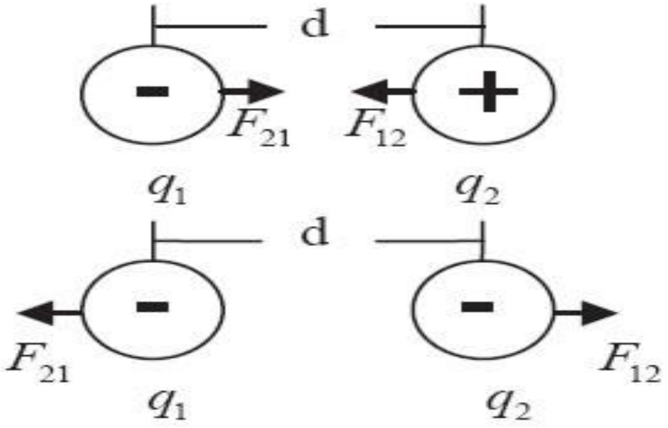
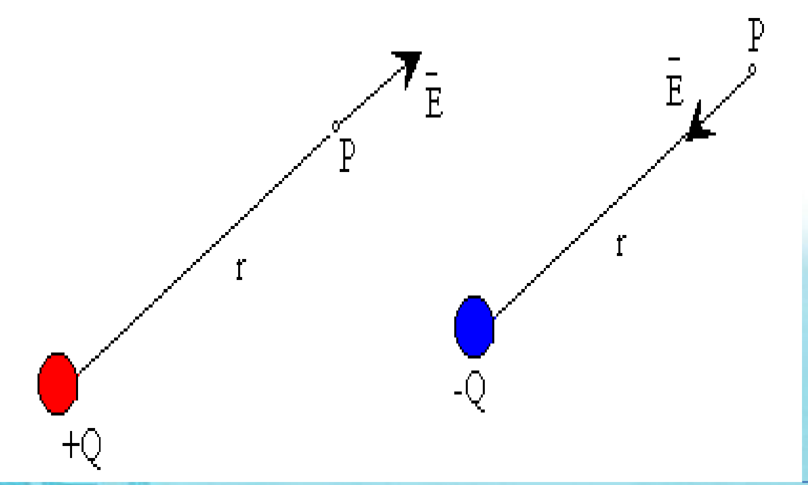
CAMPO ELECTRICO

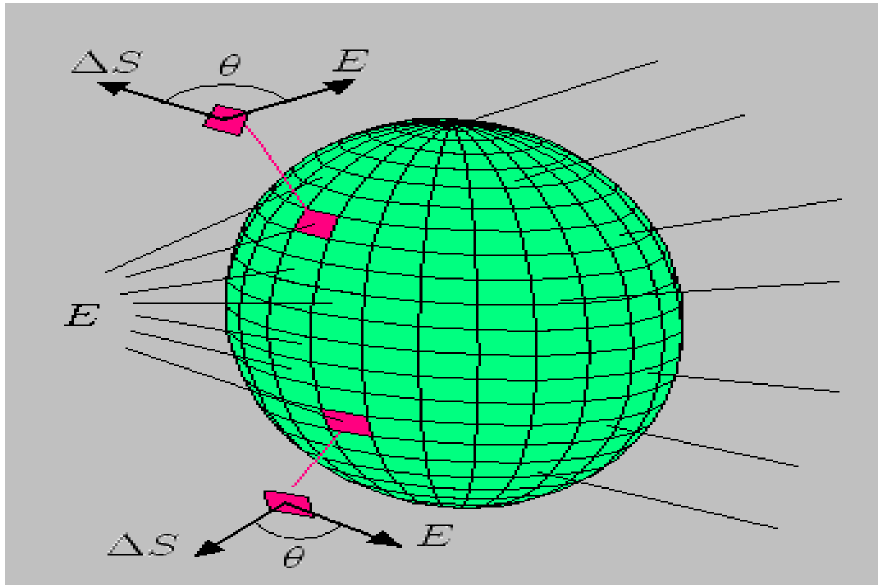
LEY DE COULOMB



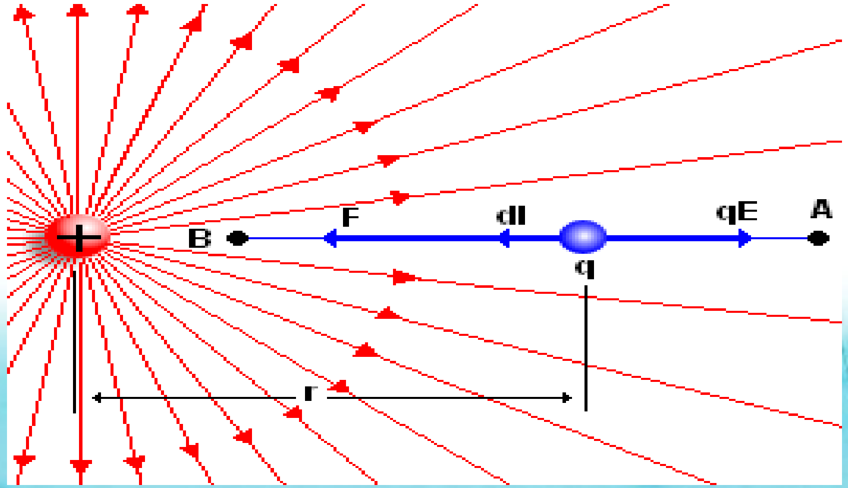
INTENSIDAD DE CAMPO ELECTRICO



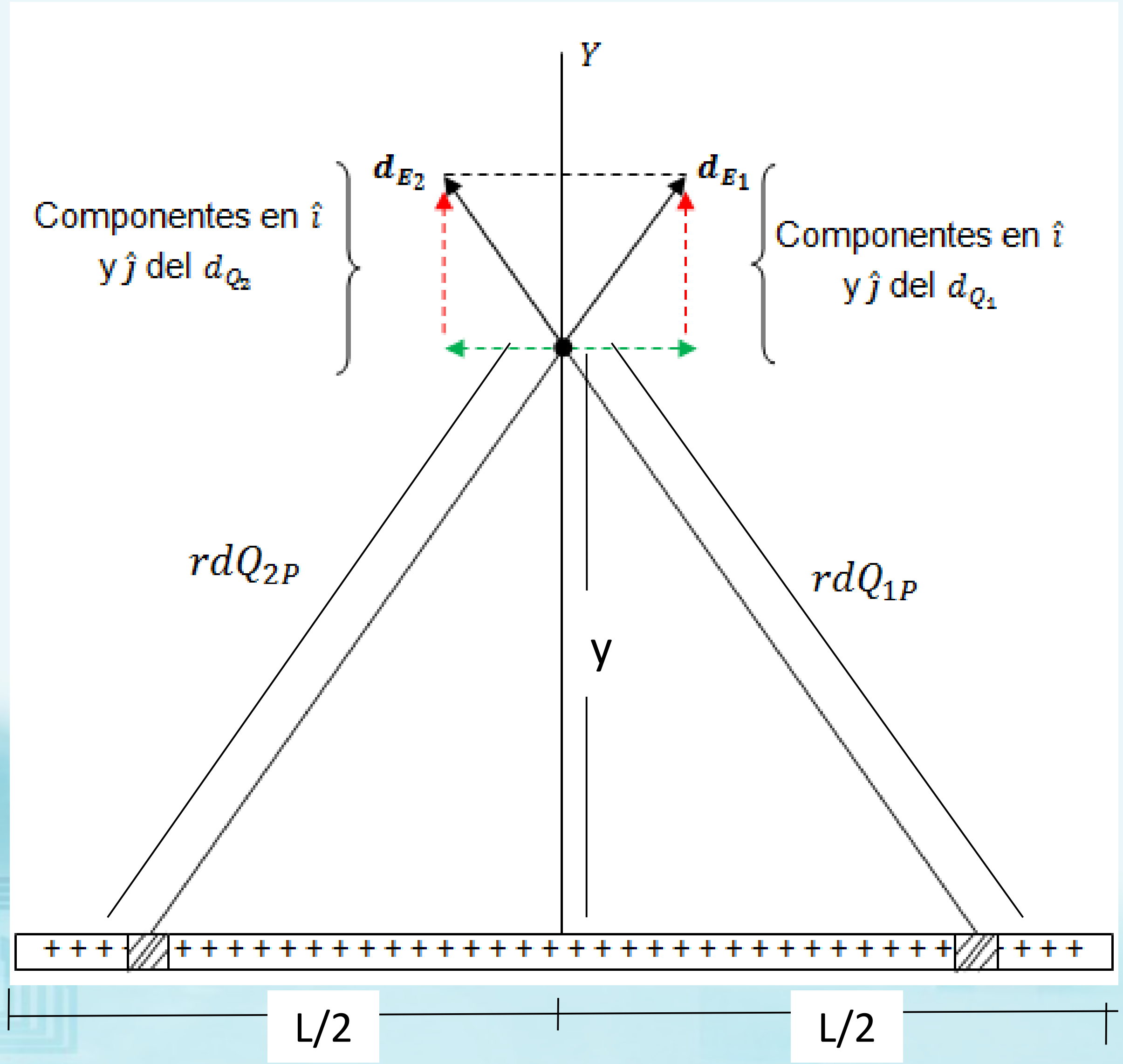
LEY (TEOREMA) DE GAUSS



ENERGIA POTENCIAL ELECTRICA



Caso 2: Campo eléctrico generado por una línea finita de carga y densidad de carga λ. Sin simetría ($\theta_1 = \theta_2$)



$$\lambda = \frac{dQ}{dl}$$

Equivalencia del elemento infinitesimal de cargas con respecto al eje de simetría.

Por simetría:

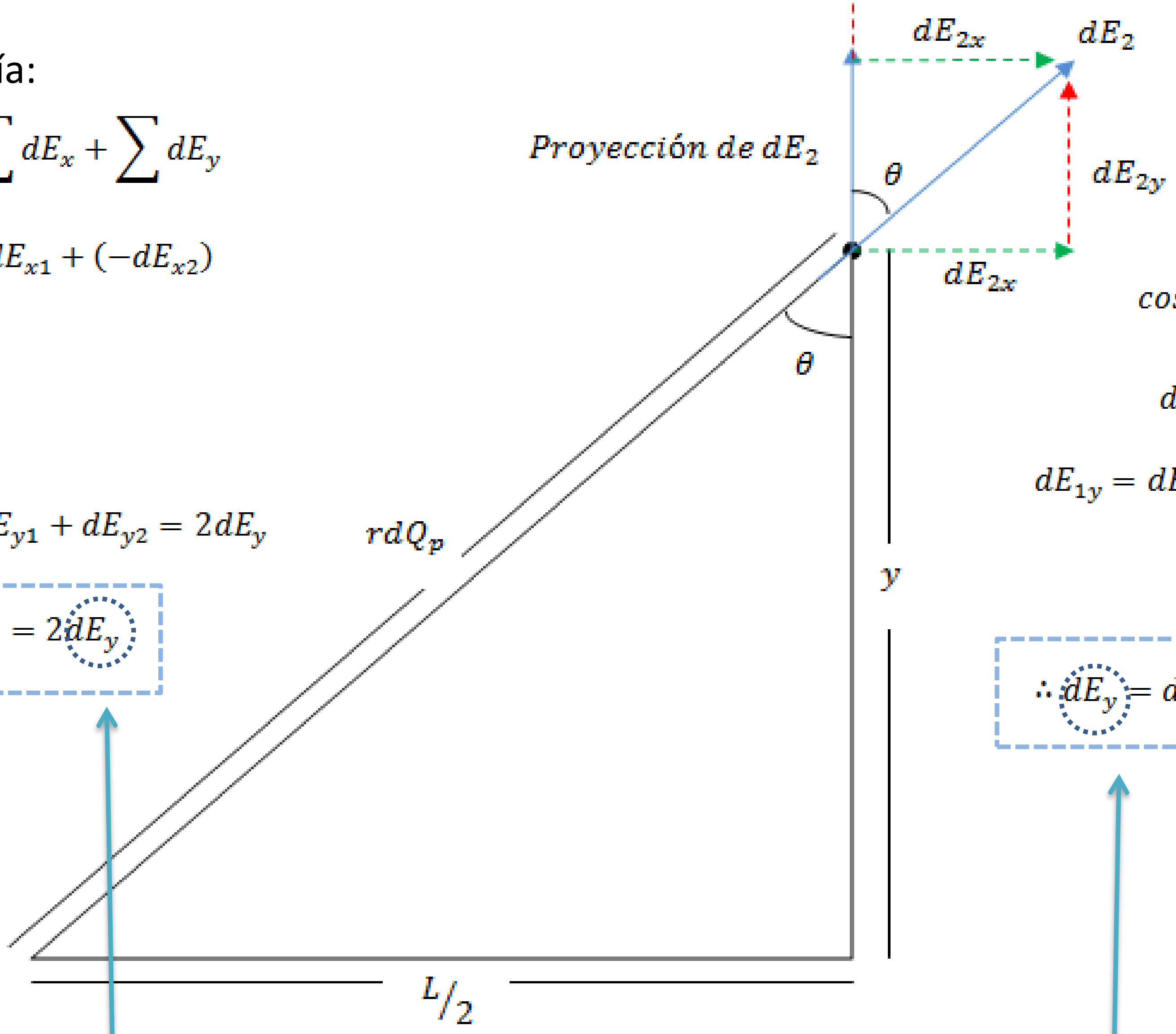
$$dE_{TOTAL} = \sum dE_x + \sum dE_y$$

$$\sum dE_x = +dE_{x1} + (-dE_{x2})$$

$$\sum dE_x = 0$$

$$\sum dE_y = +dE_{y1} + dE_{y2} = 2dE_y$$

$$\boxed{\therefore dE_{TOTAL} = 2dE_y}$$



$$\cos\theta = \frac{CA}{Hip} = \frac{dE_{2y}}{dE_2}$$

$$dE_{2y} = dE_2.\cos\theta$$

$$dE_{1y} = dE_1.\cos\theta$$

$$\boxed{\therefore dE_y = dE.\cos\theta}$$

Sustituyendo en la ecuación $dE_{TOTAL} = 2dE_y$

$$dE_{TOTAL} = 2dE.\cos\theta$$

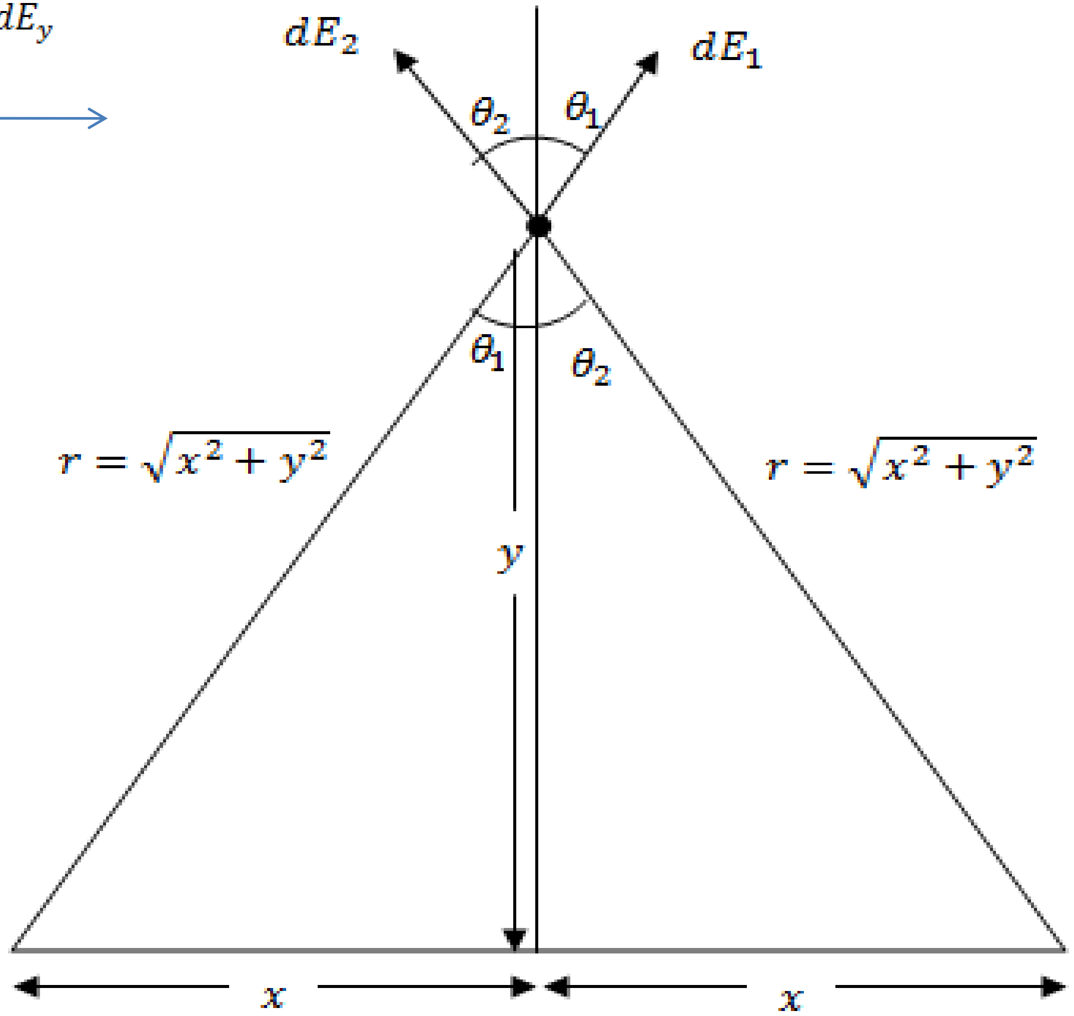
Viene dada por:

$$dE = k.\frac{dQ}{r^2}$$

$$\lambda = \frac{dQ}{dl} \rightarrow dQ = \lambda dl$$

$$dQ = \lambda dl$$

$$dQ = \lambda dx$$



$$\cos\theta_1 = \cos\theta_2 = \frac{y}{r} \qquad \cos\theta_1 = \cos\theta_2 = \frac{y}{\sqrt{x^2 + y^2}}$$

Resolviendo la integral:

$$dE_{TOTAL} = 2dEcos\theta$$

$$E_{TOTAL} = \int_0^{L/2} 2dEcos\theta$$

$$E_{TOTAL} = 2\int_0^{L/2} dEcos\theta$$

$$E_{TOTAL} = 2\int_0^{L/2} k.\frac{dQ}{r^2}cos\theta$$

$$cos\theta_1 = cos\theta_2 = \frac{y}{\sqrt{x^2+y^2}}$$

$$E_{TOTAL} = 2k\int_0^{L/2} \frac{\lambda dx}{\left(\sqrt{x^2+y^2}\right)^2}.\frac{y}{\sqrt{x^2+y^2}}$$

$$E_{TOTAL} = 2k\int_0^{L/2} \frac{\lambda y dx}{\left(\sqrt{x^2+y^2}\right)^3}$$

$$E_{TOTAL} = 2.\frac{1}{4\pi\varepsilon_0}\lambda y\int_0^{L/2} \frac{dx}{\left(\sqrt{x^2+y^2}\right)^3}$$

$$\boxed{E_{TOTAL} = \frac{\lambda y}{2\pi\varepsilon_0}\int_0^{L/2} \frac{dx}{\left(\sqrt{x^2+y^2}\right)^3}}$$

Resolviendo por sustitución trigonométrica:

$$tan\theta = \frac{x}{y}$$

$$x = ytan\theta$$

$$dx = ysec^2\theta d\theta$$

$$cos\theta = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\sqrt{x^2 + y^2} = \frac{y}{cos\theta} = ysec\theta$$

$$E_{TOTAL} = \frac{\lambda y}{2\pi\varepsilon_0}\int_0^{L/2} \frac{dx}{\left(\sqrt{x^2 + y^2}\right)^3}$$

$$E_{TOTAL} = \frac{\lambda y}{2\pi\varepsilon_0}\int_0^{L/2} \frac{ysec^2\theta d\theta}{(ysec\theta)^3}$$

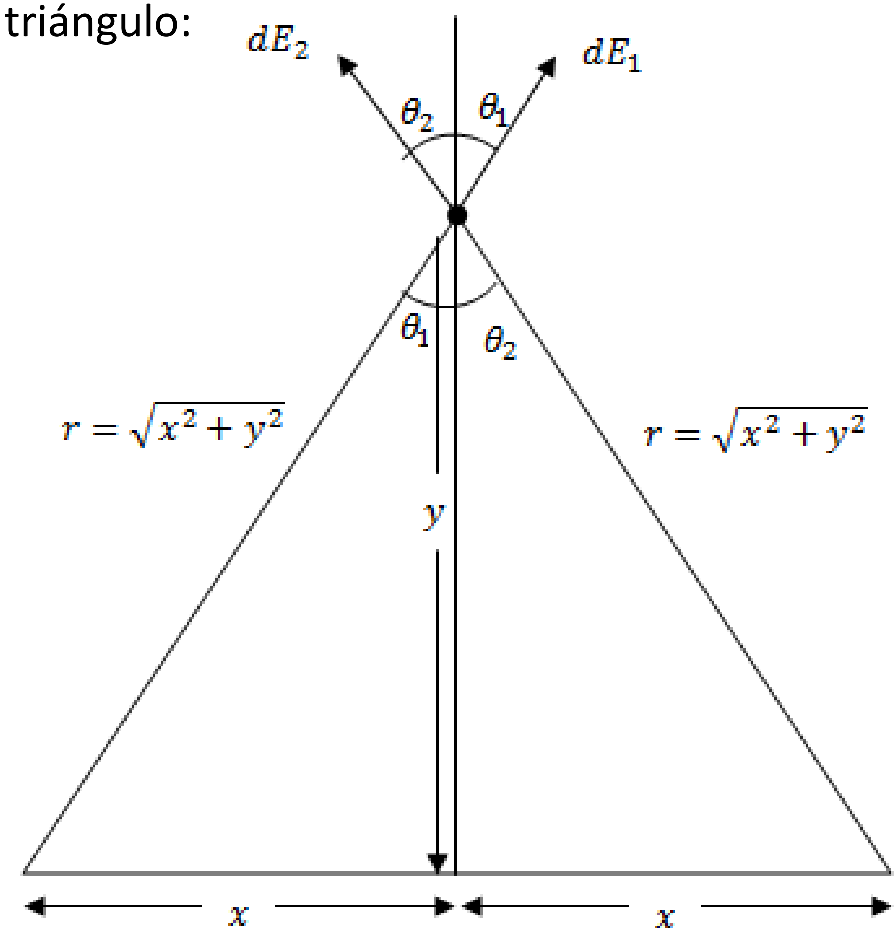
$$E_{TOTAL} = \frac{\lambda y}{2\pi\varepsilon_0}\int_0^{L/2} \frac{ysec^2\theta d\theta}{y^3 sec^3\theta}$$

$$E_{TOTAL} = \frac{\lambda}{2\pi\varepsilon_0 y}\int_0^{L/2} \frac{d\theta}{sec\theta} = \frac{\lambda}{2\pi\varepsilon_0 y}\int_0^{L/2} cos\theta d\theta = \frac{\lambda}{2\pi\varepsilon_0 y}.\, sen\theta\Big|_0^{L/2}$$

$$E_{TOTAL} = \frac{\lambda}{2\pi\varepsilon_0 y}.\, sen\theta\Big|_0^{L/2}$$

Antes de evaluar la integral se debe calcular el $sen\theta$ ya que los limites de integración no se corresponden con la expresión $sen\theta$

Del triángulo:



$$sen\theta = \frac{C.O}{Hip} = \frac{x}{\sqrt{x^2 + y^2}}$$

$$Sustituyendo\ en\ \frac{\lambda}{2\pi\varepsilon_0}.sen\theta\Big|_0^{L/2}$$

$$E_{TOTAL} = \frac{\lambda}{2\pi\varepsilon_0}.\frac{x}{\sqrt{x^2 + y^2}}\Bigg|_0^{L/2}$$

$$E_{TOTAL} = \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{\frac{L}{2}}{\sqrt{\left(\frac{L}{2}\right)^2 + y^2}} - \frac{0}{\sqrt{0^2 + y^2}}\right]$$

$$E_{TOTAL} = \frac{\lambda}{2\pi\varepsilon_0 y}.sen\theta\Big|_0^{L/2}$$

$$E_{TOTAL} = \frac{\lambda L}{4\pi\varepsilon_0\sqrt{\frac{L^2}{4} + y^2}}$$

$$E_{TOTAL} = \frac{\lambda L}{4\pi\varepsilon_0\sqrt{\frac{L^2+4y^2}{4}}} = \frac{\lambda L}{4\pi\varepsilon_0\frac{\sqrt{L^2+4y^2}}{2}}$$

$$E_{TOTAL} = \frac{\lambda L}{2\pi\varepsilon_0\sqrt{L^2+4y^2}} \quad pero\ \lambda = \frac{Q}{L}$$

$$E_{TOTAL} = \frac{QL}{L2\pi\varepsilon_0\sqrt{L^2+4y^2}}$$

$$\boxed{E_{TOTAL} = \frac{Q}{2\pi\varepsilon_0\sqrt{L^2+4y^2}}\ Escalarmente}$$

$$\boxed{\overrightarrow{E_{TOTAL}} = \frac{Q}{2\pi\varepsilon_0\sqrt{L^2+4y^2}}(\hat{j})\ Vectorialmente}$$

Caso 3: Campo eléctrico generado por una línea finita de carga y densidad de carga λ. Sin simetría ($\theta_1 \neq \theta_2$)

Escalar:

$$dE = k.\frac{dQ}{(r_{dQP})^2}$$

Vectorial:

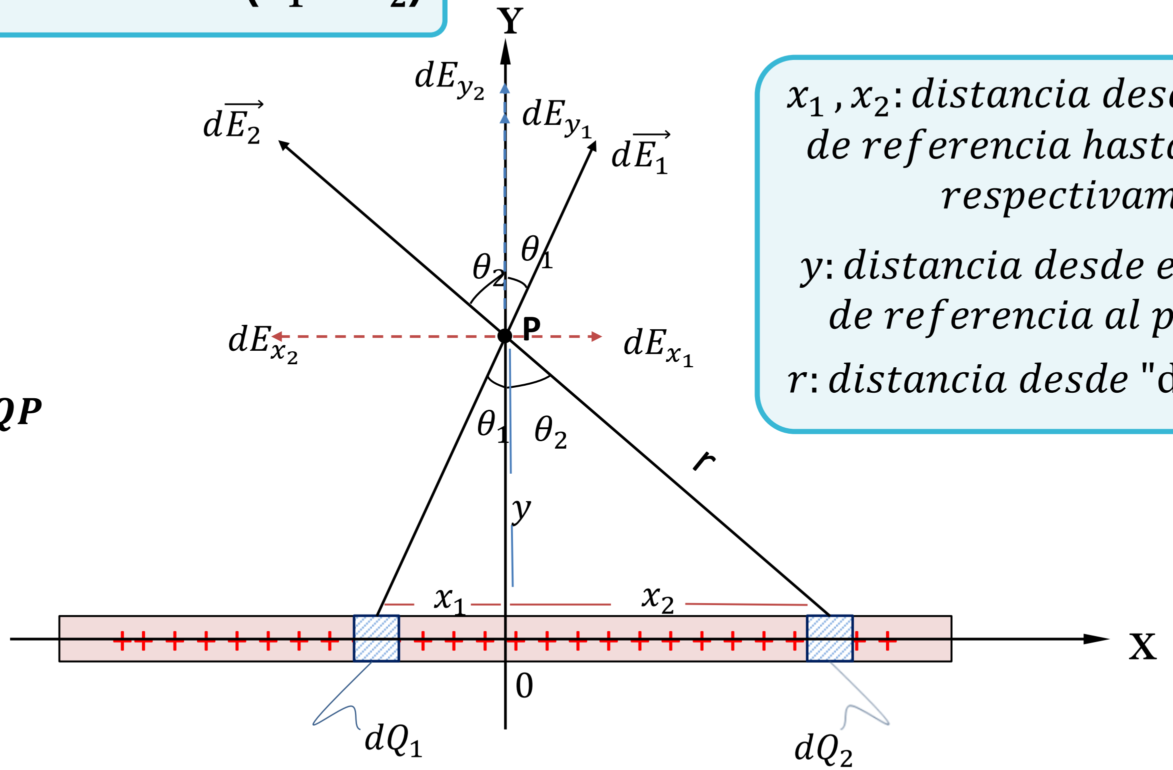
$$d\vec{E} = k\cdot \frac{dQ}{(r_{dQP})^2} \cdot \hat{r}_{dQP}$$

dQ: *Es el elemento diferencial de carga;*

$$\lambda = \frac{dQ}{dl}$$

$$dQ = \lambda . dl$$

$$\boxed{dQ = \lambda . dx}$$



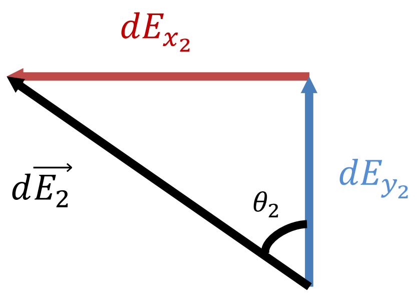
x_1, x_2: *distancia desde el sistema de referencia hasta el* dQ_1, dQ_2 *respectivamente*

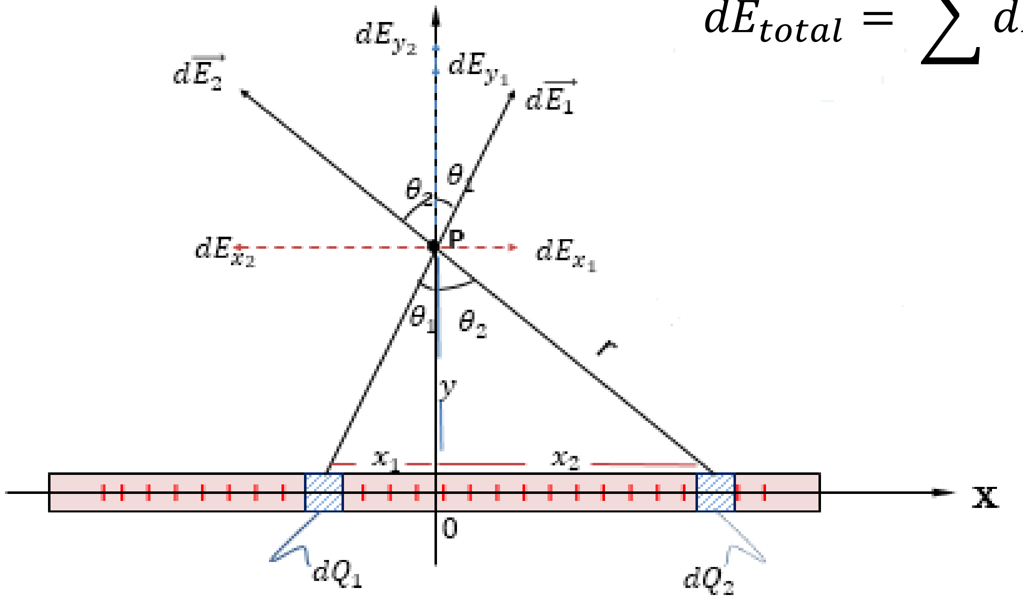
y: *distancia desde el sistema de referencia al punto "P"*

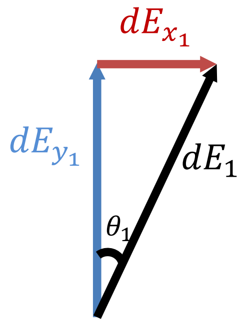
r: *distancia desde* "dQ" al punto "P"

$$dE_{total} = \sum dE_x + \sum dE_y$$

$$dE_{total} = \sum dE_x + \sum dE_y$$







$$\sum dE_x = dE_{x_1} + (-dE_{x_2})$$

$$\boxed{\theta_1 \neq \theta_2} \Rightarrow \boxed{dE_{x_1} \neq dE_{x_2}}$$

$$\boxed{\sum dE_x = dE_{x_1} - dE_{x_2}}$$

$$\underbrace{\sin\theta_2 = \frac{dE_{x_2}}{dE_2}} \qquad \underbrace{\cos\theta_2 = \frac{dE_{y_2}}{dE_2}}$$

$$\boxed{dE_{x_2} = dE_2 \cdot \sin\theta_2} \qquad \boxed{dE_{y_2} = dE_2 \cdot \cos\theta_2}$$

$$\sum dE_y = dE_{y_1} + dE_{y_2}$$

$$\boxed{\theta_1 \neq \theta_2} \Rightarrow \boxed{dE_{y_1} \neq dE_{y_2}}$$

$$\boxed{\sum dE_y = dE_{y_1} + dE_{y_2}}$$

$$\underbrace{\sin\theta_1 = \frac{dE_{x_1}}{dE_1}} \qquad \underbrace{\cos\theta_1 = \frac{dE_{y_1}}{dE_1}}$$

$$\boxed{dE_{x_1} = dE_1 \cdot \sin\theta_1} \qquad \boxed{dE_{y_1} = dE_1 \cdot \cos\theta_1}$$

En el eje "x"

$$\boxed{\sum dE_x = dE_{x_1} - dE_{x_2}}$$

$$\boxed{dE_{x_1} = dE \cdot \sin\theta_1}$$

$$\boxed{dE_{x_2} = dE \cdot \sin\theta_2}$$

$$dE_x = dE \cdot \sin\theta_1 - dE \cdot \sin\theta_2$$

$$Ex = \int_0^{\theta_1} dE.\sin\theta_1 - \int_0^{\theta_2} dE \cdot \sin\theta_2$$

$$\boxed{\boldsymbol{dQ = \lambda . dx}}$$

$$Ex = \int_0^{\theta_1} k\frac{dQ}{r^2}.\sin\theta_1 - \int_0^{\theta_2} k\frac{dQ}{r^2} \cdot \sin\theta_2$$

?

?

$$Ex = \int_0^{\theta_1} k\frac{\lambda.dx}{r^2}.\sin\theta_1 - \int_0^{\theta_2} k\frac{\lambda.dx}{r^2} \cdot \sin\theta_2$$

$$Ex = \int_0^{\theta_1} k\frac{\lambda.dx}{r^2}.\sin\theta_1 - \int_0^{\theta_2} k\frac{\lambda.dx}{r^2}\cdot\sin\theta_2$$

$$\text{Si } \tan\boldsymbol{\theta} = \frac{\boldsymbol{x}}{\boldsymbol{y}} \Rightarrow \boldsymbol{x} = \boldsymbol{y}\tan\boldsymbol{\theta} \quad \Rightarrow \quad \boxed{\boldsymbol{dx} = \boldsymbol{y}\sec\boldsymbol{\theta}\ \boldsymbol{d\theta}}$$

$$\boldsymbol{Si}\ \cos\boldsymbol{\theta} = \frac{\boldsymbol{y}}{\boldsymbol{r}} \Rightarrow \boldsymbol{r} = \frac{\boldsymbol{y}}{\cos\boldsymbol{\theta}} \quad \Rightarrow \quad \boxed{\boldsymbol{r} = \boldsymbol{y}\sec\boldsymbol{\theta}}$$

Sustituyendo

$$Ex = k\lambda\int_0^{\theta_1}\frac{y\sec^2\theta}{(y\sec\theta)^2}\sin\theta_1\,d\theta - k\lambda\int_0^{\theta_2}\frac{y\sec^2\theta}{(y\sec\theta)^2}\sin\theta_2\,d\theta$$

$$Ex = \frac{k\lambda}{y}\int_0^{\theta_1}\sin\theta_1\,d\theta - \frac{k\lambda}{y}\int_0^{\theta_2}\sin\theta_2\,d\theta$$

$$Ex = -\frac{k\lambda}{y}\cos\theta_1|_0^{\ \theta_1} + \frac{k\lambda}{y}\cos\theta_2|_0^{\ \theta_2} \quad \Rightarrow \quad \boxed{\boldsymbol{Ex} = \frac{\boldsymbol{k\lambda}}{\boldsymbol{y}}(\cos\boldsymbol{\theta_2} - \cos\boldsymbol{\theta_1})}$$

En el eje "y"

$$\boxed{\sum dE_y = dE_{y_1} + dE_{y_2}}$$

$$\boxed{dE_{y_1} = dE \cdot \cos\theta_1}$$

$$\boxed{dE_{y_2} = dE \cdot \cos\theta_2}$$

$$dE_y = dE \cdot \cos\theta_1 + dE \cdot \cos\theta_2$$

$$E_y = \int_0^{\theta_1} dE . \cos\theta_1 + \int_0^{\theta_2} dE \cdot \cos\theta_2$$

$$E_y = \int_0^{\theta_1} k\frac{dQ}{r^2} . \cos\theta_1 + \int_0^{\theta_2} k\frac{dQ}{r^2} \cdot \cos\theta_2$$

$$Ey = \int_0^{\theta_1} k\frac{\lambda . dx}{r^2} \cos\theta_1 + \int_0^{\theta_2} k\frac{\lambda . dx}{r^2} \cdot \cos\theta_2$$

$$Ey = \frac{k\lambda}{y}\int_0^{\theta_1}\cos\theta_1\, d\theta + \frac{k\lambda}{y}\int_0^{\theta_2}\cos\theta_2\, d\theta$$

$$Ey = \frac{k\lambda}{y}\sin\theta_1\Big|_0^{\theta_1} + \frac{k\lambda}{y}\sin\theta_2\Big|_0^{\theta_2}$$

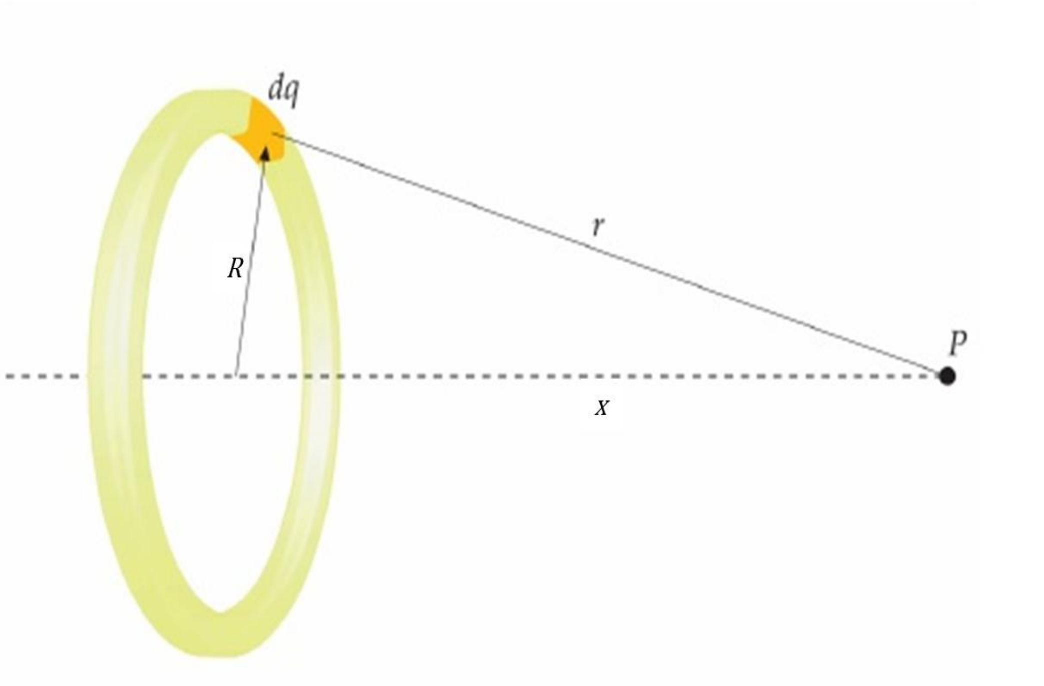
$$\boxed{\boldsymbol{Ey = \frac{k\lambda}{y}(sin\,\theta_1 + sin\,\theta_2)}}$$

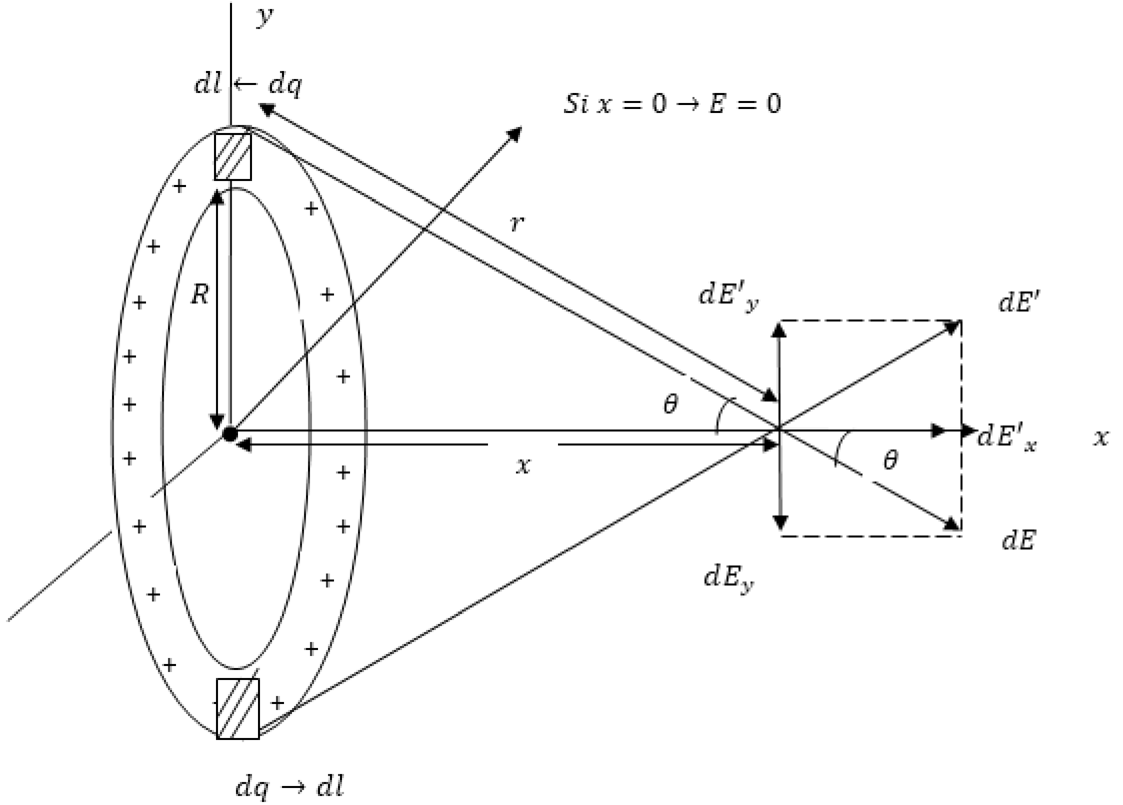
$$\vec{E}_{total} = \overrightarrow{E_x} + \overrightarrow{E_y}$$

$$\boxed{\overrightarrow{E_{total}} = \frac{k\lambda}{y}(\cos\theta_2 - \cos\theta_1)(\hat{\imath}) + \frac{k\lambda}{y}(\sin\theta_1 + \sin\theta_2)\,(\hat{\jmath})}$$

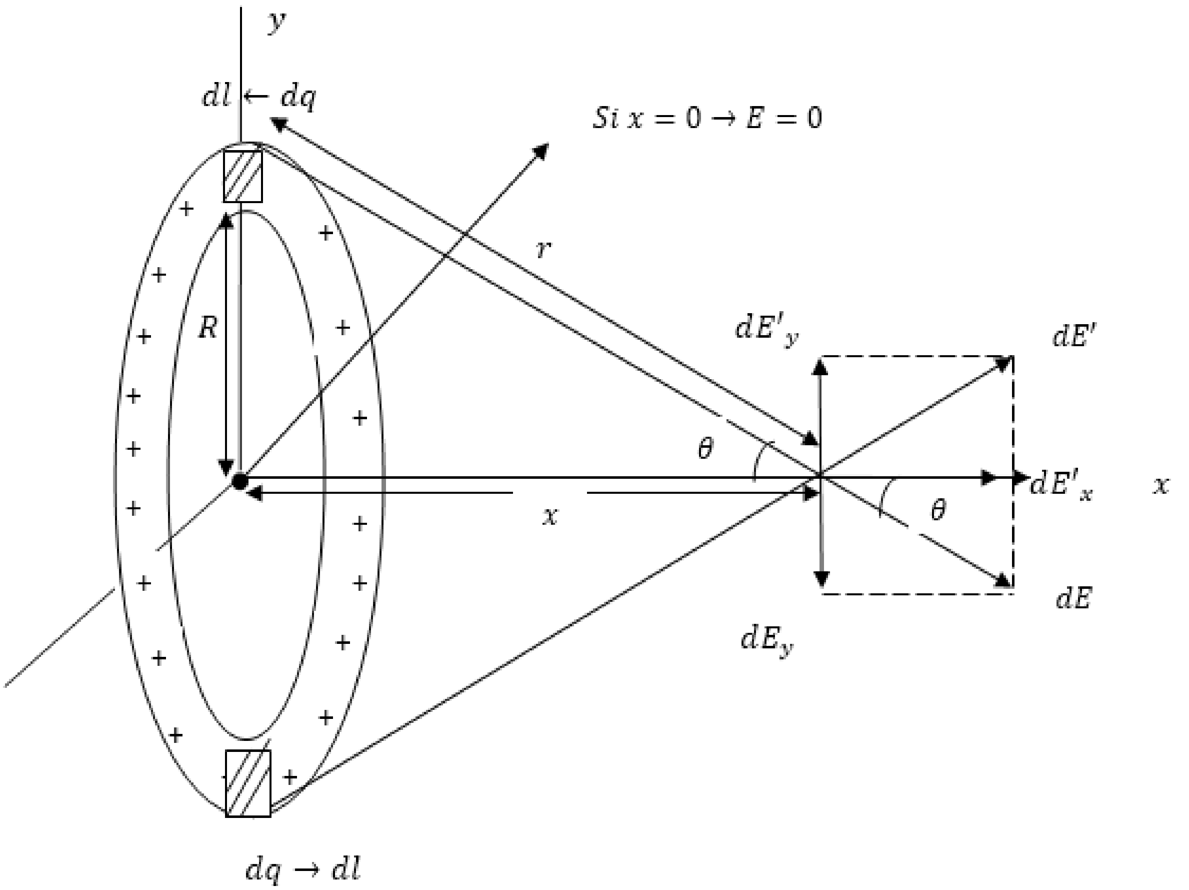
CASO 4: Campo Eléctrico generado por un Anillo (aro) de Densidad de Carga Uniforme σ sobre los puntos de su Eje Axial.

Dado un anillo de radio R que posee una distribución de carga **q** uniforme. Determinar el valor del campo creado en los puntos de su eje del anillo situado a x distancia del su centro.





CASO 4: Campo Eléctrico generado por un Anillo (aro) de Densidad de Carga Uniforme σ sobre los puntos de su Eje Axial.



$$\vec{E}_{(x,0)} = ?$$

$$E = k.\frac{q \rightarrow carga\ puntual}{r^2 \rightarrow distancia\ P\ al\ elemento\ infinitesimal\ carga\ dq}$$

$$\bar{E}_T = \sum_{i=1}^{n} E_i \qquad \lambda = \frac{dq}{dl} \qquad dq \rightarrow dE \rightarrow dE = k.\frac{dq}{r^2}$$

$$\left|d\vec{E'}\right| = \left|d\vec{E}\right|$$

$$dE_y\ , dE'_y \perp al\ eje\ x\ se\ anulan \qquad \sum E_y = 0$$

$$dE_x\ , dE'_x\ //\ al\ eje\ x\ se\ suman$$

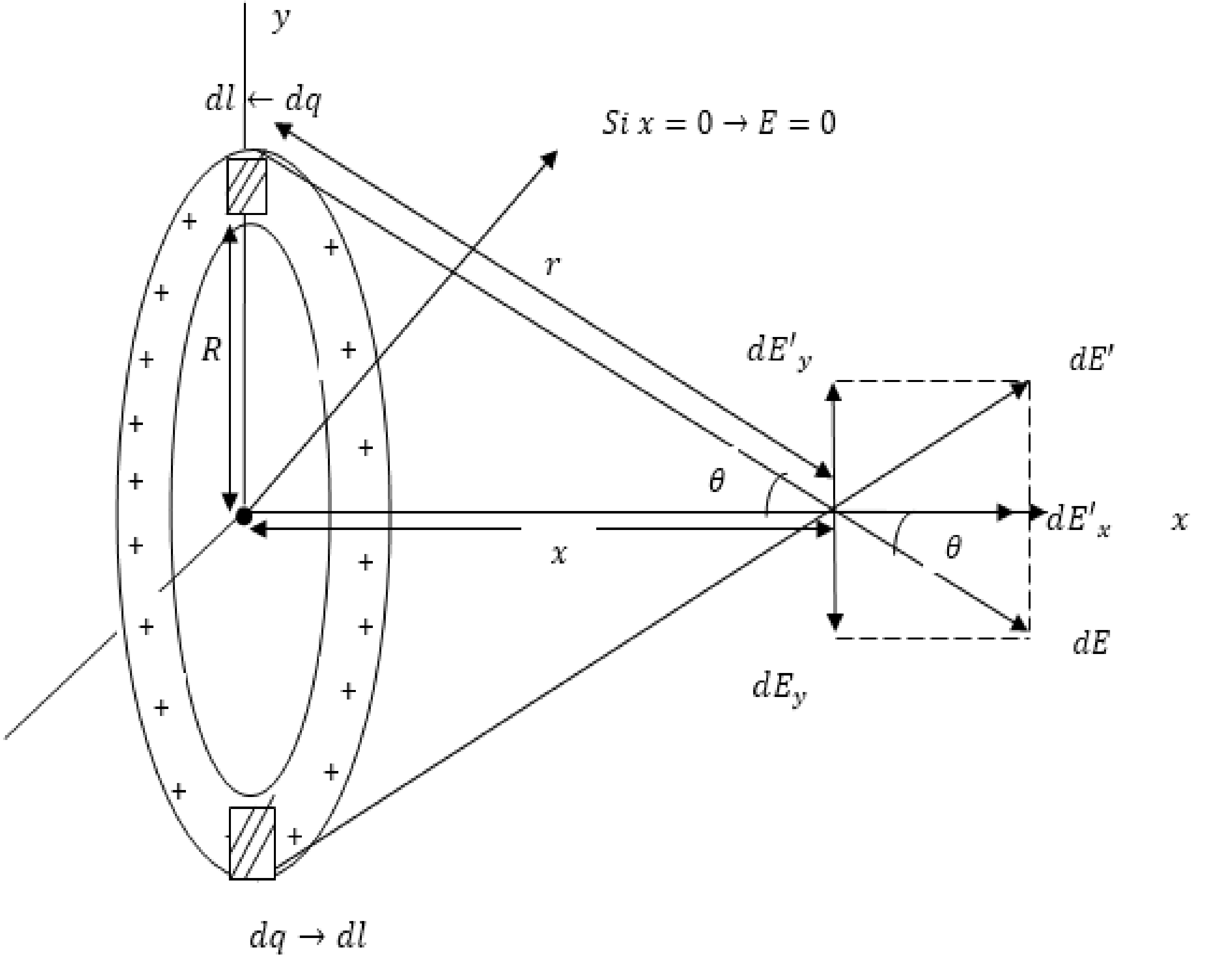
$$dE_x + dE'_x = 2dE_x \rightarrow integra \rightarrow E_T = 2E_x$$

$$dE_x = dEcos\alpha \qquad dE_x = k\frac{dq}{r^2}cos\alpha \qquad E_x = \int dE_x$$

$$E_x = \frac{k}{r^2}cos\alpha\int dq\ ; \qquad Como\ dq = \lambda dl;$$

$$Campo\ en\ el\ eje\ x \leftarrow E_x = \frac{kcos\alpha}{r^2}\lambda\int_0^{\pi R} dl$$

CASO 4: Campo Eléctrico generado por un Anillo (aro) de Densidad de Carga Uniforme σ sobre los puntos de su Eje Axial.



$$E_x = \frac{k\cos\alpha\lambda}{r^2}\pi R$$

$$r = (x^2 + R^2)^{1/2} \qquad \cos\alpha = \frac{x}{(x^2 + R^2)^{1/2}}$$

$$\lambda = \frac{q}{2\pi R}\ La\ carga\ contenida\ ó\ distribuida\ en\ el\ aro.$$

$$E_x = \frac{k.\pi R}{(x^2 + R^2)} \cdot \frac{x}{(x^2 + R^2)^{1/2}} \cdot \frac{q}{2\pi k}$$

$$E_x = \frac{2kqx}{2(x^2 + R^2)^{3/2}}$$

$$E_T = \frac{kqx}{(x^2 + R^2)^{3/2}}\ \hat{\imath}$$

$\rightarrow$ *Está en el semieje positivo de* x: *el vector es* $\hat{\imath}$

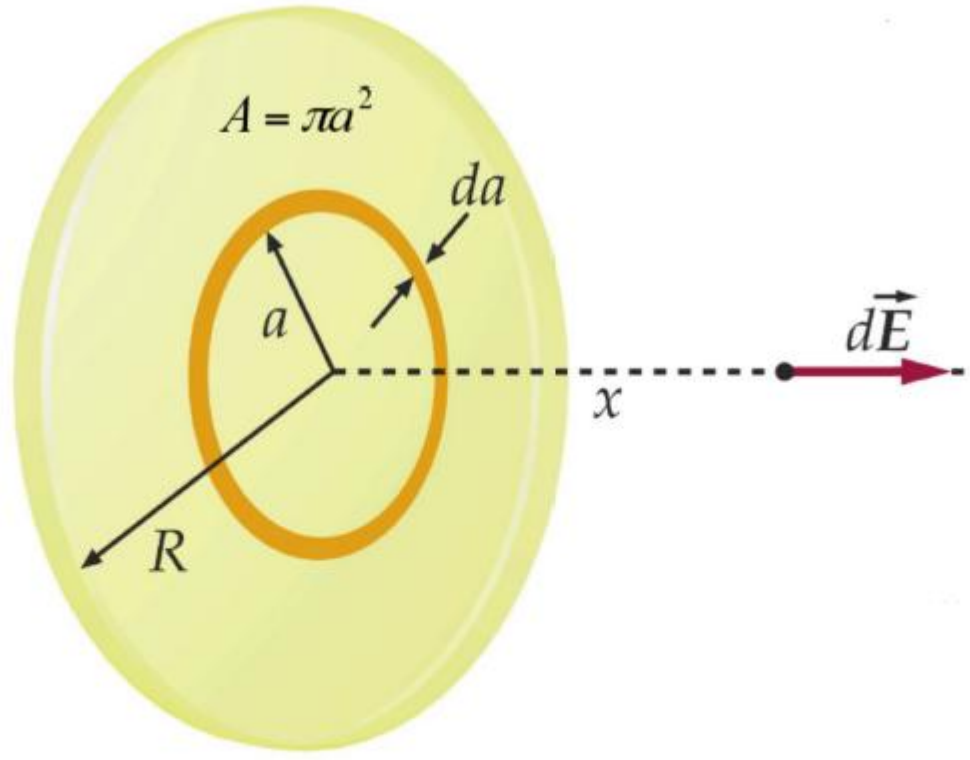
$x = 0 \rightarrow E = 0 \rightarrow$ *Se anula por simetría*

$$x^2 \lll R^2 \rightarrow x^2\ es\ despreciable\ \therefore E = \frac{kqx}{R^3}$$

$$x \gg R \quad x^2 \ggg R^2$$

$$E = k.\frac{q}{x^2}\ se\ comporta\ como\ una\ carga\ puntual.$$

CASO 5: Campo Eléctrico generado por un Disco de Densidad de Carga Uniforme σ sobre los puntos de su Eje Axial.



Un disco esta formado por infinitos aros;

por lo tanto, partimos de $dE = \frac{kxdq}{(x^2+R^2)^{3/2}}$

$$E = \int dE \qquad E = kx\int \frac{dq}{(x^2+y^2)^{3/2}} \qquad a = y$$

$$dq = \sigma dA \qquad A = \pi y^2\,; \qquad dA = \pi 2ydy$$

$$E = kx\int \frac{\sigma\pi 2ydy}{(x^2+y^2)^{3/2}}$$

$$E = k\sigma\pi\int \frac{2ydy}{(x^2+y^2)^{3/2}} \qquad v = x^2+y^2\,; dv = 2ydy$$

$$E = (-2)\frac{1}{(x^2+y^2)^{1/2}}\Bigg|_0^R$$

$$\vec{E}_{(p)} = 2\pi k\sigma\left[1 - \frac{x}{(x^2+R^2)^{1/2}}\right]\hat{\imath}$$

$$x = 0 \qquad E = 2\pi k\sigma = 2\pi\frac{1}{4\pi\varepsilon_0}\sigma \qquad E = \frac{\sigma}{2\varepsilon_0} \rightarrow plano\ \infty$$

$$E_x = \frac{kQ}{x^2}, \quad x >> R$$

$$E_x = 2\pi k\sigma, \quad x > 0$$
$$E_x = -2\pi k\sigma, \quad x < 0$$