2P_Test$

Flow f_0

Total Flow Analysis

Arrival Bounds

$(s_1, \{f_1\}, \emptyset) =: \alpha_{\underline{s}}^{\underline{s}}$ $(s_2, \{f_0\}, \emptyset) =: \alpha_{\underline{s}}^{\underline{s}}$ $=: \alpha_{\underline{s}_i}^{f_n} \text{ with } (n, i) \in \{(1, 1)\}$	f_0^-	FIFO Multiplexing	Arbitrary Multiplexing
		$\alpha_{s_i}^{f_n} = \alpha^{f_n} \oslash \beta_{s_0}^{\text{l.o.}f_n}$	
$\alpha_{s_0}^{x(f_n)}$		$=\gamma$	5,25
$\beta_{s_0}^{\text{l.o.}f_n} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_n)}$	$R_{s_0}^{\mathrm{l.o.}f_n}$	I	15
		$\beta_{s_0} = b_{s_0}^{x(f_n)}$	$\beta_{s_0} = \alpha_{s_0}^{x(f_n)}$
$= \beta_{R_{s_0}^{\text{l.o.}f_n}, T_{s_0}^{\text{l.o.}f_n}}$	$T_{s_0}^{\mathrm{l.o.}f_n}$	$20 \cdot \left[t - 20\right]^+ = 25$	$20 \cdot [t - 20]^+ = 5 \cdot t + 25$
	30	$t = 21\frac{1}{4}$	$t = 28\frac{1}{3}$
	=	$=\beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$
$\alpha_{s_i}^{f_n} = \alpha^{f_n} \oslash \beta_{s_0}^{\text{l.o.} f_n}$	$r_{s_i}^{f_n}$		5
$= \gamma_{r_{s_i}^{f_n}, b_{s_i}^{f_n}} $	$b_{s_i}^{f_n}$	$\alpha^{f_0}(T_{s_0}^{\text{l.o.}f_n}) = 5 \cdot 21\frac{1}{4} + 25$	$\alpha^{f_0} \left(T_{s_0}^{\text{l.o.} f_n} \right) = 5 \cdot 28 \frac{1}{3} + 25$
	o_{s_i}	$=$ $131\frac{1}{4}$	$=$ $166\frac{2}{3}$
	=	$=\gamma_{5,131\frac{1}{4}}$	$=\gamma_{5,166\frac{2}{3}}$

$(s_2, \{f_1\}, \emptyset) =: \alpha_s^f$	1 2	FIFO Multip	lexing	Arbitrary Multiplexing	
		$\alpha_{s_2}^{f_1} = \alpha^{f_1} \oslash (A)$	$\beta_{s_0}^{\mathrm{l.o.}f_1}\otimes\beta_{s_1}^{\mathrm{l.o.}f}$		
		(reuse of p	orevious resul	t)	
		$=$ $\alpha^{f_1} \oslash$	$\beta_{s_0}^{\mathrm{l.o.}f_1} \oslash \beta_{s_1}^{\mathrm{l.o.}}$	f_1	
		=	$\alpha_{s_1}^{f_1} \oslash \beta_{s_1}^{\mathrm{l.o.}}$	f_1	
$\alpha_{s_1}^{x(f_1)}$		$=\gamma_{0,0}$			
$\beta_{s_1}^{\text{l.o.}f_1} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f)}$	1)	$=\beta_{s_1}=\beta_{20,20}$			
$\alpha_{s_1}^{f_1}$		$= \gamma_{5,131\frac{1}{4}} \qquad \qquad = \gamma_{5,166\frac{2}{3}}$		$=\gamma_{5,166\frac{2}{3}}$	
$\alpha_{s_2}^{f_1} = \alpha^{f_1} \oslash \beta_{s_1}^{\text{l.o.}f_1}$	$r_{s_2}^{f_1}$		=		
$= \gamma_{r_{s_2}^{f_1}, b_{s_2}^{f_1}}$	1. f 1	$\alpha^{f_0}\left(T_{s_0}^{\text{l.o.}f_0}\right) = 5$	$\cdot 20 + 131\frac{1}{4}$	$\alpha^{f_0} \left(T_{s_0}^{\text{l.o.}f_0} \right) = 5 \cdot 20 + 166 \frac{2}{3}$	
	$o_{s_2}^{r_1}$	=	$231\frac{1}{4}$	$=$ $266\frac{2}{3}$	
	=	$= \gamma_{5,231}$	$\frac{1}{4}$	$=\gamma_{5,266\frac{2}{3}}$	

Remark:

 $\label{lower_pmoder_pmoder_pmoder_pmoder_pmoder} \because f_0 does not have cross-traffic interfering on multiple consecutive hops. \\$

Analysis

	177.A	PIPO M. I.I. I	
TFA		FIFO Multiplexing	Arbitrary Multiplexing
s_0	α_{s_0}		$\alpha^{f_0} + \alpha^{f_1}$ $_{,25} + \gamma_{5,25}$
00		=	$\gamma_{10,50}$
		$\beta_{s_0} = b_{s_0}$ $20 \cdot \left[t - 20\right]^+ = 50$	$\beta_{s_0} = \alpha_{s_0}$
	$D_{s_0}^{f_0}$	$20 \cdot [t - 20]^+ = 50$	$20 \cdot [t - 20]^+ = 10 \cdot t + 50$
		$t = 22\frac{1}{2}$	t = 45
	$\mathbf{p}f_0$	$\alpha_{s_0}\left(\bar{T}_{s_0}\right) =$	$10 \cdot 20 + 50$
	$B_{s_0}^{f_0}$	=	250
		$= \alpha_{s_2}^{f_0} + \alpha_{s_2}^{f_1}$	$= \alpha_{s_2}^{f_0} + \alpha_{s_2}^{f_1}$
	α_{s_2}	$= \gamma_{5,231\frac{1}{4}} + \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,266\frac{2}{3}} + \gamma_{5,166\frac{2}{3}}$
s_2		$= \gamma_{10,362\frac{1}{2}}$	$= \gamma_{10,433\frac{1}{3}}$
		$\beta_{s_2} = b_{s_2}$	$\beta_{s_2} = \alpha_{s_2}$
	$D_{s_2}^{f_0}$	$20 \cdot [t - 20]^+ = 362\frac{1}{2}$	$20 \cdot [t - 20]^{+} = 10 \cdot t + 433 \frac{1}{3}$
		$t = 38\frac{1}{8}$	$t = 83\frac{1}{3}$
	$B_{s_2}^{f_0}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 362 \frac{1}{2}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 433\frac{1}{3}$
	D_{s_2}	$=$ $562\frac{1}{2}$	$=$ $633\frac{1}{3}$
1	\mathcal{I}^{f_0}	$\sum_{i=\{0,2\}} D_{s_i}^{f_0} = 60\frac{5}{8}$	$\sum_{i=\{0,2\}} D_{s_i}^{f_0} = 128\frac{1}{3}$
	3^{f_0}	$\max_{i=\{0,2\}} B_{s_i}^{f_0} = 562\frac{1}{2}$	$\max_{i=\{0,2\}} B_{s_i}^{f_0} = 633\frac{1}{3}$

Separate Flow Analysis and PMOO Analysis

Arrival Bounds

$\left(s_2,\left\{f_1\right\},f_0\right) \eqqcolon \alpha_{s_2}^{f_1}$		FIFO Multiplexing	Arbitrary Multiplexing	
	$\alpha_{s_2}^{f_1}$	$=\alpha^{f_1}\oslash\left(\beta_{s_0}^{\mathrm{l.o.}f_1}\otimes\beta_{s_1}^{\mathrm{l.o.}f_1}\right)$		
$lpha_{s_0}^{x(f_0)}$		$=\gamma$	5,25	
$\beta_{s_0}^{\mathrm{l.o.}f_1} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_1)}$	$R_{s_0}^{\mathrm{l.o.}f_1}$		15	
$= \beta_{R_{s_0}^{1,o.f_1}, T_{s_0}^{1,o.f_1}}$		$\beta_{s_0} = b_{s_0}^{x(f_1)}$	$\beta_{s_0} = \alpha_{s_0}^{x(f_1)}$	
R_{s_0} -, I_{s_0} -	$T_{s_0}^{\mathrm{l.o.}f_1}$	$20 \cdot [t - 20]^+ = 25$	$20 \cdot [t - 20]^+ = 5 \cdot t + 25$	
		$t = 21\frac{1}{4}$	$t = 28\frac{1}{3}$	
	=	$=\beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$	
$lpha_{s_1}^{x(f_1)}$		= ^	Y0,0	
$\beta_{s_1}^{\mathrm{l.o.}f_1} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_1)}$		$=\beta_{s_1}=\beta_{20,20}$		
$\beta_{s_0}^{\text{l.o.}f_1} \otimes \beta_{s_1}^{\text{l.o.}f_1} = \beta$	$ l.o.f_1 \langle s_0, s_1 \rangle $	$= \beta_{s_0}^{\text{l.o.}f_1} \otimes \beta_{s_1}$	$= \beta_{s_0}^{\mathrm{l.o.}f_1} \otimes \beta_{s_1}$	
		$= \beta_{15,21\frac{1}{4}} \otimes \beta_{20,20}$	$= \beta_{15,28\frac{1}{3}} \otimes \beta_{20,20}$	
$= \beta_{R_{\langle s_0, s_1 \rangle}^{\text{l.o.} f_1}, T}$	$\langle s_0, s_1 \rangle$	$= \beta_{15,41\frac{1}{4}}$	$= \beta_{15,48\frac{1}{3}}$	
$\alpha_{s_2}^{f_1} = \alpha^{f_1} \oslash \left(\beta_{s_0}^{\text{l.o.}f_1} \otimes \beta_{s_1}^{\text{l.o.}f_1}\right)$	$lpha_{s_2}^{f_1} = lpha^{f_1} \oslash \left(eta_{s_0}^{\mathrm{l.o.}f_1} \otimes eta_{s_1}^{\mathrm{l.o.}f_1} ight) r_{\langle s_0, s_1 \rangle}^{f_1}$		5	
$= \gamma_{r_{\langle s_0, s_1 \rangle}^{f_1}, b_{\langle s_0, s_1 \rangle}^{f_1}}$	b^{f_1}	$\alpha^{f_1}\left(T_{\langle s_0, s_1\rangle}^{\text{l.o.}f_1}\right) = 5 \cdot 41\frac{1}{4} + 25$	$\alpha^{f_1}\left(T_{\langle s_0, s_1\rangle}^{\text{l.o.}f_1}\right) = 5 \cdot 48\frac{1}{3} + 25$	
	$\langle s_0, s_1 \rangle$	$=$ $231\frac{1}{4}$	$=$ $266\frac{2}{3}$	
	=	$=\gamma_{5,231\frac{1}{4}}$	$=\gamma_{5,266\frac{2}{3}}$	

Remark:

 ${\tt PmooArrivalBound.java}$ will have the same result as ${\tt PbooArrivalBound_Concatenation.java}$ because f_0 does not have cross-traffic interfering on multiple consecutive hops.

Analyses

	SFA		FIFO Multiplexing	Arbitrary Multiplexing
	$lpha_{s_0}^{x(f_0)}$		$=\alpha^{f_1}$	$=\gamma_{5,25}$
s_0	$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)}$	$R_{s_0}^{\mathrm{l.o.}f_0}$	$R_{s_0} - r_{s_0}^{x(\cdot)}$	
	$= \beta_{R_{s_0}^{1.0.f_0}, T_{s_0}^{1.0.f_0}}$		$\beta_{s_0} = b_{s_0}^{x(f_0)}$	$\beta_{s_0} = \alpha_{s_0}^{x(f_0)}$
		$T_{s_0}^{\mathrm{l.o.}f_0}$	$20 \cdot [t - 20]^+ = 25$	$20 \cdot [t - 20]^{+} = 5 \cdot t + 25$
			$t = 21\frac{1}{4}$	$t = 28\frac{1}{3}$
		=	$=\beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$
	$\alpha_{s_2}^{x(f_0)}$		$=\gamma_{5,231\frac{1}{4}}$	$=\gamma_{5,266\frac{2}{3}}$
s_2	$\beta_{s_2}^{\text{l.o.}f_0} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_0)}$	$R_{s_2}^{\mathrm{l.o.}f_0}$	$\left[R_{s_2} - r_{s_2}^{x(\cdot)}\right]$	
	$= \beta_{R_{s_2}^{1.0.f_0}, T_{s_2}^{1.0.f_0}}$		$\beta_{s_2} = b_{s_2}^{x(f_0)}$	$\beta_{s_2} = \alpha_{s_2}^{x(f_0)}$
		$T_{s_2}^{\mathrm{l.o.}f_0}$	$20 \cdot [t - 20]^+ = 231\frac{1}{4}$	$20 \cdot [t - 20]^{+} = 5 \cdot t + 266 \frac{2}{3}$
			$t = 31\frac{9}{16}$	$t = 44\frac{4}{9}$
		=	$=\beta_{15,31\frac{9}{16}}$	$=\beta_{15,44\frac{4}{9}}$
	$\beta_{\text{e2e}}^{\text{l.o.}f_0} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_0}, T_{\text{e2e}}^{\text{l.o.}f_0}}$		$\bigotimes_{i=\{0,2\}} \beta_{s_i}^{\text{l.o.}f_0} = \beta_{15,52\frac{13}{16}}$ $\beta_{\text{e2e}}^{\text{l.o.}f_0} = b^{f_0}$	$\bigotimes_{i=\{0,2\}} \beta_{s_i}^{\text{l.o.}f_0} = \beta_{15,72\frac{7}{9}}$ $\beta_{\text{e}2e}^{\text{l.o.}f_0} = b^{f_0}$
				$\beta_{\text{e2e}}^{\text{l.o.}f_0} = b^{f_0}$
	D^{f_0}		$15 \cdot \left[t - 52\frac{13}{16}\right]^+ = 25$	$15 \cdot \left[t - 72\frac{7}{9} \right]^+ = 25$
				$t = 74\frac{4}{9}$
B^{f_0}			$t = 54\frac{23}{48}$ $\alpha^{f_0} \left(T_{\text{e2e}}^{\text{l.o.}f_0} \right) = 5 \cdot 52\frac{13}{16} + 25$	$\alpha^{f_0}\left(T_{\text{e2e}}^{\text{l.o.}f_0}\right) = 5 \cdot 72\frac{7}{9} + 25$
			$=$ $289\frac{1}{16}$	$= 388\frac{8}{9}$

PMOO		Arbitrary Multiplexing
s_0	$\begin{array}{c} \alpha_{s_0}^{x(f_0)} \\ \alpha_{s_0}^{\bar{x}(f_0)} \end{array}$	$=\gamma_{5,25}$
s_2	$\begin{array}{c} \alpha_{s_2}^{x(f_0)} \\ \alpha_{s_2}^{\bar{x}(f_0)} \end{array}$	$=\gamma_{5,266\frac{2}{3}}$
$\beta_{\text{e2e}}^{\text{l.o.}f_0} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_0}, T_{\text{e2e}}^{\text{l.o.}f_0}}$	$R_{ m e2e}^{ m l.o.}f_0$	$= \bigwedge_{i \in \{0,2\}} \left(R_{s_i} - r_{s_i}^{x(f_0)} \right)$ $= (20 - 5) \wedge (20 - 5)$ $= 15$
		$= \sum_{i \in \{0,2\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_0)} + r_{s_i}^{x(f_0)} \cdot T_{s_i}}{R_{e2e}^{\text{l.o.}f_0}} \right)$
	$T_{ m e2e}^{{ m l.o.}f_0}$	$= 20 + \frac{25 + 5 \cdot 20}{15} + 20 + \frac{266\frac{2}{3} + 5 \cdot 20}{15}$
		$= 40 + \frac{491\frac{2}{3}}{15}$
		$=$ $72\frac{7}{9}$
	=	$= \beta_{15,72\frac{7}{9}}$ $\beta_{e^{2}e}^{\text{l.o.}f_{0}} = b^{f_{0}}$
		$eta_{ ext{e}2 ext{e}}^{ ext{l.o.}f_0} = b^{f_0}$
D^{f_0}		$15 \cdot \left[t - 72\frac{7}{9} \right]^+ = 25$
		$t = 74\frac{4}{9}$
B^{f_0}		$t = 74\frac{4}{9}$ $\alpha^{f_0} \left(T_{\text{e2e}}^{\text{l.o.}f_0} \right) = 5 \cdot 72\frac{7}{9} + 25$
		$= 388\frac{8}{9}$

Flow f_1

Total Flow Analysis

Arrival Bounds

$(s_1, \{f_1\}, \emptyset) =: \alpha_s^J$ $(s_2, \{f_0\}, \emptyset) =: \alpha_s^J$ $=: \alpha_{s_i}^{f_n} \text{ with } (n, i) \in \{(1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,$	$f_0 = f_2$	FIFO Multiplexing	Arbitrary Multiplexing
$lpha_{s_i}^{f_n} = lpha^{f_n} \oslash eta_{s_0}^{ ext{l.o.}f_n}$			
$\alpha_{s_0}^{x(f_n)}$		$=\gamma_{5,25}$	
$\beta_{s_0}^{\text{l.o.}f_n} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_n)}$	$R_{s_0}^{\mathrm{l.o.}f_n}$		= 15
$= \beta_{R_{s_0}^{\text{l.o.}f_n}, T_{s_0}^{\text{l.o.}f_n}}$		$\beta_{s_0} = b_{s_0}^{x(f_n)}$	$\beta_{s_0} = \alpha_{s_0}^{x(f_n)}$
R_{s_0} R_{s_0}	$T_{s_0}^{\mathrm{l.o.}f_n}$	$20 \cdot \left[t - 20\right]^+ = 25$	$20 \cdot [t - 20]^+ = 5 \cdot t + 25$
	50	$t = 21\frac{1}{4}$	$t = 28\frac{1}{3}$
	=	$=\beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$
	$r_{s_i}^{f_n}$		=5
$\alpha_{s_i}^{f_n} = \alpha^{f_n} \oslash \beta_{s_0}^{\text{l.o.} f_n}$	$\begin{array}{c} r_{s_i}^{f_n} \\ b_{s_i}^{f_n} \end{array}$	$\alpha^{f_0}\left(T_{s_0}^{\text{l.o.}f_n}\right) = 131\frac{1}{4}$	$\alpha^{f_0} \left(T_{s_0}^{\text{l.o.}f_n} \right) = 166\frac{2}{3}$
	=	$=\gamma_{5,131\frac{1}{4}}$	$=\gamma_{5,166\frac{2}{3}}$

$(s_2, \{f_1\}, \emptyset) =: \alpha_{s_2}^{f_1}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha_{s_2}^{f_1} = \alpha^{f_1} \oslash \left(\beta_{s_0}^{\text{l.o.}f_1} \otimes \beta_{s_1}^{\text{l.o.}f_1}\right) = \alpha^{f_1} \oslash \beta_{s_0}^{\text{l.o.}f_1} \oslash \beta_{s_1}^{\text{l.o.}f_1} = \alpha_{s_1}^{f_1} \oslash \beta_{s_1}^{\text{l.o.}f_1}$			
$\alpha_{s_1}^{x(f_1)}$		$=\gamma_{0,0}$	
$\beta_{s_1}^{\text{l.o.}f_1} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_1)}$		$=\beta_{s_1}=\beta_{20,20}$	
$\alpha_{s_1}^{f_1}$		$=\gamma_{5,131\frac{1}{4}}$	$=\gamma_{5,166\frac{2}{3}}$
$\alpha_{s_2}^{f_1} = \alpha_{s_1}^{f_1} \oslash \beta_{s_1}^{\text{l.o.}f_1} \qquad \frac{r_s^f}{r_s^f}$		=5	
$\begin{array}{ccc} \alpha_{s_2} & \alpha_{s_1} \otimes \beta_{s_1} \\ & = & \alpha^{f_1} \otimes \beta_{s_1}^{\text{l.o.}f_1} \end{array}$	$b_{s_2}^{f_1}$	$\alpha^{f_0}\left(T_{s_0}^{\text{l.o.}f_0}\right) = 231\frac{1}{4}$	$\alpha^{f_0} \left(T_{s_0}^{\text{l.o.} f_0} \right) = 266 \frac{2}{3}$
$= \alpha^{r_1} \oslash \beta_{s_1}^{r_{s_1}}$	=	$=\gamma_{5,231\frac{1}{4}}$	$=\gamma_{5,266\frac{2}{3}}$

Remark:

 $\label{lem:pmooArrivalBound_Concatenation.java} PmooArrivalBound_Concatenation.java because \ f_1 \ does \ not \ have \ cross-traffic interfering \ on \ multiple \ consecutive \ hops.$

Analysis

TFA		FIFO Multiplexing	Arbitrary Multiplexing		
α_{s_0}		$= \alpha^{f_0} + \alpha^{f_1} = \gamma_{5,25} + \gamma_{5,25} = \gamma_{10,50}$			
s_0		$\beta_{s_0} = b_{s_0}$	$\beta_{s_0} = \alpha_{s_0}$		
	$D_{s_0}^{f_0}$	$20 \cdot [t - 20]^+ = 50$	$20 \cdot [t - 20]^+ = 10 \cdot t + 50$		
		$t = 22\frac{1}{2}$ $\alpha_{s_0}(T_{s_0}) =$	t = 45		
	$\mathbf{p}f_0$	$\alpha_{s_0}\left(\tilde{T}_{s_0}\right) =$	$10 \cdot 20 + 50$		
	$B_{s_0}^{f_0}$	=	250		
	α_{s_1}	$=\alpha_{s_1}^{f_1} = \gamma_{5,131\frac{1}{4}}$	$=\alpha_{s_1}^{f_1} = \gamma_{5,166\frac{2}{3}}$		
s_1		$\beta_{s_1} = b_{s_1}$	FIFO per micro flow		
			$\beta_{s_1} = b_{s_1}$		
	$D_{s_1}^{f_1}$	$20 \cdot [t - 20]^+ = 131\frac{1}{4}$	$20 \cdot [t - 20]^+ = 166\frac{2}{3}$		
		$t = 26\frac{9}{16}$	¥		
		10	$t = 28\frac{1}{3}$		
	$\mathbf{R}f_1$	$\alpha_{s_1}(T_{s_1}) = 5 \cdot 20 + 131 \frac{1}{4}$	$\alpha_{s_1}(T_{s_1}) = 5 \cdot 20 + 166\frac{2}{3}$		
	$B_{s_1}^{f_1}$	$=$ $231\frac{1}{4}$	$=$ $266\frac{2}{3}$		
		$= \alpha_{s_2}^{f_0} + \alpha_{s_2}^{f_1}$	$= \alpha_{s_2}^{f_0} + \alpha_{s_2}^{f_1}$		
	α_{s_2}	$= \gamma_{5,231\frac{1}{4}} + \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,266\frac{2}{3}} + \gamma_{5,166\frac{2}{3}}$		
s_2		$= \gamma_{10,362\frac{1}{2}} \\ \beta_{s_2} = b_{s_2}$	$= \gamma_{10,433\frac{1}{3}}$ $\beta_{s_2} = \alpha_{s_2}$		
		$\beta_{s_2} = b_{s_2}$	$\beta_{s_2} = \alpha_{s_2}$		
	$D_{s_2}^{f_1}$	$20 \cdot [t - 20]^+ = 362\frac{1}{2}$	$20 \cdot [t - 20]^{+} = 10 \cdot t + 433\frac{1}{3}$		
		$t = 38\frac{1}{8}$	$t = 83\frac{1}{3}$		
	$B_{s_2}^{f_1}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 362\frac{1}{2}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 433\frac{1}{3}$		
		$=$ $562\frac{1}{2}$	$=$ $633\frac{1}{3}$		
	\mathcal{I}^{f_1}	$\sum_{i=0}^{2} \beta_{s_i}^{f_1} = 87 \frac{3}{16} $ $\max_{i=0}^{2} B_{s_i}^{f_1} = 562 \frac{1}{2}$	$\sum_{i=0}^{2} \beta_{s_i}^{f_1} = 156\frac{2}{3}$ $\max_{i=0}^{2} B_{s_i}^{f_1} = 633\frac{1}{3}$		
I	3^{f_1}	$\max_{i=0}^{2} B_{s_i}^{f_1} = 562\frac{1}{2}$	$\max_{i=0}^{2} B_{s_i}^{f_1} = 633\frac{1}{3}$		

Separate Flow Analysis and PMOO Analysis

Arrival Bounds

$(s_2, \{f_0\}, \emptyset) =: \alpha_s^f$	0	FIFO Multiplexing	Arbitrary Multiplexing
$lpha_{s_2}^{f_0} = lpha^{f_0} \oslash eta_{s_0}^{ ext{l.o.}f_0}$			
$lpha_{s_0}^{x(f_0)}$		$=\gamma_{5,25}$	
$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)}$	$R_{s_0}^{\mathrm{l.o.}f_0}$	= 15	
$= \beta_{R_{s_0}^{1.0.f_0}, T_{s_0}^{1.0.f_0}}$		$\beta_{s_0} = b_{s_0}^{x(f_0)}$	$\beta_{s_0} = \alpha_{s_0}^{x(f_0)}$
$R_{s_0}^{-10}, T_{s_0}^{-10}$	$T_{s_0}^{\mathrm{l.o.}f_0}$	$20 \cdot \left[t - 20\right]^+ = 25$	$20 \cdot [t - 20]^+ = 5 \cdot t + 25$
		$t = 21\frac{1}{4}$	$t = 28\frac{1}{3}$
	=	$=\beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$
$\alpha^{f_0} = \alpha^{f_0} \oslash \beta^{\text{l.o.}f_0}$	$r_{s_2}^{f_0}$		=5
$\alpha_{s_2}^{f_0} = \alpha^{f_0} \oslash \beta_{s_2}^{\text{l.o.}f_0}$ $= \alpha^{f_0} \oslash \beta_{s_2}^{\text{l.o.}f_0}$	$b_{s_2}^{f_0}$	$\alpha^{f_0}\left(T_{s_0}^{\text{l.o.}f_0}\right) = 131\frac{1}{4}$ $\alpha^{f_0}\left(T_{s_0}^{\text{l.o.}f_0}\right) = 166\frac{2}{3}$	
$= \alpha^{s_s} \oslash \rho_{s_2}$	=	$=\gamma_{5,131\frac{1}{4}}$	$=\gamma_{5,166\frac{2}{3}}$

Remark:

 $\label{lower} {\tt PmooArrivalBound_Goncatenation.java} \ \ {\tt because} \ f_1 \ \ {\tt does} \ \ {\tt not} \ \ {\tt have} \ \ {\tt cross-traffic} \ \ {\tt interfering} \ \ {\tt on} \ \ {\tt multiple} \ \ {\tt consecutive} \ \ {\tt hops}.$

Analyses

SFA		FIFO Multiplexing	Arbitrary Multiplexing	
$\alpha_{s_0}^{x(f_1)}$		$=\alpha^{f_0}=\gamma_{5,25}$		
$ s_0 $	n(f)	$R_{s_0}^{\mathrm{l.o.}f_1}$	$\left[R_{s_0} - r_{s_0}^{x(j)}\right]$	
	$\beta_{s_0}^{\text{l.o.}f_1} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_1)}$		$\beta_{s_0} = b_{s_0}^{x(f_1)}$	$\beta_{s_0} = \alpha_{s_0}^{x(f_1)}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 25$
		$T_{s_0}^{\mathrm{l.o.}f_1}$	$20 \cdot [t - 20]^+ = 25$	$20 \cdot [t - 20]^+ = 5 \cdot t + 25$
			$t = 21\frac{1}{4}$	$t = 28\frac{1}{3}$
		=	$=\beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$
s_1	$\alpha_{s_1}^{x(f_1)}$, .	$=\gamma$	′0,0
	$\beta_{s_1}^{\text{l.o.}f_1} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_1)}$		$=\beta_{s_1}=\beta_{20,20}$	
	$\alpha_{s_2}^{x(f_1)}$		$=\gamma_{5,131\frac{1}{4}}$	$=\gamma_{5,166\frac{2}{3}}$
s_2		$R_{s_2}^{\mathrm{l.o.}f_1}$	$ \begin{cases} R_{s_2} - r_{s_2}^{x(f_2)} \\ \beta_{s_2} = b_{s_2}^{x(f_1)} \end{cases} $	f_{1}) $= 15$
	$\beta_{s_2}^{\text{l.o.}f_1} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_1)}$		$\beta_{s_2} = b_{s_2}^{x(f_1)}$	$\beta_{s_2} = \alpha_{s_2}^{x(f_1)}$
		$T_{s_2}^{\mathrm{l.o.}f_1}$	$20 \cdot [t - 20]^+ = 131\frac{1}{4}$	$20 \cdot [t - 20]^{+} = 5 \cdot t + 166 \frac{2}{3}$
			$t = 26\frac{9}{16}$ $= \beta_{15,26\frac{9}{16}}$	$t = 37\frac{7}{9}$
		=	$=\beta_{15,26\frac{9}{16}}$	$=\beta_{15,37\frac{7}{9}}$
	$\beta_{\text{e2e}}^{\text{l.o.}f_1} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_1}, T_{\text{e2e}}^{\text{l.o.}f}}$	1	$\bigotimes_{i=0}^{2} \beta_{s_{i}}^{\text{l.o.}f_{1}} = \beta_{15,67\frac{13}{16}}$ $\beta_{\text{e}2e}^{\text{l.o.}f_{1}} = b^{f_{1}}$	$\bigotimes_{i=0}^{2} \beta_{s_{i}}^{\text{l.o.}f_{1}} = \beta_{15,86\frac{1}{9}}$ $\beta_{\text{e2e}}^{\text{l.o.}f_{1}} = b^{f_{1}}$
			$\beta_{\text{e2e}}^{\text{l.o.}f_1} = b^{f_1}$	$\beta_{\text{e2e}}^{\text{l.o.}f_1} = b^{f_1}$
	D^{f_1}		$15 \cdot \left[t - 67\frac{13}{16}\right]^+ = 25$	$15 \cdot \left[t - 86 \frac{1}{9} \right]^+ = 25$
		$t = 69 \frac{23}{48}$ $\alpha^{f_1} \left(T_{\text{e2e}}^{\text{l.o.}f_1} \right) = 5 \cdot 67 \frac{13}{16} + 25$	$t = 87\frac{7}{9}$	
	B^{f_1}			$\alpha^{f_1}\left(T_{\text{e2e}}^{\text{l.o.}f_1}\right) = 5 \cdot 86\frac{1}{9} + 25$
	Z.		$=$ $364\frac{1}{16}$	$=$ $455\frac{5}{9}$

PMOO		Arbitrary Multiplexing	
s_0	$\frac{\alpha_{s_0}^{x(f_1)}}{\alpha_{s_0}^{\bar{x}(f_1)}}$	$=\gamma_{5,25}$	
s_1	$\alpha_{s_0}^{x(f_1)}$ $\alpha_{s_0}^{\bar{x}(f_1)}$	$=\gamma_{0,0}$	
s_2	$\alpha_{s_2}^{x(f_1)}$ $\alpha_{s_2}^{\bar{x}(f_1)}$	$=\gamma_{5,166\frac{2}{3}}$	
$\beta_{\text{e2e}}^{\text{l.o.}f_1} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_1}, T_{\text{e2e}}^{\text{l.o.}f_1}} \begin{vmatrix} & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & &$		$= (20-5) \wedge (20-0) \wedge (20-5)$	
	$T_{ m e2e}^{ m l.o.}f_3$	$= \sum_{i \in \{0,1,2\}} \left(T_{s_i} + \frac{b_{s_i}^{\overline{x}(f_1)} + r_{s_i}^{x(f_1)} \cdot T_{s_i}}{R_{\text{e}2e}^{\text{l.o.}f_1}} \right)$ $= 20 + \frac{25 + 5 \cdot 20}{15} + 20 + \frac{0 + 0 \cdot 20}{15} + 20 + \frac{166\frac{2}{3} + 5 \cdot 20}{15}$ $= 60 + \frac{391\frac{2}{3}}{15}$	
	=	$= 86\frac{1}{9}$ $= \beta_{15,86\frac{1}{9}}$	
D^{f_1}		$= \beta_{15,86\frac{1}{9}}$ $\beta_{e2e}^{l.o.f_1} = b^{f_1}$ $15 \cdot \left[t - 86\frac{1}{9} \right]^+ = 25$ $t = 87\frac{7}{9}$ $\alpha^{f_1} \left(T_{e2e}^{l.o.f_1} \right) = 5 \cdot 86\frac{1}{9} + 25$	
B^{f_1}		$\alpha^{f_1} \left(T_{\text{e2e}}^{\text{l.o.}f_1} \right) = 5 \cdot 86 \frac{1}{9} + 25$ $= 455 \frac{5}{9}$	